Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/7.4.2-Exponentials-of-inverse-hyperbolic-cotangent-functions

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [935]. This is test number [199].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (935)	% 0.00 (0)
Mathematica	% 97.75 (914)	% 2.25 (21)
Maple	% 83.85 (784)	% 16.15 (151)
Maxima	% 53.69 (502)	% 46.31 (433)
Fricas	% 89.63 (838)	% 10.37 (97)
Sympy	% 21.39 (200)	% 78.61 (735)
Giac	% 52.41 (490)	% 47.59 (445)
Mupad	% 55.40 (518)	% 44.60 (417)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

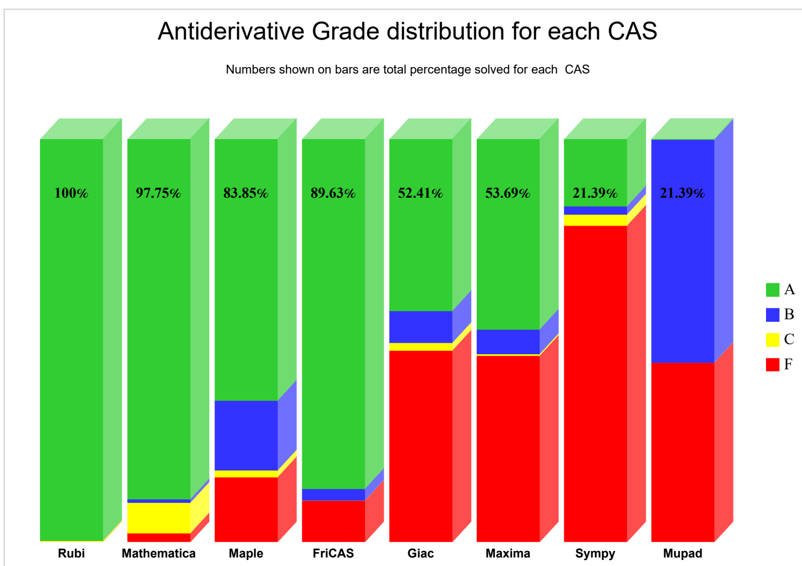
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

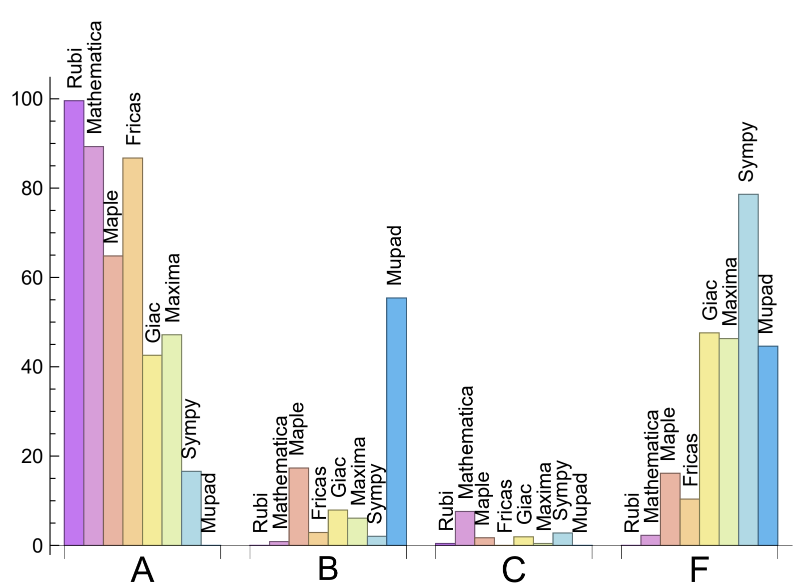
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.57	0.00	0.43	0.00
Mathematica	89.30	0.86	7.59	2.25
Maple	64.81	17.33	1.71	16.15
Maxima	47.17	6.10	0.43	46.31
Fricas	86.74	2.89	0.00	10.37
Sympy	16.58	2.03	2.78	78.61
Giac	42.57	7.91	1.93	47.59
Mupad	0.00	55.40	0.00	44.60

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	21	80.95 %	19.05 %	0.00 %
Maple	151	100.00 %	0.00 %	0.00 %
Maxima	433	100.00 %	0.00 %	0.00 %
Fricas	97	100.00 %	0.00 %	0.00 %
Sympy	735	63.67 %	35.78 %	0.54 %
Giac	445	68.76 %	0.00 %	31.24 %
Mupad	417	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

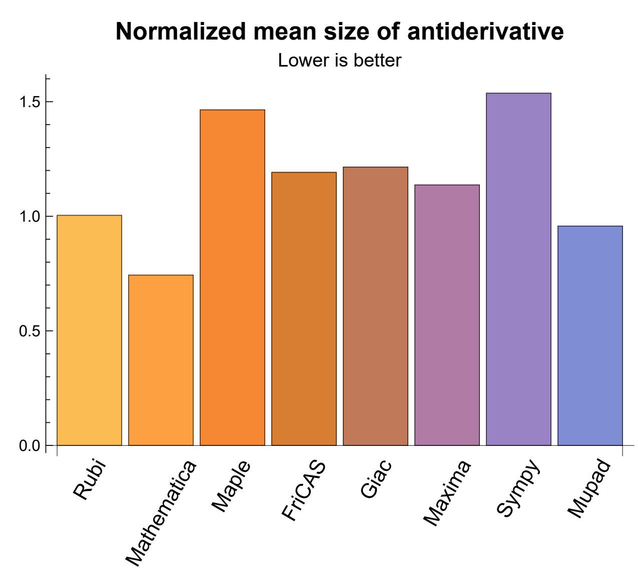
1.3 Performance

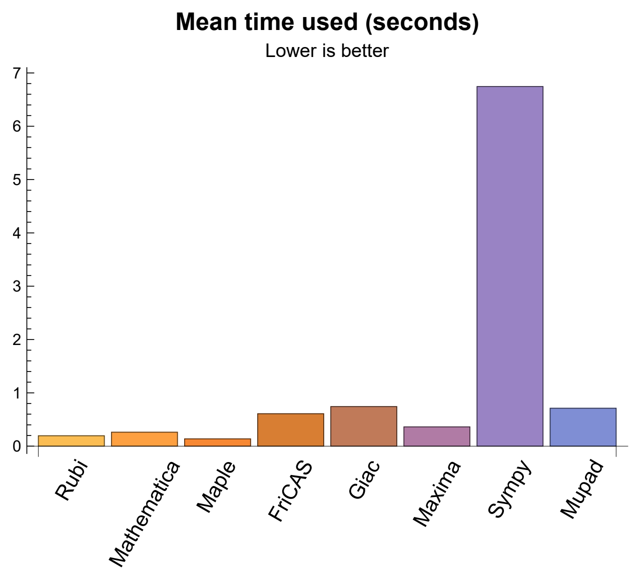
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.20	133.81	1.00	110.00	1.00
Mathematica	0.26	80.60	0.74	70.50	0.67
Maple	0.14	174.71	1.46	93.00	0.93
Maxima	0.36	116.15	1.14	97.00	1.03
Fricas	0.61	153.13	1.19	99.50	1.08
Sympy	6.74	122.91	1.54	58.00	1.00
Giac	0.74	128.24	1.21	106.00	1.03
Mupad	0.71	97.81	0.96	81.00	0.90

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {228, 229, 252, 253, 254, 255, 256, 336, 337, 338, 438, 439, 440, 542, 751, 756, 928, 929, 930, 931}

Mathematica {1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 136, 138, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 178, 179, 180, 181, 182, 198, 199, 200, 201, 206, 217, 218, 219, 220, 228, 229, 230, 246, 252, 253, 254, 255, 256, 257, 267, 268, 269, 276, 279, 280, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 298, 326, 331, 332, 334, 336, 337, 338, 355, 356, 357, 358, 361, 365, 366, 367, 368, 379, 380, 381, 382, 383, 387, 388, 389, 396, 397, 398, 399, 400, 404, 406, 407, 413, 414, 415, 416, 421, 422, 423, 426, 429, 430, 431, 432, 433, 438, 439, 440, 441, 442, 451, 452, 453, 454, 457, 458, 466, 467, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 490, 491, 492, 493, 498, 499, 500, 507, 508, 509, 511, 512, 513, 514, 515, 517, 518, 519, 520, 533, 534, 535, 537, 542, 543, 556, 557, 558, 559, 561, 562, 563, 573, 574, 575, 576, 578, 579, 580, 590, 591, 592, 593, 595, 596, 597, 606, 607, 608, 609, 611, 612, 613, 623, 624, 625, 626, 627, 628, 629, 636, 651, 652, 653, 654, 655, 662, 676, 677, 678, 679, 680, 681, 715, 716, 717, 718, 719, 720, 731, 734, 736, 737, 738, 739, 744, 745, 746, 748, 749, 750, 751, 752, 755, 756, 757, 758, 759, 760, 761, 762, 766, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 789, 790, 791, 792, 793, 794, 795, 796, 806, 807, 808, 809, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 829, 832, 838, 839, 840, 841, 842, 843, 844, 845, 848, 857, 863, 864, 865, 866, 867, 868, 869, 870, 873, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 928, 929, 930, 931, 932}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

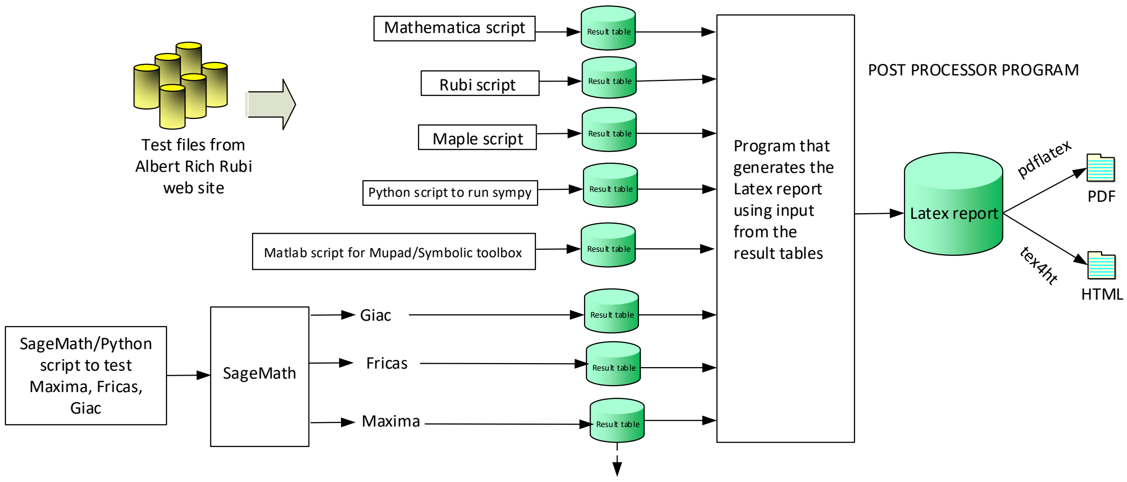
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 929, 930, 932, 933, 934, 935 }

B grade: { }

C grade: { 172, 542, 928, 931 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 65, 66, 68, 69, 70, 71, 77, 78, 79, 80, 81, 83, 84, 86, 87, 89, 95, 96, 98, 99, 101, 102, 104, 105, 106, 107, 113, 114, 121, 122, 124, 127, 132, 134, 137, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 484, 489, 490, 491, 492, 493, 494, 495, 496, 497, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 550, 552, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 732, 733, 735, 738, 739, 740, 741, 742, 743, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade: { 365, 381, 398, 426, 584, 736, 737, 744 }

C grade: { 61, 64, 67, 72, 73, 74, 75, 76, 82, 85, 88, 90, 91, 92, 93, 94, 97, 100, 103, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 123, 125, 126, 128, 129, 130, 131, 133, 135, 136, 138, 172, 267, 268, 269, 429, 430, 431, 432, 451, 452, 453, 454, 458, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 498, 499, 569, 602, 731, 734 }

F grade: { 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 545, 546, 547, 548, 549, 551, 553, 554, 933, 934, 935 }

2.1.3 Maple

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 404, 405, 406, 407, 408, 409, 410, 411, 412, 417, 421, 422, 423, 424, 425, 426, 427, 428, 435, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 475, 476, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 676, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 715, 716, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 830, 831, 832, 833, 834, 835, 836, 837, 841, 842, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 866, 867, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 161, 162, 174, 178, 181, 182, 199, 200, 201, 212, 217, 218, 219, 220, 289, 290, 291, 293, 379, 380, 381, 382, 383, 384, 385, 386, 396, 398, 399, 400, 401, 402, 403, 413, 414, 415, 416, 418, 419, 420, 429, 430, 431, 432, 433, 434, 436, 437, 450, 451, 452, 453, 454, 471, 472, 473, 474, 477, 478, 479, 500, 501, 502, 526, 527, 528, 529, 530, 531, 532, 586, 623, 624, 625, 626, 651, 674, 675, 677, 678, 679, 680, 681, 714, 717, 718, 719, 720, 776, 777, 778, 779, 792, 793, 794, 795, 796, 810, 811, 812, 813, 825, 826, 827, 828, 829, 838, 839, 840, 843, 844, 845, 863, 864, 865, 868, 869, 870, 890, 891, 892, 893, 894, 914, 915, 916, 917, 918 }

C grade: { 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 132, 134, 137 }

F grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 126, 127, 128, 129, 130, 131, 133, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.4 Maxima

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 195, 196, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 279, 280, 283, 284, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 323, 324, 327, 328, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 673, 674, 675, 676, 677, 712, 713, 714, 715, 716, 732, 733, 740, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 880, 904 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 53, 54, 158, 159, 160, 161, 178, 179, 180, 181, 187, 194, 198, 199, 200, 201, 212, 222, 281, 282, 285, 286, 379, 380, 381, 382, 396, 397, 398, 399, 414, 416, 417, 435, 584, 587, 636, 662 }

C grade: { 321, 322, 325, 326 }

F grade: { 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 277, 278, 295, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 329, 330, 331, 332, 333, 334, 335, 339, 340, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 614, 615, 616, 617, 618, 619, 620, 621, 622, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 369, 370, 371, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 578, 579, 580, 581, 582, 583, 585, 586, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade: { 5, 6, 21, 37, 38, 129, 130, 131, 183, 194, 201, 242, 290, 381, 442, 466, 467, 492, 493, 519, 520, 572, 577, 584, 587, 588, 605 }

C grade: { }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.6 Sympy

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 167, 169, 170, 171, 172, 173, 174, 175, 187, 188, 189, 190, 191, 192, 193, 195, 196, 207, 208, 209, 210, 211, 212, 213, 214, 215, 235, 236, 237, 238, 239, 240, 241, 242, 261, 262, 263, 264, 265, 266, 267, 268, 269, 288, 292, 301, 302, 303, 304, 305, 324, 327, 328, 341, 342, 343, 344, 345, 387, 388, 389, 390, 391, 392, 393, 394, 395, 404, 405, 406, 407, 408, 409, 410, 411, 412, 421, 422, 423, 424, 425, 427, 428, 502, 528, 564, 565, 566, 567, 568, 569, 570, 571, 572, 585, 588, 589, 598, 599, 600, 601, 602, 603, 604, 605, 780, 781, 782, 783, 784, 785, 786, 787, 788, 797, 798, 799, 800, 801, 802, 803, 804, 805, 814, 815, 816, 817, 818, 819, 820, 821 }

B grade: { 134, 168, 176, 194, 306, 307, 308, 309, 346, 347, 348, 349, 426, 581, 582, 583, 584, 586, 587 }

C grade: { 137, 256, 296, 297, 319, 323, 325, 326, 337, 338, 447, 552, 623, 624, 625, 626, 651, 652, 767, 770, 838, 839, 840, 863, 864, 865 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 293, 294, 295, 298, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 396, 397, 398, 399, 400, 401, 402, 403, 413, 414, 415, 416, 417, 418, 419, 420, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 573, 574, 575, 576, 577, 578, 579, 580, 590, 591, 592, 593, 594, 595, 596, 597, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 768, 769, 771, 772, 773, 774, 775, 776, 777, 778, 779, 789, 790, 791, 792, 793, 794, 795, 796, 806, 807, 808, 809, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.7 Giac

A grade: { 1, 4, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 34, 35, 36, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 158, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 175, 176, 182, 183, 184, 185, 186, 188, 189, 190, 192, 193, 195, 196, 198, 199, 200, 201, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 222, 226, 227, 228, 229, 230, 232, 233, 234, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 275, 276, 278, 279, 280, 283, 284, 287, 288, 290, 291, 292, 293, 294, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 333, 334, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 400, 401, 402, 403, 409, 410, 411, 412, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 435, 452, 453, 454, 475, 476, 477, 478, 479, 498, 499, 500, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 585, 587, 588, 589, 590, 591, 592, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 623, 624, 625, 626, 627, 651, 652, 653, 674, 675, 676, 677, 678, 713, 714, 715, 716, 717, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 838, 839, 840, 863, 864, 865, 886, 887, 888, 890, 892, 910, 911, 912, 914, 916 }

B grade: { 2, 3, 5, 6, 7, 8, 9, 20, 21, 29, 37, 39, 41, 159, 160, 161, 174, 178, 179, 180, 181, 191, 194, 212, 235, 236, 237, 281, 282, 285, 286, 289, 301, 302, 379, 380, 382, 396, 397, 398, 399, 404, 405, 406, 407, 408, 413, 414, 450, 451, 501, 530, 531, 532, 583, 584, 586, 610, 629, 655, 679, 680, 681, 718, 719, 720, 799, 800, 891, 893, 894, 915, 917, 918 }

C grade: { 231, 247, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 321, 322, 325, 326, 328 }

F grade: { 33, 38, 40, 50, 51, 52, 53, 54, 55, 56, 57, 58, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 177, 187, 197, 202, 206, 216, 217, 218, 219, 220, 221, 223, 224, 225, 252, 253, 254, 255, 257, 259, 270, 271, 272, 273, 274, 277, 295, 319, 320, 323, 324, 327, 329, 330, 331, 332, 335, 336, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 418, 419, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 594, 595, 596, 597, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 628, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 654, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 810, 811, 812, 813, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 889, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 913, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 359, 360, 361, 369, 370, 371, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 494, 495, 496, 497, 503, 504, 505, 506, 521, 522, 523, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 629, 630, 631, 632, 639, 655, 656, 657, 658, 665, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 881, 882, 885, 905, 906, 909 }

C grade: { }

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638, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 907, 908, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935
}

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	68	193	203	92	0	182	171
normalized size	1	1.00	0.60	1.69	1.78	0.81	0.00	1.60	1.50
time (sec)	N/A	0.123	0.075	0.067	0.315	0.530	0.000	0.154	1.257

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	60	173	166	84	0	151	133
normalized size	1	1.00	0.67	1.92	1.84	0.93	0.00	1.68	1.48
time (sec)	N/A	0.093	0.050	0.046	0.320	0.712	0.000	0.157	0.063

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	152	128	73	0	118	98
normalized size	1	1.00	0.78	2.41	2.03	1.16	0.00	1.87	1.56
time (sec)	N/A	0.064	0.037	0.046	0.314	0.514	0.000	0.142	0.063

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	97	90	64	0	57	58
normalized size	1	1.00	1.14	2.69	2.50	1.78	0.00	1.58	1.61
time (sec)	N/A	0.036	0.024	0.040	0.313	0.660	0.000	0.170	0.044

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	36	131	69	57	0	70	37
normalized size	1	1.00	1.64	5.95	3.14	2.59	0.00	3.18	1.68
time (sec)	N/A	0.043	0.014	0.049	0.415	0.668	0.000	0.161	1.180

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	27	220	53	46	0	53	55
normalized size	1	1.00	1.12	9.17	2.21	1.92	0.00	2.21	2.29
time (sec)	N/A	0.025	0.020	0.051	0.413	0.563	0.000	0.142	0.055

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	260	91	60	0	87	81
normalized size	1	1.00	1.11	6.84	2.39	1.58	0.00	2.29	2.13
time (sec)	N/A	0.031	0.045	0.053	0.413	0.628	0.000	0.144	1.203

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	284	136	68	0	130	105
normalized size	1	1.00	0.68	3.79	1.81	0.91	0.00	1.73	1.40
time (sec)	N/A	0.062	0.089	0.057	0.416	0.513	0.000	0.146	0.064

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	59	308	172	76	0	164	129
normalized size	1	1.00	0.67	3.50	1.95	0.86	0.00	1.86	1.47
time (sec)	N/A	0.083	0.101	0.059	0.416	0.548	0.000	0.153	0.085

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	43	42	37	47	38
normalized size	1	1.00	1.00	0.91	1.00	0.98	0.86	1.09	0.88
time (sec)	N/A	0.053	0.020	0.035	0.313	0.572	0.093	0.135	0.036

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	34	33	27	38	30
normalized size	1	1.00	1.00	0.94	1.03	1.00	0.82	1.15	0.91
time (sec)	N/A	0.049	0.015	0.036	0.316	0.572	0.089	0.126	0.036

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	26	25	20	30	23
normalized size	1	1.00	1.00	0.92	1.00	0.96	0.77	1.15	0.88
time (sec)	N/A	0.033	0.013	0.035	0.301	0.495	0.083	0.116	0.041

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	16	10	14	13
normalized size	1	1.00	1.00	1.00	0.93	1.14	0.71	1.00	0.93
time (sec)	N/A	0.014	0.011	0.034	0.306	0.449	0.073	0.114	1.171

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	13	10	15	14
normalized size	1	1.00	1.00	1.00	0.93	0.93	0.71	1.07	1.00
time (sec)	N/A	0.040	0.008	0.040	0.307	0.448	0.114	0.118	0.043

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	22	15	20	14
normalized size	1	1.00	1.00	1.00	0.95	1.16	0.79	1.05	0.74
time (sec)	N/A	0.045	0.011	0.041	0.310	0.807	0.132	0.118	1.190

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	30	35	26	32	23
normalized size	1	1.00	1.00	0.94	0.91	1.06	0.79	0.97	0.70
time (sec)	N/A	0.047	0.014	0.040	0.313	0.525	0.150	0.140	0.044

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	38	38	43	34	40	30
normalized size	1	1.00	1.00	0.95	0.95	1.08	0.85	1.00	0.75
time (sec)	N/A	0.051	0.016	0.042	0.308	0.593	0.167	0.134	0.042

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	75	471	182	112	0	171	154
normalized size	1	1.00	0.64	3.99	1.54	0.95	0.00	1.45	1.31
time (sec)	N/A	1.071	0.088	0.058	0.310	0.512	0.000	0.151	1.257

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	421	145	103	0	140	117
normalized size	1	1.00	0.72	4.58	1.58	1.12	0.00	1.52	1.27
time (sec)	N/A	0.872	0.068	0.055	0.309	0.493	0.000	0.172	0.065

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	248	110	92	0	119	59
normalized size	1	1.00	0.87	4.00	1.77	1.48	0.00	1.92	0.95
time (sec)	N/A	0.811	0.050	0.049	0.303	0.516	0.000	0.165	1.293

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	363	90	104	0	91	54
normalized size	1	1.00	1.15	7.89	1.96	2.26	0.00	1.98	1.17
time (sec)	N/A	0.783	0.063	0.055	0.411	0.588	0.000	0.156	0.037

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	593	72	74	0	85	57
normalized size	1	1.00	0.80	11.63	1.41	1.45	0.00	1.67	1.12
time (sec)	N/A	0.074	0.091	0.057	0.409	0.511	0.000	0.162	0.049

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	641	110	88	0	108	83
normalized size	1	1.00	0.62	7.04	1.21	0.97	0.00	1.19	0.91
time (sec)	N/A	0.453	0.102	0.056	0.409	0.384	0.000	0.136	0.076

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	66	666	154	96	0	150	152
normalized size	1	1.00	0.71	7.16	1.66	1.03	0.00	1.61	1.63
time (sec)	N/A	0.743	0.124	0.062	0.415	0.557	0.000	0.134	1.240

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	58	66	49	78	57
normalized size	1	1.00	1.00	0.91	1.02	1.16	0.86	1.37	1.00
time (sec)	N/A	0.066	0.050	0.042	0.312	0.635	0.155	0.141	0.044

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	49	57	39	69	49
normalized size	1	1.00	1.00	0.94	1.04	1.21	0.83	1.47	1.04
time (sec)	N/A	0.060	0.037	0.040	0.312	0.552	0.147	0.144	1.167

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	41	49	31	64	38
normalized size	1	1.00	1.00	0.92	1.05	1.26	0.79	1.64	0.97
time (sec)	N/A	0.041	0.031	0.041	0.305	0.564	0.133	0.138	0.042

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	26	38	19	46	25
normalized size	1	1.00	0.96	0.96	0.96	1.41	0.70	1.70	0.93
time (sec)	N/A	0.019	0.019	0.039	0.305	0.686	0.117	0.141	0.036

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	18	8	57	12
normalized size	1	1.00	1.00	1.00	0.92	1.38	0.62	4.38	0.92
time (sec)	N/A	0.039	0.011	0.041	0.307	0.548	0.145	0.145	0.034

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	55	26	40	28
normalized size	1	1.00	1.00	0.97	1.06	1.72	0.81	1.25	0.88
time (sec)	N/A	0.051	0.027	0.043	0.314	0.456	0.211	0.143	0.052

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	48	73	41	62	41
normalized size	1	1.00	1.00	0.93	1.04	1.59	0.89	1.35	0.89
time (sec)	N/A	0.054	0.032	0.040	0.303	0.586	0.238	0.141	1.196

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	56	81	49	74	49
normalized size	1	1.00	1.00	0.94	1.04	1.50	0.91	1.37	0.91
time (sec)	N/A	0.061	0.047	0.043	0.303	0.610	0.264	0.119	0.062

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	68	193	203	91	0	0	172
normalized size	1	1.00	0.60	1.69	1.78	0.80	0.00	0.00	1.51
time (sec)	N/A	0.127	0.072	0.051	0.307	0.474	0.000	0.000	1.213

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	60	173	166	83	0	86	134
normalized size	1	1.00	0.67	1.92	1.84	0.92	0.00	0.96	1.49
time (sec)	N/A	0.100	0.053	0.052	0.305	0.617	0.000	0.141	0.054

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	49	152	130	73	0	71	97
normalized size	1	1.00	0.77	2.38	2.03	1.14	0.00	1.11	1.52
time (sec)	N/A	0.068	0.050	0.046	0.308	0.762	0.000	0.149	0.057

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	98	90	64	0	52	58
normalized size	1	1.00	1.14	2.65	2.43	1.73	0.00	1.41	1.57
time (sec)	N/A	0.039	0.023	0.042	0.305	0.652	0.000	0.141	1.189

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	34	133	70	57	0	59	37
normalized size	1	1.00	1.70	6.65	3.50	2.85	0.00	2.95	1.85
time (sec)	N/A	0.048	0.016	0.051	0.402	0.571	0.000	0.160	0.031

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	26	220	55	47	0	0	55
normalized size	1	1.00	1.04	8.80	2.20	1.88	0.00	0.00	2.20
time (sec)	N/A	0.027	0.022	0.052	0.405	0.571	0.000	0.000	1.198

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	260	93	60	0	157	82
normalized size	1	1.00	1.02	6.50	2.32	1.50	0.00	3.92	2.05
time (sec)	N/A	0.037	0.050	0.054	0.411	0.656	0.000	0.138	1.201

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	52	284	137	68	0	0	105
normalized size	1	1.00	0.68	3.74	1.80	0.89	0.00	0.00	1.38
time (sec)	N/A	0.066	0.084	0.059	0.407	0.520	0.000	0.000	1.204

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	59	308	173	77	0	258	129
normalized size	1	1.00	0.67	3.50	1.97	0.88	0.00	2.93	1.47
time (sec)	N/A	0.090	0.103	0.066	0.409	1.335	0.000	0.163	1.221

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	43	42	37	47	38
normalized size	1	1.00	1.00	0.93	1.02	1.00	0.88	1.12	0.90
time (sec)	N/A	0.057	0.021	0.035	0.311	0.437	0.094	0.143	1.171

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	34	33	27	38	31
normalized size	1	1.00	1.00	0.97	1.03	1.00	0.82	1.15	0.94
time (sec)	N/A	0.051	0.015	0.035	0.309	0.444	0.091	0.121	0.037

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	26	25	20	30	23
normalized size	1	1.00	1.00	0.96	1.04	1.00	0.80	1.20	0.92
time (sec)	N/A	0.032	0.013	0.034	0.304	0.451	0.084	0.133	0.039

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	16	10	14	13
normalized size	1	1.00	1.00	1.08	1.00	1.23	0.77	1.08	1.00
time (sec)	N/A	0.013	0.011	0.032	0.310	0.472	0.075	0.125	0.029

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	14
normalized size	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	1.08
time (sec)	N/A	0.040	0.007	0.038	0.311	0.498	0.110	0.118	0.043

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	23	15	20	14
normalized size	1	1.00	1.00	1.06	1.00	1.28	0.83	1.11	0.78
time (sec)	N/A	0.043	0.010	0.044	0.309	0.523	0.128	0.113	1.196

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	30	35	26	32	24
normalized size	1	1.00	1.00	0.97	0.94	1.09	0.81	1.00	0.75
time (sec)	N/A	0.046	0.013	0.044	0.311	0.498	0.147	0.146	0.044

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	38	43	34	40	31
normalized size	1	1.00	1.00	0.98	0.95	1.08	0.85	1.00	0.78
time (sec)	N/A	0.050	0.015	0.042	0.304	0.672	0.167	0.145	1.211

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	83	539	223	92	0	0	192
normalized size	1	1.00	0.61	3.96	1.64	0.68	0.00	0.00	1.41
time (sec)	N/A	1.019	0.095	0.058	0.318	0.674	0.000	0.000	0.076

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	75	471	186	84	0	0	156
normalized size	1	1.00	0.65	4.06	1.60	0.72	0.00	0.00	1.34
time (sec)	N/A	0.869	0.083	0.057	0.319	0.401	0.000	0.000	0.058

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	66	421	151	75	0	0	120
normalized size	1	1.00	0.73	4.68	1.68	0.83	0.00	0.00	1.33
time (sec)	N/A	0.842	0.068	0.054	0.315	0.437	0.000	0.000	1.223

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	248	111	66	0	0	78
normalized size	1	1.00	0.90	4.13	1.85	1.10	0.00	0.00	1.30
time (sec)	N/A	0.779	0.051	0.050	0.319	0.461	0.000	0.000	0.040

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	369	89	74	0	0	54
normalized size	1	1.00	1.20	8.02	1.93	1.61	0.00	0.00	1.17
time (sec)	N/A	0.761	0.057	0.053	0.415	0.806	0.000	0.000	0.033

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	41	592	72	49	0	0	59
normalized size	1	1.00	0.77	11.17	1.36	0.92	0.00	0.00	1.11
time (sec)	N/A	0.078	0.073	0.056	0.424	0.626	0.000	0.000	0.048

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	56	642	112	61	0	0	118
normalized size	1	1.00	0.64	7.38	1.29	0.70	0.00	0.00	1.36
time (sec)	N/A	0.445	0.145	0.057	0.412	0.481	0.000	0.000	0.059

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	666	157	69	0	0	153
normalized size	1	1.00	0.69	6.94	1.64	0.72	0.00	0.00	1.59
time (sec)	N/A	0.766	0.113	0.061	0.418	0.500	0.000	0.000	1.232

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	75	690	193	77	0	0	190
normalized size	1	1.00	0.56	5.19	1.45	0.58	0.00	0.00	1.43
time (sec)	N/A	0.839	0.057	0.062	0.415	0.671	0.000	0.000	1.236

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
normalized size	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.154	5.224	0.133	0.416	0.529	0.000	0.269	0.107

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	192
normalized size	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.119	5.192	0.081	0.416	0.623	0.000	0.271	1.242

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	399	0	187	103	0	172	157
normalized size	1	1.00	2.23	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.095	9.064	0.080	0.418	0.424	0.000	0.235	0.081

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	66	0	149	93	0	139	120
normalized size	1	1.00	0.46	0.00	1.05	0.65	0.00	0.98	0.85
time (sec)	N/A	0.060	0.183	0.079	0.410	0.554	0.000	0.241	0.077

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	51	0	111	84	0	108	78
normalized size	1	1.00	0.53	0.00	1.16	0.88	0.00	1.12	0.81
time (sec)	N/A	0.037	0.098	0.079	0.410	0.576	0.000	0.213	0.065

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	291	0	232	101
normalized size	1	1.00	0.10	0.00	0.77	1.00	0.00	0.80	0.35
time (sec)	N/A	0.246	0.041	0.089	0.420	0.697	0.000	0.212	0.083

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	148	0	186	396	0	186	87
normalized size	1	1.00	0.55	0.00	0.70	1.48	0.00	0.70	0.33
time (sec)	N/A	0.225	0.258	0.107	0.413	0.466	0.000	0.191	1.195

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	173	0	226	413	0	223	132
normalized size	1	1.00	0.54	0.00	0.71	1.29	0.00	0.70	0.41
time (sec)	N/A	0.249	0.231	0.092	0.411	0.645	0.000	0.186	0.075

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	270	427	0	271	168
normalized size	1	1.00	0.26	0.00	0.76	1.20	0.00	0.76	0.47
time (sec)	N/A	0.292	0.114	0.094	0.413	0.418	0.000	0.217	1.211

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
normalized size	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.140	5.255	0.120	0.404	0.638	0.000	0.403	0.095

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	192
normalized size	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.118	5.204	0.084	0.413	0.647	0.000	0.390	1.215

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	125	0	187	103	0	172	157
normalized size	1	1.00	0.70	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.094	5.175	0.082	0.407	0.564	0.000	0.330	0.085

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	152	95	0	141	120
normalized size	1	1.00	0.49	0.00	1.07	0.67	0.00	0.99	0.85
time (sec)	N/A	0.059	0.187	0.080	0.404	0.551	0.000	0.292	1.206

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	56	0	112	86	0	109	79
normalized size	1	1.00	0.57	0.00	1.14	0.88	0.00	1.11	0.81
time (sec)	N/A	0.037	0.067	0.077	0.402	0.640	0.000	0.237	1.186

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	291	0	232	101
normalized size	1	1.00	0.10	0.00	0.77	1.00	0.00	0.80	0.35
time (sec)	N/A	0.228	0.050	0.075	0.404	0.565	0.000	0.255	0.052

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	46	0	187	376	0	187	88
normalized size	1	1.00	0.17	0.00	0.70	1.40	0.00	0.70	0.33
time (sec)	N/A	0.215	0.083	0.104	0.405	0.458	0.000	0.221	0.078

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	76	0	229	405	0	225	132
normalized size	1	1.00	0.24	0.00	0.72	1.27	0.00	0.71	0.41
time (sec)	N/A	0.246	0.086	0.090	0.410	0.499	0.000	0.245	0.089

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	277	413	0	271	168
normalized size	1	1.00	0.26	0.00	0.78	1.16	0.00	0.76	0.47
time (sec)	N/A	0.282	0.132	0.092	0.412	0.558	0.000	0.280	1.212

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	198	0	275	152	0	254	248
normalized size	1	1.00	0.69	0.00	0.96	0.53	0.00	0.89	0.86
time (sec)	N/A	0.165	5.283	0.362	0.419	0.551	0.000	0.281	1.264

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	161	0	238	144	0	223	211
normalized size	1	1.00	0.64	0.00	0.95	0.58	0.00	0.89	0.84
time (sec)	N/A	0.136	5.246	0.360	0.417	0.560	0.000	0.272	0.143

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	137	0	203	136	0	192	176
normalized size	1	1.00	0.64	0.00	0.95	0.64	0.00	0.90	0.83
time (sec)	N/A	0.112	5.213	0.362	0.410	0.580	0.000	0.253	0.092

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	80	0	166	128	0	161	139
normalized size	1	1.00	0.45	0.00	0.94	0.73	0.00	0.91	0.79
time (sec)	N/A	0.068	0.226	0.373	0.406	0.625	0.000	0.246	1.210

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	67	0	131	117	0	141	98
normalized size	1	1.00	0.52	0.00	1.01	0.90	0.00	1.08	0.75
time (sec)	N/A	0.044	0.143	0.365	0.404	0.711	0.000	0.203	0.064

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	30	0	244	358	0	252	118
normalized size	1	1.00	0.09	0.00	0.76	1.12	0.00	0.79	0.37
time (sec)	N/A	0.297	0.075	0.089	0.406	0.555	0.000	0.215	1.180

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	173	0	204	451	0	217	107
normalized size	1	1.00	0.58	0.00	0.68	1.51	0.00	0.73	0.36
time (sec)	N/A	0.257	0.396	0.375	0.403	0.667	0.000	0.185	1.217

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	186	0	244	469	0	243	152
normalized size	1	1.00	0.53	0.00	0.70	1.34	0.00	0.69	0.43
time (sec)	N/A	0.285	0.285	0.371	0.407	0.482	0.000	0.194	0.084

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	104	0	288	477	0	291	188
normalized size	1	1.00	0.27	0.00	0.75	1.24	0.00	0.76	0.49
time (sec)	N/A	0.313	0.154	0.367	0.414	0.597	0.000	0.248	1.264

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
normalized size	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.140	5.302	0.088	0.407	0.698	0.000	0.243	0.080

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	193
normalized size	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.115	5.256	0.082	0.407	0.607	0.000	0.222	1.210

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	389	0	187	102	0	172	157
normalized size	1	1.00	2.17	0.00	1.04	0.57	0.00	0.96	0.88
time (sec)	N/A	0.092	9.029	0.079	0.413	0.585	0.000	0.200	1.202

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	66	0	151	93	0	140	121
normalized size	1	1.00	0.46	0.00	1.06	0.65	0.00	0.99	0.85
time (sec)	N/A	0.060	0.169	0.080	0.413	1.582	0.000	0.218	0.061

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	33	0	111	84	0	108	79
normalized size	1	1.00	0.34	0.00	1.14	0.87	0.00	1.11	0.81
time (sec)	N/A	0.036	0.046	0.077	0.410	0.592	0.000	0.197	1.181

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	291	0	232	101
normalized size	1	1.00	0.10	0.00	0.77	1.00	0.00	0.80	0.35
time (sec)	N/A	0.228	0.053	0.074	0.411	0.913	0.000	0.193	1.176

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	33	0	186	368	0	186	88
normalized size	1	1.00	0.12	0.00	0.69	1.37	0.00	0.69	0.33
time (sec)	N/A	0.218	0.052	0.093	0.405	0.575	0.000	0.156	1.186

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	56	0	227	396	0	223	132
normalized size	1	1.00	0.18	0.00	0.71	1.24	0.00	0.70	0.41
time (sec)	N/A	0.250	0.078	0.092	0.405	0.565	0.000	0.198	0.065

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	277	412	0	271	169
normalized size	1	1.00	0.26	0.00	0.78	1.16	0.00	0.76	0.47
time (sec)	N/A	0.277	0.134	0.095	0.405	0.522	0.000	0.213	0.069

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
normalized size	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.140	5.339	0.089	0.412	0.555	0.000	0.273	1.214

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	193
normalized size	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.116	5.271	0.082	0.404	0.575	0.000	0.291	1.180

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	389	0	187	103	0	172	157
normalized size	1	1.00	2.17	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.092	8.652	0.082	0.409	0.501	0.000	0.225	0.057

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	152	95	0	141	121
normalized size	1	1.00	0.49	0.00	1.07	0.67	0.00	0.99	0.85
time (sec)	N/A	0.059	0.204	0.078	0.408	0.529	0.000	0.245	1.186

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	55	0	112	86	0	109	79
normalized size	1	1.00	0.56	0.00	1.14	0.88	0.00	1.11	0.81
time (sec)	N/A	0.035	0.123	0.078	0.409	0.476	0.000	0.167	1.183

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	28	0	224	291	0	232	101
normalized size	1	1.00	0.10	0.00	0.77	1.00	0.00	0.80	0.35
time (sec)	N/A	0.226	0.075	0.074	0.416	0.531	0.000	0.175	1.181

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	149	0	187	402	0	187	88
normalized size	1	1.00	0.55	0.00	0.70	1.49	0.00	0.70	0.33
time (sec)	N/A	0.214	0.301	0.092	0.457	0.566	0.000	0.159	0.052

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	174	0	228	419	0	225	132
normalized size	1	1.00	0.55	0.00	0.71	1.31	0.00	0.71	0.41
time (sec)	N/A	0.247	0.224	0.087	0.416	0.531	0.000	0.226	1.182

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	270	427	0	271	169
normalized size	1	1.00	0.26	0.00	0.76	1.20	0.00	0.76	0.47
time (sec)	N/A	0.281	0.151	0.092	0.411	0.618	0.000	0.252	0.064

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	198	0	279	119	0	254	253
normalized size	1	1.00	0.69	0.00	0.97	0.41	0.00	0.89	0.88
time (sec)	N/A	0.162	5.428	0.362	0.406	1.170	0.000	0.213	0.081

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	161	0	244	111	0	223	217
normalized size	1	1.00	0.64	0.00	0.98	0.44	0.00	0.89	0.87
time (sec)	N/A	0.140	5.323	0.363	0.407	0.652	0.000	0.229	0.080

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	137	0	207	103	0	192	181
normalized size	1	1.00	0.64	0.00	0.97	0.48	0.00	0.90	0.85
time (sec)	N/A	0.110	5.285	0.365	0.405	0.813	0.000	0.211	0.074

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	121	0	172	95	0	161	145
normalized size	1	1.00	0.69	0.00	0.98	0.54	0.00	0.91	0.82
time (sec)	N/A	0.071	0.267	0.364	0.414	0.901	0.000	0.194	1.198

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	31	0	132	86	0	129	103
normalized size	1	1.00	0.24	0.00	1.02	0.66	0.00	0.99	0.79
time (sec)	N/A	0.046	0.076	0.374	0.413	0.474	0.000	0.162	1.188

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	28	0	244	308	0	252	118
normalized size	1	1.00	0.09	0.00	0.76	0.96	0.00	0.79	0.37
time (sec)	N/A	0.285	0.103	0.084	0.421	0.675	0.000	0.173	1.144

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	31	0	204	377	0	204	106
normalized size	1	1.00	0.10	0.00	0.68	1.26	0.00	0.68	0.35
time (sec)	N/A	0.245	0.084	0.375	0.421	0.545	0.000	0.176	1.180

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	101	0	247	405	0	243	153
normalized size	1	1.00	0.29	0.00	0.70	1.15	0.00	0.69	0.44
time (sec)	N/A	0.281	0.152	0.367	0.419	0.508	0.000	0.179	0.070

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	104	0	297	413	0	291	188
normalized size	1	1.00	0.27	0.00	0.77	1.07	0.00	0.76	0.49
time (sec)	N/A	0.309	0.206	0.371	0.420	0.579	0.000	0.189	0.073

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	189	2891	220	173	0	215	168
normalized size	1	1.00	0.66	10.14	0.77	0.61	0.00	0.75	0.59
time (sec)	N/A	0.249	5.288	4.641	0.425	0.477	0.000	0.186	0.132

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	167	2886	194	168	0	191	142
normalized size	1	1.00	0.65	11.19	0.75	0.65	0.00	0.74	0.55
time (sec)	N/A	0.203	0.406	4.202	0.397	0.666	0.000	0.161	1.221

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	35	1700	167	160	0	168	115
normalized size	1	1.00	0.16	7.62	0.75	0.72	0.00	0.75	0.52
time (sec)	N/A	0.180	0.050	4.095	0.431	0.609	0.000	0.155	0.098

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	26	2320	0	340	0	261	167
normalized size	1	1.00	0.06	5.77	0.00	0.85	0.00	0.65	0.42
time (sec)	N/A	0.525	0.045	14.615	0.000	0.556	0.000	0.165	1.267

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	39	3474	152	223	0	152	109
normalized size	1	1.00	0.17	14.91	0.65	0.96	0.00	0.65	0.47
time (sec)	N/A	0.367	0.060	7.821	0.409	0.607	0.000	0.155	1.230

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	124	2997	178	240	0	175	136
normalized size	1	1.00	0.48	11.53	0.68	0.92	0.00	0.67	0.52
time (sec)	N/A	0.379	0.798	20.934	0.423	0.517	0.000	0.156	0.106

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	133	3484	205	246	0	199	161
normalized size	1	1.00	0.46	12.14	0.71	0.86	0.00	0.69	0.56
time (sec)	N/A	0.398	0.225	7.217	0.415	0.416	0.000	0.185	1.255

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	340	408	149	100	0	144	171
normalized size	1	1.00	2.17	2.60	0.95	0.64	0.00	0.92	1.09
time (sec)	N/A	0.070	7.772	0.701	0.417	0.398	0.000	0.178	1.191

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	165	403	123	95	0	120	145
normalized size	1	1.00	1.27	3.10	0.95	0.73	0.00	0.92	1.12
time (sec)	N/A	0.047	0.486	0.584	0.402	0.584	0.000	0.172	0.049

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	85	605	96	87	0	97	118
normalized size	1	1.00	0.89	6.30	1.00	0.91	0.00	1.01	1.23
time (sec)	N/A	0.029	0.177	0.544	0.404	0.542	0.000	0.139	0.046

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	26	1038	140	86	0	79	82
normalized size	1	1.00	0.17	6.70	0.90	0.55	0.00	0.51	0.53
time (sec)	N/A	0.051	0.047	0.923	0.418	0.648	0.000	0.175	1.416

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	87	501	98	97	0	99	118
normalized size	1	1.00	0.88	5.06	0.99	0.98	0.00	1.00	1.19
time (sec)	N/A	0.043	0.170	0.791	0.411	0.677	0.000	0.143	0.025

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	134	738	124	111	0	122	145
normalized size	1	1.00	1.03	5.68	0.95	0.85	0.00	0.94	1.12
time (sec)	N/A	0.053	0.315	0.650	0.407	0.575	0.000	0.140	0.027

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	167	0	341	457	0	333	227
normalized size	1	1.00	0.39	0.00	0.79	1.07	0.00	0.78	0.53
time (sec)	N/A	0.343	5.342	0.147	0.411	0.489	0.000	0.245	1.307

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	319	0	304	448	0	300	190
normalized size	1	1.00	0.81	0.00	0.78	1.14	0.00	0.77	0.48
time (sec)	N/A	0.247	0.803	0.092	0.418	0.500	0.000	0.230	1.275

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	56	0	265	423	0	269	149
normalized size	1	1.00	0.16	0.00	0.75	1.20	0.00	0.76	0.42
time (sec)	N/A	0.200	0.055	0.093	0.421	0.518	0.000	0.207	1.264

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	919	919	30	0	0	2289	0	660	648
normalized size	1	1.00	0.03	0.00	0.00	2.49	0.00	0.72	0.71
time (sec)	N/A	0.899	0.047	0.109	0.000	0.767	0.000	1.849	1.398

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	46	0	0	2874	0	432	162
normalized size	1	1.00	0.07	0.00	0.00	4.25	0.00	0.64	0.24
time (sec)	N/A	0.605	0.070	0.263	0.000	0.784	0.000	0.399	1.249

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	72	0	0	2931	0	473	210
normalized size	1	1.00	0.10	0.00	0.00	4.01	0.00	0.65	0.29
time (sec)	N/A	0.659	0.099	0.124	0.000	0.936	0.000	0.465	1.263

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	201	0	0	0	0	-1
normalized size	1	1.00	1.04	4.47	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.026	0.513	0.000	0.566	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	228	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.231	0.355	0.051	0.000	0.547	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	106	0	0	100	0	-1
normalized size	1	1.00	0.74	3.03	0.00	0.00	2.86	0.00	-0.03
time (sec)	N/A	0.041	0.010	0.413	0.000	0.477	2.709	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	128	0	0	0	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.423	0.047	0.000	0.593	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	115	0	0	0	0	0	-1
normalized size	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.257	0.044	0.000	0.608	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	93	0	0	119	0	-1
normalized size	1	1.00	0.75	2.58	0.00	0.00	3.31	0.00	-0.03
time (sec)	N/A	0.041	0.013	0.388	0.000	0.561	2.690	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	192	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.051	0.265	0.047	0.000	0.889	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.647	0.042	0.000	0.564	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.609	0.044	0.000	0.454	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.541	0.044	0.000	0.634	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.634	0.043	0.000	0.583	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.609	0.046	0.000	0.619	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.686	0.043	0.000	0.550	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.852	0.039	0.000	0.454	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.834	0.043	0.000	0.772	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.690	0.041	0.000	0.624	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.362	0.074	0.000	0.692	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	118	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.626	0.059	0.000	0.594	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	98	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.324	0.055	0.000	0.542	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.196	0.049	0.000	0.524	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	142	0	0	0	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.159	0.050	0.000	0.911	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.039	0.055	0.000	0.677	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	107	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.389	0.063	0.000	0.645	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	132	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.666	0.064	0.000	0.596	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	148	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.545	0.071	0.000	0.552	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	131	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.095	0.349	0.000	0.662	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	80	183	259	125	0	216	214
normalized size	1	1.00	0.61	1.39	1.96	0.95	0.00	1.64	1.62
time (sec)	N/A	0.303	0.200	0.048	0.326	0.475	0.000	0.178	0.126

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	73	141	221	115	0	185	177
normalized size	1	1.00	0.70	1.34	2.10	1.10	0.00	1.76	1.69
time (sec)	N/A	0.226	0.169	0.047	0.309	0.499	0.000	0.175	0.079

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	121	181	103	0	154	138
normalized size	1	1.00	0.82	1.55	2.32	1.32	0.00	1.97	1.77
time (sec)	N/A	0.136	0.108	0.044	0.310	0.514	0.000	0.148	1.210

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	51	93	132	77	0	154	94
normalized size	1	1.00	1.09	1.98	2.81	1.64	0.00	3.28	2.00
time (sec)	N/A	0.071	0.056	0.040	0.308	0.542	0.000	0.196	1.198

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	249	78	87	0	79	48
normalized size	1	1.00	1.18	4.88	1.53	1.71	0.00	1.55	0.94
time (sec)	N/A	0.206	0.060	0.048	0.304	0.468	0.000	0.178	0.073

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	34	36	23	57	0	35	23
normalized size	1	1.00	1.03	1.09	0.70	1.73	0.00	1.06	0.70
time (sec)	N/A	0.099	0.055	0.037	0.312	0.462	0.000	0.127	1.181

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	42	41	39	77	0	53	39
normalized size	1	1.00	0.63	0.61	0.58	1.15	0.00	0.79	0.58
time (sec)	N/A	0.125	0.060	0.037	0.308	0.484	0.000	0.132	1.170

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	51	50	55	96	0	69	56
normalized size	1	1.00	0.51	0.50	0.55	0.96	0.00	0.69	0.56
time (sec)	N/A	0.230	0.066	0.040	0.303	0.677	0.000	0.138	0.043

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	59	58	71	116	0	85	72
normalized size	1	1.00	0.44	0.44	0.53	0.87	0.00	0.64	0.54
time (sec)	N/A	0.342	0.070	0.036	0.304	0.419	0.000	0.159	1.174

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	29	49	28	124	0	28
normalized size	1	1.00	0.67	0.69	1.17	0.67	2.95	0.00	0.67
time (sec)	N/A	0.067	0.025	0.034	0.320	0.530	0.648	0.000	1.214

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	47	60	60	66	60	60
normalized size	1	1.00	0.62	1.27	1.62	1.62	1.78	1.62	1.62
time (sec)	N/A	0.060	0.022	0.030	0.307	0.514	0.078	0.121	0.035

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	30	37	37	36	37	37
normalized size	1	1.00	0.81	0.81	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.056	0.018	0.033	0.304	0.634	0.072	0.137	0.049

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	31	38	38	37	38	38
normalized size	1	1.00	0.81	0.84	1.03	1.03	1.00	1.03	1.03
time (sec)	N/A	0.059	0.016	0.032	0.304	0.925	0.070	0.141	0.046

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	17	18	18	15	18	15
normalized size	1	1.00	0.85	0.85	0.90	0.90	0.75	0.90	0.75
time (sec)	N/A	0.048	0.009	0.032	0.307	0.543	0.063	0.134	0.029

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	26	26	13	12	12	12	12	9
normalized size	1	1.86	1.86	0.93	0.86	0.86	0.86	0.86	0.64
time (sec)	N/A	0.012	0.008	0.030	0.321	0.543	0.055	0.120	0.023

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	31	30	29	20	31	29
normalized size	1	1.00	0.94	0.97	0.94	0.91	0.62	0.97	0.91
time (sec)	N/A	0.064	0.017	0.039	0.312	0.468	0.133	0.127	1.195

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	25	30	26	26	24	34	13
normalized size	1	1.00	1.79	2.14	1.86	1.86	1.71	2.43	0.93
time (sec)	N/A	0.051	0.009	0.038	0.309	0.650	0.162	0.137	1.191

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	47	47	49	21	46
normalized size	1	1.00	0.62	0.81	1.27	1.27	1.32	0.57	1.24
time (sec)	N/A	0.062	0.017	0.037	0.299	0.448	0.225	0.136	1.201

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	57	57	60	21	56
normalized size	1	1.00	0.62	0.81	1.54	1.54	1.62	0.57	1.51
time (sec)	N/A	0.061	0.018	0.038	0.318	0.458	0.276	0.139	0.087

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	155	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.148	0.334	0.000	0.484	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	192	259	126	0	216	214
normalized size	1	1.00	0.76	1.83	2.47	1.20	0.00	2.06	2.04
time (sec)	N/A	0.166	0.212	0.049	0.334	0.523	0.000	0.185	0.092

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	124	221	109	0	196	176
normalized size	1	1.00	0.82	1.59	2.83	1.40	0.00	2.51	2.26
time (sec)	N/A	0.141	0.153	0.043	0.318	0.549	0.000	0.180	1.202

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	130	181	103	0	154	139
normalized size	1	1.00	0.82	1.67	2.32	1.32	0.00	1.97	1.78
time (sec)	N/A	0.170	0.117	0.044	0.316	0.398	0.000	0.182	1.201

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	162	135	81	0	121	97
normalized size	1	1.00	0.82	2.49	2.08	1.25	0.00	1.86	1.49
time (sec)	N/A	0.185	0.085	0.051	0.309	0.509	0.000	0.172	1.204

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	345	95	120	0	108	63
normalized size	1	1.00	0.79	4.31	1.19	1.50	0.00	1.35	0.79
time (sec)	N/A	0.284	0.096	0.052	0.313	0.472	0.000	0.150	0.066

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	36	36	23	77	0	37	23
normalized size	1	1.00	1.09	1.09	0.70	2.33	0.00	1.12	0.70
time (sec)	N/A	0.105	0.063	0.037	0.314	0.519	0.000	0.165	0.037

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	41	41	39	95	0	53	39
normalized size	1	1.00	0.61	0.61	0.58	1.42	0.00	0.79	0.58
time (sec)	N/A	0.134	0.069	0.038	0.316	0.686	0.000	0.159	1.176

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	50	50	55	116	0	69	56
normalized size	1	1.00	0.53	0.53	0.59	1.23	0.00	0.73	0.60
time (sec)	N/A	0.275	0.069	0.037	0.304	0.701	0.000	0.145	1.186

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	58	58	71	134	0	85	72
normalized size	1	1.00	0.46	0.46	0.57	1.07	0.00	0.68	0.58
time (sec)	N/A	0.389	0.077	0.038	0.307	0.894	0.000	0.164	0.045

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	74	153	81	530	0	57
normalized size	1	1.00	0.76	1.12	2.32	1.23	8.03	0.00	0.86
time (sec)	N/A	0.081	0.096	0.037	0.332	0.656	1.181	0.000	1.354

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	31	45	59	59	63	42	59
normalized size	1	1.00	0.58	0.85	1.11	1.11	1.19	0.79	1.11
time (sec)	N/A	0.068	0.025	0.030	0.299	0.424	0.088	0.127	1.190

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	23	28	28	29	42	24
normalized size	1	1.00	0.81	0.72	0.88	0.88	0.91	1.31	0.75
time (sec)	N/A	0.055	0.017	0.030	0.304	0.484	0.080	0.127	0.044

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	29	37	37	37	42	37
normalized size	1	1.00	0.86	0.83	1.06	1.06	1.06	1.20	1.06
time (sec)	N/A	0.059	0.016	0.033	0.304	0.471	0.078	0.139	0.046

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	21	16	25	25	24	40	19
normalized size	1	1.00	1.24	0.94	1.47	1.47	1.41	2.35	1.12
time (sec)	N/A	0.046	0.014	0.031	0.305	0.592	0.070	0.137	0.031

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	25	24	28	26	50	26
normalized size	1	1.00	0.96	0.93	0.89	1.04	0.96	1.85	0.96
time (sec)	N/A	0.037	0.013	0.033	0.307	0.573	0.115	0.145	1.181

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	46	44	49	37	57	42
normalized size	1	1.00	0.75	0.96	0.92	1.02	0.77	1.19	0.88
time (sec)	N/A	0.068	0.024	0.038	0.305	0.432	0.208	0.136	0.058

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	42	51	51	51	50	25
normalized size	1	1.00	1.00	1.68	2.04	2.04	2.04	2.00	1.00
time (sec)	N/A	0.050	0.010	0.037	0.309	0.549	0.244	0.138	1.192

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	41	65	65	70	42	29
normalized size	1	1.00	0.60	0.79	1.25	1.25	1.35	0.81	0.56
time (sec)	N/A	0.068	0.020	0.040	0.305	0.626	0.313	0.143	1.230

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	31	42	77	77	80	42	29
normalized size	1	1.00	0.58	0.79	1.45	1.45	1.51	0.79	0.55
time (sec)	N/A	0.067	0.021	0.039	0.306	0.695	0.351	0.140	1.238

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	76	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.049	0.378	0.000	2.265	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	72	196	221	114	0	109	176
normalized size	1	1.00	0.57	1.54	1.74	0.90	0.00	0.86	1.39
time (sec)	N/A	0.352	0.207	0.048	0.322	1.838	0.000	0.163	1.228

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	176	181	104	0	90	140
normalized size	1	1.00	0.64	1.76	1.81	1.04	0.00	0.90	1.40
time (sec)	N/A	0.286	0.126	0.049	0.308	0.486	0.000	0.175	1.211

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	153	135	81	0	68	96
normalized size	1	1.00	0.82	2.35	2.08	1.25	0.00	1.05	1.48
time (sec)	N/A	0.167	0.083	0.045	0.308	0.604	0.000	0.146	0.064

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	34	76	55	47	0	33	24
normalized size	1	1.00	1.48	3.30	2.39	2.04	0.00	1.43	1.04
time (sec)	N/A	0.101	0.043	0.051	0.309	0.491	0.000	0.135	0.062

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	36	23	39	0	0	23
normalized size	1	1.00	0.96	1.29	0.82	1.39	0.00	0.00	0.82
time (sec)	N/A	0.101	0.062	0.035	0.304	0.566	0.000	0.000	0.035

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	34	41	39	57	0	45	38
normalized size	1	1.00	0.55	0.66	0.63	0.92	0.00	0.73	0.61
time (sec)	N/A	0.125	0.062	0.036	0.306	0.595	0.000	0.198	0.035

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	43	50	55	77	0	65	55
normalized size	1	1.00	0.45	0.53	0.58	0.81	0.00	0.68	0.58
time (sec)	N/A	0.229	0.067	0.037	0.312	0.581	0.000	0.206	0.040

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	51	58	71	95	0	85	71
normalized size	1	1.00	0.40	0.45	0.55	0.74	0.00	0.66	0.55
time (sec)	N/A	0.255	0.074	0.036	0.312	0.645	0.000	0.303	1.192

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.017	0.506	0.000	0.553	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	64	63	68	68	75	63
normalized size	1	1.00	0.62	0.70	0.69	0.75	0.75	0.82	0.69
time (sec)	N/A	0.071	0.023	0.036	0.307	0.813	0.162	0.144	1.168

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	53	52	57	56	64	52
normalized size	1	1.00	0.66	0.73	0.71	0.78	0.77	0.88	0.71
time (sec)	N/A	0.065	0.021	0.034	0.302	0.547	0.141	0.137	0.038

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	39	42	41	45	41	52	41
normalized size	1	1.00	0.71	0.76	0.75	0.82	0.75	0.95	0.75
time (sec)	N/A	0.058	0.016	0.035	0.304	0.637	0.122	0.128	0.044

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	24	28	24	35	26
normalized size	1	1.00	1.00	0.96	0.92	1.08	0.92	1.35	1.00
time (sec)	N/A	0.035	0.011	0.034	0.301	0.465	0.106	0.133	0.039

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	15	14
normalized size	1	1.00	1.00	1.07	1.00	1.00	0.86	1.07	1.00
time (sec)	N/A	0.055	0.010	0.039	0.304	0.461	0.062	0.123	0.039

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	30	29	23	20	25	12
normalized size	1	1.00	1.00	2.50	2.42	1.92	1.67	2.08	1.00
time (sec)	N/A	0.049	0.012	0.039	0.306	0.703	0.138	0.143	0.055

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	45	48	46	39	46	31
normalized size	1	1.00	0.97	1.36	1.45	1.39	1.18	1.39	0.94
time (sec)	N/A	0.063	0.022	0.043	0.303	0.731	0.217	0.146	0.066

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	63	76	54	51	46
normalized size	1	1.00	0.69	1.18	1.24	1.49	1.06	1.00	0.90
time (sec)	N/A	0.068	0.028	0.041	0.306	0.510	0.276	0.138	0.074

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	44	75	84	113	78	89	65
normalized size	1	1.00	0.64	1.09	1.22	1.64	1.13	1.29	0.94
time (sec)	N/A	0.080	0.034	0.042	0.307	0.458	0.364	0.136	1.195

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	96	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.069	0.366	0.000	0.533	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	86	542	244	114	0	0	199
normalized size	1	1.00	0.57	3.57	1.61	0.75	0.00	0.00	1.31
time (sec)	N/A	0.436	0.245	0.058	0.319	0.493	0.000	0.000	0.087

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	78	474	204	104	0	0	163
normalized size	1	1.00	0.60	3.67	1.58	0.81	0.00	0.00	1.26
time (sec)	N/A	0.350	0.170	0.056	0.307	0.570	0.000	0.000	1.194

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	68	422	156	81	0	0	117
normalized size	1	1.00	0.74	4.59	1.70	0.88	0.00	0.00	1.27
time (sec)	N/A	0.244	0.150	0.052	0.308	0.530	0.000	0.000	0.067

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	248	78	63	0	0	48
normalized size	1	1.00	1.02	4.68	1.47	1.19	0.00	0.00	0.91
time (sec)	N/A	0.198	0.069	0.047	0.305	0.670	0.000	0.000	1.167

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	22	22	0	0	22
normalized size	1	1.00	0.93	1.25	0.79	0.79	0.00	0.00	0.79
time (sec)	N/A	0.104	0.055	0.039	0.306	0.610	0.000	0.000	0.029

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	33	33	48	31	0	22	38
normalized size	1	1.00	1.57	1.57	2.29	1.48	0.00	1.05	1.81
time (sec)	N/A	0.101	0.061	0.035	0.307	0.450	0.000	0.145	1.168

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	50	65	58	0	0	50
normalized size	1	1.00	0.82	0.82	1.07	0.95	0.00	0.00	0.82
time (sec)	N/A	0.137	0.068	0.037	0.308	0.408	0.000	0.000	1.232

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	57	57	82	77	0	0	51
normalized size	1	1.00	0.61	0.61	0.87	0.82	0.00	0.00	0.54
time (sec)	N/A	0.313	0.071	0.036	0.307	0.498	0.000	0.000	1.207

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	66	66	97	96	0	0	60
normalized size	1	1.00	0.53	0.53	0.78	0.77	0.00	0.00	0.48
time (sec)	N/A	0.411	0.080	0.039	0.312	0.616	0.000	0.000	0.066

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	83	72	99	105	0	147	76
normalized size	1	1.00	0.33	0.28	0.39	0.41	0.00	0.58	0.30
time (sec)	N/A	0.221	0.060	0.039	0.330	0.582	0.000	0.207	1.456

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	75	64	83	94	0	124	102
normalized size	1	1.00	0.38	0.32	0.42	0.48	0.00	0.63	0.52
time (sec)	N/A	0.190	0.048	0.038	0.333	0.390	0.000	0.169	1.433

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	137	67	56	67	83	0	99	60
normalized size	1	1.19	0.58	0.49	0.58	0.72	0.00	0.86	0.52
time (sec)	N/A	0.171	0.041	0.037	0.326	0.459	0.000	0.158	1.408

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	89	57	48	45	64	0	74	50
normalized size	1	1.16	0.74	0.62	0.58	0.83	0.00	0.96	0.65
time (sec)	N/A	0.158	0.035	0.035	0.327	0.526	0.000	0.144	1.372

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	26	50	0	48	43
normalized size	1	1.00	1.48	1.21	0.90	1.72	0.00	1.66	1.48
time (sec)	N/A	0.034	0.022	0.033	0.330	0.625	0.000	0.159	1.310

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	82	0	239	0	91	-1
normalized size	1	1.00	0.84	0.69	0.00	2.03	0.00	0.77	-0.01
time (sec)	N/A	0.166	0.069	0.056	0.000	0.515	0.000	0.149	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	116	118	0	281	0	71	-1
normalized size	1	1.00	0.91	0.92	0.00	2.20	0.00	0.55	-0.01
time (sec)	N/A	0.185	0.115	0.059	0.000	0.465	0.000	0.183	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	123	167	0	337	0	89	-1
normalized size	1	1.00	0.64	0.87	0.00	1.75	0.00	0.46	-0.01
time (sec)	N/A	0.202	0.164	0.065	0.000	0.548	0.000	0.178	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	139	219	0	393	0	116	-1
normalized size	1	1.00	0.56	0.88	0.00	1.57	0.00	0.46	-0.00
time (sec)	N/A	0.221	0.174	0.064	0.000	1.149	0.000	0.203	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	32	60	170	205	32
normalized size	1	1.00	0.85	0.52	0.80	1.50	4.25	5.12	0.80
time (sec)	N/A	0.088	0.044	0.035	0.310	0.456	11.840	0.157	0.039

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	32	49	80	141	32
normalized size	1	1.00	0.85	0.52	0.80	1.22	2.00	3.52	0.80
time (sec)	N/A	0.088	0.044	0.042	0.307	0.694	7.829	0.150	0.034

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	21	32	32	61	71	32
normalized size	1	1.00	0.75	0.52	0.80	0.80	1.52	1.78	0.80
time (sec)	N/A	0.089	0.035	0.033	0.303	0.502	5.433	0.135	0.033

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	30	19	31	44	32
normalized size	1	1.00	0.61	0.53	0.79	0.50	0.82	1.16	0.84
time (sec)	N/A	0.078	0.029	0.034	0.314	0.580	2.886	0.152	0.029

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	30	29	49	32	19
normalized size	1	1.00	0.58	0.56	0.83	0.81	1.36	0.89	0.53
time (sec)	N/A	0.084	0.025	0.036	0.306	0.622	14.069	0.173	1.207

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	21	26	44	29	36	20
normalized size	1	1.00	0.89	0.55	0.68	1.16	0.76	0.95	0.53
time (sec)	N/A	0.086	0.050	0.038	0.304	0.520	29.050	0.130	0.030

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	24	56	31	34	20
normalized size	1	1.00	0.85	0.52	0.60	1.40	0.78	0.85	0.50
time (sec)	N/A	0.089	0.063	0.035	0.305	0.513	21.487	0.137	1.209

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	21	26	66	31	36	20
normalized size	1	1.00	0.85	0.52	0.65	1.65	0.78	0.90	0.50
time (sec)	N/A	0.086	0.061	0.035	0.319	0.677	40.688	0.133	1.190

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	77	64	106	105	0	132	110
normalized size	1	1.00	0.39	0.32	0.54	0.53	0.00	0.67	0.56
time (sec)	N/A	0.193	0.052	0.038	0.350	0.503	0.000	0.199	1.445

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	69	56	90	94	0	107	102
normalized size	1	1.00	0.50	0.41	0.66	0.69	0.00	0.78	0.74
time (sec)	N/A	0.173	0.043	0.039	0.337	0.595	0.000	0.206	1.437

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	59	48	74	83	0	84	93
normalized size	1	1.00	0.66	0.54	0.83	0.93	0.00	0.94	1.04
time (sec)	N/A	0.158	0.043	0.036	0.342	0.523	0.000	0.188	1.386

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	43	35	41	61	0	56	81
normalized size	1	1.00	1.39	1.13	1.32	1.97	0.00	1.81	2.61
time (sec)	N/A	0.037	0.034	0.036	0.343	0.584	0.000	0.152	1.385

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	105	107	0	250	0	105	-1
normalized size	1	1.00	0.64	0.66	0.00	1.53	0.00	0.64	-0.01
time (sec)	N/A	0.177	0.076	0.055	0.000	0.855	0.000	0.171	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	116	135	0	288	0	85	-1
normalized size	1	1.00	0.66	0.76	0.00	1.63	0.00	0.48	-0.01
time (sec)	N/A	0.180	0.154	0.067	0.000	0.505	0.000	0.167	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	125	174	0	341	0	89	-1
normalized size	1	1.00	0.67	0.93	0.00	1.82	0.00	0.48	-0.01
time (sec)	N/A	0.194	0.157	0.065	0.000	0.666	0.000	0.172	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	142	226	0	393	0	116	-1
normalized size	1	1.00	0.57	0.90	0.00	1.57	0.00	0.46	-0.00
time (sec)	N/A	0.218	0.154	0.070	0.000	0.612	0.000	0.218	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	147	278	0	449	0	140	-1
normalized size	1	1.00	0.48	0.91	0.00	1.46	0.00	0.46	-0.00
time (sec)	N/A	0.239	0.207	0.069	0.000	0.502	0.000	0.215	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	311	86	80	128	105	0	0	110
normalized size	1	1.60	0.44	0.41	0.66	0.54	0.00	0.00	0.57
time (sec)	N/A	0.226	0.054	0.039	0.333	0.527	0.000	0.000	1.408

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	254	78	72	112	94	0	0	102
normalized size	1	1.58	0.48	0.45	0.70	0.58	0.00	0.00	0.63
time (sec)	N/A	0.222	0.053	0.039	0.337	0.590	0.000	0.000	1.363

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	197	70	64	96	83	0	0	94
normalized size	1	1.54	0.55	0.50	0.75	0.65	0.00	0.00	0.73
time (sec)	N/A	0.203	0.045	0.039	0.336	0.563	0.000	0.000	1.371

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	137	60	56	72	64	0	0	82
normalized size	1	1.44	0.63	0.59	0.76	0.67	0.00	0.00	0.86
time (sec)	N/A	0.186	0.038	0.038	0.333	0.597	0.000	0.000	1.342

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	89	50	47	54	50	66	43	71
normalized size	1	1.44	0.81	0.76	0.87	0.81	1.06	0.69	1.15
time (sec)	N/A	0.153	0.030	0.036	0.330	0.576	42.840	0.145	1.276

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	35	29	44	0	0	34
normalized size	1	1.00	0.97	1.21	1.00	1.52	0.00	0.00	1.17
time (sec)	N/A	0.038	0.029	0.035	0.328	0.720	0.000	0.000	1.250

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	78	0	141	0	64	-1
normalized size	1	1.00	1.00	1.03	0.00	1.86	0.00	0.84	-0.01
time (sec)	N/A	0.180	0.064	0.057	0.000	0.592	0.000	0.144	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	116	123	0	281	0	0	-1
normalized size	1	1.00	0.85	0.90	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.137	0.059	0.000	0.485	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	125	172	0	341	0	88	-1
normalized size	1	1.00	0.65	0.89	0.00	1.77	0.00	0.46	-0.01
time (sec)	N/A	0.203	0.176	0.059	0.000	0.532	0.000	0.234	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	88	101	123	204	129	161	112
normalized size	1	1.00	0.64	0.74	0.90	1.49	0.94	1.18	0.82
time (sec)	N/A	0.156	0.086	0.039	0.405	0.509	76.389	0.165	0.082

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	87	109	182	110	134	95
normalized size	1	1.00	0.69	0.75	0.94	1.57	0.95	1.16	0.82
time (sec)	N/A	0.132	0.066	0.037	0.406	0.521	51.780	0.164	0.065

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	71	73	95	146	92	107	78
normalized size	1	1.00	0.75	0.77	1.00	1.54	0.97	1.13	0.82
time (sec)	N/A	0.120	0.047	0.040	0.405	0.596	32.540	0.141	1.248

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	79	119	73	77	61
normalized size	1	1.00	0.80	0.78	1.04	1.57	0.96	1.01	0.80
time (sec)	N/A	0.102	0.036	0.041	0.406	0.854	3.923	0.154	1.238

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	45	68	118	60	51	47
normalized size	1	1.00	1.00	0.78	1.17	2.03	1.03	0.88	0.81
time (sec)	N/A	0.099	0.028	0.040	0.410	0.564	18.118	0.158	0.074

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	29	52	88	41	36	28
normalized size	1	1.00	1.00	0.78	1.41	2.38	1.11	0.97	0.76
time (sec)	N/A	0.093	0.019	0.040	0.405	0.880	15.746	0.159	0.083

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	37	50	71	146	61	54	47
normalized size	1	1.00	0.65	0.88	1.25	2.56	1.07	0.95	0.82
time (sec)	N/A	0.103	0.024	0.042	0.416	0.707	14.832	0.137	1.234

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	39	64	81	196	82	73	65
normalized size	1	1.00	0.47	0.77	0.98	2.36	0.99	0.88	0.78
time (sec)	N/A	0.116	0.029	0.046	0.406	0.645	28.536	0.155	0.095

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	39	78	101	252	100	93	79
normalized size	1	1.00	0.38	0.75	0.97	2.42	0.96	0.89	0.76
time (sec)	N/A	0.126	0.037	0.046	0.414	0.482	21.753	0.134	0.094

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	84	88	152	105	0	0	110
normalized size	1	1.00	0.23	0.24	0.41	0.29	0.00	0.00	0.30
time (sec)	N/A	0.267	0.062	0.039	0.342	0.485	0.000	0.000	1.453

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	76	80	136	94	0	0	102
normalized size	1	1.00	0.24	0.26	0.44	0.30	0.00	0.00	0.33
time (sec)	N/A	0.236	0.050	0.038	0.340	0.708	0.000	0.000	1.410

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	68	72	120	83	0	0	94
normalized size	1	1.00	0.27	0.28	0.47	0.33	0.00	0.00	0.37
time (sec)	N/A	0.213	0.041	0.039	0.347	0.688	0.000	0.000	1.378

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	57	63	93	63	0	0	81
normalized size	1	1.00	0.29	0.32	0.48	0.32	0.00	0.00	0.42
time (sec)	N/A	0.192	0.036	0.039	0.340	0.715	0.000	0.000	1.356

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	48	55	75	50	0	0	71
normalized size	1	1.00	0.35	0.40	0.55	0.36	0.00	0.00	0.52
time (sec)	N/A	0.155	0.030	0.036	0.332	0.766	0.000	0.000	1.300

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	48	47	48	44	0	42	34
normalized size	1	1.00	0.56	0.55	0.56	0.52	0.00	0.49	0.40
time (sec)	N/A	0.143	0.033	0.036	0.338	0.612	0.000	0.157	1.398

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	41	35	45	43	0	41	32
normalized size	1	1.00	1.41	1.21	1.55	1.48	0.00	1.41	1.10
time (sec)	N/A	0.038	0.034	0.033	0.340	0.494	0.000	0.160	1.353

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	122	85	0	235	0	0	-1
normalized size	1	1.00	1.02	0.71	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.096	0.059	0.000	0.580	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	140	129	0	285	0	90	-1
normalized size	1	1.00	0.76	0.70	0.00	1.55	0.00	0.49	-0.01
time (sec)	N/A	0.198	0.157	0.067	0.000	0.636	0.000	0.216	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	41	67	110	57	0	105	94
normalized size	1	1.00	0.41	0.68	1.11	0.58	0.00	1.06	0.95
time (sec)	N/A	0.070	0.047	0.043	0.314	0.803	0.000	0.157	1.226

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	40	57	87	54	0	84	68
normalized size	1	1.00	0.51	0.72	1.10	0.68	0.00	1.06	0.86
time (sec)	N/A	0.047	0.029	0.041	0.312	0.521	0.000	0.148	0.044

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	21	22	50	24	0	29	18
normalized size	1	1.00	1.17	1.22	2.78	1.33	0.00	1.61	1.00
time (sec)	N/A	0.053	0.022	0.035	0.311	0.550	0.000	0.151	1.207

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	39	48	83	51	0	110	63
normalized size	1	1.00	1.11	1.37	2.37	1.46	0.00	3.14	1.80
time (sec)	N/A	0.047	0.023	0.039	0.319	0.445	0.000	0.136	0.035

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	52	79	138	66	0	130	118
normalized size	1	1.00	0.39	0.59	1.04	0.50	0.00	0.98	0.89
time (sec)	N/A	0.111	0.049	0.044	0.311	0.458	0.000	0.140	0.048

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	47	69	112	61	0	107	94
normalized size	1	1.00	0.44	0.65	1.06	0.58	0.00	1.01	0.89
time (sec)	N/A	0.090	0.037	0.040	0.317	0.450	0.000	0.135	0.046

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	52	70	138	66	0	130	118
normalized size	1	1.00	0.73	0.99	1.94	0.93	0.00	1.83	1.66
time (sec)	N/A	0.116	0.047	0.040	0.318	0.476	0.000	0.132	1.187

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	60	112	61	0	107	90
normalized size	1	1.00	0.89	1.13	2.11	1.15	0.00	2.02	1.70
time (sec)	N/A	0.090	0.034	0.040	0.306	0.550	0.000	0.155	1.183

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	15	16	26	15	0	29	26
normalized size	1	1.00	0.68	0.73	1.18	0.68	0.00	1.32	1.18
time (sec)	N/A	0.054	0.026	0.035	0.311	0.440	0.000	0.141	0.055

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	35	31	31	29	32	14
normalized size	1	1.00	0.82	1.59	1.41	1.41	1.32	1.45	0.64
time (sec)	N/A	0.066	0.011	0.048	0.321	0.611	4.033	0.126	0.029

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	106	74	66	0	84	43
normalized size	1	1.00	0.87	2.26	1.57	1.40	0.00	1.79	0.91
time (sec)	N/A	0.135	0.059	0.049	0.308	0.579	0.000	0.131	1.190

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	38	106	44	61	0	45	28
normalized size	1	1.00	1.15	3.21	1.33	1.85	0.00	1.36	0.85
time (sec)	N/A	0.132	0.026	0.045	0.308	0.543	0.000	0.137	1.193

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	110	44	44	0	45	28
normalized size	1	1.00	0.80	2.44	0.98	0.98	0.00	1.00	0.62
time (sec)	N/A	0.083	0.048	0.050	0.312	0.427	0.000	0.126	0.028

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	21	11	11	8	11	11
normalized size	1	1.00	0.86	1.00	0.52	0.52	0.38	0.52	0.52
time (sec)	N/A	0.065	0.012	0.037	0.308	0.635	5.384	0.127	0.175

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	43	146	56	84	0	65	40
normalized size	1	1.00	0.78	2.65	1.02	1.53	0.00	1.18	0.73
time (sec)	N/A	0.155	0.065	0.051	0.319	0.606	0.000	0.151	0.044

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	13	31	0	21	13
normalized size	1	1.00	1.00	0.92	0.54	1.29	0.00	0.88	0.54
time (sec)	N/A	0.071	0.016	0.034	0.309	0.514	0.000	0.138	0.021

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.192	0.031	0.361	0.000	0.662	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	64	49	55	69	105	102	57
normalized size	1	1.00	0.46	0.35	0.39	0.49	0.75	0.73	0.41
time (sec)	N/A	0.215	0.037	0.038	0.323	0.506	50.078	0.173	1.379

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	56	41	41	61	71	78	49
normalized size	1	1.00	0.61	0.45	0.45	0.66	0.77	0.85	0.53
time (sec)	N/A	0.173	0.028	0.036	0.337	0.743	28.092	0.150	1.367

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	35	26	50	0	48	43
normalized size	1	1.00	1.48	1.21	0.90	1.72	0.00	1.66	1.48
time (sec)	N/A	0.033	0.020	0.033	0.341	1.174	0.000	0.164	0.002

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	70	0	207	0	86	-1
normalized size	1	1.00	0.80	0.74	0.00	2.20	0.00	0.91	-0.01
time (sec)	N/A	0.195	0.056	0.053	0.000	0.532	0.000	0.186	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	76	78	0	229	0	100	-1
normalized size	1	1.00	0.78	0.80	0.00	2.36	0.00	1.03	-0.01
time (sec)	N/A	0.195	0.043	0.059	0.000	0.441	0.000	0.202	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	48	45	74	44	83	189	83
normalized size	1	1.00	0.48	0.45	0.73	0.44	0.82	1.87	0.82
time (sec)	N/A	0.225	0.075	0.039	0.325	0.447	3.535	0.145	0.040

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	40	37	60	36	68	142	66
normalized size	1	1.00	0.50	0.46	0.75	0.45	0.85	1.78	0.82
time (sec)	N/A	0.224	0.057	0.037	0.314	0.493	3.305	0.142	0.055

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	31	28	44	27	48	92	46
normalized size	1	1.00	0.54	0.49	0.77	0.47	0.84	1.61	0.81
time (sec)	N/A	0.152	0.048	0.033	0.314	0.496	3.894	0.133	0.050

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	30	19	31	44	32
normalized size	1	1.00	0.61	0.53	0.79	0.50	0.82	1.16	0.84
time (sec)	N/A	0.081	0.029	0.033	0.316	0.462	2.896	0.136	0.002

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	49	82	39	40	31
normalized size	1	1.00	1.00	0.82	1.26	2.10	1.00	1.03	0.79
time (sec)	N/A	0.198	0.032	0.039	0.399	0.463	4.095	0.140	1.191

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	45	62	97	119	48	34
normalized size	1	1.00	1.00	1.07	1.48	2.31	2.83	1.14	0.81
time (sec)	N/A	0.203	0.032	0.045	0.419	0.581	8.453	0.136	0.060

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	65	103	117	270	76	54
normalized size	1	1.00	0.81	0.96	1.51	1.72	3.97	1.12	0.79
time (sec)	N/A	0.210	0.053	0.046	0.618	0.564	18.116	0.139	1.232

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	80	134	133	439	104	74
normalized size	1	1.00	0.71	0.90	1.51	1.49	4.93	1.17	0.83
time (sec)	N/A	0.219	0.066	0.047	1.079	0.635	17.811	0.147	1.202

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	71	93	163	149	639	131	91
normalized size	1	1.00	0.65	0.85	1.48	1.35	5.81	1.19	0.83
time (sec)	N/A	0.234	0.074	0.046	0.451	0.527	28.190	0.127	0.066

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	130	161	0	304	0	185	-1
normalized size	1	1.00	0.42	0.52	0.00	0.98	0.00	0.60	-0.00
time (sec)	N/A	0.324	0.131	0.059	0.000	0.718	0.000	0.199	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	122	143	0	288	0	139	-1
normalized size	1	1.00	0.47	0.55	0.00	1.10	0.00	0.53	-0.00
time (sec)	N/A	0.300	0.094	0.060	0.000	0.566	0.000	0.196	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	114	125	0	272	0	137	-1
normalized size	1	1.00	0.54	0.59	0.00	1.29	0.00	0.65	-0.00
time (sec)	N/A	0.236	0.081	0.057	0.000	0.533	0.000	0.179	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	105	107	0	250	0	105	-1
normalized size	1	1.00	0.64	0.66	0.00	1.53	0.00	0.64	-0.01
time (sec)	N/A	0.187	0.066	0.053	0.000	0.574	0.000	0.179	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	120	107	0	352	0	136	-1
normalized size	1	1.00	0.71	0.63	0.00	2.07	0.00	0.80	-0.01
time (sec)	N/A	0.240	0.068	0.061	0.000	1.227	0.000	0.207	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	120	117	0	390	0	164	-1
normalized size	1	1.00	0.70	0.68	0.00	2.27	0.00	0.95	-0.01
time (sec)	N/A	0.251	0.084	0.066	0.000	0.583	0.000	0.227	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	132	144	0	428	0	194	-1
normalized size	1	1.00	0.59	0.64	0.00	1.91	0.00	0.87	-0.00
time (sec)	N/A	0.265	0.165	0.066	0.000	0.484	0.000	0.209	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	140	165	0	444	0	222	-1
normalized size	1	1.00	0.51	0.60	0.00	1.62	0.00	0.81	-0.00
time (sec)	N/A	0.282	0.228	0.072	0.000	0.722	0.000	0.218	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	148	186	0	460	0	249	-1
normalized size	1	1.00	0.46	0.58	0.00	1.43	0.00	0.77	-0.00
time (sec)	N/A	0.302	0.272	0.072	0.000	0.657	0.000	0.222	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	46	37	27	33	197	0	48
normalized size	1	1.00	0.32	0.26	0.19	0.23	1.37	0.00	0.33
time (sec)	N/A	0.128	0.024	0.037	0.318	0.634	91.982	0.000	1.325

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	41	32	22	28	0	0	38
normalized size	1	1.00	0.38	0.30	0.21	0.26	0.00	0.00	0.36
time (sec)	N/A	0.107	0.018	0.037	1.028	0.540	0.000	0.000	1.259

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	46	34	22	45	0	58	35
normalized size	1	1.00	0.44	0.33	0.21	0.43	0.00	0.56	0.34
time (sec)	N/A	0.126	0.024	0.036	0.990	0.496	0.000	0.172	1.309

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	29	17	40	0	44	30
normalized size	1	1.00	0.60	0.43	0.25	0.59	0.00	0.65	0.44
time (sec)	N/A	0.099	0.018	0.037	0.461	0.412	0.000	0.158	1.250

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	39	30	20	26	133	0	38
normalized size	1	1.00	0.36	0.28	0.19	0.24	1.24	0.00	0.36
time (sec)	N/A	0.106	0.016	0.036	0.456	0.407	7.926	0.000	1.252

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	34	25	15	21	39	0	21
normalized size	1	1.00	0.49	0.36	0.21	0.30	0.56	0.00	0.30
time (sec)	N/A	0.081	0.017	0.036	0.889	0.524	4.832	0.000	1.235

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	41	29	17	40	46	44	30
normalized size	1	1.00	0.58	0.41	0.24	0.56	0.65	0.62	0.42
time (sec)	N/A	0.103	0.016	0.033	1.276	0.630	16.155	0.179	1.267

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	C	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	34	24	12	33	29	27	25
normalized size	1	1.00	1.70	1.20	0.60	1.65	1.45	1.35	1.25
time (sec)	N/A	0.020	0.014	0.034	0.500	0.597	6.801	0.152	1.272

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	26	25	13	21	48	0	21
normalized size	1	1.00	0.36	0.34	0.18	0.29	0.66	0.00	0.29
time (sec)	N/A	0.097	0.017	0.035	0.997	0.627	7.823	0.000	1.242

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	22	7	18	19	13	18
normalized size	1	1.00	0.64	0.67	0.21	0.55	0.58	0.39	0.55
time (sec)	N/A	0.075	0.010	0.034	0.374	0.589	7.493	0.142	1.224

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	69	65	0	72	0	0	-1
normalized size	1	1.00	0.55	0.52	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.053	0.047	0.000	0.522	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	63	54	0	66	0	0	-1
normalized size	1	1.00	0.70	0.60	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.030	0.045	0.000	0.477	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	65	47	0	46	0	0	-1
normalized size	1	1.00	0.70	0.51	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.043	0.047	0.000	0.874	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	37	0	26	0	0	-1
normalized size	1	1.00	0.71	0.64	0.00	0.45	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.021	0.046	0.000	0.638	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	75	90	0	84	0	54	-1
normalized size	1	1.00	0.58	0.69	0.00	0.65	0.00	0.42	-0.01
time (sec)	N/A	0.132	0.081	0.056	0.000	0.514	0.000	0.134	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	58	79	0	76	0	44	-1
normalized size	1	1.00	0.64	0.88	0.00	0.84	0.00	0.49	-0.01
time (sec)	N/A	0.110	0.100	0.053	0.000	0.513	0.000	0.139	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	102	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.084	0.358	0.000	0.555	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	185	67	64	83	69	0	0	88
normalized size	1	1.30	0.47	0.45	0.58	0.49	0.00	0.00	0.62
time (sec)	N/A	0.237	0.039	0.038	0.409	0.575	0.000	0.000	1.286

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	137	58	55	69	60	105	69	57
normalized size	1	1.32	0.56	0.53	0.66	0.58	1.01	0.66	0.55
time (sec)	N/A	0.194	0.036	0.036	0.338	0.658	90.917	0.139	1.287

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	89	50	47	54	50	66	43	71
normalized size	1	1.44	0.81	0.76	0.87	0.81	1.06	0.69	1.15
time (sec)	N/A	0.153	0.027	0.033	0.338	0.526	43.137	0.131	0.002

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	74	80	0	206	0	40	-1
normalized size	1	1.00	0.79	0.85	0.00	2.19	0.00	0.43	-0.01
time (sec)	N/A	0.209	0.053	0.055	0.000	0.453	0.000	0.136	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	78	92	0	232	0	48	-1
normalized size	1	1.00	0.81	0.96	0.00	2.42	0.00	0.50	-0.01
time (sec)	N/A	0.204	0.044	0.063	0.000	0.696	0.000	0.161	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	85	101	123	168	126	159	114
normalized size	1	1.00	0.61	0.73	0.88	1.21	0.91	1.14	0.82
time (sec)	N/A	0.264	0.138	0.043	0.406	0.479	7.535	0.146	0.084

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	75	97	153	95	105	80
normalized size	1	1.00	0.80	0.77	1.00	1.58	0.98	1.08	0.82
time (sec)	N/A	0.248	0.101	0.040	0.415	0.555	6.248	0.140	1.258

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	70	73	95	136	92	105	80
normalized size	1	1.00	0.72	0.75	0.98	1.40	0.95	1.08	0.82
time (sec)	N/A	0.170	0.094	0.042	0.420	0.628	5.291	0.123	0.092

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	79	119	73	77	61
normalized size	1	1.00	0.80	0.78	1.04	1.57	0.96	1.01	0.80
time (sec)	N/A	0.099	0.033	0.038	0.417	0.504	3.852	0.144	0.002

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	58	98	157	80	67	57
normalized size	1	1.00	1.00	0.78	1.32	2.12	1.08	0.91	0.77
time (sec)	N/A	0.232	0.034	0.046	0.414	0.507	5.491	0.129	0.084

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	71	111	176	162	71	61
normalized size	1	1.00	1.00	0.91	1.42	2.26	2.08	0.91	0.78
time (sec)	N/A	0.226	0.044	0.047	0.419	0.601	8.103	0.137	1.244

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	93	95	152	204	352	106	88
normalized size	1	1.00	0.88	0.90	1.43	1.92	3.32	1.00	0.83
time (sec)	N/A	0.260	0.084	0.050	0.422	0.592	14.857	0.157	1.235

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	101	110	183	220	614	133	105
normalized size	1	1.00	0.80	0.87	1.44	1.73	4.83	1.05	0.83
time (sec)	N/A	0.278	0.103	0.047	0.427	0.572	18.384	0.132	0.128

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	109	123	212	236	991	160	122
normalized size	1	1.00	0.74	0.83	1.43	1.59	6.70	1.08	0.82
time (sec)	N/A	0.305	0.126	0.049	0.418	0.407	32.761	0.153	1.262

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	73	80	117	77	0	0	74
normalized size	1	1.00	0.26	0.28	0.42	0.27	0.00	0.00	0.26
time (sec)	N/A	0.286	0.047	0.039	0.345	0.517	0.000	0.000	1.415

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	65	72	104	69	0	0	88
normalized size	1	1.00	0.28	0.31	0.45	0.30	0.00	0.00	0.38
time (sec)	N/A	0.261	0.039	0.037	0.346	0.650	0.000	0.000	1.340

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	57	64	91	61	0	0	58
normalized size	1	1.00	0.31	0.35	0.50	0.34	0.00	0.00	0.32
time (sec)	N/A	0.216	0.035	0.036	0.344	0.719	0.000	0.000	1.331

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	48	55	75	50	0	0	71
normalized size	1	1.00	0.35	0.40	0.55	0.36	0.00	0.00	0.52
time (sec)	N/A	0.163	0.029	0.037	0.344	0.530	0.000	0.000	0.002

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	78	80	0	207	0	0	-1
normalized size	1	1.00	0.56	0.57	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.122	0.060	0.000	0.641	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	79	86	0	233	0	0	-1
normalized size	1	1.00	0.56	0.61	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.085	0.067	0.000	0.426	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	90	103	0	262	0	0	-1
normalized size	1	1.00	0.47	0.54	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.091	0.068	0.000	0.626	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	98	114	0	278	0	0	-1
normalized size	1	1.00	0.41	0.48	0.00	1.17	0.00	0.00	-0.00
time (sec)	N/A	0.246	0.100	0.067	0.000	0.537	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	106	125	0	294	0	0	-1
normalized size	1	1.00	0.37	0.44	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.264	0.101	0.069	0.000	0.599	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	116	104	0	0	0	0	223
normalized size	1	1.00	0.42	0.37	0.00	0.00	0.00	0.00	0.80
time (sec)	N/A	0.268	0.081	0.043	0.000	0.548	0.000	0.000	1.791

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	78	61	0	0	0	0	140
normalized size	1	1.00	0.61	0.48	0.00	0.00	0.00	0.00	1.10
time (sec)	N/A	0.157	0.051	0.036	0.000	0.544	0.000	0.000	1.339

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	58	34	0	0	0	0	55
normalized size	1	1.00	1.61	0.94	0.00	0.00	0.00	0.00	1.53
time (sec)	N/A	0.030	0.019	0.036	0.000	0.499	0.000	0.000	1.263

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.029	0.368	0.000	0.493	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.042	0.367	0.000	0.516	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.055	0.367	0.000	0.597	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.976	0.357	0.000	0.585	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	0	0	0	-1
normalized size	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	1.228	0.356	0.000	0.561	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.549	0.058	0.000	0.526	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	87	0	0	0	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.208	0.359	0.000	0.558	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	33	0	59	0	0	32
normalized size	1	1.00	0.69	0.69	0.00	1.23	0.00	0.00	0.67
time (sec)	N/A	0.109	0.180	0.038	0.000	0.441	0.000	0.000	1.527

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	64	46	0	125	0	0	113
normalized size	1	1.00	0.62	0.44	0.00	1.20	0.00	0.00	1.09
time (sec)	N/A	0.149	0.247	0.038	0.000	0.535	0.000	0.000	1.647

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	83	68	0	229	0	0	180
normalized size	1	1.00	0.37	0.30	0.00	1.02	0.00	0.00	0.80
time (sec)	N/A	0.262	0.312	0.039	0.000	0.909	0.000	0.000	1.806

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	103	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.103	0.369	0.000	0.528	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	101	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.076	0.385	0.000	0.609	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.065	0.388	0.000	0.514	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.059	0.358	0.000	0.695	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	94	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.042	0.354	0.000	0.625	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	117	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.116	0.347	0.000	0.615	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	138	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	0.172	0.350	0.000	0.573	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	175	224	224	156	0	211	183
normalized size	1	1.00	1.54	1.96	1.96	1.37	0.00	1.85	1.61
time (sec)	N/A	0.266	0.200	0.053	0.406	0.436	0.000	0.181	1.317

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	167	200	201	146	0	180	163
normalized size	1	1.00	1.90	2.27	2.28	1.66	0.00	2.05	1.85
time (sec)	N/A	0.191	0.161	0.055	0.416	0.472	0.000	0.160	0.110

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	158	168	125	119	0	113	90
normalized size	1	1.00	2.55	2.71	2.02	1.92	0.00	1.82	1.45
time (sec)	N/A	0.096	0.174	0.047	0.405	0.417	0.000	0.162	1.223

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	63	66	48	0	86	60
normalized size	1	1.00	1.15	2.33	2.44	1.78	0.00	3.19	2.22
time (sec)	N/A	0.029	0.043	0.044	0.401	0.559	0.000	0.132	0.063

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	250	116	94	0	128	62
normalized size	1	1.00	0.90	3.57	1.66	1.34	0.00	1.83	0.89
time (sec)	N/A	0.196	0.103	0.055	0.304	0.539	0.000	0.152	1.222

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	94	339	137	134	0	148	104
normalized size	1	1.00	0.90	3.23	1.30	1.28	0.00	1.41	0.99
time (sec)	N/A	0.293	0.070	0.057	0.306	0.733	0.000	0.174	0.100

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	431	153	170	0	166	121
normalized size	1	1.00	0.75	3.12	1.11	1.23	0.00	1.20	0.88
time (sec)	N/A	0.393	0.082	0.057	0.307	0.580	0.000	0.177	1.216

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	112	523	169	204	0	182	137
normalized size	1	1.00	0.65	3.06	0.99	1.19	0.00	1.06	0.80
time (sec)	N/A	0.501	0.095	0.062	0.302	0.570	0.000	0.178	0.109

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	63	60	57	67	63	58	51
normalized size	1	1.00	1.03	0.98	0.93	1.10	1.03	0.95	0.84
time (sec)	N/A	0.137	0.241	0.039	0.303	0.533	0.270	0.119	0.074

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	39	37	43	39	38	35
normalized size	1	1.00	1.05	0.98	0.92	1.08	0.98	0.95	0.88
time (sec)	N/A	0.130	0.173	0.040	0.307	0.623	0.184	0.145	0.050

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	38	37	45	37	38	35
normalized size	1	1.00	1.05	0.97	0.95	1.15	0.95	0.97	0.90
time (sec)	N/A	0.128	0.133	0.039	0.302	0.735	0.157	0.121	1.180

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	21	15	16	19
normalized size	1	1.00	1.00	1.06	1.00	1.31	0.94	1.00	1.19
time (sec)	N/A	0.121	0.103	0.037	0.302	0.490	0.091	0.134	0.034

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	11
normalized size	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	1.00
time (sec)	N/A	0.075	0.043	0.034	0.297	0.659	0.077	0.124	1.169

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	36	35	40	26	36	34
normalized size	1	1.00	0.81	0.97	0.95	1.08	0.70	0.97	0.92
time (sec)	N/A	0.124	0.032	0.041	0.301	0.562	0.148	0.138	0.060

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	51	55	70	49	42	54
normalized size	1	1.00	0.96	0.96	1.04	1.32	0.92	0.79	1.02
time (sec)	N/A	0.155	0.112	0.040	0.304	0.562	0.232	0.125	1.214

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	66	75	100	73	50	71
normalized size	1	1.00	0.86	0.90	1.03	1.37	1.00	0.68	0.97
time (sec)	N/A	0.164	0.132	0.043	0.306	0.493	0.328	0.138	0.078

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	81	93	126	94	58	90
normalized size	1	1.00	0.82	0.93	1.07	1.45	1.08	0.67	1.03
time (sec)	N/A	0.173	0.171	0.041	0.305	1.503	0.420	0.139	0.089

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	175	233	223	156	0	212	183
normalized size	1	1.00	1.70	2.26	2.17	1.51	0.00	2.06	1.78
time (sec)	N/A	0.133	0.233	0.053	0.408	0.746	0.000	0.180	0.130

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	51	105	151	85	0	165	119
normalized size	1	1.00	0.84	1.72	2.48	1.39	0.00	2.70	1.95
time (sec)	N/A	0.056	0.082	0.048	0.411	0.492	0.000	0.163	1.227

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	154	174	125	114	0	125	90
normalized size	1	1.00	2.44	2.76	1.98	1.81	0.00	1.98	1.43
time (sec)	N/A	0.131	0.163	0.050	0.410	0.529	0.000	0.173	1.213

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	73	145	114	88	0	108	82
normalized size	1	1.00	1.49	2.96	2.33	1.80	0.00	2.20	1.67
time (sec)	N/A	0.174	0.118	0.051	0.408	0.599	0.000	0.164	0.095

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	346	133	128	0	148	100
normalized size	1	1.00	0.67	3.30	1.27	1.22	0.00	1.41	0.95
time (sec)	N/A	0.302	0.137	0.058	0.318	0.576	0.000	0.177	1.239

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	438	153	170	0	166	120
normalized size	1	1.00	0.75	3.17	1.11	1.23	0.00	1.20	0.87
time (sec)	N/A	0.402	0.081	0.058	0.318	0.474	0.000	0.165	0.091

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	112	530	169	204	0	182	137
normalized size	1	1.00	0.68	3.21	1.02	1.24	0.00	1.10	0.83
time (sec)	N/A	0.521	0.091	0.064	0.311	0.602	0.000	0.181	0.110

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	120	622	185	240	0	198	153
normalized size	1	1.00	0.59	3.05	0.91	1.18	0.00	0.97	0.75
time (sec)	N/A	0.640	0.104	0.068	0.315	0.583	0.000	0.184	1.245

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	61	59	67	63	123	51
normalized size	1	1.00	1.03	0.95	0.92	1.05	0.98	1.92	0.80
time (sec)	N/A	0.137	0.299	0.041	0.308	0.464	0.276	0.121	0.064

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	31	36	31	59	27
normalized size	1	1.00	1.00	0.90	1.03	1.20	1.03	1.97	0.90
time (sec)	N/A	0.126	0.201	0.038	0.306	0.449	0.154	0.124	0.047

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	37	34	43	37	98	31
normalized size	1	1.00	1.05	0.97	0.89	1.13	0.97	2.58	0.82
time (sec)	N/A	0.127	0.151	0.041	0.309	0.474	0.161	0.150	1.184

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	28	27	32	26	94	25
normalized size	1	1.00	1.07	1.04	1.00	1.19	0.96	3.48	0.93
time (sec)	N/A	0.121	0.114	0.040	0.313	0.542	0.116	0.120	1.181

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	24	23	17	55	24
normalized size	1	1.00	1.00	1.00	0.96	0.92	0.68	2.20	0.96
time (sec)	N/A	0.082	0.042	0.039	0.306	0.655	0.214	0.118	0.069

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	51	49	64	41	74	48
normalized size	1	1.00	0.96	0.96	0.92	1.21	0.77	1.40	0.91
time (sec)	N/A	0.133	0.035	0.039	0.305	0.540	0.233	0.126	0.062

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	66	75	100	73	94	71
normalized size	1	1.00	0.89	0.93	1.06	1.41	1.03	1.32	1.00
time (sec)	N/A	0.158	0.120	0.042	0.311	0.516	0.326	0.140	0.075

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	81	93	126	94	109	90
normalized size	1	1.00	0.80	0.91	1.04	1.42	1.06	1.22	1.01
time (sec)	N/A	0.172	0.157	0.040	0.311	0.460	0.423	0.123	1.225

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	79	96	113	154	114	124	109
normalized size	1	1.00	0.75	0.91	1.08	1.47	1.09	1.18	1.04
time (sec)	N/A	0.183	0.188	0.043	0.306	0.505	0.531	0.138	1.252

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	175	290	223	156	0	265	185
normalized size	1	1.00	1.30	2.15	1.65	1.16	0.00	1.96	1.37
time (sec)	N/A	0.438	0.155	0.055	0.413	0.581	0.000	0.175	0.121

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	167	266	201	143	0	232	163
normalized size	1	1.00	1.58	2.51	1.90	1.35	0.00	2.19	1.54
time (sec)	N/A	0.331	0.132	0.056	0.414	0.545	0.000	0.181	1.255

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	55	227	126	113	0	130	90
normalized size	1	1.00	0.71	2.95	1.64	1.47	0.00	1.69	1.17
time (sec)	N/A	0.238	0.218	0.054	0.418	0.539	0.000	0.170	1.224

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	73	137	114	88	0	85	82
normalized size	1	1.00	1.49	2.80	2.33	1.80	0.00	1.73	1.67
time (sec)	N/A	0.145	0.110	0.052	0.411	0.518	0.000	0.148	0.071

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	28	44	27	0	24	39
normalized size	1	1.00	1.00	1.47	2.32	1.42	0.00	1.26	2.05
time (sec)	N/A	0.040	0.073	0.036	0.315	1.500	0.000	0.138	0.053

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	256	120	97	0	0	62
normalized size	1	1.00	0.95	3.51	1.64	1.33	0.00	0.00	0.85
time (sec)	N/A	0.111	0.041	0.056	0.319	0.799	0.000	0.000	1.227

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	94	344	137	134	0	0	105
normalized size	1	1.00	0.90	3.28	1.30	1.28	0.00	0.00	1.00
time (sec)	N/A	0.293	0.071	0.057	0.312	0.556	0.000	0.000	1.246

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	436	153	170	0	59	121
normalized size	1	1.00	0.75	3.16	1.11	1.23	0.00	0.43	0.88
time (sec)	N/A	0.390	0.088	0.061	0.308	0.604	0.000	0.186	1.238

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	64	60	71	56	62	61
normalized size	1	1.00	1.03	0.98	0.92	1.09	0.86	0.95	0.94
time (sec)	N/A	0.145	0.194	0.042	0.304	0.469	0.422	0.143	0.095

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	53	51	62	42	53	51
normalized size	1	1.00	1.04	0.98	0.94	1.15	0.78	0.98	0.94
time (sec)	N/A	0.141	0.148	0.040	0.311	0.492	0.349	0.138	0.078

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	41	40	43	31	42	40
normalized size	1	1.00	1.05	1.02	1.00	1.08	0.78	1.05	1.00
time (sec)	N/A	0.135	0.113	0.041	0.310	0.659	0.274	0.141	1.223

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	22	17	25	23
normalized size	1	1.00	1.00	1.04	1.00	0.96	0.74	1.09	1.00
time (sec)	N/A	0.082	0.044	0.041	0.306	0.498	0.207	0.138	0.064

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	21	20	19	17	21	19
normalized size	1	1.00	1.10	1.05	1.00	0.95	0.85	1.05	0.95
time (sec)	N/A	0.115	0.019	0.033	0.309	0.934	0.099	0.134	1.201

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	39	35	34	27	34	36	17
normalized size	1	1.00	2.17	1.94	1.89	1.50	1.89	2.00	0.94
time (sec)	N/A	0.137	0.091	0.038	0.310	0.571	0.154	0.119	1.238

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	50	53	59	56	51	52
normalized size	1	1.00	0.98	0.88	0.93	1.04	0.98	0.89	0.91
time (sec)	N/A	0.150	0.125	0.043	0.303	0.749	0.305	0.127	0.090

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	65	69	93	73	57	68
normalized size	1	1.00	0.97	0.87	0.92	1.24	0.97	0.76	0.91
time (sec)	N/A	0.162	0.171	0.043	0.309	0.546	0.385	0.143	0.096

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	567	672	246	157	0	0	211
normalized size	1	1.00	3.46	4.10	1.50	0.96	0.00	0.00	1.29
time (sec)	N/A	0.552	1.192	0.061	0.431	0.577	0.000	0.000	0.137

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	663	450	225	146	0	0	190
normalized size	1	1.00	4.91	3.33	1.67	1.08	0.00	0.00	1.41
time (sec)	N/A	0.430	0.459	0.058	0.408	0.809	0.000	0.000	1.273

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	424	600	149	120	0	0	117
normalized size	1	1.00	4.04	5.71	1.42	1.14	0.00	0.00	1.11
time (sec)	N/A	0.326	0.424	0.054	0.418	0.634	0.000	0.000	0.102

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	234	376	135	92	0	0	107
normalized size	1	1.00	3.12	5.01	1.80	1.23	0.00	0.00	1.43
time (sec)	N/A	0.224	0.544	0.053	0.421	0.646	0.000	0.000	0.093

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	250	120	69	0	0	87
normalized size	1	1.00	0.85	3.47	1.67	0.96	0.00	0.00	1.21
time (sec)	N/A	0.173	0.106	0.053	0.305	0.583	0.000	0.000	0.056

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	69	250	125	67	0	0	90
normalized size	1	1.00	0.93	3.38	1.69	0.91	0.00	0.00	1.22
time (sec)	N/A	0.101	0.045	0.057	0.304	0.603	0.000	0.000	1.188

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	44	92	42	0	42	41
normalized size	1	1.00	0.73	0.98	2.04	0.93	0.00	0.93	0.91
time (sec)	N/A	0.049	0.025	0.036	0.304	0.532	0.000	0.130	0.069

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	94	523	160	134	0	0	128
normalized size	1	1.00	0.85	4.71	1.44	1.21	0.00	0.00	1.15
time (sec)	N/A	0.150	0.069	0.065	0.311	0.514	0.000	0.000	1.235

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	615	176	170	0	0	144
normalized size	1	1.00	0.75	4.46	1.28	1.23	0.00	0.00	1.04
time (sec)	N/A	0.390	0.093	0.068	0.319	0.606	0.000	0.000	0.087

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	279	109	166	0	437	0	0	-1
normalized size	1	1.19	0.46	0.71	0.00	1.86	0.00	0.00	-0.00
time (sec)	N/A	0.164	0.125	0.067	0.000	0.559	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	221	101	149	0	415	0	0	-1
normalized size	1	1.13	0.52	0.76	0.00	2.12	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.092	0.069	0.000	0.715	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	189	89	132	0	381	0	0	-1
normalized size	1	1.20	0.57	0.84	0.00	2.43	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.087	0.068	0.000	0.729	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	70	105	0	313	0	0	-1
normalized size	1	1.00	0.60	0.90	0.00	2.68	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.058	0.062	0.000	0.605	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	87	0	295	0	0	-1
normalized size	1	1.00	0.85	1.12	0.00	3.78	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.049	0.053	0.000	0.667	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	93	151	0	517	0	0	-1
normalized size	1	1.00	0.61	0.99	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.072	0.081	0.000	0.760	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	122	259	0	594	0	0	-1
normalized size	1	1.00	0.57	1.20	0.00	2.76	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.103	0.066	0.000	1.636	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	135	366	0	668	0	0	-1
normalized size	1	1.00	0.49	1.32	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.171	0.128	0.072	0.000	0.738	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	91	163	0	234	0	0	-1
normalized size	1	1.00	0.64	1.14	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.167	0.052	0.000	0.701	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	83	144	0	212	777	0	-1
normalized size	1	1.00	0.70	1.22	0.00	1.80	6.58	0.00	-0.01
time (sec)	N/A	0.225	0.129	0.053	0.000	0.466	14.123	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	108	0	182	0	0	-1
normalized size	1	1.00	0.79	1.14	0.00	1.92	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.072	0.052	0.000	0.600	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	55	103	0	137	0	0	-1
normalized size	1	1.00	0.79	1.47	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.051	0.048	0.000	0.676	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	120	0	124	0	96	-1
normalized size	1	1.00	1.00	2.40	0.00	2.48	0.00	1.92	-0.02
time (sec)	N/A	0.159	0.035	0.051	0.000	0.399	0.000	0.193	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	194	0	176	0	124	-1
normalized size	1	1.00	0.61	2.77	0.00	2.51	0.00	1.77	-0.01
time (sec)	N/A	0.179	0.032	0.052	0.000	0.517	0.000	0.158	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	55	260	0	238	0	136	-1
normalized size	1	1.00	0.58	2.74	0.00	2.51	0.00	1.43	-0.01
time (sec)	N/A	0.200	0.036	0.056	0.000	0.544	0.000	0.170	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	58	328	0	294	0	165	-1
normalized size	1	1.00	0.49	2.78	0.00	2.49	0.00	1.40	-0.01
time (sec)	N/A	0.216	0.039	0.056	0.000	0.483	0.000	0.143	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	46	396	0	346	0	187	-1
normalized size	1	1.00	0.32	2.73	0.00	2.39	0.00	1.29	-0.01
time (sec)	N/A	0.230	0.046	0.060	0.000	0.601	0.000	0.145	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	109	178	0	437	0	0	-1
normalized size	1	1.00	0.41	0.66	0.00	1.63	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.107	0.069	0.000	0.819	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	101	161	0	415	0	0	-1
normalized size	1	1.00	0.43	0.68	0.00	1.75	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.087	0.069	0.000	0.638	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	89	144	0	381	0	0	-1
normalized size	1	1.00	0.57	0.92	0.00	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.070	0.067	0.000	0.528	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	66	118	0	315	0	0	-1
normalized size	1	1.00	0.56	1.00	0.00	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.050	0.064	0.000	0.894	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	93	160	0	512	0	0	-1
normalized size	1	1.00	0.61	1.05	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.082	0.064	0.000	0.729	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	115	259	0	581	0	0	-1
normalized size	1	1.00	0.53	1.20	0.00	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.103	0.068	0.000	0.720	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	135	373	0	668	0	0	-1
normalized size	1	1.00	0.49	1.36	0.00	2.43	0.00	0.00	-0.00
time (sec)	N/A	0.171	0.133	0.071	0.000	0.821	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	143	480	0	736	0	0	-1
normalized size	1	1.00	0.43	1.43	0.00	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.191	0.163	0.075	0.000	1.634	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	101	161	0	415	0	0	-1
normalized size	1	1.00	0.46	0.73	0.00	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.111	0.070	0.000	0.588	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	89	144	0	381	0	0	-1
normalized size	1	1.00	0.55	0.89	0.00	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.070	0.070	0.000	0.659	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	70	118	0	315	0	0	-1
normalized size	1	1.00	0.50	0.84	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.055	0.066	0.000	0.467	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	65	101	0	297	0	0	-1
normalized size	1	1.00	0.82	1.28	0.00	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.043	0.060	0.000	0.496	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	104	0	299	0	0	-1
normalized size	1	1.00	0.85	1.33	0.00	3.83	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.051	0.051	0.000	0.745	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	91	162	0	522	0	0	-1
normalized size	1	1.00	0.60	1.07	0.00	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.077	0.068	0.000	0.942	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	123	264	0	596	0	0	-1
normalized size	1	1.00	0.56	1.21	0.00	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.112	0.074	0.000	0.795	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	135	371	0	668	0	0	-1
normalized size	1	1.00	0.49	1.34	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.140	0.076	0.000	0.699	0.000	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	125	281	0	323	0	0	-1
normalized size	1	1.00	0.77	1.72	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.273	0.076	0.000	0.585	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	116	257	0	285	0	0	-1
normalized size	1	1.00	0.84	1.86	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.111	0.051	0.000	0.726	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	229	0	235	0	0	-1
normalized size	1	1.00	0.84	2.03	0.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.089	0.055	0.000	0.433	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	190	0	219	0	0	-1
normalized size	1	1.00	1.00	2.07	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.054	0.050	0.000	0.870	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	136	0	234	0	131	-1
normalized size	1	1.00	1.00	1.43	0.00	2.46	0.00	1.38	-0.01
time (sec)	N/A	0.206	0.051	0.043	0.000	0.608	0.000	0.164	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	134	0	231	0	130	-1
normalized size	1	1.00	1.00	1.43	0.00	2.46	0.00	1.38	-0.01
time (sec)	N/A	0.215	0.057	0.048	0.000	0.579	0.000	0.149	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	70	370	0	287	0	166	-1
normalized size	1	1.00	0.60	3.19	0.00	2.47	0.00	1.43	-0.01
time (sec)	N/A	0.235	0.058	0.057	0.000	0.604	0.000	0.138	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	79	497	0	359	0	187	-1
normalized size	1	1.00	0.54	3.38	0.00	2.44	0.00	1.27	-0.01
time (sec)	N/A	0.264	0.063	0.056	0.000	0.526	0.000	0.143	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	82	626	0	431	0	207	-1
normalized size	1	1.00	0.48	3.64	0.00	2.51	0.00	1.20	-0.01
time (sec)	N/A	0.293	0.073	0.061	0.000	0.670	0.000	0.136	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	140	229	0	437	0	0	-1
normalized size	1	1.00	0.42	0.68	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.132	0.076	0.000	0.688	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	132	212	0	415	0	0	-1
normalized size	1	1.00	0.48	0.77	0.00	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.171	0.114	0.075	0.000	0.512	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	124	195	0	381	0	0	-1
normalized size	1	1.00	0.57	0.89	0.00	1.74	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.102	0.075	0.000	0.778	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	71	169	0	315	0	0	-1
normalized size	1	1.00	0.45	1.07	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.046	0.080	0.000	0.804	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	67	146	0	299	0	0	-1
normalized size	1	1.00	0.48	1.04	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.049	0.068	0.000	0.665	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	69	149	0	303	0	0	-1
normalized size	1	1.00	0.58	1.26	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.045	0.060	0.000	1.158	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	64	149	0	311	0	0	-1
normalized size	1	1.00	0.55	1.27	0.00	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.050	0.060	0.000	0.670	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	90	262	0	524	0	0	-1
normalized size	1	1.00	0.45	1.32	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.068	0.071	0.000	0.687	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	121	290	0	594	0	0	-1
normalized size	1	1.00	0.45	1.09	0.00	2.22	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.088	0.074	0.000	0.766	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.222	0.034	0.049	0.000	0.517	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	147	121	0	337	0	0	-1
normalized size	1	1.00	0.90	0.74	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.342	0.508	0.055	0.000	0.646	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	148	102	0	317	0	0	-1
normalized size	1	1.00	1.19	0.82	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.345	0.053	0.000	0.668	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	87	0	295	0	0	-1
normalized size	1	1.00	0.85	1.12	0.00	3.78	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.049	0.053	0.000	0.734	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	132	88	0	275	0	0	-1
normalized size	1	1.00	1.74	1.16	0.00	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.241	0.055	0.000	0.653	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	41	0	58	0	0	47
normalized size	1	1.00	1.22	1.11	0.00	1.57	0.00	0.00	1.27
time (sec)	N/A	0.148	0.093	0.036	0.000	0.679	0.000	0.000	1.367

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	58	47	0	68	0	0	53
normalized size	1	1.00	0.75	0.61	0.00	0.88	0.00	0.00	0.69
time (sec)	N/A	0.189	0.109	0.037	0.000	0.512	0.000	0.000	1.394

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	66	55	0	77	0	0	100
normalized size	1	1.00	0.56	0.47	0.00	0.66	0.00	0.00	0.85
time (sec)	N/A	0.246	0.121	0.038	0.000	0.528	0.000	0.000	1.415

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	74	63	0	84	0	0	108
normalized size	1	1.00	0.47	0.40	0.00	0.53	0.00	0.00	0.68
time (sec)	N/A	0.306	0.129	0.037	0.000	0.479	0.000	0.000	1.441

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	50	172	0	179	0	142	-1
normalized size	1	1.00	0.38	1.32	0.00	1.38	0.00	1.09	-0.01
time (sec)	N/A	0.377	0.040	0.047	0.000	0.511	0.000	0.174	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	50	155	0	163	0	127	-1
normalized size	1	1.00	0.48	1.48	0.00	1.55	0.00	1.21	-0.01
time (sec)	N/A	0.352	0.038	0.046	0.000	0.436	0.000	0.437	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	139	0	147	0	112	-1
normalized size	1	1.00	0.96	1.74	0.00	1.84	0.00	1.40	-0.01
time (sec)	N/A	0.241	0.081	0.042	0.000	0.624	0.000	0.177	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	118	0	124	0	96	-1
normalized size	1	1.00	1.00	2.36	0.00	2.48	0.00	1.92	-0.02
time (sec)	N/A	0.146	0.044	0.048	0.000	0.642	0.000	0.192	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	99	0	111	39	0	-1
normalized size	1	1.00	1.00	2.11	0.00	2.36	0.83	0.00	-0.02
time (sec)	N/A	0.330	0.037	0.053	0.000	0.632	10.694	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	27	0	28	0	0	24
normalized size	1	1.00	0.67	0.64	0.00	0.67	0.00	0.00	0.57
time (sec)	N/A	0.324	0.039	0.036	0.000	0.441	0.000	0.000	1.262

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	36	35	0	36	0	0	32
normalized size	1	1.00	0.52	0.51	0.00	0.52	0.00	0.00	0.46
time (sec)	N/A	0.333	0.047	0.038	0.000	0.422	0.000	0.000	1.286

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	44	43	0	44	0	0	77
normalized size	1	1.00	0.46	0.45	0.00	0.46	0.00	0.00	0.80
time (sec)	N/A	0.352	0.055	0.038	0.000	0.662	0.000	0.000	1.278

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	52	51	0	52	0	0	98
normalized size	1	1.00	0.43	0.42	0.00	0.43	0.00	0.00	0.81
time (sec)	N/A	0.378	0.068	0.037	0.000	0.543	0.000	0.000	1.370

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	252	224	0	568	0	0	-1
normalized size	1	1.00	0.81	0.72	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.988	0.063	0.000	0.883	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	244	202	0	552	0	0	-1
normalized size	1	1.00	0.93	0.77	0.00	2.11	0.00	0.00	-0.00
time (sec)	N/A	0.317	0.660	0.061	0.000	0.851	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	236	180	0	536	0	0	-1
normalized size	1	1.00	1.13	0.86	0.00	2.56	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.576	0.062	0.000	0.889	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	93	160	0	512	0	0	-1
normalized size	1	1.00	0.61	1.05	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.086	0.066	0.000	0.751	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	218	161	0	490	0	0	-1
normalized size	1	1.00	1.49	1.10	0.00	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.395	0.065	0.000	1.519	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	155	141	0	353	0	0	-1
normalized size	1	1.00	1.24	1.13	0.00	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.248	0.083	0.000	0.593	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	162	165	0	381	0	0	-1
normalized size	1	1.00	0.95	0.97	0.00	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.281	0.069	0.000	0.529	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	170	187	0	397	0	0	-1
normalized size	1	1.00	0.81	0.89	0.00	1.90	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.309	0.072	0.000	0.945	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	178	209	0	413	0	0	-1
normalized size	1	1.00	0.59	0.69	0.00	1.36	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.315	0.073	0.000	0.747	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	93	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.067	0.049	0.000	0.533	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	147	133	0	337	0	0	-1
normalized size	1	1.00	0.90	0.81	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.360	0.511	0.060	0.000	0.720	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	139	116	0	321	0	0	-1
normalized size	1	1.00	1.12	0.94	0.00	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.487	0.056	0.000	0.873	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	65	101	0	297	0	0	-1
normalized size	1	1.00	0.82	1.28	0.00	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.046	0.061	0.000	0.635	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	132	100	0	275	0	0	-1
normalized size	1	1.00	1.74	1.32	0.00	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.252	0.065	0.000	0.448	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	46	54	0	59	0	0	54
normalized size	1	1.00	0.66	0.77	0.00	0.84	0.00	0.00	0.77
time (sec)	N/A	0.185	0.089	0.035	0.000	0.554	0.000	0.000	1.331

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	58	62	0	69	0	0	62
normalized size	1	1.00	0.51	0.55	0.00	0.61	0.00	0.00	0.55
time (sec)	N/A	0.224	0.119	0.037	0.000	0.507	0.000	0.000	1.353

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	66	70	0	77	0	0	100
normalized size	1	1.00	0.44	0.47	0.00	0.52	0.00	0.00	0.67
time (sec)	N/A	0.303	0.128	0.037	0.000	0.510	0.000	0.000	1.408

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	116	259	0	271	0	0	-1
normalized size	1	1.00	0.67	1.51	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.470	0.163	0.050	0.000	0.895	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	108	237	0	259	0	0	-1
normalized size	1	1.00	0.73	1.61	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.451	0.117	0.046	0.000	0.530	0.000	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	100	216	0	239	0	0	-1
normalized size	1	1.00	0.82	1.77	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.324	0.095	0.042	0.000	0.470	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	190	0	219	0	0	-1
normalized size	1	1.00	1.00	2.07	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.052	0.050	0.000	0.424	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	228	0	203	80	0	-1
normalized size	1	1.00	1.00	2.65	0.00	2.36	0.93	0.00	-0.01
time (sec)	N/A	0.373	0.044	0.055	0.000	0.525	12.749	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	254	0	161	0	0	-1
normalized size	1	1.00	0.84	3.10	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.376	0.073	0.052	0.000	0.576	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	79	278	0	181	0	278	-1
normalized size	1	1.00	0.70	2.46	0.00	1.60	0.00	2.46	-0.01
time (sec)	N/A	0.394	0.086	0.053	0.000	0.440	0.000	0.876	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	87	302	0	201	0	356	-1
normalized size	1	1.00	0.77	2.67	0.00	1.78	0.00	3.15	-0.01
time (sec)	N/A	0.416	0.142	0.059	0.000	0.604	0.000	1.063	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	95	326	0	213	0	434	-1
normalized size	1	1.00	0.58	2.00	0.00	1.31	0.00	2.66	-0.01
time (sec)	N/A	0.458	0.148	0.058	0.000	0.470	0.000	1.327	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	306	167	197	0	353	0	0	-1
normalized size	1	1.01	0.55	0.65	0.00	1.17	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.633	0.069	0.000	0.728	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	252	159	180	0	337	0	0	-1
normalized size	1	1.00	0.63	0.72	0.00	1.34	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.541	0.070	0.000	0.599	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	151	163	0	321	0	0	-1
normalized size	1	1.00	0.76	0.82	0.00	1.61	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.541	0.066	0.000	0.531	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	67	146	0	299	0	0	-1
normalized size	1	1.00	0.48	1.04	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.053	0.070	0.000	0.836	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	131	151	0	277	0	0	-1
normalized size	1	1.00	0.98	1.13	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.408	0.069	0.000	0.645	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	58	62	0	59	0	0	54
normalized size	1	1.00	0.53	0.57	0.00	0.54	0.00	0.00	0.50
time (sec)	N/A	0.216	0.105	0.040	0.000	0.498	0.000	0.000	1.361

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	70	70	0	69	0	0	62
normalized size	1	1.00	0.47	0.47	0.00	0.46	0.00	0.00	0.41
time (sec)	N/A	0.266	0.119	0.039	0.000	0.496	0.000	0.000	1.442

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	78	78	0	77	0	0	100
normalized size	1	1.00	0.41	0.41	0.00	0.41	0.00	0.00	0.53
time (sec)	N/A	0.414	0.134	0.041	0.000	0.818	0.000	0.000	1.415

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	86	86	0	85	0	0	108
normalized size	1	1.00	0.30	0.30	0.00	0.29	0.00	0.00	0.37
time (sec)	N/A	0.293	0.151	0.041	0.000	0.548	0.000	0.000	1.410

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	81	155	0	0	0	0	0	-1
normalized size	1	0.44	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.380	0.055	0.000	0.694	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.408	0.055	0.000	0.655	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	113	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.072	0.056	0.000	0.473	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	180.005	0.050	0.000	0.586	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	180.005	0.050	0.000	0.567	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	180.008	0.051	0.000	0.535	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	180.005	0.050	0.000	0.608	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.966	0.097	0.000	0.754	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.028	0.087	0.000	0.412	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.659	0.093	0.000	0.493	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	272	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	4.77	0.00	-0.02
time (sec)	N/A	0.107	0.024	0.365	0.000	0.570	7.830	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	1.076	0.046	0.000	0.468	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.230	0.048	0.000	0.510	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.051	0.354	0.000	0.519	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	111	279	415	169	0	340	362
normalized size	1	1.00	0.28	0.71	1.06	0.43	0.00	0.87	0.92
time (sec)	N/A	0.337	0.200	0.064	0.320	0.550	0.000	0.186	1.367

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	95	231	337	148	0	278	289
normalized size	1	1.00	0.30	0.74	1.08	0.47	0.00	0.89	0.92
time (sec)	N/A	0.251	0.183	0.052	0.315	0.567	0.000	0.187	1.308

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	79	183	259	125	0	216	214
normalized size	1	1.00	0.34	0.79	1.11	0.54	0.00	0.93	0.92
time (sec)	N/A	0.194	0.105	0.046	0.312	0.574	0.000	0.185	1.261

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	119	171	91	0	152	131
normalized size	1	1.00	0.42	0.82	1.18	0.63	0.00	1.05	0.90
time (sec)	N/A	0.116	0.102	0.042	0.301	0.564	0.000	0.163	0.070

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	23	22	34	0	22	22
normalized size	1	1.00	1.00	1.77	1.69	2.62	0.00	1.69	1.69
time (sec)	N/A	0.029	0.046	0.035	0.300	0.788	0.000	0.149	1.225

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	49	65	58	0	70	50
normalized size	1	1.00	0.98	0.96	1.27	1.14	0.00	1.37	0.98
time (sec)	N/A	0.060	0.159	0.038	0.305	0.484	0.000	0.134	0.071

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	99	86	0	117	60
normalized size	1	1.00	0.78	0.76	1.16	1.01	0.00	1.38	0.71
time (sec)	N/A	0.093	0.197	0.039	0.313	0.524	0.000	0.157	0.073

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	82	81	132	134	0	164	142
normalized size	1	1.00	0.69	0.68	1.11	1.13	0.00	1.38	1.19
time (sec)	N/A	0.128	0.279	0.041	0.308	0.555	0.000	0.157	0.062

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	47	85	113	113	119	113	113
normalized size	1	1.00	0.56	1.01	1.35	1.35	1.42	1.35	1.35
time (sec)	N/A	0.099	0.041	0.033	0.311	0.447	0.098	0.138	1.251

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	63	82	82	87	82	82
normalized size	1	1.00	0.57	0.91	1.19	1.19	1.26	1.19	1.19
time (sec)	N/A	0.085	0.031	0.036	0.306	0.532	0.088	0.138	0.042

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	54	70	70	70	70	70
normalized size	1	1.00	0.60	1.04	1.35	1.35	1.35	1.35	1.35
time (sec)	N/A	0.077	0.028	0.033	0.303	0.501	0.081	0.138	0.035

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	31	38	38	36	38	38
normalized size	1	1.00	0.66	0.89	1.09	1.09	1.03	1.09	1.09
time (sec)	N/A	0.067	0.018	0.030	0.304	0.524	0.071	0.125	0.053

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	20	14	21	21	20	21	17
normalized size	1	1.00	1.33	0.93	1.40	1.40	1.33	1.40	1.13
time (sec)	N/A	0.034	0.014	0.032	0.310	0.524	0.059	0.118	0.035

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	13	13	10	14	14
normalized size	1	1.00	1.12	0.94	0.81	0.81	0.62	0.88	0.88
time (sec)	N/A	0.065	0.016	0.032	0.296	0.494	0.119	0.141	0.035

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	60	63	76	54	51	46
normalized size	1	1.00	0.69	1.18	1.24	1.49	1.06	1.00	0.90
time (sec)	N/A	0.079	0.028	0.041	0.307	0.428	0.269	0.144	1.263

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	63	90	91	121	85	74	73
normalized size	1	1.00	0.73	1.05	1.06	1.41	0.99	0.86	0.85
time (sec)	N/A	0.100	0.041	0.044	0.306	0.432	0.404	0.128	0.091

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	82	120	140	217	141	91	121
normalized size	1	1.00	0.68	0.99	1.16	1.79	1.17	0.75	1.00
time (sec)	N/A	0.123	0.071	0.043	0.304	0.605	0.563	0.121	1.302

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	111	288	415	170	0	340	362
normalized size	1	1.00	0.28	0.73	1.06	0.43	0.00	0.87	0.92
time (sec)	N/A	0.333	0.203	0.064	0.319	0.715	0.000	0.199	0.215

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	95	240	337	147	0	278	289
normalized size	1	1.00	0.30	0.77	1.08	0.47	0.00	0.89	0.92
time (sec)	N/A	0.254	0.148	0.053	0.319	0.676	0.000	0.199	0.123

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	79	192	259	126	0	216	214
normalized size	1	1.00	0.34	0.82	1.11	0.54	0.00	0.93	0.92
time (sec)	N/A	0.213	0.110	0.049	0.315	0.472	0.000	0.161	1.246

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	183	171	91	0	152	133
normalized size	1	1.00	0.42	1.26	1.18	0.63	0.00	1.05	0.92
time (sec)	N/A	0.119	0.083	0.048	0.301	0.603	0.000	0.166	0.070

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	51	0	35	23
normalized size	1	1.00	1.00	1.33	1.28	2.83	0.00	1.94	1.28
time (sec)	N/A	0.031	0.049	0.038	0.302	0.594	0.000	0.141	0.032

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	43	49	55	77	0	69	55
normalized size	1	1.00	0.78	0.89	1.00	1.40	0.00	1.25	1.00
time (sec)	N/A	0.065	0.162	0.040	0.299	0.594	0.000	0.149	0.040

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	65	97	96	0	104	60
normalized size	1	1.00	0.73	0.71	1.07	1.05	0.00	1.14	0.66
time (sec)	N/A	0.100	0.224	0.040	0.301	0.471	0.000	0.184	1.264

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	82	81	131	124	0	149	76
normalized size	1	1.00	0.65	0.64	1.03	0.98	0.00	1.17	0.60
time (sec)	N/A	0.139	0.284	0.042	0.307	0.625	0.000	0.161	1.260

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	39	75	101	101	109	102	101
normalized size	1	1.00	0.59	1.14	1.53	1.53	1.65	1.55	1.53
time (sec)	N/A	0.085	0.039	0.032	0.301	0.546	0.899	0.128	1.243

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	69	92	92	100	90	92
normalized size	1	1.00	0.60	1.33	1.77	1.77	1.92	1.73	1.77
time (sec)	N/A	0.079	0.029	0.033	0.301	0.535	0.107	0.124	0.051

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	45	59	59	63	78	59
normalized size	1	1.00	0.66	1.29	1.69	1.69	1.80	2.23	1.69
time (sec)	N/A	0.063	0.022	0.031	0.304	0.419	0.092	0.125	0.034

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	16	47	47	48	64	47
normalized size	1	1.00	2.18	0.94	2.76	2.76	2.82	3.76	2.76
time (sec)	N/A	0.057	0.023	0.033	0.297	0.441	0.089	0.139	0.032

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	34	33	37	36	60	33
normalized size	1	1.00	0.78	0.74	0.72	0.80	0.78	1.30	0.72
time (sec)	N/A	0.048	0.014	0.038	0.298	0.635	0.171	0.138	0.051

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	28	19	19	17	27	12
normalized size	1	1.00	1.92	2.15	1.46	1.46	1.31	2.08	0.92
time (sec)	N/A	0.063	0.012	0.037	0.306	1.013	0.185	0.137	0.050

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	16	41	41	42	15	40
normalized size	1	1.00	0.94	0.89	2.28	2.28	2.33	0.83	2.22
time (sec)	N/A	0.060	0.020	0.031	0.301	0.688	0.229	0.122	0.062

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	52	90	102	147	99	91	83
normalized size	1	1.00	0.60	1.03	1.17	1.69	1.14	1.05	0.95
time (sec)	N/A	0.093	0.039	0.043	0.303	0.562	0.500	0.139	0.092

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	80	120	130	191	129	127	111
normalized size	1	1.00	0.66	0.98	1.07	1.57	1.06	1.04	0.91
time (sec)	N/A	0.118	0.060	0.045	0.322	0.582	0.596	0.142	1.298

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	111	279	415	170	0	196	362
normalized size	1	1.00	0.28	0.71	1.06	0.43	0.00	0.50	0.92
time (sec)	N/A	0.322	0.204	0.066	0.332	0.571	0.000	0.179	1.343

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	95	231	337	147	0	161	289
normalized size	1	1.00	0.30	0.74	1.08	0.47	0.00	0.51	0.92
time (sec)	N/A	0.255	0.199	0.053	0.325	0.582	0.000	0.154	0.113

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	79	183	259	126	0	126	214
normalized size	1	1.00	0.34	0.79	1.11	0.54	0.00	0.54	0.92
time (sec)	N/A	0.199	0.109	0.050	0.335	1.244	0.000	0.151	0.079

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	119	171	91	0	82	132
normalized size	1	1.00	0.42	0.82	1.18	0.63	0.00	0.57	0.91
time (sec)	N/A	0.115	0.096	0.046	0.308	0.460	0.000	0.167	1.243

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	23	0	0	23
normalized size	1	1.00	1.00	1.50	1.44	1.44	0.00	0.00	1.44
time (sec)	N/A	0.030	0.047	0.036	0.309	0.900	0.000	0.000	0.031

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	67	50	0	0	55
normalized size	1	1.00	0.87	0.89	1.22	0.91	0.00	0.00	1.00
time (sec)	N/A	0.062	0.156	0.040	0.320	0.641	0.000	0.000	0.051

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	64	65	102	76	0	0	109
normalized size	1	1.00	0.70	0.71	1.12	0.84	0.00	0.00	1.20
time (sec)	N/A	0.097	0.214	0.038	0.310	0.509	0.000	0.000	1.242

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	80	81	135	104	0	0	148
normalized size	1	1.00	0.63	0.64	1.06	0.82	0.00	0.00	1.17
time (sec)	N/A	0.136	0.289	0.042	0.312	0.634	0.000	0.000	0.038

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	39	61	80	80	87	80	80
normalized size	1	1.00	0.53	0.84	1.10	1.10	1.19	1.10	1.10
time (sec)	N/A	0.081	0.034	0.033	0.302	0.720	0.307	0.136	0.047

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	31	54	70	70	70	70	70
normalized size	1	1.00	0.56	0.98	1.27	1.27	1.27	1.27	1.27
time (sec)	N/A	0.075	0.028	0.033	0.310	0.520	0.089	0.134	0.039

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	30	37	37	36	37	37
normalized size	1	1.00	0.81	0.81	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.063	0.018	0.035	0.307	0.588	0.078	0.138	0.051

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	14	20	20	19	20	17
normalized size	1	1.00	1.31	0.88	1.25	1.25	1.19	1.25	1.06
time (sec)	N/A	0.034	0.014	0.034	0.299	0.473	0.068	0.136	0.035

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	12	12	10	14	12
normalized size	1	1.00	1.29	1.07	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.061	0.016	0.033	0.303	0.455	0.124	0.146	0.050

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	60	63	76	54	51	46
normalized size	1	1.00	0.67	1.22	1.29	1.55	1.10	1.04	0.94
time (sec)	N/A	0.076	0.031	0.041	0.302	0.595	0.280	0.126	0.065

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	61	90	91	121	85	74	73
normalized size	1	1.00	0.73	1.07	1.08	1.44	1.01	0.88	0.87
time (sec)	N/A	0.100	0.045	0.044	0.300	0.545	0.418	0.150	1.261

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	80	120	140	217	141	91	121
normalized size	1	1.00	0.67	1.01	1.18	1.82	1.18	0.76	1.02
time (sec)	N/A	0.120	0.068	0.047	0.314	0.579	0.595	0.148	1.284

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	111	288	415	169	0	198	362
normalized size	1	1.00	0.28	0.73	1.06	0.43	0.00	0.50	0.92
time (sec)	N/A	0.337	0.209	0.066	1.388	0.936	0.000	0.212	0.171

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	95	240	337	148	0	162	289
normalized size	1	1.00	0.30	0.77	1.08	0.47	0.00	0.52	0.92
time (sec)	N/A	0.264	0.156	0.052	0.971	0.533	0.000	0.206	0.130

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	79	192	259	125	0	126	214
normalized size	1	1.00	0.34	0.82	1.11	0.54	0.00	0.54	0.92
time (sec)	N/A	0.204	0.117	0.048	0.869	0.523	0.000	0.180	1.269

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	183	171	92	0	82	133
normalized size	1	1.00	0.42	1.26	1.18	0.63	0.00	0.57	0.92
time (sec)	N/A	0.120	0.110	0.046	0.308	0.560	0.000	0.183	0.066

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	34	0	49	23
normalized size	1	1.00	1.00	1.33	1.28	1.89	0.00	2.72	1.28
time (sec)	N/A	0.032	0.050	0.040	0.444	0.407	0.000	0.195	1.209

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	43	49	60	58	0	65	60
normalized size	1	1.00	0.78	0.89	1.09	1.05	0.00	1.18	1.09
time (sec)	N/A	0.066	0.156	0.040	0.304	0.654	0.000	0.228	0.049

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	65	103	86	0	0	116
normalized size	1	1.00	0.73	0.71	1.13	0.95	0.00	0.00	1.27
time (sec)	N/A	0.101	0.218	0.038	0.302	0.544	0.000	0.000	1.206

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	82	81	136	134	0	0	155
normalized size	1	1.00	0.65	0.64	1.07	1.06	0.00	0.00	1.22
time (sec)	N/A	0.138	0.280	0.040	0.475	0.476	0.000	0.000	0.039

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	79	116	0	117	0	0	-1
normalized size	1	1.00	0.34	0.51	0.00	0.51	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.072	0.036	0.000	0.486	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	71	100	0	95	0	0	-1
normalized size	1	1.00	0.39	0.55	0.00	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.061	0.040	0.000	0.538	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	63	84	0	73	0	0	-1
normalized size	1	1.00	0.46	0.62	0.00	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.047	0.039	0.000	0.628	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	53	68	0	43	0	0	-1
normalized size	1	1.00	0.57	0.73	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.034	0.040	0.000	0.569	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	44	0	22	0	0	-1
normalized size	1	1.00	0.60	0.65	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.020	0.035	0.000	0.457	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	51	0	22	0	0	-1
normalized size	1	1.00	1.00	1.34	0.00	0.58	0.00	0.00	-0.03
time (sec)	N/A	0.146	0.020	0.065	0.000	0.630	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	84	0	86	0	0	-1
normalized size	1	1.00	0.62	0.92	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.042	0.058	0.000	1.057	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	83	169	0	136	0	0	-1
normalized size	1	1.00	0.45	0.92	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.064	0.062	0.000	0.500	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	101	241	0	191	0	0	-1
normalized size	1	1.00	0.36	0.87	0.00	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.090	0.064	0.000	0.591	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	167	350	192	329	1341	164	-1
normalized size	1	1.00	0.95	1.99	1.09	1.87	7.62	0.93	-0.01
time (sec)	N/A	0.165	0.164	0.045	0.414	0.701	36.347	0.195	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	151	296	173	286	1091	141	-1
normalized size	1	1.00	0.99	1.93	1.13	1.87	7.13	0.92	-0.01
time (sec)	N/A	0.153	0.141	0.044	0.422	0.591	21.833	0.191	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	135	242	154	241	478	116	-1
normalized size	1	1.00	1.04	1.86	1.18	1.85	3.68	0.89	-0.01
time (sec)	N/A	0.138	0.129	0.046	0.422	0.639	12.016	0.183	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	117	188	131	180	342	85	-1
normalized size	1	1.00	1.09	1.76	1.22	1.68	3.20	0.79	-0.01
time (sec)	N/A	0.127	0.100	0.044	0.422	0.603	7.800	0.157	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	134	47	134	0	62	-1
normalized size	1	1.00	0.88	1.56	0.55	1.56	0.00	0.72	-0.01
time (sec)	N/A	0.110	0.060	0.042	0.418	0.423	0.000	0.145	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	82	79	40	153	0	0	-1
normalized size	1	1.00	1.39	1.34	0.68	2.59	0.00	0.00	-0.02
time (sec)	N/A	0.105	0.041	0.043	0.404	0.456	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	64	31	61	47	0	148	33
normalized size	1	1.00	1.25	0.61	1.20	0.92	0.00	2.90	0.65
time (sec)	N/A	0.105	0.040	0.037	0.304	0.879	0.000	0.228	1.294

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	53	47	80	75	0	0	56
normalized size	1	1.00	0.72	0.64	1.08	1.01	0.00	0.00	0.76
time (sec)	N/A	0.110	0.043	0.038	0.320	0.737	0.000	0.000	1.381

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	64	99	124	0	0	134
normalized size	1	1.00	0.99	0.66	1.02	1.28	0.00	0.00	1.38
time (sec)	N/A	0.120	0.059	0.040	0.317	0.819	0.000	0.000	1.453

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	112	80	118	152	0	0	177
normalized size	1	1.00	0.93	0.67	0.98	1.27	0.00	0.00	1.48
time (sec)	N/A	0.131	0.067	0.045	0.325	1.842	0.000	0.000	1.512

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	71	100	204	95	0	0	-1
normalized size	1	1.00	0.38	0.54	1.10	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.065	0.046	0.362	0.671	0.000	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	63	100	172	95	0	0	-1
normalized size	1	1.00	0.45	0.72	1.24	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.054	0.041	0.341	0.399	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	55	84	140	73	0	0	-1
normalized size	1	1.00	0.59	0.90	1.51	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.045	0.040	0.344	0.551	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	58	60	97	42	0	0	-1
normalized size	1	1.00	1.26	1.30	2.11	0.91	0.00	0.00	-0.02
time (sec)	N/A	0.176	0.035	0.038	0.336	0.907	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	57	67	0	33	0	0	-1
normalized size	1	1.00	0.50	0.59	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.030	0.053	0.000	0.656	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	53	64	0	39	0	0	-1
normalized size	1	1.00	0.67	0.81	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.033	0.057	0.000	0.557	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	51	39	0	39	0	0	90
normalized size	1	1.00	1.09	0.83	0.00	0.83	0.00	0.00	1.91
time (sec)	N/A	0.172	0.049	0.037	0.000	0.464	0.000	0.000	1.555

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	71	169	0	139	0	0	-1
normalized size	1	1.00	0.38	0.91	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.066	0.069	0.000	0.501	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	99	241	0	190	0	0	-1
normalized size	1	1.00	0.36	0.87	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.092	0.068	0.000	0.611	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	79	116	0	117	0	0	-1
normalized size	1	1.00	0.34	0.50	0.00	0.50	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.066	0.041	0.000	0.649	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	71	100	0	95	0	0	-1
normalized size	1	1.00	0.38	0.53	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.055	0.040	0.000	0.615	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	63	84	0	73	0	0	-1
normalized size	1	1.00	0.45	0.60	0.00	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.046	0.040	0.000	0.428	0.000	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	53	68	0	43	0	0	-1
normalized size	1	1.00	0.56	0.72	0.00	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.036	0.039	0.000	0.768	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	41	44	0	22	0	0	-1
normalized size	1	1.00	0.59	0.64	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.023	0.039	0.000	1.171	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	0	22	0	0	-1
normalized size	1	1.00	1.00	1.38	0.00	0.59	0.00	0.00	-0.03
time (sec)	N/A	0.153	0.021	0.051	0.000	0.489	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	54	84	0	83	0	0	-1
normalized size	1	1.00	0.60	0.93	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.054	0.062	0.000	0.594	0.000	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	81	169	0	137	0	0	-1
normalized size	1	1.00	0.44	0.92	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.073	0.069	0.000	0.521	0.000	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	99	241	0	186	0	0	-1
normalized size	1	1.00	0.36	0.87	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.099	0.069	0.000	0.432	0.000	0.000	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	136	226	154	241	478	117	-1
normalized size	1	1.00	1.04	1.73	1.18	1.84	3.65	0.89	-0.01
time (sec)	N/A	0.141	0.152	0.051	0.415	0.756	10.879	0.168	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	117	176	130	180	340	85	-1
normalized size	1	1.00	1.08	1.63	1.20	1.67	3.15	0.79	-0.01
time (sec)	N/A	0.127	0.099	0.046	0.405	0.630	8.160	0.178	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	126	47	134	0	62	-1
normalized size	1	1.00	1.15	1.45	0.54	1.54	0.00	0.71	-0.01
time (sec)	N/A	0.111	0.065	0.048	0.408	0.481	0.000	0.152	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	100	73	39	151	0	0	-1
normalized size	1	1.00	1.67	1.22	0.65	2.52	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.068	0.044	0.403	0.572	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	63	31	60	47	0	148	33
normalized size	1	1.00	1.21	0.60	1.15	0.90	0.00	2.85	0.63
time (sec)	N/A	0.104	0.041	0.039	0.313	0.577	0.000	0.231	1.278

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	47	79	75	0	0	56
normalized size	1	1.00	1.05	0.63	1.05	1.00	0.00	0.00	0.75
time (sec)	N/A	0.115	0.054	0.039	0.319	0.565	0.000	0.000	1.400

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	64	98	124	0	0	134
normalized size	1	1.00	0.98	0.65	1.00	1.27	0.00	0.00	1.37
time (sec)	N/A	0.124	0.066	0.040	0.320	1.199	0.000	0.000	1.453

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	112	80	117	152	0	0	177
normalized size	1	1.00	0.93	0.66	0.97	1.26	0.00	0.00	1.46
time (sec)	N/A	0.141	0.076	0.044	0.328	1.319	0.000	0.000	1.494

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	71	100	204	95	0	0	-1
normalized size	1	1.00	0.38	0.53	1.08	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.062	0.038	0.336	0.979	0.000	0.000	0.000

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	63	100	172	95	0	0	-1
normalized size	1	1.00	0.44	0.70	1.21	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.054	0.039	0.338	0.624	0.000	0.000	0.000

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	55	84	140	73	0	0	-1
normalized size	1	1.00	0.58	0.88	1.47	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.043	0.035	0.340	0.543	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	58	60	97	42	0	0	-1
normalized size	1	1.00	1.23	1.28	2.06	0.89	0.00	0.00	-0.02
time (sec)	N/A	0.168	0.035	0.041	0.333	0.492	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	56	67	0	33	0	0	-1
normalized size	1	1.00	0.50	0.60	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.030	0.049	0.000	0.587	0.000	0.000	0.000

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	52	62	0	38	0	0	-1
normalized size	1	1.00	0.68	0.81	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.036	0.058	0.000	0.623	0.000	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	39	0	39	0	0	58
normalized size	1	1.00	1.11	0.85	0.00	0.85	0.00	0.00	1.26
time (sec)	N/A	0.169	0.055	0.036	0.000	0.526	0.000	0.000	1.472

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	71	169	0	136	0	0	-1
normalized size	1	1.00	0.39	0.93	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.068	0.062	0.000	0.559	0.000	0.000	0.000

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	99	241	0	193	0	0	-1
normalized size	1	1.00	0.36	0.88	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.102	0.067	0.000	1.296	0.000	0.000	0.000

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	47	0	25	0	0	-1
normalized size	1	1.00	0.59	0.62	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.025	0.039	0.000	0.682	0.000	0.000	0.000

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	43	47	0	25	0	0	-1
normalized size	1	1.00	0.58	0.64	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.023	0.034	0.000	0.464	0.000	0.000	0.000

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	44	0	22	0	0	-1
normalized size	1	1.00	0.60	0.65	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.020	0.034	0.000	0.542	0.000	0.000	0.000

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	42	44	0	18	0	0	-1
normalized size	1	1.00	0.61	0.64	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.023	0.055	0.000	0.522	0.000	0.000	0.000

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	43	48	0	22	0	0	-1
normalized size	1	1.00	0.59	0.66	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.025	0.054	0.000	0.594	0.000	0.000	0.000

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	96	210	117	184	0	0	-1
normalized size	1	1.00	0.70	1.53	0.85	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.413	0.131	0.054	0.422	0.720	0.000	0.000	0.000

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	186	93	168	0	84	-1
normalized size	1	1.00	0.79	1.66	0.83	1.50	0.00	0.75	-0.01
time (sec)	N/A	0.387	0.106	0.050	0.423	0.571	0.000	0.148	0.000

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	79	162	70	150	0	72	-1
normalized size	1	1.00	0.93	1.91	0.82	1.76	0.00	0.85	-0.01
time (sec)	N/A	0.251	0.089	0.046	0.424	0.694	0.000	0.144	0.000

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	134	47	134	0	62	-1
normalized size	1	1.00	0.88	1.56	0.55	1.56	0.00	0.72	-0.01
time (sec)	N/A	0.114	0.059	0.043	0.421	0.576	0.000	0.148	0.000

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	97	129	90	191	0	95	-1
normalized size	1	1.00	1.29	1.72	1.20	2.55	0.00	1.27	-0.01
time (sec)	N/A	0.345	0.082	0.049	0.437	0.706	0.000	0.167	0.000

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	104	208	0	209	0	134	-1
normalized size	1	1.00	1.27	2.54	0.00	2.55	0.00	1.63	-0.01
time (sec)	N/A	0.348	0.100	0.052	0.000	0.540	0.000	0.164	0.000

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	76	239	0	148	0	200	-1
normalized size	1	1.00	0.97	3.06	0.00	1.90	0.00	2.56	-0.01
time (sec)	N/A	0.339	0.125	0.057	0.000	0.690	0.000	0.163	0.000

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	261	0	164	0	250	-1
normalized size	1	1.00	0.83	2.64	0.00	1.66	0.00	2.53	-0.01
time (sec)	N/A	0.378	0.127	0.057	0.000	0.491	0.000	0.162	0.000

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	95	287	0	180	0	324	-1
normalized size	1	1.00	0.73	2.21	0.00	1.38	0.00	2.49	-0.01
time (sec)	N/A	0.403	0.152	0.061	0.000	0.663	0.000	0.149	0.000

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	88	92	0	58	0	0	-1
normalized size	1	1.00	0.39	0.40	0.00	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.270	0.064	0.061	0.000	0.536	0.000	0.000	0.000

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	74	83	0	49	0	0	-1
normalized size	1	1.00	0.40	0.45	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.048	0.056	0.000	0.561	0.000	0.000	0.000

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	66	76	0	42	0	0	-1
normalized size	1	1.00	0.43	0.50	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.041	0.053	0.000	0.545	0.000	0.000	0.000

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	57	67	0	33	0	0	-1
normalized size	1	1.00	0.50	0.59	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.027	0.052	0.000	0.627	0.000	0.000	0.000

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	53	59	0	28	0	0	-1
normalized size	1	1.00	0.46	0.52	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.029	0.058	0.000	0.680	0.000	0.000	0.000

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	55	65	0	33	0	0	-1
normalized size	1	1.00	0.48	0.57	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.036	0.062	0.000	0.546	0.000	0.000	0.000

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	69	77	0	90	0	0	-1
normalized size	1	1.00	0.45	0.50	0.00	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.042	0.058	0.000	0.583	0.000	0.000	0.000

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	79	85	0	98	0	0	-1
normalized size	1	1.00	0.41	0.44	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.045	0.058	0.000	0.641	0.000	0.000	0.000

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	84	93	0	106	0	0	-1
normalized size	1	1.00	0.37	0.41	0.00	0.46	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.063	0.062	0.000	0.554	0.000	0.000	0.000

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	77	106	0	76	0	0	-1
normalized size	1	1.00	0.36	0.50	0.00	0.36	0.00	0.00	-0.00
time (sec)	N/A	0.230	0.091	0.060	0.000	0.610	0.000	0.000	0.000

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	71	97	0	69	0	0	-1
normalized size	1	1.00	0.41	0.56	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.061	0.058	0.000	0.589	0.000	0.000	0.000

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	75	86	0	56	0	0	-1
normalized size	1	1.00	0.58	0.66	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.049	0.057	0.000	0.569	0.000	0.000	0.000

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	59	84	0	86	0	0	-1
normalized size	1	1.00	0.68	0.97	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.047	0.058	0.000	0.571	0.000	0.000	0.000

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	84	0	86	0	0	-1
normalized size	1	1.00	0.62	0.92	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.034	0.060	0.000	0.584	0.000	0.000	0.000

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	68	92	0	63	0	0	-1
normalized size	1	1.00	0.38	0.52	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.069	0.060	0.000	0.495	0.000	0.000	0.000

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	79	118	0	92	0	0	-1
normalized size	1	1.00	0.37	0.55	0.00	0.43	0.00	0.00	-0.00
time (sec)	N/A	0.265	0.070	0.067	0.000	0.625	0.000	0.000	0.000

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	94	138	0	113	0	0	-1
normalized size	1	1.00	0.37	0.55	0.00	0.45	0.00	0.00	-0.00
time (sec)	N/A	0.272	0.092	0.066	0.000	0.544	0.000	0.000	0.000

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	89	185	0	138	0	0	-1
normalized size	1	1.00	0.34	0.71	0.00	0.53	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.113	0.068	0.000	0.508	0.000	0.000	0.000

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	84	169	0	122	0	0	-1
normalized size	1	1.00	0.39	0.78	0.00	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.112	0.063	0.000	0.465	0.000	0.000	0.000

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	86	169	0	136	0	0	-1
normalized size	1	1.00	0.49	0.96	0.00	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.070	0.065	0.000	0.615	0.000	0.000	0.000

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	85	164	0	134	0	0	-1
normalized size	1	1.00	0.46	0.89	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.062	0.066	0.000	0.576	0.000	0.000	0.000

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	60	164	0	134	0	0	-1
normalized size	1	1.00	0.44	1.20	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.073	0.066	0.000	0.669	0.000	0.000	0.000

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	83	169	0	136	0	0	-1
normalized size	1	1.00	0.45	0.92	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.057	0.066	0.000	0.714	0.000	0.000	0.000

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	88	196	0	145	0	0	-1
normalized size	1	1.00	0.32	0.72	0.00	0.54	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.110	0.063	0.000	0.502	0.000	0.000	0.000

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	99	225	0	174	0	0	-1
normalized size	1	1.00	0.32	0.73	0.00	0.57	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.104	0.064	0.000	0.692	0.000	0.000	0.000

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	47	0	25	0	0	-1
normalized size	1	1.00	0.59	0.62	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.031	0.037	0.000	0.692	0.000	0.000	0.000

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	43	47	0	25	0	0	-1
normalized size	1	1.00	0.58	0.64	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.026	0.036	0.000	1.419	0.000	0.000	0.000

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	41	44	0	22	0	0	-1
normalized size	1	1.00	0.59	0.64	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.022	0.034	0.000	0.660	0.000	0.000	0.000

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	46	0	20	0	0	-1
normalized size	1	1.00	0.63	0.66	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.022	0.048	0.000	0.575	0.000	0.000	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	44	48	0	22	0	0	-1
normalized size	1	1.00	0.61	0.67	0.00	0.31	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.025	0.056	0.000	0.706	0.000	0.000	0.000

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	96	202	117	184	0	0	-1
normalized size	1	1.00	0.70	1.47	0.85	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.430	0.144	0.056	0.411	0.534	0.000	0.000	0.000

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	178	93	168	0	84	-1
normalized size	1	1.00	0.79	1.59	0.83	1.50	0.00	0.75	-0.01
time (sec)	N/A	0.396	0.106	0.053	0.414	1.294	0.000	0.168	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	156	70	150	0	73	-1
normalized size	1	1.00	0.94	1.86	0.83	1.79	0.00	0.87	-0.01
time (sec)	N/A	0.254	0.091	0.046	0.418	0.601	0.000	0.150	0.000

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	126	47	134	0	62	-1
normalized size	1	1.00	1.15	1.45	0.54	1.54	0.00	0.71	-0.01
time (sec)	N/A	0.113	0.060	0.053	0.409	0.531	0.000	0.166	0.000

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	97	121	86	191	0	95	-1
normalized size	1	1.00	1.29	1.61	1.15	2.55	0.00	1.27	-0.01
time (sec)	N/A	0.349	0.086	0.051	0.428	0.895	0.000	0.166	0.000

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	104	200	0	210	0	134	-1
normalized size	1	1.00	1.27	2.44	0.00	2.56	0.00	1.63	-0.01
time (sec)	N/A	0.347	0.103	0.053	0.000	0.750	0.000	0.155	0.000

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	76	231	0	149	0	200	-1
normalized size	1	1.00	0.97	2.96	0.00	1.91	0.00	2.56	-0.01
time (sec)	N/A	0.347	0.130	0.052	0.000	0.588	0.000	0.142	0.000

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	82	254	0	165	0	250	-1
normalized size	1	1.00	0.81	2.51	0.00	1.63	0.00	2.48	-0.01
time (sec)	N/A	0.368	0.136	0.066	0.000	0.716	0.000	0.157	0.000

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	95	279	0	181	0	324	-1
normalized size	1	1.00	0.73	2.15	0.00	1.39	0.00	2.49	-0.01
time (sec)	N/A	0.402	0.156	0.066	0.000	0.543	0.000	0.149	0.000

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	87	92	0	58	0	0	-1
normalized size	1	1.00	0.38	0.41	0.00	0.26	0.00	0.00	-0.00
time (sec)	N/A	0.261	0.063	0.056	0.000	0.691	0.000	0.000	0.000

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	72	83	0	49	0	0	-1
normalized size	1	1.00	0.39	0.45	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.258	0.049	0.054	0.000	0.472	0.000	0.000	0.000

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	65	76	0	42	0	0	-1
normalized size	1	1.00	0.43	0.50	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.043	0.051	0.000	0.583	0.000	0.000	0.000

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	56	67	0	33	0	0	-1
normalized size	1	1.00	0.50	0.60	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.026	0.049	0.000	0.573	0.000	0.000	0.000

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	50	57	0	26	0	0	-1
normalized size	1	1.00	0.45	0.51	0.00	0.23	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.028	0.057	0.000	0.643	0.000	0.000	0.000

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	56	65	0	33	0	0	-1
normalized size	1	1.00	0.49	0.57	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.032	0.057	0.000	0.623	0.000	0.000	0.000

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	68	77	0	88	0	0	-1
normalized size	1	1.00	0.45	0.51	0.00	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.038	0.059	0.000	0.767	0.000	0.000	0.000

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	78	85	0	98	0	0	-1
normalized size	1	1.00	0.40	0.44	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.045	0.058	0.000	0.577	0.000	0.000	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	83	93	0	104	0	0	-1
normalized size	1	1.00	0.37	0.41	0.00	0.46	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.057	0.059	0.000	0.423	0.000	0.000	0.000

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	74	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.057	0.386	0.000	0.628	0.000	0.000	0.000

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	129	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.387	0.227	0.514	0.000	0.622	0.000	0.000	0.000

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	56	62	54	74	0	0	93
normalized size	1	1.00	0.68	0.76	0.66	0.90	0.00	0.00	1.13
time (sec)	N/A	0.213	0.034	0.037	0.339	0.600	0.000	0.000	1.593

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	64	57	75	0	0	94
normalized size	1	1.00	0.70	0.77	0.69	0.90	0.00	0.00	1.13
time (sec)	N/A	0.227	0.037	0.038	0.330	0.529	0.000	0.000	1.452

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	110	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.378	0.163	0.503	0.000	0.558	0.000	0.000	0.000

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	75	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.056	0.403	0.000	0.592	0.000	0.000	0.000

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	267	0	0	0	0	0	-1
normalized size	1	1.00	3.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	2.459	0.393	0.000	0.538	0.000	0.000	0.000

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	1.424	0.378	0.000	0.682	0.000	0.000	0.000

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	111	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.771	0.057	0.000	0.629	0.000	0.000	0.000

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.038	0.033	0.000	0.475	0.000	0.000	0.000

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	31	28	0	0	39
normalized size	1	1.00	1.00	1.00	1.72	1.56	0.00	0.00	2.17
time (sec)	N/A	0.035	0.054	0.043	0.322	0.819	0.000	0.000	1.449

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	55	0	80	0	0	106
normalized size	1	1.00	0.76	0.76	0.00	1.11	0.00	0.00	1.47
time (sec)	N/A	0.078	0.177	0.037	0.000	0.574	0.000	0.000	1.586

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	97	101	0	175	0	0	192
normalized size	1	1.00	0.76	0.80	0.00	1.38	0.00	0.00	1.51
time (sec)	N/A	0.130	0.227	0.039	0.000	0.600	0.000	0.000	1.729

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	152	167	0	310	0	0	314
normalized size	1	1.00	0.77	0.85	0.00	1.57	0.00	0.00	1.59
time (sec)	N/A	0.181	0.316	0.045	0.000	0.625	0.000	0.000	1.759

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	280	0	0	0	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	2.410	0.377	0.000	0.509	0.000	0.000	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	101	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.598	0.378	0.000	0.659	0.000	0.000	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	81	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.196	0.399	0.000	0.764	0.000	0.000	0.000

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	78	0	0	78
normalized size	1	1.00	0.93	1.07	0.00	1.70	0.00	0.00	1.70
time (sec)	N/A	0.054	0.201	0.038	0.000	0.493	0.000	0.000	1.492

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	163	0	0	173
normalized size	1	1.00	1.08	0.82	0.00	1.60	0.00	0.00	1.70
time (sec)	N/A	0.112	0.629	0.040	0.000	0.775	0.000	0.000	1.597

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	299	140	0	289	0	0	289
normalized size	1	1.00	1.80	0.84	0.00	1.74	0.00	0.00	1.74
time (sec)	N/A	0.177	1.568	0.040	0.000	0.854	0.000	0.000	1.737

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	260	218	0	451	0	0	441
normalized size	1	1.00	1.09	0.91	0.00	1.89	0.00	0.00	1.85
time (sec)	N/A	0.254	1.679	0.043	0.000	0.582	0.000	0.000	1.984

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	363	133	0	0	0	0	0	-1
normalized size	1	1.01	0.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.608	0.388	0.000	0.411	0.000	0.000	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	127	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	0.436	0.377	0.000	0.608	0.000	0.000	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	83	0	0	81
normalized size	1	1.00	0.93	1.07	0.00	1.80	0.00	0.00	1.76
time (sec)	N/A	0.092	0.197	0.037	0.000	1.203	0.000	0.000	1.435

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	78	0	0	78
normalized size	1	1.00	0.93	1.07	0.00	1.70	0.00	0.00	1.70
time (sec)	N/A	0.052	0.195	0.038	0.000	0.549	0.000	0.000	0.002

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	127	0	0	0	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.374	0.889	0.400	0.000	0.662	0.000	0.000	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	463	467	201	0	0	0	0	0	-1
normalized size	1	1.01	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	2.169	0.382	0.000	0.629	0.000	0.000	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	110	93	0	176	0	0	175
normalized size	1	1.00	0.33	0.28	0.00	0.53	0.00	0.00	0.53
time (sec)	N/A	0.367	0.730	0.040	0.000	0.535	0.000	0.000	1.556

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	96	0	179	0	0	175
normalized size	1	1.00	1.07	0.94	0.00	1.75	0.00	0.00	1.72
time (sec)	N/A	0.189	0.757	0.039	0.000	0.565	0.000	0.000	1.556

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	108	86	0	170	0	0	176
normalized size	1	1.00	1.11	0.89	0.00	1.75	0.00	0.00	1.81
time (sec)	N/A	0.147	0.659	0.038	0.000	0.531	0.000	0.000	1.551

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	163	0	0	173
normalized size	1	1.00	1.08	0.82	0.00	1.60	0.00	0.00	1.70
time (sec)	N/A	0.106	0.240	0.040	0.000	0.504	0.000	0.000	0.002

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	944	944	220	0	0	0	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	1.688	0.404	0.000	0.520	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	83	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.200	0.408	0.000	0.542	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	38	36	42	0	0	59
normalized size	1	1.00	0.71	0.75	0.71	0.82	0.00	0.00	1.16
time (sec)	N/A	0.123	0.072	0.032	0.334	0.745	0.000	0.000	1.336

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	40	34	44	0	0	59
normalized size	1	1.00	0.69	0.77	0.65	0.85	0.00	0.00	1.13
time (sec)	N/A	0.122	0.066	0.033	0.336	0.837	0.000	0.000	1.300

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.026	0.631	0.000	0.559	0.000	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.279	0.383	0.000	0.763	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	67	0	0	0	651	0	-1
normalized size	1	1.00	1.24	0.00	0.00	0.00	12.06	0.00	-0.02
time (sec)	N/A	0.088	0.019	0.532	0.000	0.653	19.966	0.000	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.222	0.391	0.000	0.732	0.000	0.000	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.238	0.418	0.000	0.651	0.000	0.000	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	73	0	0	0	651	0	-1
normalized size	1	1.00	1.33	0.00	0.00	0.00	11.84	0.00	-0.02
time (sec)	N/A	0.086	0.027	0.599	0.000	0.485	17.797	0.000	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	119	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.283	0.399	0.000	0.666	0.000	0.000	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	120	320	380	201	0	335	332
normalized size	1	1.00	0.35	0.94	1.11	0.59	0.00	0.98	0.97
time (sec)	N/A	0.246	0.318	0.072	0.423	0.627	0.000	0.185	1.419

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	104	272	302	179	0	273	258
normalized size	1	1.00	0.39	1.01	1.13	0.67	0.00	1.02	0.96
time (sec)	N/A	0.188	0.247	0.063	0.413	0.515	0.000	0.181	1.319

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	94	224	223	157	0	211	183
normalized size	1	1.00	0.48	1.15	1.15	0.81	0.00	1.09	0.94
time (sec)	N/A	0.134	0.164	0.052	0.412	0.516	0.000	0.170	1.334

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	53	163	117	104	0	123	84
normalized size	1	1.00	0.50	1.52	1.09	0.97	0.00	1.15	0.79
time (sec)	N/A	0.071	0.124	0.049	0.416	0.716	0.000	0.154	0.074

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	56	251	116	93	0	127	62
normalized size	1	1.00	0.54	2.41	1.12	0.89	0.00	1.22	0.60
time (sec)	N/A	0.075	0.138	0.059	0.317	0.495	0.000	0.173	0.078

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	83	530	160	134	0	171	128
normalized size	1	1.00	0.46	2.94	0.89	0.74	0.00	0.95	0.71
time (sec)	N/A	0.116	0.215	0.066	0.314	0.522	0.000	0.163	0.068

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	99	714	194	178	0	232	171
normalized size	1	1.00	0.39	2.81	0.76	0.70	0.00	0.91	0.67
time (sec)	N/A	0.166	0.275	0.071	0.320	0.580	0.000	0.176	0.097

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	115	898	230	274	0	286	210
normalized size	1	1.00	0.35	2.74	0.70	0.84	0.00	0.87	0.64
time (sec)	N/A	0.228	0.351	0.076	0.327	0.684	0.000	0.174	0.069

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	116	114	122	124	115	89
normalized size	1	1.00	1.00	0.91	0.90	0.96	0.98	0.91	0.70
time (sec)	N/A	0.175	0.052	0.042	0.311	1.516	0.658	0.135	0.103

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	83	81	89	88	82	65
normalized size	1	1.00	1.00	0.92	0.90	0.99	0.98	0.91	0.72
time (sec)	N/A	0.160	0.036	0.045	0.316	0.414	0.441	0.123	1.244

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	71	70	78	76	71	56
normalized size	1	1.00	1.00	0.93	0.92	1.03	1.00	0.93	0.74
time (sec)	N/A	0.159	0.028	0.041	0.314	0.774	0.327	0.119	1.218

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	35	43	39	36	32
normalized size	1	1.00	1.00	0.97	0.90	1.10	1.00	0.92	0.82
time (sec)	N/A	0.147	0.020	0.041	0.307	0.451	0.178	0.137	0.051

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	23
normalized size	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.10
time (sec)	N/A	0.085	0.016	0.037	0.309	0.498	0.106	0.121	0.041

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	36	35	40	36	36	33
normalized size	1	1.00	0.78	1.00	0.97	1.11	1.00	1.00	0.92
time (sec)	N/A	0.160	0.034	0.039	0.304	0.606	0.145	0.134	0.049

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	65	69	93	73	57	68
normalized size	1	1.00	1.00	0.87	0.92	1.24	0.97	0.76	0.91
time (sec)	N/A	0.176	0.053	0.045	0.310	0.544	0.369	0.143	0.094

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	95	97	137	102	80	94
normalized size	1	1.00	0.75	0.86	0.88	1.25	0.93	0.73	0.85
time (sec)	N/A	0.199	0.080	0.047	0.308	0.459	0.592	0.132	1.281

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	98	125	145	233	156	96	142
normalized size	1	1.00	0.68	0.86	1.00	1.61	1.08	0.66	0.98
time (sec)	N/A	0.228	0.120	0.048	0.313	0.856	0.842	0.140	1.445

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	126	329	380	201	0	335	332
normalized size	1	1.00	0.37	0.96	1.11	0.59	0.00	0.98	0.97
time (sec)	N/A	0.257	0.273	0.073	0.431	0.491	0.000	0.187	1.378

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	110	281	302	179	0	273	258
normalized size	1	1.00	0.41	1.04	1.12	0.67	0.00	1.01	0.96
time (sec)	N/A	0.192	0.203	0.061	0.421	0.614	0.000	0.175	1.412

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	94	233	223	156	0	212	183
normalized size	1	1.00	0.48	1.19	1.14	0.80	0.00	1.09	0.94
time (sec)	N/A	0.135	0.146	0.057	0.419	0.594	0.000	0.175	0.137

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	235	118	106	0	111	84
normalized size	1	1.00	0.75	3.09	1.55	1.39	0.00	1.46	1.11
time (sec)	N/A	0.058	0.099	0.056	0.416	0.483	0.000	0.154	0.088

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	69	346	133	128	0	148	100
normalized size	1	1.00	0.48	2.40	0.92	0.89	0.00	1.03	0.69
time (sec)	N/A	0.101	0.156	0.060	0.319	0.546	0.000	0.153	1.345

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	78	438	153	170	0	166	121
normalized size	1	1.00	0.43	2.42	0.85	0.94	0.00	0.92	0.67
time (sec)	N/A	0.128	0.198	0.065	0.319	0.705	0.000	0.163	0.091

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	101	714	192	204	0	205	160
normalized size	1	1.00	0.40	2.80	0.75	0.80	0.00	0.80	0.63
time (sec)	N/A	0.172	0.236	0.072	0.329	0.550	0.000	0.167	1.406

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	117	766	226	248	0	264	203
normalized size	1	1.00	0.36	2.33	0.69	0.75	0.00	0.80	0.62
time (sec)	N/A	0.236	0.300	0.078	0.323	0.545	0.000	0.171	0.110

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	105	103	111	112	184	81
normalized size	1	1.00	1.00	0.91	0.89	0.96	0.97	1.59	0.70
time (sec)	N/A	0.171	0.047	0.043	0.304	0.442	0.620	0.152	0.086

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	93	92	100	100	160	72
normalized size	1	1.00	1.00	0.93	0.92	1.00	1.00	1.60	0.72
time (sec)	N/A	0.166	0.036	0.041	0.315	0.549	0.476	0.142	1.290

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	60	59	67	65	136	48
normalized size	1	1.00	1.00	0.95	0.94	1.06	1.03	2.16	0.76
time (sec)	N/A	0.152	0.027	0.041	0.313	0.475	0.294	0.129	0.058

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	50	46	56	53	112	43
normalized size	1	1.00	1.00	0.98	0.90	1.10	1.04	2.20	0.84
time (sec)	N/A	0.148	0.024	0.042	0.310	0.406	0.205	0.121	0.060

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	33	32	35	26	66	32
normalized size	1	1.00	1.00	1.00	0.97	1.06	0.79	2.00	0.97
time (sec)	N/A	0.093	0.023	0.043	0.302	0.593	0.265	0.141	0.067

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	51	49	64	41	74	48
normalized size	1	1.00	1.00	0.96	0.92	1.21	0.77	1.40	0.91
time (sec)	N/A	0.168	0.037	0.043	0.302	0.455	0.231	0.118	1.256

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	66	75	100	83	93	71
normalized size	1	1.00	0.89	0.93	1.06	1.41	1.17	1.31	1.00
time (sec)	N/A	0.172	0.042	0.041	0.316	0.644	0.323	0.134	1.267

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	95	107	163	114	130	104
normalized size	1	1.00	0.80	0.86	0.96	1.47	1.03	1.17	0.94
time (sec)	N/A	0.196	0.071	0.044	0.305	0.533	0.614	0.143	0.110

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	98	125	135	207	144	170	131
normalized size	1	1.00	0.67	0.86	0.92	1.42	0.99	1.16	0.90
time (sec)	N/A	0.221	0.107	0.048	0.321	0.419	0.920	0.127	0.148

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	120	320	380	201	0	524	332
normalized size	1	1.00	0.35	0.93	1.11	0.59	0.00	1.53	0.97
time (sec)	N/A	0.257	0.308	0.071	0.420	0.614	0.000	0.189	1.368

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	110	272	302	179	0	394	258
normalized size	1	1.00	0.41	1.01	1.12	0.67	0.00	1.46	0.96
time (sec)	N/A	0.186	0.209	0.061	0.413	0.514	0.000	0.177	0.109

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	94	224	223	156	0	264	183
normalized size	1	1.00	0.48	1.15	1.14	0.80	0.00	1.35	0.94
time (sec)	N/A	0.132	0.153	0.054	0.418	0.636	0.000	0.184	1.299

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	166	117	107	0	121	84
normalized size	1	1.00	0.51	1.54	1.08	0.99	0.00	1.12	0.78
time (sec)	N/A	0.076	0.091	0.051	0.408	0.588	0.000	0.143	1.278

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	57	250	121	67	0	0	86
normalized size	1	1.00	0.54	2.38	1.15	0.64	0.00	0.00	0.82
time (sec)	N/A	0.077	0.121	0.062	0.315	0.528	0.000	0.000	0.054

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	85	530	163	119	0	0	137
normalized size	1	1.00	0.47	2.96	0.91	0.66	0.00	0.00	0.77
time (sec)	N/A	0.118	0.183	0.066	0.305	0.436	0.000	0.000	1.296

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	101	714	197	161	0	0	178
normalized size	1	1.00	0.40	2.80	0.77	0.63	0.00	0.00	0.70
time (sec)	N/A	0.166	0.232	0.069	0.318	0.788	0.000	0.000	0.062

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	117	898	231	205	0	0	217
normalized size	1	1.00	0.36	2.73	0.70	0.62	0.00	0.00	0.66
time (sec)	N/A	0.225	0.319	0.078	0.329	0.583	0.000	0.000	1.282

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	83	81	89	88	82	67
normalized size	1	1.00	1.00	0.92	0.90	0.99	0.98	0.91	0.74
time (sec)	N/A	0.153	0.042	0.043	0.302	1.166	0.458	0.124	1.318

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	71	70	78	76	71	56
normalized size	1	1.00	1.00	0.93	0.92	1.03	1.00	0.93	0.74
time (sec)	N/A	0.156	0.033	0.042	0.301	0.466	0.326	0.120	1.251

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	37	43	39	38	35
normalized size	1	1.00	1.00	0.98	0.92	1.08	0.98	0.95	0.88
time (sec)	N/A	0.141	0.027	0.042	0.313	0.433	0.183	0.128	0.050

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	25
normalized size	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.19
time (sec)	N/A	0.085	0.018	0.040	0.310	0.506	0.107	0.133	1.227

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	34	38	36	36	33
normalized size	1	1.00	0.80	1.03	0.97	1.09	1.03	1.03	0.94
time (sec)	N/A	0.155	0.040	0.041	0.306	0.558	0.148	0.120	0.052

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	65	69	92	75	57	68
normalized size	1	1.00	0.96	0.89	0.95	1.26	1.03	0.78	0.93
time (sec)	N/A	0.174	0.060	0.044	0.311	0.538	0.379	0.131	1.307

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	104	95	97	137	102	80	94
normalized size	1	1.00	0.96	0.88	0.90	1.27	0.94	0.74	0.87
time (sec)	N/A	0.202	0.101	0.050	0.309	0.571	0.610	0.150	0.120

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	124	125	145	233	156	96	142
normalized size	1	1.00	0.87	0.87	1.01	1.63	1.09	0.67	0.99
time (sec)	N/A	0.231	0.131	0.050	0.317	0.592	0.846	0.137	0.153

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	126	329	379	201	0	525	332
normalized size	1	1.00	0.37	0.96	1.10	0.59	0.00	1.53	0.97
time (sec)	N/A	0.250	0.305	0.074	0.426	0.487	0.000	0.250	0.173

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	110	281	301	179	0	395	258
normalized size	1	1.00	0.41	1.04	1.12	0.67	0.00	1.47	0.96
time (sec)	N/A	0.184	0.252	0.060	0.419	0.445	0.000	0.202	1.263

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	94	233	224	156	0	264	183
normalized size	1	1.00	0.48	1.19	1.15	0.80	0.00	1.35	0.94
time (sec)	N/A	0.131	0.172	0.053	0.422	0.517	0.000	0.185	0.093

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	234	118	103	0	122	84
normalized size	1	1.00	0.75	3.08	1.55	1.36	0.00	1.61	1.11
time (sec)	N/A	0.057	0.103	0.054	0.421	1.015	0.000	0.166	0.061

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	69	346	140	96	0	59	114
normalized size	1	1.00	0.48	2.40	0.97	0.67	0.00	0.41	0.79
time (sec)	N/A	0.098	0.149	0.059	0.311	0.595	0.000	0.159	0.069

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	78	438	161	135	0	59	141
normalized size	1	1.00	0.43	2.42	0.89	0.75	0.00	0.33	0.78
time (sec)	N/A	0.124	0.181	0.067	0.314	0.503	0.000	0.202	0.046

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	101	714	199	179	0	0	183
normalized size	1	1.00	0.40	2.82	0.79	0.71	0.00	0.00	0.72
time (sec)	N/A	0.167	0.243	0.072	0.308	0.460	0.000	0.000	0.054

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	117	766	231	275	0	0	224
normalized size	1	1.00	0.36	2.34	0.71	0.84	0.00	0.00	0.69
time (sec)	N/A	0.222	0.294	0.080	0.328	0.606	0.000	0.000	0.063

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	96	112	0	96	0	0	-1
normalized size	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.083	0.106	0.000	0.694	0.000	0.000	0.000

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	75	96	0	74	0	0	-1
normalized size	1	1.00	0.32	0.41	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.064	0.066	0.000	0.554	0.000	0.000	0.000

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	64	80	0	42	0	0	-1
normalized size	1	1.00	0.44	0.55	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.050	0.062	0.000	0.649	0.000	0.000	0.000

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	50	0	17	0	0	-1
normalized size	1	1.00	0.58	0.75	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.025	0.055	0.000	0.484	0.000	0.000	0.000

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	44	57	0	24	0	0	-1
normalized size	1	1.00	0.61	0.79	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.032	0.052	0.000	0.642	0.000	0.000	0.000

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	68	101	0	68	0	0	-1
normalized size	1	1.00	0.39	0.58	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.077	0.067	0.000	0.567	0.000	0.000	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	86	175	0	137	0	0	-1
normalized size	1	1.00	0.33	0.67	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.117	0.069	0.000	0.659	0.000	0.000	0.000

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	104	247	0	205	0	0	-1
normalized size	1	1.00	0.29	0.69	0.00	0.57	0.00	0.00	-0.00
time (sec)	N/A	0.195	0.165	0.072	0.000	0.598	0.000	0.000	0.000

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	150	795	0	438	1059	561	-1
normalized size	1	1.00	0.40	2.14	0.00	1.18	2.85	1.51	-0.00
time (sec)	N/A	0.542	0.176	0.117	0.000	0.761	33.428	113.348	0.000

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	134	625	0	394	500	416	-1
normalized size	1	1.00	0.46	2.13	0.00	1.34	1.70	1.41	-0.00
time (sec)	N/A	0.495	0.146	0.062	0.000	0.630	20.073	7.326	0.000

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	115	455	0	317	376	266	-1
normalized size	1	1.00	0.54	2.14	0.00	1.49	1.77	1.25	-0.00
time (sec)	N/A	0.459	0.113	0.056	0.000	0.810	14.141	0.504	0.000

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	-1
normalized size	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.360	0.087	0.054	0.000	0.599	0.000	0.000	0.000

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	68	177	0	216	0	0	-1
normalized size	1	1.00	0.61	1.59	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.307	0.077	0.051	0.000	0.527	0.000	0.000	0.000

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	326	0	280	0	0	-1
normalized size	1	1.00	0.77	2.65	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.436	0.090	0.052	0.000	0.635	0.000	0.000	0.000

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	105	462	0	352	0	0	-1
normalized size	1	1.00	0.52	2.28	0.00	1.73	0.00	0.00	-0.00
time (sec)	N/A	0.463	0.110	0.061	0.000	0.669	0.000	0.000	0.000

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	133	572	0	496	0	0	-1
normalized size	1	1.00	0.47	2.02	0.00	1.75	0.00	0.00	-0.00
time (sec)	N/A	0.502	0.133	0.075	0.000	0.591	0.000	0.000	0.000

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	96	0	0	-1
normalized size	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.093	0.063	0.000	0.831	0.000	0.000	0.000

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	94	112	0	96	0	0	-1
normalized size	1	1.00	0.29	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.092	0.063	0.000	0.760	0.000	0.000	0.000

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	87	96	0	72	0	0	-1
normalized size	1	1.00	0.37	0.41	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.142	0.058	0.065	0.000	0.595	0.000	0.000	0.000

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	59	69	0	44	0	0	-1
normalized size	1	1.00	0.40	0.47	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.048	0.064	0.000	0.473	0.000	0.000	0.000

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	50	65	0	27	0	0	-1
normalized size	1	1.00	0.46	0.60	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.034	0.063	0.000	0.731	0.000	0.000	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	56	85	0	49	0	0	-1
normalized size	1	1.00	0.49	0.74	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.052	0.061	0.000	0.497	0.000	0.000	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	64	102	0	81	0	0	-1
normalized size	1	1.00	0.37	0.60	0.00	0.47	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.082	0.066	0.000	0.498	0.000	0.000	0.000

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	86	175	0	138	0	0	-1
normalized size	1	1.00	0.32	0.66	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.177	0.117	0.070	0.000	0.629	0.000	0.000	0.000

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	140	247	0	207	0	0	-1
normalized size	1	1.00	0.39	0.69	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.137	0.074	0.000	0.520	0.000	0.000	0.000

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	96	0	0	-1
normalized size	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.075	0.063	0.000	0.525	0.000	0.000	0.000

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	77	96	0	74	0	0	-1
normalized size	1	1.00	0.32	0.40	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.065	0.062	0.000	1.622	0.000	0.000	0.000

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	65	80	0	44	0	0	-1
normalized size	1	1.00	0.44	0.54	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.049	0.063	0.000	0.821	0.000	0.000	0.000

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	52	0	19	0	0	-1
normalized size	1	1.00	0.60	0.76	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.027	0.054	0.000	0.454	0.000	0.000	0.000

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	45	59	0	26	0	0	-1
normalized size	1	1.00	0.62	0.82	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.035	0.052	0.000	0.495	0.000	0.000	0.000

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	65	100	0	66	0	0	-1
normalized size	1	1.00	0.38	0.58	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.069	0.068	0.000	0.825	0.000	0.000	0.000

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	85	175	0	135	0	0	-1
normalized size	1	1.00	0.32	0.67	0.00	0.51	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.127	0.073	0.000	0.546	0.000	0.000	0.000

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	104	247	0	201	0	0	-1
normalized size	1	1.00	0.29	0.69	0.00	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.165	0.072	0.000	0.801	0.000	0.000	0.000

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	150	795	0	438	1059	561	-1
normalized size	1	1.00	0.40	2.12	0.00	1.17	2.82	1.50	-0.00
time (sec)	N/A	0.541	0.187	0.119	0.000	0.639	32.677	113.467	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	134	625	0	394	500	416	-1
normalized size	1	1.00	0.46	2.13	0.00	1.34	1.71	1.42	-0.00
time (sec)	N/A	0.483	0.150	0.064	0.000	0.524	19.521	7.131	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	115	454	0	316	376	266	-1
normalized size	1	1.00	0.54	2.13	0.00	1.48	1.77	1.25	-0.00
time (sec)	N/A	0.446	0.116	0.059	0.000	0.506	13.215	0.474	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	196	0	267	0	0	-1
normalized size	1	1.00	0.69	1.69	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.357	0.091	0.054	0.000	0.789	0.000	0.000	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	68	179	0	212	0	0	-1
normalized size	1	1.00	0.61	1.60	0.00	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.291	0.077	0.049	0.000	0.580	0.000	0.000	0.000

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	95	326	0	279	0	0	-1
normalized size	1	1.00	0.77	2.63	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.422	0.092	0.052	0.000	0.590	0.000	0.000	0.000

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	105	462	0	351	0	0	-1
normalized size	1	1.00	0.54	2.37	0.00	1.80	0.00	0.00	-0.01
time (sec)	N/A	0.451	0.106	0.061	0.000	1.095	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	131	572	0	495	0	0	-1
normalized size	1	1.00	0.49	2.12	0.00	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.489	0.128	0.076	0.000	0.741	0.000	0.000	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	96	0	0	-1
normalized size	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.091	0.066	0.000	0.613	0.000	0.000	0.000

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	94	112	0	96	0	0	-1
normalized size	1	1.00	0.29	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.091	0.066	0.000	0.466	0.000	0.000	0.000

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	81	96	0	74	0	0	-1
normalized size	1	1.00	0.34	0.41	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.071	0.065	0.000	0.429	0.000	0.000	0.000

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	59	69	0	42	0	0	-1
normalized size	1	1.00	0.40	0.47	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.069	0.063	0.000	0.626	0.000	0.000	0.000

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	47	63	0	25	0	0	-1
normalized size	1	1.00	0.44	0.59	0.00	0.23	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.063	0.066	0.000	0.638	0.000	0.000	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	54	87	0	47	0	0	-1
normalized size	1	1.00	0.48	0.77	0.00	0.42	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.054	0.059	0.000	0.587	0.000	0.000	0.000

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	63	102	0	81	0	0	-1
normalized size	1	1.00	0.38	0.61	0.00	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.093	0.064	0.000	0.469	0.000	0.000	0.000

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	85	175	0	137	0	0	-1
normalized size	1	1.00	0.32	0.66	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.136	0.069	0.000	0.618	0.000	0.000	0.000

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	105	247	0	208	0	0	-1
normalized size	1	1.00	0.29	0.69	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.239	0.073	0.000	0.635	0.000	0.000	0.000

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	63	44	72	0	0	-1
normalized size	1	1.00	0.66	0.79	0.55	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.047	0.037	0.380	0.421	0.000	0.000	0.000

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	53	0	24	0	0	46
normalized size	1	1.00	0.59	0.70	0.00	0.32	0.00	0.00	0.61
time (sec)	N/A	0.261	0.035	0.035	0.000	0.394	0.000	0.000	1.442

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	43	52	0	21	0	0	45
normalized size	1	1.00	0.61	0.73	0.00	0.30	0.00	0.00	0.63
time (sec)	N/A	0.172	0.035	0.034	0.000	0.664	0.000	0.000	1.400

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	50	0	17	0	0	-1
normalized size	1	1.00	0.58	0.75	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.030	0.047	0.000	0.538	0.000	0.000	0.000

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	50	0	21	0	0	-1
normalized size	1	1.00	0.61	0.71	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.038	0.058	0.000	0.789	0.000	0.000	0.000

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	53	0	21	0	0	63
normalized size	1	1.00	1.02	1.15	0.00	0.46	0.00	0.00	1.37
time (sec)	N/A	0.244	0.048	0.044	0.000	0.480	0.000	0.000	1.402

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	93	196	0	222	0	128	-1
normalized size	1	1.00	0.58	1.22	0.00	1.39	0.00	0.80	-0.01
time (sec)	N/A	0.558	0.123	0.051	0.000	0.567	0.000	0.182	0.000

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	84	174	0	204	0	116	-1
normalized size	1	1.00	0.68	1.41	0.00	1.66	0.00	0.94	-0.01
time (sec)	N/A	0.472	0.092	0.052	0.000	0.568	0.000	0.185	0.000

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	77	147	0	188	0	106	-1
normalized size	1	1.00	0.79	1.50	0.00	1.92	0.00	1.08	-0.01
time (sec)	N/A	0.310	0.083	0.044	0.000	0.757	0.000	0.161	0.000

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	-1
normalized size	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.341	0.100	0.053	0.000	0.812	0.000	0.000	0.000

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	82	306	0	252	0	127	-1
normalized size	1	1.00	0.70	2.62	0.00	2.15	0.00	1.09	-0.01
time (sec)	N/A	0.551	0.108	0.056	0.000	0.499	0.000	0.306	0.000

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	78	347	0	177	0	194	-1
normalized size	1	1.00	0.70	3.13	0.00	1.59	0.00	1.75	-0.01
time (sec)	N/A	0.515	0.095	0.056	0.000	0.730	0.000	0.335	0.000

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	86	378	0	201	0	231	-1
normalized size	1	1.00	0.63	2.76	0.00	1.47	0.00	1.69	-0.01
time (sec)	N/A	0.528	0.110	0.056	0.000	0.672	0.000	1.885	0.000

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	94	410	0	217	0	316	-1
normalized size	1	1.00	0.60	2.63	0.00	1.39	0.00	2.03	-0.01
time (sec)	N/A	0.556	0.143	0.059	0.000	0.604	0.000	7.404	0.000

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	447	0	233	0	362	-1
normalized size	1	1.00	0.56	2.47	0.00	1.29	0.00	2.00	-0.01
time (sec)	N/A	0.575	0.115	0.064	0.000	0.554	0.000	8.610	0.000

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	71	89	0	48	0	0	-1
normalized size	1	1.00	0.38	0.48	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.293	0.060	0.059	0.000	0.428	0.000	0.000	0.000

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	63	82	0	41	0	0	-1
normalized size	1	1.00	0.41	0.54	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.286	0.049	0.060	0.000	0.481	0.000	0.000	0.000

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	54	73	0	32	0	0	-1
normalized size	1	1.00	0.48	0.65	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.032	0.059	0.000	0.686	0.000	0.000	0.000

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	50	65	0	27	0	0	-1
normalized size	1	1.00	0.46	0.60	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.032	0.060	0.000	0.711	0.000	0.000	0.000

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	52	67	0	32	0	0	-1
normalized size	1	1.00	0.48	0.62	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.041	0.063	0.000	0.582	0.000	0.000	0.000

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	66	82	0	85	0	0	-1
normalized size	1	1.00	0.45	0.56	0.00	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.277	0.052	0.065	0.000	0.504	0.000	0.000	0.000

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	76	90	0	93	0	0	-1
normalized size	1	1.00	0.40	0.48	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.055	0.064	0.000	0.920	0.000	0.000	0.000

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	81	98	0	101	0	0	-1
normalized size	1	1.00	0.36	0.44	0.00	0.45	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.070	0.065	0.000	0.418	0.000	0.000	0.000

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	90	106	0	109	0	0	-1
normalized size	1	1.00	0.34	0.40	0.00	0.41	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.080	0.076	0.000	0.433	0.000	0.000	0.000

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	50	65	46	73	0	0	-1
normalized size	1	1.00	0.62	0.80	0.57	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.047	0.037	0.388	0.447	0.000	0.000	0.000

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	53	0	24	0	0	46
normalized size	1	1.00	0.59	0.70	0.00	0.32	0.00	0.00	0.61
time (sec)	N/A	0.263	0.030	0.034	0.000	0.402	0.000	0.000	1.363

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	42	52	0	21	0	0	45
normalized size	1	1.00	0.58	0.72	0.00	0.29	0.00	0.00	0.62
time (sec)	N/A	0.176	0.027	0.037	0.000	0.409	0.000	0.000	1.319

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	52	0	19	0	0	-1
normalized size	1	1.00	0.60	0.76	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.021	0.050	0.000	0.395	0.000	0.000	0.000

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	41	50	0	21	0	0	-1
normalized size	1	1.00	0.59	0.72	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.027	0.056	0.000	0.449	0.000	0.000	0.000

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	53	0	21	0	0	63
normalized size	1	1.00	1.00	1.13	0.00	0.45	0.00	0.00	1.34
time (sec)	N/A	0.246	0.027	0.035	0.000	0.436	0.000	0.000	1.362

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	93	196	0	222	0	128	-1
normalized size	1	1.00	0.57	1.20	0.00	1.36	0.00	0.79	-0.01
time (sec)	N/A	0.547	0.097	0.051	0.000	0.465	0.000	0.159	0.000

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	84	173	0	204	0	117	-1
normalized size	1	1.00	0.68	1.40	0.00	1.65	0.00	0.94	-0.01
time (sec)	N/A	0.470	0.085	0.048	0.000	0.411	0.000	0.172	0.000

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	147	0	188	0	106	-1
normalized size	1	1.00	1.01	1.48	0.00	1.90	0.00	1.07	-0.01
time (sec)	N/A	0.305	0.068	0.047	0.000	0.478	0.000	0.176	0.000

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	196	0	267	0	0	-1
normalized size	1	1.00	0.69	1.69	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.340	0.085	0.050	0.000	0.442	0.000	0.000	0.000

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	82	306	0	252	0	127	-1
normalized size	1	1.00	0.70	2.62	0.00	2.15	0.00	1.09	-0.01
time (sec)	N/A	0.525	0.093	0.055	0.000	0.455	0.000	0.281	0.000

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	78	348	0	176	0	194	-1
normalized size	1	1.00	0.70	3.11	0.00	1.57	0.00	1.73	-0.01
time (sec)	N/A	0.523	0.075	0.060	0.000	0.477	0.000	0.350	0.000

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	86	378	0	200	0	231	-1
normalized size	1	1.00	0.61	2.70	0.00	1.43	0.00	1.65	-0.01
time (sec)	N/A	0.534	0.084	0.057	0.000	0.443	0.000	1.915	0.000

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	94	410	0	216	0	316	-1
normalized size	1	1.00	0.60	2.63	0.00	1.38	0.00	2.03	-0.01
time (sec)	N/A	0.554	0.089	0.058	0.000	0.441	0.000	7.728	0.000

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	447	0	232	0	362	-1
normalized size	1	1.00	0.56	2.47	0.00	1.28	0.00	2.00	-0.01
time (sec)	N/A	0.585	0.101	0.063	0.000	0.448	0.000	9.781	0.000

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	69	89	0	48	0	0	-1
normalized size	1	1.00	0.37	0.48	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.057	0.058	0.000	0.427	0.000	0.000	0.000

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	62	82	0	41	0	0	-1
normalized size	1	1.00	0.41	0.54	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.046	0.056	0.000	0.456	0.000	0.000	0.000

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	53	73	0	32	0	0	-1
normalized size	1	1.00	0.47	0.65	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.033	0.056	0.000	0.412	0.000	0.000	0.000

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	47	63	0	25	0	0	-1
normalized size	1	1.00	0.44	0.59	0.00	0.23	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.030	0.053	0.000	0.414	0.000	0.000	0.000

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	53	67	0	32	0	0	-1
normalized size	1	1.00	0.49	0.62	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.039	0.063	0.000	0.450	0.000	0.000	0.000

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	65	82	0	83	0	0	-1
normalized size	1	1.00	0.45	0.56	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.278	0.049	0.063	0.000	0.409	0.000	0.000	0.000

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	75	90	0	91	0	0	-1
normalized size	1	1.00	0.40	0.48	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.053	0.064	0.000	0.527	0.000	0.000	0.000

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	80	98	0	99	0	0	-1
normalized size	1	1.00	0.36	0.44	0.00	0.45	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.061	0.065	0.000	0.543	0.000	0.000	0.000

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	89	106	0	107	0	0	-1
normalized size	1	1.00	0.34	0.40	0.00	0.41	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.077	0.066	0.000	0.574	0.000	0.000	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	81	123	0	0	0	0	0	-1
normalized size	1	0.53	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.263	0.058	0.000	0.629	0.000	0.000	0.000

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	164	94	0	0	0	0	0	-1
normalized size	1	1.09	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.275	0.056	0.000	0.560	0.000	0.000	0.000

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	303	180	0	0	0	0	0	-1
normalized size	1	1.05	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.930	0.064	0.000	0.695	0.000	0.000	0.000

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	111	146	0	0	0	0	0	-1
normalized size	1	0.38	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.517	0.050	0.000	0.757	0.000	0.000	0.000

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	112	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.412	0.056	0.000	0.644	0.000	0.000	0.000

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.593	0.128	0.000	0.658	0.000	0.000	0.000

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.625	0.105	0.000	0.672	0.000	0.000	0.000

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.604	0.097	0.000	0.710	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [129] had the largest ratio of [1.429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.00	10	0.600
2	A	7	6	1.00	10	0.600
3	A	6	6	1.00	8	0.750
4	A	5	5	1.00	6	0.833
5	A	6	6	1.00	10	0.600
6	A	3	3	1.00	10	0.300
7	A	3	3	1.00	10	0.300
8	A	7	5	1.00	10	0.500
9	A	5	4	1.00	10	0.400
10	A	4	3	1.00	12	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	10	0.300
13	A	4	3	1.00	8	0.375
14	A	4	3	1.00	12	0.250
15	A	4	3	1.00	12	0.250
16	A	4	3	1.00	12	0.250
17	A	4	3	1.00	12	0.250
18	A	14	9	1.00	12	0.750
19	A	12	8	1.00	10	0.800
20	A	8	7	1.00	8	0.875
21	A	8	7	1.00	12	0.583
22	A	5	5	1.00	12	0.417
23	A	9	7	1.00	12	0.583
24	A	10	9	1.00	12	0.750
25	A	4	3	1.00	12	0.250
26	A	4	3	1.00	12	0.250
27	A	4	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	4	3	1.00	8	0.375
29	A	4	3	1.00	12	0.250
30	A	4	3	1.00	12	0.250
31	A	4	3	1.00	12	0.250
32	A	4	3	1.00	12	0.250
33	A	8	6	1.00	12	0.500
34	A	7	6	1.00	12	0.500
35	A	6	6	1.00	10	0.600
36	A	5	5	1.00	8	0.625
37	A	6	6	1.00	12	0.500
38	A	3	3	1.00	12	0.250
39	A	3	3	1.00	12	0.250
40	A	7	5	1.00	12	0.417
41	A	5	4	1.00	12	0.333
42	A	4	3	1.00	12	0.250
43	A	4	3	1.00	12	0.250
44	A	4	3	1.00	10	0.300
45	A	4	3	1.00	8	0.375
46	A	4	3	1.00	12	0.250
47	A	4	3	1.00	12	0.250
48	A	4	3	1.00	12	0.250
49	A	4	3	1.00	12	0.250
50	A	19	9	1.00	12	0.750
51	A	14	9	1.00	12	0.750
52	A	12	8	1.00	10	0.800
53	A	8	7	1.00	8	0.875
54	A	8	7	1.00	12	0.583
55	A	5	5	1.00	12	0.417
56	A	9	7	1.00	12	0.583
57	A	10	9	1.00	12	0.750
58	A	14	11	1.00	12	0.917
59	A	11	8	1.00	14	0.571
60	A	10	8	1.00	14	0.571
61	A	9	8	1.00	14	0.571
62	A	7	7	1.00	12	0.583
63	A	6	6	1.00	10	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	17	14	1.00	14	1.000
65	A	13	10	1.00	14	0.714
66	A	14	11	1.00	14	0.786
67	A	15	12	1.00	14	0.857
68	A	11	8	1.00	14	0.571
69	A	10	8	1.00	14	0.571
70	A	9	8	1.00	14	0.571
71	A	7	7	1.00	12	0.583
72	A	6	6	1.00	10	0.600
73	A	17	14	1.00	14	1.000
74	A	13	10	1.00	14	0.714
75	A	14	11	1.00	14	0.786
76	A	15	12	1.00	14	0.857
77	A	12	9	1.00	14	0.643
78	A	11	9	1.00	14	0.643
79	A	10	9	1.00	14	0.643
80	A	8	7	1.00	12	0.583
81	A	7	6	1.00	10	0.600
82	A	19	16	1.00	14	1.143
83	A	14	11	1.00	14	0.786
84	A	15	11	1.00	14	0.786
85	A	16	12	1.00	14	0.857
86	A	11	8	1.00	14	0.571
87	A	10	8	1.00	14	0.571
88	A	9	8	1.00	14	0.571
89	A	7	7	1.00	12	0.583
90	A	6	6	1.00	10	0.600
91	A	17	14	1.00	14	1.000
92	A	13	10	1.00	14	0.714
93	A	14	11	1.00	14	0.786
94	A	15	12	1.00	14	0.857
95	A	11	8	1.00	14	0.571
96	A	10	8	1.00	14	0.571
97	A	9	8	1.00	14	0.571
98	A	7	7	1.00	12	0.583
99	A	6	6	1.00	10	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	17	14	1.00	14	1.000
101	A	13	10	1.00	14	0.714
102	A	14	11	1.00	14	0.786
103	A	15	12	1.00	14	0.857
104	A	12	9	1.00	14	0.643
105	A	11	9	1.00	14	0.643
106	A	10	9	1.00	14	0.643
107	A	8	7	1.00	12	0.583
108	A	7	6	1.00	10	0.600
109	A	19	16	1.00	14	1.143
110	A	14	11	1.00	14	0.786
111	A	15	11	1.00	14	0.786
112	A	16	12	1.00	14	0.857
113	A	16	11	1.00	12	0.917
114	A	14	10	1.00	10	1.000
115	A	13	9	1.00	8	1.125
116	A	25	13	1.00	12	1.083
117	A	14	10	1.00	12	0.833
118	A	15	11	1.00	12	0.917
119	A	16	12	1.00	12	1.000
120	A	6	5	1.00	12	0.417
121	A	4	4	1.00	10	0.400
122	A	3	3	1.00	8	0.375
123	A	4	4	1.00	12	0.333
124	A	3	3	1.00	12	0.250
125	A	4	4	1.00	12	0.333
126	A	19	15	1.00	14	1.071
127	A	17	14	1.00	12	1.167
128	A	16	13	1.00	10	1.300
129	A	39	20	1.00	14	1.429
130	A	25	11	1.00	14	0.786
131	A	26	12	1.00	14	0.857
132	A	5	5	1.00	12	0.417
133	A	9	5	1.00	12	0.417
134	A	4	4	1.00	12	0.333
135	A	4	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
136	A	4	3	1.00	12	0.250
137	A	4	4	1.00	12	0.333
138	A	9	5	1.00	12	0.417
139	A	2	2	1.00	14	0.143
140	A	2	2	1.00	14	0.143
141	A	2	2	1.00	14	0.143
142	A	2	2	1.00	14	0.143
143	A	2	2	1.00	14	0.143
144	A	2	2	1.00	14	0.143
145	A	2	2	1.00	12	0.167
146	A	2	2	1.00	12	0.167
147	A	2	2	1.00	14	0.143
148	A	2	2	1.00	12	0.167
149	A	5	5	1.00	12	0.417
150	A	3	3	1.00	10	0.300
151	A	2	2	1.00	8	0.250
152	A	4	4	1.00	12	0.333
153	A	2	2	1.00	12	0.167
154	A	3	3	1.00	12	0.250
155	A	4	4	1.00	12	0.333
156	A	4	4	1.00	12	0.333
157	A	4	4	1.00	16	0.250
158	A	9	8	1.00	16	0.500
159	A	8	8	1.00	16	0.500
160	A	7	7	1.00	16	0.438
161	A	6	6	1.00	14	0.429
162	A	7	7	1.00	16	0.438
163	A	3	3	1.00	16	0.188
164	A	4	4	1.00	16	0.250
165	A	6	6	1.00	16	0.375
166	A	8	6	1.00	16	0.375
167	A	5	4	1.00	18	0.222
168	A	4	3	1.00	18	0.167
169	A	4	3	1.00	18	0.167
170	A	4	3	1.00	18	0.167
171	A	4	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	C	1	1	1.86	16	0.062
173	A	4	3	1.00	18	0.167
174	A	3	3	1.00	18	0.167
175	A	4	3	1.00	18	0.167
176	A	4	3	1.00	18	0.167
177	A	5	4	1.00	18	0.222
178	A	8	7	1.00	18	0.389
179	A	7	6	1.00	18	0.333
180	A	8	8	1.00	18	0.444
181	A	8	8	1.00	16	0.500
182	A	9	8	1.00	18	0.444
183	A	3	3	1.00	18	0.167
184	A	4	4	1.00	18	0.222
185	A	6	6	1.00	18	0.333
186	A	7	6	1.00	18	0.333
187	A	5	4	1.00	18	0.222
188	A	4	3	1.00	18	0.167
189	A	5	4	1.00	18	0.222
190	A	4	3	1.00	18	0.167
191	A	3	3	1.00	18	0.167
192	A	4	3	1.00	16	0.188
193	A	4	3	1.00	18	0.167
194	A	3	3	1.00	18	0.167
195	A	4	3	1.00	18	0.167
196	A	4	3	1.00	18	0.167
197	A	3	3	1.00	18	0.167
198	A	9	7	1.00	18	0.389
199	A	8	7	1.00	18	0.389
200	A	7	7	1.00	16	0.438
201	A	5	5	1.00	18	0.278
202	A	3	3	1.00	18	0.167
203	A	4	4	1.00	18	0.222
204	A	6	6	1.00	18	0.333
205	A	7	6	1.00	18	0.333
206	A	4	4	1.00	18	0.222
207	A	4	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	4	3	1.00	18	0.167
209	A	4	3	1.00	18	0.167
210	A	4	3	1.00	16	0.188
211	A	3	3	1.00	18	0.167
212	A	4	4	1.00	18	0.222
213	A	5	4	1.00	18	0.222
214	A	5	4	1.00	18	0.222
215	A	5	4	1.00	18	0.222
216	A	3	3	1.00	18	0.167
217	A	10	8	1.00	18	0.444
218	A	9	8	1.00	18	0.444
219	A	8	8	1.00	16	0.500
220	A	6	6	1.00	18	0.333
221	A	3	3	1.00	18	0.167
222	A	3	3	1.00	18	0.167
223	A	5	5	1.00	18	0.278
224	A	6	5	1.00	18	0.278
225	A	7	5	1.00	18	0.278
226	A	7	6	1.00	18	0.333
227	A	6	6	1.00	18	0.333
228	A	5	5	1.19	18	0.278
229	A	4	4	1.16	18	0.222
230	A	1	1	1.00	18	0.056
231	A	5	5	1.00	18	0.278
232	A	5	5	1.00	18	0.278
233	A	6	5	1.00	18	0.278
234	A	7	5	1.00	18	0.278
235	A	5	4	1.00	20	0.200
236	A	5	4	1.00	20	0.200
237	A	5	4	1.00	20	0.200
238	A	5	4	1.00	20	0.200
239	A	5	4	1.00	20	0.200
240	A	5	4	1.00	20	0.200
241	A	5	4	1.00	20	0.200
242	A	5	4	1.00	20	0.200
243	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
244	A	5	5	1.00	20	0.250
245	A	4	4	1.00	20	0.200
246	A	1	1	1.00	20	0.050
247	A	6	5	1.00	20	0.250
248	A	6	5	1.00	20	0.250
249	A	6	5	1.00	20	0.250
250	A	7	5	1.00	20	0.250
251	A	8	5	1.00	20	0.250
252	A	8	6	1.60	20	0.300
253	A	7	6	1.58	20	0.300
254	A	6	6	1.54	20	0.300
255	A	5	5	1.44	20	0.250
256	A	4	4	1.44	20	0.200
257	A	1	1	1.00	20	0.050
258	A	4	4	1.00	20	0.200
259	A	5	5	1.00	20	0.250
260	A	6	5	1.00	20	0.250
261	A	10	6	1.00	20	0.300
262	A	9	6	1.00	20	0.300
263	A	8	6	1.00	20	0.300
264	A	7	6	1.00	20	0.300
265	A	6	6	1.00	20	0.300
266	A	5	5	1.00	20	0.250
267	A	6	6	1.00	20	0.300
268	A	7	6	1.00	20	0.300
269	A	8	6	1.00	20	0.300
270	A	9	6	1.00	20	0.300
271	A	8	6	1.00	20	0.300
272	A	7	6	1.00	20	0.300
273	A	6	6	1.00	20	0.300
274	A	5	5	1.00	20	0.250
275	A	4	4	1.00	20	0.200
276	A	1	1	1.00	20	0.050
277	A	5	5	1.00	20	0.250
278	A	6	5	1.00	20	0.250
279	A	7	6	1.00	9	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	6	5	1.00	8	0.625
281	A	3	3	1.00	11	0.273
282	A	6	6	1.00	10	0.600
283	A	8	6	1.00	11	0.546
284	A	7	5	1.00	10	0.500
285	A	8	8	1.00	13	0.615
286	A	7	7	1.00	12	0.583
287	A	3	3	1.00	11	0.273
288	A	4	4	1.00	10	0.400
289	A	8	8	1.00	13	0.615
290	A	7	7	1.00	12	0.583
291	A	5	5	1.00	11	0.454
292	A	3	3	1.00	10	0.300
293	A	9	9	1.00	13	0.692
294	A	3	3	1.00	12	0.250
295	A	3	3	1.00	21	0.143
296	A	5	4	1.00	21	0.190
297	A	4	4	1.00	19	0.210
298	A	1	1	1.00	18	0.056
299	A	5	5	1.00	21	0.238
300	A	5	5	1.00	21	0.238
301	A	5	4	1.00	23	0.174
302	A	5	4	1.00	23	0.174
303	A	5	4	1.00	21	0.190
304	A	5	4	1.00	20	0.200
305	A	6	6	1.00	23	0.261
306	A	6	6	1.00	23	0.261
307	A	7	7	1.00	23	0.304
308	A	8	7	1.00	23	0.304
309	A	9	7	1.00	23	0.304
310	A	10	7	1.00	23	0.304
311	A	9	7	1.00	23	0.304
312	A	7	6	1.00	21	0.286
313	A	6	5	1.00	20	0.250
314	A	8	8	1.00	23	0.348
315	A	8	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	9	9	1.00	23	0.391
317	A	10	9	1.00	23	0.391
318	A	11	9	1.00	23	0.391
319	A	6	6	1.00	13	0.462
320	A	5	5	1.00	12	0.417
321	A	5	5	1.00	15	0.333
322	A	4	4	1.00	14	0.286
323	A	5	5	1.00	13	0.385
324	A	4	4	1.00	12	0.333
325	A	4	4	1.00	15	0.267
326	A	1	1	1.00	14	0.071
327	A	4	4	1.00	13	0.308
328	A	3	3	1.00	12	0.250
329	A	6	6	1.00	15	0.400
330	A	5	5	1.00	14	0.357
331	A	5	5	1.00	13	0.385
332	A	4	4	1.00	12	0.333
333	A	6	6	1.00	15	0.400
334	A	5	5	1.00	14	0.357
335	A	4	4	1.00	23	0.174
336	A	6	5	1.30	23	0.217
337	A	5	5	1.32	21	0.238
338	A	4	4	1.44	20	0.200
339	A	5	5	1.00	23	0.217
340	A	5	5	1.00	23	0.217
341	A	9	7	1.00	23	0.304
342	A	9	7	1.00	23	0.304
343	A	8	7	1.00	21	0.333
344	A	7	6	1.00	20	0.300
345	A	9	8	1.00	23	0.348
346	A	9	8	1.00	23	0.348
347	A	10	9	1.00	23	0.391
348	A	11	9	1.00	23	0.391
349	A	12	9	1.00	23	0.391
350	A	8	6	1.00	23	0.261
351	A	7	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	6	6	1.00	21	0.286
353	A	5	5	1.00	20	0.250
354	A	6	6	1.00	23	0.261
355	A	6	6	1.00	23	0.261
356	A	7	7	1.00	23	0.304
357	A	8	7	1.00	23	0.304
358	A	9	7	1.00	23	0.304
359	A	6	6	1.00	24	0.250
360	A	4	4	1.00	24	0.167
361	A	1	1	1.00	22	0.045
362	A	3	3	1.00	24	0.125
363	A	3	3	1.00	24	0.125
364	A	3	3	1.00	18	0.167
365	A	3	3	1.00	18	0.167
366	A	3	3	1.00	18	0.167
367	A	3	3	1.00	16	0.188
368	A	3	3	1.00	18	0.167
369	A	3	3	1.00	18	0.167
370	A	4	4	1.00	18	0.222
371	A	6	6	1.00	18	0.333
372	A	3	3	1.00	20	0.150
373	A	3	3	1.00	20	0.150
374	A	3	3	1.00	20	0.150
375	A	3	3	1.00	20	0.150
376	A	3	3	1.00	20	0.150
377	A	4	4	1.00	20	0.200
378	A	5	4	1.00	20	0.200
379	A	9	9	1.00	20	0.450
380	A	8	8	1.00	20	0.400
381	A	7	7	1.00	20	0.350
382	A	3	3	1.00	18	0.167
383	A	7	7	1.00	20	0.350
384	A	8	7	1.00	20	0.350
385	A	9	7	1.00	20	0.350
386	A	10	7	1.00	20	0.350
387	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
388	A	5	4	1.00	22	0.182
389	A	5	4	1.00	22	0.182
390	A	6	5	1.00	22	0.227
391	A	5	4	1.00	20	0.200
392	A	5	4	1.00	22	0.182
393	A	5	4	1.00	22	0.182
394	A	5	4	1.00	22	0.182
395	A	5	4	1.00	22	0.182
396	A	8	8	1.00	22	0.364
397	A	4	4	1.00	22	0.182
398	A	8	8	1.00	22	0.364
399	A	8	8	1.00	20	0.400
400	A	8	7	1.00	22	0.318
401	A	9	7	1.00	22	0.318
402	A	10	7	1.00	22	0.318
403	A	11	7	1.00	22	0.318
404	A	5	4	1.00	22	0.182
405	A	6	5	1.00	22	0.227
406	A	5	4	1.00	22	0.182
407	A	5	4	1.00	22	0.182
408	A	5	4	1.00	20	0.200
409	A	5	4	1.00	22	0.182
410	A	5	4	1.00	22	0.182
411	A	5	4	1.00	22	0.182
412	A	5	4	1.00	22	0.182
413	A	10	8	1.00	22	0.364
414	A	9	8	1.00	22	0.364
415	A	8	8	1.00	22	0.364
416	A	7	7	1.00	20	0.350
417	A	2	2	1.00	22	0.091
418	A	6	6	1.00	22	0.273
419	A	8	7	1.00	22	0.318
420	A	9	7	1.00	22	0.318
421	A	5	4	1.00	22	0.182
422	A	5	4	1.00	22	0.182
423	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
424	A	5	4	1.00	20	0.200
425	A	5	4	1.00	22	0.182
426	A	6	5	1.00	22	0.227
427	A	5	4	1.00	22	0.182
428	A	5	4	1.00	22	0.182
429	A	11	9	1.00	22	0.409
430	A	10	9	1.00	22	0.409
431	A	9	9	1.00	22	0.409
432	A	8	8	1.00	20	0.400
433	A	6	6	1.00	22	0.273
434	A	6	6	1.00	22	0.273
435	A	3	3	1.00	22	0.136
436	A	7	7	1.00	22	0.318
437	A	9	7	1.00	22	0.318
438	A	8	7	1.19	22	0.318
439	A	7	7	1.13	22	0.318
440	A	7	7	1.20	22	0.318
441	A	5	5	1.00	22	0.227
442	A	4	4	1.00	22	0.182
443	A	8	7	1.00	22	0.318
444	A	9	8	1.00	22	0.364
445	A	10	8	1.00	22	0.364
446	A	12	9	1.00	24	0.375
447	A	11	9	1.00	24	0.375
448	A	10	9	1.00	24	0.375
449	A	9	9	1.00	24	0.375
450	A	8	8	1.00	24	0.333
451	A	9	9	1.00	24	0.375
452	A	10	9	1.00	24	0.375
453	A	11	9	1.00	24	0.375
454	A	12	9	1.00	24	0.375
455	A	8	8	1.00	24	0.333
456	A	8	7	1.00	24	0.292
457	A	6	5	1.00	24	0.208
458	A	5	5	1.00	24	0.208
459	A	8	7	1.00	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
460	A	9	8	1.00	24	0.333
461	A	10	8	1.00	24	0.333
462	A	11	8	1.00	24	0.333
463	A	7	7	1.00	24	0.292
464	A	6	6	1.00	24	0.250
465	A	6	6	1.00	24	0.250
466	A	4	4	1.00	24	0.167
467	A	4	4	1.00	24	0.167
468	A	9	8	1.00	24	0.333
469	A	10	9	1.00	24	0.375
470	A	11	10	1.00	24	0.417
471	A	14	10	1.00	24	0.417
472	A	13	10	1.00	24	0.417
473	A	12	10	1.00	24	0.417
474	A	11	9	1.00	24	0.375
475	A	11	9	1.00	24	0.375
476	A	12	10	1.00	24	0.417
477	A	12	10	1.00	24	0.417
478	A	13	10	1.00	24	0.417
479	A	14	10	1.00	24	0.417
480	A	9	8	1.00	24	0.333
481	A	8	8	1.00	24	0.333
482	A	7	7	1.00	24	0.292
483	A	6	6	1.00	24	0.250
484	A	6	6	1.00	24	0.250
485	A	5	5	1.00	24	0.208
486	A	5	5	1.00	24	0.208
487	A	9	8	1.00	24	0.333
488	A	10	9	1.00	24	0.375
489	A	3	3	1.00	25	0.120
490	A	6	5	1.00	25	0.200
491	A	5	5	1.00	23	0.217
492	A	4	4	1.00	22	0.182
493	A	4	4	1.00	25	0.160
494	A	2	2	1.00	25	0.080
495	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
496	A	4	4	1.00	25	0.160
497	A	5	4	1.00	25	0.160
498	A	11	9	1.00	27	0.333
499	A	10	9	1.00	27	0.333
500	A	9	9	1.00	25	0.360
501	A	8	8	1.00	24	0.333
502	A	8	8	1.00	27	0.296
503	A	7	6	1.00	27	0.222
504	A	7	6	1.00	27	0.222
505	A	7	6	1.00	27	0.222
506	A	7	6	1.00	27	0.222
507	A	11	8	1.00	27	0.296
508	A	10	8	1.00	27	0.296
509	A	9	8	1.00	25	0.320
510	A	8	7	1.00	24	0.292
511	A	8	7	1.00	27	0.259
512	A	5	4	1.00	27	0.148
513	A	6	5	1.00	27	0.185
514	A	7	6	1.00	27	0.222
515	A	8	6	1.00	27	0.222
516	A	4	4	1.00	27	0.148
517	A	6	5	1.00	27	0.185
518	A	5	5	1.00	25	0.200
519	A	4	4	1.00	24	0.167
520	A	4	4	1.00	27	0.148
521	A	3	3	1.00	27	0.111
522	A	4	4	1.00	27	0.148
523	A	5	5	1.00	27	0.185
524	A	14	10	1.00	27	0.370
525	A	13	10	1.00	27	0.370
526	A	12	10	1.00	25	0.400
527	A	11	9	1.00	24	0.375
528	A	11	9	1.00	27	0.333
529	A	9	8	1.00	27	0.296
530	A	10	9	1.00	27	0.333
531	A	11	9	1.00	27	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
532	A	11	9	1.00	27	0.333
533	A	9	7	1.01	27	0.259
534	A	8	7	1.00	27	0.259
535	A	7	7	1.00	25	0.280
536	A	6	6	1.00	24	0.250
537	A	6	5	1.00	27	0.185
538	A	4	3	1.00	27	0.111
539	A	5	4	1.00	27	0.148
540	A	6	5	1.00	27	0.185
541	A	4	3	1.00	27	0.111
542	C	2	2	0.44	20	0.100
543	A	3	3	1.00	22	0.136
544	A	5	5	1.00	22	0.227
545	A	3	3	1.00	24	0.125
546	A	3	3	1.00	24	0.125
547	A	3	3	1.00	24	0.125
548	A	3	3	1.00	24	0.125
549	A	3	3	1.00	22	0.136
550	A	3	3	1.00	23	0.130
551	A	3	3	1.00	23	0.130
552	A	7	7	1.00	22	0.318
553	A	3	3	1.00	20	0.150
554	A	3	3	1.00	22	0.136
555	A	9	9	1.00	22	0.409
556	A	13	5	1.00	20	0.250
557	A	11	5	1.00	20	0.250
558	A	9	5	1.00	20	0.250
559	A	7	5	1.00	18	0.278
560	A	1	1	1.00	20	0.050
561	A	2	2	1.00	20	0.100
562	A	3	2	1.00	20	0.100
563	A	4	2	1.00	20	0.100
564	A	4	3	1.00	22	0.136
565	A	4	3	1.00	22	0.136
566	A	4	3	1.00	22	0.136
567	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	3	3	1.00	20	0.150
569	A	3	3	1.00	22	0.136
570	A	5	4	1.00	22	0.182
571	A	5	4	1.00	22	0.182
572	A	5	4	1.00	22	0.182
573	A	13	5	1.00	22	0.227
574	A	11	5	1.00	22	0.227
575	A	9	5	1.00	22	0.227
576	A	7	5	1.00	20	0.250
577	A	1	1	1.00	22	0.045
578	A	2	2	1.00	22	0.091
579	A	3	2	1.00	22	0.091
580	A	4	2	1.00	22	0.091
581	A	4	3	1.00	22	0.136
582	A	4	3	1.00	22	0.136
583	A	4	3	1.00	22	0.136
584	A	3	3	1.00	22	0.136
585	A	4	3	1.00	20	0.150
586	A	3	3	1.00	22	0.136
587	A	3	3	1.00	22	0.136
588	A	5	4	1.00	22	0.182
589	A	5	4	1.00	22	0.182
590	A	13	5	1.00	22	0.227
591	A	11	5	1.00	22	0.227
592	A	9	5	1.00	22	0.227
593	A	7	5	1.00	20	0.250
594	A	1	1	1.00	22	0.045
595	A	2	2	1.00	22	0.091
596	A	3	2	1.00	22	0.091
597	A	4	2	1.00	22	0.091
598	A	4	3	1.00	22	0.136
599	A	4	3	1.00	22	0.136
600	A	4	3	1.00	22	0.136
601	A	3	3	1.00	20	0.150
602	A	3	3	1.00	22	0.136
603	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
604	A	5	4	1.00	22	0.182
605	A	5	4	1.00	22	0.182
606	A	13	5	1.00	22	0.227
607	A	11	5	1.00	22	0.227
608	A	9	5	1.00	22	0.227
609	A	7	5	1.00	20	0.250
610	A	1	1	1.00	22	0.045
611	A	2	2	1.00	22	0.091
612	A	3	2	1.00	22	0.091
613	A	4	2	1.00	22	0.091
614	A	4	3	1.00	22	0.136
615	A	4	3	1.00	22	0.136
616	A	4	3	1.00	22	0.136
617	A	4	3	1.00	22	0.136
618	A	3	2	1.00	22	0.091
619	A	3	3	1.00	22	0.136
620	A	5	4	1.00	22	0.182
621	A	5	4	1.00	22	0.182
622	A	5	4	1.00	22	0.182
623	A	10	7	1.00	24	0.292
624	A	9	7	1.00	24	0.292
625	A	8	7	1.00	24	0.292
626	A	7	7	1.00	24	0.292
627	A	6	6	1.00	24	0.250
628	A	5	5	1.00	24	0.208
629	A	4	4	1.00	24	0.167
630	A	5	5	1.00	24	0.208
631	A	6	5	1.00	24	0.208
632	A	7	5	1.00	24	0.208
633	A	4	3	1.00	24	0.125
634	A	4	3	1.00	24	0.125
635	A	4	3	1.00	24	0.125
636	A	3	3	1.00	24	0.125
637	A	4	3	1.00	24	0.125
638	A	4	3	1.00	24	0.125
639	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
640	A	5	4	1.00	24	0.167
641	A	5	4	1.00	24	0.167
642	A	4	3	1.00	24	0.125
643	A	4	3	1.00	24	0.125
644	A	4	3	1.00	24	0.125
645	A	4	3	1.00	24	0.125
646	A	3	2	1.00	24	0.083
647	A	3	3	1.00	24	0.125
648	A	5	4	1.00	24	0.167
649	A	5	4	1.00	24	0.167
650	A	5	4	1.00	24	0.167
651	A	8	7	1.00	24	0.292
652	A	7	7	1.00	24	0.292
653	A	6	6	1.00	24	0.250
654	A	5	5	1.00	24	0.208
655	A	4	4	1.00	24	0.167
656	A	5	5	1.00	24	0.208
657	A	6	5	1.00	24	0.208
658	A	7	5	1.00	24	0.208
659	A	4	3	1.00	24	0.125
660	A	4	3	1.00	24	0.125
661	A	4	3	1.00	24	0.125
662	A	3	3	1.00	24	0.125
663	A	4	3	1.00	24	0.125
664	A	4	3	1.00	24	0.125
665	A	3	3	1.00	24	0.125
666	A	5	4	1.00	24	0.167
667	A	5	4	1.00	24	0.167
668	A	4	3	1.00	25	0.120
669	A	4	3	1.00	23	0.130
670	A	3	2	1.00	22	0.091
671	A	4	3	1.00	25	0.120
672	A	4	3	1.00	25	0.120
673	A	8	7	1.00	27	0.259
674	A	7	7	1.00	27	0.259
675	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
676	A	6	6	1.00	24	0.250
677	A	9	9	1.00	27	0.333
678	A	9	9	1.00	27	0.333
679	A	7	7	1.00	27	0.259
680	A	8	8	1.00	27	0.296
681	A	9	8	1.00	27	0.296
682	A	4	3	1.00	27	0.111
683	A	4	3	1.00	27	0.111
684	A	4	3	1.00	25	0.120
685	A	4	3	1.00	24	0.125
686	A	4	3	1.00	27	0.111
687	A	4	3	1.00	27	0.111
688	A	4	3	1.00	27	0.111
689	A	4	3	1.00	27	0.111
690	A	4	3	1.00	27	0.111
691	A	4	3	1.00	25	0.120
692	A	4	3	1.00	25	0.120
693	A	4	3	1.00	25	0.120
694	A	5	4	1.00	23	0.174
695	A	5	4	1.00	22	0.182
696	A	4	3	1.00	25	0.120
697	A	4	3	1.00	25	0.120
698	A	4	3	1.00	25	0.120
699	A	4	3	1.00	25	0.120
700	A	4	3	1.00	25	0.120
701	A	5	4	1.00	25	0.160
702	A	5	4	1.00	25	0.160
703	A	5	4	1.00	23	0.174
704	A	5	4	1.00	22	0.182
705	A	4	3	1.00	25	0.120
706	A	4	3	1.00	25	0.120
707	A	4	3	1.00	27	0.111
708	A	4	3	1.00	25	0.120
709	A	3	2	1.00	24	0.083
710	A	4	3	1.00	27	0.111
711	A	4	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
712	A	8	7	1.00	27	0.259
713	A	7	7	1.00	27	0.259
714	A	6	6	1.00	25	0.240
715	A	6	6	1.00	24	0.250
716	A	9	9	1.00	27	0.333
717	A	9	9	1.00	27	0.333
718	A	7	7	1.00	27	0.259
719	A	8	8	1.00	27	0.296
720	A	9	8	1.00	27	0.296
721	A	4	3	1.00	27	0.111
722	A	4	3	1.00	27	0.111
723	A	4	3	1.00	25	0.120
724	A	4	3	1.00	24	0.125
725	A	4	3	1.00	27	0.111
726	A	4	3	1.00	27	0.111
727	A	4	3	1.00	27	0.111
728	A	4	3	1.00	27	0.111
729	A	4	3	1.00	27	0.111
730	A	5	4	1.00	27	0.148
731	A	8	6	1.00	27	0.222
732	A	4	3	1.00	25	0.120
733	A	4	3	1.00	27	0.111
734	A	8	6	1.00	27	0.222
735	A	5	4	1.00	27	0.148
736	A	3	3	1.00	22	0.136
737	A	3	3	1.00	22	0.136
738	A	3	3	1.00	20	0.150
739	A	2	2	1.00	8	0.250
740	A	1	1	1.00	22	0.045
741	A	2	2	1.00	22	0.091
742	A	3	2	1.00	22	0.091
743	A	4	2	1.00	22	0.091
744	A	3	3	1.00	24	0.125
745	A	3	3	1.00	24	0.125
746	A	3	3	1.00	24	0.125
747	A	1	1	1.00	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
748	A	2	2	1.00	24	0.083
749	A	3	2	1.00	24	0.083
750	A	4	2	1.00	24	0.083
751	A	7	6	1.01	27	0.222
752	A	4	4	1.00	27	0.148
753	A	1	1	1.00	25	0.040
754	A	1	1	1.00	24	0.042
755	A	5	5	1.00	27	0.185
756	A	8	6	1.01	27	0.222
757	A	6	4	1.00	27	0.148
758	A	2	2	1.00	27	0.074
759	A	2	2	1.00	25	0.080
760	A	2	2	1.00	24	0.083
761	A	15	6	1.00	27	0.222
762	A	3	3	1.00	22	0.136
763	A	3	3	1.00	23	0.130
764	A	3	3	1.00	23	0.130
765	A	4	4	1.00	22	0.182
766	A	3	3	1.00	22	0.136
767	A	4	4	1.00	22	0.182
768	A	3	3	1.00	20	0.150
769	A	3	3	1.00	22	0.136
770	A	4	4	1.00	22	0.182
771	A	3	3	1.00	22	0.136
772	A	14	8	1.00	20	0.400
773	A	12	8	1.00	20	0.400
774	A	10	8	1.00	20	0.400
775	A	9	9	1.00	18	0.500
776	A	6	6	1.00	20	0.300
777	A	8	6	1.00	20	0.300
778	A	10	6	1.00	20	0.300
779	A	12	6	1.00	20	0.300
780	A	5	4	1.00	22	0.182
781	A	5	4	1.00	22	0.182
782	A	5	4	1.00	22	0.182
783	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
784	A	5	4	1.00	20	0.200
785	A	5	4	1.00	22	0.182
786	A	5	4	1.00	22	0.182
787	A	5	4	1.00	22	0.182
788	A	5	4	1.00	22	0.182
789	A	14	8	1.00	22	0.364
790	A	12	8	1.00	22	0.364
791	A	10	8	1.00	22	0.364
792	A	8	8	1.00	20	0.400
793	A	7	6	1.00	22	0.273
794	A	9	8	1.00	22	0.364
795	A	10	6	1.00	22	0.273
796	A	12	6	1.00	22	0.273
797	A	5	4	1.00	22	0.182
798	A	5	4	1.00	22	0.182
799	A	5	4	1.00	22	0.182
800	A	5	4	1.00	22	0.182
801	A	5	4	1.00	20	0.200
802	A	5	4	1.00	22	0.182
803	A	5	4	1.00	22	0.182
804	A	5	4	1.00	22	0.182
805	A	5	4	1.00	22	0.182
806	A	14	8	1.00	22	0.364
807	A	12	8	1.00	22	0.364
808	A	10	8	1.00	22	0.364
809	A	9	9	1.00	20	0.450
810	A	6	6	1.00	22	0.273
811	A	8	6	1.00	22	0.273
812	A	10	6	1.00	22	0.273
813	A	12	6	1.00	22	0.273
814	A	5	4	1.00	22	0.182
815	A	5	4	1.00	22	0.182
816	A	5	4	1.00	22	0.182
817	A	5	4	1.00	20	0.200
818	A	5	4	1.00	22	0.182
819	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
820	A	5	4	1.00	22	0.182
821	A	5	4	1.00	22	0.182
822	A	14	8	1.00	22	0.364
823	A	12	8	1.00	22	0.364
824	A	10	8	1.00	22	0.364
825	A	8	8	1.00	20	0.400
826	A	7	6	1.00	22	0.273
827	A	9	8	1.00	22	0.364
828	A	10	6	1.00	22	0.273
829	A	12	6	1.00	22	0.273
830	A	4	3	1.00	22	0.136
831	A	4	3	1.00	22	0.136
832	A	4	3	1.00	22	0.136
833	A	4	3	1.00	22	0.136
834	A	4	3	1.00	22	0.136
835	A	4	3	1.00	22	0.136
836	A	4	3	1.00	22	0.136
837	A	4	3	1.00	22	0.136
838	A	15	11	1.00	24	0.458
839	A	13	11	1.00	24	0.458
840	A	11	11	1.00	24	0.458
841	A	9	9	1.00	24	0.375
842	A	7	7	1.00	24	0.292
843	A	7	7	1.00	24	0.292
844	A	9	8	1.00	24	0.333
845	A	11	8	1.00	24	0.333
846	A	4	3	1.00	24	0.125
847	A	4	3	1.00	24	0.125
848	A	4	3	1.00	24	0.125
849	A	4	3	1.00	24	0.125
850	A	4	3	1.00	24	0.125
851	A	4	3	1.00	24	0.125
852	A	4	3	1.00	24	0.125
853	A	4	3	1.00	24	0.125
854	A	4	3	1.00	24	0.125
855	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
856	A	4	3	1.00	24	0.125
857	A	4	3	1.00	24	0.125
858	A	4	3	1.00	24	0.125
859	A	4	3	1.00	24	0.125
860	A	4	3	1.00	24	0.125
861	A	4	3	1.00	24	0.125
862	A	4	3	1.00	24	0.125
863	A	15	11	1.00	24	0.458
864	A	13	11	1.00	24	0.458
865	A	11	11	1.00	24	0.458
866	A	9	9	1.00	24	0.375
867	A	7	7	1.00	24	0.292
868	A	7	7	1.00	24	0.292
869	A	9	8	1.00	24	0.333
870	A	11	8	1.00	24	0.333
871	A	4	3	1.00	24	0.125
872	A	4	3	1.00	24	0.125
873	A	4	3	1.00	24	0.125
874	A	4	3	1.00	24	0.125
875	A	4	3	1.00	24	0.125
876	A	4	3	1.00	24	0.125
877	A	4	3	1.00	24	0.125
878	A	4	3	1.00	24	0.125
879	A	4	3	1.00	24	0.125
880	A	4	3	1.00	25	0.120
881	A	4	3	1.00	25	0.120
882	A	3	2	1.00	23	0.087
883	A	4	3	1.00	22	0.136
884	A	4	3	1.00	25	0.120
885	A	3	3	1.00	25	0.120
886	A	9	8	1.00	27	0.296
887	A	8	7	1.00	27	0.259
888	A	7	6	1.00	25	0.240
889	A	9	9	1.00	24	0.375
890	A	9	9	1.00	27	0.333
891	A	7	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
892	A	8	7	1.00	27	0.259
893	A	10	8	1.00	27	0.296
894	A	11	8	1.00	27	0.296
895	A	4	3	1.00	27	0.111
896	A	4	3	1.00	27	0.111
897	A	4	3	1.00	25	0.120
898	A	4	3	1.00	24	0.125
899	A	4	3	1.00	27	0.111
900	A	4	3	1.00	27	0.111
901	A	4	3	1.00	27	0.111
902	A	4	3	1.00	27	0.111
903	A	4	3	1.00	27	0.111
904	A	4	3	1.00	27	0.111
905	A	4	3	1.00	27	0.111
906	A	3	2	1.00	25	0.080
907	A	4	3	1.00	24	0.125
908	A	4	3	1.00	27	0.111
909	A	3	3	1.00	27	0.111
910	A	9	8	1.00	27	0.296
911	A	8	7	1.00	27	0.259
912	A	7	6	1.00	25	0.240
913	A	9	9	1.00	24	0.375
914	A	9	9	1.00	27	0.333
915	A	7	6	1.00	27	0.222
916	A	8	7	1.00	27	0.259
917	A	10	8	1.00	27	0.296
918	A	11	8	1.00	27	0.296
919	A	4	3	1.00	27	0.111
920	A	4	3	1.00	27	0.111
921	A	4	3	1.00	25	0.120
922	A	4	3	1.00	24	0.125
923	A	4	3	1.00	27	0.111
924	A	4	3	1.00	27	0.111
925	A	4	3	1.00	27	0.111
926	A	4	3	1.00	27	0.111
927	A	4	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
928	C	2	2	0.53	20	0.100
929	A	5	5	1.09	22	0.227
930	A	7	5	1.05	22	0.227
931	C	3	3	0.38	24	0.125
932	A	4	4	1.00	24	0.167
933	A	3	3	1.00	22	0.136
934	A	3	3	1.00	23	0.130
935	A	3	3	1.00	23	0.130

Chapter 3

Listing of integrals

3.1 $\int e^{\coth^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=114

$$\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}$$

```
[Out] 3/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4+2/3*x*(1-1/a^2/x^2)^(1/2)/a^3+3/8*x^2*
(1-1/a^2/x^2)^(1/2)/a^2+1/3*x^3*(1-1/a^2/x^2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)
^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^3,x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2)
 + (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{1}{4}\text{Subst}\left(\int \frac{-\frac{4}{a}-\frac{3x}{a^2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4-\frac{1}{12}\text{Subst}\left(\int \frac{\frac{9}{a^2}+\frac{8x}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{1}{24}\text{Subst}\left(\int \frac{-\frac{16}{a^3}-\frac{9x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4-\frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\right.}{8a^4}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4-\frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\right.}{16a^4}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{3\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\right.}{8a^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.60

$$\frac{9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3+8a^2x^2+9ax+16\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^3,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

fricas [A] time = 0.53, size = 92, normalized size = 0.81

$$\frac{\left(6\,a^4x^4+14\,a^3x^3+17\,a^2x^2+25\,ax+16\right)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^4*x^4 + 14*a^3*x^3 + 17*a^2*x^2 + 25*a*x + 16)*sqrt((a*x - 1)/(a
*x + 1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^4
```

giac [A] time = 0.15, size = 182, normalized size = 1.60

$$\frac{1}{24}a\left(\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}-\frac{9\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^5}-\frac{2\left(\frac{31\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{49\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{9\,(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}-39\,\sqrt{\frac{ax}{ax}}\right.}{a^5\left(\frac{ax-1}{ax+1}-1\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/24*a*(9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^5 - 9*log(abs(sqrt((a*x - 1)
/(a*x + 1)) - 1))/a^5 - 2*(31*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)
 - 49*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 9*(a*x - 1)^3*sqr
t((a*x - 1)/(a*x + 1))/(a*x + 1)^3 - 39*sqrt((a*x - 1)/(a*x + 1)))/(a^5*((a
*x - 1)/(a*x + 1) - 1)^4))
```

maple [B] time = 0.07, size = 193, normalized size = 1.69

$$\frac{(ax-1)\left(6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;xa+8\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\;+15\sqrt{a^2x^2-1}\;\sqrt{a^2}\;xa+24\sqrt{(ax-1)(ax+1)}\;\sqrt{a}\right.}{24\sqrt{\frac{ax-1}{ax+1}}\;\sqrt{(ax-1)(ax+1)}\;a^4\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x)
```

```
[Out] 1/24*(a*x-1)*(6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+8*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)+15*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+24*((a*x-1)*(a*x+1))^(1/2
)*(a^2)^(1/2)-15*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+24
*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a
*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^4/(a^2)^(1/2)
```

maxima [B] time = 0.32, size = 203, normalized size = 1.78

$$\frac{1}{24}\,a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-49\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-39\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(2*(9*((a*x - 1)/(a*x + 1))^(7/2) - 49*((a*x - 1)/(a*x + 1))^(5/2) +
 31*((a*x - 1)/(a*x + 1))^(3/2) - 39*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1
)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x +
1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 9*log(sqrt((a*x - 1)/(a*x + 1))
 + 1)/a^5 - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

mupad [B] time = 1.26, size = 171, normalized size = 1.50

$$\frac{\frac{13\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12}+\frac{49\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{12}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((13*((a*x - 1)/(a*x + 1))^(1/2))/4 - (31*((a*x - 1)/(a*x + 1))^(3/2))/12 +
 (49*((a*x - 1)/(a*x + 1))^(5/2))/12 - (3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(
a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (
a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (3*atanh(((a*
x - 1)/(a*x + 1))^(1/2)))/(4*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3,x)
```

```
[Out] Integral(x**3/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.2 $\int e^{\coth^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=90

$$\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+2/3*x*(1-1/a^2/x^2)^(1/2)/a^2+1/2*x^2*
(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2,x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}x^2\,dx &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{1}{3}\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{2x}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{1}{6}\text{Subst}\left(\int \frac{\frac{4}{a^2}+\frac{3x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^3}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.67

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+3ax+4\right)+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^2,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + 3*a*x + 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/(6*a^3)
```

fricas [A] time = 0.71, size = 84, normalized size = 0.93

$$\frac{\left(2\,a^3x^3+5\,a^2x^2+7\,ax+4\right)\sqrt{\frac{ax-1}{ax+1}}+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 + 5*a^2*x^2 + 7*a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) + 3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

giac [B] time = 0.16, size = 151, normalized size = 1.68

$$\frac{1}{6}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^4}+\frac{2\left(\frac{4\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{3\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-9\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 - 3*log(abs(sqrt((a*x - 1)/
(a*x + 1)) - 1))/a^4 + 2*(4*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) -
 3*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 9*sqrt((a*x - 1)/(a*
x + 1)))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [B] time = 0.05, size = 173, normalized size = 1.92

$$\frac{(ax-1)\left(3\sqrt{a^2x^2-1}\;\sqrt{a^2}\;xa+2\,((ax-1)\,(ax+1))^{\frac{3}{2}}\;\sqrt{a^2}\;-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\;\sqrt{a^2}}{\sqrt{a^2}}\right)a+6\sqrt{(ax-1)\,(ax+1)}\;\sqrt{a}}{6\sqrt{\frac{ax-1}{ax+1}}\;\sqrt{(ax-1)\,(ax+1)}\;a^3\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x)
```

```
[Out] 1/6*(a*x-1)*(3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+2*((a*x-1)*(a*x+1))^(3/2)*
(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+6*((a
*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+6*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^3/
(a^2)^(1/2)
```

maxima [B] time = 0.32, size = 166, normalized size = 1.84

$$-\frac{1}{6}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(3*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
9*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4
/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^4 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

mupad [B] time = 0.06, size = 133, normalized size = 1.48

$$\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a^3+\frac{3\,a^3\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{a^3\,{(a\,x-1)}^3}{{(a\,x+1)}^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + ((a*x
- 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x -
 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*x +
 1))^(1/2))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2,x)
```

```
[Out] Integral(x**2/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.3 $\int e^{\coth^{-1}(ax)} x \, dx$

Optimal. Leaf size=63

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2+x*(1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a
^2/x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]]/(2*a^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}x\,dx &= -\operatorname{Subst}\left(\int \frac{1+\frac{x}{a}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{2}\operatorname{Subst}\left(\int \frac{-\frac{2}{a}-\frac{x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.78

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax+2)+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/
(2*a^2)
```

fricas [A] time = 0.51, size = 73, normalized size = 1.16

$$\frac{\left(a^2x^2+3\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 + 3*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(
a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

giac [B] time = 0.14, size = 118, normalized size = 1.87

$$\frac{1}{2}\,a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^3}-\frac{2\left(\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-3\sqrt{\frac{ax-1}{ax+1}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - log(abs(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^3 - 2*((a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) - 3*sqr
t((a*x - 1)/(a*x + 1)))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

maple [B] time = 0.05, size = 152, normalized size = 2.41

$$\frac{(ax-1)\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+2a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x,x)
```

```
[Out] 1/2*(a*x-1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+2*((a*x-1)*(a*x+1))^(1/2)*(a
^2)^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+2*a*ln((a
^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(
1/2)/((a*x-1)*(a*x+1))^(1/2)/a^2/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 128, normalized size = 2.03

$$\frac{1}{2}\,a\left(\frac{2\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(((a*x - 1)/(a*x + 1))^(3/2) - 3*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^3 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

mupad [B] time = 0.06, size = 98, normalized size = 1.56

$$\frac{3\sqrt{\frac{a\,x-1}{a\,x+1}}-\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*(
a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(
a*x + 1))^(1/2))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x,x)
```

```
[Out] Integral(x/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.4 $\int e^{\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=36

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] arctanh((1-1/a^2/x^2)^(1/2))/a+x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6168, 807, 266, 63, 208}

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x],x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6168

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$
$$= \sqrt{1-\frac{1}{a^2x^2}}\,x + a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.14

$$x\sqrt{1-\frac{1}{a^2x^2}} + \frac{\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x],x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

fricas [A] time = 0.66, size = 64, normalized size = 1.78

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}} + \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] ((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

giac [A] time = 0.17, size = 57, normalized size = 1.58

$$-\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)}{|a|\text{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2-1}}{a\text{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x
^2 - 1)/(a*sgn(a*x + 1))
```

maple [B] time = 0.04, size = 97, normalized size = 2.69

$$\frac{(ax-1)\left(\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2} + a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] (a*x-1)*(((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+a*ln((a^2*x+((a*x-1)*(a*x+1))^
(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^
(1/2)/a/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 90, normalized size = 2.50

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 0.04, size = 58, normalized size = 1.61

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.5 $\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=22

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

```
[Out] -arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6169, 844, 216, 266, 63, 208}

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x,x]
```

```
[Out] -ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\csc^{-1}(ax) + a^2\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.64

$$\log\left(x\left(\sqrt{\frac{a^2x^2-1}{a^2x^2}}+1\right)\right)-\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x,x]
[Out] -ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

fricas [B] time = 0.67, size = 57, normalized size = 2.59

$$2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
[Out] 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

giac [B] time = 0.16, size = 70, normalized size = 3.18

$$a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a)
```

maple [B] time = 0.05, size = 131, normalized size = 5.95

$$\frac{(ax-1)\left(\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}-\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}+a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] (a*x-1)*(((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-
arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)
*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)
/(a^2)^(1/2)
```

maxima [B] time = 0.42, size = 69, normalized size = 3.14

$$a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

mupad [B] time = 1.18, size = 37, normalized size = 1.68

$$2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(1/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.6 $\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=24

$$a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)$$

```
[Out] -a*arccsc(a*x)+a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6169, 641, 216}

$$a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^2,x]
```

```
[Out] a*Sqrt[1 - 1/(a^2*x^2)] - a*ArcCsc[a*x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x^2} \, dx &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right)\\
&= a\sqrt{1-\frac{1}{a^2x^2}}-\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right)\\
&= a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.12

$$a\left(\sqrt{1-\frac{1}{a^2x^2}}-\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^2,x]
```

```
[Out] a*(Sqrt[1 - 1/(a^2*x^2)] - ArcSin[1/(a*x)])
```

fricas [B] time = 0.56, size = 46, normalized size = 1.92

$$\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/x
```

giac [B] time = 0.14, size = 53, normalized size = 2.21

$$2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}+\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) + arctan(sqrt((a*x
 - 1)/(a*x + 1))))
```

maple [B] time = 0.05, size = 220, normalized size = 9.17

$$\frac{(ax-1)\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}\,xa-\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa-ax\sqrt{a^2}\,\text{arc}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)\,(ax+1)}\,x\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] (a*x-1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2)*x*a-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a
-a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a
^2)^(1/2))/(a^2)^(1/2))*x*a^2+ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)
)/(a^2)^(1/2))*x*a^2)/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/x/(a^
2)^(1/2)
```

maxima [B] time = 0.41, size = 53, normalized size = 2.21

$$2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}+\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) + arctan(sqrt((a*x
 - 1)/(a*x + 1))))
```

mupad [B] time = 0.05, size = 55, normalized size = 2.29

$$2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (2*a*((a*x - 1)/(a*x + 1))^(1/2))/(
(a*x - 1)/(a*x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.7 $\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=38

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[Out] -1/2*a^2*arccsc(a*x)+1/2*a*(2*a+1/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6169, 780, 216}

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^3,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a + x^(-1)))/2 - (a^2*ArcCsc[a*x])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx &= -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 1.11

$$\frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(2ax+1)-ax\sin^{-1}\left(\frac{1}{ax}\right)\right)}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^3,x]
```

```
[Out] (a*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a*x) - a*x*ArcSin[1/(a*x)]))/(2*x)
```

fricas [A] time = 0.63, size = 60, normalized size = 1.58

$$\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(2\,a^2x^2+3\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (2*a^2*x^2 + 3*a*x + 1)*
sqrt((a*x - 1)/(a*x + 1)))/x^2
```

giac [B] time = 0.14, size = 87, normalized size = 2.29

$$\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{(ax-1)a\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+3\,a\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt((a*x - 1)/(a*x + 1))) + ((a*x - 1)*a*sqrt((a*x - 1)/(a*x + 1
))/(a*x + 1) + 3*a*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)/(a*x + 1) + 1)^2)*
a
```

maple [B] time = 0.05, size = 260, normalized size = 6.84

$$\frac{(ax-1)\left(2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3+2\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}\,x^2a^2-2\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,xa-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2\right.}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

```
[Out] 1/2*(a*x-1)*(2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+2*((a*x-1)*(a*x+1))^(1
/2)*(a^2)^(1/2)*x^2*a^2-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-(a^2*x^2-1)^(1/
2)*(a^2)^(1/2)*x^2*a^2-a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-2*ln
((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+2*ln((a^2*x+((a
*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-(a^2*x^2-1)^(3/2)*(a
^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/x^2/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 91, normalized size = 2.39

$$\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] (a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*((a*x - 1)/(a*x + 1))^(3/2) + 3*a
*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^
2 + 1))*a
```

mupad [B] time = 1.20, size = 81, normalized size = 2.13

$$a^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2}+a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{3\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] a^2*((a*x - 1)/(a*x + 1))^(1/2) + ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) + a^2
*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (3*a*((a*x - 1)/(a*x + 1))^(1/2))/(2*x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.8 $\int \frac{e^{\coth^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=75

$$-\frac{1}{2}a^3\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] -1/3*a^3*(1-1/a^2/x^2)^(3/2)-1/2*a^3*arccsc(a*x)+a^3*(1-1/a^2/x^2)^(1/2)+1/
2*a^2*(1-1/a^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6169, 797, 641, 195, 216}

$$-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^4,x]
```

```
[Out] a^3*Sqrt[1 - 1/(a^2*x^2)] - (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[1 -
 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(a^2\,\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + a^2\,\text{Subst}\left(\int \left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)$$
$$= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} - a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) + a^2\,\text{Subst}\left(\int \sqrt{1-\frac{x^2}{a^2}}\,d\right.$$
$$= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - a^3\csc^{-1}(ax) + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx\right.$$
$$= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.68

$$\frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(4a^2x^2+3ax+2\right)}{x^2} - 3a^2\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^4,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x + 4*a^2*x^2))/x^2 - 3*a^2*ArcSin[1/(a
*x)]))/6
```

fricas [A] time = 0.51, size = 68, normalized size = 0.91

$$\frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \left(4\,a^3x^3 + 7\,a^2x^2 + 5\,ax + 2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (4*a^3*x^3 + 7*a^2*x^2 +
 5*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

giac [B] time = 0.15, size = 130, normalized size = 1.73

$$\frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{\frac{4\,(ax-1)a^2\sqrt{\frac{ax-1}{ax+1}}}{ax+1} + \frac{3\,(ax-1)^2a^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2} + 9\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (4*(a*x - 1)*a^2*sqrt((a*x -
 1)/(a*x + 1))/(a*x + 1) + 3*(a*x - 1)^2*a^2*sqrt((a*x - 1)/(a*x + 1))/(a*x
 + 1)^2 + 9*a^2*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

maple [B] time = 0.06, size = 284, normalized size = 3.79

$$-\frac{(ax-1)\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{a^2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

```
[Out] -1/6*(a*x-1)*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+6*(a^2*x^2-1)^(3/2)*
(a^2)^(1/2)*x^2*a^2+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+3*a^3*x^3*(a^2)
^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*x^3*a^4-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-6*ln(
(a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+3*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1)
)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/x^3/(a^2)^(1/2)
```

maxima [B] time = 0.42, size = 136, normalized size = 1.81

$$\frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{3\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (3*a^2*((a*x - 1)/(a*x + 1))
^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 9*a^2*sqrt((a*x - 1)/(a*x + 1)
))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x +
1)^3 + 1))*a
```

mupad [B] time = 0.06, size = 105, normalized size = 1.40

$$\frac{2\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^3}+a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{7\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x}+\frac{5\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3)
 + a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (7*a^2*((a*x - 1)/(a*x + 1))^(1/
2))/(6*x) + (5*a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Integral(1/(x**4*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.9 $\int \frac{e^{\coth^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=88

$$-\frac{3}{8}a^4\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)$$

```
[Out] -3/8*a^4*arccsc(a*x)+1/24*a^3*(16*a+9/x)*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^2
/x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6169, 833, 780, 216}

$$\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{3}{8}a^4\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^5,x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a + 9/x))/24 + (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) - (3*a^4*ArcCsc[a*x])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5} dx = -\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)$$
$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{1}{4}a^2 \text{Subst}\left(\int \frac{x^2\left(-\frac{3}{a}-\frac{4x}{a^2}\right)}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)$$
$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{12}a^4 \text{Subst}\left(\int \frac{x\left(\frac{8}{a^2}+\frac{9x}{a^3}\right)}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right) + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} dx,\right.$$
$$= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right) + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{3}{8}a^4\csc^{-1}(ax)$$

Mathematica [A] time = 0.10, size = 59, normalized size = 0.67

$$\frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(16a^3x^3+9a^2x^2+8ax+6\right)}{x^3}-9a^3\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^5,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(6 + 8*a*x + 9*a^2*x^2 + 16*a^3*x^3))/x^3 - 9*a^
3*ArcSin[1/(a*x)]))/24
```

fricas [A] time = 0.55, size = 76, normalized size = 0.86

$$\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(16\,a^4x^4+25\,a^3x^3+17\,a^2x^2+14\,ax+6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) + (16*a^4*x^4 + 25*a^3*x
^3 + 17*a^2*x^2 + 14*a*x + 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

giac [B] time = 0.15, size = 164, normalized size = 1.86

$$\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{31\,(ax-1)a^3\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{49\,(ax-1)^2a^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{9\,(ax-1)^3a^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}+39\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (31*(a*x - 1)*a^3*sqrt((a*x
 - 1)/(a*x + 1))/(a*x + 1) + 49*(a*x - 1)^2*a^3*sqrt((a*x - 1)/(a*x + 1))/(
```

```
a*x + 1)^2 + 9*(a*x - 1)^3*a^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 39*a
^3*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)/(a*x + 1) + 1)^4)*a
```

maple [B] time = 0.06, size = 308, normalized size = 3.50

$$-\frac{(ax-1)\left(-24\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5+24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^3a^3+9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+9a^4x^4\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x)
```

```
[Out] -1/24*(a*x-1)*(-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+24*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^3*a^3+9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+9*a^4*x^4*(a
^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+24*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^
(1/2))/(a^2)^(1/2))*x^4*a^5-24*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-
24*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+15*(
a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+8*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+6*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1
/2)/x^4/(a^2)^(1/2)
```

maxima [B] time = 0.42, size = 172, normalized size = 1.95

$$\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{9\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+39\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1}+\frac{6\,(ax-1)^2}{(ax+1)^2}+\frac{4\,(ax-1)^3}{(ax+1)^3}+\frac{(ax-1)^4}{(ax+1)^4}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (9*a^3*((a*x - 1)/(a*x + 1)
)^(7/2) + 49*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 31*a^3*((a*x - 1)/(a*x + 1))
^(3/2) + 39*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x
- 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 +
1))*a
```

mupad [B] time = 0.08, size = 129, normalized size = 1.47

$$\frac{2\,a^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4}+\frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}+\frac{17\,a^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2}+\frac{25\,a^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x}+\frac{7\,a\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4)
 + (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 + (17*a^2*((a*x - 1)/(a*x +
1))^(1/2))/(24*x^2) + (25*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (7*a*((
a*x - 1)/(a*x + 1))^(1/2))/(12*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

```
[Out] Integral(1/(x**5*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.10 $\int e^{2\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=43

$$\frac{2\log(1-ax)}{a^4}+\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

```
[Out] 2*x/a^3+x^2/a^2+2/3*x^3/a+1/4*x^4+2*ln(-a*x+1)/a^4
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\log(1-ax)}{a^4}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3,x]
```

```
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^3\,dx &= -\int e^{2\tanh^{-1}(ax)}x^3\,dx\\ &= -\int \frac{x^3(1+ax)}{1-ax}\,dx\\ &= -\int \left(-\frac{2}{a^3}-\frac{2x}{a^2}-\frac{2x^2}{a}-x^3-\frac{2}{a^3(-1+ax)}\right)dx\\ &= \frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{2\log(1-ax)}{a^4}+\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3,x]
```

```
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

fricas [A] time = 0.57, size = 42, normalized size = 0.98

$$\frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax + 24\,\log\left(ax - 1\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x + 24*log(a*x - 1))/a^4
```

giac [A] time = 0.14, size = 47, normalized size = 1.09

$$\frac{3\,a^4x^4 + 8\,a^3x^3 + 12\,a^2x^2 + 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax - 1|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x)/a^4 + 2*log(abs(a*x - 1)
)/a^4
```

maple [A] time = 0.04, size = 39, normalized size = 0.91

$$\frac{x^4}{4} + \frac{2x^3}{3a} + \frac{x^2}{a^2} + \frac{2x}{a^3} + \frac{2\ln\left(ax - 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3,x)
```

```
[Out] 1/4*x^4+2/3*x^3/a+x^2/a^2+2*x/a^3+2/a^4*ln(a*x-1)
```

maxima [A] time = 0.31, size = 43, normalized size = 1.00

$$\frac{3\,a^3x^4 + 8\,a^2x^3 + 12\,ax^2 + 24\,x}{12\,a^3} + \frac{2\,\log\left(ax - 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^4 + 8*a^2*x^3 + 12*a*x^2 + 24*x)/a^3 + 2*log(a*x - 1)/a^4
```

mupad [B] time = 0.04, size = 38, normalized size = 0.88

$$\frac{2\,\ln\left(a\,x - 1\right)}{a^4} + \frac{2\,x}{a^3} + \frac{x^4}{4} + \frac{2\,x^3}{3\,a} + \frac{x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*log(a*x - 1))/a^4 + (2*x)/a^3 + x^4/4 + (2*x^3)/(3*a) + x^2/a^2
```

sympy [A] time = 0.09, size = 37, normalized size = 0.86

$$\frac{x^4}{4} + \frac{2x^3}{3a} + \frac{x^2}{a^2} + \frac{2x}{a^3} + \frac{2\log\left(ax - 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3,x)
```

```
[Out] x**4/4 + 2*x**3/(3*a) + x**2/a**2 + 2*x/a**3 + 2*log(a*x - 1)/a**4
```

3.11 $\int e^{2\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=33

$$\frac{2\log(1-ax)}{a^3}+\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}$$

```
[Out] 2*x/a^2+x^2/a+1/3*x^3+2*ln(-a*x+1)/a^3
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$\frac{2x}{a^2}+\frac{2\log(1-ax)}{a^3}+\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^2\,dx &= -\int e^{2\tanh^{-1}(ax)}x^2\,dx\\ &= -\int \frac{x^2(1+ax)}{1-ax}\,dx\\ &= -\int\left(-\frac{2}{a^2}-\frac{2x}{a}-x^2-\frac{2}{a^2(-1+ax)}\right)dx\\ &= \frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{2\log(1-ax)}{a^3}+\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2,x]
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

fricas [A] time = 0.57, size = 33, normalized size = 1.00

$$\frac{a^3x^3 + 3\,a^2x^2 + 6\,ax + 6\,\log(ax - 1)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="fricas")
[Out] 1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^3
```

giac [A] time = 0.13, size = 38, normalized size = 1.15

$$\frac{a^3x^3 + 3\,a^2x^2 + 6\,ax}{3\,a^3} + \frac{2\,\log(|ax - 1|)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="giac")
[Out] 1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x)/a^3 + 2*log(abs(a*x - 1))/a^3
```

maple [A] time = 0.04, size = 31, normalized size = 0.94

$$\frac{x^3}{3} + \frac{x^2}{a} + \frac{2x}{a^2} + \frac{2\ln(ax - 1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2,x)
[Out] 1/3*x^3+x^2/a+2*x/a^2+2/a^3*ln(a*x-1)
```

maxima [A] time = 0.32, size = 34, normalized size = 1.03

$$\frac{a^2x^3 + 3\,ax^2 + 6\,x}{3\,a^2} + \frac{2\,\log(ax - 1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="maxima")
[Out] 1/3*(a^2*x^3 + 3*a*x^2 + 6*x)/a^2 + 2*log(a*x - 1)/a^3
```

mupad [B] time = 0.04, size = 30, normalized size = 0.91

$$\frac{2\,\ln(a\,x - 1)}{a^3} + \frac{2\,x}{a^2} + \frac{x^3}{3} + \frac{x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/(a*x - 1),x)
[Out] (2*log(a*x - 1))/a^3 + (2*x)/a^2 + x^3/3 + x^2/a
```

sympy [A] time = 0.09, size = 27, normalized size = 0.82

$$\frac{x^3}{3} + \frac{x^2}{a} + \frac{2x}{a^2} + \frac{2\log(ax - 1)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2,x)
[Out] x**3/3 + x**2/a + 2*x/a**2 + 2*log(a*x - 1)/a**3
```

3.12 $\int e^{2\coth^{-1}(ax)} x \, dx$

Optimal. Leaf size=26

$$\frac{2\log(1-ax)}{a^2} + \frac{2x}{a} + \frac{x^2}{2}$$

```
[Out] 2*x/a+1/2*x^2+2*ln(-a*x+1)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6126, 77}

$$\frac{2\log(1-ax)}{a^2} + \frac{2x}{a} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x,x]
```

```
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)} x \, dx &= -\int e^{2\tanh^{-1}(ax)} x \, dx \\
&= -\int \frac{x(1+ax)}{1-ax} \, dx \\
&= -\int \left(-\frac{2}{a} - x - \frac{2}{a(-1+ax)}\right) dx \\
&= \frac{2x}{a} + \frac{x^2}{2} + \frac{2\log(1-ax)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{2\log(1-ax)}{a^2} + \frac{2x}{a} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x,x]
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

fricas [A] time = 0.50, size = 25, normalized size = 0.96

$$\frac{a^2x^2 + 4\,ax + 4\,\log\left(ax - 1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="fricas")
[Out] 1/2*(a^2*x^2 + 4*a*x + 4*log(a*x - 1))/a^2
```

giac [A] time = 0.12, size = 30, normalized size = 1.15

$$\frac{a^2x^2 + 4\,ax}{2\,a^2} + \frac{2\,\log\left(|ax - 1|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="giac")
[Out] 1/2*(a^2*x^2 + 4*a*x)/a^2 + 2*log(abs(a*x - 1))/a^2
```

maple [A] time = 0.04, size = 24, normalized size = 0.92

$$\frac{x^2}{2} + \frac{2x}{a} + \frac{2\ln\left(ax - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x,x)
[Out] 1/2*x^2+2*x/a+2/a^2*ln(a*x-1)
```

maxima [A] time = 0.30, size = 26, normalized size = 1.00

$$\frac{ax^2 + 4\,x}{2\,a} + \frac{2\,\log\left(ax - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="maxima")
[Out] 1/2*(a*x^2 + 4*x)/a + 2*log(a*x - 1)/a^2
```

mupad [B] time = 0.04, size = 23, normalized size = 0.88

$$\frac{2\,\ln\left(a\,x - 1\right)}{a^2} + \frac{2\,x}{a} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/(a*x - 1),x)
[Out] (2*log(a*x - 1))/a^2 + (2*x)/a + x^2/2
```

sympy [A] time = 0.08, size = 20, normalized size = 0.77

$$\frac{x^2}{2} + \frac{2x}{a} + \frac{2\log\left(ax - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x)
[Out] x**2/2 + 2*x/a + 2*log(a*x - 1)/a**2
```

3.13 $\int e^{2\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=14

$$\frac{2\log(1-ax)}{a}+x$$

```
[Out] x+2*ln(-a*x+1)/a
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6125, 43}

$$\frac{2\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x]),x]
[Out] x + (2*Log[1 - a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\,dx &= -\int e^{2\tanh^{-1}(ax)}\,dx\\ &= -\int \frac{1+ax}{1-ax}\,dx\\ &= -\int \left(-1-\frac{2}{-1+ax}\right)dx\\ &= x+\frac{2\log(1-ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x]),x]
[Out] x + (2*Log[1 - a*x])/a
```

fricas [A] time = 0.45, size = 16, normalized size = 1.14

$$\frac{ax + 2\log(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="fricas")
[Out] (a*x + 2*log(a*x - 1))/a
```

giac [A] time = 0.11, size = 14, normalized size = 1.00

$$x + \frac{2\log(|ax - 1|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="giac")
[Out] x + 2*log(abs(a*x - 1))/a
```

maple [A] time = 0.03, size = 14, normalized size = 1.00

$$x + \frac{2\ln(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1),x)
[Out] x+2/a*ln(a*x-1)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.93

$$x + \frac{2\log(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="maxima")
[Out] x + 2*log(a*x - 1)/a
```

mupad [B] time = 1.17, size = 13, normalized size = 0.93

$$x + \frac{2\ln(a\,x - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(a*x - 1),x)
[Out] x + (2*log(a*x - 1))/a
```

sympy [A] time = 0.07, size = 10, normalized size = 0.71

$$x + \frac{2\log(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x)
[Out] x + 2*log(a*x - 1)/a
```

3.14 $\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=14

$$2\log(1-ax)-\log(x)$$

```
[Out] -ln(x)+2*ln(-a*x+1)
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 72}

$$2\log(1-ax)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/x,x]
[Out] -Log[x] + 2*Log[1 - a*x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x}\,dx\\ &= -\int \frac{1+ax}{x(1-ax)}\,dx\\ &= -\int \left(\frac{1}{x}-\frac{2a}{-1+ax}\right)dx\\ &= -\log(x)+2\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$2\log(1-ax)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x,x]
[Out] -Log[x] + 2*Log[1 - a*x]
```

fricas [A] time = 0.45, size = 13, normalized size = 0.93

$$2 \log(ax - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="fricas")
[Out] 2*log(a*x - 1) - log(x)
```

giac [A] time = 0.12, size = 15, normalized size = 1.07

$$2 \log(|ax - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="giac")
[Out] 2*log(abs(a*x - 1)) - log(abs(x))
```

maple [A] time = 0.04, size = 14, normalized size = 1.00

$$-\ln(x) + 2\ln(ax - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/x,x)
[Out] -ln(x)+2*ln(a*x-1)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.93

$$2 \log(ax - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="maxima")
[Out] 2*log(a*x - 1) - log(x)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.00

$$2 \ln(3 - 3\, a\, x) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(a*x - 1)),x)
[Out] 2*log(3 - 3*a*x) - log(x)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.71

$$-\log(x) + 2\log\left(x - \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x)
[Out] -log(x) + 2*log(x - 1/a)
```

3.15 $\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=19

$$-2a\log(x) + 2a\log(1 - ax) + \frac{1}{x}$$

[Out] 1/x-2*a*ln(x)+2*a*ln(-a*x+1)

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$-2a\log(x) + 2a\log(1 - ax) + \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/x^2,x]

[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x^2}\,dx \\
&= -\int \frac{1 + ax}{x^2(1 - ax)}\,dx \\
&= -\int \left(\frac{1}{x^2} + \frac{2a}{x} - \frac{2a^2}{-1 + ax}\right)dx \\
&= \frac{1}{x} - 2a\log(x) + 2a\log(1 - ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-2a\log(x) + 2a\log(1 - ax) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^2,x]
```

```
[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

fricas [A] time = 0.81, size = 22, normalized size = 1.16

$$\frac{2\,ax\log\left(ax-1\right)-2\,ax\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*log(a*x - 1) - 2*a*x*log(x) + 1)/x
```

giac [A] time = 0.12, size = 20, normalized size = 1.05

$$2\,a\log\left(|ax-1|\right)-2\,a\log\left(|x|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*log(abs(a*x - 1)) - 2*a*log(abs(x)) + 1/x
```

maple [A] time = 0.04, size = 19, normalized size = 1.00

$$\frac{1}{x}-2a\ln(x)+2a\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/x^2,x)
```

```
[Out] 1/x-2*a*ln(x)+2*a*ln(a*x-1)
```

maxima [A] time = 0.31, size = 18, normalized size = 0.95

$$2\,a\log\left(ax-1\right)-2\,a\log(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*log(a*x - 1) - 2*a*log(x) + 1/x
```

mupad [B] time = 1.19, size = 14, normalized size = 0.74

$$\frac{1}{x}-4\,a\,\mathrm{atanh}\left(2\,a\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(a*x - 1)),x)
```

```
[Out] 1/x - 4*a*atanh(2*a*x - 1)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.79

$$2a\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**2,x)
```

```
[Out] 2*a*(-log(x) + log(x - 1/a)) + 1/x
```

3.16 $\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=33

$$-2a^2\log(x)+2a^2\log(1-ax)+\frac{2a}{x}+\frac{1}{2x^2}$$

```
[Out] 1/2/x^2+2*a/x-2*a^2*ln(x)+2*a^2*ln(-a*x+1)
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$-2a^2\log(x)+2a^2\log(1-ax)+\frac{2a}{x}+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/x^3,x]
```

```
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x^3}\,dx\\
&= -\int \frac{1+ax}{x^3(1-ax)}\,dx\\
&= -\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{2a^2}{x}-\frac{2a^3}{-1+ax}\right)dx\\
&= \frac{1}{2x^2}+\frac{2a}{x}-2a^2\log(x)+2a^2\log(1-ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-2a^2\log(x)+2a^2\log(1-ax)+\frac{2a}{x}+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^3,x]
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

fricas [A] time = 0.52, size = 35, normalized size = 1.06

$$\frac{4\,a^2x^2\log\left(ax-1\right)-4\,a^2x^2\log(x)+4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="fricas")
[Out] 1/2*(4*a^2*x^2*log(a*x - 1) - 4*a^2*x^2*log(x) + 4*a*x + 1)/x^2
```

giac [A] time = 0.14, size = 32, normalized size = 0.97

$$2\,a^2\log\left(|ax-1|\right)-2\,a^2\log\left(|x|\right)+\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="giac")
[Out] 2*a^2*log(abs(a*x - 1)) - 2*a^2*log(abs(x)) + 1/2*(4*a*x + 1)/x^2
```

maple [A] time = 0.04, size = 31, normalized size = 0.94

$$\frac{1}{2x^2}+\frac{2a}{x}-2a^2\ln(x)+2a^2\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/x^3,x)
[Out] 1/2/x^2+2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x-1)
```

maxima [A] time = 0.31, size = 30, normalized size = 0.91

$$2\,a^2\log\left(ax-1\right)-2\,a^2\log(x)+\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="maxima")
[Out] 2*a^2*log(a*x - 1) - 2*a^2*log(x) + 1/2*(4*a*x + 1)/x^2
```

mupad [B] time = 0.04, size = 23, normalized size = 0.70

$$\frac{2\,a\,x+\frac{1}{2}}{x^2}-4\,a^2\,\mathrm{atanh}\left(2\,a\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(a*x - 1)),x)
[Out] (2*a*x + 1/2)/x^2 - 4*a^2*atanh(2*a*x - 1)
```

sympy [A] time = 0.15, size = 26, normalized size = 0.79

$$2a^2\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{4ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**3,x)
```

```
[Out] 2*a**2*(-log(x) + log(x - 1/a)) + (4*a*x + 1)/(2*x**2)
```

3.17 $\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=40

$$-2a^3\log(x)+2a^3\log(1-ax)+\frac{2a^2}{x}+\frac{a}{x^2}+\frac{1}{3x^3}$$

[Out] 1/3/x^3+a/x^2+2*a^2/x-2*a^3*ln(x)+2*a^3*ln(-a*x+1)

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)+\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/x^4,x]

[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x^4}\,dx\\
&= -\int \frac{1+ax}{x^4(1-ax)}\,dx\\
&= -\int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}-\frac{2a^4}{-1+ax}\right)dx\\
&= \frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$-2a^3\log(x)+2a^3\log(1-ax)+\frac{2a^2}{x}+\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^4,x]
```

```
[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

fricas [A] time = 0.59, size = 43, normalized size = 1.08

$$\frac{6\,a^3x^3\log\left(ax-1\right)-6\,a^3x^3\log(x)+6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 1)/x^3
```

giac [A] time = 0.13, size = 40, normalized size = 1.00

$$2\,a^3\log\left(|ax-1|\right)-2\,a^3\log\left(|x|\right)+\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*log(abs(a*x - 1)) - 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x
^3
```

maple [A] time = 0.04, size = 38, normalized size = 0.95

$$\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\ln(x)+2a^3\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/x^4,x)
```

```
[Out] 1/3/x^3+a/x^2+2*a^2/x-2*a^3*ln(x)+2*a^3*ln(a*x-1)
```

maxima [A] time = 0.31, size = 38, normalized size = 0.95

$$2\,a^3\log\left(ax-1\right)-2\,a^3\log(x)+\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] 2*a^3*log(a*x - 1) - 2*a^3*log(x) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x^3
```

mupad [B] time = 0.04, size = 30, normalized size = 0.75

$$\frac{2\,a^2\,x^2+a\,x+\frac{1}{3}}{x^3}-4\,a^3\,\mathrm{atanh}\left(2\,a\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(a*x - 1)),x)
```

```
[Out] (a*x + 2*a^2*x^2 + 1/3)/x^3 - 4*a^3*atanh(2*a*x - 1)
```

sympy [A] time = 0.17, size = 34, normalized size = 0.85

$$2a^3\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{6a^2x^2+3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**4,x)
```

```
[Out] 2*a**3*(-log(x) + log(x - 1/a)) + (6*a**2*x**2 + 3*a*x + 1)/(3*x**3)
```

3.18 $\int e^{3\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=118

$$\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] 11/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3-4*(1-1/a^2/x^2)^(1/2)/a^2/(a-1/x)+14/
3*x*(1-1/a^2/x^2)^(1/2)/a^2+3/2*x^2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/
x^2)^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6169, 6742, 651, 271, 264, 266, 51, 63, 208}

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2,x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x
)/(3*a^2) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^
3)/3 + (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^4\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{4}{a^3(a-x)\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{ax^3\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{a^2x^2\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{a^3x\sqrt{1-\frac{x^2}{a^2}}}\right.\right.$$

$$= -\frac{4\,\text{Subst}\left(\int\frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^3}-\frac{4\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^3}-\frac{4\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x\right.}{a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^2}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{2\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{a^3}-\frac{2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{3\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,\right.}{4a^3}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^3}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.64

$$\frac{33\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3+7a^2x^2+19ax-52\right)}{ax-1}}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-52 + 19*a*x + 7*a^2*x^2 + 2*a^3*x^3))/(-1 + a
*x) + 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

fricas [A] time = 0.51, size = 112, normalized size = 0.95

$$\frac{33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4x^4+9\,a^3x^3+26\,a^2x^2-33\,ax-52\right)\sqrt{\frac{ax-1}{ax+1}}}{6\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(33*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 33*(a*x - 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) + (2*a^4*x^4 + 9*a^3*x^3 + 26*a^2*x^2 - 33*a*x
- 52)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)
```

giac [A] time = 0.15, size = 171, normalized size = 1.45

$$\frac{1}{6}a\left(\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{33\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^4}-\frac{24}{a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{2\left(\frac{52(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{21(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-39\sqrt{\frac{ax-1}{ax+1}}\right.}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*a*(33*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 - 33*log(abs(sqrt((a*x - 1
)/(a*x + 1)) - 1))/a^4 - 24/(a^4*sqrt((a*x - 1)/(a*x + 1))) + 2*(52*(a*x -
1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) - 21*(a*x - 1)^2*sqrt((a*x - 1)/(a*x
 + 1))/(a*x + 1)^2 - 39*sqrt((a*x - 1)/(a*x + 1)))/(a^4*((a*x - 1)/(a*x + 1
) - 1)^3))
```

maple [B] time = 0.06, size = 471, normalized size = 3.99

$$\frac{9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3+2\sqrt{a^2}\,((ax-1)(ax+1))^{\frac{3}{2}}x^2a^2-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x)
```

```
[Out] 1/6/a^3*(9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+2*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x^2*a^2-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-9*ln((a^2*x+(a
^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-4*(a^2)^(1/2)*((a*x-1)*(a
*x+1))^(3/2)*x*a+42*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+42*ln((a^2*
x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+9*(a^2*x^2-1)^(
1/2)*(a^2)^(1/2)*x*a+18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2
))*x*a^2-10*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-84*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2)*x*a-84*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/
2))*x*a^2-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+42*((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+42*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2)))/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)
/(a*x+1))^(3/2)
```

maxima [A] time = 0.31, size = 182, normalized size = 1.54

$$-\frac{1}{6}a\left(\frac{2\left(\frac{75(ax-1)}{ax+1}-\frac{88(ax-1)^2}{(ax+1)^2}+\frac{33(ax-1)^3}{(ax+1)^3}-12\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-3a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(75*(a*x - 1)/(a*x + 1) - 88*(a*x - 1)^2/(a*x + 1)^2 + 33*(a*x -
1)^3/(a*x + 1)^3 - 12)/(a^4*((a*x - 1)/(a*x + 1))^(7/2) - 3*a^4*((a*x - 1)/
(a*x + 1))^(5/2) + 3*a^4*((a*x - 1)/(a*x + 1))^(3/2) - a^4*sqrt((a*x - 1)/(
a*x + 1))) - 33*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 + 33*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^4)
```

mupad [B] time = 1.26, size = 154, normalized size = 1.31

$$\frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}-\frac{\frac{88\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{11\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{25\,\left(a\,x-1\right)}{a\,x+1}+4}{a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3 - ((88*(a*x - 1)^2)/(3*(a*x + 1
)^2) - (11*(a*x - 1)^3)/(a*x + 1)^3 - (25*(a*x - 1))/(a*x + 1) + 4)/(a^3*((
a*x - 1)/(a*x + 1))^(1/2) - 3*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^3*((a*x
 - 1)/(a*x + 1))^(5/2) - a^3*((a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.19 $\int e^{3\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-4*(1-1/a^2/x^2)^(1/2)/a/(a-1/x)+3*x*(1
-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6169, 6742, 651, 266, 51, 63, 208, 264}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x,x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) + (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^3\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{4}{a^2(a-x)\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{ax^2\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{a^2x\sqrt{1-\frac{x^2}{a^2}}}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}-\frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}-\frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\right.}{a}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)-\frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x\right.}{a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+4\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}+\frac{1}{2}\text{Subst}\left(\int \frac{}{a^2-}\right.$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.72

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(a^2x^2+5ax-14\right)}{ax-1}+9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 + 5*a*x + a^2*x^2))/(-1 + a*x) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

fricas [A] time = 0.49, size = 103, normalized size = 1.12

$$\frac{9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^3x^3+6\,a^2x^2-9\,ax-14\right)\sqrt{\frac{ax-1}{ax+1}}}{2\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(9*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x - 1)*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1) + (a^3*x^3 + 6*a^2*x^2 - 9*a*x - 14)*sqrt((a*x -
1)/(a*x + 1)))/(a^3*x - a^2)
```

giac [A] time = 0.17, size = 140, normalized size = 1.52

$$\frac{1}{2}\,a\left(\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{9\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^3}-\frac{8}{a^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{2\left(\frac{5\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-7\sqrt{\frac{ax-1}{ax+1}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*a*(9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - 9*log(abs(sqrt((a*x - 1)/
(a*x + 1)) - 1))/a^3 - 8/(a^3*sqrt((a*x - 1)/(a*x + 1))) - 2*(5*(a*x - 1)*s
qrt((a*x - 1)/(a*x + 1))/(a*x + 1) - 7*sqrt((a*x - 1)/(a*x + 1)))/(a^3*((a*
x - 1)/(a*x + 1) - 1)^2))
```

maple [B] time = 0.06, size = 421, normalized size = 4.58

$$-\frac{-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-10\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}\,x^2a^2+2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x,x)
```

```
[Out] -1/2/a^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-10*((a*x-1)*(a*x+1))^(1/2)
*(a^2)^(1/2)*x^2*a^2+2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-10*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+4*((a*x-1)*(a*x+1))^(3/2)*(a^2)
^(1/2)+20*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-(a^2*x^2-1)^(1/2)*(a^2)^(
1/2)*x*a-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+20*l
```

```
n((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-10*((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2)+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)
^(1/2))*a-10*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))
/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.31, size = 145, normalized size = 1.58

$$\frac{1}{2}\,a\left(\frac{2\left(\frac{15\,(ax-1)}{ax+1}-\frac{9\,(ax-1)^2}{(ax+1)^2}-4\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-2\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(15*(a*x - 1)/(a*x + 1) - 9*(a*x - 1)^2/(a*x + 1)^2 - 4)/(a^3*((a*
x - 1)/(a*x + 1))^(5/2) - 2*a^3*((a*x - 1)/(a*x + 1))^(3/2) + a^3*sqrt((a*x
 - 1)/(a*x + 1))) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - 9*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^3)
```

mupad [B] time = 0.06, size = 117, normalized size = 1.27

$$\frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{\frac{9\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{15\,(a\,x-1)}{a\,x+1}+4}{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-2\,a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}+a^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - ((9*(a*x - 1)^2)/(a*x + 1)^2 -
 (15*(a*x - 1))/(a*x + 1) + 4)/(a^2*((a*x - 1)/(a*x + 1))^(1/2) - 2*a^2*((a
*x - 1)/(a*x + 1))^(3/2) + a^2*((a*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.20 $\int e^{3\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=62

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 3*arctanh((1-1/a^2/x^2)^(1/2))/a-4*(1-1/a^2/x^2)^(1/2)/(a-1/x)+x*(1-1/a^2/x
^2)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6168, 6742, 651, 264, 266, 63, 208}

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x + (3*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6168

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^2\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{4}{a(a-x)\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{ax\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}-\frac{4\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}-\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x\right.\\
&= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}\\
&= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x+(3a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.87

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-5)}{ax-1}+\frac{3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + a*x))/(-1 + a*x) + (3*Log[a*(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/a
```

fricas [A] time = 0.52, size = 92, normalized size = 1.48

$$\frac{3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-4\,ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a*x - 1)*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 4*a*x - 5)*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*x - a)
```

giac [B] time = 0.16, size = 119, normalized size = 1.92

$$a\left(\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{3(ax-1)}{ax+1}-2\right)}{a^2\left(\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\sqrt{\frac{ax-1}{ax+1}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(abs(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^2 - 2*(3*(a*x - 1)/(a*x + 1) - 2)/(a^2*((a*x - 1)*sqrt((a*x
- 1)/(a*x + 1))/(a*x + 1) - sqrt((a*x - 1)/(a*x + 1)))))
```

maple [B] time = 0.05, size = 248, normalized size = 4.00

$$-\frac{-3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2-3\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+2\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}+6\sqrt{(ax-1)}}{a\sqrt{a^2}\,\sqrt{(ax-1)(ax+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/a*(-3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-3*ln((a^2*x+((a*x-1)*(
a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+2*((a*x-1)*(a*x+1))^(3/2)*(
a^2)^(1/2)+6*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+6*ln((a^2*x+((a*x-1)*(
a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-3*((a*x-1)*(a*x+1))^(1/2)*(a^
2)^(1/2)-3*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/(
a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.30, size = 110, normalized size = 1.77

$$-a\left(\frac{2\left(\frac{3(ax-1)}{ax+1}-2\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*(3*(a*x - 1)/(a*x + 1) - 2)/(a^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*sq
rt((a*x - 1)/(a*x + 1))) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*log
(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 1.29, size = 59, normalized size = 0.95

$$\frac{2\,a\,x+12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\sqrt{\frac{a\,x-1}{a\,x+1}}-10}{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*a*x + 12*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 10)/(2*a*((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-3/2), x)
```

3.21 $\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=46

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

```
[Out] arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))-4*a*(1-1/a^2/x^2)^(1/2)/(a-1/x)
```

Rubi [A] time = 0.78, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6169, 6742, 216, 651, 266, 63, 208}

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x,x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + ArcCsc[a*x] + ArcTanh[Sqrt[1 -
1/(a^2*x^2)]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
```

```
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{1}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}-\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right.$$
$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\csc^{-1}(ax)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\csc^{-1}(ax)+a^2\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\csc^{-1}(ax)+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

Mathematica [A] time = 0.06, size = 53, normalized size = 1.15

$$-\frac{4ax\sqrt{1-\frac{1}{a^2x^2}}}{ax-1}+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x,x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]
```

fricas [B] time = 0.59, size = 104, normalized size = 2.26

$$-\frac{2(ax-1)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -(2*(a*x - 1)*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x - 1)*log(sqrt((a*x -
 1)/(a*x + 1)) + 1) + (a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + 4*(a*x
 + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)
```

giac [B] time = 0.16, size = 91, normalized size = 1.98

$$-a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a}+\frac{4}{a\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a + 4/(a*sqrt((a*x - 1)/(a*
x + 1))))
```

maple [B] time = 0.06, size = 363, normalized size = 7.89

$$\frac{\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] ((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^
2-1)^(1/2))+ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2
*a^3+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-2*(a^2*x^2-1)^(1/2)*(a^2)^
(1/2)*x*a-2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-2*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-2*((a*x-1)*(a*x+1))^(3/2)*
(a^2)^(1/2)-2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+(a^2*x^2-1)^(1/2)*(a^
2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+a*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a
^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [B] time = 0.41, size = 90, normalized size = 1.96

$$-a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}+\frac{4}{a\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a + 4/(a*sqrt((a*x - 1)/(a*x + 1
))))
```

mupad [B] time = 0.04, size = 54, normalized size = 1.17

$$2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{4}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] 2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
- 4/((a*x - 1)/(a*x + 1))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.22 $\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=51

$$-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}-3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

```
[Out] 3*a*arccsc(a*x)-2*(a+1/x)^2/a/(1-1/a^2/x^2)^(1/2)-3*a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6169, 853, 669, 641, 216}

$$-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}-3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^2,x]
```

```
[Out] -3*a*Sqrt[1 - 1/(a^2*x^2)] - (2*(a + x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
 3*a*ArcCsc[a*x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\,\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -3a\sqrt{1-\frac{1}{a^2x^2}} - \frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -3a\sqrt{1-\frac{1}{a^2x^2}} - \frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3a\csc^{-1}(ax)$$

Mathematica [A] time = 0.09, size = 41, normalized size = 0.80

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,(1-5ax)}{ax-1} + 3a\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^2,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x))/(-1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

fricas [A] time = 0.51, size = 74, normalized size = 1.45

$$-\frac{6\left(a^2x^2-ax\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \left(5\,a^2x^2+4\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*(a^2*x^2 - a*x)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (5*a^2*x^2 + 4*a*x
- 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)
```

giac [A] time = 0.16, size = 85, normalized size = 1.67

$$-2\,a\left(\frac{\frac{3\,(ax-1)}{ax+1}+2}{\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\sqrt{\frac{ax-1}{ax+1}}} + 3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -2*a*((3*(a*x - 1)/(a*x + 1) + 2)/((a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x
 + 1) + sqrt((a*x - 1)/(a*x + 1))) + 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

maple [B] time = 0.06, size = 593, normalized size = 11.63

$$-\sqrt{a^2x^2-1}\ \sqrt{a^2}\ x^4a^4+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\ x^2a^2+5\sqrt{a^2x^2-1}\ \sqrt{a^2}\ x^3a^3+3a^3x^3\sqrt{a^2}\ \arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] (-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a
^2+5*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+3*a^3*x^3*(a^2)^(1/2)*arctan(1/(
a^2*x^2-1)^(1/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3
*a^4-(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-ln((a^2*x+((a*x-1)*(a*x+1)
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x
*a-7*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-6*a^2*x^2*(a^2)^(1/2)*arctan(1/(
a^2*x^2-1)^(1/2))-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x
^2*a^3-2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+2*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2)*x^2*a^2+2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*x^2*a^3+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2
)*x*a+3*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+ln((a^2*x+(a^2*x^2-1)^(
1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*
a-ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/(a^2)^
(1/2)/x/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.41, size = 72, normalized size = 1.41

$$-2\,a\left(\frac{\frac{3\,(ax-1)}{ax+1}+2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}+3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*((3*(a*x - 1)/(a*x + 1) + 2)/(((a*x - 1)/(a*x + 1))^(3/2) + sqrt((a*x
 - 1)/(a*x + 1))) + 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

mupad [B] time = 0.05, size = 57, normalized size = 1.12

$$\frac{1}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}-6\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{5\,a}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] 1/(x*((a*x - 1)/(a*x + 1))^(1/2)) - 6*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) -
 (5*a)/((a*x - 1)/(a*x + 1))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.23 $\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=91

$$-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{9}{2}a^2\csc^{-1}(ax)-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}$$

```
[Out] -a^5*(1-1/a^2/x^2)^(5/2)/(a-1/x)^3-3/2*a^3*(1-1/a^2/x^2)^(3/2)/(a-1/x)+9/2*
a^2*arccsc(a*x)-9/2*a^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6169, 1633, 1593, 12, 793, 665, 216}

$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{9}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^3,x]
```

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a - x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a - x^(-1))) + (9*a^2*ArcCsc[a*x
])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\left(-ax-x^2\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} + (3a)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{1}{2}(9a)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{1}{2}(9a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{9}{2}a^2\csc^{-1}(ax)$$

Mathematica [A] time = 0.10, size = 56, normalized size = 0.62

$$\frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(-14a^2x^2+5ax+1\right)}{x(ax-1)} + 9a\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^3,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x - 14*a^2*x^2))/(x*(-1 + a*x)) + 9*a*A
rcSin[1/(a*x)]))/2
```

fricas [A] time = 0.38, size = 88, normalized size = 0.97

$$-\frac{18\left(a^3x^3-a^2x^2\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \left(14\,a^3x^3+9\,a^2x^2-6\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(18*(a^3*x^3 - a^2*x^2)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (14*a^3*x^
3 + 9*a^2*x^2 - 6*a*x - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)
```

giac [A] time = 0.14, size = 108, normalized size = 1.19

$$-\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{4\,a}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\frac{5\,(ax-1)a\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+7\,a\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*a/sqrt((a*x - 1)/(a*x + 1)) + (
5*(a*x - 1)*a*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 7*a*sqrt((a*x - 1)/(a*x
 + 1)))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

maple [B] time = 0.06, size = 641, normalized size = 7.04

$$\frac{-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^3a^3+21\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+9a^4x^4\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] 1/2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*x^3*a^3+21*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+9*a^4*x^4*(a^2)^(1/2)*a
rctan(1/(a^2*x^2-1)^(1/2))+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)
^(1/2))*x^4*a^5-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-6*ln((a^2*x+(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-11*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^2*a^2-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-18*a^3*x^3*
(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-12*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2))*x^3*a^4-4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2
+12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+12*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+4*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)
*x*a+9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+9*a^2*x^2*(a^2)^(1/2)*arctan(1
/(a^2*x^2-1)^(1/2))+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*x^2*a^3-6*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-6*ln((a^2*x+((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+(a^2*x^2-1)^(3/2)*(a^2)^(
1/2))/(a^2)^(1/2)/x^2/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/
2)
```

maxima [A] time = 0.41, size = 110, normalized size = 1.21

$$-\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{15\,(ax-1)a}{ax+1}+\frac{9\,(ax-1)^2a}{(ax+1)^2}+4\,a}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (15*(a*x - 1)*a/(a*x + 1) + 9*(a*
x - 1)^2*a/(a*x + 1)^2 + 4*a)/(((a*x - 1)/(a*x + 1))^(5/2) + 2*((a*x - 1)/(
a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

mupad [B] time = 0.08, size = 83, normalized size = 0.91

$$\frac{1}{2\,x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}-\frac{7\,a^2}{\sqrt{\frac{a\,x-1}{a\,x+1}}}-9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{5\,a}{2\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] 1/(2*x^2*((a*x - 1)/(a*x + 1))^(1/2)) - (7*a^2)/((a*x - 1)/(a*x + 1))^(1/2)
 - 9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (5*a)/(2*x*((a*x - 1)/(a*x + 1
))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.24 $\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=93

$$\frac{11}{2}a^3\csc^{-1}(ax)-\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right)-\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2-\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 11/2*a^3*arccsc(a*x)-(a+1/x)^3/(1-1/a^2/x^2)^(1/2)-1/3*a*(3*a+1/x)^2*(1-1/a
^2/x^2)^(1/2)-1/6*a^2*(28*a+3/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6169, 1633, 1593, 12, 852, 1635, 1654, 780, 216}

$$-\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right)-\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2-\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{11}{2}a^3\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^4,x]
```

```
[Out] -((a + x^(-1))^3/Sqrt[1 - 1/(a^2*x^2)]) - (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a + x
^(-1))^2)/3 - (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a + 3/x))/6 + (11*a^3*ArcCsc[a
*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\,(-ax^2-x^3)}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2\left(3a^2+ax\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-5-\frac{3x}{a}\right)\left(3a^2+ax\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{1}{2}\left(11a^2\right)\text{Subst}\Bigg(\int$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{11}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] time = 0.12, size = 66, normalized size = 0.71

$$\frac{1}{6}a\left(33a^2\sin^{-1}\left(\frac{1}{ax}\right) + \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(-52a^3x^3+19a^2x^2+7ax+2\right)}{x^2(ax-1)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^4,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 7*a*x + 19*a^2*x^2 - 52*a^3*x^3))/(x^2*(-1
+ a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

fricas [A] time = 0.56, size = 96, normalized size = 1.03

$$-\frac{66\left(a^4x^4-a^3x^3\right)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(52\,a^4x^4+33\,a^3x^3-26\,a^2x^2-9\,ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(66*(a^4*x^4 - a^3*x^3)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (52*a^4*x^
4 + 33*a^3*x^3 - 26*a^2*x^2 - 9*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4
- x^3)
```

giac [A] time = 0.13, size = 150, normalized size = 1.61

$$-\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{12\,a^2}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\frac{52\,(ax-1)a^2\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{21\,(ax-1)^2a^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+39\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 12*a^2/sqrt((a*x - 1)/(a*x
 + 1)) + (52*(a*x - 1)*a^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 21*(a*x -
1)^2*a^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 39*a^2*sqrt((a*x - 1)/(a*x
 + 1)))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

maple [B] time = 0.06, size = 666, normalized size = 7.16

$$\frac{-30\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^6a^6+30\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4+93\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5+33\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,x^5a^5+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] 1/6*(-30*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^6*a^6+30*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^4*a^4+93*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+33*arctan(1/(a^2*x^2-
1)^(1/2))*(a^2)^(1/2)*x^5*a^5+30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(
a^2)^(1/2))*x^5*a^6-30*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-30*ln((a
^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6-51*(a^2*x^2-
1)^(3/2)*(a^2)^(1/2)*x^3*a^3-96*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4-66*a^
4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-60*ln((a^2*x+(a^2*x^2-1)^(1/2
)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*
x^3*a^3+60*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+60*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+14*(a^2*x^2-1)^(3/2)*(a^
2)^(1/2)*x^2*a^2+33*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+33*a^3*x^3*(a^2)^
(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*x^3*a^4-30*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-30*l
n((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+5*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/(a^2)^(1/2)/x^
3/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.42, size = 154, normalized size = 1.66

$$-\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{75\,(ax-1)a^2}{ax+1}+\frac{88\,(ax-1)^2a^2}{(ax+1)^2}+\frac{33\,(ax-1)^3a^2}{(ax+1)^3}+12\,a^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (75*(a*x - 1)*a^2/(a*x + 1
) + 88*(a*x - 1)^2*a^2/(a*x + 1)^2 + 33*(a*x - 1)^3*a^2/(a*x + 1)^3 + 12*a^
2)/(((a*x - 1)/(a*x + 1))^(7/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + 3*((a*x -
 1)/(a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

mupad [B] time = 1.24, size = 152, normalized size = 1.63

$$-\frac{4\,a^3+\frac{88\,a^3\,(a\,x-1)^2}{3\,(a\,x+1)^2}+\frac{11\,a^3\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{25\,a^3\,(a\,x-1)}{a\,x+1}}{\sqrt{\frac{a\,x-1}{a\,x+1}}+3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}+3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}+\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}-11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] - (4*a^3 + (88*a^3*(a*x - 1)^2)/(3*(a*x + 1)^2) + (11*a^3*(a*x - 1)^3)/(a*x
 + 1)^3 + (25*a^3*(a*x - 1))/(a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/2) + 3*((
a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + ((a*x - 1)/(a*x
 + 1))^(7/2)) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.25 $\int e^{4\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=57

$$\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

```
[Out] 12*x/a^3+4*x^2/a^2+4/3*x^3/a+1/4*x^4+4/a^4/(-a*x+1)+16*ln(-a*x+1)/a^4
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6167, 6126, 88}

$$\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}x^3\,dx &= \int e^{4\tanh^{-1}(ax)}x^3\,dx\\ &= \int \frac{x^3(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(\frac{12}{a^3}+\frac{8x}{a^2}+\frac{4x^2}{a}+x^3+\frac{4}{a^3(-1+ax)^2}+\frac{16}{a^3(-1+ax)}\right)dx\\ &= \frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 1.00

$$\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

fricas [A] time = 0.64, size = 66, normalized size = 1.16

$$\frac{3\,a^5x^5 + 13\,a^4x^4 + 32\,a^3x^3 + 96\,a^2x^2 - 144\,ax + 192\,(ax-1)\log\left(ax-1\right) - 48}{12\,\left(a^5x - a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^5*x^5 + 13*a^4*x^4 + 32*a^3*x^3 + 96*a^2*x^2 - 144*a*x + 192*(a*x
 - 1)*log(a*x - 1) - 48)/(a^5*x - a^4)
```

giac [A] time = 0.14, size = 78, normalized size = 1.37

$$\frac{(ax-1)^4\left(\frac{28}{ax-1} + \frac{114}{(ax-1)^2} + \frac{300}{(ax-1)^3} + 3\right)}{12\,a^4} - \frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^4} - \frac{4}{(ax-1)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(a*x - 1)^4*(28/(a*x - 1) + 114/(a*x - 1)^2 + 300/(a*x - 1)^3 + 3)/a^4
 - 16*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a^4 - 4/((a*x - 1)*a^4)
```

maple [A] time = 0.04, size = 52, normalized size = 0.91

$$\frac{x^4}{4} + \frac{4x^3}{3a} + \frac{4x^2}{a^2} + \frac{12x}{a^3} + \frac{16\ln\left(ax-1\right)}{a^4} - \frac{4}{a^4\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^3,x)
```

```
[Out] 1/4*x^4+4/3*x^3/a+4*x^2/a^2+12*x/a^3+16/a^4*ln(a*x-1)-4/a^4/(a*x-1)
```

maxima [A] time = 0.31, size = 58, normalized size = 1.02

$$-\frac{4}{a^5x - a^4} + \frac{3\,a^3x^4 + 16\,a^2x^3 + 48\,ax^2 + 144\,x}{12\,a^3} + \frac{16\,\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -4/(a^5*x - a^4) + 1/12*(3*a^3*x^4 + 16*a^2*x^3 + 48*a*x^2 + 144*x)/a^3 + 1
6*log(a*x - 1)/a^4
```

mupad [B] time = 0.04, size = 57, normalized size = 1.00

$$\frac{16\,\ln\left(a\,x-1\right)}{a^4} - \frac{4}{a\,\left(a^4\,x - a^3\right)} + \frac{12\,x}{a^3} + \frac{x^4}{4} + \frac{4\,x^3}{3\,a} + \frac{4\,x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (16*log(a*x - 1))/a^4 - 4/(a*(a^4*x - a^3)) + (12*x)/a^3 + x^4/4 + (4*x^3)/
(3*a) + (4*x^2)/a^2
```

sympy [A] time = 0.15, size = 49, normalized size = 0.86

$$\frac{x^4}{4} - \frac{4}{a^5x - a^4} + \frac{4x^3}{3a} + \frac{4x^2}{a^2} + \frac{12x}{a^3} + \frac{16\log(ax - 1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**3,x)
```

```
[Out] x**4/4 - 4/(a**5*x - a**4) + 4*x**3/(3*a) + 4*x**2/a**2 + 12*x/a**3 + 16*lo
g(a*x - 1)/a**4
```

3.26 $\int e^{4\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=47

$$\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}$$

```
[Out] 8*x/a^2+2*x^2/a+1/3*x^3+4/a^3/(-a*x+1)+12*ln(-a*x+1)/a^3
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 88}

$$\frac{8x}{a^2}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{2x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}x^2\,dx&=\int e^{4\tanh^{-1}(ax)}x^2\,dx\\&=\int\frac{x^2(1+ax)^2}{(1-ax)^2}\,dx\\&=\int\left(\frac{8}{a^2}+\frac{4x}{a}+x^2+\frac{4}{a^2(-1+ax)^2}+\frac{12}{a^2(-1+ax)}\right)dx\\&=\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.00

$$\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

fricas [A] time = 0.55, size = 57, normalized size = 1.21

$$\frac{a^4x^4 + 5\,a^3x^3 + 18\,a^2x^2 - 24\,ax + 36\,(ax-1)\log\left(ax-1\right) - 12}{3\left(a^4x - a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 24*a*x + 36*(a*x - 1)*log(a*x - 1)
- 12)/(a^4*x - a^3)
```

giac [A] time = 0.14, size = 69, normalized size = 1.47

$$\frac{(ax-1)^3\left(\frac{9}{ax-1} + \frac{39}{(ax-1)^2} + 1\right)}{3\,a^3} - \frac{12\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^3} - \frac{4}{(ax-1)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(a*x - 1)^3*(9/(a*x - 1) + 39/(a*x - 1)^2 + 1)/a^3 - 12*log(abs(a*x - 1
)/((a*x - 1)^2*abs(a)))/a^3 - 4/((a*x - 1)*a^3)
```

maple [A] time = 0.04, size = 44, normalized size = 0.94

$$\frac{x^3}{3} + \frac{2x^2}{a} + \frac{8x}{a^2} + \frac{12\ln\left(ax-1\right)}{a^3} - \frac{4}{a^3\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^2,x)
```

```
[Out] 1/3*x^3+2*x^2/a+8*x/a^2+12/a^3*ln(a*x-1)-4/a^3/(a*x-1)
```

maxima [A] time = 0.31, size = 49, normalized size = 1.04

$$-\frac{4}{a^4x - a^3} + \frac{a^2x^3 + 6\,ax^2 + 24\,x}{3\,a^2} + \frac{12\,\log\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="maxima")
```

```
[Out] -4/(a^4*x - a^3) + 1/3*(a^2*x^3 + 6*a*x^2 + 24*x)/a^2 + 12*log(a*x - 1)/a^3
```

mupad [B] time = 1.17, size = 49, normalized size = 1.04

$$\frac{12\,\ln\left(a\,x-1\right)}{a^3} - \frac{4}{a\,\left(a^3\,x-a^2\right)} + \frac{8\,x}{a^2} + \frac{x^3}{3} + \frac{2\,x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (12*log(a*x - 1))/a^3 - 4/(a*(a^3*x - a^2)) + (8*x)/a^2 + x^3/3 + (2*x^2)/a
```

sympy [A] time = 0.15, size = 39, normalized size = 0.83

$$\frac{x^3}{3} - \frac{4}{a^4x - a^3} + \frac{2x^2}{a} + \frac{8x}{a^2} + \frac{12\log\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**2,x)
```

```
[Out] x**3/3 - 4/(a**4*x - a**3) + 2*x**2/a + 8*x/a**2 + 12*log(a*x - 1)/a**3
```

3.27 $\int e^{4\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=39

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

```
[Out] 4*x/a+1/2*x^2+4/a^2/(-a*x+1)+8*ln(-a*x+1)/a^2
```

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6126, 77}

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}x\,dx &= \int e^{4\tanh^{-1}(ax)}x\,dx\\
&= \int \frac{x(1+ax)^2}{(1-ax)^2}\,dx\\
&= \int \left(\frac{4}{a}+x+\frac{4}{a(-1+ax)^2}+\frac{8}{a(-1+ax)}\right)dx\\
&= \frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.00

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

fricas [A] time = 0.56, size = 49, normalized size = 1.26

$$\frac{a^{3} x^{3} + 7 \, a^{2} x^{2} - 8 \, a x + 16 \, {\left(a x - 1\right)} \log\left(a x - 1\right) - 8}{2 \, {\left(a^{3} x - a^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*x^3 + 7*a^2*x^2 - 8*a*x + 16*(a*x - 1)*log(a*x - 1) - 8)/(a^3*x -
a^2)
```

giac [A] time = 0.14, size = 64, normalized size = 1.64

$$\frac{\frac{{\left(a x - 1\right)}^{2} {\left(\frac{10}{a x - 1} + 1\right)}}{a} - \frac{16 \, \log\left(\frac{{\left| a x - 1 \right|}}{{\left(a x - 1\right)}^{2} {\left| a \right|}}\right)}{a} - \frac{8}{{\left(a x - 1\right)} a}}{2 \, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="giac")
```

```
[Out] 1/2*((a*x - 1)^2*(10/(a*x - 1) + 1)/a - 16*log(abs(a*x - 1)/((a*x - 1)^2*ab
s(a)))/a - 8/((a*x - 1)*a))/a
```

maple [A] time = 0.04, size = 36, normalized size = 0.92

$$\frac{x^{2}}{2}+\frac{4 x}{a}+\frac{8 \ln \left(a x -1\right)}{a^{2}}-\frac{4}{a^{2} \left(a x -1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x,x)
```

```
[Out] 1/2*x^2+4*x/a+8/a^2*ln(a*x-1)-4/a^2/(a*x-1)
```

maxima [A] time = 0.31, size = 41, normalized size = 1.05

$$\frac{a x^{2} + 8 \, x}{2 \, a} - \frac{4}{a^{3} x - a^{2}} + \frac{8 \, \log\left(a x - 1\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 + 8*x)/a - 4/(a^3*x - a^2) + 8*log(a*x - 1)/a^2
```

mupad [B] time = 0.04, size = 38, normalized size = 0.97

$$\frac{8\,\ln\left(a\,x-1\right)}{a^2}+\frac{4\,x}{a}+\frac{x^2}{2}+\frac{4}{a\,\left(a-a^2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (8*log(a*x - 1))/a^2 + (4*x)/a + x^2/2 + 4/(a*(a - a^2*x))
```

sympy [A] time = 0.13, size = 31, normalized size = 0.79

$$\frac{x^2}{2} - \frac{4}{a^3x - a^2} + \frac{4x}{a} + \frac{8\log(ax - 1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x,x)
```

```
[Out] x**2/2 - 4/(a**3*x - a**2) + 4*x/a + 8*log(a*x - 1)/a**2
```

3.28 $\int e^{4 \coth^{-1}(ax)} dx$

Optimal. Leaf size=27

$$\frac{4}{a(1-ax)} + \frac{4\log(1-ax)}{a} + x$$

```
[Out] x+4/a/(-a*x+1)+4*ln(-a*x+1)/a
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6125, 43}

$$\frac{4}{a(1-ax)} + \frac{4\log(1-ax)}{a} + x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x]),x]
```

```
[Out] x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)} dx &= \int e^{4\tanh^{-1}(ax)} dx \\
&= \int \frac{(1+ax)^2}{(1-ax)^2} dx \\
&= \int \left(1 + \frac{4}{(-1+ax)^2} + \frac{4}{-1+ax}\right) dx \\
&= x + \frac{4}{a(1-ax)} + \frac{4\log(1-ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$-\frac{4}{a(ax-1)} + \frac{4\log(1-ax)}{a} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x]),x]
```

```
[Out] x - 4/(a*(-1 + a*x)) + (4*Log[1 - a*x])/a
```

fricas [A] time = 0.69, size = 38, normalized size = 1.41

$$\frac{a^2x^2 - ax + 4\,(ax-1)\log{(ax-1)} - 4}{a^2x - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 4*(a*x - 1)*log(a*x - 1) - 4)/(a^2*x - a)
```

giac [A] time = 0.14, size = 46, normalized size = 1.70

$$\frac{ax-1}{a} - \frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{4}{(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="giac")
[Out] (a*x - 1)/a - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4/((a*x - 1)*a)
```

maple [A] time = 0.04, size = 26, normalized size = 0.96

$$x + \frac{4\ln{(ax-1)}}{a} - \frac{4}{a\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2,x)
[Out] x+4/a*ln(a*x-1)-4/a/(a*x-1)
```

maxima [A] time = 0.30, size = 26, normalized size = 0.96

$$x + \frac{4\,\log{(ax-1)}}{a} - \frac{4}{a^2x - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="maxima")
[Out] x + 4*log(a*x - 1)/a - 4/(a^2*x - a)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.93

$$x - \frac{4}{a\,(a\,x-1)} + \frac{4\,\ln{(a\,x-1)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(a*x - 1)^2,x)
[Out] x - 4/(a*(a*x - 1)) + (4*log(a*x - 1))/a
```

sympy [A] time = 0.12, size = 19, normalized size = 0.70

$$x - \frac{4}{a^2x - a} + \frac{4\log{(ax-1)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2,x)
[Out] x - 4/(a**2*x - a) + 4*log(a*x - 1)/a
```

3.29 $\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=13

$$\frac{4}{1-ax}+\log(x)$$

```
[Out] 4/(-a*x+1)+ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 88}

$$\frac{4}{1-ax}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{x}\,dx\\
&= \int \frac{(1+ax)^2}{x(1-ax)^2}\,dx\\
&= \int \left(\frac{1}{x}+\frac{4a}{(-1+ax)^2}\right)dx\\
&= \frac{4}{1-ax}+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{4}{1-ax}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x,x]
[Out] 4/(1 - a*x) + Log[x]
```

fricas [A] time = 0.55, size = 18, normalized size = 1.38

$$\frac{(ax-1)\log(x)-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="fricas")
[Out] ((a*x - 1)*log(x) - 4)/(a*x - 1)
```

giac [B] time = 0.15, size = 57, normalized size = 4.38

$$-a\left(\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{4}{(ax-1)a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="giac")
[Out] -a*(log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - log(abs(-1/(a*x - 1) - 1))/a
 + 4/((a*x - 1)*a))
```

maple [A] time = 0.04, size = 13, normalized size = 1.00

$$\ln(x)-\frac{4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x,x)
[Out] ln(x)-4/(a*x-1)
```

maxima [A] time = 0.31, size = 12, normalized size = 0.92

$$-\frac{4}{ax-1}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="maxima")
[Out] -4/(a*x - 1) + log(x)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.92

$$\ln(x)-\frac{4}{a\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(x*(a*x - 1)^2),x)
[Out] log(x) - 4/(a*x - 1)
```

sympy [A] time = 0.15, size = 8, normalized size = 0.62

$$\log(x)-\frac{4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x,x)
```

```
[Out] log(x) - 4/(a*x - 1)
```

3.30 $\int \frac{e^{4\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=32

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

```
[Out] -1/x+4*a/(-a*x+1)+4*a*ln(x)-4*a*ln(-a*x+1)
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 88}

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{x^2} dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{x^2} dx \\
&= \int \frac{(1+ax)^2}{x^2(1-ax)^2} dx \\
&= \int \left(\frac{1}{x^2}+\frac{4a}{x}+\frac{4a^2}{(-1+ax)^2}-\frac{4a^2}{-1+ax}\right) dx \\
&= -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.00

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^2,x]
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

fricas [A] time = 0.46, size = 55, normalized size = 1.72

$$-\frac{5\,ax+4\,\left(a^2x^2-ax\right)\log\left(ax-1\right)-4\,\left(a^2x^2-ax\right)\log(x)-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="fricas")
[Out] -(5*a*x + 4*(a^2*x^2 - a*x)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 1)/(a
*x^2 - x)
```

giac [A] time = 0.14, size = 40, normalized size = 1.25

$$4\,a\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a}{ax-1}+\frac{a}{\frac{1}{ax-1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="giac")
[Out] 4*a*log(abs(-1/(a*x - 1) - 1)) - 4*a/(a*x - 1) + a/(1/(a*x - 1) + 1)
```

maple [A] time = 0.04, size = 31, normalized size = 0.97

$$-\frac{1}{x}+4a\ln(x)-\frac{4a}{ax-1}-4a\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^2,x)
[Out] -1/x+4*a*ln(x)-4*a/(a*x-1)-4*a*ln(a*x-1)
```

maxima [A] time = 0.31, size = 34, normalized size = 1.06

$$-4\,a\log\left(ax-1\right)+4\,a\log(x)-\frac{5\,ax-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="maxima")
[Out] -4*a*log(a*x - 1) + 4*a*log(x) - (5*a*x - 1)/(a*x^2 - x)
```

mupad [B] time = 0.05, size = 28, normalized size = 0.88

$$8\,a\,\mathrm{atanh}\left(2\,a\,x-1\right)+\frac{5\,a\,x-1}{x-a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(x^2*(a*x - 1)^2),x)
[Out] 8*a*atanh(2*a*x - 1) + (5*a*x - 1)/(x - a*x^2)
```

sympy [A] time = 0.21, size = 26, normalized size = 0.81

$$4a\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-5ax+1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**2,x)
[Out] 4*a*(log(x) - log(x - 1/a)) + (-5*a*x + 1)/(a*x**2 - x)
```

3.31 $\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

```
[Out] -1/2/x^2-4*a/x+4*a^2/(-a*x+1)+8*a^2*ln(x)-8*a^2*ln(-a*x+1)
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 88}

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/(2*x^2) - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{x^3}\,dx\\ &= \int \frac{(1+ax)^2}{x^3(1-ax)^2}\,dx\\ &= \int \left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{(-1+ax)^2}-\frac{8a^3}{-1+ax}\right)dx\\ &= -\frac{1}{2x^2}-\frac{4a}{x}+\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

fricas [A] time = 0.59, size = 73, normalized size = 1.59

$$-\frac{16\,a^2x^2-7\,ax+16\left(a^3x^3-a^2x^2\right)\log\left(ax-1\right)-16\left(a^3x^3-a^2x^2\right)\log(x)-1}{2\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(16*a^2*x^2 - 7*a*x + 16*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 16*(a^3*x^
3 - a^2*x^2)*log(x) - 1)/(a*x^3 - x^2)
```

giac [A] time = 0.14, size = 62, normalized size = 1.35

$$8\,a^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^2}{ax-1}+\frac{9\,a^2+\frac{10\,a^2}{ax-1}}{2\left(\frac{1}{ax-1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="giac")
```

```
[Out] 8*a^2*log(abs(-1/(a*x - 1) - 1)) - 4*a^2/(a*x - 1) + 1/2*(9*a^2 + 10*a^2/(a
*x - 1))/(1/(a*x - 1) + 1)^2
```

maple [A] time = 0.04, size = 43, normalized size = 0.93

$$-\frac{1}{2x^2}-\frac{4a}{x}+8a^2\ln(x)-\frac{4a^2}{ax-1}-8a^2\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^3,x)
```

```
[Out] -1/2/x^2-4*a/x+8*a^2*ln(x)-4*a^2/(a*x-1)-8*a^2*ln(a*x-1)
```

maxima [A] time = 0.30, size = 48, normalized size = 1.04

$$-8\,a^2\log\left(ax-1\right)+8\,a^2\log(x)-\frac{16\,a^2x^2-7\,ax-1}{2\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*log(a*x - 1) + 8*a^2*log(x) - 1/2*(16*a^2*x^2 - 7*a*x - 1)/(a*x^3 -
x^2)
```

mupad [B] time = 1.20, size = 41, normalized size = 0.89

$$16\,a^2\,\mathrm{atanh}\left(2\,a\,x-1\right)+\frac{-8\,a^2\,x^2+\frac{7\,a\,x}{2}+\frac{1}{2}}{a\,x^3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(x^3*(a*x - 1)^2),x)
```

```
[Out] 16*a^2*atanh(2*a*x - 1) + ((7*a*x)/2 - 8*a^2*x^2 + 1/2)/(a*x^3 - x^2)
```

sympy [A] time = 0.24, size = 41, normalized size = 0.89

$$8a^2\left(\log(x) - \log\left(x - \frac{1}{a}\right)\right) + \frac{-16a^2x^2 + 7ax + 1}{2ax^3 - 2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**3,x)
```

```
[Out] 8*a**2*(log(x) - log(x - 1/a)) + (-16*a**2*x**2 + 7*a*x + 1)/(2*a*x**3 - 2*
x**2)
```

3.32 $\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=54

$$\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)-\frac{8a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+4*a^3/(-a*x+1)+12*a^3*ln(x)-12*a^3*ln(-a*x+1)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 88}

$$\frac{4a^3}{1-ax}-\frac{8a^2}{x}+12a^3\log(x)-12a^3\log(1-ax)-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/(3*x^3) - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{x^4}\,dx\\ &= \int \frac{(1+ax)^2}{x^4(1-ax)^2}\,dx\\ &= \int \left(\frac{1}{x^4}+\frac{4a}{x^3}+\frac{8a^2}{x^2}+\frac{12a^3}{x}+\frac{4a^4}{(-1+ax)^2}-\frac{12a^4}{-1+ax}\right)dx\\ &= -\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.00

$$\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)-\frac{8a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

fricas [A] time = 0.61, size = 81, normalized size = 1.50

$$-\frac{36\,a^3x^3 - 18\,a^2x^2 - 5\,ax + 36\left(a^4x^4 - a^3x^3\right)\log\left(ax-1\right) - 36\left(a^4x^4 - a^3x^3\right)\log(x) - 1}{3\left(ax^4 - x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x + 36*(a^4*x^4 - a^3*x^3)*log(a*x - 1)
 - 36*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*x^4 - x^3)
```

giac [A] time = 0.12, size = 74, normalized size = 1.37

$$12\,a^3\log\left(\left|-\frac{1}{ax-1} - 1\right|\right) - \frac{4\,a^3}{ax-1} + \frac{31\,a^3 + \frac{69\,a^3}{ax-1} + \frac{39\,a^3}{(ax-1)^2}}{3\left(\frac{1}{ax-1} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 12*a^3*log(abs(-1/(a*x - 1) - 1)) - 4*a^3/(a*x - 1) + 1/3*(31*a^3 + 69*a^3/
(a*x - 1) + 39*a^3/(a*x - 1)^2)/(1/(a*x - 1) + 1)^3
```

maple [A] time = 0.04, size = 51, normalized size = 0.94

$$-\frac{1}{3x^3} - \frac{2a}{x^2} - \frac{8a^2}{x} + 12a^3\ln(x) - \frac{4a^3}{ax-1} - 12a^3\ln\left(ax-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^4,x)
```

```
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+12*a^3*ln(x)-4*a^3/(a*x-1)-12*a^3*ln(a*x-1)
```

maxima [A] time = 0.30, size = 56, normalized size = 1.04

$$-12\,a^3\log\left(ax-1\right) + 12\,a^3\log(x) - \frac{36\,a^3x^3 - 18\,a^2x^2 - 5\,ax - 1}{3\left(ax^4 - x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] -12*a^3*log(a*x - 1) + 12*a^3*log(x) - 1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x
 - 1)/(a*x^4 - x^3)
```

mupad [B] time = 0.06, size = 49, normalized size = 0.91

$$24\,a^3\,\mathrm{atanh}\left(2\,a\,x-1\right) + \frac{-12\,a^3\,x^3 + 6\,a^2\,x^2 + \frac{5\,a\,x}{3} + \frac{1}{3}}{a\,x^4 - x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(x^4*(a*x - 1)^2),x)
```

```
[Out] 24*a^3*atanh(2*a*x - 1) + ((5*a*x)/3 + 6*a^2*x^2 - 12*a^3*x^3 + 1/3)/(a*x^4
 - x^3)
```

sympy [**A**] time = 0.26, size = 49, normalized size = 0.91

$$12a^3\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-36a^3x^3+18a^2x^2+5ax+1}{3ax^4-3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**4,x)
```

```
[Out] 12*a**3*(log(x) - log(x - 1/a)) + (-36*a**3*x**3 + 18*a**2*x**2 + 5*a*x + 1
)/(3*a*x**4 - 3*x**3)
```

3.33 $\int e^{-\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=114

$$\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}$$

```
[Out] 3/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4-2/3*x*(1-1/a^2/x^2)^(1/2)/a^3+3/8*x^2*
(1-1/a^2/x^2)^(1/2)/a^2-1/3*x^3*(1-1/a^2/x^2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)
^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcCoth[a*x],x]
```

```
[Out] (-2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2
) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{4}{a}-\frac{3x}{a^2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{9}{a^2}-\frac{8x}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{24}\text{Subst}\left(\int \frac{\frac{16}{a^3}-\frac{9x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right.}{8a^4}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\right.}{16a^4}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{3\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\right.}{8a^2}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right.}{8a^4}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 0.60

$$\frac{9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3-8a^2x^2+9ax-16\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^ArcCoth[a*x],x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-16 + 9*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
 + Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

fricas [A] time = 0.47, size = 91, normalized size = 0.80

$$\frac{\left(6\,a^4x^4-2\,a^3x^3+a^2x^2-7\,ax-16\right)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 7*a*x - 16)*sqrt((a*x - 1)/(a*x +
1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) - 1))/a^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [B] time = 0.05, size = 193, normalized size = 1.69

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(6\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,xa+15\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,xa-8\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^{2}}-15\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{\vphantom{a}}}{\sqrt{a^{2}}}\right.\right.}{24\sqrt{(ax-1)(ax+1)}\,a^{4}\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+1
5*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-8*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-1
5*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+24*a*ln((a^2*x+((
a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-24*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^4/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 203, normalized size = 1.78

$$-\frac{1}{24}\,a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-31\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^{5}}{ax+1}-\frac{6(ax-1)^{2}a^{5}}{(ax+1)^{2}}+\frac{4(ax-1)^{3}a^{5}}{(ax+1)^{3}}-\frac{(ax-1)^{4}a^{5}}{(ax+1)^{4}}-a^{5}}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{5}}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*a*(2*(39*((a*x - 1)/(a*x + 1))^(7/2) - 31*((a*x - 1)/(a*x + 1))^(5/2)
 + 49*((a*x - 1)/(a*x + 1))^(3/2) - 9*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x -
1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x +
 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^5 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

mupad [B] time = 1.21, size = 172, normalized size = 1.51

$$\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^{4}}-\frac{\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{49\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{31\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{13\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^{4}+\frac{6\,a^{4}\,{\left(a\,x-1\right)}^{2}}{{\left(a\,x+1\right)}^{2}}-\frac{4\,a^{4}\,{\left(a\,x-1\right)}^{3}}{{\left(a\,x+1\right)}^{3}}+\frac{a^{4}\,{\left(a\,x-1\right)}^{4}}{{\left(a\,x+1\right)}^{4}}-\frac{4\,a^{4}\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((3*((a*x - 1)/(a*x + 1))^
(1/2))/4 - (49*((a*x - 1)/(a*x + 1))^(3/2))/12 + (31*((a*x - 1)/(a*x + 1))^
(5/2))/12 - (13*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1)^2)/
(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)
^4 - (4*a^4*(a*x - 1))/(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.34 $\int e^{-\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=90

$$-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] -1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+2/3*x*(1-1/a^2/x^2)^(1/2)/a^2-1/2*x^2
*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}x^2\,dx &= -\operatorname{Subst}\left(\int \frac{1-\frac{x}{a}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{1}{3}\operatorname{Subst}\left(\int \frac{\frac{3}{a}-\frac{2x}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{1}{6}\operatorname{Subst}\left(\int \frac{\frac{4}{a^2}-\frac{3x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a^3}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a^3}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}\\
&= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.67

$$
\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-3ax+4\right)-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{6a^3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^ArcCoth[a*x],x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 - 3*a*x + 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/(6*a^3)
```

fricas [A] time = 0.62, size = 83, normalized size = 0.92

$$
\frac{\left(2\,a^3x^3-a^2x^2+ax+4\right)\sqrt{\frac{ax-1}{ax+1}}-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 - a^2*x^2 + a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt
((a*x - 1)/(a*x + 1)) + 1) + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

giac [**A**] time = 0.14, size = 86, normalized size = 0.96

$$\frac{1}{6}\sqrt{a^2x^2-1}\left(x\left(\frac{2\,x\mathrm{sgn}\left(ax+1\right)}{a}-\frac{3\,\mathrm{sgn}\left(ax+1\right)}{a^2}\right)+\frac{4\,\mathrm{sgn}\left(ax+1\right)}{a^3}\right)+\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{2\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 - 1)*(x*(2*x*sgn(a*x + 1)/a - 3*sgn(a*x + 1)/a^2) + 4*sgn(
a*x + 1)/a^3) + 1/2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a
^2*abs(a))
```

maple [**B**] time = 0.05, size = 173, normalized size = 1.92

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa-2\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-6\sqrt{(ax-1)(ax+}\right.}{6\sqrt{(ax-1)(ax+1)}\,a^3\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-2
*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1
/2))/(a^2)^(1/2))*a-6*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+6*a*ln((a^2*x+((a
*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^(1/2)/a^3
/(a^2)^(1/2)
```

maxima [**B**] time = 0.31, size = 166, normalized size = 1.84

$$-\frac{1}{6}\,a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(9*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) +
3*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4
/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^4 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)
```

mupad [**B**] time = 0.05, size = 134, normalized size = 1.49

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + 3*((a*x
- 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x -
 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - atanh(((a*x - 1)/(a*x +
 1))^(1/2))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.35 $\int e^{-\coth^{-1}(ax)} x \, dx$

Optimal. Leaf size=64

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-x*(1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a
^2/x^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6169, 835, 807, 266, 63, 208}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/a) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]]/(2*a^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}x\,dx &= -\operatorname{Subst}\left(\int \frac{1-\frac{x}{a}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\frac{2}{a}-\frac{x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.77

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-2)+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^ArcCoth[a*x],x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])
/(2*a^2)
```

fricas [A] time = 0.76, size = 73, normalized size = 1.14

$$\frac{\left(a^2x^2-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

giac [A] time = 0.15, size = 71, normalized size = 1.11

$$\frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x\mathrm{sgn}(ax+1)}{a}-\frac{2\,\mathrm{sgn}(ax+1)}{a^2}\right)-\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2*x^2 - 1)*(x*sgn(a*x + 1)/a - 2*sgn(a*x + 1)/a^2) - 1/2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a*abs(a))
```

maple [B] time = 0.05, size = 152, normalized size = 2.38

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-2a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax}}{\sqrt{a^2}}\right.\right.}{2\sqrt{(ax-1)(ax+1)}\,a^2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+2*
((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)
)/(a^2)^(1/2))*a-2*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2)))/((a*x-1)*(a*x+1))^(1/2)/a^2/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 130, normalized size = 2.03

$$-\frac{1}{2}a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(3*((a*x - 1)/(a*x + 1))^(3/2) - sqrt((a*x - 1)/(a*x + 1)))/(2*(a
*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^3 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

mupad [B] time = 0.06, size = 97, normalized size = 1.52

$$\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] atanh(((a*x - 1)/(a*x + 1))^(1/2))/a^2 - (((a*x - 1)/(a*x + 1))^(1/2) - 3*(
(a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(
a*x - 1))/(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.36 $\int e^{-\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=37

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a+x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6168, 807, 266, 63, 208}

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcCoth[a*x]),x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6168

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$
$$= \sqrt{1-\frac{1}{a^2x^2}}\,x - a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.14

$$x\sqrt{1-\frac{1}{a^2x^2}} - \frac{\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcCoth[a*x]),x]
[Out] Sqrt[1 - 1/(a^2*x^2)]*x - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

fricas [A] time = 0.65, size = 64, normalized size = 1.73

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}} - \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] ((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

giac [A] time = 0.14, size = 52, normalized size = 1.41

$$\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}\,\operatorname{sgn}(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2
- 1)*sgn(a*x + 1)/a
```

maple [B] time = 0.04, size = 98, normalized size = 2.65

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2} - a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-a*ln((
a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^
(1/2)/a/(a^2)^(1/2)
```

maxima [B] time = 0.30, size = 90, normalized size = 2.43

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 1.19, size = 58, normalized size = 1.57

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1)), x)
```

3.37 $\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=20

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

```
[Out] arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6169, 844, 216, 266, 63, 208}

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x),x]
```

```
[Out] ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx &= -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right) \\
&= \csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}} \, dx, x, \frac{1}{x^2}\right) \\
&= \csc^{-1}(ax) + a^2\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2} \, dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right) \\
&= \csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.70

$$\log\left(x\left(\sqrt{\frac{a^2x^2-1}{a^2x^2}}+1\right)\right)+\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x),x]
[Out] ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

fricas [B] time = 0.57, size = 57, normalized size = 2.85

$$-2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
[Out] -2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

giac [B] time = 0.16, size = 59, normalized size = 2.95

$$-2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\text{sgn}(ax+1)-\frac{a\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\text{sgn}(ax+1)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] -2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) - a*log(abs(-x*abs(a)
 + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a)
```

maple [B] time = 0.05, size = 133, normalized size = 6.65

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}-\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}}{\sqrt{a^2}}\right.\right.}{\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-(a^2*
x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-a*ln((a^2*
x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^(1/2
)/(a^2)^(1/2)
```

maxima [B] time = 0.40, size = 70, normalized size = 3.50

$$-a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

mupad [B] time = 0.03, size = 37, normalized size = 1.85

$$2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x,x)
```

```
[Out] 2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x, x)
```

3.38 $\int \frac{e^{-\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=25

$$a\left(-\csc^{-1}(ax)\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] -a*arccsc(a*x)-a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6169, 641, 216}

$$a\left(-\csc^{-1}(ax)\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -(a*Sqrt[1 - 1/(a^2*x^2)]) - a*ArcCsc[a*x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{x^2} dx &= -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)\\
&= -a\sqrt{1-\frac{1}{a^2x^2}}-\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)\\
&= -a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.04

$$-a\left(\sqrt{1-\frac{1}{a^2x^2}}+\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -(a*(Sqrt[1 - 1/(a^2*x^2)] + ArcSin[1/(a*x)]))
```

fricas [B] time = 0.57, size = 47, normalized size = 1.88

$$\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [B] time = 0.05, size = 220, normalized size = 8.80

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\ (ax+1)\left(-\sqrt{a^2x^2-1}\ \sqrt{a^2}\ x^2a^2+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}\ xa+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\ -\sqrt{a^2x^2-1}\ \sqrt{a^2}\ xa\ -}{\sqrt{(ax-1)(ax+1)}\ x\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+((a
*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-(a^2*x^2
-1)^(1/2)*(a^2)^(1/2)*x*a-a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+ln((a
^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-ln((a^2*x+((a*x-1)*(
a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/((a*x-1)*(a*x+1))^(1/2)/x/(a
^2)^(1/2)
```

maxima [B] time = 0.40, size = 55, normalized size = 2.20

$$-2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}-\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) - arctan(sqrt((a*
x - 1)/(a*x + 1))))
```

mupad [B] time = 1.20, size = 55, normalized size = 2.20

$$2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^2,x)
```

```
[Out] 2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (2*a*((a*x - 1)/(a*x + 1))^(1/2))/(
(a*x - 1)/(a*x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**2, x)
```

3.39 $\int \frac{e^{-\coth^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=40

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[Out] 1/2*a^2*arccsc(a*x)+1/2*a*(2*a-1/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6169, 780, 216}

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a - x^(-1)))/2 + (a^2*ArcCsc[a*x])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{x^3} \, dx &= -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)\end{aligned}$$

Mathematica [A] time = 0.05, size = 41, normalized size = 1.02

$$\frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(2ax-1)+ax\sin^{-1}\left(\frac{1}{ax}\right)\right)}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (a*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + 2*a*x) + a*x*ArcSin[1/(a*x)]))/(2*x)
```

fricas [A] time = 0.66, size = 60, normalized size = 1.50

$$-\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(2\,a^2x^2+ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^2*x^2 + a*x - 1)*s
qrt((a*x - 1)/(a*x + 1)))/x^2
```

giac [B] time = 0.14, size = 157, normalized size = 3.92

$$-a^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)+\frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3a^2\operatorname{sgn}(ax+1)+2\left(x|a|-\sqrt{a^2x^2-1}\right)^2a|a|\operatorname{sgn}}{\left(\left(x|a|-\sqrt{a^2x^2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + ((x*abs(a) - sqrt
(a^2*x^2 - 1))^3*a^2*sgn(a*x + 1) + 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*ab
s(a)*sgn(a*x + 1) - (x*abs(a) - sqrt(a^2*x^2 - 1))*a^2*sgn(a*x + 1) + 2*a*a
bs(a)*sgn(a*x + 1))/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2
```

maple [B] time = 0.05, size = 260, normalized size = 6.50

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3+2\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2+2\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,xa-\sqrt{a^2x^2-1}\right.}{2\sqrt{(ax-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*
a^3+2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+2*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2)*x*a-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-a^2*x^2*(a^2)^(1/2)*arctan
(1/(a^2*x^2-1)^(1/2))+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2
))*x^2*a^3-2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^
2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/x^2/(a^2)^(1/2
)
```

maxima [B] time = 0.41, size = 93, normalized size = 2.32

$$-\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -(a*arctan(sqrt((a*x - 1)/(a*x + 1))) - (3*a*((a*x - 1)/(a*x + 1))^(3/2) +
a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)
^2 + 1))*a
```

mupad [B] time = 1.20, size = 82, normalized size = 2.05

$$a^2\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2}-a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^3,x)
```

```
[Out] a^2*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) - a^2
*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**3, x)
```

3.40 $\int \frac{e^{-\coth^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=76

$$-\frac{1}{2}a^3\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-a^3\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] 1/3*a^3*(1-1/a^2/x^2)^(3/2)-1/2*a^3*arccsc(a*x)-a^3*(1-1/a^2/x^2)^(1/2)+1/2
*a^2*(1-1/a^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = $\{6169, 797, 641, 195, 216\}$

$$\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] -(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[
1 - 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 797

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx = -\operatorname{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(a^2 \operatorname{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + a^2 \operatorname{Subst}\left(\int \left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -a^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} - a^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) + a^2 \operatorname{Subst}\left(\int \sqrt{1-\right.$$
$$= -a^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - a^3\csc^{-1}(ax) + \frac{1}{2}a^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\right.$$
$$= -a^3\sqrt{1-\frac{1}{a^2x^2}} + \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] time = 0.08, size = 52, normalized size = 0.68

$$-\frac{1}{2}a^3\sin^{-1}\left(\frac{1}{ax}\right) - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(4a^2x^2-3ax+2\right)}{6x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] -1/6*(a*Sqrt[1 - 1/(a^2*x^2)]*(2 - 3*a*x + 4*a^2*x^2))/x^2 - (a^3*ArcSin[1/
(a*x)])/2
```

fricas [A] time = 0.52, size = 68, normalized size = 0.89

$$\frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \left(4\,a^3x^3 + a^2x^2 - ax + 2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (4*a^3*x^3 + a^2*x^2 - a
*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [B] time = 0.06, size = 284, normalized size = 3.74

$$\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-3a^3x^3\sqrt{a^2}\,\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a
^4+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
x^3*a^3-3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+6*ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+6*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(1/2)*x^3*a^3-6*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))*x^3*a^4-3*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2))/((a*x-1)*(a*x+1))^(1/2)/x^3/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 137, normalized size = 1.80

$$\frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (9*a^2*((a*x - 1)/(a*x + 1))
^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^2*sqrt((a*x - 1)/(a*x + 1)
))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x +
1)^3 + 1))*a
```

mupad [B] time = 1.20, size = 105, normalized size = 1.38

$$a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^3}-\frac{2\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}-\frac{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x}+\frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^4,x)
```

```
[Out] a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) - ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3)
 - (2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 - (a^2*((a*x - 1)/(a*x + 1))^(1/2)
)/(6*x) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**4, x)
```

3.41 $\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{3}{8}a^4\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)$$

```
[Out] 3/8*a^4*arccsc(a*x)+1/24*a^3*(16*a-9/x)*(1-1/a^2/x^2)^(1/2)-1/4*a*(1-1/a^2/
x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6169, 833, 780, 216}

$$\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{3}{8}a^4\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^5),x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a - 9/x))/24 - (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) + (3*a^4*ArcCsc[a*x])/8
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx = -\operatorname{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{1}{4}a^2\operatorname{Subst}\left(\int \frac{x^2\left(\frac{3}{a}-\frac{4x}{a^2}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{12}a^4\operatorname{Subst}\left(\int \frac{x\left(\frac{8}{a^2}-\frac{9x}{a^3}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right) - \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} + \frac{1}{8}\left(3a^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x\right.$$
$$= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right) - \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} + \frac{3}{8}a^4\csc^{-1}(ax)$$

Mathematica [A] time = 0.10, size = 59, normalized size = 0.67

$$\frac{1}{24}a\left(9a^3\sin^{-1}\left(\frac{1}{ax}\right) + \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(16a^3x^3-9a^2x^2+8ax-6\right)}{x^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^5),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(-6 + 8*a*x - 9*a^2*x^2 + 16*a^3*x^3))/x^3 + 9*a
^3*ArcSin[1/(a*x)]))/24
```

fricas [A] time = 1.34, size = 77, normalized size = 0.88

$$-\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \left(16\,a^4x^4+7\,a^3x^3-a^2x^2+2\,ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (16*a^4*x^4 + 7*a^3*x
^3 - a^2*x^2 + 2*a*x - 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

giac [B] time = 0.16, size = 258, normalized size = 2.93

$$-\frac{3}{4}a^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1) + \frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^7 a^4\operatorname{sgn}(ax+1) + 33\left(x|a|-\sqrt{a^2x^2-1}\right)^5 a^4\operatorname{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + 1/12*(9*(x*ab
s(a) - sqrt(a^2*x^2 - 1))^7*a^4*sgn(a*x + 1) + 33*(x*abs(a) - sqrt(a^2*x^2
```

```
- 1))^5*a^4*sgn(a*x + 1) + 48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a)*s
gn(a*x + 1) - 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*a^4*sgn(a*x + 1) + 64*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a)*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(
a^2*x^2 - 1))*a^4*sgn(a*x + 1) + 16*a^3*abs(a)*sgn(a*x + 1))/((x*abs(a) - s
qrt(a^2*x^2 - 1))^2 + 1)^4
```

maple [B] time = 0.07, size = 308, normalized size = 3.50

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-24\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5+24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^3a^3-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4-9a^4x^4\sqrt{a^2}\,\mathrm{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^5,x)
```

```
[Out] -1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^
5*a^5+24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-9*(a^2*x^2-1)^(1/2)*(a^2)^(1
/2)*x^4*a^4-9*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+24*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+24*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2)*x^4*a^4-24*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(
a^2)^(1/2))*x^4*a^5-15*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+8*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2)*x*a-6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1
/2)/x^4/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 173, normalized size = 1.97

$$-\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{39\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1}+\frac{6\,(ax-1)^2}{(ax+1)^2}+\frac{4\,(ax-1)^3}{(ax+1)^3}+\frac{(ax-1)^4}{(ax+1)^4}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (39*a^3*((a*x - 1)/(a*x +
1))^(7/2) + 31*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 49*a^3*((a*x - 1)/(a*x + 1
))^(3/2) + 9*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x
 - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 +
 1))*a
```

mupad [B] time = 1.22, size = 129, normalized size = 1.47

$$\frac{2\,a^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4}-\frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}-\frac{a^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2}+\frac{7\,a^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x}+\frac{a\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^5,x)
```

```
[Out] (2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 - ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4)
 - (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - (a^2*((a*x - 1)/(a*x + 1))
^(1/2))/(24*x^2) + (7*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (a*((a*x -
1)/(a*x + 1))^(1/2))/(12*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**5, x)
```

3.42 $\int e^{-2\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=42

$$\frac{2\log(ax+1)}{a^4}-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

```
[Out] -2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$\frac{x^2}{a^2}-\frac{2x}{a^3}+\frac{2\log(ax+1)}{a^4}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^3\,dx\\
&= -\int \frac{x^3(1-ax)}{1+ax}\,dx\\
&= -\int \left(\frac{2}{a^3}-\frac{2x}{a^2}+\frac{2x^2}{a}-x^3-\frac{2}{a^3(1+ax)}\right)dx\\
&= -\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{2\log(ax+1)}{a^4}-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(2*ArcCoth[a*x]),x]
[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

fricas [A] time = 0.44, size = 42, normalized size = 1.00

$$\frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax + 24\,\log\left(ax + 1\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x + 24*log(a*x + 1))/a^4
```

giac [A] time = 0.14, size = 47, normalized size = 1.12

$$\frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax + 1|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x)/a^4 + 2*log(abs(a*x + 1)
)/a^4
```

maple [A] time = 0.04, size = 39, normalized size = 0.93

$$-\frac{2x}{a^3} + \frac{x^2}{a^2} - \frac{2x^3}{3a} + \frac{x^4}{4} + \frac{2\ln\left(ax + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)*(a*x-1),x)
[Out] -2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4
```

maxima [A] time = 0.31, size = 43, normalized size = 1.02

$$\frac{3\,a^3x^4 - 8\,a^2x^3 + 12\,ax^2 - 24\,x}{12\,a^3} + \frac{2\,\log\left(ax + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/12*(3*a^3*x^4 - 8*a^2*x^3 + 12*a*x^2 - 24*x)/a^3 + 2*log(a*x + 1)/a^4
```

mupad [B] time = 1.17, size = 38, normalized size = 0.90

$$\frac{2\,\ln\left(a\,x + 1\right)}{a^4} - \frac{2\,x}{a^3} + \frac{x^4}{4} - \frac{2\,x^3}{3\,a} + \frac{x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x - 1))/(a*x + 1),x)
[Out] (2*log(a*x + 1))/a^4 - (2*x)/a^3 + x^4/4 - (2*x^3)/(3*a) + x^2/a^2
```

sympy [A] time = 0.09, size = 37, normalized size = 0.88

$$\frac{x^4}{4} - \frac{2x^3}{3a} + \frac{x^2}{a^2} - \frac{2x}{a^3} + \frac{2\log\left(ax + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*x-1)/(a*x+1),x)
```

```
[Out] x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**3 + 2*log(a*x + 1)/a**4
```

3.43 $\int e^{-2\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=33

$$-\frac{2\log(ax+1)}{a^3}+\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}$$

```
[Out] 2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$\frac{2x}{a^2}-\frac{2\log(ax+1)}{a^3}-\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x^2\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^2\,dx\\ &= -\int \frac{x^2(1-ax)}{1+ax}\,dx\\ &= -\int \left(-\frac{2}{a^2}+\frac{2x}{a}-x^2+\frac{2}{a^2(1+ax)}\right)dx\\ &= \frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{2\log(ax+1)}{a^3}+\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*ArcCoth[a*x]),x]
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

fricas [A] time = 0.44, size = 33, normalized size = 1.00

$$\frac{a^3x^3 - 3\,a^2x^2 + 6\,ax - 6\,\log\left(ax + 1\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x - 6*log(a*x + 1))/a^3
```

giac [A] time = 0.12, size = 38, normalized size = 1.15

$$\frac{a^3x^3 - 3\,a^2x^2 + 6\,ax}{3\,a^3} - \frac{2\,\log\left(|ax + 1|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x)/a^3 - 2*log(abs(a*x + 1))/a^3
```

maple [A] time = 0.04, size = 32, normalized size = 0.97

$$\frac{2x}{a^2} - \frac{x^2}{a} + \frac{x^3}{3} - \frac{2\ln\left(ax + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)*(a*x-1),x)
[Out] 2*x/a^2-x^2/a+1/3*x^3-2/a^3*ln(a*x+1)
```

maxima [A] time = 0.31, size = 34, normalized size = 1.03

$$\frac{a^2x^3 - 3\,ax^2 + 6\,x}{3\,a^2} - \frac{2\,\log\left(ax + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/3*(a^2*x^3 - 3*a*x^2 + 6*x)/a^2 - 2*log(a*x + 1)/a^3
```

mupad [B] time = 0.04, size = 31, normalized size = 0.94

$$\frac{2\,x}{a^2} - \frac{2\,\ln\left(a\,x + 1\right)}{a^3} + \frac{x^3}{3} - \frac{x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x - 1))/(a*x + 1),x)
[Out] (2*x)/a^2 - (2*log(a*x + 1))/a^3 + x^3/3 - x^2/a
```

sympy [A] time = 0.09, size = 27, normalized size = 0.82

$$\frac{x^3}{3} - \frac{x^2}{a} + \frac{2x}{a^2} - \frac{2\log\left(ax + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x-1)/(a*x+1),x)
[Out] x**3/3 - x**2/a + 2*x/a**2 - 2*log(a*x + 1)/a**3
```

3.44 $\int e^{-2\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=25

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

```
[Out] -2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6126, 77}

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x\,dx&=-\int e^{-2\tanh^{-1}(ax)}x\,dx\\&=-\int\frac{x(1-ax)}{1+ax}\,dx\\&=-\int\left(\frac{2}{a}-x-\frac{2}{a(1+ax)}\right)dx\\&=-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(2*ArcCoth[a*x]),x]
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

fricas [A] time = 0.45, size = 25, normalized size = 1.00

$$\frac{a^2x^2 - 4\,ax + 4\,\log\left(ax + 1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/2*(a^2*x^2 - 4*a*x + 4*log(a*x + 1))/a^2
```

giac [A] time = 0.13, size = 30, normalized size = 1.20

$$\frac{a^2x^2 - 4\,ax}{2\,a^2} + \frac{2\,\log\left(|ax + 1|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/2*(a^2*x^2 - 4*a*x)/a^2 + 2*log(abs(a*x + 1))/a^2
```

maple [A] time = 0.03, size = 24, normalized size = 0.96

$$-\frac{2x}{a} + \frac{x^2}{2} + \frac{2\ln\left(ax + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)*(a*x-1),x)
[Out] -2*x/a+1/2*x^2+2/a^2*ln(a*x+1)
```

maxima [A] time = 0.30, size = 26, normalized size = 1.04

$$\frac{ax^2 - 4\,x}{2\,a} + \frac{2\,\log\left(ax + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/2*(a*x^2 - 4*x)/a + 2*log(a*x + 1)/a^2
```

mupad [B] time = 0.04, size = 23, normalized size = 0.92

$$\frac{2\,\ln\left(a\,x + 1\right)}{a^2} - \frac{2\,x}{a} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x - 1))/(a*x + 1),x)
[Out] (2*log(a*x + 1))/a^2 - (2*x)/a + x^2/2
```

sympy [A] time = 0.08, size = 20, normalized size = 0.80

$$\frac{x^2}{2} - \frac{2x}{a} + \frac{2\log\left(ax + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x)
[Out] x**2/2 - 2*x/a + 2*log(a*x + 1)/a**2
```

3.45 $\int e^{-2\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=13

$$x-\frac{2\log(ax+1)}{a}$$

[Out] x-2*ln(a*x+1)/a

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6125, 43}

$$x-\frac{2\log(ax+1)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^(-2*ArcCoth[a*x]),x]

[Out] x - (2*Log[1 + a*x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6125

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\,dx\\
&= -\int \frac{1-ax}{1+ax}\,dx\\
&= -\int \left(-1+\frac{2}{1+ax}\right)dx\\
&= x-\frac{2\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$x-\frac{2\log(ax+1)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-2*ArcCoth[a*x]),x]

[Out] x - (2*Log[1 + a*x])/a

fricas [A] time = 0.47, size = 16, normalized size = 1.23

$$\frac{ax - 2 \log(ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a*x - 2*log(a*x + 1))/a
```

giac [A] time = 0.13, size = 14, normalized size = 1.08

$$x - \frac{2 \log(|ax + 1|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="giac")
[Out] x - 2*log(abs(a*x + 1))/a
```

maple [A] time = 0.03, size = 14, normalized size = 1.08

$$x - \frac{2 \ln(ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1),x)
[Out] x-2*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 13, normalized size = 1.00

$$x - \frac{2 \log(ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] x - 2*log(a*x + 1)/a
```

mupad [B] time = 0.03, size = 13, normalized size = 1.00

$$x - \frac{2 \ln(a\,x + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(a*x + 1),x)
[Out] x - (2*log(a*x + 1))/a
```

sympy [A] time = 0.07, size = 10, normalized size = 0.77

$$x - \frac{2 \log(ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x)
[Out] x - 2*log(a*x + 1)/a
```

3.46 $\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=13

$$2\log(ax+1)-\log(x)$$

```
[Out] -ln(x)+2*ln(a*x+1)
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 72}

$$2\log(ax+1)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] -Log[x] + 2*Log[1 + a*x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x}\,dx\\
&= -\int \frac{1-ax}{x(1+ax)}\,dx\\
&= -\int \left(\frac{1}{x}-\frac{2a}{1+ax}\right)dx\\
&= -\log(x)+2\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$2\log(ax+1)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] -Log[x] + 2*Log[1 + a*x]
```

fricas [A] time = 0.50, size = 13, normalized size = 1.00

$$2 \log(ax+1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="fricas")
[Out] 2*log(a*x + 1) - log(x)
```

giac [A] time = 0.12, size = 15, normalized size = 1.15

$$2 \log(|ax+1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="giac")
[Out] 2*log(abs(a*x + 1)) - log(abs(x))
```

maple [A] time = 0.04, size = 14, normalized size = 1.08

$$-\ln(x) + 2\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/x,x)
[Out] -ln(x)+2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 13, normalized size = 1.00

$$2 \log(ax+1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="maxima")
[Out] 2*log(a*x + 1) - log(x)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.08

$$2 \ln(-3\,a\,x - 3) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(x*(a*x + 1)),x)
[Out] 2*log(- 3*a*x - 3) - log(x)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$-\log(x) + 2\log\left(x + \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x)
[Out] -log(x) + 2*log(x + 1/a)
```

3.47 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=18

$$2a\log(x) - 2a\log(ax + 1) + \frac{1}{x}$$

[Out] 1/x+2*a*ln(x)-2*a*ln(a*x+1)

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$2a\log(x) - 2a\log(ax + 1) + \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcCoth[a*x])*x^2),x]

[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^2} dx \\
&= -\int \frac{1 - ax}{x^2(1 + ax)} dx \\
&= -\int \left(\frac{1}{x^2} - \frac{2a}{x} + \frac{2a^2}{1 + ax}\right) dx \\
&= \frac{1}{x} + 2a\log(x) - 2a\log(1 + ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$2a\log(x) - 2a\log(ax + 1) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^2),x]
[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

fricas [A] time = 0.52, size = 23, normalized size = 1.28

$$-\frac{2\,ax\log\left(ax+1\right)-2\,ax\log(x)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
[Out] -(2*a*x*log(a*x + 1) - 2*a*x*log(x) - 1)/x
```

giac [A] time = 0.11, size = 20, normalized size = 1.11

$$-2\,a\log\left(|ax+1|\right)+2\,a\log\left(|x|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
[Out] -2*a*log(abs(a*x + 1)) + 2*a*log(abs(x)) + 1/x
```

maple [A] time = 0.04, size = 19, normalized size = 1.06

$$\frac{1}{x}+2a\ln(x)-2a\ln\left(ax+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/x^2,x)
[Out] 1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

maxima [A] time = 0.31, size = 18, normalized size = 1.00

$$-2\,a\log\left(ax+1\right)+2\,a\log(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
[Out] -2*a*log(a*x + 1) + 2*a*log(x) + 1/x
```

mupad [B] time = 1.20, size = 14, normalized size = 0.78

$$\frac{1}{x}-4\,a\,\mathrm{atanh}\left(2\,a\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(x^2*(a*x + 1)),x)
[Out] 1/x - 4*a*atanh(2*a*x + 1)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.83

$$2a\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**2,x)
[Out] 2*a*(log(x) - log(x + 1/a)) + 1/x
```

3.48 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=32

$$-2a^2\log(x)+2a^2\log(ax+1)-\frac{2a}{x}+\frac{1}{2x^2}$$

[Out] 1/2/x^2-2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x+1)

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$-2a^2\log(x)+2a^2\log(ax+1)-\frac{2a}{x}+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcCoth[a*x])*x^3),x]

[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^3}\,dx\\
&= -\int \frac{1-ax}{x^3(1+ax)}\,dx\\
&= -\int \left(\frac{1}{x^3}-\frac{2a}{x^2}+\frac{2a^2}{x}-\frac{2a^3}{1+ax}\right)dx\\
&= \frac{1}{2x^2}-\frac{2a}{x}-2a^2\log(x)+2a^2\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-2a^2\log(x)+2a^2\log(ax+1)-\frac{2a}{x}+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^3),x]
[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

fricas [A] time = 0.50, size = 35, normalized size = 1.09

$$\frac{4\,a^2x^2\log\left(ax+1\right)-4\,a^2x^2\log(x)-4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
[Out] 1/2*(4*a^2*x^2*log(a*x + 1) - 4*a^2*x^2*log(x) - 4*a*x + 1)/x^2
```

giac [A] time = 0.15, size = 32, normalized size = 1.00

$$2\,a^2\log\left(|ax+1|\right)-2\,a^2\log\left(|x|\right)-\frac{4\,ax-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
[Out] 2*a^2*log(abs(a*x + 1)) - 2*a^2*log(abs(x)) - 1/2*(4*a*x - 1)/x^2
```

maple [A] time = 0.04, size = 31, normalized size = 0.97

$$\frac{1}{2x^2}-\frac{2a}{x}-2a^2\ln(x)+2a^2\ln\left(ax+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/x^3,x)
[Out] 1/2/x^2-2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

maxima [A] time = 0.31, size = 30, normalized size = 0.94

$$2\,a^2\log\left(ax+1\right)-2\,a^2\log(x)-\frac{4\,ax-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
[Out] 2*a^2*log(a*x + 1) - 2*a^2*log(x) - 1/2*(4*a*x - 1)/x^2
```

mupad [B] time = 0.04, size = 24, normalized size = 0.75

$$4\,a^2\,\mathrm{atanh}\left(2\,a\,x+1\right)-\frac{2\,a\,x-\frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(x^3*(a*x + 1)),x)
[Out] 4*a^2*atanh(2*a*x + 1) - (2*a*x - 1/2)/x^2
```

sympy [A] time = 0.15, size = 26, normalized size = 0.81

$$2a^2\left(-\log\left(x\right)+\log\left(x+\frac{1}{a}\right)\right)+\frac{-4ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] 2*a**2*(-log(x) + log(x + 1/a)) + (-4*a*x + 1)/(2*x**2)
```

3.49 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=40

$$2a^3\log(x)-2a^3\log(ax+1)+\frac{2a^2}{x}-\frac{a}{x^2}+\frac{1}{3x^3}$$

```
[Out] 1/3/x^3-a/x^2+2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6126, 77}

$$\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(ax+1)-\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^4}\,dx\\
&= -\int \frac{1-ax}{x^4(1+ax)}\,dx\\
&= -\int \left(\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^2}{x^2}-\frac{2a^3}{x}+\frac{2a^4}{1+ax}\right)dx\\
&= \frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1+ax)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$2a^3\log(x)-2a^3\log(ax+1)+\frac{2a^2}{x}-\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

fricas [A] time = 0.67, size = 43, normalized size = 1.08

$$-\frac{6\,a^3x^3\log\left(ax+1\right)-6\,a^3x^3\log\left(x\right)-6\,a^2x^2+3\,ax-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*x^3*log(a*x + 1) - 6*a^3*x^3*log(x) - 6*a^2*x^2 + 3*a*x - 1)/x^
3
```

giac [A] time = 0.15, size = 40, normalized size = 1.00

$$-2\,a^3\log\left(|ax+1|\right)+2\,a^3\log\left(|x|\right)+\frac{6\,a^2x^2-3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(a*x + 1)) + 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/
x^3
```

maple [A] time = 0.04, size = 39, normalized size = 0.98

$$\frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\ln(x)-2a^3\ln\left(ax+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] 1/3/x^3-a/x^2+2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

maxima [A] time = 0.30, size = 38, normalized size = 0.95

$$-2\,a^3\log\left(ax+1\right)+2\,a^3\log\left(x\right)+\frac{6\,a^2x^2-3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] -2*a^3*log(a*x + 1) + 2*a^3*log(x) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/x^3
```

mupad [B] time = 1.21, size = 31, normalized size = 0.78

$$\frac{2\,a^2\,x^2-a\,x+\frac{1}{3}}{x^3}-4\,a^3\,\mathrm{atanh}\left(2\,a\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(x^4*(a*x + 1)),x)
```

```
[Out] (2*a^2*x^2 - a*x + 1/3)/x^3 - 4*a^3*atanh(2*a*x + 1)
```

sympy [A] time = 0.17, size = 34, normalized size = 0.85

$$2a^3\left(\log(x)-\log\left(x+\frac{1}{a}\right)\right)+\frac{6a^2x^2-3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] 2*a**3*(log(x) - log(x + 1/a)) + (6*a**2*x**2 - 3*a*x + 1)/(3*x**3)
```

3.50 $\int e^{-3\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=136

$$\frac{19x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{51\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}-\frac{6x\sqrt{1-\frac{1}{a^2x^2}}}{a^3}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}$$

```
[Out] 51/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4-4*(1-1/a^2/x^2)^(1/2)/a^3/(a+1/x)-6*x
*(1-1/a^2/x^2)^(1/2)/a^3+19/8*x^2*(1-1/a^2/x^2)^(1/2)/a^2-x^3*(1-1/a^2/x^2)
^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6169, 6742, 266, 51, 63, 208, 271, 264, 651}

$$\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{19x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}-\frac{6x\sqrt{1-\frac{1}{a^2x^2}}}{a^3}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}+\frac{51\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^3*(a + x^(-1))) - (6*Sqrt[1 - 1/(a^2*x^2)]*x)
/a^3 + (19*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)
/a + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (51*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8
*a^4)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^5\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{1}{x^5\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^4\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^3\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^3x^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^4x\sqrt{1-\frac{x^2}{a^2}}} -\right.\right.$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4} + \frac{4\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4} + \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,\right.}{a^3}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\,\text{Sub}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{4\tanh}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{6\tanh}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{51\tan}{}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.61

$$\frac{51\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right) + \frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^4x^4-6a^3x^3+11a^2x^2-29ax-80\right)}{ax+1}}{8a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-80 - 29*a*x + 11*a^2*x^2 - 6*a^3*x^3 + 2*a^4*
x^4))/(1 + a*x) + 51*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(8*a^4)
```

fricas [A] time = 0.67, size = 92, normalized size = 0.68

$$\frac{\left(2\,a^4x^4 - 6\,a^3x^3 + 11\,a^2x^2 - 29\,ax - 80\right)\sqrt{\frac{ax-1}{ax+1}} + 51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) - 51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{8\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^4*x^4 - 6*a^3*x^3 + 11*a^2*x^2 - 29*a*x - 80)*sqrt((a*x - 1)/(a*x
 + 1)) + 51*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 539, normalized size = 3.96

$$\frac{\left(2\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^3a^3+4\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^2a^2+21\sqrt{a^2x^2-1}\;\sqrt{a^2}\;x^3a^3-8\sqrt{a^2}\;\left((ax-1)(ax+1)\right)^{\frac{3}{2}}x^2a^2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/8*(2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+4*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
)*x^2*a^2+21*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-8*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(3/2)*x^2*a^2+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+42*(a^2*x^2-1)^(1
/2)*(a^2)^(1/2)*x^2*a^2-21*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*x^2*a^3-16*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-72*((a*x-1)*(a*x+1
))^(1/2)*(a^2)^(1/2)*x^2*a^2+72*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/
2))/(a^2)^(1/2))*x^2*a^3+21*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-42*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+8*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)-144*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+144*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-21*ln((a^2*x+(a^2*x^2-1
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-72*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+
72*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a^4*((a*x
-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.32, size = 223, normalized size = 1.64

$$-\frac{1}{8}a\left(\frac{2\left(77\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-149\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-35\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{51\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{51\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^5}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*a*(2*(77*((a*x - 1)/(a*x + 1))^(7/2) - 149*((a*x - 1)/(a*x + 1))^(5/2)
 + 123*((a*x - 1)/(a*x + 1))^(3/2) - 35*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x
- 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x
 + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 51*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^5 + 51*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5 + 32*sqrt((a*x -
1)/(a*x + 1))/a^5)
```

mupad [B] time = 0.08, size = 192, normalized size = 1.41

$$\frac{51\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}-\frac{\frac{35\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{123\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{149\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}-\frac{77\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}-\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (51*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((35*((a*x - 1)/(a*x + 1)
)^(1/2))/4 - (123*((a*x - 1)/(a*x + 1))^(3/2))/4 + (149*((a*x - 1)/(a*x + 1
))^(5/2))/4 - (77*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1)^2
)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x +
1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (4*((a*x - 1)/(a*x + 1))^(1/2))/a^4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.51 $\int e^{-3\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=116

$$-\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] -11/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+4*(1-1/a^2/x^2)^(1/2)/a^2/(a+1/x)+14
/3*x*(1-1/a^2/x^2)^(1/2)/a^2-3/2*x^2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2
/x^2)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6169, 6742, 271, 264, 266, 51, 63, 208, 651}

$$\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}-\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a + x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x)
/(3*a^2) - (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^3
)/3 - (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^4\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^3\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^2\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^3x\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^3(a+x)\sqrt{1}}\right.\right.$$

$$= \frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx\right.}{a^2}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^2} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{a^3} - \frac{2\,\text{Subs}}{}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\right.}{4a^3}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right.}{a^3}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x}}\right.}{2a^3}$$

Mathematica [A] time = 0.08, size = 75, normalized size = 0.65

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-7a^2x^2+19ax+52\right)}{ax+1} - 33\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(52 + 19*a*x - 7*a^2*x^2 + 2*a^3*x^3))/(1 + a*x
) - 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

fricas [A] time = 0.40, size = 84, normalized size = 0.72

$$\frac{\left(2\,a^3x^3 - 7\,a^2x^2 + 19\,ax + 52\right)\sqrt{\frac{ax-1}{ax+1}} - 33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 33\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 - 7*a^2*x^2 + 19*a*x + 52)*sqrt((a*x - 1)/(a*x + 1)) - 33*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) + 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/
a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\mathit{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 471, normalized size = 4.06

$$-\frac{\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-2\sqrt{a^2}\,\left((ax-1)(ax+1)\right)^{\frac{3}{2}}x^2a^2+18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*(9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-2*(a^2)^(1/2)*((a*x-1)*(a*x+1
))^(3/2)*x^2*a^2+18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-9*ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-4*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x*a-42*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+42*ln((a^2*x+(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+9*(a^2*x^2-1)^(1/2
)*(a^2)^(1/2)*x*a-18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*
x*a^2+10*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-84*((a*x-1)*(a*x+1))^(1/2)*(a^
2)^(1/2)*x*a+84*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*x*a^2-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-42*((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2)+42*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2)))/a^3*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1
))^(1/2)/(a*x-1)
```

maxima [A] time = 0.32, size = 186, normalized size = 1.60

$$-\frac{1}{6}a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-52\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}-\frac{24\sqrt{\frac{ax-1}{ax+1}}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(39*((a*x - 1)/(a*x + 1))^(5/2) - 52*((a*x - 1)/(a*x + 1))^(3/2)
+ 21*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*
a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^4 - 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4 - 24*sqrt
((a*x - 1)/(a*x + 1))/a^4)
```

mupad [B] time = 0.06, size = 156, normalized size = 1.34

$$\frac{7\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{52\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+13\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^3}-\frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (7*((a*x - 1)/(a*x + 1))^(1/2) - (52*((a*x - 1)/(a*x + 1))^(3/2))/3 + 13*((
a*x - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a
*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (4*((a*x - 1)/(a*x
+ 1))^(1/2))/a^3 - (11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.52 $\int e^{-3\coth^{-1}(ax)} x \, dx$

Optimal. Leaf size=90

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-4*(1-1/a^2/x^2)^(1/2)/a/(a+1/x)-3*x*(1
-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6169, 6742, 266, 51, 63, 208, 264, 651}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a + x^(-1))) - (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^3\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}-\frac{3}{ax^2\sqrt{1-\frac{x^2}{a^2}}}+\frac{4}{a^2x\sqrt{1-\frac{x^2}{a^2}}}-\frac{4}{a^2(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}+\frac{4\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,d\right.}{a}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)-\frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\right.}{a^2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+4\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x}}\right.$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2}+\frac{1}{2}\text{Subst}\left(\int\right.$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.73

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(a^2x^2-5ax-14\right)}{ax+1}+9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(3*ArcCoth[a*x]),x]
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 - 5*a*x + a^2*x^2))/(1 + a*x) + 9*Log[(1 +
 Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

fricas [A] time = 0.44, size = 75, normalized size = 0.83

$$\frac{\left(a^2x^2-5\,ax-14\right)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
[Out] 1/2*((a^2*x^2 - 5*a*x - 14)*sqrt((a*x - 1)/(a*x + 1)) + 9*log(sqrt((a*x - 1
)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] undef
```

maple [B] time = 0.05, size = 421, normalized size = 4.68

$$\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-10\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2+2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(3/2),x)
[Out] 1/2*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-10*((a*x-1)*(a*x+1))^(1/2)*(a^2)
^(1/2)*x^2*a^2+2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-ln((a^2*x+(a^2*x^2-1
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+10*ln((a^2*x+((a*x-1)*(a*x+1))^(1
/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+4*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-20*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x
*a-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+20*ln((a^2
*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-10*((a*x-1)*(a*x
+1))^(1/2)*(a^2)^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)
)*a+10*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a^2*(
(a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.31, size = 151, normalized size = 1.68

$$-\frac{1}{2}\,a\left(\frac{2\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}+\frac{8\sqrt{\frac{ax-1}{ax+1}}}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(7*((a*x - 1)/(a*x + 1))^(3/2) - 5*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 9*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^3 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3 + 8
*sqrt((a*x - 1)/(a*x + 1))/a^3)
```

mupad [B] time = 1.22, size = 120, normalized size = 1.33

$$\frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^2}-\frac{5\,\sqrt{\frac{a\,x-1}{a\,x+1}}-7\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - (4*((a*x - 1)/(a*x + 1))^(1/2)
)/a^2 - (5*((a*x - 1)/(a*x + 1))^(1/2) - 7*((a*x - 1)/(a*x + 1))^(3/2))/(a^
2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.53 $\int e^{-3\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -3*arctanh((1-1/a^2/x^2)^(1/2))/a+4*(1-1/a^2/x^2)^(1/2)/(a+1/x)+x*(1-1/a^2/
x^2)^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6168, 6742, 264, 266, 63, 208, 651}

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x - (3*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6168

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \frac{4\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx\right.\\
&= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}\\
&= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x - (3a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.90

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax+5)}{ax+1} - \frac{3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(5 + a*x))/(1 + a*x) - (3*Log[a*(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/a
```

fricas [A] time = 0.46, size = 66, normalized size = 1.10

$$\frac{(ax+5)\sqrt{\frac{ax-1}{ax+1}} - 3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] ((a*x + 5)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.05, size = 248, normalized size = 4.13

$$-\frac{\left(-3\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}\ x^2a^2+3\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+2\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\ -6\sqrt{(ax-1)(a}{a\sqrt{a^2}\ (ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -(-3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+3*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+2*((a*x-1)*(a*x+1))^(3/2)*(a^2)
^(1/2)-6*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+6*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(
1/2)+3*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a
*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

maxima [B] time = 0.32, size = 111, normalized size = 1.85

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + 3*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
- 4*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

mupad [B] time = 0.04, size = 78, normalized size = 1.30

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (4*((a*x -
1)/(a*x + 1))^(1/2))/a - (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.54 $\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=46

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

```
[Out] -arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))-4*a*(1-1/a^2/x^2)^(1/2)/(a+1/x)
```

Rubi [A] time = 0.76, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6169, 6742, 216, 266, 63, 208, 651}

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ArcCsc[a*x] + ArcTanh[Sqrt[1 - 
1/(a^2*x^2)]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
```

```
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}-\frac{4}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}-\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right.\\
&= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\csc^{-1}(ax)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\csc^{-1}(ax)+a^2\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\csc^{-1}(ax)+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 1.20

$$-\frac{4ax\sqrt{1-\frac{1}{a^2x^2}}}{ax+1}+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - ArcSin[1/(a*x)] + Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]
```

fricas [A] time = 0.81, size = 74, normalized size = 1.61

$$-4\sqrt{\frac{ax-1}{ax+1}}+2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -4*sqrt((a*x - 1)/(a*x + 1)) + 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sq
rt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.05, size = 369, normalized size = 8.02

$$\frac{\left(-\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2-a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] (-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)
*x^2*a^2-a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+ln((a^2*x+((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+2*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)-2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-2*(a^2*x^2-1)^(1/2)*
(a^2)^(1/2)*x*a-2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+2*ln((a^2*x+(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(1
/2)*(a^2)^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(
a^2)^(1/2)+a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))*(
(a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

maxima [B] time = 0.42, size = 89, normalized size = 1.93

$$a\left(\frac{2\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}-\frac{4\,\sqrt{\frac{ax-1}{ax+1}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a - 4*sqrt((a*x - 1)/(a*x + 1))/a
)
```

mupad [B] time = 0.03, size = 54, normalized size = 1.17

$$2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-4\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
- 4*((a*x - 1)/(a*x + 1))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x, x)
```

3.55 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=53

$$\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

```
[Out] 3*a*arccsc(a*x)+2*(a-1/x)^2/a/(1-1/a^2/x^2)^(1/2)+3*a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6169, 853, 669, 641, 216}

$$\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] 3*a*Sqrt[1 - 1/(a^2*x^2)] + (2*(a - x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
3*a*ArcCsc[a*x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 853

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p))
/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\,\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= 3a\sqrt{1-\frac{1}{a^2x^2}} + \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= 3a\sqrt{1-\frac{1}{a^2x^2}} + \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3a\csc^{-1}(ax)$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.77

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,(5ax+1)}{ax+1} + 3a\sin^{-1}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^2),x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x))/(1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

fricas [A] time = 0.63, size = 49, normalized size = 0.92

$$-\frac{6\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - (5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
[Out] -(6*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (5*a*x + 1)*sqrt((a*x - 1)/(a*x
 + 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
[Out] undef
```

maple [B] time = 0.06, size = 592, normalized size = 11.17

$$-\frac{\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4 + \sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^3a^3 + \left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2 - 5\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3 - 3a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] -(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
)*x^3*a^3+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-5*(a^2*x^2-1)^(1/2)*(a^2)^(
1/2)*x^3*a^3-3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+ln((a^2*x+(a
^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-ln((a^2*x+((a*x-1)*(a*x+1
))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(3/2)*x*a+2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+2*(a^2*x^2-1)^(3/2)
*(a^2)^(1/2)*x*a-7*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-6*a^2*x^2*(a^2)^(1
/2)*arctan(1/(a^2*x^2-1)^(1/2))+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/
(a^2)^(1/2))*x^2*a^3-2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)
^(1/2))*x^2*a^3+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)-3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-3*a*x*(a^2)^(1/2)*arctan(1/(
a^2*x^2-1)^(1/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a
^2-ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)*((a*x
-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

maxima [A] time = 0.42, size = 72, normalized size = 1.36

$$2\,a\left(2\,\sqrt{\frac{ax-1}{ax+1}}+\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}-3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*(2*sqrt((a*x - 1)/(a*x + 1)) + sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*
x + 1) + 1) - 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

mupad [B] time = 0.05, size = 59, normalized size = 1.11

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}+5\,a\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}-6\,a\,x\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^2,x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2) + 5*a*x*((a*x - 1)/(a*x + 1))^(1/2) - 6*a*x*at
an(((a*x - 1)/(a*x + 1))^(1/2)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**2, x)
```

3.56 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=87

$$-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{2}a^2\csc^{-1}(ax)-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}$$

```
[Out] -a^5*(1-1/a^2/x^2)^(5/2)/(a+1/x)^3-3/2*a^3*(1-1/a^2/x^2)^(3/2)/(a+1/x)-9/2*
a^2*arccsc(a*x)-9/2*a^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6169, 1633, 1593, 12, 793, 665, 216}

$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a + x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a + x^(-1))) - (9*a^2*ArcCsc[a*x
])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(ax-x^2\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - (3a)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{1}{2}(9a)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{1+\frac{x}{a}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{1}{2}(9a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x,\right.$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{9}{2}a^2\csc^{-1}(ax)$$

Mathematica [A] time = 0.15, size = 56, normalized size = 0.64

$$\frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(-14a^2x^2-5ax+1\right)}{x(ax+1)} - 9a\sin^{-1}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x - 14*a^2*x^2))/(x*(1 + a*x)) - 9*a*Ar
cSin[1/(a*x)]))/2
```

fricas [A] time = 0.48, size = 61, normalized size = 0.70

$$\frac{18\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - \left(14\,a^2x^2+5\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(18*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (14*a^2*x^2 + 5*a*x - 1
)*sqrt((a*x - 1)/(a*x + 1)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 642, normalized size = 7.38

$$\frac{\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5+6\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^4a^4+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^3a^3-21\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4-9a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] 1/2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+6*(a^2)^(1/2)*((a*x-1)*(a*x+1
))^(1/2)*x^4*a^4+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-21*(a^2*x^2-1)^(1/
2)*(a^2)^(1/2)*x^4*a^4-9*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+6*
ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-6*ln((a^2*x+(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+4*(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(3/2)*x^2*a^2+12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+
11*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x
^3*a^3-18*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+12*ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-12*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+6*((a*x-1)*(a*x+1))^(1/2)*(a^2)
^(1/2)*x^2*a^2+4*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-9*(a^2*x^2-1)^(1/2)*(a^2
)^(1/2)*x^2*a^2-9*a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+6*ln((a^2
*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-6*ln((a^2*x+((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(
1/2))*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x^2/(a*x-1)/((a*x-1)*(a*x+1))^(1/
2)
```

maxima [A] time = 0.41, size = 112, normalized size = 1.29

$$\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-4\,a\sqrt{\frac{ax-1}{ax+1}}-\frac{7\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+5\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] (9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) - 4*a*sqrt((a*x - 1)/(a*x + 1)) - (7
*a*((a*x - 1)/(a*x + 1))^(3/2) + 5*a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1
)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

mupad [B] time = 0.06, size = 118, normalized size = 1.36

$$9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{5\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+7\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{\frac{{(a\,x-1)}^2}{{(a\,x+1)}^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}-4\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^3,x)
```

```
[Out] 9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (5*a^2*((a*x - 1)/(a*x + 1))^(1/2
) + 7*a^2*((a*x - 1)/(a*x + 1))^(3/2))/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x -
 1))/(a*x + 1) + 1) - 4*a^2*((a*x - 1)/(a*x + 1))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**3, x)
```

3.57 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=96

$$\frac{11}{2}a^3\csc^{-1}(ax)+\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 11/2*a^3*arccsc(a*x)+(a-1/x)^3/(1-1/a^2/x^2)^(1/2)+1/6*a^2*(28*a-3/x)*(1-1/
a^2/x^2)^(1/2)+1/3*a*(3*a-1/x)^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6169, 1633, 1593, 12, 852, 1635, 1654, 780, 216}

$$\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{11}{2}a^3\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a - 3/x))/6 + (a - x^(-1))^3/Sqrt[1 - 1/(a^2
*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a - x^(-1))^2)/3 + (11*a^3*ArcCsc[a*x]
)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(ax^2-x^3\right)}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\left(3a^2-ax\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-5+\frac{3x}{a}\right)\left(3a^2-ax\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right) + \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right) + \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 + \frac{11}{2}a^3\csc^{-1}(ax)$$

Mathematica [A] time = 0.11, size = 66, normalized size = 0.69

$$\frac{1}{6}a\left(33a^2\sin^{-1}\left(\frac{1}{ax}\right) + \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(52a^3x^3+19a^2x^2-7ax+2\right)}{x^2(ax+1)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 - 7*a*x + 19*a^2*x^2 + 52*a^3*x^3))/(x^2*(1 +
 a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

fricas [A] time = 0.50, size = 69, normalized size = 0.72

$$-\frac{66\,a^{3}x^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(52\,a^{3}x^{3}+19\,a^{2}x^{2}-7\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(66*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (52*a^3*x^3 + 19*a^2*x
^2 - 7*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 666, normalized size = 6.94

$$-\frac{\left(-30\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,x^{6}a^{6}+30\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,x^{4}a^{4}-93\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,x^{5}a^{5}-33\arctan\left(\frac{1}{\sqrt{a^{2}x^{2}-1}}\right)\sqrt{a^{2}}\,x^{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] -1/6*(-30*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^6*a^6+30*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2)*x^4*a^4-93*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5-33*arctan(1/(a^2*x^2
-1)^(1/2))*(a^2)^(1/2)*x^5*a^5+30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/
(a^2)^(1/2))*x^5*a^6+30*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-30*ln((
a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+51*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-96*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4-66*a
^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+60*ln((a^2*x+(a^2*x^2-1)^(1/
2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)
*x^3*a^3+60*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-60*ln((a^2*x+((a*x-
1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+14*(a^2*x^2-1)^(3/2)*(a
^2)^(1/2)*x^2*a^2-33*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-33*a^3*x^3*(a^2)
^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/
2))/(a^2)^(1/2))*x^3*a^4+30*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-30*
ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-5*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))*((a*x-1)/(a*x
+1))^(3/2)/(a^2)^(1/2)/x^3/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

maxima [A] time = 0.42, size = 157, normalized size = 1.64

$$-\frac{1}{3}\left(33\,a^{2}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-12\,a^{2}\sqrt{\frac{ax-1}{ax+1}}-\frac{39\,a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+52\,a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\,a^{2}\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^{2}}{(ax+1)^{2}}+\frac{(ax-1)^{3}}{(ax+1)^{3}}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 12*a^2*sqrt((a*x - 1)/(a*x
 + 1)) - (39*a^2*((a*x - 1)/(a*x + 1))^(5/2) + 52*a^2*((a*x - 1)/(a*x + 1))
^(3/2) + 21*a^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x
- 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

mupad [B] time = 1.23, size = 153, normalized size = 1.59

$$\frac{7\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{52\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+13\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1}+4\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^4,x)
```

```
[Out] (7*a^3*((a*x - 1)/(a*x + 1))^(1/2) + (52*a^3*((a*x - 1)/(a*x + 1))^(3/2))/3
 + 13*a^3*((a*x - 1)/(a*x + 1))^(5/2))/((3*(a*x - 1)^2)/(a*x + 1)^2 + (a*x
- 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 4*a^3*((a*x - 1)/(a*x +
 1))^(1/2) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**4, x)
```

3.58 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=133

$$-\frac{51}{8}a^4\csc^{-1}(ax)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)$$

```
[Out] -51/8*a^4*arccsc(a*x)-a*(a-1/x)^3/(1-1/a^2/x^2)^(1/2)-27/4*a^4*(1-1/a^2/x^2
)^(1/2)-9/8*a^3*(2*a-3/x)*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^2/x^2)^(1/2)/x^3
-a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A] time = 0.84, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6169, 1633, 1593, 12, 852, 1635, 1815, 27, 743, 641, 216}

$$-\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{51}{8}a^4\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-27*a^4*Sqrt[1 - 1/(a^2*x^2)])/4 - (9*a^3*Sqrt[1 - 1/(a^2*x^2)]*(2*a - 3/x
))/8 - (a*(a - x^(-1))^3)/Sqrt[1 - 1/(a^2*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)])
/(4*x^3) - (a^2*Sqrt[1 - 1/(a^2*x^2)])/x^2 - (51*a^4*ArcCsc[a*x])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
```

```
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 6169

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\,(ax^3-x^4)}{\left(1+\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x^3\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\left(-3a^3+a^2x-ax^2\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{12a-28x+\frac{27x^2}{a}-\frac{12x^3}{a^2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{12}a^4\,\text{Subst}\left(\int \frac{-\frac{36}{a}+\frac{108x}{a^2}-\frac{81x^2}{a^3}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{12}a^4\,\text{Subst}\left(\int -\frac{9(2a-3x)^2}{a^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} - \frac{1}{4}(3a)\,\text{Subst}\left(\int \frac{(2a-3x)^2}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right) - \frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{8}\left(3a^3\right)\text{Subst}\Bigg(\int$$

$$= -\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}} - \frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right) - \frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}$$

$$= -\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}} - \frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right) - \frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.56

$$-\frac{51}{8}a^4\sin^{-1}\left(\frac{1}{ax}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(80a^4x^4+29a^3x^3-11a^2x^2+6ax-2\right)}{8x^3(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/8*(a*Sqrt[1 - 1/(a^2*x^2)]*(-2 + 6*a*x - 11*a^2*x^2 + 29*a^3*x^3 + 80*a^
4*x^4))/(x^3*(1 + a*x)) - (51*a^4*ArcSin[1/(a*x)])/8
```

fricas [A] time = 0.67, size = 77, normalized size = 0.58

$$\frac{102\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(80\,a^4x^4+29\,a^3x^3-11\,a^2x^2+6\,ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(102*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (80*a^4*x^4 + 29*a^3*x
^3 - 11*a^2*x^2 + 6*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 690, normalized size = 5.19

$$\frac{\left(-56\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^7a^7+56\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^5a^5-163\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^6a^6-51\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}\,x^6a^6 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] 1/8*(-56*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^7*a^7+56*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^5*a^5-163*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^6*a^6-51*arctan(1/(a^2*x^2
-1)^(1/2))*(a^2)^(1/2)*x^6*a^6+56*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/
(a^2)^(1/2))*x^6*a^7+56*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^6*a^6-56*ln((
a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6*a^7+91*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)*x^4*a^4-158*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5-102
*arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)*x^5*a^5+112*ln((a^2*x+(a^2*x^2-1)^
(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+16*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3
/2)*x^4*a^4+112*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-112*ln((a^2*x+(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+22*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^3*a^3-51*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4-51*a^4*x^4*
(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+56*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2))*x^4*a^5+56*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^
4-56*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-7*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+4*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-2
*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x^4/(a*
x-1)/((a*x-1)*(a*x+1))^(1/2)
```

maxima [A] time = 0.42, size = 193, normalized size = 1.45

$$\frac{1}{4}\left(51\,a^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-16\,a^{3}\sqrt{\frac{ax-1}{ax+1}}-\frac{77\,a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+149\,a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\,a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+35\,a^{3}\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1}+\frac{6\,(ax-1)^{2}}{(ax+1)^{2}}+\frac{4\,(ax-1)^{3}}{(ax+1)^{3}}+\frac{(ax-1)^{4}}{(ax+1)^{4}}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(51*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 16*a^3*sqrt((a*x - 1)/(a*x
+ 1)) - (77*a^3*((a*x - 1)/(a*x + 1))^(7/2) + 149*a^3*((a*x - 1)/(a*x + 1))
^(5/2) + 123*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 35*a^3*sqrt((a*x - 1)/(a*x +
 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a
*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + 1))*a
```

mupad [B] time = 1.24, size = 190, normalized size = 1.43

$$\frac{51\,a^{4}\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}-4\,a^{4}\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{\frac{35\,a^{4}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{123\,a^{4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{149\,a^{4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}+\frac{77\,a^{4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{\frac{6\,{\left(a\,x-1\right)}^{2}}{{\left(a\,x+1\right)}^{2}}+\frac{4\,{\left(a\,x-1\right)}^{3}}{{\left(a\,x+1\right)}^{3}}+\frac{{\left(a\,x-1\right)}^{4}}{{\left(a\,x+1\right)}^{4}}+\frac{4\,\left(a\,x-1\right)}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^5,x)
```

```
[Out] (51*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - 4*a^4*((a*x - 1)/(a*x + 1))^
(1/2) - ((35*a^4*((a*x - 1)/(a*x + 1))^(1/2))/4 + (123*a^4*((a*x - 1)/(a*x
+ 1))^(3/2))/4 + (149*a^4*((a*x - 1)/(a*x + 1))^(5/2))/4 + (77*a^4*((a*x -
1)/(a*x + 1))^(7/2))/4)/((6*(a*x - 1)^2)/(a*x + 1)^2 + (4*(a*x - 1)^3)/(a*x
 + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + (4*(a*x - 1))/(a*x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**5, x)
```

3.59 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{31\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{611x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{1920a^4}+\frac{269x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{960a^3}+\frac{11x^3\left(1-\frac{1}{ax}\right)^{3/4}}{48a^2}$$

```
[Out] 611/1920*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4+269/960*(1-1/a/x)^(3/4)*(1+1
/a/x)^(1/4)*x^2/a^3+11/48*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2+9/40*(1-1
/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5+3
1/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+31/128*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A] time = 0.15, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{11x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{48a^2}+\frac{269x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{960a^3}+\frac{611x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{1920a^4}+\frac{31\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\tanh^{-1}\left(\right.}{128}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^4,x]
```

```
[Out] (611*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(1920*a^4) + (269*(1 - 1/(a
*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(3/4)*(1
+ 1/(a*x))^(1/4)*x^3)/(48*a^2) + (9*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^6\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{\frac{9}{2a}+\frac{4x}{a^2}}{x^5\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{55}{4a^2}-\frac{27x}{2a^3}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\right.$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 - \frac{1}{60}\text{Sub}$$

$$= \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\right.$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\right.}{}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\right.}{}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\right.}{}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\right.}{}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\right.}{}$$

Mathematica [A] time = 5.22, size = 173, normalized size = 0.68

$$\frac{\frac{9620e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{34000e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{64640e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{62976e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} + \frac{24576e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^5} - 465\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 4}{3840a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^4,x]
```

```
[Out] ((24576*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (62976*E^(ArcCoth
[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^(ArcCoth[a*x]/2))/(-1 + E^
(2*ArcCoth[a*x]))^3 + (34000*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^
2 + (9620*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 930*ArcTan[E^(Arc
Coth[a*x]/2)] - 465*Log[1 - E^(ArcCoth[a*x]/2)] + 465*Log[1 + E^(ArcCoth[a*
x]/2)])/(3840*a^5)
```

fricas [A] time = 0.53, size = 119, normalized size = 0.47

$$\frac{2\left(384\,a^5x^5+816\,a^4x^4+872\,a^3x^3+978\,a^2x^2+1149\,ax+611\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+465\,\log\left(\left(\right.\right.}{3840\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 + 816*a^4*x^4 + 872*a^3*x^3 + 978*a^2*x^2 + 1149*a*x
 + 611)*((a*x - 1)/(a*x + 1))^(3/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 465*log(((a*x - 1)/(a*x + 1
))^(1/4) - 1))/a^5
```

giac [A] time = 0.27, size = 234, normalized size = 0.92

$$-\frac{1}{3840}\,a\left(\frac{930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{465\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{465\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{1120\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{5}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="giac")
```

```
[Out] -1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^6 + 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^6 - 4*(1120*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5090*(a*x -
 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 696*(a*x - 1)^3*((a*x - 1)/
(a*x + 1))^(3/4)/(a*x + 1)^3 - 465*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/
(a*x + 1)^4 - 2405*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) -
 1)^5))
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

maxima [A] time = 0.42, size = 259, normalized size = 1.02

$$-\frac{1}{3840}\,a\left(\frac{4\left(465\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+2405\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\,\arctan\left(\left(\right.\right.}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(465*((a*x - 1)/(a*x + 1))^(19/4) - 696*((a*x - 1)/(a*x + 1))^
(15/4) + 5090*((a*x - 1)/(a*x + 1))^(11/4) - 1120*((a*x - 1)/(a*x + 1))^(7/
4) + 2405*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
 - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^
6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x - 1)/
(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 465*
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

mupad [B] time = 0.11, size = 229, normalized size = 0.91

$$\frac{\frac{481\left(\frac{ax-1}{ax+1}\right)^{3/4}}{192}-\frac{7\left(\frac{ax-1}{ax+1}\right)^{7/4}}{6}+\frac{509\left(\frac{ax-1}{ax+1}\right)^{11/4}}{96}-\frac{29\left(\frac{ax-1}{ax+1}\right)^{15/4}}{40}+\frac{31\left(\frac{ax-1}{ax+1}\right)^{19/4}}{64}}{a^5+\frac{10a^5(ax-1)^2}{(ax+1)^2}-\frac{10a^5(ax-1)^3}{(ax+1)^3}+\frac{5a^5(ax-1)^4}{(ax+1)^4}-\frac{a^5(ax-1)^5}{(ax+1)^5}-\frac{5a^5(ax-1)}{ax+1}}-\frac{31\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128a^5}+\frac{31\,\mathrm{atanh}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((481*((a*x - 1)/(a*x + 1))^(3/4))/192 - (7*((a*x - 1)/(a*x + 1))^(7/4))/6
+ (509*((a*x - 1)/(a*x + 1))^(11/4))/96 - (29*((a*x - 1)/(a*x + 1))^(15/4))
/40 + (31*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*
x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)
^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (31*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (31*atanh(((a*x - 1)/(a*x + 1))
^(1/4)))/(128*a^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.60 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$\frac{11\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{83x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{192a^3}+\frac{29x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{96a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}$$

```
[Out] 83/192*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+29/96*(1-1/a/x)^(3/4)*(1+1/a/x
)^(1/4)*x^2/a^2+7/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^(3
/4)*(1+1/a/x)^(1/4)*x^4+11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+1
1/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] time = 0.12, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{29x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{96a^2}+\frac{83x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{192a^3}+\frac{11\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^3,x]
```

```
[Out] (83*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(192*a^3) + (29*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(96*a^2) + (7*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
 (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^5\sqrt[4]{1-\frac{x}{a}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{3x}{a^2}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}dx, x, \frac{1}{x}\right)$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{29}{4a^2}-\frac{7x}{a^3}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3}}\right.$$

$$= \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^4 - \frac{1}{24}\text{S}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\right.$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\right.$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\right.$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\right.$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}x^3}{24a} + \frac{1}{4}\left(1-\right.$$

Mathematica [A] time = 5.19, size = 149, normalized size = 0.69

$$\frac{\frac{980e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{2512e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{3200e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} - 33\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 33\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right.}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^3,x]
```

```
[Out] ((1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (3200*E^(ArcCoth[a
*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^(ArcCoth[a*x]/2))/(-1 + E^(2*
ArcCoth[a*x]))^2 + (980*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 66*
ArcTan[E^(ArcCoth[a*x]/2)] - 33*Log[1 - E^(ArcCoth[a*x]/2)] + 33*Log[1 + E^
(ArcCoth[a*x]/2)])/(384*a^4)
```

fricas [A] time = 0.62, size = 111, normalized size = 0.51

$$\frac{2\left(48\,a^4x^4+104\,a^3x^3+114\,a^2x^2+141\,ax+83\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 33\,\text{l}}{384\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 + 104*a^3*x^3 + 114*a^2*x^2 + 141*a*x + 83)*((a*x - 1)
/(a*x + 1))^(3/4) - 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x -
 1)/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

giac [A] time = 0.27, size = 203, normalized size = 0.94

$$-\frac{1}{384}\,a\left(\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{33\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{107\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{279\,(ax-1)^2\left(\right.}{(ax+1)}\right.}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="giac")
```

```
[Out] -1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^5 + 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5
+ 4*(107*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 279*(a*x - 1)^2*
((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 33*(a*x - 1)^3*((a*x - 1)/(a*x +
1))^(3/4)/(a*x + 1)^3 - 245*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a
*x + 1) - 1)^4))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

maxima [A] time = 0.42, size = 224, normalized size = 1.04

$$\frac{1}{384}\,a\left(\frac{4\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-279\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+107\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-245\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/384*a*(4*(33*((a*x - 1)/(a*x + 1))^(15/4) - 279*((a*x - 1)/(a*x + 1))^(11
/4) + 107*((a*x - 1)/(a*x + 1))^(7/4) - 245*((a*x - 1)/(a*x + 1))^(3/4))/(4
*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^
5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 33*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

mupad [B] time = 1.24, size = 192, normalized size = 0.89

$$\frac{\frac{245\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{96}-\frac{107\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{96}+\frac{93\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32}-\frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{32}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}-\frac{11\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}+\frac{11\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((245*((a*x - 1)/(a*x + 1))^(3/4))/96 - (107*((a*x - 1)/(a*x + 1))^(7/4))/9
6 + (93*((a*x - 1)/(a*x + 1))^(11/4))/32 - (11*((a*x - 1)/(a*x + 1))^(15/4)
)/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1
)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (11*at
an(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (11*atanh(((a*x - 1)/(a*x + 1))
^(1/4)))/(64*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.61 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{11x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{5x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}\cdots}}{12a}$$

```
[Out] 11/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2+5/12*(1-1/a/x)^(3/4)*(1+1/a/x)^
(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3+3/8*arctan((1+1/a/x)^(1
/4)/(1-1/a/x)^(1/4))/a^3+3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{11x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{5x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}\cdots}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^2,x]
```

```
[Out] (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) + (5*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
+ (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^4\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{5}{2a}+\frac{2x}{a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{11}{4a^2}-\frac{5x}{2a^3}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,d\right.$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{1}{6}\text{Subst}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{3\,\text{Subst}}{}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{3\,\text{Subst}}{}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{3\,\text{Subst}}{}$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{3\tan^{-1}}{8}$$

Mathematica [C] time = 9.06, size = 399, normalized size = 2.23

$$-\frac{e^{-\frac{7}{2}\coth^{-1}(ax)}\left(1280e^{8\coth^{-1}(ax)}\left(1346e^{2\coth^{-1}(ax)}+557e^{4\coth^{-1}(ax)}+821\right){}_4F_3\left(2,2,2,\frac{9}{4};1,1,\frac{21}{4};e^{2\coth^{-1}(ax)}\right)+1024\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^2,x]
```

```
[Out] -1/1909440*(-1070609085 - 946471617*E^(2*ArcCoth[a*x]) + 369641285*E^(4*Arc
Coth[a*x]) + 351173641*E^(6*ArcCoth[a*x]) - 23818496*E^(8*ArcCoth[a*x]) + 1
070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 732349800*E^
(2*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] - 63506
7810*E^(4*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])]
- 384831720*E^(6*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[
a*x])] + 60913125*E^(8*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*Ar
cCoth[a*x])] + 1280*E^(8*ArcCoth[a*x])*(821 + 1346*E^(2*ArcCoth[a*x]) + 557
*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(2*A
rcCoth[a*x])] + 10240*E^(8*ArcCoth[a*x])*(23 + 42*E^(2*ArcCoth[a*x]) + 19*E
^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 21/4}, E^
(2*ArcCoth[a*x])] + 20480*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2,
 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])] + 40960*E^(10*ArcCoth[a*x
])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth
```

[a*x])] + 20480*E^(12*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((7*ArcCoth[a*x])/2))

fricas [A] time = 0.42, size = 103, normalized size = 0.58

$$\frac{2\left(8a^3x^3+18a^2x^2+21ax+11\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 + 18*a^2*x^2 + 21*a*x + 11)*((a*x - 1)/(a*x + 1))^(3/4)
- 18*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

giac [A] time = 0.24, size = 172, normalized size = 0.96

$$-\frac{1}{48}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{9(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}\cdots\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] -1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^4 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 4
*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 9*(a*x - 1)^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 29*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a
*x - 1)/(a*x + 1) - 1)^3))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

maxima [A] time = 0.42, size = 187, normalized size = 1.04

$$-\frac{1}{48}a\left(\frac{4\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+29\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-}{ax+}\cdots\right.\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(9*((a*x - 1)/(a*x + 1))^(11/4) - 6*((a*x - 1)/(a*x + 1))^(7/4)
+ 29*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^
2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 9*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

mupad [B] time = 0.08, size = 157, normalized size = 0.88

$$\frac{\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((29*((a*x - 1)/(a*x + 1))^(3/4))/12 - ((a*x - 1)/(a*x + 1))^(7/4)/2 + (3*(
(a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a
^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a*x
- 1)/(a*x + 1))^(1/4)))/(8*a^3) + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8
*a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.62 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

```
[Out] 1/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4)
*x^2+1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x,x]
```

```
[Out] ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(3/4)*(1
 + 1/(a*x))^(5/4)*x^2)/2 + ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/
(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{1}{2}\coth^{-1}(ax)}x\,dx &= -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^3\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^2\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\text{Subs}}{}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 66, normalized size = 0.46

$$
\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(5e^{2\coth^{-1}(ax)}-1\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x,x]
```

```
[Out] ((2*E^(ArcCoth[a*x]/2)*(-1 + 5*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x]
))^2 + ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

fricas [A] time = 0.55, size = 93, normalized size = 0.65

$$\frac{2\left(2\,a^{2}x^{2}+5\,ax+3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 + 5*a*x + 3)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x
 - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x
- 1)/(a*x + 1))^(1/4) - 1))/a^2
```

giac [A] time = 0.24, size = 139, normalized size = 0.98

$$-\frac{1}{8}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{3}}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{3}}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^{3}}+\frac{4\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^{3}\left(\frac{ax-1}{ax+1}-1\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="giac")
```

```
[Out] -1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^3 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*((a*x
 - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5*((a*x - 1)/(a*x + 1))^(3/4)
)/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

maxima [A] time = 0.41, size = 149, normalized size = 1.05

$$\frac{1}{8}\,a\left(\frac{4\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2\,(ax-1)a^{3}}{ax+1}-\frac{(ax-1)^{2}a^{3}}{(ax+1)^{2}}-a^{3}}-\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{3}}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{3}}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(((a*x - 1)/(a*x + 1))^(7/4) - 5*((a*x - 1)/(a*x + 1))^(3/4))/(2*(
a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

mupad [B] time = 0.08, size = 120, normalized size = 0.85

$$\frac{\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((5*((a*x - 1)/(a*x + 1))^(3/4))/2 - ((a*x - 1)/(a*x + 1))^(7/4)/2)/(a^2 +
(a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - atan(((a*x -
 1)/(a*x + 1))^(1/4))/(4*a^2) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.63 $\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=96

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
+arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A] time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6170, 94, 93, 212, 206, 203}

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2),x]
```

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^2\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\\
&= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\\
&= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.53

$$\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + ArcTan[E^(ArcCoth[a*x]/
2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

fricas [A] time = 0.58, size = 84, normalized size = 0.88

$$\frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1))/a
```

giac [A] time = 0.21, size = 108, normalized size = 1.12

$$-\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^2 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a*x
 - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

maxima [A] time = 0.41, size = 111, normalized size = 1.16

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) +
1)/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

mupad [B] time = 0.06, size = 78, normalized size = 0.81

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
- 1)/(a*x + 1))^(1/4))/a + atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-1/4), x)
```

3.64 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{\frac{1}{ax}}}\right.$$

[Out] 2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)

Rubi [A] time = 0.25, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6171, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{\frac{1}{ax}}}\right.$$

Antiderivative was successfully verified.

[In] Int[E^(ArcCoth[a*x]/2)/x,x]

[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx &= -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\,\text{Subst}\Bigg(\int \frac{\ }{1\ }\\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\,\text{Subs}\\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\text{Subst}\Big(}{}\\
&= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\Big(1+\frac{\sqrt{1-}}{\sqrt{1+}}}{}\\
&= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{ta}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 30, normalized size = 0.10

$$\frac{8}{5}e^{\frac{5}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{5}{8},1;\frac{13}{8};e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x,x]
```

```
[Out] (8*E^((5*ArcCoth[a*x])/2)*Hypergeometric2F1[5/8, 1, 13/8, E^(4*ArcCoth[a*x]
)])/5
```

fricas [A] time = 0.70, size = 291, normalized size = 1.00

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(
((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

giac [A] time = 0.21, size = 232, normalized size = 0.80

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

maxima [A] time = 0.42, size = 224, normalized size = 0.77

$$\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a)
```

mupad [B] time = 0.08, size = 101, normalized size = 0.35

$$-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(1-\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 - 1i) - 2
*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2
i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

3.65 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=267

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}$$

```
[Out] a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(
1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1
/4))*2^(1/2)+1/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(
1/2)/(1+1/a/x)^(1/2))*2^(1/2)-1/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6171, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (a*ArcTan[1 - (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 - 1/
(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*Log[1 + Sqrt[1 - 1/(a*x)]/
Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*
Sqrt[2]) - (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (2a)\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (2a)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - a\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + a\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{}{1+\sqrt{}}\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} +$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{2}$$

Mathematica [A] time = 0.26, size = 148, normalized size = 0.55

$$a\left(\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} + \frac{\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} - \frac{\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1-\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^2,x]
```

```
[Out] a*((2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + ArcTan[1 - Sqrt[2]*E^(
ArcCoth[a*x]/2)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)]/Sqrt[2] +
 Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]) - Log[1 +
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]))
```

fricas [A] time = 0.47, size = 396, normalized size = 1.48

$$-\frac{4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^4}\,a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^4\right)^{\frac{1}{4}}}{a^4}\right)+4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*a^3*((a*x
- 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a
^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4))
/a^4) + 4*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(1/4)*a^3*((a*x
 - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(
a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4)
)/a^4) + sqrt(2)*(a^4)^(1/4)*x*log(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^4
)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - sqrt(2)*(a^4
)^(1/4)*x*log(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^4)*a^4 - sqrt(2)*(a^4)
^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))
^(3/4))/x
```

giac [A] time = 0.19, size = 186, normalized size = 0.70

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax}{ax}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

maxima [A] time = 0.41, size = 186, normalized size = 0.70

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax}{ax}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

mupad [B] time = 1.19, size = 87, normalized size = 0.33

$$(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-(-1)^{1/4}\,a\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)+\frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (-1)^(1/4)*a*at
anh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) + (2*a*((a*x - 1)/(a*x + 1))^(3
/4))/((a*x - 1)/(a*x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

3.66 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}}{}\right.}{8\sqrt{2}}$$

```
[Out] 1/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(
5/4)+1/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)-1/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}}{}\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(3/4)*
(1 + 1/(a*x))^(5/4))/2 - (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{4}a\,\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{8}a\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/}}\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{8}a^2\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,d\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}}$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{a^2\,\text{t}}{}$$

Mathematica [A] time = 0.23, size = 173, normalized size = 0.54

$$\frac{1}{16}a^2\left(\frac{40e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} - \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}+1\right)^2} + \sqrt{2}\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right) - \sqrt{2}\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^3,x]
```

```
[Out] (a^2*((-32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (40*E^(ArcCoth[
a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCoth
[a*x]/2)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[
1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] - Sqrt[2]*Log[1 + Sqrt[2]*
E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

fricas [A] time = 0.64, size = 413, normalized size = 1.29

$$-\frac{4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - \sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}\,a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right) + 4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(1/4)*a^6*((a
*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1)) + sq
rt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8)^(1
/4))/a^8) + 4*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(1/4)*a^6
*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1))
+ sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8
)^(1/4))/a^8) + sqrt(2)*(a^8)^(1/4)*x^2*log(a^12*sqrt((a*x - 1)/(a*x + 1))
+ sqrt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - sq
rt(2)*(a^8)^(1/4)*x^2*log(a^12*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^8)*a^8 -
sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(3*a^2*x^2 + 5*a*x
 + 2)*((a*x - 1)/(a*x + 1))^(3/4))/x^2
```

giac [A] time = 0.19, size = 223, normalized size = 0.70

$$\frac{1}{16}\left(2\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{a}{a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a
*x + 1) + 5*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

maxima [A] time = 0.41, size = 226, normalized size = 0.71

$$\frac{1}{16}\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax}{ax}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1))*a + 8*(a*((a*x - 1)/(a*x + 1))^(7/4) + 5*a*((a*x - 1)
/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

mupad [B] time = 0.07, size = 132, normalized size = 0.41

$$\frac{\frac{5\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{2}+\frac{a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] ((5*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (a^2*((a*x - 1)/(a*x + 1))^(7/4))/
2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*a^
2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - ((-1)^(1/4)*a^2*atanh((
-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \sqrt[4]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

3.67 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}}{}\right.}{16\sqrt{2}}$$

```
[Out] 3/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/12*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^
(5/4)+1/3*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4)/x+3/16*a^3*arctan(-1+(1-1/a/x
)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+(1-1/a/x)^(1/4)*
2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1
/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-3/32*a^3*ln(1+(1-1/a/x
)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}+\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^4,x]
```

```
[Out] (3*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 + (a^3*(1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(5/4))/12 + (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/(
3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x
))^(1/4)])/(8*Sqrt[2]) + (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (3*a^
3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{\left(-1-\frac{x}{2a}\right)\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x,\right.$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{16}\left(3\right.$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{4}\left(3a\right.$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{4}\left(3a\right.$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{8}\left(3a\right.$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{16}\left(3\right.$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{3a^3\,1}{}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{3a^3\,\mathrm{t}}{}$$

Mathematica [C] time = 0.11, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(9\text{RootSum}\left[\#1^4+1\&, \frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right] + \frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(6e^{2\coth^{-1}(ax)}+29e^{4\coth^{-1}(a}\right.}{\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^4,x]
```

```
[Out] (a^3*((8*E^(ArcCoth[a*x]/2)*(9 + 6*E^(2*ArcCoth[a*x]) + 29*E^(4*ArcCoth[a*x
])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x] - 2*
Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ]))/96
```

fricas [A] time = 0.42, size = 427, normalized size = 1.20

$$-\frac{36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(36*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(1/4)*a^9
*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(a*x + 1))
+ sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4))*(
a^12)^(1/4))/a^12) + 36*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^
12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^18*sqrt((a*x - 1
)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x +
 1))^(1/4))*(a^12)^(1/4))/a^12) + 9*sqrt(2)*(a^12)^(1/4)*x^3*log(729*a^18*s
qrt((a*x - 1)/(a*x + 1)) + 729*sqrt(a^12)*a^12 + 729*sqrt(2)*(a^12)^(3/4)*a
^9*((a*x - 1)/(a*x + 1))^(1/4)) - 9*sqrt(2)*(a^12)^(1/4)*x^3*log(729*a^18*s
qrt((a*x - 1)/(a*x + 1)) + 729*sqrt(a^12)*a^12 - 729*sqrt(2)*(a^12)^(3/4)*a
^9*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(11*a^3*x^3 + 21*a^2*x^2 + 18*a*x + 8)*
((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

giac [A] time = 0.22, size = 271, normalized size = 0.76

$$\frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-9\sqrt{2}a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) - 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(6*(a*x - 1)*a^2*((a*x - 1)/(
a*x + 1))^(3/4)/(a*x + 1) + 9*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(
a*x + 1)^2 + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^
3)*a
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

maxima [A] time = 0.41, size = 270, normalized size = 0.76

$$\frac{1}{96}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{a}{a}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))*a^2 + 8*(9*a^2*((a*x - 1)/(a*x + 1))^(11/4) + 6*a^2
*((a*x - 1)/(a*x + 1))^(7/4) + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x
- 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*
a
```

mupad [B] time = 1.21, size = 168, normalized size = 0.47

$$\frac{\frac{29\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}+\frac{a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1}+\frac{3\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}-\frac{3\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] ((29*a^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (a^3*((a*x - 1)/(a*x + 1))^(7/4)
)/2 + (3*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^2
+ (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + (3*(-1)^(1/4)*a^
3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (3*(-1)^(1/4)*a^3*atanh
((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

3.68 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$-\frac{237\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{557x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{640a^4}+\frac{157x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{320a^3}+\frac{5x^3\sqrt[4]{1-}}{}$$

```
[Out] 557/640*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4+157/320*(1-1/a/x)^(1/4)*(1+1/
a/x)^(3/4)*x^2/a^3+5/16*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2+11/40*(1-1/
a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^5-23
7/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+237/128*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A] time = 0.14, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = $\{6171, 99, 151, 12, 93, 298, 203, 206\}$

$$\frac{5x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{16a^2}+\frac{157x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{320a^3}+\frac{557x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{640a^4}-\frac{237\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\tanh}{1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (557*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(640*a^4) + (157*(1 - 1/(a*
x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(1/4)*(1 +
1/(a*x))^(3/4)*x^3)/(16*a^2) + (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*
x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 - (237*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (237*ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^6\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{\frac{11}{2a}+\frac{4x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{75}{4a^2}-\frac{33x}{2a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1-}}\right.$$

$$= \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{60}\text{S}$$

$$= \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11}{}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11}{}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11}{}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11}{}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11}{}$$

Mathematica [A] time = 5.26, size = 173, normalized size = 0.68

$$\frac{\frac{5500e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{14032e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{23936e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{22016e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} + \frac{8192e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^5} - 1185\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{1280a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] ((8192*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (22016*E^((3*A
rcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((3*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^3 + (14032*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*
ArcCoth[a*x]))^2 + (5500*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))
- 2370*ArcTan[E^(ArcCoth[a*x]/2)] - 1185*Log[1 - E^(ArcCoth[a*x]/2)] + 1185
*Log[1 + E^(ArcCoth[a*x]/2)])/(1280*a^5)
```

fricas [A] time = 0.64, size = 119, normalized size = 0.47

$$\frac{2\left(128\,a^5x^5+304\,a^4x^4+376\,a^3x^3+514\,a^2x^2+871\,ax+557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1185\,\log\left(\left(\frac{ax}{ax}\right.\right.}{1280\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(2*(128*a^5*x^5 + 304*a^4*x^4 + 376*a^3*x^3 + 514*a^2*x^2 + 871*a*x
+ 557)*((a*x - 1)/(a*x + 1))^(1/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1185*log(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a^5
```

giac [A] time = 0.40, size = 234, normalized size = 0.92

$$\frac{1}{1280}\,a\left(\frac{2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{1185\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{4\left(\frac{1992\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{37}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="giac")
```

```
[Out] 1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 - 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 + 4*(1992*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 3710*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1440*(a*x - 1)^3*((a*x -
1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 395*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/
4)/(a*x + 1)^4 - 1375*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1
) - 1)^5))
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

maxima [A] time = 0.40, size = 259, normalized size = 1.02

$$-\frac{1}{1280}\,a\left(\frac{4\left(395\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+1375\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\,\arctan\left(\left(\frac{ax}{ax}\right.\right.}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/1280*a*(4*(395*((a*x - 1)/(a*x + 1))^(17/4) - 1440*((a*x - 1)/(a*x + 1))
^(13/4) + 3710*((a*x - 1)/(a*x + 1))^(9/4) - 1992*((a*x - 1)/(a*x + 1))^(5/
4) + 1375*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
 - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^
6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x - 1)
/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 11
85*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

mupad [B] time = 0.09, size = 229, normalized size = 0.91

$$\frac{\frac{275\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{249\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{371\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{32}-\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{2}+\frac{79\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{64}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}+\frac{237\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+\frac{237\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1}\right.}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((275*((a*x - 1)/(a*x + 1))^(1/4))/64 - (249*((a*x - 1)/(a*x + 1))^(5/4))/4
0 + (371*((a*x - 1)/(a*x + 1))^(9/4))/32 - (9*((a*x - 1)/(a*x + 1))^(13/4))
/2 + (79*((a*x - 1)/(a*x + 1))^(17/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*x
 + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)^
4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (237*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (237*atanh(((a*x - 1)/(a*x + 1)
)^(1/4)))/(128*a^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.69 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$-\frac{123\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{63x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{64a^3}+\frac{15x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{32a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\,($$

```
[Out] 63/64*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+15/32*(1-1/a/x)^(1/4)*(1+1/a/x)
^(3/4)*x^2/a^2+3/8*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)^(1/4
)*(1+1/a/x)^(3/4)*x^4-123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+12
3/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] time = 0.12, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{15x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{32a^2}+\frac{63x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{64a^3}-\frac{123\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (63*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(64*a^3) + (15*(1 - 1/(a*x))
^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(32*a^2) + (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a
*x))^(3/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - (
123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^5\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\frac{9}{2a}+\frac{3x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{45}{4a^2}-\frac{9x}{a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\right.$$

$$= \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{24}\text{Sub}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-}$$

Mathematica [A] time = 5.20, size = 149, normalized size = 0.69

$$\frac{\frac{532e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{1008e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{1152e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{512e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} - 123\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 123\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right.}{128a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] ((512*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (1152*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((3*ArcCoth[a*x])/2))/
(-1 + E^(2*ArcCoth[a*x]))^2 + (532*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCo
th[a*x])) - 246*ArcTan[E^(ArcCoth[a*x]/2)] - 123*Log[1 - E^(ArcCoth[a*x]/2)
] + 123*Log[1 + E^(ArcCoth[a*x]/2)])/(128*a^4)
```

fricas [A] time = 0.65, size = 111, normalized size = 0.51

$$\frac{2\left(16a^4x^4+40a^3x^3+54a^2x^2+93ax+63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 123\log}{128a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="fricas")
```

```
[Out] 1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 54*a^2*x^2 + 93*a*x + 63)*((a*x - 1)/(a
*x + 1))^(1/4) + 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x -
1)/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

giac [A] time = 0.39, size = 203, normalized size = 0.94

$$\frac{1}{128}a\left(\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{123\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{147(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{183(a\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="giac")
```

```
[Out] 1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^5 - 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
5 - 4*(147*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 183*(a*x - 1)^
2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 41*(a*x - 1)^3*((a*x - 1)/(a*x
+ 1))^(1/4)/(a*x + 1)^3 - 133*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/
(a*x + 1) - 1)^4))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

maxima [A] time = 0.41, size = 224, normalized size = 1.04

$$\frac{1}{128}a\left(\frac{4\left(41\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-183\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+147\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-133\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right.\right.}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/128*a*(4*(41*((a*x - 1)/(a*x + 1))^(13/4) - 183*((a*x - 1)/(a*x + 1))^(9/
4) + 147*((a*x - 1)/(a*x + 1))^(5/4) - 133*((a*x - 1)/(a*x + 1))^(1/4))/(4*
(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5
/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 123*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

mupad [B] time = 1.21, size = 192, normalized size = 0.89

$$\frac{\frac{133\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{147\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{183\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{32}-\frac{41\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{32}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}+\frac{123\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}+\frac{123\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((133*((a*x - 1)/(a*x + 1))^(1/4))/32 - (147*((a*x - 1)/(a*x + 1))^(5/4))/3
2 + (183*((a*x - 1)/(a*x + 1))^(9/4))/32 - (41*((a*x - 1)/(a*x + 1))^(13/4)
)/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1
)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (123*a
tan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (123*atanh(((a*x - 1)/(a*x + 1
))^(1/4)))/(64*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.70 $\int e^{\frac{3}{2}\coth^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=179

$$-\frac{17\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{23x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{7x^2\sqrt[4]{1-\frac{1}{ax}}\left(\right.}{12a}$$

```
[Out] 23/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2+7/12*(1-1/a/x)^(1/4)*(1+1/a/x)^
(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3-17/8*arctan((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^3+17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{23x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}-\frac{17\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{7x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (23*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) + (7*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 + (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^4\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{2x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{23}{4a^2}-\frac{7x}{2a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\right.$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{6}\text{Su}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\,\text{Su}}{}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\,\text{Su}}{}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{17\,\text{Su}}{}$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{17\,\text{ta}}{}$$

Mathematica [A] time = 5.18, size = 125, normalized size = 0.70

$$\frac{\frac{180e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{240e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{128e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} - 51\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 51\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}+1\right) - 102\tan^{-1}}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] ((128*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (240*E^((3*ArcC
oth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (180*E^((3*ArcCoth[a*x])/2))/(-
1 + E^(2*ArcCoth[a*x])) - 102*ArcTan[E^(ArcCoth[a*x]/2)] - 51*Log[1 - E^(Ar
cCoth[a*x]/2)] + 51*Log[1 + E^(ArcCoth[a*x]/2)])/(48*a^3)
```

fricas [A] time = 0.56, size = 103, normalized size = 0.58

$$\frac{2\left(8\,a^3x^3+22\,a^2x^2+37\,ax+23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 37*a*x + 23)*((a*x - 1)/(a*x + 1))^(1/4)
+ 102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 51*log(((a*x - 1)/(a*x + 1))^(1
/4) + 1) - 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

giac [A] time = 0.33, size = 172, normalized size = 0.96

$$\frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{17(ax-1)^2\left(\frac{ax-1}{ax+1}\right.}{(ax+1)^2}\right.}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="giac")
```

```
[Out] 1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 - 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 +
 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 17*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 45*((a*x - 1)/(a*x + 1))^(1/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

maxima [A] time = 0.41, size = 187, normalized size = 1.04

$$-\frac{1}{48}a\left(\frac{4\left(17\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+45\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{51\log\left(\left(\frac{a}{a}\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(17*((a*x - 1)/(a*x + 1))^(9/4) - 30*((a*x - 1)/(a*x + 1))^(5/4)
 + 45*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)
^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 +
51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

mupad [B] time = 0.08, size = 157, normalized size = 0.88

$$\frac{\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{17\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{17\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((15*((a*x - 1)/(a*x + 1))^(1/4))/4 - (5*((a*x - 1)/(a*x + 1))^(5/4))/2 + (
17*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2
- (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*atan((
(a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) + (17*atanh(((a*x - 1)/(a*x + 1))^(1/4
)))/(8*a^3)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.71 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$-\frac{9\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

```
[Out] 3/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)
*x^2-9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 298, 203, 206}

$$-\frac{9\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x,x]
```

```
[Out] (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(4*a) + ((1 - 1/(a*x))^(1/4)*
(1 + 1/(a*x))^(7/4)*x^2)/2 - (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1
/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2
)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{3\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 + \frac{9\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} - 9$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right)}{4a^2}$$

Mathematica [A] time = 0.19, size = 70, normalized size = 0.49

$$\frac{\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(7e^{2\coth^{-1}(ax)}-3\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} - 9\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 9\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x,x]
```

```
[Out] ((2*E^((3*ArcCoth[a*x])/2)*(-3 + 7*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[
a*x]))^2 - 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4
*a^2)
```

fricas [A] time = 0.55, size = 95, normalized size = 0.67

$$\frac{2\left(2a^2x^2+7ax+5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 + 7*a*x + 5)*((a*x - 1)/(a*x + 1))^(1/4) + 18*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log((
(a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

giac [A] time = 0.29, size = 141, normalized size = 0.99

$$\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{3(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="giac")
```

```
[Out] 1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1)/a^3 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4*(
3*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 7*((a*x - 1)/(a*x + 1))
^(1/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

maxima [A] time = 0.40, size = 152, normalized size = 1.07

$$\frac{1}{8}a\left(\frac{4\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(3*((a*x - 1)/(a*x + 1))^(5/4) - 7*((a*x - 1)/(a*x + 1))^(1/4))/(2
*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan((
(a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^
3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

mupad [B] time = 1.21, size = 120, normalized size = 0.85

$$\frac{\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}+\frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((7*((a*x - 1)/(a*x + 1))^(1/4))/2 - (3*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^
2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + (9*atan(
((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) + (9*atanh(((a*x - 1)/(a*x + 1))^(1/4
)))/(4*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.72 $\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
/a+3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6170, 94, 93, 298, 203, 206}

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a + (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx &= -\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{3\operatorname{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{6\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\\
&= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{3\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\\
&= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{3\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 56, normalized size = 0.57

$$\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(\left(e^{2\coth^{-1}(ax)}-1\right){}_2F_1\left(\frac{3}{4},2;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)+1\right)}{a\left(e^{2\coth^{-1}(ax)}-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (8*E^((3*ArcCoth[a*x])/2)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[
3/4, 2, 7/4, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

fricas [A] time = 0.64, size = 86, normalized size = 0.88

$$\frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) + 6*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 3*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a
```

giac [A] time = 0.24, size = 109, normalized size = 1.11

$$\frac{1}{2}\,a\left(\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 - 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*((
a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

maxima [A] time = 0.40, size = 112, normalized size = 1.14

$$-\frac{1}{2}\,a\left(\frac{4\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

mupad [B] time = 1.19, size = 79, normalized size = 0.81

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((a
*x - 1)/(a*x + 1))^(1/4)))/a + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-3/4), x)
```

3.73 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}$$

```
[Out] -2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)
/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*
2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6171, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x,x]
```

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\,\text{Subst}\left(\int \frac{1}{1+x}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\,\text{Subst}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{a}}}{\sqrt{1+\frac{1}{a}}}\right.}{\sqrt{}}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh$$

Mathematica [C] time = 0.05, size = 30, normalized size = 0.10

$$\frac{8}{7}e^{\frac{7}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x,x]
```

```
[Out] (8*E^((7*ArcCoth[a*x])/2)*Hypergeometric2F1[7/8, 1, 15/8, E^(4*ArcCoth[a*x]
)])/7
```

fricas [A] time = 0.56, size = 291, normalized size = 1.00

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax}{ax}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(
((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

giac [A] time = 0.25, size = 232, normalized size = 0.80

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

maxima [A] time = 0.40, size = 224, normalized size = 0.77

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a)
```

mupad [B] time = 0.05, size = 101, normalized size = 0.35

$$2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)2\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(1+1\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x}{a\,x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

3.74 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=268

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}}$$

```
[Out] a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+3/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(
1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1
/4))*2^(1/2)-3/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(
1/2)/(1+1/a/x)^(1/2))*2^(1/2)+3/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6171, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{\frac{1}{ax}}}\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) - (3*a*ArcTan[1 - (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (3*a*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(2*Sqrt[2]) + (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2
]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (6a)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (6a)\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (3a)\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + (3a)\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,d\right.$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}(3a)\operatorname{Subst}\left(\right.$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.}{2\sqrt{2}}$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{3a\log\left(1\right.}{}$$

Mathematica [C] time = 0.08, size = 46, normalized size = 0.17

$$-8ae^{\frac{3}{2}\coth^{-1}(ax)}\left({}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right) - \frac{1}{e^{2\coth^{-1}(ax)}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] -8*a*E^((3*ArcCoth[a*x])/2)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometri
c2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])])
```

fricas [A] time = 0.46, size = 376, normalized size = 1.40

$$-\frac{12\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}\sqrt{a^2\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}}}{a^4}\right)+12\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4\ldots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1
)) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
```

```
12*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a
*x + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - s
qrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - 3*sqr
t(2)*(a^4)^(1/4)*x*log(9*a^2*sqrt((a*x - 1)/(a*x + 1)) + 9*sqrt(2)*(a^4)^(1
/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt(a^4)) + 3*sqrt(2)*(a^4)^(1/4)*x*
log(9*a^2*sqrt((a*x - 1)/(a*x + 1)) - 9*sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a
*x + 1))^(1/4) + 9*sqrt(a^4)) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

giac [A] time = 0.22, size = 187, normalized size = 0.70

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax-}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

maxima [A] time = 0.40, size = 187, normalized size = 0.70

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax-}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

mupad [B] time = 0.08, size = 88, normalized size = 0.33

$$\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] (2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1) - (-1)^(1/4)*a*
atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i - (-1)^(1/4)*a*atan((-1)^(
1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

3.75 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1\cdots}}{\sqrt[4]{\frac{1}{ax}\cdots}}\right.}{8\sqrt{2}}$$

```
[Out] 3/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(
7/4)+9/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+9/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)+9/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1\cdots}}{\sqrt[4]{\frac{1}{ax}\cdots}}\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (3*a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(7/4))/2 - (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\mathrm{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{1}{4}(3a)\,\mathrm{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{1}{8}(9a)\,\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,d\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{2}\left(9a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{2}\left(9a^2\right)\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1-}}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{4}\left(9a^2\right)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1-}}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{8}\left(9a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} + $$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} + \frac{9a^2\tan}{}$$

Mathematica [C] time = 0.09, size = 76, normalized size = 0.24

$$-\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)-3\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)+2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCot
h[a*x])] - 3*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])] + 2*Hyperg
eometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

fricas [A] time = 0.50, size = 405, normalized size = 1.27

$$-\frac{36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")

[Out] -1/16*(36*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*x + 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))/a^8) + 36*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))/a^8) - 9*sqrt(2)*(a^8)^(1/4)*x^2*log(81*a^4*sqrt((a*x - 1)/(a*x + 1)) + 81*sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 81*sqrt(a^8)) + 9*sqrt(2)*(a^8)^(1/4)*x^2*log(81*a^4*sqrt((a*x - 1)/(a*x + 1)) - 81*sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 81*sqrt(a^8)) - 4*(5*a^2*x^2 + 7*a*x + 2)*((a*x - 1)/(a*x + 1))^(1/4))/x^2

giac [A] time = 0.25, size = 225, normalized size = 0.71

$$\frac{1}{16}\left(18\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}\,a\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")

[Out] 1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 7*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)

[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)

maxima [A] time = 0.41, size = 229, normalized size = 0.72

$$\frac{1}{16}\left(18\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}\,a\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")

[Out] 1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))

```
^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*a*((a*x - 1)/(a*x + 1))^(5/4) + 7*
a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x +
1)^2 + 1))*a
```

mupad [B] time = 0.09, size = 132, normalized size = 0.41

$$\frac{\frac{7a^2\left(\frac{ax-1}{ax+1}\right)^{1/4}}{2}+\frac{3a^2\left(\frac{ax-1}{ax+1}\right)^{5/4}}{2}}{\frac{(ax-1)^2}{(ax+1)^2}+\frac{2(ax-1)}{ax+1}+1}-\frac{(-1)^{1/4}a^2\operatorname{atan}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)9i}{4}-\frac{(-1)^{1/4}a^2\operatorname{atanh}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)9i}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] ((7*a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (3*a^2*((a*x - 1)/(a*x + 1))^(5/4)
)/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*
a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4 - ((-1)^(1/4)*a^2*at
anh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

3.76 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\right.}{16\sqrt{2}}$$

```
[Out] 17/24*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/4*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)
^(7/4)+1/3*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)/x+17/16*a^3*arctan(-1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+(1-1/a/x)^(1/
4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-17/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+17/32*a^3*ln(1+(1-
1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/
2)
```

Rubi [A] time = 0.28, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}-\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (17*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/24 + (a^3*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(7/4))/4 + (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/
(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(
a*x))^(1/4)])/(8*Sqrt[2]) - (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a
*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (
17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))
^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{\left(-1-\frac{3x}{2a}\right)\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{1}{24}\left(17a^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x\right.$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{1}{16}\left(17\right.$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{4}\left(17a\right.$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{4}\left(17a\right.$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{8}\left(17a\right.$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{16}\left(17\right.$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{17a^3\,\mathrm{lo}}{}$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{17a^3\,\mathrm{ta}}{}$$

Mathematica [C] time = 0.13, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(51\text{RootSum}\left[\#1^4+1\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]+\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(30e^{2\coth^{-1}(ax)}+45e^{4\coth^{-1}}\right.}{\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (a^3*((8*E^((3*ArcCoth[a*x])/2)*(17 + 30*E^(2*ArcCoth[a*x]) + 45*E^(4*ArcCo
th[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*
x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1 & ]))/96
```

fricas [A] time = 0.56, size = 413, normalized size = 1.16

$$-\frac{204\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+204\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^
3*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1
)/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
a^12)))/a^12) + 204*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^
(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt(
(a*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt(a^12)))/a^12) - 51*sqrt(2)*(a^12)^(1/4)*x^3*log(289*a^6*sqrt((a*x -
 1)/(a*x + 1)) + 289*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) +
 289*sqrt(a^12)) + 51*sqrt(2)*(a^12)^(1/4)*x^3*log(289*a^6*sqrt((a*x - 1)/(
a*x + 1)) - 289*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 289*
sqrt(a^12)) - 4*(23*a^3*x^3 + 37*a^2*x^2 + 22*a*x + 8)*((a*x - 1)/(a*x + 1)
)^(1/4))/x^3
```

giac [A] time = 0.28, size = 271, normalized size = 0.76

$$\frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+51\sqrt{2}a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(30*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(1/4)/(a*x + 1) + 17*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
1/4)/(a*x + 1)^2 + 45*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1)
 + 1)^3)*a
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

maxima [A] time = 0.41, size = 277, normalized size = 0.78

$$\frac{1}{96}\left(102\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+51\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(17*a^2*((a*x - 1)/(a*x +
 1))^(9/4) + 30*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 45*a^2*((a*x - 1)/(a*x +
1))^(1/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3
/(a*x + 1)^3 + 1))*a
```

mupad [B] time = 1.21, size = 168, normalized size = 0.47

$$\frac{\frac{15\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}+\frac{5\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1}-\frac{(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,17\mathrm{i}}{8}-\frac{(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a}{a}\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] ((15*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (5*a^3*((a*x - 1)/(a*x + 1))^(5/4
))/2 + (17*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1)^
2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^
3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8 - ((-1)^(1/4)*a^3*ata
nh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

3.77 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=287

$$-\frac{26111\sqrt[4]{\frac{1}{ax}+1}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{1003\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{5533x\sqrt[4]{\frac{1}{ax}+1}}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189x^2\sqrt[4]{\frac{1}{ax}+1}}{960a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{181x^3\sqrt[4]{\frac{1}{ax}+1}}{240a^2\sqrt[4]{1-\frac{1}{ax}}}$$

```
[Out] -26111/1920*(1+1/a/x)^(1/4)/a^5/(1-1/a/x)^(1/4)+5533/1920*(1+1/a/x)^(1/4)*x
/a^4/(1-1/a/x)^(1/4)+1189/960*(1+1/a/x)^(1/4)*x^2/a^3/(1-1/a/x)^(1/4)+181/2
40*(1+1/a/x)^(1/4)*x^3/a^2/(1-1/a/x)^(1/4)+21/40*(1+1/a/x)^(1/4)*x^4/a/(1-1
/a/x)^(1/4)+1/5*(1+1/a/x)^(1/4)*x^5/(1-1/a/x)^(1/4)+1003/128*arctan((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^5+1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1
/4))/a^5
```

Rubi [A] time = 0.17, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{181x^3\sqrt[4]{\frac{1}{ax}+1}}{240a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189x^2\sqrt[4]{\frac{1}{ax}+1}}{960a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{5533x\sqrt[4]{\frac{1}{ax}+1}}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}-\frac{26111\sqrt[4]{\frac{1}{ax}+1}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{1003\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (-26111*(1 + 1/(a*x))^(1/4))/(1920*a^5*(1 - 1/(a*x))^(1/4)) + (5533*(1 + 1/
(a*x))^(1/4)*x)/(1920*a^4*(1 - 1/(a*x))^(1/4)) + (1189*(1 + 1/(a*x))^(1/4)*
x^2)/(960*a^3*(1 - 1/(a*x))^(1/4)) + (181*(1 + 1/(a*x))^(1/4)*x^3)/(240*a^2
*(1 - 1/(a*x))^(1/4)) + (21*(1 + 1/(a*x))^(1/4)*x^4)/(40*a*(1 - 1/(a*x))^(1
/4)) + ((1 + 1/(a*x))^(1/4)*x^5)/(5*(1 - 1/(a*x))^(1/4)) + (1003*ArcTan[(1
+ 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (1003*ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^6\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{21}{2a}-\frac{10x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{20}\text{Subst}\left(\int \frac{\frac{181}{4a^2}+\frac{42x}{a^3}}{x^4\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{60}\text{Subst}\left(\int \frac{-\frac{1189}{8a^3}-\frac{543x}{4a^4}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x\right.$$

$$= \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{120}\text{Subst}\left(\int \frac{553}{16a}\ldots\right.$$

$$= \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{120}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}}$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}}$$

Mathematica [A] time = 5.28, size = 198, normalized size = 0.69

$$\frac{-8e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{4117e^{\frac{1}{2}\coth^{-1}(ax)}}{192\left(e^{2\coth^{-1}(ax)}-1\right)} + \frac{1661e^{\frac{1}{2}\coth^{-1}(ax)}}{48\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{233e^{\frac{1}{2}\coth^{-1}(ax)}}{6\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{122e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(e^{2\coth^{-1}(ax)}-1\right)^4} + \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(e^{2\coth^{-1}(ax)}-1\right)^5} - \frac{1003}{256}\log\ldots}{a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (-8*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]
))^5) + (122*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]))^4) + (233*E^(
ArcCoth[a*x]/2))/(6*(-1 + E^(2*ArcCoth[a*x]))^3) + (1661*E^(ArcCoth[a*x]/2)
)/(48*(-1 + E^(2*ArcCoth[a*x]))^2) + (4117*E^(ArcCoth[a*x]/2))/(192*(-1 + E
^(2*ArcCoth[a*x]))) + (1003*ArcTan[E^(ArcCoth[a*x]/2)])/128 - (1003*Log[1 -
 E^(ArcCoth[a*x]/2)])/256 + (1003*Log[1 + E^(ArcCoth[a*x]/2)])/256)/a^5
```

fricas [A] time = 0.55, size = 152, normalized size = 0.53

$$-\frac{30090\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\,(}{3840\left(a^6x-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="fricas")
```

```
[Out] -1/3840*(30090*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 15045*(a*x -
 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*(a*x - 1)*log(((a*x - 1)/(
a*x + 1))^(1/4) - 1) - 2*(384*a^6*x^6 + 1392*a^5*x^5 + 2456*a^4*x^4 + 3826*
a^3*x^3 + 7911*a^2*x^2 - 20578*a*x - 26111)*((a*x - 1)/(a*x + 1))^(3/4))/(a
^6*x - a^5)
```

giac [A] time = 0.28, size = 254, normalized size = 0.89

$$-\frac{1}{3840}\,a\left(\frac{30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{15045\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{30720}{a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="giac")
```

```
[Out] -1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4)
- 1))/a^6 + 30720/(a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(49120*(a*x - 1)*((
a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x + 1
))^(3/4)/(a*x + 1)^2 + 33816*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1)^3 - 7365*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^4 - 20585*((
a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

maxima [A] time = 0.42, size = 275, normalized size = 0.96

$$-\frac{1}{3840}\,a\left(\frac{4\left(\frac{58985\,(ax-1)}{ax+1}-\frac{125920\,(ax-1)^2}{(ax+1)^2}+\frac{137930\,(ax-1)^3}{(ax+1)^3}-\frac{72216\,(ax-1)^4}{(ax+1)^4}+\frac{15045\,(ax-1)^5}{(ax+1)^5}-7680\right)}{a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{21}{4}}-5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}+10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{30090\,\mathrm{arcta}}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(58985*(a*x - 1)/(a*x + 1) - 125920*(a*x - 1)^2/(a*x + 1)^2 +
137930*(a*x - 1)^3/(a*x + 1)^3 - 72216*(a*x - 1)^4/(a*x + 1)^4 + 15045*(a*x
 - 1)^5/(a*x + 1)^5 - 7680)/(a^6*((a*x - 1)/(a*x + 1))^(21/4) - 5*a^6*((a*x
 - 1)/(a*x + 1))^(17/4) + 10*a^6*((a*x - 1)/(a*x + 1))^(13/4) - 10*a^6*((a*
x - 1)/(a*x + 1))^(9/4) + 5*a^6*((a*x - 1)/(a*x + 1))^(5/4) - a^6*((a*x - 1
)/(a*x + 1))^(1/4)) + 30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(((a*x - 1)/(a*x + 1))
^(1/4) - 1)/a^6)
```

mupad [B] time = 1.26, size = 248, normalized size = 0.86

$$\frac{1003\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{\frac{787\,(a\,x-1)^2}{6\,(a\,x+1)^2}-\frac{13793\,(a\,x-1)^3}{96\,(a\,x+1)^3}+\frac{3009\,(a\,x-1)^4}{40\,(a\,x+1)^4}-\frac{1003\,(a\,x-1)^5}{64\,(a\,x+1)^5}}{a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-5\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+10\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}-10\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (1003*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (1003*atan(((a*x - 1)
/(a*x + 1))^(1/4)))/(128*a^5) - ((787*(a*x - 1)^2)/(6*(a*x + 1)^2) - (13793
*(a*x - 1)^3)/(96*(a*x + 1)^3) + (3009*(a*x - 1)^4)/(40*(a*x + 1)^4) - (100
3*(a*x - 1)^5)/(64*(a*x + 1)^5) - (11797*(a*x - 1))/(192*(a*x + 1)) + 8)/(a
^5*((a*x - 1)/(a*x + 1))^(1/4) - 5*a^5*((a*x - 1)/(a*x + 1))^(5/4) + 10*a^5
*((a*x - 1)/(a*x + 1))^(9/4) - 10*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 5*a^5*
((a*x - 1)/(a*x + 1))^(17/4) - a^5*((a*x - 1)/(a*x + 1))^(21/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.78 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=250

$$-\frac{2467\sqrt[4]{\frac{1}{ax}+1}}{192a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{475\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{521x\sqrt[4]{\frac{1}{ax}+1}}{192a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{113x^2\sqrt[4]{\frac{1}{ax}+1}}{96a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}+\frac{1}{2}$$

```
[Out] -2467/192*(1+1/a/x)^(1/4)/a^4/(1-1/a/x)^(1/4)+521/192*(1+1/a/x)^(1/4)*x/a^3
/(1-1/a/x)^(1/4)+113/96*(1+1/a/x)^(1/4)*x^2/a^2/(1-1/a/x)^(1/4)+17/24*(1+1/
a/x)^(1/4)*x^3/a/(1-1/a/x)^(1/4)+1/4*(1+1/a/x)^(1/4)*x^4/(1-1/a/x)^(1/4)+47
5/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] time = 0.14, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{113x^2\sqrt[4]{\frac{1}{ax}+1}}{96a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{521x\sqrt[4]{\frac{1}{ax}+1}}{192a^3\sqrt[4]{1-\frac{1}{ax}}}-\frac{2467\sqrt[4]{\frac{1}{ax}+1}}{192a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{475\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}+\frac{17}{24}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (-2467*(1 + 1/(a*x))^(1/4))/(192*a^4*(1 - 1/(a*x))^(1/4)) + (521*(1 + 1/(a*
x))^(1/4)*x)/(192*a^3*(1 - 1/(a*x))^(1/4)) + (113*(1 + 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 - 1/(a*x))^(1/4)) + (17*(1 + 1/(a*x))^(1/4)*x^3)/(24*a*(1 - 1/(a
*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) + (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^5\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{17}{2a}-\frac{8x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{113}{4a^2}+\frac{51x}{2a^3}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{24}\text{Subst}\left(\int \frac{-\frac{521}{8a^3}-\frac{113x}{2a^4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx\right.$$

$$= \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{24}\text{Subst}\left(\int \frac{\frac{14}{16}}{x\left(1-\frac{x}{a}\right.}\right.$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{12}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{47}{}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{47}{}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{47}{}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{47}{}$$

Mathematica [A] time = 5.25, size = 161, normalized size = 0.64

$$\frac{-3072e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{6292e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{7376e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{5248e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} - 1425\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right.}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (-3072*E^(ArcCoth[a*x]/2) + (1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*
x]))^4 + (5248*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (7376*E^(A
rcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (6292*E^(ArcCoth[a*x]/2))/(-1
 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(ArcCoth[a*x]/2)] - 1425*Log[1 - E^(
ArcCoth[a*x]/2)] + 1425*Log[1 + E^(ArcCoth[a*x]/2)])/(384*a^4)
```

fricas [A] time = 0.56, size = 144, normalized size = 0.58

$$-\frac{2850\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(48\,a^5x^5\right.}{384\left(a^5x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="fricas")
```

```
[Out] -1/384*(2850*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 1425*(a*x - 1)
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1425*(a*x - 1)*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1) - 2*(48*a^5*x^5 + 184*a^4*x^4 + 362*a^3*x^3 + 747*a^2*x^2
- 1946*a*x - 2467)*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*x - a^4)
```

giac [A] time = 0.27, size = 223, normalized size = 0.89

$$-\frac{1}{384}\,a\left(\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{1425\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{3072}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{4\left(\frac{2875\,(ax\right.}{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="giac")
```

```
[Out] -1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^5 + 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^5 + 3072/(a^5*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(2875*(a*x - 1)*((a*x -
1)/(a*x + 1))^(3/4)/(a*x + 1) - 2343*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4
)/(a*x + 1)^2 + 657*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1
573*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

maxima [A] time = 0.42, size = 238, normalized size = 0.95

$$\frac{1}{384}\,a\left(\frac{4\left(\frac{4645\,(ax-1)}{ax+1}-\frac{7483\,(ax-1)^2}{(ax+1)^2}+\frac{5415\,(ax-1)^3}{(ax+1)^3}-\frac{1425\,(ax-1)^4}{(ax+1)^4}-768\right)}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+6\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{1425\,\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/384*a*(4*(4645*(a*x - 1)/(a*x + 1) - 7483*(a*x - 1)^2/(a*x + 1)^2 + 5415*
(a*x - 1)^3/(a*x + 1)^3 - 1425*(a*x - 1)^4/(a*x + 1)^4 - 768)/(a^5*((a*x -
1)/(a*x + 1))^(17/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 6*a^5*((a*x - 1
)/(a*x + 1))^(9/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(5/4) + a^5*((a*x - 1)/(a*
x + 1))^(1/4)) - 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((
a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1)/a^5)
```

mupad [B] time = 0.14, size = 211, normalized size = 0.84

$$\frac{475\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{7483\,{\left(a\,x-1\right)}^2}{96\,{\left(a\,x+1\right)}^2}-\frac{1805\,{\left(a\,x-1\right)}^3}{32\,{\left(a\,x+1\right)}^3}+\frac{475\,{\left(a\,x-1\right)}^4}{32\,{\left(a\,x+1\right)}^4}-\frac{4645\,\left(a\,x-1\right)}{96\,\left(a\,x+1\right)}+}{a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-4\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+6\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}-4\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (475*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (475*atan(((a*x - 1)/(a
*x + 1))^(1/4)))/(64*a^4) - ((7483*(a*x - 1)^2)/(96*(a*x + 1)^2) - (1805*(a
*x - 1)^3)/(32*(a*x + 1)^3) + (475*(a*x - 1)^4)/(32*(a*x + 1)^4) - (4645*(a
*x - 1))/(96*(a*x + 1)) + 8)/(a^4*((a*x - 1)/(a*x + 1))^(1/4) - 4*a^4*((a*x
 - 1)/(a*x + 1))^(5/4) + 6*a^4*((a*x - 1)/(a*x + 1))^(9/4) - 4*a^4*((a*x -
1)/(a*x + 1))^(13/4) + a^4*((a*x - 1)/(a*x + 1))^(17/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.79 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=213

$$-\frac{287\sqrt[4]{\frac{1}{ax}+1}}{24a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{55\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{61x\sqrt[4]{\frac{1}{ax}+1}}{24a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}+\frac{13x^2\sqrt[4]{\frac{1}{ax}+1}}{12a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[Out] -287/24*(1+1/a/x)^(1/4)/a^3/(1-1/a/x)^(1/4)+61/24*(1+1/a/x)^(1/4)*x/a^2/(1-
1/a/x)^(1/4)+13/12*(1+1/a/x)^(1/4)*x^2/a/(1-1/a/x)^(1/4)+1/3*(1+1/a/x)^(1/4
)*x^3/(1-1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3+55/8
*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] time = 0.11, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6171, 98, 151, 155, 12, 93, 212, 206, 203}

$$\frac{61x\sqrt[4]{\frac{1}{ax}+1}}{24a^2\sqrt[4]{1-\frac{1}{ax}}}-\frac{287\sqrt[4]{\frac{1}{ax}+1}}{24a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{55\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}+\frac{13x^2\sqrt[4]{\frac{1}{ax}+1}}{12a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (-287*(1 + 1/(a*x))^(1/4))/(24*a^3*(1 - 1/(a*x))^(1/4)) + (61*(1 + 1/(a*x))
^(1/4)*x)/(24*a^2*(1 - 1/(a*x))^(1/4)) + (13*(1 + 1/(a*x))^(1/4)*x^2)/(12*a
*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) +
 (55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) + (55*ArcTanh
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^4\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{13}{2a}-\frac{6x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{61}{4a^2}+\frac{13x}{a^3}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{165}{8a^3}-\frac{61x}{4a^4}}{x\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{16a^4x\sqrt[4]{1-}}\right.$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{55\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/}}\right.}{16a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{55\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x,\right.}{4a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.}{8a^3}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} + \cdots$$

Mathematica [A] time = 5.21, size = 137, normalized size = 0.64

$$\frac{-384e^{\frac{1}{2}\coth^{-1}(ax)} + \frac{548e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{400e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{128e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} - 165\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 165\log\left(e^{\frac{1}{2}\coth^{-1}(a\ldots}\right.}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (-384*E^(ArcCoth[a*x]/2) + (128*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]
))^3 + (400*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (548*E^(ArcCo
th[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 330*ArcTan[E^(ArcCoth[a*x]/2)] - 16
5*Log[1 - E^(ArcCoth[a*x]/2)] + 165*Log[1 + E^(ArcCoth[a*x]/2)])/(48*a^3)
```

fricas [A] time = 0.58, size = 136, normalized size = 0.64

$$-\frac{330\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(8\,a^4x^4+\cdots\right.}{48\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="fricas")
```

```
[Out] -1/48*(330*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*(a*x - 1)*lo
g(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 165*(a*x - 1)*log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1) - 2*(8*a^4*x^4 + 34*a^3*x^3 + 87*a^2*x^2 - 226*a*x - 287)*((a*
x - 1)/(a*x + 1))^(3/4))/(a^4*x - a^3)
```

giac [A] time = 0.25, size = 192, normalized size = 0.90

$$-\frac{1}{48}\,a\left(\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{165\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{384}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{4\left(\frac{174\,(ax-1)\left(\cdots\right.}{ax+\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="giac")
```

```
[Out] -1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^4 + 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
4 + 384/(a^4*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(174*(a*x - 1)*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1) - 69*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1)^2 - 137*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

maxima [A] time = 0.41, size = 203, normalized size = 0.95

$$-\frac{1}{48}\,a\left(\frac{4\left(\frac{425\,(ax-1)}{ax+1}-\frac{462\,(ax-1)^2}{(ax+1)^2}+\frac{165\,(ax-1)^3}{(ax+1)^3}-96\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\cdots\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(425*(a*x - 1)/(a*x + 1) - 462*(a*x - 1)^2/(a*x + 1)^2 + 165*(a*
x - 1)^3/(a*x + 1)^3 - 96)/(a^4*((a*x - 1)/(a*x + 1))^(13/4) - 3*a^4*((a*x
```

```
- 1)/(a*x + 1))^(9/4) + 3*a^4*((a*x - 1)/(a*x + 1))^(5/4) - a^4*((a*x - 1)/
(a*x + 1))^(1/4)) + 330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1)/a^4)
```

mupad [B] time = 0.09, size = 176, normalized size = 0.83

$$\frac{55\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{\frac{77\,{\left(a\,x-1\right)}^2}{2\,{\left(a\,x+1\right)}^2}-\frac{55\,{\left(a\,x-1\right)}^3}{4\,{\left(a\,x+1\right)}^3}-\frac{425\,\left(a\,x-1\right)}{12\,\left(a\,x+1\right)}+8}{a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}-a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (55*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (55*atan(((a*x - 1)/(a*x
+ 1))^(1/4)))/(8*a^3) - ((77*(a*x - 1)^2)/(2*(a*x + 1)^2) - (55*(a*x - 1)^3
)/(4*(a*x + 1)^3) - (425*(a*x - 1))/(12*(a*x + 1)) + 8)/(a^3*((a*x - 1)/(a*
x + 1))^(1/4) - 3*a^3*((a*x - 1)/(a*x + 1))^(5/4) + 3*a^3*((a*x - 1)/(a*x +
 1))^(9/4) - a^3*((a*x - 1)/(a*x + 1))^(13/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.80 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=176

$$-\frac{25\sqrt[4]{\frac{1}{ax}+1}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5x\left(\frac{1}{ax}+1\right)^{5/4}}{4a\sqrt[4]{1-\frac{1}{ax}}}$$

```
[Out] -25/2*(1+1/a/x)^(1/4)/a^2/(1-1/a/x)^(1/4)+5/4*(1+1/a/x)^(5/4)*x/a/(1-1/a/x)
^(1/4)+1/2*(1+1/a/x)^(9/4)*x^2/(1-1/a/x)^(1/4)+25/4*arctan((1+1/a/x)^(1/4)/
(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] time = 0.07, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 212, 206, 203}

$$-\frac{25\sqrt[4]{\frac{1}{ax}+1}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5x\left(\frac{1}{ax}+1\right)^{5/4}}{4a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x,x]
```

```
[Out] (-25*(1 + 1/(a*x))^(1/4))/(2*a^2*(1 - 1/(a*x))^(1/4)) + (5*(1 + 1/(a*x))^(5
/4)*x)/(4*a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(9/4)*x^2)/(2*(1 - 1/(a*x
))^(1/4)) + (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^3\left(1-\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{5\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{4a}\\
&= \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{25\,\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{8a^2}\\
&= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{25\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{8a^2}\\
&= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}-\frac{25\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}\\
&= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+25\,\text{Sub}\\
&= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 80, normalized size = 0.45

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-45e^{2\coth^{-1}(ax)}+16e^{4\coth^{-1}(ax)}+25\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}+25\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+25\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x,x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(25 - 45*E^(2*ArcCoth[a*x]) + 16*E^(4*ArcCoth[a*x])
))/(-1 + E^(2*ArcCoth[a*x]))^2 + 25*ArcTan[E^(ArcCoth[a*x]/2)] + 25*ArcTanh
[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

fricas [A] time = 0.63, size = 128, normalized size = 0.73

$$-\frac{50\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(2\,a^3x^3+11\right.}{8\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="fricas")
```

```
[Out] -1/8*(50*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 25*(a*x - 1)*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) + 25*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1) - 2*(2*a^3*x^3 + 11*a^2*x^2 - 34*a*x - 43)*((a*x - 1)/(a*x + 1))^(
3/4))/(a^3*x - a^2)
```

giac [A] time = 0.25, size = 161, normalized size = 0.91

$$-\frac{1}{8}\,a\left(\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{64}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{4\left(\frac{9\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}\right.}{a^3\left(\frac{ax-1}{ax+1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="giac")
```

```
[Out] -1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^3 + 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 +
64/(a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(9*(a*x - 1)*((a*x - 1)/(a*x + 1))
^(3/4)/(a*x + 1) - 13*((a*x - 1)/(a*x + 1))^(3/4))/(a^3*((a*x - 1)/(a*x + 1
) - 1)^2))
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

maxima [A] time = 0.41, size = 166, normalized size = 0.94

$$\frac{1}{8}\,a\left(\frac{4\left(\frac{45\,(ax-1)}{ax+1}-\frac{25\,(ax-1)^2}{(ax+1)^2}-16\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-2\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\right.}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(45*(a*x - 1)/(a*x + 1) - 25*(a*x - 1)^2/(a*x + 1)^2 - 16)/(a^3*((
a*x - 1)/(a*x + 1))^(9/4) - 2*a^3*((a*x - 1)/(a*x + 1))^(5/4) + a^3*((a*x -
 1)/(a*x + 1))^(1/4)) - 50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4)
- 1)/a^3)
```

mupad [B] time = 1.21, size = 139, normalized size = 0.79

$$\frac{25\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{25\,(a\,x-1)^2}{2\,(a\,x+1)^2}-\frac{45\,(a\,x-1)}{2\,(a\,x+1)}+8}{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-2\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (25*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (25*atan(((a*x - 1)/(a*x
+ 1))^(1/4)))/(4*a^2) - ((25*(a*x - 1)^2)/(2*(a*x + 1)^2) - (45*(a*x - 1))/
(2*(a*x + 1)) + 8)/(a^2*((a*x - 1)/(a*x + 1))^(1/4) - 2*a^2*((a*x - 1)/(a*x
 + 1))^(5/4) + a^2*((a*x - 1)/(a*x + 1))^(9/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.81 $\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=130

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{10\sqrt[4]{\frac{1}{ax}+1}}{a\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] -10*(1+1/a/x)^(1/4)/a/(1-1/a/x)^(1/4)+(1+1/a/x)^(5/4)*x/(1-1/a/x)^(1/4)+5*a
rctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a+5*arctanh((1+1/a/x)^(1/4)/(1-1/a/x
)^(1/4))/a
```

Rubi [A] time = 0.04, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6170, 94, 93, 212, 206, 203}

$$\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{10\sqrt[4]{\frac{1}{ax}+1}}{a\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-10*(1 + 1/(a*x))^(1/4))/(a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(5/4)*x)
/(1 - 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/
a + (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{10\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.14, size = 67, normalized size = 0.52

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(4e^{2\coth^{-1}(ax)}-5\right)}{e^{2\coth^{-1}(ax)}-1} + 5\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 5\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(-5 + 4*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
])) + 5*ArcTan[E^(ArcCoth[a*x]/2)] + 5*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

fricas [A] time = 0.71, size = 117, normalized size = 0.90

$$-\frac{10\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(a^2x^2-8\,ax-\right.}{2\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(10*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 5*(a*x - 1)*log(((
a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1) - 2*(a^2*x^2 - 8*a*x - 9)*((a*x - 1)/(a*x + 1))^(3/4))/(a^2*x - a)
```

giac [A] time = 0.20, size = 141, normalized size = 1.08

$$-\frac{1}{2}\,a\left(\frac{10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{5\,(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 5*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^2 + 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*
(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*
x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

maxima [A] time = 0.40, size = 131, normalized size = 1.01

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{5\,(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)/(a*x + 1))^(5/4) - a^
2*((a*x - 1)/(a*x + 1))^(1/4)) + 10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2
 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^2 + 5*log(((a*x - 1)/(a*x + 1))
^(1/4) - 1)/a^2)
```

mupad [B] time = 0.06, size = 98, normalized size = 0.75

$$\frac{\frac{10\,(a\,x-1)}{a\,x+1}-8}{a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}-\frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{5\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] ((10*(a*x - 1))/(a*x + 1) - 8)/(a*((a*x - 1)/(a*x + 1))^(1/4) - a*((a*x - 1
)/(a*x + 1))^(5/4)) - (5*atan(((a*x - 1)/(a*x + 1))^(1/4)))/a + (5*atanh(((
a*x - 1)/(a*x + 1))^(1/4)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-5/4), x)
```

3.82 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=320

$$-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\text{ t}$$

```
[Out] -8*(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/
2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-
arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6171, 98, 21, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\text{ t}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x,x]
```

```
[Out] (-8*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + (4a)\,\text{Subst}\left(\int \frac{-\frac{1}{4a}+\frac{x}{4a^2}}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} - \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} - 4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right.$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\,\text{S}$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.}{\sqrt{2}}$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \cdots$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

Mathematica [C] time = 0.07, size = 30, normalized size = 0.09

$$8e^{\frac{1}{2}\coth^{-1}(ax)}\left({}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{4\coth^{-1}(ax)}\right)-1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x,x]
```

```
[Out] 8*E^(ArcCoth[a*x]/2)*(-1 + Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x]
)])
```

fricas [A] time = 0.55, size = 358, normalized size = 1.12

$$\frac{4\sqrt{2}(ax-1)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}(ax-1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{\cdots}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*(a*x - 1)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^
(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) - 1) + 4*sqrt(2)*(a*x - 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a
*x + 1))^(1/4) + 1) + sqrt(2)*(a*x - 1)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*(a*x - 1)*log(-4*sqrt(2
)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - 4*(a*x -
 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 2*(a*x - 1)*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1) - 2*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) - 1) - 16*(a*
x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x - 1)
```

giac [A] time = 0.22, size = 252, normalized size = 0.79

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\cdots\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

maxima [A] time = 0.41, size = 244, normalized size = 0.76

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)\cdots\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

mupad [B] time = 1.18, size = 118, normalized size = 0.37

$$-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\frac{8}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,2\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1+1\mathrm{i}\right)+\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(1
/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 -
1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1
i) - 8/((a*x - 1)/(a*x + 1))^(1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

3.83 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=299

$$-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+$$

```
[Out] -5*a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-4*a*(1+1/a/x)^(5/4)/(1-1/a/x)^(1/4)-5/
2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arctan
(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/4*a*ln(1-(1-1/a/x)^(1
/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+5/4*a*
ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2
))*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = $\{6171, 47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628\}$

$$-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] -5*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (4*a*(1 + 1/(a*x))^(5/4))/(1
 - 1/(a*x))^(1/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a
*x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1
/(a*x))^(1/4)])/Sqrt[2] - (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
- (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) + (5*a*Lo
g[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + 5\,\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - (10a)\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - (10a)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + (5a)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - (5a$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}(5a)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{1+\frac{1}{a}}}\right.$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{5a\log\left(1\right.}{}$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\tan^{-1}\left(1+\frac{\sqrt{2}}{\right.}{\sqrt{2}}$$

Mathematica [A] time = 0.40, size = 173, normalized size = 0.58

$$a\left(-\frac{10e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} - \frac{8e^{\frac{5}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} - \frac{5\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} + \frac{5\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth\right.}{2\sqrt{2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] a*((-10*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - (8*E^((5*ArcCoth[a*x
])/2))/(1 + E^(2*ArcCoth[a*x])) - (5*ArcTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)]
)/Sqrt[2] + (5*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (5*Log[1 -
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]) + (5*Log[1 + Sqr
t[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

fricas [A] time = 0.67, size = 451, normalized size = 1.51

$$\frac{20\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\left(ax^2-x\right)\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^4}\,a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^4\right)^{\frac{1}{4}}}{a^4}\right)+20\sqrt{2}\left(a^4\right)^{\frac{1}{4}}\left(ax^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*
a^3*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)
) + sqrt(a^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a
^4)^(1/4))/a^4) + 20*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*arctan((a^4 - sqrt(2)*
(a^4)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x -
1)/(a*x + 1)) + sqrt(a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1
))^(1/4))*(a^4)^(1/4))/a^4) + 5*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*log(15625*a
^6*sqrt((a*x - 1)/(a*x + 1)) + 15625*sqrt(a^4)*a^4 + 15625*sqrt(2)*(a^4)^(3
/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 5*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*lo
g(15625*a^6*sqrt((a*x - 1)/(a*x + 1)) + 15625*sqrt(a^4)*a^4 - 15625*sqrt(2)
*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(9*a^2*x^2 + 8*a*x - 1)*(
(a*x - 1)/(a*x + 1))^(3/4))/(a*x^2 - x)
```

giac [A] time = 0.19, size = 217, normalized size = 0.73

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-5\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/((a*x - 1)*((a*x
- 1)/(a*x + 1))^(1/4)/(a*x + 1) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

maxima [A] time = 0.40, size = 204, normalized size = 0.68

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-5\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/((((a*x - 1)/(a*x
+ 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

mupad [B] time = 1.22, size = 107, normalized size = 0.36

$$5\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-5\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-\frac{8\,a+\frac{10\,a\,\left(a\,x-1\right)}{a\,x+1}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] 5*(-1)^(1/4)*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 5*(-1)^(1/4)
*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (8*a + (10*a*(a*x - 1))/(
a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/4) + ((a*x - 1)/(a*x + 1))^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

3.84 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=351

$$-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\dots$$

```
[Out] -25/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-5/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)
^(5/4)-2*a^2*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-25/8*a^2*arctan(-1+(1-1/a/x)^(
1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(
1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/16*a^2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a
/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+25/16*a^2*ln(1+(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 78, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\dots$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (-25*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 - (5*a^2*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(5/4))/2 - (2*a^2*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1
/4) + (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) - (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + (5a)\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}(25a)\,\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{8}(25a)\,\text{Subst}$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}\left(25a^2\right)\text{Sub}$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}\left(25a^2\right)\text{Sub}$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}\left(25a^2\right)\text{Sub}$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{8}\left(25a^2\right)\text{Sub}$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{25a^2\log\left(1+\right.}{}$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{25a^2\tan^{-1}\left(1\right.}{4\sqrt{}}$$

Mathematica [A] time = 0.29, size = 186, normalized size = 0.53

$$\frac{1}{16}a^2\left(-128e^{\frac{1}{2}\coth^{-1}(ax)} - \frac{104e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} + \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}+1\right)^2} - 25\sqrt{2}\log\left(-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)} + e^{\coth^{-1}(ax)} + 1\right) + 25\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (a^2*(-128*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a
*x]))^2 - (104*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 50*Sqrt[2]*Ar
```

```
cTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)] + 50*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(Arc
Coth[a*x]/2)] - 25*Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a
*x]] + 25*Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

fricas [A] time = 0.48, size = 469, normalized size = 1.34

$$100\sqrt{2}\left(a^{8}\right)^{\frac{1}{4}}\left(ax^{3}-x^{2}\right)\arctan\left(-\frac{a^{8}+\sqrt{2}\left(a^{8}\right)^{\frac{1}{4}}a^{6}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{8}}a^{8}+\sqrt{2}\left(a^{8}\right)^{\frac{3}{4}}a^{6}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{8}\right)^{\frac{1}{4}}}{a^{8}}\right)+100\sqrt{2}\left(a^{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(100*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)*arctan(-(a^8 + sqrt(2)*(a^8)^(1
/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x
 + 1)) + sqrt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4
))*(a^8)^(1/4))/a^8) + 100*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)*arctan((a^8 -
sqrt(2)*(a^8)^(1/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqr
t((a*x - 1)/(a*x + 1)) + sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)
/(a*x + 1))^(1/4))*(a^8)^(1/4))/a^8) + 25*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)
*log(244140625*a^12*sqrt((a*x - 1)/(a*x + 1)) + 244140625*sqrt(a^8)*a^8 + 2
44140625*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 25*sqrt(2)*
(a^8)^(1/4)*(a*x^3 - x^2)*log(244140625*a^12*sqrt((a*x - 1)/(a*x + 1)) + 24
4140625*sqrt(a^8)*a^8 - 244140625*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x +
 1))^(1/4)) - 4*(43*a^3*x^3 + 34*a^2*x^2 - 11*a*x - 2)*((a*x - 1)/(a*x + 1)
)^(3/4))/(a*x^3 - x^2)
```

giac [A] time = 0.19, size = 243, normalized size = 0.69

$$-\frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-25\sqrt{2}a\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 128*a/((a*x - 1)/(a*x + 1))^(1/4) +
8*(9*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 13*a*((a*x - 1)/(a
*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

maxima [A] time = 0.41, size = 244, normalized size = 0.70

$$-\frac{1}{16}\left(25\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(25*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1))*a + 8*(45*(a*x - 1)*a/(a*x + 1) + 25*(a*x - 1)^2*
a/(a*x + 1)^2 + 16*a)/(((a*x - 1)/(a*x + 1))^(9/4) + 2*((a*x - 1)/(a*x + 1)
)^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

mupad [B] time = 0.08, size = 152, normalized size = 0.43

$$\frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4}-\frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4}-\frac{8\,a^2+\frac{25\,a^2\,(a\,x-1)^2}{2\,(a\,x+1)^2}+\frac{45\,a^2\,(a\,x-1)}{2\,(a\,x+1)}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}+2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] (25*(-1)^(1/4)*a^2*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (25*(
-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (8*a^2 + (2
5*a^2*(a*x - 1)^2)/(2*(a*x + 1)^2) + (45*a^2*(a*x - 1))/(2*(a*x + 1)))/(((a
*x - 1)/(a*x + 1))^(1/4) + 2*((a*x - 1)/(a*x + 1))^(5/4) + ((a*x - 1)/(a*x
+ 1))^(9/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

3.85 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=385

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{55a}{}$$

```
[Out] -55/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-11/4*a^3*(1-1/a/x)^(3/4)*(1+1/a/x
)^(5/4)-2*a^3*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-1/3*a^3*(1-1/a/x)^(3/4)*(1+1/
a/x)^(9/4)-55/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(
1/2)-55/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-55
/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a
/x)^(1/2))*2^(1/2)+55/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 89, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{55a}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (-55*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 - (11*a^3*(1 - 1/(a*x))
^(3/4)*(1 + 1/(a*x))^(5/4))/4 - (2*a^3*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(
1/4) - (a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(9/4))/3 + (55*a^3*ArcTan[1 -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*
ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
- (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*
x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (55*a^3*Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)])/(16*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \left(2a^3\right)\text{Subst}\left(\int \frac{\left(\frac{5}{2a}+\frac{x}{2a^2}\right)\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{8}\left(55a^2\right)\text{Su}$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right.$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right.$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right.$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right.$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right.$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right.$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right.$$

Mathematica [C] time = 0.15, size = 104, normalized size = 0.27

$$a^3\left(-\frac{55}{32}\text{RootSum}\left[\#1^4+1\&, \frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right] - \frac{e^{\frac{1}{2}\coth^{-1}(ax)}\left(462e^{2\coth^{-1}(ax)}+425e^{4\coth^{-1}}\right.}{12\left(e^{2\coth^{-1}(ax)}+\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] a^3*(-1/12*(E^(ArcCoth[a*x]/2)*(165 + 462*E^(2*ArcCoth[a*x]) + 425*E^(4*Arc
Coth[a*x]) + 96*E^(6*ArcCoth[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 - (55*RootS
um[1 + #1^4 & , (ArcCoth[a*x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ])/3
2)
```

fricas [A] time = 0.60, size = 477, normalized size = 1.24

$$660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}\left(ax^4-x^3\right)\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+660\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*arctan(-(a^12 + sqrt(2)*(a^12)
^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(
a*x + 1)) + sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1)
)^(1/4))*(a^12)^(1/4))/a^12) + 660*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*arcta
n((a^12 - sqrt(2)*(a^12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sq
rt(a^18*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*
a^9*((a*x - 1)/(a*x + 1))^(1/4))*(a^12)^(1/4))/a^12) + 165*sqrt(2)*(a^12)^(
1/4)*(a*x^4 - x^3)*log(27680640625*a^18*sqrt((a*x - 1)/(a*x + 1)) + 2768064
0625*sqrt(a^12)*a^12 + 27680640625*sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x
 + 1))^(1/4)) - 165*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*log(27680640625*a^18
*sqrt((a*x - 1)/(a*x + 1)) + 27680640625*sqrt(a^12)*a^12 - 27680640625*sqrt
(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(287*a^4*x^4 + 226*a^
3*x^3 - 87*a^2*x^2 - 34*a*x - 8)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x^4 - x^3)
```

giac [A] time = 0.25, size = 291, normalized size = 0.76

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-165\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) - 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) + 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 768*a^2/((a*x - 1)/(a*x
+ 1))^(1/4) + 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) +
69*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 137*a^2*((a*x
- 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

maxima [A] time = 0.41, size = 288, normalized size = 0.75

$$-\frac{1}{96}\left(165\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(165*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))*a^2 + 8*(425*(a*x - 1)*a^2/(a*x + 1) + 462*(a*x
- 1)^2*a^2/(a*x + 1)^2 + 165*(a*x - 1)^3*a^2/(a*x + 1)^3 + 96*a^2)/(((a*x -
 1)/(a*x + 1))^(13/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + 3*((a*x - 1)/(a*x +
 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

mupad [B] time = 1.26, size = 188, normalized size = 0.49

$$\frac{55\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}-\frac{55\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}-\frac{8\,a^3+\frac{77\,a^3\,(a\,x-1)^2}{2\,(a\,x+1)^2}+\frac{55\,a^3\,(a\,x-1)^3}{4\,(a\,x+1)^3}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}+3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+3\left(\frac{a\,x-1}{a\,x+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] (55*(-1)^(1/4)*a^3*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (55*(
-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (8*a^3 + (7
7*a^3*(a*x - 1)^2)/(2*(a*x + 1)^2) + (55*a^3*(a*x - 1)^3)/(4*(a*x + 1)^3) +
 (425*a^3*(a*x - 1))/(12*(a*x + 1)))/(((a*x - 1)/(a*x + 1))^(1/4) + 3*((a*x
 - 1)/(a*x + 1))^(5/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + ((a*x - 1)/(a*x +
1))^(13/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

3.86 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{31\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{611x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{1920a^4}-\frac{269x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{960a^3}+\frac{11x^3\sqrt[4]{1-\frac{1}{ax}}\left(\right.}{48a^2}$$

```
[Out] 611/1920*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4-269/960*(1-1/a/x)^(1/4)*(1+1
/a/x)^(3/4)*x^2/a^3+11/48*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2-9/40*(1-1
/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^5+3
1/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-31/128*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A] time = 0.14, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{11x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{48a^2}-\frac{269x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{960a^3}+\frac{611x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{1920a^4}+\frac{31\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\tanh^{-1}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (611*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(1920*a^4) - (269*(1 - 1/(a
*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4)*x^3)/(48*a^2) - (9*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^6\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{9}{2a}+\frac{4x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{55}{4a^2}+\frac{27x}{2a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1}}\right.$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{60}$$

$$= -\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9}{}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9}{}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9}{}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9}{}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9}{}$$

Mathematica [A] time = 5.30, size = 173, normalized size = 0.68

$$\frac{\frac{9620e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} - \frac{34000e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{64640e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} - \frac{62976e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} + \frac{24576e^{\frac{19}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^5} + 465\log\left(1-e^{-\frac{1}{2}\coth^{-1}(a}\right.}{3840a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((24576*E^((19*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (62976*E^((1
5*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^((11*ArcCoth[a*x
])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (34000*E^((7*ArcCoth[a*x])/2))/(-1 + E
^(2*ArcCoth[a*x]))^2 + (9620*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x
])) - 930*ArcTan[E^(-1/2*ArcCoth[a*x])] + 465*Log[1 - E^(-1/2*ArcCoth[a*x])
] - 465*Log[1 + E^(-1/2*ArcCoth[a*x])])/(3840*a^5)
```

fricas [A] time = 0.70, size = 119, normalized size = 0.47

$$\frac{2\left(384\,a^{5}x^{5}-48\,a^{4}x^{4}+8\,a^{3}x^{3}-98\,a^{2}x^{2}+73\,ax+611\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-465\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right.}{3840\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 - 48*a^4*x^4 + 8*a^3*x^3 - 98*a^2*x^2 + 73*a*x + 611
)*((a*x - 1)/(a*x + 1))^(1/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 4
65*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 465*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1))/a^5
```

giac [A] time = 0.24, size = 234, normalized size = 0.92

$$-\frac{1}{3840}\,a\left(\frac{930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{6}}+\frac{465\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{6}}-\frac{465\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^{6}}-\frac{4\left(\frac{696\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{5090\,(\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^6 - 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^6 - 4*(696*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 5090*(a*x -
1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1120*(a*x - 1)^3*((a*x - 1)/
(a*x + 1))^(1/4)/(a*x + 1)^3 - 2405*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)
/(a*x + 1)^4 - 465*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) -
 1)^5))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

maxima [A] time = 0.41, size = 259, normalized size = 1.02

$$-\frac{1}{3840}\,a\left(\frac{4\left(2405\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+465\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5\,(ax-1)a^{6}}{ax+1}-\frac{10\,(ax-1)^{2}a^{6}}{(ax+1)^{2}}+\frac{10\,(ax-1)^{3}a^{6}}{(ax+1)^{3}}-\frac{5\,(ax-1)^{4}a^{6}}{(ax+1)^{4}}+\frac{(ax-1)^{5}a^{6}}{(ax+1)^{5}}-a^{6}}+\frac{930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right.\right.}{a^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(2405*((a*x - 1)/(a*x + 1))^(17/4) - 1120*((a*x - 1)/(a*x + 1)
)^(13/4) + 5090*((a*x - 1)/(a*x + 1))^(9/4) - 696*((a*x - 1)/(a*x + 1))^(5/
4) + 465*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
- 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^6
/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 465*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

mupad [B] time = 0.08, size = 229, normalized size = 0.91

$$\frac{\frac{31\left(\frac{ax-1}{ax+1}\right)^{1/4}}{64}-\frac{29\left(\frac{ax-1}{ax+1}\right)^{5/4}}{40}+\frac{509\left(\frac{ax-1}{ax+1}\right)^{9/4}}{96}-\frac{7\left(\frac{ax-1}{ax+1}\right)^{13/4}}{6}+\frac{481\left(\frac{ax-1}{ax+1}\right)^{17/4}}{192}}{a^5+\frac{10a^5(ax-1)^2}{(ax+1)^2}-\frac{10a^5(ax-1)^3}{(ax+1)^3}+\frac{5a^5(ax-1)^4}{(ax+1)^4}-\frac{a^5(ax-1)^5}{(ax+1)^5}-\frac{5a^5(ax-1)}{ax+1}}-\frac{31\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128a^5}-\frac{31\,\mathrm{atanh}\left(\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)}{128a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((31*((a*x - 1)/(a*x + 1))^(1/4))/64 - (29*((a*x - 1)/(a*x + 1))^(5/4))/40
+ (509*((a*x - 1)/(a*x + 1))^(9/4))/96 - (7*((a*x - 1)/(a*x + 1))^(13/4))/6
 + (481*((a*x - 1)/(a*x + 1))^(17/4))/192)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*x
 + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)^
4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (31*atan
(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (31*atanh(((a*x - 1)/(a*x + 1))^
(1/4)))/(128*a^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**4*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.87 $\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^3\,dx$

Optimal. Leaf size=216

$$-\frac{11\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{83x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{192a^3}+\frac{29x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{96a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}\right.$$

```
[Out] -83/192*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+29/96*(1-1/a/x)^(1/4)*(1+1/a/
x)^(3/4)*x^2/a^2-7/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)^(
1/4)*(1+1/a/x)^(3/4)*x^4-11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+
11/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] time = 0.11, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{29x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{96a^2}-\frac{83x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{192a^3}-\frac{11\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (-83*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(192*a^3) + (29*(1 - 1/(a*x
))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(96*a^2) - (7*(1 - 1/(a*x))^(1/4)*(1 + 1/
(a*x))^(3/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4
- (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^5\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{3x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{29}{4a^2}+\frac{7x}{a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\right.}}\right.$$

$$= \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{24}\text{Su}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1}$$

Mathematica [A] time = 5.26, size = 149, normalized size = 0.69

$$\frac{-\frac{980e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{2512e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} - \frac{3200e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} - 33\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right) + 33\log\left(e^{-\frac{1}{2}\coth^{-1}(a}\right.}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (3200*E^((11*
ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^((7*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^2 - (980*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*Ar
cCoth[a*x])) + 66*ArcTan[E^(-1/2*ArcCoth[a*x])] - 33*Log[1 - E^(-1/2*ArcCot
h[a*x])] + 33*Log[1 + E^(-1/2*ArcCoth[a*x])])/(384*a^4)
```

fricas [A] time = 0.61, size = 111, normalized size = 0.51

$$\frac{2\left(48a^4x^4 - 8a^3x^3 + 2a^2x^2 - 25ax - 83\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 33\log\left(\left(\frac{ax-}{ax+}\right.\right.}{384a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 - 8*a^3*x^3 + 2*a^2*x^2 - 25*a*x - 83)*((a*x - 1)/(a*x
 + 1))^(1/4) + 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

giac [A] time = 0.22, size = 203, normalized size = 0.94

$$\frac{1}{384}\,a\left(\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{33\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{279\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{107\,(ax-1)}{(ax+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] 1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^5 - 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5 +
 4*(279*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 107*(a*x - 1)^2*(
(a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 245*(a*x - 1)^3*((a*x - 1)/(a*x +
1))^(1/4)/(a*x + 1)^3 - 33*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*
x + 1) - 1)^4))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

maxima [A] time = 0.41, size = 224, normalized size = 1.04

$$-\frac{1}{384}\,a\left(\frac{4\left(245\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-107\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+279\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-33\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right.\right.}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/384*a*(4*(245*((a*x - 1)/(a*x + 1))^(13/4) - 107*((a*x - 1)/(a*x + 1))^(
9/4) + 279*((a*x - 1)/(a*x + 1))^(5/4) - 33*((a*x - 1)/(a*x + 1))^(1/4))/(4
*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^
5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 33*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

mupad [B] time = 1.21, size = 193, normalized size = 0.89

$$\frac{11\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{93\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{107\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{245\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}+\frac{11\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] (11*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - ((11*((a*x - 1)/(a*x + 1)
)^(1/4))/32 - (93*((a*x - 1)/(a*x + 1))^(5/4))/32 + (107*((a*x - 1)/(a*x +
1))^(9/4))/96 - (245*((a*x - 1)/(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*x -
 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a
*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (11*atanh(((a*x - 1)/(a*x + 1))^
(1/4)))/(64*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.88 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{11x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{5x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+\right.}{12a}$$

```
[Out] 11/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2-5/12*(1-1/a/x)^(1/4)*(1+1/a/x)^
(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3+3/8*arctan((1+1/a/x)^(1
/4)/(1-1/a/x)^(1/4))/a^3-3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 298, 203, 206}

$$\frac{11x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{5x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+\right.}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) - (5*(1 - 1/(a*x))^
(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^
(3/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
- (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^4\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{5}{2a}+\frac{2x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{11}{4a^2}+\frac{5x}{2a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1\cdot}}\right.$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{6}\text{Su}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\,\text{Su}}{}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\,\text{Su}}{}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{3\,\text{Su}}{}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{3\tan}{}$$

Mathematica [C] time = 9.03, size = 389, normalized size = 2.17

$$-\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\left(256e^{6\coth^{-1}(ax)}\left(1090e^{2\coth^{-1}(ax)}+437e^{4\coth^{-1}(ax)}+685\right){}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{2\coth^{-1}(ax)}\right)+20}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(ArcCoth[a*x]/2),x]
```

```
[Out] -1/221760*(-22034705 - 26688365*E^(2*ArcCoth[a*x]) - 3731255*E^(4*ArcCoth[a
*x]) + 3122405*E^(6*ArcCoth[a*x]) + 22034705*Hypergeometric2F1[3/4, 1, 7/4,
 E^(2*ArcCoth[a*x])] + 17244920*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1
, 7/4, E^(2*ArcCoth[a*x])] - 9077530*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3
/4, 1, 7/4, E^(2*ArcCoth[a*x])] - 7043960*E^(6*ArcCoth[a*x])*Hypergeometric
2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 446985*E^(8*ArcCoth[a*x])*Hypergeome
tric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 256*E^(6*ArcCoth[a*x])*(685 + 10
90*E^(2*ArcCoth[a*x]) + 437*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7/4, 2,
2, 2}, {1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 2048*E^(6*ArcCoth[a*x])*(21 + 38
*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7/4, 2, 2,
2, 2}, {1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(6*ArcCoth[a*x])*Hyper
geometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])]
+ 8192*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1,
 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(10*ArcCoth[a*x])*HypergeometricPFQ
```

```
[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((5*
ArcCoth[a*x])/2))
```

fricas [A] time = 0.59, size = 102, normalized size = 0.57

$$\frac{2\left(8a^3x^3-2a^2x^2+ax+11\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 - 2*a^2*x^2 + a*x + 11)*((a*x - 1)/(a*x + 1))^(1/4) - 18
*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) +
1) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

giac [A] time = 0.20, size = 172, normalized size = 0.96

$$-\frac{1}{48}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{29(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}\right.}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^4 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 4
*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 29*(a*x - 1)^2*((a*x
- 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a
*x - 1)/(a*x + 1) - 1)^3))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

maxima [A] time = 0.41, size = 187, normalized size = 1.04

$$-\frac{1}{48}a\left(\frac{4\left(29\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(29*((a*x - 1)/(a*x + 1))^(9/4) - 6*((a*x - 1)/(a*x + 1))^(5/4)
+ 9*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2
```

```
*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x - 1
)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 - 9*lo
g(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

mupad [B] time = 1.20, size = 157, normalized size = 0.88

$$\frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((3*((a*x - 1)/(a*x + 1))^(1/4))/4 - ((a*x - 1)/(a*x + 1))^(5/4)/2 + (29*((
a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^
3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a*x -
 1)/(a*x + 1))^(1/4)))/(8*a^3) - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8*
a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.89 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

```
[Out] -1/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4
)*x^2-1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 298, 203, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(ArcCoth[a*x]/2),x]
```

```
[Out] -((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(4*a) + ((1 - 1/(a*x))^(5/4)*(
1 + 1/(a*x))^(3/4)*x^2)/2 - ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^3\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 + \frac{\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^2\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} -$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.}{4a^2}$$

Mathematica [A] time = 0.17, size = 66, normalized size = 0.46

$$\frac{-\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}-5\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} - \tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((-2*E^((3*ArcCoth[a*x])/2)*(-5 + E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a
*x]))^2 - ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

fricas [A] time = 1.58, size = 93, normalized size = 0.65

$$\frac{2\left(2a^2x^2-ax-3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - a*x - 3)*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x -
 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x -
1)/(a*x + 1))^(1/4) - 1))/a^2
```

giac [A] time = 0.22, size = 140, normalized size = 0.99

$$\frac{1}{8}a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\left(\frac{5(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] 1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1)
)^(1/4) + 1)/a^3 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*(5*(a*
x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))
/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

maxima [A] time = 0.41, size = 151, normalized size = 1.06

$$-\frac{1}{8}a\left(\frac{4\left(5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(5*((a*x - 1)/(a*x + 1))^(5/4) - ((a*x - 1)/(a*x + 1))^(1/4))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((a
*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 +
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

mupad [B] time = 0.06, size = 121, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}+\frac{\operatorname{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] atan(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2) - (((a*x - 1)/(a*x + 1))^(1/4)/2
- (5*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 -
 (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.90 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=97

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
-arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6170, 94, 93, 298, 203, 206}

$$x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-ArcCoth[a*x]/2),x]
```

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^2\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] time = 0.05, size = 33, normalized size = 0.34

$$-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-1/2*ArcCoth[a*x]),x]
```

```
[Out] (-8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, E^(2*ArcCoth[a*x]
)])/(3*a)
```

fricas [A] time = 0.59, size = 84, normalized size = 0.87

$$\frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) - 2*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1))/a
```

giac [A] time = 0.20, size = 108, normalized size = 1.11

$$-\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^2 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a*x
 - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4),x)
```

maxima [A] time = 0.41, size = 111, normalized size = 1.14

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) +
1)/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

mupad [B] time = 1.18, size = 79, normalized size = 0.81

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}-\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
- 1)/(a*x + 1))^(1/4))/a - atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.91 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{\frac{1}{ax}+1}}\ldots\right)$$

```
[Out] -2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))+1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)
/(1+1/a/x)^(1/2))*2^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*
2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6171, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{\frac{1}{ax}+1}}\ldots\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x),x]
```

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqr
t[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)\right) - 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\,\text{Subst}\left(\int \frac{1}{1+ \right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\,\text{Subs}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{}$$

$$= \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{}$$

Mathematica [C] time = 0.05, size = 30, normalized size = 0.10

$$\frac{8}{3}e^{\frac{3}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{8},1;\frac{11}{8};e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x),x]
```

```
[Out] (8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/8, 1, 11/8, E^(4*ArcCoth[a*x]
)])/3
```

fricas [A] time = 0.91, size = 291, normalized size = 1.00

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-}{ax+}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 2*sqrt
(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt
((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/2
*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
+ 1)) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

giac [A] time = 0.19, size = 232, normalized size = 0.80

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

maxima [A] time = 0.41, size = 224, normalized size = 0.77

$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\,\log\left(\sqrt{2}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a)
```

mupad [B] time = 1.18, size = 101, normalized size = 0.35

$$2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,2\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x, x)
```

3.92 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=268

$$-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{\frac{1}{ax}+1}}\right.}{\sqrt{2}}$$

```
[Out] -a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(
1/4))*2^(1/2)-1/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)
^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6171, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{\frac{1}{ax}+1}}\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

```
[Out] -(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)) - (a*ArcTan[1 - (Sqrt[2]*(1 -
1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (a*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(2*Sqrt[2]) + (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (2a)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (2a)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + a\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + a\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x\right.$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}a\,\text{Subst}\left(\int \right.$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\log\left(1+\right.}{}$$

Mathematica [C] time = 0.05, size = 33, normalized size = 0.12

$$-\frac{8}{3}ae^{\frac{3}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

```
[Out] (-8*a*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a
*x])])/3
```

fricas [A] time = 0.57, size = 368, normalized size = 1.37

$$-\frac{4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}\sqrt{a^2\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}}}{a^4}\right)+4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)
```

```
) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
4*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - sqr
t(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - sqrt(2)
*(a^4)^(1/4)*x*log(a^2*sqrt((a*x - 1)/(a*x + 1)) + sqrt(2)*(a^4)^(1/4)*a*((
a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)) + sqrt(2)*(a^4)^(1/4)*x*log(a^2*sqrt
((a*x - 1)/(a*x + 1)) - sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt(a^4)) + 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

giac [A] time = 0.16, size = 186, normalized size = 0.69

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

maxima [A] time = 0.41, size = 186, normalized size = 0.69

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-}{ax+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

mupad [B] time = 1.19, size = 88, normalized size = 0.33

$$-\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,1{}\mathrm{i}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^2,x)
```

```
[Out] - (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (-1)^(1/4)
*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (2*a*((a*x - 1)/(a*x
+ 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**2, x)
```

3.93 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{\frac{1}{ax}+}}\right.}{8\sqrt{2}}$$

```
[Out] 1/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(
3/4)-1/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-1/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)-1/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{\frac{1}{ax}+}}\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(5/4)*
(1 + 1/(a*x))^(3/4))/2 + (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}a\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,\right.$$
$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1\right.$$
$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.$$
$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.$$
$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{8}a^2\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,\right.$$
$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{8\sqrt{2}} -$$
$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{a^2\tan}{}$$

Mathematica [C] time = 0.08, size = 56, normalized size = 0.18

$$-\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left({}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)-2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

```
[Out] (-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCot
h[a*x])] - 2*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

fricas [A] time = 0.57, size = 396, normalized size = 1.24

$$\frac{4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*((a*
x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*x
+ 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))/a
^8) + 4*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*((a*x
 - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*x +
 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))/a^
8) - sqrt(2)*(a^8)^(1/4)*x^2*log(a^4*sqrt((a*x - 1)/(a*x + 1)) + sqrt(2)*(a
^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)) + sqrt(2)*(a^8)^(1/4
)*x^2*log(a^4*sqrt((a*x - 1)/(a*x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1
)/(a*x + 1))^(1/4) + sqrt(a^8)) + 4*(3*a^2*x^2 + a*x - 2)*((a*x - 1)/(a*x +
 1))^(1/4))/x^2
```

giac [A] time = 0.20, size = 223, normalized size = 0.70

$$-\frac{1}{16}\left(2\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\,a\log\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)
/(a*x + 1) + a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

maxima [A] time = 0.41, size = 227, normalized size = 0.71

$$-\frac{1}{16}\left(2\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\,a\log\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
```

```
rt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*a*((a*x - 1)/(a*x + 1))^(5/4) + a*((a*x
 - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
1))*a
```

mupad [B] time = 0.07, size = 132, normalized size = 0.41

$$\frac{\frac{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{5\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}+\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}}{4}+\frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^3,x)
```

```
[Out] ((a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (5*a^2*((a*x - 1)/(a*x + 1))^(5/4))/
2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*a^
2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4 + ((-1)^(1/4)*a^2*atan
h((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**3, x)
```

3.94 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=356

$$-\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}\right.}{16}$$

```
[Out] -3/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-1/12*a^3*(1-1/a/x)^(5/4)*(1+1/a/x)
^(3/4)+1/3*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)/x+3/16*a^3*arctan(-1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+(1-1/a/x)^(1/4)
*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-3/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+
1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+3/32*a^3*ln(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 90, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}-\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{\frac{1}{ax}+}}\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

```
[Out] (-3*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/8 - (a^3*(1 - 1/(a*x))^(5/
4)*(1 + 1/(a*x))^(3/4))/12 + (a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/
(3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(8*Sqrt[2]) - (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)
] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (3*a
^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}\left(-1+\frac{x}{2a}\right)}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,d\right.$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} -$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} +$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} +$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} +$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} +$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} -$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} -$$

Mathematica [C] time = 0.13, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(9\text{RootSum}\left[\#1^4+1\&, \frac{2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)+\coth^{-1}(ax)}{\#1^3}\&\right] - \frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(6e^{2\coth^{-1}(ax)}+9e^{4\coth^{-1}}\right.}{\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

```
[Out] (a^3*((-8*E^((3*ArcCoth[a*x])/2)*(29 + 6*E^(2*ArcCoth[a*x]) + 9*E^(4*ArcCot
h[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x]
 + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ]))/96
```

fricas [A] time = 0.52, size = 412, normalized size = 1.16

$$-\frac{36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\operatorname{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(36*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^3
*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1)
/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a
^12)))/a^12) + 36*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^(3
/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a
*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt(a^12)))/a^12) - 9*sqrt(2)*(a^12)^(1/4)*x^3*log(9*a^6*sqrt((a*x - 1)/(
a*x + 1)) + 9*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt
(a^12)) + 9*sqrt(2)*(a^12)^(1/4)*x^3*log(9*a^6*sqrt((a*x - 1)/(a*x + 1)) -
9*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt(a^12)) + 4*
(11*a^3*x^3 + a^2*x^2 - 2*a*x + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

giac [A] time = 0.21, size = 271, normalized size = 0.76

$$\frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}a^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(6*(a*x - 1)*a^2*((a*x - 1)/(
a*x + 1))^(1/4)/(a*x + 1) + 29*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/
(a*x + 1)^2 + 9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^
3)*a
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

maxima [A] time = 0.40, size = 277, normalized size = 0.78

$$\frac{1}{96}\left(18\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}a^2\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(29*a^2*((a*x - 1)/(a*x + 1))
^(9/4) + 6*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 9*a^2*((a*x - 1)/(a*x + 1))^(1
/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x
+ 1)^3 + 1))*a
```

mupad [B] time = 0.07, size = 169, normalized size = 0.47

$$-\frac{\frac{3a^3\left(\frac{ax-1}{ax+1}\right)^{1/4}}{4}+\frac{a^3\left(\frac{ax-1}{ax+1}\right)^{5/4}}{2}+\frac{29a^3\left(\frac{ax-1}{ax+1}\right)^{9/4}}{12}}{\frac{3(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+\frac{3(ax-1)}{ax+1}+1}-\frac{(-1)^{1/4}a^3\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)3\mathrm{i}}{8}-\frac{(-1)^{1/4}a^3\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{ax-}{ax+}\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^4,x)
```

```
[Out] - ((3*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (a^3*((a*x - 1)/(a*x + 1))^(5/4)
)/2 + (29*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1)^2
 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^3
*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8 - ((-1)^(1/4)*a^3*atanh
((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**4, x)
```

3.95 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$-\frac{237\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{557x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{640a^4}-\frac{157x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{320a^3}+\frac{5x^3\left(1-\frac{1}{ax}\right)}{16}$$

```
[Out] 557/640*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4-157/320*(1-1/a/x)^(3/4)*(1+1/
a/x)^(1/4)*x^2/a^3+5/16*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2-11/40*(1-1/
a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5-23
7/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-237/128*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A] time = 0.14, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{5x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{16a^2}-\frac{157x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{320a^3}+\frac{557x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{640a^4}-\frac{237\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\tanh^{-1}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (557*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(640*a^4) - (157*(1 - 1/(a*
x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(3/4)*(1 +
1/(a*x))^(1/4)*x^3)/(16*a^2) - (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*
x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 - (237*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (237*ArcTanh[(1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^6\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{11}{2a}+\frac{4x}{a^2}}{x^5\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{75}{4a^2}+\frac{33x}{2a^3}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)\right.$$

$$= \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 - \frac{1}{60}\text{Sul}$$

$$= -\frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1\right.$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1\right.}{}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1\right.}{}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1\right.}{}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1\right.}{}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1\right.}{}$$

Mathematica [A] time = 5.34, size = 173, normalized size = 0.68

$$\frac{\frac{5500e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} - \frac{14032e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{23936e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} - \frac{22016e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} + \frac{8192e^{\frac{17}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^5} + 1185\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)}{1280a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((8192*E^((17*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (22016*E^((13
*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((9*ArcCoth[a*x])
/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (14032*E^((5*ArcCoth[a*x])/2))/(-1 + E^(
2*ArcCoth[a*x]))^2 + (5500*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) +
2370*ArcTan[E^(-1/2*ArcCoth[a*x])] + 1185*Log[1 - E^(-1/2*ArcCoth[a*x])] -
1185*Log[1 + E^(-1/2*ArcCoth[a*x])])/(1280*a^5)
```

fricas [A] time = 0.56, size = 119, normalized size = 0.47

$$\frac{2\left(128\,a^5x^5-48\,a^4x^4+24\,a^3x^3-114\,a^2x^2+243\,ax+557\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1185\,\log\left(\left(\frac{ax}{ax}\right.\right.}{1280\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/1280*(2*(128*a^5*x^5 - 48*a^4*x^4 + 24*a^3*x^3 - 114*a^2*x^2 + 243*a*x +
557)*((a*x - 1)/(a*x + 1))^(3/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))
 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1185*log(((a*x - 1)/(a*x + 1
))^(1/4) - 1))/a^5
```

giac [A] time = 0.27, size = 234, normalized size = 0.92

$$\frac{1}{1280}\,a\left(\frac{2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{1185\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{4\left(\frac{1440\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 + 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 + 4*(1440*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3710*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 1992*(a*x - 1)^3*((a*x -
1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1375*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3
/4)/(a*x + 1)^4 - 395*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1
) - 1)^5))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

maxima [A] time = 0.41, size = 259, normalized size = 1.02

$$-\frac{1}{1280}\,a\left(\frac{4\left(1375\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+395\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\,\arctan\left(\right.}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/1280*a*(4*(1375*((a*x - 1)/(a*x + 1))^(19/4) - 1992*((a*x - 1)/(a*x + 1)
)^(15/4) + 3710*((a*x - 1)/(a*x + 1))^(11/4) - 1440*((a*x - 1)/(a*x + 1))^(
7/4) + 395*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*
x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a
^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x - 1
)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 1
185*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

mupad [B] time = 1.21, size = 229, normalized size = 0.91

$$\frac{\frac{79\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{64}-\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{371\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32}-\frac{249\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{40}+\frac{275\left(\frac{a\,x-1}{a\,x+1}\right)^{19/4}}{64}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}+\frac{237\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}-\frac{237\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((79*((a*x - 1)/(a*x + 1))^(3/4))/64 - (9*((a*x - 1)/(a*x + 1))^(7/4))/2 +
(371*((a*x - 1)/(a*x + 1))^(11/4))/32 - (249*((a*x - 1)/(a*x + 1))^(15/4))/
40 + (275*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*
x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)
^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (237*at
an(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (237*atanh(((a*x - 1)/(a*x + 1
))^(1/4)))/(128*a^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Timed out
```

3.96 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$\frac{123\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{63x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{64a^3}+\frac{15x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{32a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)$$

```
[Out] -63/64*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+15/32*(1-1/a/x)^(3/4)*(1+1/a/x
)^(1/4)*x^2/a^2-3/8*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^(3/
4)*(1+1/a/x)^(1/4)*x^4+123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+1
23/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] time = 0.12, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{15x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{32a^2}-\frac{63x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{64a^3}+\frac{123\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (-63*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(64*a^3) + (15*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(32*a^2) - (3*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
(123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^5\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{9}{2a}+\frac{3x}{a^2}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{45}{4a^2}+\frac{9x}{a^3}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\right.}\right.$$

$$= \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4 - \frac{1}{24}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(\right.$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(\right.$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(\right.$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(\right.$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(\right.$$

Mathematica [A] time = 5.27, size = 149, normalized size = 0.69

$$\frac{-\frac{532e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{1008e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} - \frac{1152e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{512e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} - 123\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right) + 123\log\left(e^{-\frac{1}{2}\coth}\right.}{128a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((512*E^((13*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (1152*E^((9*Ar
cCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((5*ArcCoth[a*x])/2))
/(-1 + E^(2*ArcCoth[a*x]))^2 - (532*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[
a*x])) - 246*ArcTan[E^(-1/2*ArcCoth[a*x])] - 123*Log[1 - E^(-1/2*ArcCoth[a*
x])] + 123*Log[1 + E^(-1/2*ArcCoth[a*x])])/(128*a^4)
```

fricas [A] time = 0.57, size = 111, normalized size = 0.51

$$\frac{2\left(16\,a^4x^4 - 8\,a^3x^3 + 6\,a^2x^2 - 33\,ax - 63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} - 246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) - 123\,\log}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/128*(2*(16*a^4*x^4 - 8*a^3*x^3 + 6*a^2*x^2 - 33*a*x - 63)*((a*x - 1)/(a*x
 + 1))^(3/4) - 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

giac [A] time = 0.29, size = 203, normalized size = 0.94

$$-\frac{1}{128}\,a\left(\frac{246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{123\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{183\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{147\,(ax-\right.}{(a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] -1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^5 + 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a
^5 - 4*(183*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 147*(a*x - 1)
^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 133*(a*x - 1)^3*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1)^3 - 41*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)
/(a*x + 1) - 1)^4))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

maxima [A] time = 0.40, size = 224, normalized size = 1.04

$$-\frac{1}{128}\,a\left(\frac{4\left(133\,\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-147\,\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+183\,\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-41\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123\,\log\left(\left(\frac{ax-1}{ax+1}\right.\right.}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/128*a*(4*(133*((a*x - 1)/(a*x + 1))^(15/4) - 147*((a*x - 1)/(a*x + 1))^(
11/4) + 183*((a*x - 1)/(a*x + 1))^(7/4) - 41*((a*x - 1)/(a*x + 1))^(3/4))/(
4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a
^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x - 1)
/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 123
*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

mupad [B] time = 1.18, size = 193, normalized size = 0.89

$$\frac{123\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{123\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{41\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{32}-\frac{183\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{32}+\frac{147\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32}-\frac{133\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{32}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (123*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (123*atan(((a*x - 1)/(a
*x + 1))^(1/4)))/(64*a^4) - ((41*((a*x - 1)/(a*x + 1))^(3/4))/32 - (183*((a
*x - 1)/(a*x + 1))^(7/4))/32 + (147*((a*x - 1)/(a*x + 1))^(11/4))/32 - (133
*((a*x - 1)/(a*x + 1))^(15/4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 -
 (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(
a*x - 1))/(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.97 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$-\frac{17\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{23x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{7x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{12a}$$

```
[Out] 23/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2-7/12*(1-1/a/x)^(3/4)*(1+1/a/x)^
(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3-17/8*arctan((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^3-17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6171, 99, 151, 12, 93, 212, 206, 203}

$$\frac{23x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}-\frac{17\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{7x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (23*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) - (7*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 - (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^4\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{1}{3}\mathrm{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{2x}{a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{1}{6}\mathrm{Subst}\left(\int \frac{-\frac{23}{4a^2}+\frac{7x}{2a^3}}{x^2\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\right.$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{1}{6}\mathrm{Subs}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{17\,\mathrm{Sub}}{}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{17\,\mathrm{Sub}}{}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{17\,\mathrm{Sub}}{}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{17\tan}{}$$

Mathematica [C] time = 8.65, size = 389, normalized size = 2.17

$$-\frac{e^{-\frac{7}{2}\coth^{-1}(ax)}\left(256e^{6\coth^{-1}(ax)}\left(850e^{2\coth^{-1}(ax)}+325e^{4\coth^{-1}(ax)}+557\right){}_4F_3\left(\frac{5}{4},2,2,2;1,1,\frac{17}{4};e^{2\coth^{-1}(ax)}\right)+2048e\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] -1/112320*(-15779205 - 17312841*E^(2*ArcCoth[a*x]) - 1213875*E^(4*ArcCoth[a
*x]) + 2199249*E^(6*ArcCoth[a*x]) + 15779205*Hypergeometric2F1[1/4, 1, 5/4,
 E^(2*ArcCoth[a*x])] + 14157000*E^(2*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1
, 5/4, E^(2*ArcCoth[a*x])] - 2472210*E^(4*ArcCoth[a*x])*Hypergeometric2F1[1
/4, 1, 5/4, E^(2*ArcCoth[a*x])] - 3598920*E^(6*ArcCoth[a*x])*Hypergeometric
2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 21645*E^(8*ArcCoth[a*x])*Hypergeomet
ric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 256*E^(6*ArcCoth[a*x])*(557 + 850
*E^(2*ArcCoth[a*x]) + 325*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{5/4, 2, 2,
 2}, {1, 1, 17/4}, E^(2*ArcCoth[a*x])] + 2048*E^(6*ArcCoth[a*x])*(19 + 34*E
^(2*ArcCoth[a*x]) + 15*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{5/4, 2, 2, 2,
 2}, {1, 1, 1, 17/4}, E^(2*ArcCoth[a*x])] + 4096*E^(6*ArcCoth[a*x])*Hyperge
ometricPFQ[{5/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 17/4}, E^(2*ArcCoth[a*x])] +
8192*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{5/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1
, 17/4}, E^(2*ArcCoth[a*x])] + 4096*E^(10*ArcCoth[a*x])*HypergeometricPFQ[{
5/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 17/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((7*Ar
cCoth[a*x])/2))
```

fricas [A] time = 0.50, size = 103, normalized size = 0.58

$$\frac{2\left(8a^3x^3-6a^2x^2+9ax+23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right.}{48a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 - 6*a^2*x^2 + 9*a*x + 23)*((a*x - 1)/(a*x + 1))^(3/4) +
102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 51*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1) + 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

giac [A] time = 0.22, size = 172, normalized size = 0.96

$$\frac{1}{48}a\left(\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{51\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{45(ax-1)^2\left(\right.}{(ax+1)}\right.}{a^4\left(\frac{ax-1}{ax+1}-1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 + 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 +
 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 45*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 17*((a*x - 1)/(a*x + 1))^(3/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

maxima [A] time = 0.41, size = 187, normalized size = 1.04

$$-\frac{1}{48}a\left(\frac{4\left(45\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+17\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{51\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(45*((a*x - 1)/(a*x + 1))^(11/4) - 30*((a*x - 1)/(a*x + 1))^(7/4
) + 17*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1
)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a*x
```

```
- 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 -
51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

mupad [B] time = 0.06, size = 157, normalized size = 0.88

$$\frac{\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}}+\frac{17\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8\,a^3}-\frac{17\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((17*((a*x - 1)/(a*x + 1))^(3/4))/12 - (5*((a*x - 1)/(a*x + 1))^(7/4))/2 +
(15*((a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2
 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*atan(
((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (17*atanh(((a*x - 1)/(a*x + 1))^(1/
4)))/(8*a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.98 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$\frac{9\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

```
[Out] -3/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4
)*x^2+9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 212, 206, 203}

$$\frac{9\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (-3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(7/4)
*(1 + 1/(a*x))^(1/4)*x^2)/2 + (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^
2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^3\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 + \frac{3\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^2\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 - \frac{9\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 - \frac{9\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 + \frac{9\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9}{}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 + \frac{9\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right)}{4a^2}$$

Mathematica [A] time = 0.20, size = 70, normalized size = 0.49

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(3e^{2\coth^{-1}(ax)}-7\right)}{\left(e^{2\coth^{-1}(ax)}-1\right)^2}+9\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+9\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(-7 + 3*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
]))^2 + 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^
2)
```

fricas [A] time = 0.53, size = 95, normalized size = 0.67

$$\frac{2\left(2a^2x^2-3ax-5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 3*a*x - 5)*((a*x - 1)/(a*x + 1))^(3/4) - 18*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log((
(a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

giac [A] time = 0.25, size = 141, normalized size = 0.99

$$-\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{7(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4*
(7*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3*((a*x - 1)/(a*x + 1)
)^(3/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

maxima [A] time = 0.41, size = 152, normalized size = 1.07

$$-\frac{1}{8}a\left(\frac{4\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(7*((a*x - 1)/(a*x + 1))^(7/4) - 3*((a*x - 1)/(a*x + 1))^(3/4))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan(
((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

mupad [B] time = 1.19, size = 121, normalized size = 0.85

$$\frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}-\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (9*atan(((a*x - 1)/(a*x +
1))^(1/4)))/(4*a^2) - ((3*((a*x - 1)/(a*x + 1))^(3/4))/2 - (7*((a*x - 1)/(a
*x + 1))^(7/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))
/(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.99 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
/a-3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A] time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6170, 94, 93, 212, 206, 203}

$$x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*ArcCoth[a*x])/2),x]
```

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a - (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^2\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)\\
&= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x+\frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x+\frac{6\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\\
&= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x-\frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}\\
&= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.56

$$\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}-3\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-3\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-3*ArcCoth[a*x])/2),x]
```

```
[Out] ((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) - 3*ArcTan[E^(ArcCoth[a*x
]/2)] - 3*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

fricas [A] time = 0.48, size = 86, normalized size = 0.88

$$\frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) + 6*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 3*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a
```

giac [A] time = 0.17, size = 109, normalized size = 1.11

$$\frac{1}{2}\,a\left(\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{2}}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{2}}+\frac{3\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^{2}}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^{2}\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 + 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*((
a*x - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4),x)
```

maxima [A] time = 0.41, size = 112, normalized size = 1.14

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^{2}}{ax+1}-a^{2}}-\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{2}}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{2}}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

mupad [B] time = 1.18, size = 79, normalized size = 0.81

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}-\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((a
*x - 1)/(a*x + 1))^(1/4)))/a - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.100 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}$$

```
[Out] 2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/
x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/
(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(
1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2
^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6171, 105, 63, 331, 297, 1162, 617, 204, 1165, 628, 93, 212, 206, 203}

$$-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqr
t[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)
/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt
[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)\right) - 4\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\,\text{Subst}\left(\int \right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\,\text{Sub}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\text{Subst}}{}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\left(1+\frac{\sqrt{1}}{\sqrt{1}}\right.}{}$$

$$= \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{ta}$$

Mathematica [C] time = 0.07, size = 28, normalized size = 0.10

$$8e^{\frac{1}{2}\coth^{-1}(ax)}{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

```
[Out] 8*E^(ArcCoth[a*x]/2)*Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x])]
```

fricas [A] time = 0.53, size = 291, normalized size = 1.00

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 2*sqrt
(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt
((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/2
*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
+ 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

giac [A] time = 0.17, size = 232, normalized size = 0.80

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

maxima [A] time = 0.42, size = 224, normalized size = 0.77

$$-\frac{1}{2}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a)
```

mupad [B] time = 1.18, size = 101, normalized size = 0.35

$$-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(-1+1\mathrm{i})+\sqrt{2}\,\mathrm{atan}\left(\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x,x)
```

```
[Out] - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(1
/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 -
1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1
i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x, x)
```

3.101 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=269

$$-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}$$

```
[Out] -a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+3/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(
1/4))*2^(1/2)+3/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-3/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)
^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6171, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] -(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) - (3*a*ArcTan[1 - (Sqrt[2]*(1
 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*Log[1 + Sqrt[1 -
1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1
/4)])/(2*Sqrt[2]) - (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqr
t[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$
$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$
$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (6a)\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$
$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (6a)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$
$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - (3a)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + (3a)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,d\right.$$
$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}(3a)\,\text{Subst}\left(\right.$$
$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}}$$
$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\log\left(1\right.}{}$$

Mathematica [A] time = 0.30, size = 149, normalized size = 0.55

$$a\left(-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} + \frac{3\log\left(-\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} - \frac{3\log\left(\sqrt{2}e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}+1\right)}{2\sqrt{2}} + \frac{3\tan^{-1}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] a*((-2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + (3*ArcTan[1 - Sqrt[2]
*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/
Sqrt[2] + (3*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[
2]) - (3*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

fricas [A] time = 0.57, size = 402, normalized size = 1.49

$$-\frac{12\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^4}\,a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^4\right)^{\frac{1}{4}}}{a^4}\right)+12\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4\ldots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*a^3*((a*x
 - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(
a^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4)
)/a^4) + 12*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(1/4)*a^3*((a
*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqr
t(a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/
4))/a^4) + 3*sqrt(2)*(a^4)^(1/4)*x*log(729*a^6*sqrt((a*x - 1)/(a*x + 1)) +
729*sqrt(a^4)*a^4 + 729*sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
) - 3*sqrt(2)*(a^4)^(1/4)*x*log(729*a^6*sqrt((a*x - 1)/(a*x + 1)) + 729*sqr
t(a^4)*a^4 - 729*sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(
a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/x
```

giac [A] time = 0.16, size = 187, normalized size = 0.70

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

maxima [A] time = 0.46, size = 187, normalized size = 0.70

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

mupad [B] time = 0.05, size = 88, normalized size = 0.33

$$3\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-3\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^2,x)
```

```
[Out] 3*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 3*(-1)^(1/4)*
a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (2*a*((a*x - 1)/(a*x + 1)
)^(3/4))/((a*x - 1)/(a*x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**2, x)
```

3.102 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}}{}\right.}{8\sqrt{2}}$$

```
[Out] 3/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^(
1/4)-9/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)+9/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}}{}\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (3*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4
)*(1 + 1/(a*x))^(1/4))/2 + (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{4}(3a)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{8}(9a)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right.}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,d\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x,\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x,\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{8}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.}{8\sqrt{2}}$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{9a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}} - \frac{9a}{}$$

Mathematica [A] time = 0.22, size = 174, normalized size = 0.55

$$\frac{1}{16}a^2\left(\frac{24e^{\frac{1}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}+1} + \frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}+1\right)^2} - 9\sqrt{2}\log\left(-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)} + e^{\coth^{-1}(ax)} + 1\right) + 9\sqrt{2}\log\left(\sqrt{2}\,e^{\frac{1}{2}\coth}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (a^2*((32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (24*E^(ArcCoth[a
*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 18*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCoth
[a*x]/2)] + 18*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] - 9*Sqrt[2]*L
og[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] + 9*Sqrt[2]*Log[1 + Sqr
t[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

fricas [A] time = 0.53, size = 419, normalized size = 1.31

$$36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - \sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}} + \sqrt{a^8}\,a^8 + \sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right) + 36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(36*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(1/4)*a^6*((a
*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1)) + sq
rt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8)^(1
/4))/a^8) + 36*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(1/4)*a^
6*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1))
 + sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^
8)^(1/4))/a^8) + 9*sqrt(2)*(a^8)^(1/4)*x^2*log(531441*a^12*sqrt((a*x - 1)/(
a*x + 1)) + 531441*sqrt(a^8)*a^8 + 531441*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1
)/(a*x + 1))^(1/4)) - 9*sqrt(2)*(a^8)^(1/4)*x^2*log(531441*a^12*sqrt((a*x -
 1)/(a*x + 1)) + 531441*sqrt(a^8)*a^8 - 531441*sqrt(2)*(a^8)^(3/4)*a^6*((a*
x - 1)/(a*x + 1))^(1/4)) + 4*(5*a^2*x^2 + 3*a*x - 2)*((a*x - 1)/(a*x + 1))^
(3/4))/x^2
```

giac [A] time = 0.23, size = 225, normalized size = 0.71

$$-\frac{1}{16}\left(18\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-9\sqrt{2}\,a\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(7*(a*x - 1)*a*((a*x - 1)/(a*x + 1))
^(3/4)/(a*x + 1) + 3*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) +
1)^2)*a
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

maxima [A] time = 0.42, size = 228, normalized size = 0.71

$$-\frac{1}{16}\left(9\left(2\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\,\log\left(\sqrt{2}\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1))*a - 8*(7*a*((a*x - 1)/(a*x + 1))^(7/4) + 3*a*((a*x
 - 1)/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
1))*a
```

mupad [B] time = 1.18, size = 132, normalized size = 0.41

$$\frac{\frac{3\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}+\frac{7\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}-\frac{9\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4}+\frac{9\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^3,x)
```

```
[Out] ((3*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (7*a^2*((a*x - 1)/(a*x + 1))^(7/4)
)/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - (9*(-1)^(1/4
)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 + (9*(-1)^(1/4)*a^2*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**3, x)
```

3.103 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\right.}{16\sqrt{2}}$$

```
[Out] -17/24*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-1/4*a^3*(1-1/a/x)^(7/4)*(1+1/a/x
)^(1/4)+1/3*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4)/x+17/16*a^3*arctan(-1+(1-1/
a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+(1-1/a/x)^(1
/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)
/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-17/32*a^3*ln(1+(1
-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1
/2)
```

Rubi [A] time = 0.28, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 90, 80, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}+\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (-17*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/24 - (a^3*(1 - 1/(a*x))^(
7/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))
/(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/
4)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/(8*Sqrt[2]) + (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(
a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) -
(17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}\left(-1+\frac{3x}{2a}\right)}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{1}{24}\left(17a^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x\right.$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{1}{16}(17$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{4}(17$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{4}(17$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{1}{8}(17$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{16}(17$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{17a^3\,1}{}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{17a^3\,\mathrm{t}}{}$$

Mathematica [C] time = 0.15, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(51\text{RootSum}\left[\#1^4+1\&,\frac{2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)+\coth^{-1}(ax)}{\#1}\&\right]-\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(30e^{2\coth^{-1}(ax)}+17e^{4\coth^{-1}(ax)}\right.}{\left(e^{2\coth^{-1}(ax)}+1\right)^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (a^3*((-8*E^(ArcCoth[a*x]/2)*(45 + 30*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth[
a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*x]
+ 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1 & ]))/96
```

fricas [A] time = 0.62, size = 427, normalized size = 1.20

$$-\frac{204\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+204\sqrt{2}\left(a^{12}\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(1/4)*a^
9*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(a*x + 1))
 + sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4))*
(a^12)^(1/4))/a^12) + 204*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(
a^12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^18*sqrt((a*x -
 1)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x
 + 1))^(1/4))*(a^12)^(1/4))/a^12) + 51*sqrt(2)*(a^12)^(1/4)*x^3*log(2413756
9*a^18*sqrt((a*x - 1)/(a*x + 1)) + 24137569*sqrt(a^12)*a^12 + 24137569*sqrt
(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) - 51*sqrt(2)*(a^12)^(1/4)
*x^3*log(24137569*a^18*sqrt((a*x - 1)/(a*x + 1)) + 24137569*sqrt(a^12)*a^12
 - 24137569*sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(23*a
^3*x^3 + 9*a^2*x^2 - 6*a*x + 8)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

giac [A] time = 0.25, size = 271, normalized size = 0.76

$$\frac{1}{96}\left(102\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-51\sqrt{2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) - 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(30*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1) + 45*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
3/4)/(a*x + 1)^2 + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1)
 + 1)^3)*a
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

maxima [A] time = 0.41, size = 270, normalized size = 0.76

$$\frac{1}{96}\left(51\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(51*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1))*a^2 - 8*(45*a^2*((a*x - 1)/(a*x + 1))^(11/4) + 30*
a^2*((a*x - 1)/(a*x + 1))^(7/4) + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a
*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1
))*a
```

mupad [B] time = 0.06, size = 169, normalized size = 0.47

$$\frac{17\,(-1)^{1/4}\,a^3\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8}-\frac{\frac{17\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}+\frac{5\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\,a^3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{(a\,x-1)^3}{(a\,x+1)^3}+\frac{3\,(a\,x-1)}{a\,x+1}+1}-\frac{17\,(-1)^{1/4}\,a^3\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x}{a\,x}\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^4,x)
```

```
[Out] (17*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - ((17*a
^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (5*a^3*((a*x - 1)/(a*x + 1))^(7/4))/2
+ (15*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^2 + (
a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - (17*(-1)^(1/4)*a^3*
atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**4, x)
```

3.104 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=287

$$\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{\frac{1}{ax}+1}}+\frac{1003\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{5533x\sqrt[4]{1-\frac{1}{ax}}}{1920a^4\sqrt[4]{\frac{1}{ax}+1}}-\frac{1189x^2\sqrt[4]{1-\frac{1}{ax}}}{960a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{181x^3\sqrt[4]{1\cdots}}{240a^2\sqrt[4]{\cdots}}$$

```
[Out] 26111/1920*(1-1/a/x)^(1/4)/a^5/(1+1/a/x)^(1/4)+5533/1920*(1-1/a/x)^(1/4)*x/
a^4/(1+1/a/x)^(1/4)-1189/960*(1-1/a/x)^(1/4)*x^2/a^3/(1+1/a/x)^(1/4)+181/24
0*(1-1/a/x)^(1/4)*x^3/a^2/(1+1/a/x)^(1/4)-21/40*(1-1/a/x)^(1/4)*x^4/a/(1+1/
a/x)^(1/4)+1/5*(1-1/a/x)^(1/4)*x^5/(1+1/a/x)^(1/4)+1003/128*arctan((1+1/a/x
)^(1/4)/(1-1/a/x)^(1/4))/a^5-1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/
4))/a^5
```

Rubi [A] time = 0.16, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{181x^3\sqrt[4]{1-\frac{1}{ax}}}{240a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{1189x^2\sqrt[4]{1-\frac{1}{ax}}}{960a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{5533x\sqrt[4]{1-\frac{1}{ax}}}{1920a^4\sqrt[4]{\frac{1}{ax}+1}}+\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{\frac{1}{ax}+1}}+\frac{1003\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{1\cdots}}\right)}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (26111*(1 - 1/(a*x))^(1/4))/(1920*a^5*(1 + 1/(a*x))^(1/4)) + (5533*(1 - 1/(
a*x))^(1/4)*x)/(1920*a^4*(1 + 1/(a*x))^(1/4)) - (1189*(1 - 1/(a*x))^(1/4)*x
^2)/(960*a^3*(1 + 1/(a*x))^(1/4)) + (181*(1 - 1/(a*x))^(1/4)*x^3)/(240*a^2*
(1 + 1/(a*x))^(1/4)) - (21*(1 - 1/(a*x))^(1/4)*x^4)/(40*a*(1 + 1/(a*x))^(1/
4)) + ((1 - 1/(a*x))^(1/4)*x^5)/(5*(1 + 1/(a*x))^(1/4)) + (1003*ArcTan[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (1003*ArcTanh[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^6\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{5}\text{Subst}\left(\int \frac{\frac{21}{2a}-\frac{10x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{20}\text{Subst}\left(\int \frac{\frac{181}{4a^2}-\frac{42x}{a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{60}\text{Subst}\left(\int \frac{\frac{1189}{8a^3}-\frac{543x}{4a^4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,d\right.$$

$$= -\frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{120}\text{Subst}\left(\int \frac{}{x^2}\right.$$

$$= \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} +$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x}{40a\sqrt[4]{1+\frac{1}{ax}}}$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x}{40a\sqrt[4]{1+\frac{1}{ax}}}$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x}{40a\sqrt[4]{1+\frac{1}{ax}}}$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x}{40a\sqrt[4]{1+\frac{1}{ax}}}$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x}{40a\sqrt[4]{1+\frac{1}{ax}}}$$

<u>**Mathematica**</u> **[A]** time = 5.43, size = 198, normalized size = 0.69

$$\frac{8e^{-\frac{1}{2}\coth^{-1}(ax)} - \frac{4117e^{-\frac{1}{2}\coth^{-1}(ax)}}{192\left(e^{-2\coth^{-1}(ax)}-1\right)} - \frac{1661e^{-\frac{1}{2}\coth^{-1}(ax)}}{48\left(e^{-2\coth^{-1}(ax)}-1\right)^2} - \frac{233e^{-\frac{1}{2}\coth^{-1}(ax)}}{6\left(e^{-2\coth^{-1}(ax)}-1\right)^3} - \frac{122e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(e^{-2\coth^{-1}(ax)}-1\right)^4} - \frac{32e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(e^{-2\coth^{-1}(ax)}-1\right)^5} + \cdots}{a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (8/E^(ArcCoth[a*x]/2) - 32/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))
^5) - 122/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^4) - 233/(6*E^(A
rcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^3) - 1661/(48*E^(ArcCoth[a*x]/2)*
(-1 + E^(-2*ArcCoth[a*x]))^2) - 4117/(192*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*Ar
cCoth[a*x]))) - (1003*ArcTan[E^(-1/2*ArcCoth[a*x])])/128 + (1003*Log[1 - E^
(-1/2*ArcCoth[a*x])])/256 - (1003*Log[1 + E^(-1/2*ArcCoth[a*x])])/256)/a^5
```

fricas [A] time = 1.17, size = 119, normalized size = 0.41

$$\frac{2\left(384\,a^5x^5-1008\,a^4x^4+1448\,a^3x^3-2378\,a^2x^2+5533\,ax+26111\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-15045}{3840\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 - 1008*a^4*x^4 + 1448*a^3*x^3 - 2378*a^2*x^2 + 5533*
a*x + 26111)*((a*x - 1)/(a*x + 1))^(1/4) - 30090*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*log(((a*x -
1)/(a*x + 1))^(1/4) - 1))/a^5
```

giac [A] time = 0.21, size = 254, normalized size = 0.89

$$-\frac{1}{3840}\,a\left(\frac{30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{15045\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{30720\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^6}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^6 - 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4)
- 1))/a^6 - 30720*((a*x - 1)/(a*x + 1))^(1/4)/a^6 - 4*(33816*(a*x - 1)*((a*
x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x + 1))
^(1/4)/(a*x + 1)^2 + 49120*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1
)^3 - 20585*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^4 - 7365*((a*
x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

maxima [A] time = 0.41, size = 279, normalized size = 0.97

$$-\frac{1}{3840}\,a\left(\frac{4\left(20585\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-49120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+61130\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-33816\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+7365\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{30090\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(20585*((a*x - 1)/(a*x + 1))^(17/4) - 49120*((a*x - 1)/(a*x +
1))^(13/4) + 61130*((a*x - 1)/(a*x + 1))^(9/4) - 33816*((a*x - 1)/(a*x + 1)
)^(5/4) + 7365*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10
*(a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)
^4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 30090*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^6 - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6 - 30720*((a*x - 1)/(a*x
 + 1))^(1/4)/a^6)
```

mupad [B] time = 0.08, size = 253, normalized size = 0.88

$$\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^5}+\frac{\frac{491\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{1409\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{6113\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{307\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{6}+\frac{4117\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{192}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}-\frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*1003i)/(128*a^5) + (8*((a*x - 1)/(a*x
 + 1))^(1/4))/a^5 + ((491*((a*x - 1)/(a*x + 1))^(1/4))/64 - (1409*((a*x - 1
)/(a*x + 1))^(5/4))/40 + (6113*((a*x - 1)/(a*x + 1))^(9/4))/96 - (307*((a*x
 - 1)/(a*x + 1))^(13/4))/6 + (4117*((a*x - 1)/(a*x + 1))^(17/4))/192)/(a^5
+ (10*a^5*(a*x - 1)^2)/(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*
a^5*(a*x - 1)^4)/(a*x + 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x
- 1))/(a*x + 1)) - (1003*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

3.105 $\int e^{-\frac{5}{2}\coth^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=250

$$-\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{\frac{1}{ax}+1}}-\frac{475\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{521x\sqrt[4]{1-\frac{1}{ax}}}{192a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{113x^2\sqrt[4]{1-\frac{1}{ax}}}{96a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}-\frac{17x^3}{24a}$$

```
[Out] -2467/192*(1-1/a/x)^(1/4)/a^4/(1+1/a/x)^(1/4)-521/192*(1-1/a/x)^(1/4)*x/a^3
/(1+1/a/x)^(1/4)+113/96*(1-1/a/x)^(1/4)*x^2/a^2/(1+1/a/x)^(1/4)-17/24*(1-1/
a/x)^(1/4)*x^3/a/(1+1/a/x)^(1/4)+1/4*(1-1/a/x)^(1/4)*x^4/(1+1/a/x)^(1/4)-47
5/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A] time = 0.14, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6171, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{113x^2\sqrt[4]{1-\frac{1}{ax}}}{96a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{521x\sqrt[4]{1-\frac{1}{ax}}}{192a^3\sqrt[4]{\frac{1}{ax}+1}}-\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{\frac{1}{ax}+1}}-\frac{475\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}-\frac{17x^3}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-2467*(1 - 1/(a*x))^(1/4))/(192*a^4*(1 + 1/(a*x))^(1/4)) - (521*(1 - 1/(a*
x))^(1/4)*x)/(192*a^3*(1 + 1/(a*x))^(1/4)) + (113*(1 - 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 + 1/(a*x))^(1/4)) - (17*(1 - 1/(a*x))^(1/4)*x^3)/(24*a*(1 + 1/(a
*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) - (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^5\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{4}\mathrm{Subst}\left(\int \frac{\frac{17}{2a}-\frac{8x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{12}\mathrm{Subst}\left(\int \frac{\frac{113}{4a^2}-\frac{51x}{2a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{24}\mathrm{Subst}\left(\int \frac{\frac{521}{8a^3}-\frac{113x}{2a^4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x\right.$$

$$= -\frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{24}\mathrm{Subst}\left(\int \frac{\frac{142}{16a}}{x\left(1-\frac{x}{a}\right)}\right.$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{12}a$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475}{}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475}{}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{475}{}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{475}{}$$

Mathematica [A] time = 5.32, size = 161, normalized size = 0.64

$$\frac{-3072e^{-\frac{1}{2}\coth^{-1}(ax)} - \frac{6292e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} + \frac{7376e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} - \frac{5248e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + \frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^4} - 1425\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right.}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-3072/E^(ArcCoth[a*x]/2) + (1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCo
th[a*x]))^4 - (5248*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 +
(7376*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 - (6292*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(-1/2*ArcCoth[a*x]
```

```
)] - 1425*Log[1 - E^(-1/2*ArcCoth[a*x])] + 1425*Log[1 + E^(-1/2*ArcCoth[a*x
])])/(384*a^4)
```

fricas [A] time = 0.65, size = 111, normalized size = 0.44

$$\frac{2\left(48\,a^4x^4-136\,a^3x^3+226\,a^2x^2-521\,ax-2467\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right.}{384\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 - 136*a^3*x^3 + 226*a^2*x^2 - 521*a*x - 2467)*((a*x -
1)/(a*x + 1))^(1/4) + 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 1425*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^4
```

giac [A] time = 0.23, size = 223, normalized size = 0.89

$$\frac{1}{384}a\left(\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{1425\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{3072\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^5}+\frac{4\left(\frac{23}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] 1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))
/a^5 - 3072*((a*x - 1)/(a*x + 1))^(1/4)/a^5 + 4*(2343*(a*x - 1)*((a*x - 1)/
(a*x + 1))^(1/4)/(a*x + 1) - 2875*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(
a*x + 1)^2 + 1573*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 657
*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

maxima [A] time = 0.41, size = 244, normalized size = 0.98

$$-\frac{1}{384}a\left(\frac{4\left(1573\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-2875\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+2343\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-657\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/384*a*(4*(1573*((a*x - 1)/(a*x + 1))^(13/4) - 2875*((a*x - 1)/(a*x + 1))
^(9/4) + 2343*((a*x - 1)/(a*x + 1))^(5/4) - 657*((a*x - 1)/(a*x + 1))^(1/4)
)/(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^
3*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 2850*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5
+ 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5 + 3072*((a*x - 1)/(a*x + 1)
)^(1/4)/a^5)
```

mupad [B] time = 0.08, size = 217, normalized size = 0.87

$$\frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{219\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{781\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{2875\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{1573\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}-\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^4}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (475*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (8*((a*x - 1)/(a*x + 1))
^(1/4))/a^4 - ((219*((a*x - 1)/(a*x + 1))^(1/4))/32 - (781*((a*x - 1)/(a*x
+ 1))^(5/4))/32 + (2875*((a*x - 1)/(a*x + 1))^(9/4))/96 - (1573*((a*x - 1)/
(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x
 - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a
*x + 1)) - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*475i)/(64*a^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

3.106 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=213

$$\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{55\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{61x\sqrt[4]{1-\frac{1}{ax}}}{24a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}-\frac{13x^2\sqrt[4]{1-\frac{1}{ax}}}{12a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[Out] 287/24*(1-1/a/x)^(1/4)/a^3/(1+1/a/x)^(1/4)+61/24*(1-1/a/x)^(1/4)*x/a^2/(1+1
/a/x)^(1/4)-13/12*(1-1/a/x)^(1/4)*x^2/a/(1+1/a/x)^(1/4)+1/3*(1-1/a/x)^(1/4)
*x^3/(1+1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3-55/8*
arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A] time = 0.11, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6171, 98, 151, 155, 12, 93, 298, 203, 206}

$$\frac{61x\sqrt[4]{1-\frac{1}{ax}}}{24a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{55\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}-\frac{13x^2\sqrt[4]{1-\frac{1}{ax}}}{12a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (287*(1 - 1/(a*x))^(1/4))/(24*a^3*(1 + 1/(a*x))^(1/4)) + (61*(1 - 1/(a*x))^
(1/4)*x)/(24*a^2*(1 + 1/(a*x))^(1/4)) - (13*(1 - 1/(a*x))^(1/4)*x^2)/(12*a*
(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) +
(55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) - (55*ArcTanh[
(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^4\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}\text{Subst}\left(\int \frac{\frac{13}{2a}-\frac{6x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{61}{4a^2}-\frac{13x}{a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{6}\text{Subst}\left(\int \frac{\frac{165}{8a^3}-\frac{61x}{4a^4}}{x\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{}{16a^4x\,(1}\right.$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\,\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{}}\right.}{16a^3}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,\right.}{4a^3}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} - \frac{55\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.}{8a^3}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3} - $$

Mathematica [A] time = 5.29, size = 137, normalized size = 0.64

$$\frac{384e^{-\frac{1}{2}\coth^{-1}(ax)} + \frac{548e^{\frac{3}{2}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} - \frac{400e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} + \frac{128e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} + 165\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right) - 165\log\left(e^{-\frac{1}{2}\coth}\right.}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (384/E^(ArcCoth[a*x]/2) + (128*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[
a*x]))^3 - (400*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (548*
E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) - 330*ArcTan[E^(-1/2*ArcC
oth[a*x])] + 165*Log[1 - E^(-1/2*ArcCoth[a*x])] - 165*Log[1 + E^(-1/2*ArcCo
th[a*x])])/(48*a^3)
```

fricas [A] time = 0.81, size = 103, normalized size = 0.48

$$\frac{2\left(8\,a^{3}x^{3}-26\,a^{2}x^{2}+61\,ax+287\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right.}{48\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 - 26*a^2*x^2 + 61*a*x + 287)*((a*x - 1)/(a*x + 1))^(1/4)
 - 330*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*log(((a*x - 1)/(a*x + 1))^
(1/4) + 1) + 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

giac [A] time = 0.21, size = 192, normalized size = 0.90

$$-\frac{1}{48}\,a\left(\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{4}}+\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{4}}-\frac{165\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^{4}}-\frac{384\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^{4}}-\frac{4\left(\frac{174\,(ax-1)\left(\frac{a}{a}\right.}{ax+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^4 - 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
4 - 384*((a*x - 1)/(a*x + 1))^(1/4)/a^4 - 4*(174*(a*x - 1)*((a*x - 1)/(a*x
+ 1))^(1/4)/(a*x + 1) - 137*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x +
1)^2 - 69*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

maxima [A] time = 0.41, size = 207, normalized size = 0.97

$$-\frac{1}{48}\,a\left(\frac{4\left(137\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-174\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+69\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3\,(ax-1)a^{4}}{ax+1}-\frac{3\,(ax-1)^{2}a^{4}}{(ax+1)^{2}}+\frac{(ax-1)^{3}a^{4}}{(ax+1)^{3}}-a^{4}}+\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{4}}+\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{4}}-\frac{165\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(137*((a*x - 1)/(a*x + 1))^(9/4) - 174*((a*x - 1)/(a*x + 1))^(5/
4) + 69*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x -
1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 330*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4
```

```
 - 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4 - 384*((a*x - 1)/(a*x + 1))
^(1/4)/a^4)
```

mupad [B] time = 0.07, size = 181, normalized size = 0.85

$$\frac{\frac{23\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{137\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^3}-\frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,55\mathrm{i}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] ((23*((a*x - 1)/(a*x + 1))^(1/4))/4 - (29*((a*x - 1)/(a*x + 1))^(5/4))/2 +
(137*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^
2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (atan(((
a*x - 1)/(a*x + 1))^(1/4)*1i)*55i)/(8*a^3) + (8*((a*x - 1)/(a*x + 1))^(1/4)
)/a^3 - (55*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

3.107 $\int e^{-\frac{5}{2}\coth^{-1}(ax)} x \, dx$

Optimal. Leaf size=176

$$-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{25\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}-\frac{5x\left(1-\frac{1}{ax}\right)^{5/4}}{4a\sqrt[4]{\frac{1}{ax}+1}}$$

```
[Out] -25/2*(1-1/a/x)^(1/4)/a^2/(1+1/a/x)^(1/4)-5/4*(1-1/a/x)^(5/4)*x/a/(1+1/a/x)
^(1/4)+1/2*(1-1/a/x)^(9/4)*x^2/(1+1/a/x)^(1/4)-25/4*arctan((1+1/a/x)^(1/4)/
(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A] time = 0.07, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6171, 96, 94, 93, 298, 203, 206}

$$-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{\frac{1}{ax}+1}}-\frac{25\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}-\frac{5x\left(1-\frac{1}{ax}\right)^{5/4}}{4a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-25*(1 - 1/(a*x))^(1/4))/(2*a^2*(1 + 1/(a*x))^(1/4)) - (5*(1 - 1/(a*x))^(5
/4)*x)/(4*a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(9/4)*x^2)/(2*(1 + 1/(a*x
))^(1/4)) - (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^3\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^2\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{2a^2}$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} + \frac{25\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} - 25$$

$$= -\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}} - \frac{25\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{25\tanh^{-1}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right)}{4a^2}$$

Mathematica [A] time = 0.27, size = 121, normalized size = 0.69

$$-\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(-90e^{2\coth^{-1}(ax)}+50e^{4\coth^{-1}(ax)}+25e^{\frac{1}{2}\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}-1\right)^2\tan^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-25e^{\frac{1}{2}\coth^{-1}(a}\right.}{4a^2\left(e^{2\coth^{-1}(ax)}-1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] -1/4*(32 - 90*E^(2*ArcCoth[a*x]) + 50*E^(4*ArcCoth[a*x]) + 25*E^(ArcCoth[a*
x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTan[E^(ArcCoth[a*x]/2)] - 25*E^(ArcCot
h[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2*ArcTanh[E^(ArcCoth[a*x]/2)])/(a^2*E^(
ArcCoth[a*x]/2)*(-1 + E^(2*ArcCoth[a*x]))^2)
```

fricas [A] time = 0.90, size = 95, normalized size = 0.54

$$\frac{2\left(2a^2x^2-9ax-43\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 9*a*x - 43)*((a*x - 1)/(a*x + 1))^(1/4) + 50*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 25*lo
g(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

giac [A] time = 0.19, size = 161, normalized size = 0.91

$$\frac{1}{8}a\left(\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{64\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^3}+\frac{4\left(\frac{13(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\right.}{a^3\left(\frac{ax-1}{ax+1}-1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] 1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 - 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 6
4*((a*x - 1)/(a*x + 1))^(1/4)/a^3 + 4*(13*(a*x - 1)*((a*x - 1)/(a*x + 1))^(
1/4)/(a*x + 1) - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^3*((a*x - 1)/(a*x + 1) -
 1)^2))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

maxima [A] time = 0.41, size = 172, normalized size = 0.98

$$-\frac{1}{8}\,a\left(\frac{4\left(13\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{50\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(13*((a*x - 1)/(a*x + 1))^(5/4) - 9*((a*x - 1)/(a*x + 1))^(1/4))/
(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 50*arctan
(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^3 + 25*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3 + 64*((a*x - 1)/(a*x + 1
))^(1/4)/a^3)
```

mupad [B] time = 1.20, size = 145, normalized size = 0.82

$$\frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^2}-\frac{\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{13\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,25\mathrm{i}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (25*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - ((9*((a*x - 1)/(a*x + 1))^
(1/4))/2 - (13*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*
x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (8*((a*x - 1)/(a*x + 1))^(1/4))/a
^2 - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*25i)/(4*a^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

3.108 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=130

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{\frac{1}{ax}+1}}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] 10*(1-1/a/x)^(1/4)/a/(1+1/a/x)^(1/4)+(1-1/a/x)^(5/4)*x/(1+1/a/x)^(1/4)+5*ar
ctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a-5*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)
^(1/4))/a
```

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6170, 94, 93, 298, 203, 206}

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{\frac{1}{ax}+1}}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-5*ArcCoth[a*x])/2),x]
```

```
[Out] (10*(1 - 1/(a*x))^(1/4))/(a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(5/4)*x)/
(1 + 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
 - (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^2\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\,\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{10\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.}{a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{5\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] time = 0.08, size = 31, normalized size = 0.24

$$\frac{8e^{-\frac{1}{2}\coth^{-1}(ax)}\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};e^{2\coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((-5*ArcCoth[a*x])/2),x]
```

```
[Out] (8*Hypergeometric2F1[-1/4, 2, 3/4, E^(2*ArcCoth[a*x])])/(a*E^(ArcCoth[a*x]/
2))
```

fricas [A] time = 0.47, size = 86, normalized size = 0.66

$$\frac{2\,(ax+9)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) - 5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right) + 5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 9)*((a*x - 1)/(a*x + 1))^(1/4) - 10*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*log(((a*x - 1)/(a*x
 + 1))^(1/4) - 1))/a
```

giac [A] time = 0.16, size = 129, normalized size = 0.99

$$-\frac{1}{2}a\left(\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^2 - 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 16
*((a*x - 1)/(a*x + 1))^(1/4)/a^2 + 4*((a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x
 - 1)/(a*x + 1) - 1)))
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4),x)
```

maxima [A] time = 0.41, size = 132, normalized size = 1.02

$$-\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 10*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x + 1))^(1/4)
 + 1)/a^2 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2 - 16*((a*x - 1)/(a*x
 + 1))^(1/4)/a^2)
```

mupad [B] time = 1.19, size = 103, normalized size = 0.79

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a}-\frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,5\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (atan(((a*x
 - 1)/(a*x + 1))^(1/4)*1i)*5i)/a + (8*((a*x - 1)/(a*x + 1))^(1/4))/a - (5*a
tan(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4), x)
```

3.109 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=320

$$-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan$$

```
[Out] -8*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/
2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+
arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {6171, 98, 21, 105, 63, 240, 211, 1165, 628, 1162, 617, 204, 93, 298, 203, 206}

$$-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```

```
[Out] (-8*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - (4a)\,\text{Subst}\left(\int \frac{\frac{1}{4a}+\frac{x}{4a^2}}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} + 4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} + 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{\sqrt{2}}$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}$$
$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$

Mathematica [C] time = 0.10, size = 28, normalized size = 0.09

$$-8e^{-\frac{1}{2}\coth^{-1}(ax)}\,{}_2F_1\left(-\frac{1}{8},1;\frac{7}{8};e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x),x]
[Out] (-8*Hypergeometric2F1[-1/8, 1, 7/8, E^(4*ArcCoth[a*x])])/E^(ArcCoth[a*x]/2)
```

fricas [A] time = 0.68, size = 308, normalized size = 0.96

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax}{ax}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) - 8*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*
x - 1)/(a*x + 1))^(1/4) - 1)
```

giac [A] time = 0.17, size = 252, normalized size = 0.79

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

maxima [A] time = 0.42, size = 244, normalized size = 0.76

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
```

```
 - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

mupad [B] time = 1.14, size = 118, normalized size = 0.37

$$2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)-8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)2\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(1+1\mathrm{i})+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
 - 8*((a*x - 1)/(a*x + 1))^(1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x, x)
```

3.110 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=299

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+5a\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+5a\,\mathrm{ta}$$

```
[Out] 4*a*(1-1/a/x)^(5/4)/(1+1/a/x)^(1/4)+5*a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-5/2
*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arctan(
1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+5/4*a*ln(1-(1-1/a/x)^(1/
4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-5/4*a*l
n(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
)*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+5a\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+5a\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] (4*a*(1 - 1/(a*x))^(5/4))/(1 + 1/(a*x))^(1/4) + 5*a*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/Sqrt[2] + (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) - (5*a*Log
[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5\operatorname{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (10a)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (10a)\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (5a)\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - (5a)\operatorname{Su}$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}(5a)\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{5a\log\left(1+\right.}{}$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

Mathematica [C] time = 0.08, size = 31, normalized size = 0.10

$$8ae^{-\frac{1}{2}\coth^{-1}(ax)}\,{}_2F_1\left(-\frac{1}{4}, 2; \frac{3}{4}; -e^{2\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] (8*a*Hypergeometric2F1[-1/4, 2, 3/4, -E^(2*ArcCoth[a*x])])/E^(ArcCoth[a*x]/
2)
```

fricas [A] time = 0.54, size = 377, normalized size = 1.26

$$20\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}\sqrt{a^2\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}}}{a^4}\right) + 20\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{2}\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)
) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
20*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a*
x + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - sq
rt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - 5*sqrt
(2)*(a^4)^(1/4)*x*log(25*a^2*sqrt((a*x - 1)/(a*x + 1)) + 25*sqrt(2)*(a^4)^(
1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + 25*sqrt(a^4)) + 5*sqrt(2)*(a^4)^(1/4)*
x*log(25*a^2*sqrt((a*x - 1)/(a*x + 1)) - 25*sqrt(2)*(a^4)^(1/4)*a*((a*x - 1
)/(a*x + 1))^(1/4) + 25*sqrt(a^4)) + 4*(9*a*x + 1)*((a*x - 1)/(a*x + 1))^(1
/4))/x
```

giac [A] time = 0.18, size = 204, normalized size = 0.68

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+5\sqrt{2}\log\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x - 1)/(
a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

maxima [A] time = 0.42, size = 204, normalized size = 0.68

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+5\sqrt{2}\log\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
```

```
(a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x - 1)/(
a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

mupad [B] time = 1.18, size = 106, normalized size = 0.35

$$8\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}+5\,(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)+\frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}+(-1)^{1/4}\,a\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,5\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^2,x)
```

```
[Out] 8*a*((a*x - 1)/(a*x + 1))^(1/4) + (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(
a*x + 1))^(1/4))*5i + 5*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^
(1/4)*1i) + (2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**2, x)
```

3.111 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=351

$$-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{5}{2}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}-\frac{25}{4}a^2\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

```
[Out] -2*a^2*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)-25/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(
3/4)-5/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+25/8*a^2*arctan(-1+(1-1/a/x)^(
1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+25/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(
1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/16*a^2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a
/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+25/16*a^2*ln(1+(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 78, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{5}{2}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}-\frac{25}{4}a^2\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (-2*a^2*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) - (25*a^2*(1 - 1/(a*x))^(1
/4)*(1 + 1/(a*x))^(3/4))/4 - (5*a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/2 - (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) + (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - (5a)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{4}(25a)\,\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{8}(25a)\,\text{Su}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(25a^2\right)\text{S}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(25a^2\right)\text{S}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}\left(25a^2\right)\text{S}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}\left(25a^2\right)\text{S}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\log\left(\right.}{}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{25a^2\tan^{-}}{}$$

Mathematica [C] time = 0.15, size = 101, normalized size = 0.29

$$-\frac{8}{3}a^2e^{-\frac{1}{2}\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)+2e^{2\coth^{-1}(ax)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (-8*a^2*(3 + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCot
h[a*x])] + E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[
a*x])] + 2*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[
a*x])]))/(3*E^(ArcCoth[a*x]/2))
```

fricas [A] time = 0.51, size = 405, normalized size = 1.15

$$-\frac{100\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+100\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(100*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*(
(a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a
*x + 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8))
)/a^8) + 100*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*
((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(
a*x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)
))/a^8) - 25*sqrt(2)*(a^8)^(1/4)*x^2*log(625*a^4*sqrt((a*x - 1)/(a*x + 1))
+ 625*sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 625*sqrt(a^8))
+ 25*sqrt(2)*(a^8)^(1/4)*x^2*log(625*a^4*sqrt((a*x - 1)/(a*x + 1)) - 625*sq
rt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 625*sqrt(a^8)) + 4*(43*
a^2*x^2 + 9*a*x - 2)*((a*x - 1)/(a*x + 1))^(1/4))/x^2
```

giac [A] time = 0.18, size = 243, normalized size = 0.69

$$\frac{1}{16}\left(50\sqrt{2}a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+25\sqrt{2}a\log\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4) - 8
*(13*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 9*a*((a*x - 1)/(a*
x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

maxima [A] time = 0.42, size = 247, normalized size = 0.70

$$\frac{1}{16}\left(50\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+25\sqrt{2}\,a\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4) - 8
*(13*a*((a*x - 1)/(a*x + 1))^(5/4) + 9*a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a
*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

mupad [B] time = 0.07, size = 153, normalized size = 0.44

$$-8\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-\frac{\frac{9\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{13\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}-\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)25\mathrm{i}}{4}-\frac{25\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^3,x)
```

```
[Out] - 8*a^2*((a*x - 1)/(a*x + 1))^(1/4) - ((9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/
2 + (13*a^2*((a*x - 1)/(a*x + 1))^(5/4))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(
a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x
+ 1))^(1/4))*25i)/4 - (25*(-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x +
1))^(1/4)*1i))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**3, x)
```

3.112 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=385

$$\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{55a^3\log}{}$$

```
[Out] 2*a^3*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)+55/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3
/4)+11/4*a^3*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+1/3*a^3*(1-1/a/x)^(9/4)*(1+1/a
/x)^(3/4)-55/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1
/2)-55/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+55/
32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/
x)^(1/2))*2^(1/2)-55/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 89, 80, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{55a^3\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (2*a^3*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) + (55*a^3*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4))/8 + (11*a^3*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/4 + (a^3*(1 - 1/(a*x))^(9/4)*(1 + 1/(a*x))^(3/4))/3 + (55*a^3*ArcTan[1 -
(Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*A
rcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) +
 (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (55*a^3*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(16*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \left(2a^3\right)\text{Subst}\left(\int \frac{\left(-\frac{5}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}\left(55a^2\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\right.$$

Mathematica [C] time = 0.21, size = 104, normalized size = 0.27

$$a^3\left(\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(425e^{2\coth^{-1}(ax)}+462e^{4\coth^{-1}(ax)}+165e^{6\coth^{-1}(ax)}+96\right)}{12\left(e^{2\coth^{-1}(ax)}+1\right)^3} - \frac{55}{32}\text{RootSum}\left[\#1^4+1\&, \frac{2\log\left(e^{-\frac{1}{2}}\right.}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] a^3*((96 + 425*E^(2*ArcCoth[a*x]) + 462*E^(4*ArcCoth[a*x]) + 165*E^(6*ArcCo
th[a*x]))/(12*E^(ArcCoth[a*x]/2)*(1 + E^(2*ArcCoth[a*x]))^3) - (55*RootSum[
1 + #1^4 & , (ArcCoth[a*x] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ])/3
2)
```

fricas [A] time = 0.58, size = 413, normalized size = 1.07

$$\frac{660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^{3}\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}\sqrt{a^{6}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^{3}\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^3
*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1)
/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a
^12)))/a^12) + 660*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^(
3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((
a*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt(a^12)))/a^12) - 165*sqrt(2)*(a^12)^(1/4)*x^3*log(3025*a^6*sqrt((a*x
- 1)/(a*x + 1)) + 3025*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
 + 3025*sqrt(a^12)) + 165*sqrt(2)*(a^12)^(1/4)*x^3*log(3025*a^6*sqrt((a*x -
 1)/(a*x + 1)) - 3025*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
+ 3025*sqrt(a^12)) + 4*(287*a^3*x^3 + 61*a^2*x^2 - 26*a*x + 8)*((a*x - 1)/(
a*x + 1))^(1/4))/x^3
```

giac [A] time = 0.19, size = 291, normalized size = 0.76

$$-\frac{1}{96}\left(330\sqrt{2}\,a^{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}\,a^{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+165\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) +
137*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 69*a^2*((a*x
- 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

maxima [A] time = 0.42, size = 297, normalized size = 0.77

$$-\frac{1}{96}\left(330\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+165\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 8*(137*a^2*((a*x - 1)/(a*x + 1))^(9/4) + 174*a^2*((a*x - 1)/(
a*x + 1))^(5/4) + 69*a^2*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)/(a*x + 1
) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

mupad [B] time = 0.07, size = 188, normalized size = 0.49

$$\frac{\frac{23a^3\left(\frac{ax-1}{ax+1}\right)^{1/4}}{4}+\frac{29a^3\left(\frac{ax-1}{ax+1}\right)^{5/4}}{2}+\frac{137a^3\left(\frac{ax-1}{ax+1}\right)^{9/4}}{12}}{\frac{3(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+\frac{3(ax-1)}{ax+1}+1}+8a^3\left(\frac{ax-1}{ax+1}\right)^{1/4}+\frac{(-1)^{1/4}a^3\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{ax-1}{ax+1}\right)^{1/4}\right)55\mathrm{i}}{8}+\frac{55(-1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^4,x)
```

```
[Out] ((23*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (29*a^3*((a*x - 1)/(a*x + 1))^(5/
4))/2 + (137*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1
)^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 8*a^3*((a*x
- 1)/(a*x + 1))^(1/4) + ((-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1
))^(1/4))*55i)/8 + (55*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))
^(1/4)*1i))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**4, x)
```

3.113 $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

Optimal. Leaf size=285

$$\frac{1}{3}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^3+\frac{7}{18}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{11}{27}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{19}{324}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+$$

```
[Out] 11/27*(1/x+1)^(1/6)*((-1+x)/x)^(5/6)*x+7/18*(1/x+1)^(1/6)*((-1+x)/x)^(5/6)*
x^2+1/3*(1/x+1)^(1/6)*((-1+x)/x)^(5/6)*x^3+19/81*arctanh((1/x+1)^(1/6)/((-1
+x)/x)^(1/6))-19/324*ln(1+(1/x+1)^(1/3)/((-1+x)/x)^(1/3)-(1/x+1)^(1/6)/((-1
+x)/x)^(1/6))+19/324*ln(1+(1/x+1)^(1/3)/((-1+x)/x)^(1/3)+(1/x+1)^(1/6)/((-1
+x)/x)^(1/6))-19/162*arctan(1/3*(1-2*(1/x+1)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2
))*3^(1/2)+19/162*arctan(1/3*(1+2*(1/x+1)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*
3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6171, 99, 151, 12, 93, 210, 634, 618, 204, 628, 206}

$$\frac{1}{3}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^3+\frac{7}{18}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{11}{27}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{19}{324}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x^2,x]
```

```
[Out] (11*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/27 + (7*(1 + x^(-1))^(1/6)*((-
1 + x)/x)^(5/6)*x^2)/18 + ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x^3)/3 - (
19*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])/(54*Sqr
t[3]) + (19*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]]
)/(54*Sqrt[3]) + (19*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/81 - (
19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/324 + (19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (
1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/324
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{7}{3}+2x}{\sqrt[6]{1-x}\,x^3(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{22}{9}-\frac{7x}{3}}{\sqrt[6]{1-x}\,x^2(1+x)^{5/6}}\,dx$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{1}{6}\text{Subs}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{19}{162}\text{Su}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{19}{27}\text{Sub}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{19}{81}\text{Sub}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{19}{81}\tan$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{19}{81}\tan$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{19\tan}{}$$

Mathematica [A] time = 5.29, size = 189, normalized size = 0.66

$$\frac{1}{324}\left(\frac{732e^{\frac{1}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}-1} + \frac{1368e^{\frac{1}{3}\coth^{-1}(x)}}{\left(e^{2\coth^{-1}(x)}-1\right)^2} + \frac{864e^{\frac{1}{3}\coth^{-1}(x)}}{\left(e^{2\coth^{-1}(x)}-1\right)^3} - 38\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right) + 38\log\left(e^{\frac{1}{3}\coth^{-1}(x)}+1\right) -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)*x^2,x]
```

```
[Out] ((864*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^3 + (1368*E^(ArcCoth[x]/3))
/(-1 + E^(2*ArcCoth[x]))^2 + (732*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))
 + 38*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] + 38*Sqrt[3]*ArcTan
[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 38*Log[1 - E^(ArcCoth[x]/3)] + 38*Log[
1 + E^(ArcCoth[x]/3)] - 19*Log[1 - E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)]
 + 19*Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/324
```

fricas [A] time = 0.48, size = 173, normalized size = 0.61

$$\frac{1}{54}\left(18\,x^3+39\,x^2+43\,x+22\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}-\frac{19}{162}\,\sqrt{3}\,\arctan\left(\frac{2}{3}\,\sqrt{3}\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\,\sqrt{3}\right)-\frac{19}{162}\,\sqrt{3}\,\arctan\left(\frac{2}{3}\,\sqrt{3}\,\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="fricas")
```

```
[Out] 1/54*(18*x^3 + 39*x^2 + 43*x + 22)*((x - 1)/(x + 1))^(5/6) - 19/162*sqrt(3)
*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 19/162*sqrt(3)
*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 19/324*log(((x
 - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x - 1)/(
x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x + 1))
^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

giac [A] time = 0.19, size = 215, normalized size = 0.75

$$-\frac{19}{162}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{19}{162}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)+\frac{\frac{8\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-\frac{19\,(x-1)^2\left(\right.}{(x+1)^2}}{27\left(\frac{x-1}{x+1}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="giac")
```

```
[Out] -19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/16
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/27*(8*(x
- 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) - 19*(x - 1)^2*((x - 1)/(x + 1))^(5/6)
/(x + 1)^2 - 61*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1) - 1)^3 + 19/324*l
og(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x
- 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x
 + 1))^(1/6) + 1) - 19/162*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

maple [C] time = 4.64, size = 2891, normalized size = 10.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x^2,x)
```

```
[Out] 1/54*(18*x^2+21*x+22)*(-1+x)/((-1+x)/(1+x))^(1/6)+(19/162*RootOf(_Z^2-_Z+1)
*ln(-(1+4*x+6*x^2+x^4+4*x^3+6*RootOf(_Z^2-_Z+1)*x^2+RootOf(_Z^2-_Z+1)+RootO
f(_Z^2-_Z+1)*x^4+4*RootOf(_Z^2-_Z+1)*x^3+4*RootOf(_Z^2-_Z+1)*x-3*(x^6+4*x^5
+5*x^4-5*x^2-4*x-1)^(5/6)-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-6*(x^6+4*x^
5+5*x^4-5*x^2-4*x-1)^(1/2)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)+3*RootOf(_
Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+6*RootOf(_Z^2-_Z+1)*(x^6+4*
x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2
-4*x-1)^(1/3)*x^3+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x
^4+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+18*RootOf(_Z^
2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2+12*RootOf(_Z^2-_Z+1)*(x^6+4
```

```
*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x
^2-4*x-1)^(1/3)*x+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*
x^2+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x+3*RootOf(_Z^
2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)
^(2/3)*x-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2-3*(x^6+4*x^5+5*x^4-5*x^2
-4*x-1)^(1/3)*x^3+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)-1
2*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/
3)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)-9*(x^6+4*x^5
+5*x^4-5*x^2-4*x-1)^(1/3)*x+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-
1)^(1/6))/(1+x)^4)-19/162*ln((-2-8*x-12*x^2-2*x^4-8*x^3+6*RootOf(_Z^2-_Z+1)
*x^2+RootOf(_Z^2-_Z+1)+RootOf(_Z^2-_Z+1)*x^4+4*RootOf(_Z^2-_Z+1)*x^3+4*Root
Of(_Z^2-_Z+1)*x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)+3*(x^6+4*x^5+5*x^4-5*
x^2-4*x-1)^(5/6)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-3*(x^6+4*x^5+5*x^4-5
*x^2-4*x-1)^(1/3)+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x
+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2+6*RootOf(_Z^2-
_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^3+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^
5+5*x^4-5*x^2-4*x-1)^(1/6)*x^4+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-
4*x-1)^(1/2)*x+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2
+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3+18*RootOf(_Z^
2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x+18*RootOf(_Z^2-_Z+1)*(x^6+4*x
^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2
-4*x-1)^(1/6)*x+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+3*(
x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*
x^3+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)-9*(x^6+4*x^5+5*
x^4-5*x^2-4*x-1)^(1/3)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1
)^(1/3)-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x+3*RootOf(_Z^2-_Z+1)*(x^6+4*
x^5+5*x^4-5*x^2-4*x-1)^(1/6)-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x-3*(x^
6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^4-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)
*x^3-18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2)/(1+x)^4)*RootOf(_Z^2-_Z+1)
+19/162*ln((-2-8*x-12*x^2-2*x^4-8*x^3+6*RootOf(_Z^2-_Z+1)*x^2+RootOf(_Z^2-_
Z+1)+RootOf(_Z^2-_Z+1)*x^4+4*RootOf(_Z^2-_Z+1)*x^3+4*RootOf(_Z^2-_Z+1)*x-3*
(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)+3
*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)+
3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+6*RootOf(_Z^2-_Z+
1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5
*x^4-5*x^2-4*x-1)^(1/3)*x^3+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-
1)^(1/6)*x^4+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+18*
RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2+12*RootOf(_Z^2-_Z
+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5
+5*x^4-5*x^2-4*x-1)^(1/3)*x+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x
-1)^(1/6)*x^2+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x+3*
RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+3*(x^6+4*x^5+5*x^4-5*
x^2-4*x-1)^(2/3)*x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^3+6*RootOf(_Z^2-
_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(
1/3)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)-9*(x^6+4*x
^5+5*x^4-5*x^2-4*x-1)^(1/3)*x+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*
x-1)^(1/6)-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x-3*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(1/6)*x^4-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3-18*(x^6+4*x^5
+5*x^4-5*x^2-4*x-1)^(1/6)*x^2)/(1+x)^4))/((-1+x)/(1+x))^(1/6)*((-1+x)*(1+x)
^5)^(1/6)/(1+x)
```

maxima [A] time = 0.42, size = 220, normalized size = 0.77

$$-\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}-8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+\cdots}{27\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-\cdots}{(x+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="maxima")
```

```
[Out] -19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/16
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/27*(19*((
x - 1)/(x + 1))^(17/6) - 8*((x - 1)/(x + 1))^(11/6) + 61*((x - 1)/(x + 1))^
(5/6))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1
) + 19/324*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/
324*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log
(((x - 1)/(x + 1))^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

mupad [B] time = 0.13, size = 168, normalized size = 0.59

$$-\frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)19\mathrm{i}}{81}-\frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27}-\frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27}+\frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}-\operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}4952198\mathrm{i}}{14348907\left(-\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19\sqrt{}}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)/(x + 1))^(1/6),x)
```

```
[Out] - (atan(((x - 1)/(x + 1))^(1/6)*1i)*19i)/81 - ((61*((x - 1)/(x + 1))^(5/6))
/27 - (8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)/(
(3*(x - 1))/(x + 1) - (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1) -
atan((((x - 1)/(x + 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/14348
907 - 2476099/14348907)))*((19*3^(1/2))/162 - 19i/162) - atan((((x - 1)/(x
+ 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/14348907 + 2476099/1434
8907)))*((19*3^(1/2))/162 + 19i/162)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x**2,x)
```

```
[Out] Integral(x**2/((x - 1)/(x + 1))**(1/6), x)
```

3.114 $\int e^{\frac{1}{3}\coth^{-1}(x)} x\,dx$

Optimal. Leaf size=258

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{7/6}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{1}{6}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right.$$

```
[Out] 1/6*(1/x+1)^(1/6)*((-1+x)/x)^(5/6)*x+1/2*(1/x+1)^(7/6)*((-1+x)/x)^(5/6)*x^2
+1/9*arctanh((1/x+1)^(1/6)/((-1+x)/x)^(1/6))-1/36*ln(1+(1/x+1)^(1/3)/((-1+x
)/x)^(1/3)-(1/x+1)^(1/6)/((-1+x)/x)^(1/6))+1/36*ln(1+(1/x+1)^(1/3)/((-1+x)/
x)^(1/3)+(1/x+1)^(1/6)/((-1+x)/x)^(1/6))-1/18*arctan(1/3*(1-2*(1/x+1)^(1/6)
/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)+1/18*arctan(1/3*(1+2*(1/x+1)^(1/6)/((-1
+x)/x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6171, 96, 94, 93, 210, 634, 618, 204, 628, 206}

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{7/6}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{1}{6}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x,x]
```

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/6 + ((1 + x^(-1))^(7/6)*((-1 + x)
/x)^(5/6)*x^2)/2 - ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/S
qrt[3]]/(6*Sqrt[3]) + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6)
)/Sqrt[3]]/(6*Sqrt[3]) + ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/9 -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/36 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-
1))^(1/6)/((-1 + x)/x)^(1/6)]/36
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx,\right.$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{36}\text{Subst}\left(\int\right.$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{36}\log\left(1+\right.$$
$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Mathematica [A] time = 0.41, size = 167, normalized size = 0.65

$$\frac{1}{36}\left(\frac{84e^{\frac{1}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}-1} + \frac{72e^{\frac{1}{3}\coth^{-1}(x)}}{\left(e^{2\coth^{-1}(x)}-1\right)^2} - 2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right) + 2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}+1\right) - \log\left(-e^{\frac{1}{3}\coth^{-1}(x)} + e^{\frac{2}{3}\coth\cdots}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)*x,x]
```

```
[Out] ((72*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^2 + (84*E^(ArcCoth[x]/3))/(-
1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]]
 + 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E^(ArcCot
h[x]/3)] + 2*Log[1 + E^(ArcCoth[x]/3)] - Log[1 - E^(ArcCoth[x]/3) + E^((2*A
rcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/36
```

fricas [A] time = 0.67, size = 168, normalized size = 0.65

$$\frac{1}{6}\left(3x^2+7x+4\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} - \frac{1}{18}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{18}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right) + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 7*x + 4)*((x - 1)/(x + 1))^(5/6) - 1/18*sqrt(3)*arctan(2/3*sqr
t(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 1/18*sqrt(3)*arctan(2/3*sqrt(
3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 1/36*log(((x - 1)/(x + 1))^(1/3
) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x -
 1)/(x + 1))^(1/6) + 1) + 1/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(
((x - 1)/(x + 1))^(1/6) - 1)
```

giac [A] time = 0.16, size = 191, normalized size = 0.74

$$-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}-1\right)^{2}}+\frac{1}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*sq
rt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/3*((x - 1)*((
x - 1)/(x + 1))^(5/6)/(x + 1) - 7*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1)
 - 1)^2 + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) -
 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1/18*log
(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

maple [C] time = 4.20, size = 2886, normalized size = 11.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x,x)
```

```
[Out] 1/6*(4+3*x)*(-1+x)/((-1+x)/(1+x))^(1/6)+(1/18*RootOf(_Z^2-_Z+1)*ln(-(1+4*x+
6*x^2+x^4+4*x^3+6*RootOf(_Z^2-_Z+1)*x^2+RootOf(_Z^2-_Z+1)+RootOf(_Z^2-_Z+1)
*x^4+4*RootOf(_Z^2-_Z+1)*x^3+4*RootOf(_Z^2-_Z+1)*x-3*(x^6+4*x^5+5*x^4-5*x^2
-4*x-1)^(5/6)-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-6*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(1/2)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)+3*RootOf(_Z^2-_Z+1)*(x
^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*
x^2-4*x-1)^(1/2)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3
)*x^3+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^4+12*RootOf
(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+18*RootOf(_Z^2-_Z+1)*(x^6
+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5
*x^2-4*x-1)^(1/6)*x^3+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1
/3)*x+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2+12*RootO
f(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x+3*RootOf(_Z^2-_Z+1)*(x^6
+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x-6*(
x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3
)*x^3+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)-12*(x^6+4*x^5
+5*x^4-5*x^2-4*x-1)^(1/2)*x-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2+6*Roo
tOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)-9*(x^6+4*x^5+5*x^4-5*x^2
-4*x-1)^(1/3)*x+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6))/(1
+x)^4)-1/18*ln((-2-8*x-12*x^2-2*x^4-8*x^3+6*RootOf(_Z^2-_Z+1)*x^2+RootOf(_Z
^2-_Z+1)+RootOf(_Z^2-_Z+1)*x^4+4*RootOf(_Z^2-_Z+1)*x^3+4*RootOf(_Z^2-_Z+1)*
x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/
6)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1
/3)+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+6*RootOf(_Z^2
-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x
^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^3+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-
4*x-1)^(1/6)*x^4+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x
```

```
+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2+12*RootOf(_Z^
2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3+18*RootOf(_Z^2-_Z+1)*(x^6+4
*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2
-4*x-1)^(1/6)*x^2+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*
x+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+3*(x^6+4*x^5+5*x^
4-5*x^2-4*x-1)^(2/3)*x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^3+6*RootOf(_
Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-
1)^(1/3)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)-9*(x^6
+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(1/6)-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x-3*(x^6+4*x^5+5*x^4-
5*x^2-4*x-1)^(1/6)*x^4-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3-18*(x^6+4
*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2)/(1+x)^4)*RootOf(_Z^2-_Z+1)+1/18*ln((-2-8
*x-12*x^2-2*x^4-8*x^3+6*RootOf(_Z^2-_Z+1)*x^2+RootOf(_Z^2-_Z+1)+RootOf(_Z^2
-_Z+1)*x^4+4*RootOf(_Z^2-_Z+1)*x^3+4*RootOf(_Z^2-_Z+1)*x-3*(x^6+4*x^5+5*x^4
-5*x^2-4*x-1)^(1/6)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)+3*(x^6+4*x^5+5*x^
4-5*x^2-4*x-1)^(2/3)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)+3*RootOf(_Z^2-_Z
+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*
x^4-5*x^2-4*x-1)^(1/2)*x^2+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1
)^(1/3)*x^3+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^4+12*
RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+18*RootOf(_Z^2-_Z+1
)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2+12*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5
*x^4-5*x^2-4*x-1)^(1/6)*x^3+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x
-1)^(1/3)*x+18*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2+12
*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x+3*RootOf(_Z^2-_Z+1
)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)
*x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^3+6*RootOf(_Z^2-_Z+1)*(x^6+4*x^5
+5*x^4-5*x^2-4*x-1)^(1/2)-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2+6*RootO
f(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)-9*(x^6+4*x^5+5*x^4-5*x^2-4
*x-1)^(1/3)*x+3*RootOf(_Z^2-_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)-12*(x
^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x
^4-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3-18*(x^6+4*x^5+5*x^4-5*x^2-4*x
-1)^(1/6)*x^2)/(1+x)^4))/((-1+x)/(1+x))^(1/6)*((-1+x)*(1+x)^5)^(1/6)/(1+x)
```

maxima [A] time = 0.40, size = 194, normalized size = 0.75

$$-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)+\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{36}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*sq
rt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/3*(((x - 1)/(
x + 1))^(11/6) - 7*((x - 1)/(x + 1))^(5/6))/(2*(x - 1)/(x + 1) - (x - 1)^2/
(x + 1)^2 - 1) + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6)
 + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1
/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(((x - 1)/(x + 1))^(1/6) - 1
)
```

mupad [B] time = 1.22, size = 142, normalized size = 0.55

$$\frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3}-\frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}-\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)1\mathrm{i}}{9}-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}2\mathrm{i}}{243\left(-\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{\sqrt{3}}{18}-\frac{1}{18}\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}2\mathrm{i}}{243\left(\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x - 1)/(x + 1))^(1/6),x)
```

```
[Out] ((7*((x - 1)/(x + 1))^(5/6))/3 - ((x - 1)/(x + 1))^(11/6)/3)/((x - 1)^2/(x
+ 1)^2 - (2*(x - 1))/(x + 1) + 1) - (atan(((x - 1)/(x + 1))^(1/6)*1i)*1i)/9
 - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)/243 - 1/243)))*(3^(
1/2)/18 - 1i/18) - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)/243
 + 1/243)))*(3^(1/2)/18 + 1i/18)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[6]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x,x)
```

```
[Out] Integral(x/((x - 1)/(x + 1))**(1/6), x)
```

3.115 $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

Optimal. Leaf size=223

$$\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\ldots \tan^{-}$$

```
[Out] (1/x+1)^(1/6)*((-1+x)/x)^(5/6)*x+2/3*arctanh((1/x+1)^(1/6)/((-1+x)/x)^(1/6)
)-1/6*ln(1+(1/x+1)^(1/3)/((-1+x)/x)^(1/3)-(1/x+1)^(1/6)/((-1+x)/x)^(1/6))+1
/6*ln(1+(1/x+1)^(1/3)/((-1+x)/x)^(1/3)+(1/x+1)^(1/6)/((-1+x)/x)^(1/6))-1/3*
arctan(1/3*(1-2*(1/x+1)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)+1/3*arctan
(1/3*(1+2*(1/x+1)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6170, 94, 93, 210, 634, 618, 204, 628, 206}

$$\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\ldots \tan^{-}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3),x]
```

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x - ArcTan[(1 - (2*(1 + x^(-1))^(1/6)
)/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))
/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + (2*ArcTanh[(1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/3 - Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^
(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)
^(1/3) + (1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x - 2\,\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \ldots\right.$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{6}\text{Su}\ldots$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) + \frac{1}{6}\log\left(1+\ldots\right.$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{\tan^{-1}\left(\frac{-1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\text{lo}\ldots$$

Mathematica [C] time = 0.05, size = 35, normalized size = 0.16

$$2e^{\frac{1}{3}\coth^{-1}(x)}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};e^{2\coth^{-1}(x)}\right) + \frac{1}{e^{2\coth^{-1}(x)}-1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3),x]
```

```
[Out] 2*E^(ArcCoth[x]/3)*((-1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6, 1
, 7/6, E^(2*ArcCoth[x])])
```

fricas [A] time = 0.61, size = 160, normalized size = 0.72

$$(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} - \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right) + \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right.\right.\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="fricas")
```

```
[Out] (x + 1)*((x - 1)/(x + 1))^(5/6) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(
x + 1))^(1/6) + 1/3*sqrt(3)) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x +
 1))^(1/6) - 1/3*sqrt(3)) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x +
 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6)
 + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1))^(1
/6) - 1)
```

giac [A] time = 0.16, size = 168, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x + 1
))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)
/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^
(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(abs(((x - 1)/(x
 + 1))^(1/6) - 1))
```

maple [C] time = 4.10, size = 1700, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6),x)
```

```
[Out] (-1+x)/((-1+x)/(1+x))^(1/6)+(1/3*ln((-2-8*x-12*x^2-6*RootOf(_Z^2+_Z+1)*x^2-
RootOf(_Z^2+_Z+1)*x^4-4*RootOf(_Z^2+_Z+1)*x^3-4*RootOf(_Z^2+_Z+1)*x-2*x^4-8
*x^3-RootOf(_Z^2+_Z+1)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)+3*(x^6+4*x^5+5
*x^4-5*x^2-4*x-1)^(5/6)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-3*(x^6+4*x^5+
5*x^4-5*x^2-4*x-1)^(1/3)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*RootOf(_Z^2+
_Z+1)*x-6*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^3-18*Root
Of(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2-18*RootOf(_Z^2+_Z+1)*
(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x-6*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-
5*x^2-4*x-1)^(1/2)*x^2-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^2+_Z
+1)*x^4-12*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x-12*(x^6+
4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^2+_Z+1)*x^3-18*(x^6+4*x^5+5*x^4-5*
x^2-4*x-1)^(1/6)*RootOf(_Z^2+_Z+1)*x^2-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/
6)*RootOf(_Z^2+_Z+1)*x-6*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1
/2)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^2+_Z+1)-3*(x^6+4*x^5+5*
x^4-5*x^2-4*x-1)^(2/3)*RootOf(_Z^2+_Z+1)-6*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x
^4-5*x^2-4*x-1)^(1/3)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x-3*(x^6+4*x^5+
5*x^4-5*x^2-4*x-1)^(1/3)*x^3-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2-9*(x
^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*
x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^4-12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1
)^(1/6)*x^3-18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2)/(1+x)^4)+1/3*RootOf
(_Z^2+_Z+1)*ln((-1-4*x-6*x^2-12*RootOf(_Z^2+_Z+1)*x^2-2*RootOf(_Z^2+_Z+1)*x
^4-8*RootOf(_Z^2+_Z+1)*x^3-8*RootOf(_Z^2+_Z+1)*x-x^4-4*x^3-2*RootOf(_Z^2+_Z
+1)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(
5/6)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^
(1/2)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)
^(2/3)*RootOf(_Z^2+_Z+1)*x+6*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1
)^(1/3)*x^3+18*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2+18
*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x-6*(x^6+4*x^5+5*x^4
-5*x^2-4*x-1)^(2/3)*RootOf(_Z^2+_Z+1)+6*RootOf(_Z^2+_Z+1)*(x^6+4*x^5+5*x^4-
5*x^2-4*x-1)^(1/3)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x-6*(x^6+4*x^5+5*x
^4-5*x^2-4*x-1)^(1/2)*x^2+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^3-12*(x^6
+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x^2
+9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x+12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(
1/6)*x+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^4+12*(x^6+4*x^5+5*x^4-5*x^2-
```

```
4*x-1)^(1/6)*x^3+18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2)/(1+x)^4))/((-1
+x)/(1+x))^(1/6)*((-1+x)*(1+x)^5)^(1/6)/(1+x)
```

maxima [A] time = 0.43, size = 167, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x + 1
))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)
/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^
(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1
))^(1/6) - 1)
```

mupad [B] time = 0.10, size = 115, normalized size = 0.52

$$-\frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)2\mathrm{i}}{3}-\frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}-1}-\operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}64\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}64\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)/(x + 1))^(1/6),x)
```

```
[Out] - (atan(((x - 1)/(x + 1))^(1/6)*1i)*2i)/3 - (2*((x - 1)/(x + 1))^(5/6))/((x
 - 1)/(x + 1) - 1) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i - 32))
*(3^(1/2)/3 - 1i/3) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i + 32)
)*(3^(1/2)/3 + 1i/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6),x)
```

```
[Out] Integral(((x - 1)/(x + 1))**(-1/6), x)
```

3.116 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=402

$$-\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\sqrt{}$$

```
[Out] 2*arctan(((-1+x)/x)^(1/6)/(1/x+1)^(1/6))+arctan(2*((-1+x)/x)^(1/6)/(1/x+1)^
(1/6)-3^(1/2))+arctan(2*((-1+x)/x)^(1/6)/(1/x+1)^(1/6)+3^(1/2))+2*arctanh((
1/x+1)^(1/6)/((-1+x)/x)^(1/6))-1/2*ln(1+(1/x+1)^(1/3)/((-1+x)/x)^(1/3)-(1/x
+1)^(1/6)/((-1+x)/x)^(1/6))+1/2*ln(1+(1/x+1)^(1/3)/((-1+x)/x)^(1/3)+(1/x+1)
^(1/6)/((-1+x)/x)^(1/6))-arctan(1/3*(1-2*(1/x+1)^(1/6)/((-1+x)/x)^(1/6))*3^
(1/2))*3^(1/2)+arctan(1/3*(1+2*(1/x+1)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(
1/2)+1/2*ln(1+((-1+x)/x)^(1/3)/(1/x+1)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1/x+
1)^(1/6))*3^(1/2)-1/2*ln(1+((-1+x)/x)^(1/3)/(1/x+1)^(1/3)+((-1+x)/x)^(1/6)*
3^(1/2)/(1/x+1)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6171, 105, 63, 331, 295, 634, 618, 204, 628, 203, 93, 210, 206}

$$-\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x,x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])
+ Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]] -
 ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + ArcTan[Sqrt[
3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + 2*ArcTan[((-1 + x)/x)^(1/
6)/(1 + x^(-1))^(1/6)] + 2*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)] -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/2 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-1
))^(1/6)/((-1 + x)/x)^(1/6)]/2 + (Sqrt[3]*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1/
6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - (Sqrt[
3]*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(
1/3)/(1 + x^(-1))^(1/3)])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right) - \text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= 6\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right) - 6\,\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + 2\,\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + 2\,\text{Subst}\left(\int \cdots\right.$$

$$= 2\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, \cdots\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) + \frac{1}{2}\log\left(1+\cdots\right.$$

$$= -\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) + \sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) + 2\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + 2\tanh^{-1}\left(\cdots\right.$$

$$= -\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) + \sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) - \tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \tan^{-1}\left(\sqrt{3}\cdots\right.$$

Mathematica [C] time = 0.04, size = 26, normalized size = 0.06

$$\frac{12}{7}e^{\frac{7}{3}\coth^{-1}(x)}{}_2F_1\left(\frac{7}{12},1;\frac{19}{12};e^{4\coth^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)/x,x]
```

```
[Out] (12*E^((7*ArcCoth[x])/3)*Hypergeometric2F1[7/12, 1, 19/12, E^(4*ArcCoth[x])
])/7
```

fricas [A] time = 0.56, size = 340, normalized size = 0.85

$$-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - sqrt(3
)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) - 1/2*sqrt(3)*l
og(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) +
1/2*sqrt(3)*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^
(1/3) + 16) - 2*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3)*((x - 1)/(x + 1))^(1/
6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/(x + 1))^(1/6)) - 2*arct
an(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1
/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) +
 1/2*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log((
(x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1
))^(1/6) + 1) - log(((x - 1)/(x + 1))^(1/6) - 1)
```

giac [A] time = 0.16, size = 261, normalized size = 0.65

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/2*sqrt(3)*log(sqrt(3)*(
(x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*sqrt(3)*log(-sq
rt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + arctan(sqrt(
3) + 2*((x - 1)/(x + 1))^(1/6)) + arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/
6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1)/(x + 1))^(1/3) +
 ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/
(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1) - log(abs(((x - 1)/(
x + 1))^(1/6) - 1))
```

maple [C] time = 14.62, size = 2320, normalized size = 5.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x,x)
```

```
[Out] -3*ln(-(18*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+18*R
ootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(
5/6)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(_Z^2+1)*RootOf(9*_
Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+3*(-(1-x)/(1+x))^(5/6)+3*RootOf(_Z^2+1)*
(-(1-x)/(1+x))^(2/3)+18*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))
^(1/3)+6*(-(1-x)/(1+x))^(1/2)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+6*R
ootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*x+6*(-(1-x)/(1+x))^(1/2)+3*RootOf(_Z^2+
1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x+RootOf(_Z^2+1)*x+3*(-(1-x)
/(1+x))^(1/6))/x)*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)+3*RootOf(9*_Z^2+3*_Z
+1)*ln(9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(5/6
)*x+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2+3*_Z+1)*(
-(1-x)/(1+x))^(1/2)*x-3*(-(1-x)/(1+x))^(5/6)+6*(-(1-x)/(1+x))^(2/3)*x-18*Ro
otOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(
1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(2/3)-6*(-(1-x)/(1+x))^(1/2)*x+18*RootOf(9*_
Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/
6)*x-6*(-(1-x)/(1+x))^(1/2)+3*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(9*_Z^2+3*_Z+1
)*(-(1-x)/(1+x))^(1/6)+3*(-(1-x)/(1+x))^(1/3)+3*RootOf(9*_Z^2+3*_Z+1)-1)-3*
ln(-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(5/6)*x
-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1
-x)/(1+x))^(1/2)*x-3*(-(1-x)/(1+x))^(5/6)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootO
f(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x
))^(1/3)*x+3*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(
1/3)+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)*
x+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/3)+3*(-(
1-x)/(1+x))^(1/6)*x-3*RootOf(9*_Z^2+3*_Z+1)+3*(-(1-x)/(1+x))^(1/6)-2)*RootO
f(9*_Z^2+3*_Z+1)+RootOf(_Z^2+1)*ln(-(9*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)
*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x
))^(2/3)+3*(-(1-x)/(1+x))^(5/6)*x+6*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-1
8*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x-18*RootOf(_Z^2+1)*RootOf(9*_
Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+3*(-(1-x)/(1+x))^(5/6)+6*RootOf(_Z^2+1)*
(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)-18*RootO
f(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)-6*(-(1-x)/(1+x))^(1/2)
*x-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/
(1+x))^(1/6)*x+3*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*x-6*(-(1-x)/(1+x))^(1
/2)-3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(
1+x))^(1/6)-RootOf(_Z^2+1)*x)/x)-ln(-(18*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+
1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(
1+x))^(2/3)+3*(-(1-x)/(1+x))^(5/6)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*
x+18*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+3*(-(1-x)/
(1+x))^(5/6)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(_Z^2+1)*RootOf
(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)+6*(-(1-x)/(1+x))^(1/2)*x+3*RootOf(_Z^2
+1)*(-(1-x)/(1+x))^(1/3)*x+6*RootOf(_Z^2+1)*RootOf(9*_Z^2+3*_Z+1)*x+6*(-(1-
x)/(1+x))^(1/2)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6
)*x+RootOf(_Z^2+1)*x+3*(-(1-x)/(1+x))^(1/6))/x)*RootOf(_Z^2+1)-ln(-9*RootOf
(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(5/6)*x-9*RootOf(9*
_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(
1/2)*x-3*(-(1-x)/(1+x))^(5/6)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^2+3*_
Z+1)*(-(1-x)/(1+x))^(1/2)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+3
*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)+9*RootO
f(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(9
*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^
(1/6)*x-3*RootOf(9*_Z^2+3*_Z+1)+3*(-(1-x)/(1+x))^(1/6)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((x - 1)/(x + 1))^(1/6)), x)
```

mupad [B] time = 1.27, size = 167, normalized size = 0.42

$$2\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)-\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)2\mathrm{i}-\operatorname{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}1486016741376\mathrm{i}}{-743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)\left(\sqrt{3}-\mathrm{i}\right)-\operatorname{atan}\left(\frac{}{743}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] 2*atan(((x - 1)/(x + 1))^(1/6)) - atan(((x - 1)/(x + 1))^(1/6)*1i)*2i - ata
n((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(3^(1/2)*743008370688i - 7430083
70688))*(3^(1/2) - 1i) - atan((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(3^(
1/2)*743008370688i + 743008370688))*(3^(1/2) + 1i) - atan((1486016741376*((
x - 1)/(x + 1))^(1/6))/(3^(1/2)*743008370688i - 743008370688))*(3^(1/2)*1i
+ 1) - atan((1486016741376*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*743008370688i
+ 743008370688))*(3^(1/2)*1i - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x,x)
```

```
[Out] Integral(1/(x*((x - 1)/(x + 1))**(1/6)), x)
```

3.117 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=233

$$\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{3}\tan$$

```
[Out] (1/x+1)^(1/6)*((-1+x)/x)^(5/6)+2/3*arctan(((-1+x)/x)^(1/6)/(1/x+1)^(1/6))+1
/3*arctan(2*((-1+x)/x)^(1/6)/(1/x+1)^(1/6)-3^(1/2))+1/3*arctan(2*((-1+x)/x)
^(1/6)/(1/x+1)^(1/6)+3^(1/2))+1/6*ln(1+((-1+x)/x)^(1/3)/(1/x+1)^(1/3)-((-1+
x)/x)^(1/6)*3^(1/2)/(1/x+1)^(1/6))*3^(1/2)-1/6*ln(1+((-1+x)/x)^(1/3)/(1/x+1
)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1/x+1)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6171, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{3}\tan$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x^2,x]
```

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6) - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6)]/3 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x
^(-1))^(1/6)]/3 + (2*ArcTan[((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/3 + Log
[1 - (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(
1 + x^(-1))^(1/3)]/(2*Sqrt[3]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 +
x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(2*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + 2\,\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + 2\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{2}{3}\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) +$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)}{2\sqrt{3}} - \frac{\log\left(1+\frac{\sqrt{3}\sqrt[6]{}}{\sqrt[6]{1}}\right.}{2}$$
$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{3}\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{2}{3}\tan^{-1}\left(\right.$$

Mathematica [C] time = 0.06, size = 39, normalized size = 0.17

$$-2e^{\frac{1}{3}\coth^{-1}(x)}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-e^{2\coth^{-1}(x)}\right)-\frac{1}{e^{2\coth^{-1}(x)}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^2,x]
```

```
[Out] -2*E^(ArcCoth[x]/3)*(-(1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6,
1, 7/6, -E^(2*ArcCoth[x])])
```

fricas [A] time = 0.61, size = 223, normalized size = 0.96

$$-\frac{\sqrt{3}x\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)-\sqrt{3}x\log\left(-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)+4x\arctan\left(\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(3)*x*log(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1
))^(1/3) + 16) - sqrt(3)*x*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x
 - 1)/(x + 1))^(1/3) + 16) + 4*x*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3)*((x
```

```
- 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/(x + 1)
)^(1/6)) + 4*x*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^(1/6) + (
(x - 1)/(x + 1))^(1/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) - 4*x*arctan(((x -
 1)/(x + 1))^(1/6)) - 6*(x + 1)*((x - 1)/(x + 1))^(5/6))/x
```

giac [A] time = 0.15, size = 152, normalized size = 0.65

$$-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
+ 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))
^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arctan(
sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)/(x
+ 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

maple [C] time = 7.82, size = 3474, normalized size = 14.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^2,x)
```

```
[Out] (-1+x)/x/((-1+x)/(1+x))^(1/6)+(1/1594323*ln(-(-2*RootOf(_Z^4-6561*_Z^2+4304
6721)^3*x^5-8*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^4-12*RootOf(_Z^4-6561*_Z^
2+43046721)^3*x^3+6561*RootOf(_Z^4-6561*_Z^2+43046721)*x^5+26244*RootOf(_Z^
4-6561*_Z^2+43046721)*x^4+39366*RootOf(_Z^4-6561*_Z^2+43046721)*x^3-8*RootO
f(_Z^4-6561*_Z^2+43046721)^3*x^2-2*RootOf(_Z^4-6561*_Z^2+43046721)^3*x+2624
4*RootOf(_Z^4-6561*_Z^2+43046721)*x^2+6561*RootOf(_Z^4-6561*_Z^2+43046721)*
x+1594323*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)-3188646*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(1/2)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*RootOf(_Z^4-6561*_Z^2+
43046721)^3+19683*RootOf(_Z^4-6561*_Z^2+43046721)*(x^6+4*x^5+5*x^4-5*x^2-4*
x-1)^(2/3)+486*RootOf(_Z^4-6561*_Z^2+43046721)^2*(x^6+4*x^5+5*x^4-5*x^2-4*x
-1)^(1/2)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046
721)^3-39366*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+4304
6721)-243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+4304672
1)^2-3188646*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2-6377292*(x^6+4*x^5+5*x
^4-5*x^2-4*x-1)^(1/2)*x+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*RootOf(_Z^4-6
561*_Z^2+43046721)^3*x+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-65
61*_Z^2+43046721)^3*x^3+486*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*RootOf(_Z^4
-6561*_Z^2+43046721)^2*x^2+9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^
4-6561*_Z^2+43046721)^3*x^2-243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(
_Z^4-6561*_Z^2+43046721)^2*x^4+19683*RootOf(_Z^4-6561*_Z^2+43046721)*(x^6+4
*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+972*RootOf(_Z^4-6561*_Z^2+43046721)^2*(x^6+
4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*Root
Of(_Z^4-6561*_Z^2+43046721)^3*x-39366*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*R
ootOf(_Z^4-6561*_Z^2+43046721)*x^3-972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*
RootOf(_Z^4-6561*_Z^2+43046721)^2*x^3-118098*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^
(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x^2-1458*(x^6+4*x^5+5*x^4-5*x^2-4*x-1
)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^2-118098*(x^6+4*x^5+5*x^4-5*x^2
-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x-972*(x^6+4*x^5+5*x^4-5*x^2-
4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x)/(1+x)^4/x)*RootOf(_Z^4-65
61*_Z^2+43046721)^3-1/243*ln(-(-2*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^5-8*R
ootOf(_Z^4-6561*_Z^2+43046721)^3*x^4-12*RootOf(_Z^4-6561*_Z^2+43046721)^3*x
^3+6561*RootOf(_Z^4-6561*_Z^2+43046721)*x^5+26244*RootOf(_Z^4-6561*_Z^2+430
```

```
46721)*x^4+39366*RootOf(_Z^4-6561*_Z^2+43046721)*x^3-8*RootOf(_Z^4-6561*_Z^
2+43046721)^3*x^2-2*RootOf(_Z^4-6561*_Z^2+43046721)^3*x+26244*RootOf(_Z^4-6
561*_Z^2+43046721)*x^2+6561*RootOf(_Z^4-6561*_Z^2+43046721)*x+1594323*(x^6+
4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)-3188646*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)+
3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3+196
83*RootOf(_Z^4-6561*_Z^2+43046721)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+486*
RootOf(_Z^4-6561*_Z^2+43046721)^2*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)+3*(x^
6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3-39366*(x
^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)-243*(x^6+
4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2-3188646*(x
^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2-6377292*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)
^(1/2)*x+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*RootOf(_Z^4-6561*_Z^2+430467
21)^3*x+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+4304672
1)^3*x^3+486*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*RootOf(_Z^4-6561*_Z^2+4304
6721)^2*x^2+9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+430
46721)^3*x^2-243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+
43046721)^2*x^4+19683*RootOf(_Z^4-6561*_Z^2+43046721)*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(2/3)*x+972*RootOf(_Z^4-6561*_Z^2+43046721)^2*(x^6+4*x^5+5*x^4-5*x
^2-4*x-1)^(1/2)*x+9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z
^2+43046721)^3*x-39366*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561
*_Z^2+43046721)*x^3-972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-656
1*_Z^2+43046721)^2*x^3-118098*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z
^4-6561*_Z^2+43046721)*x^2-1458*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(
_Z^4-6561*_Z^2+43046721)^2*x^2-118098*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*R
ootOf(_Z^4-6561*_Z^2+43046721)*x-972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*Ro
otOf(_Z^4-6561*_Z^2+43046721)^2*x)/(1+x)^4/x)*RootOf(_Z^4-6561*_Z^2+4304672
1)+1/243*RootOf(_Z^4-6561*_Z^2+43046721)*ln(-(-RootOf(_Z^4-6561*_Z^2+430467
21)^3*x^5-4*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^4-6*RootOf(_Z^4-6561*_Z^2+4
3046721)^3*x^3-6561*RootOf(_Z^4-6561*_Z^2+43046721)*x^5-26244*RootOf(_Z^4-6
561*_Z^2+43046721)*x^4-39366*RootOf(_Z^4-6561*_Z^2+43046721)*x^3-4*RootOf(_
Z^4-6561*_Z^2+43046721)^3*x^2-RootOf(_Z^4-6561*_Z^2+43046721)^3*x-26244*Roo
tOf(_Z^4-6561*_Z^2+43046721)*x^2-6561*RootOf(_Z^4-6561*_Z^2+43046721)*x-159
4323*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)+1594323*(x^6+4*x^5+5*x^4-5*x^2-4*x
-1)^(5/6)+6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*RootOf(_Z^4-6561*_Z^2+43046
721)^3-19683*RootOf(_Z^4-6561*_Z^2+43046721)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^
(2/3)-486*RootOf(_Z^4-6561*_Z^2+43046721)^2*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(
1/2)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^
3+39366*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)
+243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2-
6377292*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x-1594323*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(1/6)*x^4-6377292*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3-9565938*
(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2+6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/
3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3
)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^3-486*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(
1/2)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^2-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^
(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^2+243*(x^6+4*x^5+5*x^4-5*x^2-4*x-
1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^4-19683*RootOf(_Z^4-6561*_Z^2+
43046721)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x-972*RootOf(_Z^4-6561*_Z^2+4
3046721)^2*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x-9*(x^6+4*x^5+5*x^4-5*x^2-4
*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x+39366*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x^3+972*(x^6+4*x^5+5*x^4-5*x
^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^3+118098*(x^6+4*x^5+5*x
^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x^2+1458*(x^6+4*x^5+5
*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^2+118098*(x^6+4
*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x+972*(x^6+4*
x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x)/(1+x)^4/x
))/((-1+x)/(1+x))^(1/6)*((-1+x)*(1+x)^5)^(1/6)/(1+x)
```

maxima [A] time = 0.41, size = 152, normalized size = 0.65

$$-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
+ 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))
^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arctan(
sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)/(x
+ 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

mupad [B] time = 1.23, size = 109, normalized size = 0.47

$$\frac{2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{3}+\frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}+1}-\mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{-32+\sqrt{3}\,32i}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1i}{3}\right)-\mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{32+\sqrt{3}\,32i}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] (2*atan(((x - 1)/(x + 1))^(1/6)))/3 + (2*((x - 1)/(x + 1))^(5/6))/((x - 1)/
(x + 1) + 1) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i - 32))*((3^(1
/2)*1i)/3 + 1/3) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i + 32))*((
3^(1/2)*1i)/3 - 1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**2,x)
```

```
[Out] Integral(1/(x**2*((x - 1)/(x + 1))**(1/6)), x)
```

3.118 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=260

$$\frac{1}{2}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{1}{6}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{1}{18}\text{ta}$$

```
[Out] 1/6*(1/x+1)^(1/6)*((-1+x)/x)^(5/6)+1/2*(1/x+1)^(7/6)*((-1+x)/x)^(5/6)+1/9*a
rctan(((-1+x)/x)^(1/6)/(1/x+1)^(1/6))+1/18*arctan(2*((-1+x)/x)^(1/6)/(1/x+1
)^(1/6)-3^(1/2))+1/18*arctan(2*((-1+x)/x)^(1/6)/(1/x+1)^(1/6)+3^(1/2))+1/36
*ln(1+((-1+x)/x)^(1/3)/(1/x+1)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1/x+1)^(1/6)
)*3^(1/2)-1/36*ln(1+((-1+x)/x)^(1/3)/(1/x+1)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)
/(1/x+1)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6171, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\frac{1}{2}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{1}{6}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{1}{18}\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x^3,x]
```

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/6 + ((1 + x^(-1))^(7/6)*((-1 + x)/x
)^(5/6))/2 - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18
 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18 + ArcTan[
((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)]/9 + Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3
]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)
^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\left(2-x^6\right)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+}{x}}\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) +$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{36}\text{Subst}\left(\int \frac{}{1-}\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log\left(1-\frac{\sqrt{3}\sqrt[6]{-\frac{1-}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right.}{12\sqrt{}}$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{18}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{18}\tan^{-1}$$

Mathematica [C] time = 0.80, size = 124, normalized size = 0.48

$$\frac{1}{54}\left(\text{RootSum}\left[\#1^4-\#1^2+1\&, \frac{\#1^2\left(-\coth^{-1}(x)\right)+3\#1^2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)-6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+2\,c}{2\#1^3-\#1}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^3,x]
```

```
[Out] ((18*E^(ArcCoth[x]/3)*(1 + 7*E^(2*ArcCoth[x])))/(1 + E^(2*ArcCoth[x]))^2 -
6*ArcTan[E^(ArcCoth[x]/3)] + RootSum[1 - #1^2 + #1^4 & , (2*ArcCoth[x] - 6*
Log[E^(ArcCoth[x]/3) - #1] - ArcCoth[x]*#1^2 + 3*Log[E^(ArcCoth[x]/3) - #1]
*#1^2)/(-#1 + 2*#1^3) & ])/54
```

fricas [A] time = 0.52, size = 240, normalized size = 0.92

$$-\frac{\sqrt{3}\,x^2\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)-\sqrt{3}\,x^2\log\left(-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)+4\,x^2\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(sqrt(3)*x^2*log(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x
+ 1))^(1/3) + 16) - sqrt(3)*x^2*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1
6*((x - 1)/(x + 1))^(1/3) + 16) + 4*x^2*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(
3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/
(x + 1))^(1/6)) + 4*x^2*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^
(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) - 4*x^2*a
rctan(((x - 1)/(x + 1))^(1/6)) - 6*(4*x^2 + 7*x + 3)*((x - 1)/(x + 1))^(5/6
))/x^2
```

giac [A] time = 0.16, size = 175, normalized size = 0.67

$$-\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}+7\left(\frac{x-1}{x+1}\right)}{3\left(\frac{x-1}{x+1}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="giac")
```

```
[Out] -1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 1/3*((x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 7*((x - 1)/(
x + 1))^(5/6))/((x - 1)/(x + 1) + 1)^2 + 1/18*arctan(sqrt(3) + 2*((x - 1)/(
x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/9*ar
ctan(((x - 1)/(x + 1))^(1/6))
```

maple [C] time = 20.93, size = 2997, normalized size = 11.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^3,x)
```

```
[Out] 1/6*(-1+x)*(4*x+3)/x^2/((-1+x)/(1+x))^(1/6)+(-1/18*RootOf(_Z^2+1)*ln(-(-6*(
x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*
RootOf(_Z^2+1)*x^2+4*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^2+RootOf(-81
*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x+RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^5
+4*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^4+6*RootOf(-81*_Z*RootOf(_Z^2+
1)+_Z^2-6561)*x^3+243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^2+1)*x^
3-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*RootOf(_Z^2+1)*RootOf(-81*_Z*RootOf
(_Z^2+1)+_Z^2-6561)-18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(-81*_Z*Ro
otOf(_Z^2+1)+_Z^2-6561)*x^2+729*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(
_Z^2+1)*x^2-18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(-81*_Z*RootOf(_Z^
2+1)+_Z^2-6561)*x+729*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x+
3*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5
*x^2-4*x-1)^(1/6)+324*RootOf(_Z^2+1)*x^2+81*RootOf(_Z^2+1)*x+324*RootOf(_Z^
2+1)*x^4+486*RootOf(_Z^2+1)*x^3+243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)-486
*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)+81*RootOf(_Z^2+1)*x^5+3*RootOf(-81*_Z*
RootOf(_Z^2+1)+_Z^2-6561)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x-486*RootOf(
_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x-6*(x^6+4*x^5+5*x^4-5*x^2-4*x-
1)^(1/3)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^3+3*(x^6+4*x^5+5*x^4-5*x
^2-4*x-1)^(1/6)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*RootOf(_Z^2+1)*x^4-
12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*RootOf(_Z^2+1)*RootOf(-81*_Z*RootOf(
_Z^2+1)+_Z^2-6561)*x+12*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(-81*_Z*R
ootOf(_Z^2+1)+_Z^2-6561)*RootOf(_Z^2+1)*x^3+18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1
```

```
)^(1/6)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*RootOf(_Z^2+1)*x^2+12*RootO
f(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*
x-1)^(1/6)*x+3*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*(x^6+4*x^5+5*x^4-5*x
^2-4*x-1)^(2/3)-486*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-6*(x
^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)+2
43*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)-486*(x^6+4*x^5+5*x^4-
5*x^2-4*x-1)^(1/2)*x^2-972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x)/(1+x)^4/x
)-1/1458*ln(-(8*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^2+2*RootOf(-81*_Z
*RootOf(_Z^2+1)+_Z^2-6561)*x+2*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^5+
8*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^4+12*RootOf(-81*_Z*RootOf(_Z^2+
1)+_Z^2-6561)*x^3-243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^2+1)*x^
3+18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-
6561)*x^2-729*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^2+1)*x^2+18*(x^
6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x-
729*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*x-324*RootOf(_Z^2+1)
*x^2-81*RootOf(_Z^2+1)*x-324*RootOf(_Z^2+1)*x^4-486*RootOf(_Z^2+1)*x^3+243*
(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)+243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)
+486*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)-81*RootOf(_Z^2+1)*x^5+6*RootOf(-81
*_Z*RootOf(_Z^2+1)+_Z^2-6561)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x-243*Roo
tOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+6*(x^6+4*x^5+5*x^4-5*x^2-
4*x-1)^(1/3)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^3+6*RootOf(-81*_Z*Ro
otOf(_Z^2+1)+_Z^2-6561)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)-243*RootOf(_Z^2
+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/
3)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)-243*RootOf(_Z^2+1)*(x^6+4*x^5+5*
x^4-5*x^2-4*x-1)^(1/3)+486*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2+972*(x^6
+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x
+243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^4+972*(x^6+4*x^5+5*x^4-5*x^2-4*x
-1)^(1/6)*x^3+1458*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2)/(1+x)^4/x)*Root
Of(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)+1/18*ln(-(8*RootOf(-81*_Z*RootOf(_Z^2+1
)+_Z^2-6561)*x^2+2*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x+2*RootOf(-81*_
Z*RootOf(_Z^2+1)+_Z^2-6561)*x^5+8*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x
^4+12*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^3-243*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(1/3)*RootOf(_Z^2+1)*x^3+18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*Ro
otOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x^2-729*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)
^(1/3)*RootOf(_Z^2+1)*x^2+18*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(-81
*_Z*RootOf(_Z^2+1)+_Z^2-6561)*x-729*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4
*x-1)^(1/3)*x-324*RootOf(_Z^2+1)*x^2-81*RootOf(_Z^2+1)*x-324*RootOf(_Z^2+1)
*x^4-486*RootOf(_Z^2+1)*x^3+243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)+243*(x^
6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)+486*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)-81
*RootOf(_Z^2+1)*x^5+6*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*(x^6+4*x^5+5*
x^4-5*x^2-4*x-1)^(2/3)*x-243*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(
2/3)*x+6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(-81*_Z*RootOf(_Z^2+1)+_
Z^2-6561)*x^3+6*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)*(x^6+4*x^5+5*x^4-5*
x^2-4*x-1)^(2/3)-243*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+6*(
x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(-81*_Z*RootOf(_Z^2+1)+_Z^2-6561)-
243*RootOf(_Z^2+1)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)+486*(x^6+4*x^5+5*x^4
-5*x^2-4*x-1)^(1/2)*x^2+972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+972*(x^6+
4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x+243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^
4+972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^3+1458*(x^6+4*x^5+5*x^4-5*x^2-4
*x-1)^(1/6)*x^2)/(1+x)^4/x)*RootOf(_Z^2+1))/((-1+x)/(1+x))^(1/6)*((-1+x)*(1
+x)^5)^(1/6)/(1+x)
```

maxima [A] time = 0.42, size = 178, normalized size = 0.68

$$-\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+7\left(\frac{x-1}{x+1}\right.}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="maxima")
```

```
[Out] -1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 1/3*(((x - 1)/(x + 1))^(11/6) + 7*((x - 1)/(x + 1))^(5/6))/
(2*(x - 1)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) + 1/18*arctan(sqrt(3) + 2*((x
 - 1)/(x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) +
 1/9*arctan(((x - 1)/(x + 1))^(1/6))
```

mupad [B] time = 0.11, size = 136, normalized size = 0.52

$$\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{9}+\frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3}+\frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{2\,(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1}-\mathrm{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\,\left(-\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\mathrm{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\,\left(\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(-\frac{1}{18}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] atan(((x - 1)/(x + 1))^(1/6))/9 + ((7*((x - 1)/(x + 1))^(5/6))/3 + ((x - 1)
/(x + 1))^(11/6)/3)/((2*(x - 1))/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - atan(
(2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 - 1/243)))*((3^(1/2)*1i)
/18 + 1/18) - atan((2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 + 1/2
43)))*((3^(1/2)*1i)/18 - 1/18)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[6]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**3,x)
```

```
[Out] Integral(1/(x**3*((x - 1)/(x + 1))**(1/6)), x)
```

3.119 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

Optimal. Leaf size=287

$$\frac{1}{18}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}+\frac{19}{54}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{108\sqrt{3}}-\frac{19\log\Big(}{}$$

```
[Out] 19/54*(1/x+1)^(1/6)*((-1+x)/x)^(5/6)+1/18*(1/x+1)^(7/6)*((-1+x)/x)^(5/6)+1/
3*(1/x+1)^(7/6)*((-1+x)/x)^(5/6)/x+19/81*arctan(((-1+x)/x)^(1/6)/(1/x+1)^(1
/6))+19/162*arctan(2*((-1+x)/x)^(1/6)/(1/x+1)^(1/6)-3^(1/2))+19/162*arctan(
2*((-1+x)/x)^(1/6)/(1/x+1)^(1/6)+3^(1/2))+19/324*ln(1+((-1+x)/x)^(1/3)/(1/x
+1)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1/x+1)^(1/6))*3^(1/2)-19/324*ln(1+((-1+
x)/x)^(1/3)/(1/x+1)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1/x+1)^(1/6))*3^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6171, 90, 80, 50, 63, 331, 295, 634, 618, 204, 628, 203}

$$\frac{1}{18}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}+\frac{19}{54}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{108\sqrt{3}}-\frac{19\log\Big(}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x^4,x]
```

```
[Out] (19*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/54 + ((1 + x^(-1))^(7/6)*((-1 +
x)/x)^(5/6))/18 + ((1 + x^(-1))^(7/6)*((-1 + x)/x)^(5/6))/(3*x) - (19*ArcTa
n[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[Sq
rt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[((-1 +
 x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/81 + (19*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt
[3]) - (19*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 +
 x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt[3])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{1}{3}\text{Subst}\left(\int \frac{\left(-1-\frac{x}{3}\right)\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{54}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{162}\text{Subst}\Bigg(\int$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{27}\text{Subst}\Bigg(\int$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{27}\text{Subst}\Bigg(\int$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\text{Subst}\Bigg(\int$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\tan^{-1}\Bigg(\frac{\sqrt[6]{}}{\sqrt[6]{}}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\tan^{-1}\Bigg(\frac{\sqrt[6]{}}{\sqrt[6]{}}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{162}\tan^{-1}\Bigg(\sqrt{}$$

Mathematica [C] time = 0.22, size = 133, normalized size = 0.46

$$\frac{1}{486}\Bigg(-19\text{RootSum}\Bigg[\#1^4-\#1^2+1\&, \frac{\#1^2\coth^{-1}(x)-3\#1^2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)-2}{2\#1^3-\#1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^4,x]
```

```
[Out] ((18*E^(ArcCoth[x]/3)*(19 + 8*E^(2*ArcCoth[x]) + 61*E^(4*ArcCoth[x])))/(1 +
 E^(2*ArcCoth[x]))^3 - 114*ArcTan[E^(ArcCoth[x]/3)] - 19*RootSum[1 - #1^2 +
 #1^4 & , (-2*ArcCoth[x] + 6*Log[E^(ArcCoth[x]/3) - #1] + ArcCoth[x]*#1^2 -
 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-#1 + 2*#1^3) & ])/486
```

fricas [A] time = 0.42, size = 246, normalized size = 0.86

$$-\frac{19\sqrt{3}\,x^3\log\left(5776\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+5776\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+5776\right)-19\sqrt{3}\,x^3\log\left(-5776\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+5776\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+577\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="fricas")
```

```
[Out] -1/324*(19*sqrt(3)*x^3*log(5776*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 5776*((x
- 1)/(x + 1))^(1/3) + 5776) - 19*sqrt(3)*x^3*log(-5776*sqrt(3)*((x - 1)/(x
+ 1))^(1/6) + 5776*((x - 1)/(x + 1))^(1/3) + 5776) + 76*x^3*arctan(sqrt(3)
+ 1/38*sqrt(-5776*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 5776*((x - 1)/(x + 1))^
(1/3) + 5776) - 2*((x - 1)/(x + 1))^(1/6)) + 76*x^3*arctan(-sqrt(3) + 2*sqr
t(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) - 2*((x -
1)/(x + 1))^(1/6)) - 76*x^3*arctan(((x - 1)/(x + 1))^(1/6)) - 6*(22*x^3 + 4
3*x^2 + 39*x + 18)*((x - 1)/(x + 1))^(5/6))/x^3
```

giac [A] time = 0.18, size = 199, normalized size = 0.69

$$-\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{\frac{8(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}+\frac{19(x-}{}}{27\left(\frac{x}{x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="giac")
```

```
[Out] -19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/
3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x
 + 1))^(1/3) + 1) + 1/27*(8*(x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 19*(x
 - 1)^2*((x - 1)/(x + 1))^(5/6)/(x + 1)^2 + 61*((x - 1)/(x + 1))^(5/6))/((x
 - 1)/(x + 1) + 1)^3 + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) +
 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*arctan(((x - 1
)/(x + 1))^(1/6))
```

maple [C] time = 7.22, size = 3484, normalized size = 12.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^4,x)
```

```
[Out] 1/54*(-1+x)*(22*x^2+21*x+18)/x^3/((-1+x)/(1+x))^(1/6)+(19/86093442*ln(-(-2*
RootOf(_Z^4-6561*_Z^2+43046721)^3*x^5-8*RootOf(_Z^4-6561*_Z^2+43046721)^3*x
^4-12*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^3+6561*RootOf(_Z^4-6561*_Z^2+4304
6721)*x^5+26244*RootOf(_Z^4-6561*_Z^2+43046721)*x^4+39366*RootOf(_Z^4-6561*
_Z^2+43046721)*x^3-8*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^2-2*RootOf(_Z^4-65
```

```
61*_Z^2+43046721)^3*x+26244*RootOf(_Z^4-6561*_Z^2+43046721)*x^2+6561*RootOf
(_Z^4-6561*_Z^2+43046721)*x+1594323*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)-318
8646*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2
/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3+19683*RootOf(_Z^4-6561*_Z^2+43046721)
*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+486*RootOf(_Z^4-6561*_Z^2+43046721)^2*
(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*R
ootOf(_Z^4-6561*_Z^2+43046721)^3-39366*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*
RootOf(_Z^4-6561*_Z^2+43046721)-243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*Roo
tOf(_Z^4-6561*_Z^2+43046721)^2-3188646*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*
x^2-6377292*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+3*(x^6+4*x^5+5*x^4-5*x^2-
4*x-1)^(2/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x+3*(x^6+4*x^5+5*x^4-5*x^2-4
*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^3+486*(x^6+4*x^5+5*x^4-5*x^
2-4*x-1)^(1/2)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^2+9*(x^6+4*x^5+5*x^4-5*x
^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^2-243*(x^6+4*x^5+5*x^4-
5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^4+19683*RootOf(_Z^4-
6561*_Z^2+43046721)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+972*RootOf(_Z^4-6
561*_Z^2+43046721)^2*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+9*(x^6+4*x^5+5*x
^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x-39366*(x^6+4*x^5+
5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x^3-972*(x^6+4*x^5
+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^3-118098*(x^6
+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x^2-1458*(x
^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^2-118
098*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x-9
72*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x)
/(1+x)^4/x)*RootOf(_Z^4-6561*_Z^2+43046721)^3-19/13122*ln(-(-2*RootOf(_Z^4-
6561*_Z^2+43046721)^3*x^5-8*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^4-12*RootOf
(_Z^4-6561*_Z^2+43046721)^3*x^3+6561*RootOf(_Z^4-6561*_Z^2+43046721)*x^5+26
244*RootOf(_Z^4-6561*_Z^2+43046721)*x^4+39366*RootOf(_Z^4-6561*_Z^2+4304672
1)*x^3-8*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^2-2*RootOf(_Z^4-6561*_Z^2+4304
6721)^3*x+26244*RootOf(_Z^4-6561*_Z^2+43046721)*x^2+6561*RootOf(_Z^4-6561*_
Z^2+43046721)*x+1594323*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)-3188646*(x^6+4*
x^5+5*x^4-5*x^2-4*x-1)^(1/2)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*RootOf(_
Z^4-6561*_Z^2+43046721)^3+19683*RootOf(_Z^4-6561*_Z^2+43046721)*(x^6+4*x^5+
5*x^4-5*x^2-4*x-1)^(2/3)+486*RootOf(_Z^4-6561*_Z^2+43046721)^2*(x^6+4*x^5+5
*x^4-5*x^2-4*x-1)^(1/2)+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6
561*_Z^2+43046721)^3-39366*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-
6561*_Z^2+43046721)-243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-656
1*_Z^2+43046721)^2-3188646*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x^2-6377292*
(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)
*RootOf(_Z^4-6561*_Z^2+43046721)^3*x+3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*
RootOf(_Z^4-6561*_Z^2+43046721)^3*x^3+486*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/
2)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^2+9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1
/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^2-243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)
^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^4+19683*RootOf(_Z^4-6561*_Z^2+43
046721)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*x+972*RootOf(_Z^4-6561*_Z^2+430
46721)^2*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*x+9*(x^6+4*x^5+5*x^4-5*x^2-4*x
-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x-39366*(x^6+4*x^5+5*x^4-5*x^2-
4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x^3-972*(x^6+4*x^5+5*x^4-5*x^2
-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^3-118098*(x^6+4*x^5+5*x^4
-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x^2-1458*(x^6+4*x^5+5*x
^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^2-118098*(x^6+4*x
^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)*x-972*(x^6+4*x^
5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x)/(1+x)^4/x)*
RootOf(_Z^4-6561*_Z^2+43046721)-19/13122*RootOf(_Z^4-6561*_Z^2+43046721)*ln
((-RootOf(_Z^4-6561*_Z^2+43046721)^3*x^5-4*RootOf(_Z^4-6561*_Z^2+43046721)^
3*x^4-6*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^3-6561*RootOf(_Z^4-6561*_Z^2+43
046721)*x^5-26244*RootOf(_Z^4-6561*_Z^2+43046721)*x^4-39366*RootOf(_Z^4-656
1*_Z^2+43046721)*x^3-4*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^2-RootOf(_Z^4-65
61*_Z^2+43046721)^3*x-26244*RootOf(_Z^4-6561*_Z^2+43046721)*x^2-6561*RootOf
```

```
(_Z^4-6561*_Z^2+43046721)*x+1594323*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)-159
4323*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(5/6)+6*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2
/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3-19683*RootOf(_Z^4-6561*_Z^2+43046721)
*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)+486*RootOf(_Z^4-6561*_Z^2+43046721)^2*
(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)-3*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*R
ootOf(_Z^4-6561*_Z^2+43046721)^3+39366*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*
RootOf(_Z^4-6561*_Z^2+43046721)-243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*Roo
tOf(_Z^4-6561*_Z^2+43046721)^2+6377292*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*
x+1594323*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^4+6377292*(x^6+4*x^5+5*x^4-
5*x^2-4*x-1)^(1/6)*x^3+9565938*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*x^2+6*(x
^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x-3*(x^
6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^3+486*
(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/2)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x^2-9
*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^3*x^2-
243*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+43046721)^2*x
^4-19683*RootOf(_Z^4-6561*_Z^2+43046721)*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(2/3
)*x+972*RootOf(_Z^4-6561*_Z^2+43046721)^2*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/
2)*x-9*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+43046721)^
3*x+39366*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+4304672
1)*x^3-972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^2+430467
21)^2*x^3+118098*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-6561*_Z^2+
43046721)*x^2-1458*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-6561*_Z^
2+43046721)^2*x^2+118098*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/3)*RootOf(_Z^4-65
61*_Z^2+43046721)*x-972*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)^(1/6)*RootOf(_Z^4-656
1*_Z^2+43046721)^2*x)/(1+x)^4/x))/((-1+x)/(1+x))^(1/6)*((-1+x)*(1+x)^5)^(1/
6)/(1+x)
```

maxima [A] time = 0.41, size = 205, normalized size = 0.71

$$-\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}+8\left(\frac{x-}{x+}\right.}{27\left(\frac{3(x-1)}{x+1}+\frac{3(x-}{(x+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="maxima")
```

```
[Out] -19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/
3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x
 + 1))^(1/3) + 1) + 1/27*(19*((x - 1)/(x + 1))^(17/6) + 8*((x - 1)/(x + 1))
^(11/6) + 61*((x - 1)/(x + 1))^(5/6))/(3*(x - 1)/(x + 1) + 3*(x - 1)^2/(x +
 1)^2 + (x - 1)^3/(x + 1)^3 + 1) + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x +
1))^(1/6)) + 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*ar
ctan(((x - 1)/(x + 1))^(1/6))
```

mupad [B] time = 1.26, size = 161, normalized size = 0.56

$$\frac{19\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{81}+\frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27}+\frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27}+\frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3(x-1)}{x+1}+\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}+1}-\mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(-\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099i}{14348907}\right)}\right)\left(\frac{19}{162}+\frac{\sqrt{3}\,1}{162}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] (19*atan(((x - 1)/(x + 1))^(1/6)))/81 + ((61*((x - 1)/(x + 1))^(5/6))/27 +
(8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)/((3*(x
- 1))/(x + 1) + (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 + 1) - atan((
4952198*((x - 1)/(x + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907 - 2
```

```
476099/14348907)))*((3^(1/2)*19i)/162 + 19/162) - atan((4952198*((x - 1)/(x
 + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907 + 2476099/14348907)))*
((3^(1/2)*19i)/162 - 19/162)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[6]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**4,x)
```

```
[Out] Integral(1/(x**4*((x - 1)/(x + 1))**(1/6)), x)
```

3.120 $\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx$

Optimal. Leaf size=157

$$\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^3+\frac{4}{9}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{14}{27}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{11}{27}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{11\log}{81}$$

```
[Out] 14/27*(1/x+1)^(1/3)*((-1+x)/x)^(2/3)*x+4/9*(1/x+1)^(1/3)*((-1+x)/x)^(2/3)*x
^2+1/3*(1/x+1)^(1/3)*((-1+x)/x)^(2/3)*x^3-11/27*ln((1/x+1)^(1/3)-((-1+x)/x)
^(1/3))-11/81*ln(x)-22/81*arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1/x+1)^(
1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = $\{6171, 99, 151, 12, 91\}$

$$\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^3+\frac{4}{9}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{14}{27}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{11}{27}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{11\log}{81}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)*x^2,x]
```

```
[Out] (14*(1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/27 + (4*(1 + x^(-1))^(1/3)*((-
1 + x)/x)^(2/3)*x^2)/9 + ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x^3)/3 - (2
2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/
(27*Sqrt[3]) - (11*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/27 - (11*L
og[x])/81
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{8}{3}+2x}{\sqrt[3]{1-x}\,x^3(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{28}{9}-\frac{8x}{3}}{\sqrt[3]{1-x}\,x^2(1+x)^{2/3}}\,d$$
$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{1}{6}\text{Su}$$
$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{22}{81}\text{S}$$
$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{22\,\text{ta}}{}$$

Mathematica [C] time = 7.77, size = 340, normalized size = 2.17

$$-\frac{e^{-\frac{10}{3}\coth^{-1}(x)}\left(54e^{8\coth^{-1}(x)}\left(782e^{2\coth^{-1}(x)}+325e^{4\coth^{-1}(x)}+475\right)\,{}_4F_3\left(2,2,2,\frac{7}{3};1,1,\frac{16}{3};e^{2\coth^{-1}(x)}\right)+162e^{8\,\text{cot}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x^2,x]
```

```
[Out] -1/49140*(-22750000 - 20915440*E^(2*ArcCoth[x]) + 7026175*E^(4*ArcCoth[x])
+ 7394140*E^(6*ArcCoth[x]) - 433485*E^(8*ArcCoth[x]) + 22750000*Hypergeomet
ric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] + 15227940*E^(2*ArcCoth[x])*Hypergeom
etric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] - 14083160*E^(4*ArcCoth[x])*Hyperge
ometric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] - 8250060*E^(6*ArcCoth[x])*Hyperg
eometric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] + 1456000*E^(8*ArcCoth[x])*Hyper
geometric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] + 54*E^(8*ArcCoth[x])*(475 + 78
2*E^(2*ArcCoth[x]) + 325*E^(4*ArcCoth[x]))*HypergeometricPFQ[{2, 2, 2, 7/3}
, {1, 1, 16/3}, E^(2*ArcCoth[x])] + 162*E^(8*ArcCoth[x])*(35 + 64*E^(2*ArcC
oth[x]) + 29*E^(4*ArcCoth[x]))*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1,
1, 16/3}, E^(2*ArcCoth[x])] + 486*E^(8*ArcCoth[x])*HypergeometricPFQ[{2, 2,
 2, 2, 2, 7/3}, {1, 1, 1, 1, 16/3}, E^(2*ArcCoth[x])] + 972*E^(10*ArcCoth[x
])*HypergeometricPFQ[{2, 2, 2, 2, 2, 7/3}, {1, 1, 1, 1, 16/3}, E^(2*ArcCoth
```

```
[x])] + 486*E^(12*ArcCoth[x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 7/3}, {1, 1
, 1, 1, 16/3}, E^(2*ArcCoth[x])])/E^((10*ArcCoth[x])/3)
```

fricas [A] time = 0.40, size = 100, normalized size = 0.64

$$\frac{1}{27}\left(9x^3+21x^2+26x+14\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{22}{81}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^3 + 21*x^2 + 26*x + 14)*((x - 1)/(x + 1))^(2/3) - 22/81*sqrt(3)*a
rctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 11/81*log(((x -
1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(((x - 1)/(x +
1))^(1/3) - 1)
```

giac [A] time = 0.18, size = 144, normalized size = 0.92

$$-\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\frac{2\left(\frac{10(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-\frac{11(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{(x+1)^2}-35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{x-1}{x+1}-1\right)^3}+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="giac")
```

```
[Out] -22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/27*(
10*(x - 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 11*(x - 1)^2*((x - 1)/(x + 1))
^(2/3)/(x + 1)^2 - 35*((x - 1)/(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)^3 + 11
/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(
abs(((x - 1)/(x + 1))^(1/3) - 1))
```

maple [C] time = 0.70, size = 408, normalized size = 2.60

$$\frac{\left(9x^2+12x+14\right)(-1+x)}{27\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{22\ln\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{1}{3}}x+4\,\mathrm{RootOf}\left(\_Z\right.}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x^2,x)
```

```
[Out] 1/27*(9*x^2+12*x+14)*(-1+x)/((-1+x)/(1+x))^(1/3)+(-22/81*ln(-(4*RootOf(_Z^2
-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)-3*RootOf(_Z^2-_Z+1)*(x
^3+x^2-x-1)^(1/3)*x+4*RootOf(_Z^2-_Z+1)^2*x-4*RootOf(_Z^2-_Z+1)*x^2-3*RootO
f(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)+3*(x^3+x^2-x-1)^(1/3)*x-2*RootOf(_Z^2-_Z+1
)*x+x^2+3*(x^3+x^2-x-1)^(1/3)+2*RootOf(_Z^2-_Z+1)-1)/(1+x))+22/81*RootOf(_Z
^2-_Z+1)*ln((2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2
/3)+2*RootOf(_Z^2-_Z+1)^2*x-5*RootOf(_Z^2-_Z+1)*x^2-3*(x^3+x^2-x-1)^(2/3)+3
*(x^3+x^2-x-1)^(1/3)*x-6*RootOf(_Z^2-_Z+1)*x+2*x^2+3*(x^3+x^2-x-1)^(1/3)-Ro
otOf(_Z^2-_Z+1)+4*x+2)/(1+x)))/((-1+x)/(1+x))^(1/3)*((-1+x)*(1+x)^2)^(1/3)/
(1+x)
```

maxima [A] time = 0.42, size = 149, normalized size = 0.95

$$-\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(11\left(\frac{x-1}{x+1}\right)^{\frac{8}{3}}-10\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-}{x+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="maxima")
```

```
[Out] -22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/27*(
11*((x - 1)/(x + 1))^(8/3) - 10*((x - 1)/(x + 1))^(5/3) + 35*((x - 1)/(x +
1))^(2/3))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3
 - 1) + 11/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) -
22/81*log(((x - 1)/(x + 1))^(1/3) - 1)
```

mupad [B] time = 1.19, size = 171, normalized size = 1.09

$$-\frac{22\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-\frac{484}{729}\right)}{81}-\frac{\frac{70\left(\frac{x-1}{x+1}\right)^{2/3}}{27}-\frac{20\left(\frac{x-1}{x+1}\right)^{5/3}}{27}+\frac{22\left(\frac{x-1}{x+1}\right)^{8/3}}{27}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}-\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(-\frac{11}{81}+\frac{\sqrt{3}\,11\mathrm{i}}{81}\right)^2\right)\left(-\frac{11}{81}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] log((484*((x - 1)/(x + 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 + 11/81)^2)*((3
^(1/2)*11i)/81 + 11/81) - ((70*((x - 1)/(x + 1))^(2/3))/27 - (20*((x - 1)/(
x + 1))^(5/3))/27 + (22*((x - 1)/(x + 1))^(8/3))/27)/((3*(x - 1))/(x + 1) -
 (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1) - log((484*((x - 1)/(x
+ 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 - 11/81)^2)*((3^(1/2)*11i)/81 - 11/8
1) - (22*log((484*((x - 1)/(x + 1))^(1/3))/729 - 484/729))/81
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x**2,x)
```

```
[Out] Integral(x**2/((x - 1)/(x + 1))**(1/3), x)
```

3.121 $\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx$

Optimal. Leaf size=130

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{4/3}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{1}{3}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{9}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/3*(1/x+1)^(1/3)*((-1+x)/x)^(2/3)*x+1/2*(1/x+1)^(4/3)*((-1+x)/x)^(2/3)*x^2
-1/3*ln((1/x+1)^(1/3)-((-1+x)/x)^(1/3))-1/9*ln(x)-2/9*arctan(1/3*3^(1/2)+2/
3*((-1+x)/x)^(1/3)/(1/x+1)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6171, 96, 94, 91}

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{4/3}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{1}{3}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{9}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)*x,x]
```

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/3 + ((1 + x^(-1))^(4/3)*((-1 + x)
/x)^(2/3)*x^2)/2 - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1
 + x^(-1))^(1/3))])/(3*Sqrt[3]) - Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/
3)]/3 - Log[x]/9
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^3}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^2}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)^{2/3}}\,dx\right.\\ &= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\end{aligned}$$

Mathematica [A] time = 0.49, size = 165, normalized size = 1.27

$$\frac{1}{9}\left(\frac{24e^{\frac{2}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}-1}+\frac{18e^{\frac{2}{3}\coth^{-1}(x)}}{\left(e^{2\coth^{-1}(x)}-1\right)^2}-2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)-2\log\left(e^{\frac{1}{3}\coth^{-1}(x)}+1\right)+\log\left(-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\cdots}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x,x]
```

```
[Out] ((18*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (24*E^((2*ArcCoth[x]
)/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/
Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E
^(ArcCoth[x]/3)] - 2*Log[1 + E^(ArcCoth[x]/3)] + Log[1 - E^(ArcCoth[x]/3) +
 E^((2*ArcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/
9
```

fricas [A] time = 0.58, size = 95, normalized size = 0.73

$$\frac{1}{6}\left(3x^2+8x+5\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{2}{9}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 8*x + 5)*((x - 1)/(x + 1))^(2/3) - 2/9*sqrt(3)*arctan(2/3*sqrt
(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 1/9*log(((x - 1)/(x + 1))^(2/3
) + ((x - 1)/(x + 1))^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

giac [A] time = 0.17, size = 120, normalized size = 0.92

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}-1\right)^2}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/3*((x
- 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 4*((x - 1)/(x + 1))^(2/3))/((x - 1)/
(x + 1) - 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3)
+ 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

maple [C] time = 0.58, size = 403, normalized size = 3.10

$$\frac{(5+3x)(-1+x)}{6\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(-\frac{4\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{1}{3}}x+4\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2x-4\,\mathrm{F}}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x,x)
```

```
[Out] 1/6*(5+3*x)*(-1+x)/((-1+x)/(1+x))^(1/3)+(-2/9*ln(-(4*RootOf(_Z^2-_Z+1)^2*x^
2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)-3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)
^(1/3)*x+4*RootOf(_Z^2-_Z+1)^2*x-4*RootOf(_Z^2-_Z+1)*x^2-3*RootOf(_Z^2-_Z+1
)*(x^3+x^2-x-1)^(1/3)+3*(x^3+x^2-x-1)^(1/3)*x-2*RootOf(_Z^2-_Z+1)*x+x^2+3*(
x^3+x^2-x-1)^(1/3)+2*RootOf(_Z^2-_Z+1)-1)/(1+x))+2/9*RootOf(_Z^2-_Z+1)*ln((
2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+2*RootOf(
_Z^2-_Z+1)^2*x-5*RootOf(_Z^2-_Z+1)*x^2-3*(x^3+x^2-x-1)^(2/3)+3*(x^3+x^2-x-1
)^(1/3)*x-6*RootOf(_Z^2-_Z+1)*x+2*x^2+3*(x^3+x^2-x-1)^(1/3)-RootOf(_Z^2-_Z+
1)+4*x+2)/(1+x)))/((-1+x)/(1+x))^(1/3)*((-1+x)*(1+x)^2)^(1/3)/(1+x)
```

maxima [A] time = 0.40, size = 123, normalized size = 0.95

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-}{x-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/3*(((x
 - 1)/(x + 1))^(5/3) - 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) - (x -
 1)^2/(x + 1)^2 - 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^
(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

mupad [B] time = 0.05, size = 145, normalized size = 1.12

$$\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}-\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}-\frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-\frac{4}{9}\right)}{9}-\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)+\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] ((8*((x - 1)/(x + 1))^(2/3))/3 - (2*((x - 1)/(x + 1))^(5/3))/3)/((x - 1)^2/
(x + 1)^2 - (2*(x - 1))/(x + 1) + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))/9
 - 4/9))/9 - log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1/2)*1i)/9 - 1/9)^2
)*((3^(1/2)*1i)/9 - 1/9) + log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1/2)*
1i)/9 + 1/9)^2)*((3^(1/2)*1i)/9 + 1/9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x,x)
```

```
[Out] Integral(x/((x - 1)/(x + 1))**(1/3), x)
```

3.122 $\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx$

Optimal. Leaf size=96

$$\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{3}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (1/x+1)^(1/3)*((-1+x)/x)^(2/3)*x-ln((1/x+1)^(1/3)-((-1+x)/x)^(1/3))-1/3*ln(
x)-2/3*arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1/x+1)^(1/3)*3^(1/2))*3^(1/
2)
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6170, 94, 91}

$$\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{3}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3),x]
```

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x
)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[(1 + x^(-1))^(1/3)
 - ((-1 + x)/x)^(1/3)] - Log[x]/3
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} x - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} x - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}} - \log\left(\sqrt[3]{1+\frac{1}{x}} - \sqrt[3]{-\frac{1-x}{x}}\right) - \frac{\log(x)}{3}$$

Mathematica [A] time = 0.18, size = 85, normalized size = 0.89

$$\frac{1}{3}\left(\frac{6e^{\frac{2}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}-1} - 2\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}\right) + \log\left(e^{\frac{2}{3}\coth^{-1}(x)} + e^{\frac{4}{3}\coth^{-1}(x)} + 1\right) + 2\sqrt{3}\tan^{-1}\left(\frac{2e^{\frac{2}{3}\coth^{-1}(x)}+1}{\sqrt{3}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3),x]
```

```
[Out] ((6*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(1 + 2
*E^((2*ArcCoth[x])/3))/Sqrt[3]] - 2*Log[1 - E^((2*ArcCoth[x])/3)] + Log[1 +
 E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)])/3
```

fricas [A] time = 0.54, size = 87, normalized size = 0.91

$$(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="fricas")
```

```
[Out] (x + 1)*((x - 1)/(x + 1))^(2/3) - 2/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(
x + 1))^(1/3) + 1/3*sqrt(3)) + 1/3*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(
x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

giac [A] time = 0.14, size = 97, normalized size = 1.01

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)\right) - \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1} - 1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) +
 ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

maple [C] time = 0.54, size = 605, normalized size = 6.30

$$\frac{-1+x}{\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(\frac{2\,\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)^{2}x^{2}+3\,\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)\left(x^{3}+x^{2}-x-1\right)^{\frac{2}{3}}+3\,\mathrm{RootOf}\left(\_Z^{2}-\_Z+1\right)\left(x^{3}+x^{2}-x-1\right)^{\frac{1}{3}}x-2\,\mathrm{RootOf}\left(\_Z^{2}-\_Z\cdots\right.}{1+\cdots}\right.}{3}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3),x)
```

```
[Out] (-1+x)/((-1+x)/(1+x))^(1/3)+(2/3*RootOf(_Z^2-_Z+1)*ln(-(-2*RootOf(_Z^2-_Z+1
)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^
2-x-1)^(1/3)*x-2*RootOf(_Z^2-_Z+1)^2*x+5*RootOf(_Z^2-_Z+1)*x^2+3*RootOf(_Z^
2-_Z+1)*(x^3+x^2-x-1)^(1/3)+4*RootOf(_Z^2-_Z+1)*x-2*x^2-RootOf(_Z^2-_Z+1)+2
)/(1+x))-2/3*ln((2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1
)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x+2*RootOf(_Z^2-_Z+1)^2*x+R
ootOf(_Z^2-_Z+1)*x^2-3*(x^3+x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1
)^(1/3)-3*(x^3+x^2-x-1)^(1/3)*x-x^2-3*(x^3+x^2-x-1)^(1/3)-RootOf(_Z^2-_Z+1)
-2*x-1)/(1+x))*RootOf(_Z^2-_Z+1)+2/3*ln((2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf
(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x+2
*RootOf(_Z^2-_Z+1)^2*x+RootOf(_Z^2-_Z+1)*x^2-3*(x^3+x^2-x-1)^(2/3)+3*RootOf
(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)-3*(x^3+x^2-x-1)^(1/3)*x-x^2-3*(x^3+x^2-x-1)
^(1/3)-RootOf(_Z^2-_Z+1)-2*x-1)/(1+x)))/((-1+x)/(1+x))^(1/3)*((-1+x)*(1+x)^
2)^(1/3)/(1+x)
```

maxima [A] time = 0.40, size = 96, normalized size = 1.00

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) +
 ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

mupad [B] time = 0.05, size = 118, normalized size = 1.23

$$-\frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-4\right)}{3}-\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)+\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 + 1/3)^2)*((3^(1/2)*1i)/3
 + 1/3) - log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 - 1/3)^2)*((3^(
1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) - 4))/3 - (2*((x - 1)/
(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3),x)
```

```
[Out] Integral(((x - 1)/(x + 1))**(-1/3), x)
```

3.123 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=155

$$-\frac{3}{2}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)-\frac{\log(x)}{2}-\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\sqrt{3}$$

```
[Out] -3/2*ln((1/x+1)^(1/3)-((-1+x)/x)^(1/3))-3/2*ln(1+((-1+x)/x)^(1/3)/(1/x+1)^(
1/3))-1/2*ln(1/x+1)-1/2*ln(x)+arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1/x
+1)^(1/3)*3^(1/2))*3^(1/2)-arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1/x+1)^
(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 105, 60, 91}

$$-\frac{3}{2}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)-\frac{\log(x)}{2}-\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\sqrt{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x,x]
```

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(
1/3))]) - Sqrt[3]*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x
^(-1))^(1/3))] - (3*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/2 - (3*Lo
g[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - Log[1 + x^(-1)]/2 - Log[x
]/2
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x}\,dx, x, \frac{1}{x}\right)$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right) - \text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)$$
$$= -\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{3}{2}\log\left(\sqrt[3]{1+\frac{1}{x}} - \sqrt[3]{-\right.$$

Mathematica [C] time = 0.05, size = 26, normalized size = 0.17

$$\frac{3}{2}e^{\frac{8}{3}\coth^{-1}(x)}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};e^{4\coth^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x,x]
[Out] (3*E^((8*ArcCoth[x])/3)*Hypergeometric2F1[2/3, 1, 5/3, E^(4*ArcCoth[x])])/2
```

fricas [A] time = 0.65, size = 86, normalized size = 0.55

$$\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \frac{1}{3}\sqrt{3}\right) - \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - 1\right) + \frac{1}{2}\log\left(\frac{(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + (x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + x + 1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="fricas")
[Out] sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(2/3) + 1/3*sqrt(3)) - log(((x
 - 1)/(x + 1))^(2/3) - 1) + 1/2*log(((x + 1)*((x - 1)/(x + 1))^(2/3) + (x -
 1)*((x - 1)/(x + 1))^(1/3) + x + 1)/(x + 1))
```

giac [A] time = 0.18, size = 79, normalized size = 0.51

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + 1\right)\right) + \frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}{x+1} + 1\right) - \log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="giac")
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(2/3) + 1)) + 1/2*log(((x -
 1)/(x + 1))^(2/3) + (x - 1)*((x - 1)/(x + 1))^(1/3)/(x + 1) + 1) - log(abs
(((x - 1)/(x + 1))^(2/3) - 1))
```

maple [C] time = 0.92, size = 1038, normalized size = 6.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x,x)
```

```
[Out] 3*RootOf(9*_Z^2-3*_Z+1)*ln((945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*
x^2+1890*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-168*(-(1-x)/(1+x))^(2
/3)*x^2+945*(-(1-x)/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2-1224*RootOf(9*_Z
^2-3*_Z+1)^2*x^2+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-336*(-(1-x)
/(1+x))^(2/3)*x-168*(-(1-x)/(1+x))^(1/3)*x^2+3060*RootOf(9*_Z^2-3*_Z+1)^2*x
+1353*RootOf(9*_Z^2-3*_Z+1)*x^2-168*(-(1-x)/(1+x))^(2/3)-945*RootOf(9*_Z^2-
3*_Z+1)*(-(1-x)/(1+x))^(1/3)-1224*RootOf(9*_Z^2-3*_Z+1)^2-1062*RootOf(9*_Z^
2-3*_Z+1)*x-304*x^2+168*(-(1-x)/(1+x))^(1/3)+1353*RootOf(9*_Z^2-3*_Z+1)+32*
x-304)/x)-3*ln(-(945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1890*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-147*(-(1-x)/(1+x))^(2/3)*x^2+945
*(-(1-x)/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1224*RootOf(9*_Z^2-3*_Z+1)^
2*x^2+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-294*(-(1-x)/(1+x))^(2/
3)*x-147*(-(1-x)/(1+x))^(1/3)*x^2-3060*RootOf(9*_Z^2-3*_Z+1)^2*x+537*RootOf
(9*_Z^2-3*_Z+1)*x^2-147*(-(1-x)/(1+x))^(2/3)-945*RootOf(9*_Z^2-3*_Z+1)*(-(1
-x)/(1+x))^(1/3)+1224*RootOf(9*_Z^2-3*_Z+1)^2+978*RootOf(9*_Z^2-3*_Z+1)*x-1
1*x^2+147*(-(1-x)/(1+x))^(1/3)+537*RootOf(9*_Z^2-3*_Z+1)-18*x-11)/x)*RootOf
(9*_Z^2-3*_Z+1)+ln(-(945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+189
0*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-147*(-(1-x)/(1+x))^(2/3)*x^2
+945*(-(1-x)/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1224*RootOf(9*_Z^2-3*_Z
+1)^2*x^2+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-294*(-(1-x)/(1+x))
^(2/3)*x-147*(-(1-x)/(1+x))^(1/3)*x^2-3060*RootOf(9*_Z^2-3*_Z+1)^2*x+537*Ro
otOf(9*_Z^2-3*_Z+1)*x^2-147*(-(1-x)/(1+x))^(2/3)-945*RootOf(9*_Z^2-3*_Z+1)*
(-(1-x)/(1+x))^(1/3)+1224*RootOf(9*_Z^2-3*_Z+1)^2+978*RootOf(9*_Z^2-3*_Z+1)
*x-11*x^2+147*(-(1-x)/(1+x))^(1/3)+537*RootOf(9*_Z^2-3*_Z+1)-18*x-11)/x)
```

maxima [A] time = 0.42, size = 140, normalized size = 0.90

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 1/2*log(((x - 1)/(x + 1))
^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*log(((x - 1)/(x + 1))^(2/3) - (
(x - 1)/(x + 1))^(1/3) + 1) - log(((x - 1)/(x + 1))^(1/3) + 1) - log(((x -
1)/(x + 1))^(1/3) - 1)
```

mupad [B] time = 1.42, size = 82, normalized size = 0.53

$$-\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}-1296\right)-\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}+648-\sqrt{3}\,648\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}+648-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)/(x + 1))^(1/3)),x)
```

```
[Out] log(3^(1/2)*648i + 1296*((x - 1)/(x + 1))^(2/3) + 648)*((3^(1/2)*1i)/2 + 1/
2) - log(1296*((x - 1)/(x + 1))^(2/3) - 3^(1/2)*648i + 648)*((3^(1/2)*1i)/2
 - 1/2) - log(1296*((x - 1)/(x + 1))^(2/3) - 1296)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x,x)
```

```
[Out] Integral(1/(x*((x - 1)/(x + 1))**(1/3)), x)
```

3.124 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=99

$$\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}-\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{3}\log\left(\frac{1}{x}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{\sqrt{3}}$$

```
[Out] (1/x+1)^(1/3)*((-1+x)/x)^(2/3)-ln(1+((-1+x)/x)^(1/3)/(1/x+1)^(1/3))-1/3*ln(
1/x+1)+2/3*arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1/x+1)^(1/3)*3^(1/2))*
3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6171, 50, 60}

$$\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}-\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{3}\log\left(\frac{1}{x}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x^2,x]
```

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3) - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/
x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[1 + ((-1 + x)/x)^(1/
3)/(1 + x^(-1))^(1/3)] - Log[1 + x^(-1)]/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}} - \log\left(1+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{1}{3}\log\left(1+\frac{1}{x}\right)$$

Mathematica [A] time = 0.17, size = 87, normalized size = 0.88

$$\frac{2e^{\frac{2}{3}\coth^{-1}(x)}}{e^{2\coth^{-1}(x)}+1} - \frac{2}{3}\log\left(e^{\frac{2}{3}\coth^{-1}(x)}+1\right) + \frac{1}{3}\log\left(-e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}+1\right) - \frac{2\tan^{-1}\left(\frac{2e^{\frac{2}{3}\coth^{-1}(x)}-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x^2,x]
```

```
[Out] (2*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x])) - (2*ArcTan[(-1 + 2*E^((2*A
rcCoth[x])/3))/Sqrt[3]])/Sqrt[3] - (2*Log[1 + E^((2*ArcCoth[x])/3)])/3 + Lo
g[1 - E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)]/3
```

fricas [A] time = 0.68, size = 97, normalized size = 0.98

$$\frac{2\sqrt{3}\,x\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - 2\,x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + 3\,(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3))
+ x*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 2*x*log(((
x - 1)/(x + 1))^(1/3) + 1) + 3*(x + 1)*((x - 1)/(x + 1))^(2/3))/x
```

giac [A] time = 0.14, size = 99, normalized size = 1.00

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)\right) + \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1} + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1
)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

maple [C] time = 0.79, size = 501, normalized size = 5.06

$$\frac{-1+x}{x\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2x^2+27\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-45\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3+x^2-x-1\right)^{\frac{1}{3}}x-8\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2x-30\,\mathrm{RootOf}\cdots}{\cdots}\right)}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x^2,x)
```

```
[Out] (-1+x)/x/((-1+x)/(1+x))^(1/3)+(-2/3*ln((8*RootOf(_Z^2-3*_Z+9)^2*x^2+27*Root
Of(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-45*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1
/3)*x-8*RootOf(_Z^2-3*_Z+9)^2*x-30*RootOf(_Z^2-3*_Z+9)*x^2-216*(x^3+x^2-x-1
)^(2/3)-45*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-81*(x^3+x^2-x-1)^(1/3)*x
-16*RootOf(_Z^2-3*_Z+9)^2-54*RootOf(_Z^2-3*_Z+9)*x-27*x^2-81*(x^3+x^2-x-1)^
(1/3)-24*RootOf(_Z^2-3*_Z+9)-36*x-9)/x/(1+x))+2/9*RootOf(_Z^2-3*_Z+9)*ln(-(
-2*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)+72*
RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x+2*RootOf(_Z^2-3*_Z+9)^2*x-27*Root
Of(_Z^2-3*_Z+9)*x^2+135*(x^3+x^2-x-1)^(2/3)+72*RootOf(_Z^2-3*_Z+9)*(x^3+x^2
-x-1)^(1/3)-81*(x^3+x^2-x-1)^(1/3)*x+4*RootOf(_Z^2-3*_Z+9)^2+6*RootOf(_Z^2-
3*_Z+9)*x-36*x^2-81*(x^3+x^2-x-1)^(1/3)+33*RootOf(_Z^2-3*_Z+9)-216*x-180)/x
/(1+x)))/((-1+x)/(1+x))^(1/3)*((-1+x)*(1+x)^2)^(1/3)/(1+x)
```

maxima [A] time = 0.41, size = 98, normalized size = 0.99

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1
)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) + 1)
```

mupad [B] time = 0.02, size = 118, normalized size = 1.19

$$\frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}+1}-\ln\left(9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)+\ln\left(9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((x - 1)/(x + 1))^(1/3)),x)
```

```
[Out] log(9*((3^(1/2)*1i)/3 + 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3^(1/2)*1i)/3
 + 1/3) - log(9*((3^(1/2)*1i)/3 - 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3^(
1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) + 4))/3 + (2*((x - 1)/
(x + 1))^(2/3))/((x - 1)/(x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x**2,x)
```

```
[Out] Integral(1/(x**2*((x - 1)/(x + 1))**(1/3)), x)
```

3.125 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=130

$$\frac{1}{2}\left(\frac{x-1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}+\frac{1}{3}\left(\frac{x-1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{1}{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{9}\log\left(\frac{1}{x}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{3\sqrt{3}}$$

```
[Out] 1/3*(1/x+1)^(1/3)*((-1+x)/x)^(2/3)+1/2*(1/x+1)^(4/3)*((-1+x)/x)^(2/3)-1/3*l
n(1+((-1+x)/x)^(1/3)/(1/x+1)^(1/3))-1/9*ln(1/x+1)+2/9*arctan(-1/3*3^(1/2)+2
/3*((-1+x)/x)^(1/3)/(1/x+1)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 80, 50, 60}

$$\frac{1}{2}\left(\frac{x-1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}+\frac{1}{3}\left(\frac{x-1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{1}{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{9}\log\left(\frac{1}{x}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x^3,x]
```

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3))/3 + ((1 + x^(-1))^(4/3)*((-1 + x)/x
)^(2/3))/2 - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(
-1))^(1/3))])/(3*Sqrt[3]) - Log[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/
3 - Log[1 + x^(-1)]/9
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(1+\right.$$

Mathematica [C] time = 0.32, size = 134, normalized size = 1.03

$$-\frac{2}{27}\left(-\text{RootSum}\left[\#1^4 - \#1^2 + 1\&, \frac{\#1^2\coth^{-1}(x) - 3\#1^2\log\left(e^{\frac{1}{3}\coth^{-1}(x)} - \#1\right) - 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)} - \#1\right) + \text{cot}}{\#1^2 - 2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x^3,x]
```

```
[Out] (-2*((27*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x]))^2 - (36*E^((2*ArcCoth
[x])/3))/(1 + E^(2*ArcCoth[x])) - 2*ArcCoth[x] + 3*Log[1 + E^((2*ArcCoth[x]
)/3)] - RootSum[1 - #1^2 + #1^4 & , (ArcCoth[x] - 3*Log[E^(ArcCoth[x]/3) -
#1] + ArcCoth[x]*#1^2 - 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-2 + #1^2) & ])
)/27
```

fricas [A] time = 0.58, size = 111, normalized size = 0.85

$$\frac{4\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + 2x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - 4x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + 3\left(5x^2 + 8\right.}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*x^2*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3
)) + 2*x^2*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 4*x
^2*log(((x - 1)/(x + 1))^(1/3) + 1) + 3*(5*x^2 + 8*x + 3)*((x - 1)/(x + 1))
^(2/3))/x^2
```

giac [A] time = 0.14, size = 122, normalized size = 0.94

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)\right) + \frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1} + 4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1} + 1\right)^2} + \frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \frac{2}{9}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*((x -
 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) + 4*((x - 1)/(x + 1))^(2/3))/((x - 1)/(
x + 1) + 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) +
 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

maple [C] time = 0.65, size = 738, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x^3,x)
```

```
[Out] 1/6*(-1+x)*(5*x+3)/x^2/((-1+x)/(1+x))^(1/3)+(2/27*RootOf(_Z^2-3*_Z+9)*ln((-
10*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-27*
RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x+10*RootOf(_Z^2-3*_Z+9)^2*x+69*Roo
tOf(_Z^2-3*_Z+9)*x^2-216*(x^3+x^2-x-1)^(2/3)-27*RootOf(_Z^2-3*_Z+9)*(x^3+x^
2-x-1)^(1/3)+216*(x^3+x^2-x-1)^(1/3)*x+20*RootOf(_Z^2-3*_Z+9)^2+36*RootOf(_
Z^2-3*_Z+9)*x-108*x^2+216*(x^3+x^2-x-1)^(1/3)-33*RootOf(_Z^2-3*_Z+9)-144*x-
36)/x/(1+x))-2/27*ln(-(10*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*
(x^3+x^2-x-1)^(2/3)-27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x-10*RootOf(
_Z^2-3*_Z+9)^2*x+9*RootOf(_Z^2-3*_Z+9)*x^2+135*(x^3+x^2-x-1)^(2/3)-27*RootO
f(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-135*(x^3+x^2-x-1)^(1/3)*x-20*RootOf(_Z^2
-3*_Z+9)^2+96*RootOf(_Z^2-3*_Z+9)*x-9*x^2-135*(x^3+x^2-x-1)^(1/3)+87*RootOf
(_Z^2-3*_Z+9)-54*x-45)/x/(1+x))*RootOf(_Z^2-3*_Z+9)+2/9*ln(-(10*RootOf(_Z^2
-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-27*RootOf(_Z^2-3*
_Z+9)*(x^3+x^2-x-1)^(1/3)*x-10*RootOf(_Z^2-3*_Z+9)^2*x+9*RootOf(_Z^2-3*_Z+9
)*x^2+135*(x^3+x^2-x-1)^(2/3)-27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-13
5*(x^3+x^2-x-1)^(1/3)*x-20*RootOf(_Z^2-3*_Z+9)^2+96*RootOf(_Z^2-3*_Z+9)*x-9
*x^2-135*(x^3+x^2-x-1)^(1/3)+87*RootOf(_Z^2-3*_Z+9)-54*x-45)/x/(1+x)))/((-1
+x)/(1+x))^(1/3)*((-1+x)*(1+x)^2)^(1/3)/(1+x)
```

maxima [A] time = 0.41, size = 124, normalized size = 0.95

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1\right)}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*(((x
- 1)/(x + 1))^(5/3) + 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) + (x -
1)^2/(x + 1)^2 + 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(
1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) + 1)
```

mupad [B] time = 0.03, size = 145, normalized size = 1.12

$$\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}+\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1}-\frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}+\frac{4}{9}\right)}{9}-\ln\left(9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)+\ln\left(9\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((x - 1)/(x + 1))^(1/3)),x)
```

```
[Out] ((8*((x - 1)/(x + 1))^(2/3))/3 + (2*((x - 1)/(x + 1))^(5/3))/3)/((2*(x - 1)
)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))/9
 + 4/9))/9 - log(9*((3^(1/2)*1i)/9 - 1/9)^2 + (4*((x - 1)/(x + 1))^(1/3))/9
)*((3^(1/2)*1i)/9 - 1/9) + log(9*((3^(1/2)*1i)/9 + 1/9)^2 + (4*((x - 1)/(x
+ 1))^(1/3))/9)*((3^(1/2)*1i)/9 + 1/9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x**3,x)
```

```
[Out] Integral(1/(x**3*((x - 1)/(x + 1))**(1/3)), x)
```

3.126 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=429

$$-\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}+\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}-\frac{11\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}a^3}+\frac{11\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right.}{64\sqrt{2}a^3}$$

```
[Out] 37/96*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a^2+3/8*(1-1/a/x)^(7/8)*(1+1/a/x)^(
1/8)*x^2/a+1/3*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x^3+11/64*arctan((1+1/a/x)^(
1/8)/(1-1/a/x)^(1/8))/a^3+11/64*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^
3-11/128*arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)+11/1
28*arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)-11/256*ln(
1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/
a^3*2^(1/2)+11/256*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+(1+1/a/x)^(1/8)*2^(
1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6171, 99, 151, 12, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{37x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{96a^2}-\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}+\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}a^3}-\frac{11\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right.}{64\sqrt{2}a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x^2,x]
```

```
[Out] (37*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(96*a^2) + (3*(1 - 1/(a*x))^
(7/8)*(1 + 1/(a*x))^(1/8)*x^2)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(
1/8)*x^3)/3 - (11*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1
/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 -
1/(a*x))^(1/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(
a*x))^(1/8)])/(64*a^3) + (11*ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8
)])/(64*a^3) - (11*Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8
) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3) + (11*Log[1
 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/
(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
```

```
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^4\sqrt[8]{1-\frac{x}{a}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{9}{4a}+\frac{2x}{a^2}}{x^3\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}dx, x, \frac{1}{x}\right)$$

$$= \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{37}{16a^2}-\frac{9x}{4a^3}}{x^2\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/}}\right.$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{1}{6}\text{Su}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\,\text{Su}}{}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\,\text{Su}}{}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\,\text{Su}}{}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\,\text{Su}}{}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\,\text{ta}}{}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 + \frac{11\,\text{ta}}{}$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}x^3 - \frac{11\,\text{ta}}{}$$

Mathematica [C] time = 5.34, size = 167, normalized size = 0.39

$$\frac{-33\text{RootSum}\left[\#1^4+1\&, \frac{\coth^{-1}(ax)-4\log\left(e^{\frac{1}{4}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right] - 4\left(-\frac{840e^{\frac{1}{4}\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1} - \frac{1600e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^2} - \frac{1024e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(e^{2\coth^{-1}(ax)}-1\right)^3} +\right.}{1536a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x^2,x]
```

```
[Out] (-4*((-1024*E^(ArcCoth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^3 - (1600*E^(ArcC
oth[a*x]/4))/(-1 + E^(2*ArcCoth[a*x]))^2 - (840*E^(ArcCoth[a*x]/4))/(-1 + E
```

```
^(2*ArcCoth[a*x])) - 66*ArcTan[E^(ArcCoth[a*x]/4)] + 33*Log[1 - E^(ArcCoth[
a*x]/4)] - 33*Log[1 + E^(ArcCoth[a*x]/4)]) - 33*RootSum[1 + #1^4 & , (ArcCo
th[a*x] - 4*Log[E^(ArcCoth[a*x]/4) - #1])/#1^3 & ])/(1536*a^3)
```

fricas [A] time = 0.49, size = 457, normalized size = 1.07

$$\frac{132\sqrt{2}\,a^3\frac{1}{a^{12}}^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^{12}}^{\frac{3}{4}}+a^6\sqrt{\frac{1}{a^{12}}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}a^3\frac{1}{a^{12}}^{\frac{1}{4}}}-\sqrt{2}\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^{12}}^{\frac{1}{4}}-1\right)+132\sqrt{2}\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="fricas")
```

```
[Out] 1/768*(132*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^9*((a*
x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-12)) + ((a*x - 1)/(a
*x + 1))^(1/4))*a^3*(a^(-12))^(1/4) - sqrt(2)*a^3*((a*x - 1)/(a*x + 1))^(1/
8)*(a^(-12))^(1/4) - 1) + 132*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*sq
rt(-sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-
12)) + ((a*x - 1)/(a*x + 1))^(1/4))*a^3*(a^(-12))^(1/4) - sqrt(2)*a^3*((a*x
 - 1)/(a*x + 1))^(1/8)*(a^(-12))^(1/4) + 1) + 33*sqrt(2)*a^3*(a^(-12))^(1/4
)*log(sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^
(-12)) + ((a*x - 1)/(a*x + 1))^(1/4)) - 33*sqrt(2)*a^3*(a^(-12))^(1/4)*log(
-sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-12)
) + ((a*x - 1)/(a*x + 1))^(1/4)) + 8*(32*a^3*x^3 + 68*a^2*x^2 + 73*a*x + 37
)*((a*x - 1)/(a*x + 1))^(7/8) - 132*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + 6
6*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 66*log(((a*x - 1)/(a*x + 1))^(1/8)
 - 1))/a^3
```

giac [A] time = 0.25, size = 333, normalized size = 0.78

$$-\frac{1}{768}a\left(\frac{66\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}+\frac{66\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}-\frac{33\sqrt{2}\log\left(\sqrt{2}\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="giac")
```

```
[Out] -1/768*a*(66*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/8)))/a^4 + 66*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/8)))/a^4 - 33*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 33*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 132*arctan(((a*x - 1)
/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*l
og(abs(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^4 - 16*(10*(a*x - 1)*((a*x - 1)/
(a*x + 1))^(7/8)/(a*x + 1) - 33*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(7/8)/(a*
x + 1)^2 - 105*((a*x - 1)/(a*x + 1))^(7/8))/(a^4*((a*x - 1)/(a*x + 1) - 1)^
3))
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

maxima [A] time = 0.41, size = 341, normalized size = 0.79

$$-\frac{1}{768}a\left(\frac{16\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{23}{8}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}+105\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="maxima")
```

```
[Out] -1/768*a*(16*(33*((a*x - 1)/(a*x + 1))^(23/8) - 10*((a*x - 1)/(a*x + 1))^(1
5/8) + 105*((a*x - 1)/(a*x + 1))^(7/8))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x
 - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2)*l
og(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(
1/4) + 1))/a^4 + 132*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x
 - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*log(((a*x - 1)/(a*x + 1))^(1/8) - 1)/a
^4)
```

mupad [B] time = 1.31, size = 227, normalized size = 0.53

$$\frac{\frac{35\left(\frac{ax-1}{ax+1}\right)^{7/8}}{16}-\frac{5\left(\frac{ax-1}{ax+1}\right)^{15/8}}{24}+\frac{11\left(\frac{ax-1}{ax+1}\right)^{23/8}}{16}}{a^3+\frac{3a^3(ax-1)^2}{(ax+1)^2}-\frac{a^3(ax-1)^3}{(ax+1)^3}-\frac{3a^3(ax-1)}{ax+1}}-\frac{\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/8}1\mathrm{i}\right)11\mathrm{i}}{64a^3}-\frac{11\,\mathrm{atan}\left(\left(\frac{ax-1}{ax+1}\right)^{1/8}\right)}{64a^3}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{1/8}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] ((35*((a*x - 1)/(a*x + 1))^(7/8))/16 - (5*((a*x - 1)/(a*x + 1))^(15/8))/24
+ (11*((a*x - 1)/(a*x + 1))^(23/8))/16)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1
)^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (atan(
((a*x - 1)/(a*x + 1))^(1/8)*1i)*11i)/(64*a^3) - (11*atan(((a*x - 1)/(a*x +
1))^(1/8)))/(64*a^3) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1
/2 - 1i/2))*(11/128 - 11i/128))/a^3 - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x
 + 1))^(1/8)*(1/2 + 1i/2))*(11/128 + 11i/128))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(1/8), x)
```

3.127 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=392

$$-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{16\sqrt{2}a^2}+\ldots$$

```
[Out] 1/8*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a+1/2*(1-1/a/x)^(7/8)*(1+1/a/x)^(9/8)
*x^2+1/16*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^2+1/16*arctanh((1+1/a/x
)^(1/8)/(1-1/a/x)^(1/8))/a^2-1/32*arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x
)^(1/8))/a^2*2^(1/2)+1/32*arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))
/a^2*2^(1/2)-1/64*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1
/2)/(1-1/a/x)^(1/8))/a^2*2^(1/2)+1/64*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+
(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^2*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6171, 96, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}a^2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}a^2}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{16\sqrt{2}a^2}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x,x]
```

```
[Out] ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1
 + 1/(a*x))^(9/8)*x^2)/2 - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/
(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(
1 - 1/(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a
*x))^(1/8)]/(16*a^2) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/(16
*a^2) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(32*Sqrt[2]*a^2) + Log[1 + (Sqrt[2]*(1 +
1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/(32*Sqrt[2]*a^2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^3\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^2\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{32a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{4a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{8a^2} + \frac{\text{Su}}{}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\text{Su}}{}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right.}{16a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right.}{16a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}\,a^2} + \frac{\tan^{-1}\left(1\right.}{16}$$

Mathematica [A] time = 0.80, size = 319, normalized size = 0.81

$$\frac{\frac{6}{e^{\frac{1}{4}\coth^{-1}(ax)}-1} + \frac{6}{e^{\frac{1}{4}\coth^{-1}(ax)}+1} - \frac{12e^{\frac{1}{4}\coth^{-1}(ax)}}{e^{\frac{1}{2}\coth^{-1}(ax)}+1} - \frac{40e^{\frac{1}{4}\coth^{-1}(ax)}}{e^{\coth^{-1}(ax)}+1} + \frac{2}{\left(e^{\frac{1}{4}\coth^{-1}(ax)}-1\right)^2} - \frac{2}{\left(e^{\frac{1}{4}\coth^{-1}(ax)}+1\right)^2} + \frac{8e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(e^{\frac{1}{2}\coth^{-1}(ax)}+1\right)^2} + \cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x,x]
```

```
[Out] (2/(-1 + E^(ArcCoth[a*x]/4))^2 + 6/(-1 + E^(ArcCoth[a*x]/4)) - 2/(1 + E^(Ar
cCoth[a*x]/4))^2 + 6/(1 + E^(ArcCoth[a*x]/4)) + (8*E^(ArcCoth[a*x]/4))/(1 +
 E^(ArcCoth[a*x]/2))^2 - (12*E^(ArcCoth[a*x]/4))/(1 + E^(ArcCoth[a*x]/2)) +
 (32*E^(ArcCoth[a*x]/4))/(1 + E^ArcCoth[a*x])^2 - (40*E^(ArcCoth[a*x]/4))/(
1 + E^ArcCoth[a*x]) + 4*ArcTan[E^(ArcCoth[a*x]/4)] - 2*Sqrt[2]*ArcTan[1 - S
qrt[2]*E^(ArcCoth[a*x]/4)] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/4
)] + 4*ArcTanh[E^(ArcCoth[a*x]/4)] - Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x
]/4) + E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/4) + E
^(ArcCoth[a*x]/2)])/(64*a^2)
```

fricas [A] time = 0.50, size = 448, normalized size = 1.14

$$4\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^8}^{\frac{3}{4}}+a^4\sqrt{\frac{1}{a^8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}a^2\frac{1}{a^8}^{\frac{1}{4}}-\sqrt{2}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\frac{1}{a^8}^{\frac{1}{4}}-1\right)+4\sqrt{2}a^2\frac{1}{a^8}^{\frac{1}{4}}\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="fricas")
```

```
[Out] 1/64*(4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^6*((a*x -
1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(a*x + 1
))^(1/4))*a^2*(a^(-8))^(1/4) - sqrt(2)*a^2*((a*x - 1)/(a*x + 1))^(1/8)*(a^(
-8))^(1/4) - 1) + 4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*sqrt(-sqrt(2)
*a^6*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x
- 1)/(a*x + 1))^(1/4))*a^2*(a^(-8))^(1/4) - sqrt(2)*a^2*((a*x - 1)/(a*x + 1
))^(1/8)*(a^(-8))^(1/4) + 1) + sqrt(2)*a^2*(a^(-8))^(1/4)*log(sqrt(2)*a^6*(
(a*x - 1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(
a*x + 1))^(1/4)) - sqrt(2)*a^2*(a^(-8))^(1/4)*log(-sqrt(2)*a^6*((a*x - 1)/(
a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(a*x + 1))^(
1/4)) + 8*(4*a^2*x^2 + 9*a*x + 5)*((a*x - 1)/(a*x + 1))^(7/8) - 4*arctan(((
a*x - 1)/(a*x + 1))^(1/8)) + 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*log
(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^2
```

giac [A] time = 0.23, size = 300, normalized size = 0.77

$$-\frac{1}{64}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right.}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="giac")
```

```
[Out] -1/64*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/8)))/a^3 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/8)))/a^3 - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^3 + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 + 4*arctan(((a*x - 1)/(a*x + 1))
^(1/8))/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 + 2*log(abs(((a*x
- 1)/(a*x + 1))^(1/8) - 1))/a^3 + 16*((a*x - 1)*((a*x - 1)/(a*x + 1))^(7/8)
/(a*x + 1) - 9*((a*x - 1)/(a*x + 1))^(7/8))/(a^3*((a*x - 1)/(a*x + 1) - 1)^
2))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

maxima [A] time = 0.42, size = 304, normalized size = 0.78

$$\frac{1}{64}a\left(\frac{16\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax}{ax}\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="maxima")
```

```
[Out] 1/64*a*(16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*((a*x - 1)/(a*x + 1))^(7/8))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - (2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2)*lo
g(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1
/4) + 1))/a^3 - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^3 + 2*log(((a*x - 1
)/(a*x + 1))^(1/8) + 1)/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1)/a^3)
```

mupad [B] time = 1.28, size = 190, normalized size = 0.48

$$\frac{\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,a^2}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{16\,a^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] ((9*((a*x - 1)/(a*x + 1))^(7/8))/4 - ((a*x - 1)/(a*x + 1))^(15/8)/4)/(a^2 +
 (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (atan(((a*x
 - 1)/(a*x + 1))^(1/8)*1i)*1i)/(16*a^2) - atan(((a*x - 1)/(a*x + 1))^(1/8))
/(16*a^2) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))
*(1/32 - 1i/32))/a^2 - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1
/2 + 1i/2))*(1/32 + 1i/32))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(1/8), x)
```

3.128 $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=352

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}\,a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}\,a}+\text{tan}$$

```
[Out] (1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x+1/2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8
))/a+1/2*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a-1/4*arctan(1-(1+1/a/x)^
(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)+1/4*arctan(1+(1+1/a/x)^(1/8)*2^(1/
2)/(1-1/a/x)^(1/8))/a*2^(1/2)-1/8*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1
/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)+1/8*ln(1+(1+1/a/x)^(1/4)/(1-
1/a/x)^(1/4)+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {6170, 94, 93, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}\,a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}\,a}+\text{tan}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4),x]
```

```
[Out] (1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x
))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*(1 + 1/(
a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)]/(2*a) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1
/8)]/(2*a) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1
 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*Sqrt[2]*a) + Log[1 + (Sqrt[2]*(1
+ 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/(4*Sqrt[2]*a)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^2\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \text{Subs}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[8]{1+}}{\sqrt[8]{1-}}\right.}{4a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4\sqrt{2}\,a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}\,a} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}\,a} + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} +$$

Mathematica [C] time = 0.06, size = 56, normalized size = 0.16

$$\frac{2e^{\frac{1}{4}\coth^{-1}(ax)}\left(\left(e^{2\coth^{-1}(ax)}-1\right){}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{2\coth^{-1}(ax)}\right)+1\right)}{a\left(e^{2\coth^{-1}(ax)}-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4),x]
```

```
[Out] (2*E^(ArcCoth[a*x]/4)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[1/8,
 1, 9/8, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

fricas [A] time = 0.52, size = 423, normalized size = 1.20

$$4\sqrt{2}\,a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+a^2\sqrt{\frac{1}{a^4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}\,a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^3*((a*x - 1)/
(a*x + 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^
(1/4))*a*(a^(-4))^(1/4) - sqrt(2)*a*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-4))^(1
/4) - 1) + 4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*sqrt(-sqrt(2)*a^3*((a*
x - 1)/(a*x + 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x
 + 1))^(1/4))*a*(a^(-4))^(1/4) - sqrt(2)*a*((a*x - 1)/(a*x + 1))^(1/8)*(a^(
-4))^(1/4) + 1) + sqrt(2)*a*(a^(-4))^(1/4)*log(sqrt(2)*a^3*((a*x - 1)/(a*x
+ 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^(1/4)
) - sqrt(2)*a*(a^(-4))^(1/4)*log(-sqrt(2)*a^3*((a*x - 1)/(a*x + 1))^(1/8)*(
a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^(1/4)) + 8*(a*x +
1)*((a*x - 1)/(a*x + 1))^(7/8) - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + 2*
log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*log(((a*x - 1)/(a*x + 1))^(1/8) -
1))/a
```

giac [A] time = 0.21, size = 269, normalized size = 0.76

$$-\frac{1}{8}\,a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^2}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^2}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="giac")
```

```
[Out] -1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
8)))/a^2 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/8)))/a^2 - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^2 + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^2 + 4*arctan(((a*x - 1)/(a*x + 1))^
(1/8))/a^2 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^2 + 2*log(abs(((a*x -
 1)/(a*x + 1))^(1/8) - 1))/a^2 + 16*((a*x - 1)/(a*x + 1))^(7/8)/(a^2*((a*x
- 1)/(a*x + 1) - 1)))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

maxima [A] time = 0.42, size = 265, normalized size = 0.75

$$-\frac{1}{8}a\left(\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)-\sqrt{2}\log}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="maxima")
```

```
[Out] -1/8*a*(16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)*a^2/(a*x + 1) - a^2) + (2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sq
rt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4)
 + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x
+ 1))^(1/4) + 1))/a^2 + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^2 - 2*log((
(a*x - 1)/(a*x + 1))^(1/8) + 1)/a^2 + 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1
)/a^2)
```

mupad [B] time = 1.26, size = 149, normalized size = 0.42

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}1\mathrm{i}\right)1\mathrm{i}}{2\,a}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{2\,a}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{a}+\frac{\sqrt{2}\,\mathrm{atan}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(7/8))/(a - (a*(a*x - 1))/(a*x + 1)) - (atan(((a*x
 - 1)/(a*x + 1))^(1/8)*1i)*1i)/(2*a) - atan(((a*x - 1)/(a*x + 1))^(1/8))/(2
*a) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/4
- 1i/4))/a - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 + 1i/2)
)*(1/4 + 1i/4))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-1/8), x)
```

3.129 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x} dx$

Optimal. Leaf size=919

$$-\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)-\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)+\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\ldots}{\sqrt{2\ldots}}\right.$$

```
[Out] 2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))+2*arctanh((1+1/a/x)^(1/8)/(1-1/a/
x)^(1/8))-1/2*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1/2)/
(1-1/a/x)^(1/8))*2^(1/2)+1/2*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+(1+1/a/x)
^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))*2^(1/2)-arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1
-1/a/x)^(1/8))*2^(1/2)+arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))*2^
(1/2)-arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1
/2))^(1/2))*(2-2^(1/2))^(1/2)+arctan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+
2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/2*ln(1+(1-1/a/x)^(1/
4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^
(1/2))^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2-2^
(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-arctan((-2*(1-1/a/x)^(1/8)/
(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+arc
tan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
)*(2+2^(1/2))^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8
)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/2*ln(1+(1-1/a/x)^(
1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+
2^(1/2))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {6171, 105, 63, 331, 299, 1122, 1169, 634, 618, 204, 628, 93, 214, 212, 206, 203, 211, 1165, 1162, 617}

$$-\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)-\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)+\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\ldots}{\sqrt{2\ldots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x,x]
```

```
[Out] -(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1
 + 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]]) - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 +
Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]]
 + Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1
 + 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + S
qrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]] -
 Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + Sq
rt[2]*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + 2*Arc
Tan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] + 2*ArcTanh[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)] + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1
 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1
/8)])/2 - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4
) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 + (Sqrt
[2 + Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + S
qrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*L
```

```
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8
))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2] +
 Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= 8\,\text{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right) - 8\,\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 8\,\text{Subst}\Bigg(\int \frac{}{1}$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\,\text{Subst}\Bigg(\int \frac{1}{1}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{\text{Subst}\Big(}{}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{\log\Big(1+\frac{\sqrt{2}}{\sqrt[8]{}}}{}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\,\text{ta}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\,\text{ta}$$

$$= -\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2}$$

Mathematica [C] time = 0.05, size = 30, normalized size = 0.03

$$\frac{16}{9}e^{\frac{9}{4}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{9}{16},1;\frac{25}{16};e^{4\coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x,x]
```

```
[Out] (16*E^((9*ArcCoth[a*x])/4)*Hypergeometric2F1[9/16, 1, 25/16, E^(4*ArcCoth[a
*x])])/9
```

fricas [B] time = 0.77, size = 2289, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-((sqr
t(2) + 2)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - sqrt(2)*sqrt(2*(sqrt(2
)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2
) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)
/(a*x + 1))^(1/4) + 4) + 2*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) - 3*sqrt(sqr
t(2) + 2))/((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 3*sqrt
(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) +
2))*arctan(((sqrt(2) + 2)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + sqrt(2
)*sqrt(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*
sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/
8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 2*sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt
(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)
*sqrt(-sqrt(2) + 2))*arctan(((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt
(2) + 2) - sqrt(2)*sqrt(2*(sqrt(2)*(sqrt(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2)
+ 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1
)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) + 2*sqrt(2)*((a*x -
 1)/(a*x + 1))^(1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2)^(3/2) - (sqrt(2)
 + 1)*sqrt(-sqrt(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2
) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-((sqrt(2) + 2)^(3/2) + (sqrt(2
) + 1)*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) +
(sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(
2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) -
 2*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2
)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/8*(s
qrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(sqrt(2)*(sqrt
(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*s
qrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x +
 1))^(1/4) + 4) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) +
2))*log(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2) + 2) - sqrt(2))
*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1
/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) -
 sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(sqrt(2)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*
(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*
((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 1/8*(sqr
t(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(-2*(sqrt(2)*(sqrt(
2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sq
rt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x +
1))^(1/4) + 4) + 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^
(1/8) - 1) + 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x +
 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - sqrt(2)*((a*x - 1)/(a*x +
 1))^(1/8) + 1) - sqrt(-sqrt(2) + 2)*arctan(-((sqrt(2) + 1)*sqrt(-sqrt(2) +
 2) - 2*sqrt((sqrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) +
 ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 2*((a*x - 1)/(a*x + 1))^(1/8))/((sqrt(2
) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))) - sqrt(-sqrt(2) + 2)*arctan(((sqrt(2)
+ 1)*sqrt(-sqrt(2) + 2) + 2*sqrt(-(sqrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x -
1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 2*((a*x - 1)/(a*x
+ 1))^(1/8))/((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))) - sqrt(sqrt(2) +
2)*arctan(-((sqrt(2) + 2)^(3/2) - 2*sqrt(((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt
(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) -
3*sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/((sqrt(2) + 1)*sqrt(-s
```

```
qrt(2) + 2))) - sqrt(sqrt(2) + 2)*arctan(((sqrt(2) + 2)^(3/2) + 2*sqrt(-((s
qrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + ((a*
x - 1)/(a*x + 1))^(1/4) + 1) - 3*sqrt(sqrt(2) + 2) - 2*((a*x - 1)/(a*x + 1)
)^(1/8))/((sqrt(2) + 1)*sqrt(-sqrt(2) + 2))) - 1/4*sqrt(sqrt(2) + 2)*log((s
qrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*
x + 1))^(1/4) + 1) + 1/4*sqrt(sqrt(2) + 2)*log(-(sqrt(2) + 1)*sqrt(-sqrt(2)
 + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/4*
sqrt(-sqrt(2) + 2)*log(((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x -
1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/4*sqrt(-sqrt(2)
+ 2)*log(-((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))
^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/2*sqrt(2)*log(4*sqrt(2)*((a*x
 - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 1/2*sqrt(2)*l
og(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) +
 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + log(((a*x - 1)/(a*x + 1))^(1/
8) + 1) - log(((a*x - 1)/(a*x + 1))^(1/8) - 1)
```

giac [A] time = 1.85, size = 660, normalized size = 0.72

$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,\left(\frac{ax-1}{ax+1}\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
8)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/8)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + (
(a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a
 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1
))^(1/8) - 1))/a - 4*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1
/8))/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sqrt(2) + 4)) - 4*arctan(-(sqrt(sqrt(2)
+ 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sqrt(2)
 + 4)) - 4*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt
(sqrt(2) + 2))/(a*sqrt(-2*sqrt(2) + 4)) - 4*arctan(-(sqrt(-sqrt(2) + 2) - 2
*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2))/(a*sqrt(-2*sqrt(2) + 4)) +
 2*log(sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1)
)^(1/4) + 1)/(a*sqrt(-2*sqrt(2) + 4)) - 2*log(-sqrt(sqrt(2) + 2)*((a*x - 1)
/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(-2*sqrt(2) + 4
)) + 2*log(sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/(a*sqrt(2*sqrt(2) + 4)) - 2*log(-sqrt(-sqrt(2) + 2)*((a*x
 - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(2*sqrt(2)
 + 4)))
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

mupad [B] time = 1.40, size = 648, normalized size = 0.71

$$-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1+1\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

```
[Out] atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (((a*x - 1)/(a*
x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8
)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2
*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i + (2^(1/2) + 2)^(1/2)*1i) -
2*atan(((a*x - 1)/(a*x + 1))^(1/8)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x
+ 1))^(1/8)*(1/2 - 1i/2))*(1 - 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x +
 1))^(1/8)*(1/2 + 1i/2))*(1 + 1i) - atan(((a*x - 1)/(a*x + 1))^(1/8)*1i)*2i
 - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (((a*x - 1)/
(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (2^(1/2)*((a*x - 1)/(a*x + 1))^(
1/8)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*1i)
/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i - (2^(1/2) + 2)^(1/2)*1i)
 - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) - 2)^(1/2) - (((a*x - 1
)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*((a*x - 1)/(a*x + 1))
^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)
*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i + (2 - 2^(1/2))^(1/
2)*1i) - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) - 2)^(1/2) + (((a
*x - 1)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*((a*x - 1)/(a*x
 + 1))^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) - (2^(1/2)*((a*x - 1)/(a*x + 1))
^(1/8)*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i - (2 - 2^(1/2
))^(1/2)*1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

3.130 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=676

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\right.$$

```
[Out] a*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)-1/4*a*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x
)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*a*arcta
n((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*
(2-2^(1/2))^(1/2)+1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8
)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-1/8*a*ln(1+(1-1/a/x)
^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(
2-2^(1/2))^(1/2)-1/4*a*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2
))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/4*a*arctan((2*(1-1/a/x)^(1
/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
+1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/
2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6171, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8) - (Sqrt[2 + Sqrt[2]]*a*ArcTan[(Sq
rt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/4 - (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x
))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqrt[2]]*a
*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/S
qrt[2 + Sqrt[2]]])/4 + (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*
(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 -
 Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 - Sqrt[2]]*a*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 + (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1
/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))
/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)
/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x)
)^(1/8)])/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + (2a)\,\text{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + (2a)\,\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{a\,\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\,\text{Subst}\left(\int \frac{x^4}{1+\sqrt{2}x^2+x^4}\,dx, x, \ldots\right)}{\sqrt{2}}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{a\,\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\,\text{Subst}\left(\int \frac{1+\sqrt{2}x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, \ldots\right)}{\sqrt{2}}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\ldots\right.}{1+\sqrt{2-\sqrt{2}}\,x\ldots}\ldots\right)}{2\sqrt{2\left(2-\ldots\right.}}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{4}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,a\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx, x, \frac{\sqrt[8]{1-\ldots}}{\sqrt[8]{1+\ldots}}\ldots\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right) - \frac{1}{8}\sqrt{2-\ldots}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\sqrt{2+\sqrt{2}}\,a\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2-\sqrt{2}}\,a\tan^{-1}\left(\ldots\right.$$

Mathematica [C] time = 0.07, size = 46, normalized size = 0.07

$$-2ae^{\frac{1}{4}\coth^{-1}(ax)}\left({}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{2\coth^{-1}(ax)}\right)-\frac{1}{e^{2\coth^{-1}(ax)}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x^2,x]
```

```
[Out] -2*a*E^(ArcCoth[a*x]/4)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometric2F1
[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])])
```

fricas [B] time = 0.78, size = 2874, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="fricas")

[Out] -1/32*(8*(a^8)^(1/8)*x*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - 2*sqrt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 + (a^8)^(7/8)*(a^7*(sqrt(2) + 2) - a^7)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2))) + 8*(a^8)^(1/8)*x*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - 2*sqrt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^8)^(7/8)*(a^7*(sqrt(2) + 2) - a^7)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2))) + 8*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*arctan(-(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8) - 2*sqrt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 + (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/((a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 8*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*arctan((a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8) + 2*sqrt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/((a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 2*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*log(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 + (a^8)^(7/8)*(a^7*(sqrt(2) + 2) - a^7)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)) - 2*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*log(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^8)^(7/8)*(a^7*(sqrt(2) + 2) - a^7)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)) + 2*(a^8)^(1/8)*x*sqrt(-sqrt(2) + 2)*log(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 + (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) - 2*(a^8)^(1/8)*x*sqrt(-sqrt(2) + 2)*log(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) + 4*(a^8)^(1/8)*(sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan(-(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - sqrt(2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)*a^7*(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan((a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)*a^7*(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan((a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - sqrt(2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan(-(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sq

```
rt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x +
 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2)
 - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + (a^8)^(1/8)*(sqrt(2)*x*
sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(4*a^14*((a*x - 1)/(a*
x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*
sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sqrt(2) + 2) - sqrt(2)*a^7)*s
qrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) - (a^8)^(1/8)*(
sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(4*a^14*((a*
x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt(2)*a^7*(sqrt(2) + 2)^
(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sqrt(2) + 2) - sqrt
(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) - (a^
8)^(1/8)*(sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(4
*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)*a^7*(sqr
t(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)*a^7*(sqrt(2) +
 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1
/8)) + (a^8)^(1/8)*(sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) +
 2))*log(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt(2
)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)*a^7*
(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*
x + 1))^(1/8)) - 32*(a*x + 1)*((a*x - 1)/(a*x + 1))^(7/8))/x
```

giac [A] time = 0.40, size = 432, normalized size = 0.64

$$\frac{1}{8}\left(2\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{\sqrt{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1
))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2)
+ 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(sqrt(2)
+ 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(
2) + 2)) + 2*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a*x - 1)/(
a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) +
2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(sq
rt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1) - sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*((a*x - 1
)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(-sqrt(2) + 2)*
log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))
^(1/4) + 1) + 16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)/(a*x + 1) + 1))*a
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

mupad [B] time = 1.25, size = 162, normalized size = 0.24

$$\frac{(-1)^{1/8}\, a\, \mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{2}+\frac{(-1)^{1/8}\, a\, \mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8} 1\mathrm{i}\right) 1\mathrm{i}}{2}+\frac{2\,a\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{\frac{a\,x-1}{a\,x+1}+1}+(-1)^{1/8}\sqrt{2}\, a\, \mathrm{atan}\Bigg((-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

```
[Out] ((-1)^(1/8)*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/2 + ((-1)^(1/8)
*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/2 + (2*a*((a*x - 1)/
(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1) + (-1)^(1/8)*2^(1/2)*a*atan((-1
)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/4 - 1i/4) + (-
1)^(1/8)*2^(1/2)*a*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2
 + 1i/2))*(1/4 + 1i/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \sqrt[8]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

3.131 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=731

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}+\frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{64}\sqrt{2-\sqrt{2}}\,a^2\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)$$

```
[Out] 1/8*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)+1/2*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^(
9/8)-1/32*a^2*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))
/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/32*a^2*arctan((2*(1-1/a/x)^(1/8)/(1
+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/64*
a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/
(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-
1/32*a^2*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/32*a^2*arctan((2*(1-1/a/x)^(1/8)/(1+1/a/
x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/64*a^2*l
n(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/
a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)
+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6171, 80, 50, 63, 331, 299, 1122, 1169, 634, 618, 204, 628}

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}+\frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{64}\sqrt{2-\sqrt{2}}\,a^2\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8))/8 + (a^2*(1 - 1/(a*x))^(7/8)*
(1 + 1/(a*x))^(9/8))/2 - (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sqrt[2 - Sqrt[2]] -
 (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]])/32 - (Sqr
t[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 +
 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sq
rt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/32 + (Sqrt[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(
a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 - Sqrt[2
]]*a^2*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]
*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 - Sqrt[2]]*a^2*Log
[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a
*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 + (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 -
1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8)
)/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 - 1/(a*x))^(
1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(
a*x))^(1/8)])/64
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{8}a\,\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{32}a\,\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{x^6}{\left(2-x^8\right)^{7/8}}\,dx, x\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{a^2\,\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}x^2+x^4}\,dx, x, \right.}{8\sqrt{2}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{a^2\,\text{Subst}\left(\int \frac{1-\sqrt{2}x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, \right.}{8\sqrt{2}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx\right.}{16\sqrt{2\left(2-\sqrt{2}\right)}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{32}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}a^2\right)\text{Subst}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{64}\sqrt{2-\sqrt{2}}\,a^2\log\left(1+\frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{32}\sqrt{2+\sqrt{2}}\,a^2\tan^{-1}\left(\frac{\sqrt{2-}}{\sqrt{}}\right.$$

Mathematica [C] time = 0.10, size = 72, normalized size = 0.10

$$-\frac{a^2 e^{\frac{1}{4}\coth^{-1}(ax)}\left(\left(e^{2\coth^{-1}(ax)}+1\right)^2 {}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{2\coth^{-1}(ax)}\right)-9e^{2\coth^{-1}(ax)}-1\right)}{4\left(e^{2\coth^{-1}(ax)}+1\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x^3,x]
```

```
[Out] -1/4*(a^2*E^(ArcCoth[a*x]/4)*(-1 - 9*E^(2*ArcCoth[a*x]) + (1 + E^(2*ArcCoth
[a*x]))^2*Hypergeometric2F1[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])]))/(1 + E^(2*A
rcCoth[a*x]))^2
```

fricas [B] time = 0.94, size = 2931, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="fricas")
```

```
[Out] -1/256*(8*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^16)^(1/8)*a^14*
((a*x - 1)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2
) - 2*sqrt(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 + (a^16)^(7
/8)*(a^14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1
/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2))) +
 8*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^16)^(1/8)*a^14*((a*x -
 1)/(a*x + 1))^(1/8) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) - 2*s
qrt(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^
14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a
^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2))) + 8*(a^1
6)^(1/8)*x^2*sqrt(sqrt(2) + 2)*arctan(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*s
qrt(sqrt(2) + 2) + 2*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) - 2*sqrt
(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 + (a^16)^(7/8)*(a^14*
(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8)
)*(a^16)^(1/8))/((a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 8*(a^16
)^(1/8)*x^2*sqrt(sqrt(2) + 2)*arctan((a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqr
t(sqrt(2) + 2) - 2*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) + 2*sqrt(a
^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14*(s
qrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*
(a^16)^(1/8))/((a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 2*(a^16)^
(1/8)*x^2*sqrt(sqrt(2) + 2)*log(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(
3/4)*a^16 + (a^16)^(7/8)*(a^14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a
*x - 1)/(a*x + 1))^(1/8)) - 2*(a^16)^(1/8)*x^2*sqrt(sqrt(2) + 2)*log(a^28*(
(a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14*(sqrt(2
) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)) + 2*(a^16)^(
1/8)*x^2*sqrt(-sqrt(2) + 2)*log(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(
3/4)*a^16 + (a^16)^(7/8)*(a^14*(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) +
2))*((a*x - 1)/(a*x + 1))^(1/8)) - 2*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*lo
g(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14
*(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8
)) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + sqrt(2)*x^2*sqrt(-sqrt
(2) + 2))*arctan(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + 2*
sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) - (a^16*(sqrt(2) + 2)
 - a^16)*sqrt(-sqrt(2) + 2) - sqrt(2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/
4) + 4*(a^16)^(3/4)*a^16 + 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2)
 - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) - (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)
*a^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16
*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + (a^16*(sqrt(2) + 2) - a^1
6)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + s
qrt(2)*x^2*sqrt(-sqrt(2) + 2))*arctan((a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sq
rt(sqrt(2) + 2) - 2*sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) -
 (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(4*a^28*((a*x
 - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 - 2*(a^16)^(7/8)*(sqrt(2)*a^14
*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) - (sqrt(2)*a^14*(sq
rt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))
*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + (a^16
*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*s
qrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*arctan((a^16*(sqrt(2) +
2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1
```

```
)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) - sqrt(
2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 + 2*(a^16)
^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2)
 + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x -
1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(
sqrt(2) + 2) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^
(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*arct
an(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^16)
^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(
-sqrt(2) + 2) + sqrt(2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^
(3/4)*a^16 - 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a
^14*sqrt(sqrt(2) + 2) + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-s
qrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)
^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(
2) + 2))) + (a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + sqrt(2)*x^2*sqrt(
-sqrt(2) + 2))*log(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16
 + 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(s
qrt(2) + 2) + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2
))*((a*x - 1)/(a*x + 1))^(1/8)) - (a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) +
2) + sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*log(4*a^28*((a*x - 1)/(a*x + 1))^(1/4)
 + 4*(a^16)^(3/4)*a^16 - 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) -
 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a
^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8)) - (a^16)^(1/8)*(sqrt(
2)*x^2*sqrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*log(4*a^28*((a*x
 - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 + 2*(a^16)^(7/8)*(sqrt(2)*a^14
*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) - (sqrt(2)*a^14*(sq
rt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))
 + (a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2)
+ 2))*log(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 - 2*(a^1
6)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) +
2) - (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x
- 1)/(a*x + 1))^(1/8)) - 32*(5*a^2*x^2 + 9*a*x + 4)*((a*x - 1)/(a*x + 1))^(
7/8))/x^2
```

giac [A] time = 0.47, size = 473, normalized size = 0.65

$$\frac{1}{64}\left(2\,a\sqrt{-\sqrt{2}+2}\,\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\,a\sqrt{-\sqrt{2}+2}\,\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(2*a*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x
+ 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqr
t(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(s
qrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqr
t(sqrt(2) + 2)) + 2*a*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a
*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - a*sqrt(sqrt(2) + 2)*log(sqrt
(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1
) + a*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)
+ ((a*x - 1)/(a*x + 1))^(1/4) + 1) - a*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2)
 + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + a*sq
rt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*
x - 1)/(a*x + 1))^(1/4) + 1) + 16*((a*x - 1)*a*((a*x - 1)/(a*x + 1))^(7/8)/
(a*x + 1) + 9*a*((a*x - 1)/(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="maxima")
[Out] integrate(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

mupad [B] time = 1.26, size = 210, normalized size = 0.29

$$\frac{\frac{9\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4}+\frac{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}+\frac{(-1)^{1/8}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{16}+\frac{(-1)^{1/8}\,a^2\,\mathrm{atan}\left((-1)^{1/8}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{16}+(-1)^{1/8}\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)),x)
[Out] ((9*a^2*((a*x - 1)/(a*x + 1))^(7/8))/4 + (a^2*((a*x - 1)/(a*x + 1))^(15/8))
/4)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/8)*a
^2*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/16 + ((-1)^(1/8)*a^2*atan(
(-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/16 + (-1)^(1/8)*2^(1/2)*a^2*
atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/32 - 1
i/32) + (-1)^(1/8)*2^(1/2)*a^2*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1)
)^(1/8)*(1/2 + 1i/2))*(1/32 + 1i/32)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \sqrt[8]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**3,x)
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

3.132 $\int e^{4\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=45

$$-4x^{m+1}{}_2F_1(1,m+1;m+2;ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

```
[Out] x^(1+m)/(1+m)+4*x^(1+m)/(-a*x+1)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6126, 89, 80, 64}

$$-4x^{m+1}{}_2F_1(1,m+1;m+2;ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, a*x]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}x^m\,dx &= \int e^{4\tanh^{-1}(ax)}x^m\,dx\\ &= \int \frac{x^m(1+ax)^2}{(1-ax)^2}\,dx\\ &= \frac{4x^{1+m}}{1-ax}-\frac{\int \frac{x^m\left(a^2(3+4m)+a^3x\right)}{1-ax}\,dx}{a^2}\\ &= \frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-ax}-(4(1+m))\int \frac{x^m}{1-ax}\,dx\\ &= \frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-ax}-4x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.04

$$\frac{x^{m+1}(-4(m+1)(ax-1)\,{}_2F_1(1,m+1;m+2;ax)+ax-4m-5)}{(m+1)(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(-5 - 4*m + a*x - 4*(1 + m)*(-1 + a*x)*Hypergeometric2F1[1, 1 +
m, 2 + m, a*x]))/((1 + m)*(-1 + a*x))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)x^m}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2x^m}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

maple [C] time = 0.51, size = 201, normalized size = 4.47

$$-\frac{(-a)^{-m}\left(\frac{x^m(-a)^m\left(a^2m\,x^2+amx+2ax-m^2-3m-2\right)}{(1+m)m(-ax+1)}+x^m\,(-a)^m\,(2+m)\,\Phi\,(ax,1,m)\right)}{a}+\frac{2\,(-a)^{-m}\left(-\frac{x^m(-a)^m(ax-m-1)}{m(-ax+1)}-x^m\,(-a)^{\cdots}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^m,x)
```

```
[Out] -(-a)^(-m)/a*(x^m*(-a)^m*(a^2*m*x^2+a*m*x+2*a*x-m^2-3*m-2)/(1+m)/m/(-a*x+1)
+x^m*(-a)^m*(2+m)*LerchPhi(a*x,1,m))+2*(-a)^(-m)/a*(-x^m*(-a)^m*(a*x-m-1)/m
/(-a*x+1)-x^m*(-a)^m*(1+m)*LerchPhi(a*x,1,m))-(-a)^(-m)/a*(1/(1+m)*x^m*(-a)
^m*(-1-m)/(-a*x+1)+x^m*(-a)^m*m*LerchPhi(a*x,1,m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2 x^m}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,(a\,x+1)^2}{(a\,x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] int((x^m*(a*x + 1)^2)/(a*x - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**m,x)
```

```
[Out] Integral(x**m*(a*x + 1)**2/(a*x - 1)**2, x)
```

3.133 $\int e^{3\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=151

$$-\frac{3x^{m+1}\,_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{4x^{m+1}\,_2F_1\left(\frac{3}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}-\frac{x^m\,_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}+\frac{4x^m\,_2F_1}{}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-x^m*hyp
ergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m+4*x^(1+m)*hypergeom([3/2, -1/
2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+4*x^m*hypergeom([3/2, -1/2*m],[1-1/2*
m],1/a^2/x^2)/a/m
```

Rubi [A] time = 1.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6172, 6742, 364, 850, 808}

$$-\frac{x^m\,_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}+\frac{4x^m\,_2F_1\left(\frac{3}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}-\frac{3x^{m+1}\,_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{4x^{m+1}\,_2F_1\left(\frac{3}{2},\frac{1}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) - (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m) + (
4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1
+ m) + (4*x^m*Hypergeometric2F1[3/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6172

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m
, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 -
 x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
!IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\left(-\frac{3x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}} - \frac{x^{-1-m}}{a\sqrt{1-\frac{x^2}{a^2}}} + \frac{4x^{-2-m}}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\right)$$

$$= \left(3\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subs}$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subs}$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-\right.}{1+}$$

Mathematica [C] time = 0.36, size = 228, normalized size = 1.51

$$\frac{x^{m+1}\left(3(m+1)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1-ax}\sqrt{\frac{ax+1}{a^2}}F_1\left(m;-\frac{1}{2},\frac{1}{2};m+1;-ax,ax\right) - 2(m+1)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1-ax}\sqrt{\frac{ax+1}{a^2}}F_1\left(\right.\right.}{m(m+1)\sqrt{ax-1}\sqrt{ax+1}\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^
2]*AppellF1[m, -1/2, 1/2, 1 + m, -(a*x), a*x] - 2*(1 + m)*Sqrt[1 - 1/(a^2*x
^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, -(a*x)
, a*x] + m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]*Hypergeometric2
F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]))/(m*(1 + m)*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*Sqrt[-a^(-2) + x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)x^m\sqrt{\frac{ax-1}{ax+1}}}{a^2x^2-2\,ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a^2*x^2 - 2*a
*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x***m,x)
```

```
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.134 $\int e^{2\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=35

$$\frac{x^{m+1}}{m+1} - \frac{2x^{m+1}\,{}_2F_1(1, m+1; m+2; ax)}{m+1}$$

```
[Out] x^(1+m)/(1+m)-2*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6126, 80, 64}

$$\frac{x^{m+1}}{m+1} - \frac{2x^{m+1}\,{}_2F_1(1, m+1; m+2; ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(
1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)} x^m \, dx &= -\int e^{2\tanh^{-1}(ax)} x^m \, dx \\
&= -\int \frac{x^m(1+ax)}{1-ax} \, dx \\
&= \frac{x^{1+m}}{1+m} - 2\int \frac{x^m}{1-ax} \, dx \\
&= \frac{x^{1+m}}{1+m} - \frac{2x^{1+m}\,{}_2F_1(1, 1+m; 2+m; ax)}{1+m}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.74

$$\frac{x^{m+1}(1 - 2\,{}_2F_1(1, m+1; m+2; ax))}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(1 + m)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)x^m}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^m/(a*x - 1), x)
```

maple [C] time = 0.41, size = 106, normalized size = 3.03

$$\frac{(-a)^{-m}\left(-\frac{x^m(-a)^m(-1-m)}{(1+m)m} - x^m(-a)^m\,\Phi(ax,1,m)\right)}{a} - \frac{(-a)^{-m}\left(-\frac{x^m(-a)^m(amx+m+1)}{(1+m)m} + x^m(-a)^m\,\Phi(ax,1,m)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^m,x)
```

```
[Out] (-a)^(-m)/a*(-1/(1+m)*x^m*(-a)^m*(-1-m)/m-x^m*(-a)^m*LerchPhi(a*x,1,m))-(-a
)^(-m)/a*(-x^m*(-a)^m*(a*m*x+m+1)/(1+m)/m+x^m*(-a)^m*LerchPhi(a*x,1,m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)x^m}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*x^m/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/(a*x - 1),x)
```

[Out] int((x^m*(a*x + 1))/(a*x - 1), x)

sympy [B] time = 2.71, size = 100, normalized size = 2.86

$$-\frac{amx^2x^m\Phi(ax,1,m+2)\Gamma(m+2)}{\Gamma(m+3)}-\frac{2ax^2x^m\Phi(ax,1,m+2)\Gamma(m+2)}{\Gamma(m+3)}-\frac{mxx^m\Phi(ax,1,m+1)\Gamma(m+1)}{\Gamma(m+2)}-\frac{xx^m\Phi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*x**m,x)

[Out] -a*m*x**2*x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - 2*a*x**2*x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - m*x*x**m*lerchphi(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) - x*x**m*lerchphi(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2)

3.135 $\int e^{\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=74

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

```
[Out] x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+x^m*hyperg
eom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6172, 808, 364}

$$\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}+\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) + (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6172

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m
, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 -
 x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
!IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right)\right)-\frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}} \, dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m}+\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}
\end{aligned}$$

Mathematica [C] time = 0.42, size = 128, normalized size = 1.73

$$x^{m+1}\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{m}{2}-\frac{1}{2};\frac{1}{2}-\frac{m}{2};\frac{1}{a^2x^2}\right)}{m+1}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{x^2-\frac{1}{a^2}}F_1\left(m;-\frac{1}{2},\frac{1}{2};m+1;-ax,ax\right)}{m\sqrt{ax-1}\sqrt{\frac{ax+1}{a^2}}\sqrt{1-a^2x^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^m,x]
[Out] x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[-a^(-2) + x^2]*AppellF1[m, -1/2, 1
/2, 1 + m, -(a*x), a*x])/(m*Sqrt[-1 + a*x]*Sqrt[(1 + a*x)/a^2]*Sqrt[1 - a^2
*x^2])) + Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 +
m))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)x^m\sqrt{\frac{ax-1}{ax+1}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="fricas")
[Out] integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="giac")
[Out] integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m,x)
```

```
[Out] Integral(x**m/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.136 $\int e^{-\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=75

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

```
[Out] x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-x^m*hyperg
eom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6172, 808, 364}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^ArcCoth[a*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) - (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 6172

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m
, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 -
 x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
!IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\right)+\frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m}-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}
\end{aligned}$$

Mathematica [C] time = 0.26, size = 115, normalized size = 1.53

$$x^{m+1}\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{m}{2}-\frac{1}{2};\frac{1}{2}-\frac{m}{2};\frac{1}{a^2x^2}\right)}{m+1}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{ax-1}{a^2}}F_1\left(m;-\frac{1}{2},\frac{1}{2};m+1;ax,-ax\right)}{m\sqrt{1-ax}\sqrt{x^2-\frac{1}{a^2}}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^ArcCoth[a*x],x]
```

```
[Out] x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[m, -1/2,
1/2, 1 + m, a*x, -(a*x)])/(m*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])) + Hypergeo
metric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 + m))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\sqrt{\frac{ax-1}{ax+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**m*sqrt((a*x - 1)/(a*x + 1)), x)
```

3.137 $\int e^{-2\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}}{m+1} - \frac{2x^{m+1}\,{}_2F_1(1, m+1; m+2; -ax)}{m+1}$$

```
[Out] x^(1+m)/(1+m)-2*x^(1+m)*hypergeom([1, 1+m],[2+m],-a*x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6167, 6126, 80, 64}

$$\frac{x^{m+1}}{m+1} - \frac{2x^{m+1}\,{}_2F_1(1, m+1; m+2; -ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(2*ArcCoth[a*x]),x]
```

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]
)/(1 + m)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6126

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[(x^m*(1 + a*x
)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)} x^m \, dx &= -\int e^{-2\tanh^{-1}(ax)} x^m \, dx \\
&= -\int \frac{x^m(1-ax)}{1+ax} \, dx \\
&= \frac{x^{1+m}}{1+m} - 2\int \frac{x^m}{1+ax} \, dx \\
&= \frac{x^{1+m}}{1+m} - \frac{2x^{1+m}\,{}_2F_1(1, 1+m; 2+m; -ax)}{1+m}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{x^{m+1}(1-2\,{}_2F_1(1,m+1;m+2;-ax))}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^(2*ArcCoth[a*x]),x]
[Out] (x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(1 + m)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax-1)x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral((a*x - 1)*x^m/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*x^m/(a*x + 1), x)
```

maple [C] time = 0.39, size = 93, normalized size = 2.58

$$a^{-1-m}\left(\frac{x^m a^m\,(amx-m-1)}{(1+m)\,m}+x^m a^m \Phi\left(-ax,1,m\right)\right)-a^{-1-m}\left(\frac{x^m a^m}{m}+\frac{x^m a^m\,(-1-m)\,\Phi\left(-ax,1,m\right)}{1+m}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x+1)*(a*x-1),x)
[Out] a^(-1-m)*(x^m*a^m*(a*m*x-m-1)/(1+m)/m+x^m*a^m*LerchPhi(-a*x,1,m))-a^(-1-m)*
(x^m*a^m/m+1/(1+m)*x^m*a^m*(-1-m)*LerchPhi(-a*x,1,m))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)x^m}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*x^m/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^m*(a*x - 1))/(a*x + 1), x)
```

sympy [C] time = 2.69, size = 119, normalized size = 3.31

$$\frac{amx^2x^m\Phi\left(axe^{i\pi},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}+\frac{2ax^2x^m\Phi\left(axe^{i\pi},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)}-\frac{mxx^m\Phi\left(axe^{i\pi},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a*x-1)/(a*x+1),x)
```

```
[Out] a*m*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/gamma(m
+ 3) + 2*a*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/g
amma(m + 3) - m*x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1)
/gamma(m + 2) - x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1)
/gamma(m + 2)
```

3.138 $\int e^{-3\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=150

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}-\frac{4x^m\,{}_2...}{...}$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+x^m*hyp
ergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m+4*x^(1+m)*hypergeom([3/2, -1/
2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-4*x^m*hypergeom([3/2, -1/2*m],[1-1/2*
m],1/a^2/x^2)/a/m
```

Rubi [A] time = 1.05, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6172, 6742, 364, 850, 808}

$$\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}-\frac{4x^m\,{}_2F_1\left(\frac{3}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},...\right)}{...}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) + (x^m*Hypergeometric2F1[1/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m) + (
4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1
+ m) - (4*x^m*Hypergeometric2F1[3/2, -m/2, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6172

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m
, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 -
 x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
!IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \left(-\frac{3x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}+\frac{x^{-1-m}}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{4x^{-2-m}}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)\right)$$

$$= \left(3\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x,$$
$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}$$
$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1}{1+m}$$

Mathematica [C] time = 0.27, size = 192, normalized size = 1.28

$$\frac{x^{m+1}\left(-3(m+1)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{ax-1}{a^2}}F_1\left(m;-\frac{1}{2},\frac{1}{2};m+1;ax,-ax\right)+2(m+1)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{ax-1}{a^2}}F_1\left(m;-\frac{1}{2},\frac{3}{2};m+1;a\right.}{m(m+1)\sqrt{1-ax}\sqrt{x^2-\frac{1}{a^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*(-3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[
m, -1/2, 1/2, 1 + m, a*x, -(a*x)] + 2*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-
1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, a*x, -(a*x)] + m*Sqrt[1 - a*x]*
Sqrt[-a^(-2) + x^2]*Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x
^2)]))/(m*(1 + m)*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax-1)x^m\sqrt{\frac{ax-1}{ax+1}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^{m} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.139 $\int e^{\frac{5}{2}\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=41

$$\frac{x^{m+1} F_1\left(-m-1; \frac{5}{4}, -\frac{5}{4}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,5/4,-5/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1} F_1\left(-m-1; \frac{5}{4}, -\frac{5}{4}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 5/4, -5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{5}{2}\coth^{-1}(ax)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}} \, dx, x, \frac{1}{x}\right)\right)\\
&= \frac{x^{1+m} F_1\left(-1-m; \frac{5}{4}, -\frac{5}{4}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}
\end{aligned}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int e^{\frac{5}{2}\coth^{-1}(ax)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((5*ArcCoth[a*x])/2)*x^m, x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a^2*x^2 - 2
*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**m,x)
[Out] Timed out
```

3.140 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;\frac{3}{4},-\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,3/4,-3/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{3}{4},-\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 3/4, -3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx&=-\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)\right)\\&=\frac{x^{1+m}F_1\left(-1-m;\frac{3}{4},-\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.61, size = 0, normalized size = 0.00

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((3*ArcCoth[a*x])/2)*x^m, x]
```

fricas [F]   time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="fricas")
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x - 1), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

maple [F]   time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(3/4),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**m,x)
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(3/4), x)
```

3.141 $\int e^{\frac{1}{2}\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=41

$$\frac{x^{m+1} F_1\left(-m-1; \frac{1}{4}, -\frac{1}{4}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/4,-1/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1} F_1\left(-m-1; \frac{1}{4}, -\frac{1}{4}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^m,x]
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/4, -1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{2}\coth^{-1}(ax)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m} \sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)\right)\\ &= \frac{x^{1+m} F_1\left(-1-m; \frac{1}{4}, -\frac{1}{4}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{2}\coth^{-1}(ax)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^m,x]
[Out] Integrate[E^(ArcCoth[a*x]/2)*x^m, x]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="fricas")
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/4),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**m,x)
```

```
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(1/4), x)
```

3.142 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{1}{4},\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

[Out] x^(1+m)*AppellF1(-1-m,-1/4,1/4,-m,1/a/x,-1/a/x)/(1+m)

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{1}{4},\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[x^m/E^(ArcCoth[a*x]/2),x]

[Out] (x^(1 + m)*AppellF1[-1 - m, -1/4, 1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int\frac{x^{-2-m}\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;-\frac{1}{4},\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.63, size = 0, normalized size = 0.00

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m/E^(ArcCoth[a*x]/2),x]

[Out] Integrate[x^m/E^(ArcCoth[a*x]/2), x]

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
[Out] int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(1/4),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(1/4),x)
[Out] Timed out
```

3.143 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,-3/4,3/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((3*ArcCoth[a*x])/2),x]
[Out] (x^(1 + m)*AppellF1[-1 - m, -3/4, 3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.61, size = 0, normalized size = 0.00

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((3*ArcCoth[a*x])/2),x]
[Out] Integrate[x^m/E^((3*ArcCoth[a*x])/2), x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
[Out] int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(\frac{a\,x-1}{a\,x+1}\right)^{3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(3/4),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(3/4),x)
[Out] Timed out
```

3.144 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,-5/4,5/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((5*ArcCoth[a*x])/2),x]
[Out] (x^(1 + m)*AppellF1[-1 - m, -5/4, 5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.69, size = 0, normalized size = 0.00

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/E^((5*ArcCoth[a*x])/2),x]
[Out] Integrate[x^m/E^((5*ArcCoth[a*x])/2), x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax-1)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
[Out] integral((a*x - 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
[Out] int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(5/4),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(5/4),x)
[Out] Timed out
```

3.145 $\int e^{\frac{2}{3}\coth^{-1}(x)} x^m \, dx$

Optimal. Leaf size=34

$$\frac{x^{m+1} F_1\left(-m-1;\frac{1}{3},-\frac{1}{3};-m;\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/3,-1/3,-m,1/x,-1/x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6173, 133}

$$\frac{x^{m+1} F_1\left(-m-1;\frac{1}{3},-\frac{1}{3};-m;\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/3, -1/3, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int e^{\frac{2}{3}\coth^{-1}(x)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\sqrt[3]{1+x}}{\sqrt[3]{1-x}} \, dx, x, \frac{1}{x}\right)\right) \\ &= \frac{x^{1+m} F_1\left(-1-m;\frac{1}{3},-\frac{1}{3};-m;\frac{1}{x},-\frac{1}{x}\right)}{1+m} \end{aligned}$$

Mathematica [F] time = 0.85, size = 0, normalized size = 0.00

$$\int e^{\frac{2}{3}\coth^{-1}(x)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x^m,x]
```

```
[Out] Integrate[E^((2*ArcCoth[x])/3)*x^m, x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(x+1)x^m \left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="fricas")
[Out] integral((x + 1)*x^m*((x - 1)/(x + 1))^(2/3)/(x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x^m,x)
[Out] int(1/((-1+x)/(1+x))^(1/3)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(\frac{x-1}{x+1}\right)}^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((x - 1)/(x + 1))^(1/3),x)
[Out] int(x^m/((x - 1)/(x + 1))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt[3]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x**m,x)
[Out] Integral(x**m/((x - 1)/(x + 1))**(1/3), x)
```

3.146 $\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx$

Optimal. Leaf size=34

$$\frac{x^{m+1} F_1\left(-m-1;\frac{1}{6},-\frac{1}{6};-m;\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/6,-1/6,-m,1/x,-1/x)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6173, 133}

$$\frac{x^{m+1} F_1\left(-m-1;\frac{1}{6},-\frac{1}{6};-m;\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/6, -1/6, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\sqrt[6]{1+x}}{\sqrt[6]{1-x}} \, dx, x, \frac{1}{x}\right)\right)\\
&= \frac{x^{1+m} F_1\left(-1-m;\frac{1}{6},-\frac{1}{6};-m;\frac{1}{x},-\frac{1}{x}\right)}{1+m}
\end{aligned}$$

Mathematica [F] time = 0.83, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcCoth[x]/3)*x^m,x]
```

```
[Out] Integrate[E^(ArcCoth[x]/3)*x^m, x]
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(x+1)x^m \left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="fricas")
[Out] integral((x + 1)*x^m*((x - 1)/(x + 1))^(5/6)/(x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{-1+x}{1+x}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x^m,x)
[Out] int(1/((-1+x)/(1+x))^(1/6)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(\frac{x-1}{x+1}\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((x - 1)/(x + 1))^(1/6),x)
[Out] int(x^m/((x - 1)/(x + 1))^(1/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x**m,x)
[Out] Integral(x**m/((x - 1)/(x + 1))**(1/6), x)
```

3.147 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{m+1}F_1\left(-m-1;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/8,-1/8,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/8, -1/8, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.69, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x^m,x]
```

```
[Out] Integrate[E^(ArcCoth[a*x]/4)*x^m, x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{ax-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="fricas")
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(7/8)/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/8),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**m,x)
```

```
[Out] Timed out
```

3.148 $\int e^{n\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=45

$$\frac{x^{m+1}F_1\left(-m-1;\frac{n}{2},-\frac{n}{2};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/2*n,-1/2*n,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6173, 133}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{n}{2},-\frac{n}{2};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, n/2, -n/2, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6173

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> -Dist[x^m*(1/x)^m,
 Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /;
FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-2-m}\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;\frac{n}{2},-\frac{n}{2};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int e^{n\coth^{-1}(ax)}x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x^m,x]
```

```
[Out] Integrate[E^(n*ArcCoth[a*x])*x^m, x]
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="fricas")
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="giac")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arccoth}(ax)} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^m,x)
[Out] int(exp(n*arccoth(a*x))*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \, e^{n \operatorname{acoth}(a\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*acoth(a*x)),x)
[Out] int(x^m*exp(n*acoth(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{n \operatorname{acoth}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**m,x)
[Out] Integral(x**m*exp(n*acoth(a*x)), x)
```

3.149 $\int e^{n \coth^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=174

$$\frac{2\left(n^2+2\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}+\frac{1}{3}x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{nx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\right.}{6a}$$

```
[Out] 1/6*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/a+1/3*(1-1/a/x)^(1-1/2*n)
*(1+1/a/x)^(1+1/2*n)*x^3+2/3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*
n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^3/(2-n)
```

Rubi [A] time = 0.09, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6171, 129, 151, 12, 131}

$$\frac{2\left(n^2+2\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}+\frac{1}{3}x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{nx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\right.}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x^2,x]
```

```
[Out] (n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/(6*a) + ((1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3)/3 + (2*(2 + n^2)*(1 - 1/(a*x
))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (a - x^(-1))/(a + x^(-1))])/(3*a^3*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 + \frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{\left(2+n^2\right)\left(1\right.}{}\right.$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 - \frac{\left(2+n^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-}}{}\right.}{6a^2}$$

$$= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3 + \frac{2\left(2+n^2\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1\right.}{}$$

Mathematica [A] time = 0.63, size = 118, normalized size = 0.68

$$\frac{e^{n\coth^{-1}(ax)}\left((n+2)\left(2a^3x^3+n\left(a^2x^2-1\right)+\left(n^2+2\right){}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+an^2x\right)+n\left(n^2+2\right)e^{2\coth^{-1}}\right.}{6a^3(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x^2,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometric2F1[1, 1
+ n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n^2*x + 2*a^3*x^3 + n*(-1
+ a^2*x^2) + (2 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x]
)])))/(6*a^3*(2 + n))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="giac")
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arccoth}(ax)} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2,x)
[Out] int(exp(n*arccoth(a*x))*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \, e^{n \operatorname{acoth}(a\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*acoth(a*x)),x)
[Out] int(x^2*exp(n*acoth(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{n \operatorname{acoth}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**2,x)
[Out] Integral(x**2*exp(n*acoth(a*x)), x)
```

3.150 $\int e^{n\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=122

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}+\frac{1}{2}x^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

```
[Out] 1/2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2+2*n*(1-1/a/x)^(1-1/2*n)*(1+
1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^2/(2-
n)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6171, 96, 131}

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}+\frac{1}{2}x^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x,x]
```

```
[Out] ((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/2 + (2*n*(1 - 1/(a*
x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, (a - x^(-1))/(a + x^(-1))])/(a^2*(2 - n))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 - \frac{n\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 + \frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}{}_2F_1\left(2, 1-\frac{n}{2}; 2-\frac{n}{2}; \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}$$

Mathematica [A] time = 0.32, size = 98, normalized size = 0.80

$$\frac{e^{n\coth^{-1}(ax)}\left((n+2)\left(a^2x^2+n\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+anx-1\right)+n^2e^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(\right.\right.}}{2a^2(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + a^2*x^2 + n*Hypergeomet
ric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(2*a^2*(2 + n))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="giac")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x,x)
```

```
[Out] int(exp(n*arccoth(a*x))*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="maxima")
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,e^{n\,\mathrm{acoth}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(n*acoth(a*x)),x)
[Out] int(x*exp(n*acoth(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x,x)
[Out] Integral(x*exp(n*acoth(a*x)), x)
```

3.151 $\int e^{n \coth^{-1}(ax)}\,dx$

Optimal. Leaf size=78

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[Out] 4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n]
,(a-1/x)/(a+1/x))/a/(2-n)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6170, 131}

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}\end{aligned}$$

Mathematica [A] time = 0.20, size = 82, normalized size = 1.05

$$\frac{e^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(ax)}\right)+(n+2)\left({}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+ax\right)\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arccoth}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x)),x)
```

```
[Out] int(exp(n*arccoth(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x)),x)
```

```
[Out] int(exp(n*acoth(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{acoth}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x)),x)
```

```
[Out] Integral(exp(n*acoth(a*x)), x)
```

3.152 $\int \frac{e^{n \coth^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=127

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2} {}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{n}-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}$$

```
[Out] -2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/n/((1
-1/a/x)^(1/2*n))+2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],1/2*(a-1/
x)/a)/n/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 105, 69, 131}

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2} {}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{n}-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x,x]
```

```
[Out] (-2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
 + x^(-1))])/(n*(1 - 1/(a*x))^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-n/2,
 -n/2, 1 - n/2, (a - x^(-1))/(2*a)])/(n*(1 - 1/(a*x))^(n/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n} + \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{a-\ldots}{2a\ldots}\right.}{n}$$

Mathematica [A] time = 0.16, size = 142, normalized size = 1.12

$$\frac{e^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\coth^{-1}(ax)}\right)+ne^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(ax)}\right)-(n+2\ldots\right.}{n(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, -E^(2*ArcCoth[a*x])] + E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 +
n/2, 2 + n/2, E^(2*ArcCoth[a*x])] - (2 + n)*(Hypergeometric2F1[1, n/2, 1 +
n/2, -E^(2*ArcCoth[a*x])] - Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth
[a*x])])))/(n*(2 + n))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x,x)
```

```
[Out] int(exp(n*acoth(a*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x, x)
```

3.153 $\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=70

$$\frac{a2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

```
[Out] 2^(1+1/2*n)*a*(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2
*(a-1/x)/a)/(2-n)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6171, 69}

$$\frac{a2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^2,x]
```

```
[Out] (2^(1 + n/2)*a*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 -
 n/2, (a - x^(-1))/(2*a)])/(2 - n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx &= -\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)\\ &= \frac{2^{1+\frac{n}{2}}a\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.63

$$-\frac{4ae^{(n+2)\coth^{-1}(ax)}\,{}_2F_1\left(2,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\coth^{-1}(ax)}\right)}{n+2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^2,x]
```

```
[Out] (-4*a*E^((2 + n)*ArcCoth[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(2
*ArcCoth[a*x])])/(2 + n)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{acoth}(a\,x)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x^2,x)
```

```
[Out] int(exp(n*acoth(a*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**2,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x**2, x)
```

3.154 $\int \frac{e^{n \coth^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=114

$$\frac{a^2 2^{n/2} n \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}+\frac{1}{2}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

```
[Out] 1/2*a^2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+2^(1/2*n)*a^2*n*(1-1/a/x)^(
1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

Rubi [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6171, 80, 69}

$$\frac{a^2 2^{n/2} n \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}+\frac{1}{2}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/2 + (2^(n/2)*a^2*n*
(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (a - x^(-
1))/(2*a)])/(2 - n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}dx = -\operatorname{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} - \frac{1}{2}(an)\operatorname{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{2^{n/2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

Mathematica [A] time = 0.39, size = 107, normalized size = 0.94

$$-\frac{a^2e^{n\coth^{-1}(ax)}\left((n+2)\left(\frac{1}{a^2x^2}+n\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;-e^{2\coth^{-1}(ax)}\right)+\frac{n}{ax}-1\right)-n^2e^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\coth^{-1}(ax)}\right)\right)}{2(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/2*(a^2*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1,
 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-1 + 1/(a^2*x^2) + n/(a
*x) + n*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 + n)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x^3,x)
```

```
[Out] int(exp(n*acoth(a*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**3,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x**3, x)
```

3.155 $\int \frac{e^{n \coth^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=167

$$\frac{a^3 2^{n/2} (n^2+2) \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}+\frac{1}{6}a^3 n \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{a^2 \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3x}$$

```
[Out] 1/6*a^3*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+1/3*a^2*(1-1/a/x)^(1-1/2*
n)*(1+1/a/x)^(1+1/2*n)/x+1/3*2^(1/2*n)*a^3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*hype
rgeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 90, 80, 69}

$$\frac{a^3 2^{n/2} (n^2+2) \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}+\frac{1}{6}a^3 n \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{a^2 \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^4,x]
```

```
[Out] (a^3*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/6 + (a^2*(1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(3*x) + (2^(n/2)*a^3*(2 + n^2)*(
1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -n/2, 2 - n/2, (a - x^(-1
))/(2*a)])/(3*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x} + \frac{1}{3}a^2\,\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\left(-1-\frac{nx}{a}\right)dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x} - \frac{1}{6}\left(a^2\left(2+n^2\right)\right)\text{Subst}\left(\int\left(\right.\right.$$

$$= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x} + \frac{2^{n/2}a^3\left(2+n^2\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3(2}$$

Mathematica [A] time = 0.67, size = 132, normalized size = 0.79

$$-\frac{a^3e^{n\coth^{-1}(ax)}\left((n+2)\left(-\left(1-\frac{1}{a^2x^2}\right)\left(\frac{2}{ax}+n\right)+\left(n^2+2\right){}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;-e^{2\coth^{-1}(ax)}\right)+\frac{n^2+2}{ax}\right)-n\left(n^2+2\right)e^2}{6(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/6*(a^3*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometr
ic2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-((1 - 1/(a^2*x
^2))*(n + 2/(a*x))) + (2 + n^2)/(a*x) + (2 + n^2)*Hypergeometric2F1[1, n/2,
 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 + n)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^4,x)
[Out] int(exp(n*arccoth(a*x))/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{acoth}(a\, x)}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x^4,x)
[Out] int(exp(n*acoth(a*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}{(ax)}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**4,x)
[Out] Integral(exp(n*acoth(a*x))/x**4, x)
```

3.156 $\int \frac{e^{n \coth^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=183

$$\frac{a^4 2^{\frac{n}{2}-2} n (n^2+8) \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}+\frac{1}{24}a^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a(n^2+6)+\frac{2n}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+$$

```
[Out] 1/24*a^3*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(a*(n^2+6)+2*n/x)+1/4*a^2*
(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)/x^2+1/3*2^(-2+1/2*n)*a^4*n*(n^2+8)*
(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2
-n)
```

Rubi [A] time = 0.12, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6171, 100, 147, 69}

$$\frac{a^4 2^{\frac{n}{2}-2} n (n^2+8) \left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}+\frac{1}{24}a^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a(n^2+6)+\frac{2n}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^5,x]
```

```
[Out] (a^3*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(a*(6 + n^2) + (2*n)
/x))/24 + (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4*x^2) +
 (2^(-2 + n/2)*a^4*n*(8 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1
- n/2, -n/2, 2 - n/2, (a - x^(-1))/(2*a)])/(3*(2 - n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
```

```
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 6171

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx = -\operatorname{Subst}\left(\int x^3\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$
$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} + \frac{1}{4}a^2\operatorname{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\left(-2-\frac{nx}{a}\right)dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a\left(6+n^2\right)+\frac{2n}{x}\right) + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} - \frac{1}{24}\left(a^3n\left(8\right.\right.$$
$$= \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a\left(6+n^2\right)+\frac{2n}{x}\right) + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} + \frac{2^{-2+\frac{n}{2}}a^4n\left(\right.}{}$$

Mathematica [A] time = 0.55, size = 148, normalized size = 0.81

$$-\frac{1}{24}a^4e^{n\coth^{-1}(ax)}\left(\frac{6}{a^4x^4}+\frac{2n}{a^3x^3}+\frac{n^2}{a^2x^2}-\frac{\left(n^2+8\right)n^2e^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\coth^{-1}(ax)}\right)}{n+2}+\left(n^2+8\right)n\,{}_2F\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^5,x]
```

```
[Out] -1/24*(a^4*E^(n*ArcCoth[a*x])*(-6 - n^2 + 6/(a^4*x^4) + (2*n)/(a^3*x^3) + n
^2/(a^2*x^2) + (6*n)/(a*x) + n^3/(a*x) - (E^(2*ArcCoth[a*x])*n^2*(8 + n^2)*
Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])])/(2 + n) + n*(8
 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])]))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^5, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^5,x)
[Out] int(exp(n*arccoth(a*x))/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{acoth}(a\,x)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x^5,x)
[Out] int(exp(n*acoth(a*x))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}{(ax)}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**5,x)
[Out] Integral(exp(n*acoth(a*x))/x**5, x)
```

3.157 $\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}(c-acx)^p\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\,(c-acx)^p}{p+1}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2-p)*(-a*c*x+c)^p*hypergeom([-p, 1/2-p],[1-p],2/(a+1/x
)/x)*(1+1/a/x)^(1/2)/a/p/(1+p)/(1-1/a/x)^(1/2)+x*(-a*c*x+c)^p*(1-1/a/x)^(1/
2)*(1+1/a/x)^(1/2)/(1+p)
```

Rubi [A] time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 132}

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}(c-acx)^p\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\,(c-acx)^p}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^p)/(1 + p) + (((a - x^(-
1))/(a + x^(-1)))^(1/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric2
F1[1/2 - p, -p, 1 - p, 2/((a + x^(-1))*x)])/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx\\
&= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p}{1+p}-\frac{\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-1-p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}}{\sqrt{1+\frac{x}{a}}}\right.}{a(1+p)}\\
&= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p}{1+p}+\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\right.}{ap(1+p)\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 131, normalized size = 0.92

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{ax-1}{ax+1}\right)^{-p}(c-acx)^p\left(\sqrt{\frac{ax-1}{ax+1}}\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\frac{2}{ax+1}\right)+p(ax-1)\left(\frac{ax-1}{ax+1}\right)^p\right)}{ap(p+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*(p*(-1 + a*x)*((-1 + a*x)/(1 + a*x))^p + S
qrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1/2 - p, -p, 1 - p, 2/(1 + a*x)
]))/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^p)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-acx+c)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(-acx + c)^{p}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-acx + c)^{p}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c - a\,c\,x)^{p}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^{p}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.158 $\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=132

$$-\frac{7}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{17}{15}a^2c^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3}{4}a^3c^4x^4\left(1-\right.$$

```
[Out] 17/15*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3-3/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1
/5*a^4*c^4*(1-1/a^2/x^2)^(3/2)*x^5+7/8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a-7
/8*a*c^4*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6175, 6178, 1807, 807, 266, 47, 63, 208}

$$\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{17}{15}a^2c^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{7}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{7c^4\tanh^{-1}\left(\sqrt{1-\right.}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (-7*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (17*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2
)*x^3)/15 - (3*a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (a^4*c^4*(1 - 1/(a^
2*x^2))^(3/2)*x^5)/5 + (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \left(a^4c^4\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^4 x^4\,dx$$

$$= -\left(\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{5}\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(\frac{15}{a}-\frac{17x}{a^2}+\frac{5x^2}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5-\frac{1}{20}\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(\frac{68}{a^2}-\frac{35x}{a^3}\right.}{x}\right.$$

$$= \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{4}\left(7a\right.$$

$$= \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{1}{8}\left(7a\right.$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(\right.$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(\right.$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(\right.$$

Mathematica [A] time = 0.20, size = 80, normalized size = 0.61

$$\frac{c^4\left(105\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
 24*a^4*x^4) + 105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

fricas [A] time = 0.48, size = 125, normalized size = 0.95

$$\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(24\,a^5c^4x^5-66\,a^4c^4x^4+22\,a^3c^4x^3+97\,a^2c^4x^2-151\,ac^4x\right.}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^4*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (24*a^5*c^4*x^5 - 66*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 9
7*a^2*c^4*x^2 - 151*a*c^4*x - 136*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.18, size = 216, normalized size = 1.64

$$\frac{1}{120}\,ac^4\left(\frac{105\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{490\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{896\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{790\,(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/120*a*c^4*(105*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*log(abs(sqrt(
(a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(490*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))
/(a*x + 1) - 896*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 790*(a
*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 105*(a*x - 1)^4*sqrt((a*x
 - 1)/(a*x + 1))/(a*x + 1)^4 - 105*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x -
1)/(a*x + 1) - 1)^5))
```

maple [A] time = 0.05, size = 183, normalized size = 1.39

$$\frac{(ax-1)\,c^4\left(24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^2a^2-90\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;xa+16\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;-105\sqrt{a^2x^2-1}\;\sqrt{a^2}\;xa+12\right.}{120a\sqrt{\frac{ax-1}{ax+1}}\;\sqrt{(ax-1)\,(ax+1)}\;\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x)
```

```
[Out] 1/120*(a*x-1)*c^4/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-90*(a^2*x^2-1
)^(3/2)*(a^2)^(1/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-105*(a^2*x^2-1)^(1
/2)*(a^2)^(1/2)*x*a+120*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+105*ln((a^2*x+(
a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x
-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [B] time = 0.33, size = 259, normalized size = 1.96

$$\frac{1}{120}\left(\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1}{(ax+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^4*((a*x - 1)/(a*x + 1))^(9/2) + 790*
c^4*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 490
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a
*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2
/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5
- a^2))*a
```

mupad [B] time = 0.13, size = 214, normalized size = 1.62

$$\frac{\frac{49\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{7\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{79\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}+\frac{7\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}+\frac{7\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((49*c^4*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^4*((a*x - 1)/(a*x + 1))^(1/2
))/4 - (224*c^4*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^4*((a*x - 1)/(a*x +
 1))^(7/2))/6 + (7*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*c^4*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(6*a*
*2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**3*x**3/sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**4*x**4/sqrt(a*x/(a*x + 1) -
1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.159 $\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=105

$$-\frac{5}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{5c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{2}{3}a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[Out] 2/3*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3-1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4+5/8
*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a-5/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6175, 6178, 1807, 807, 266, 47, 63, 208}

$$-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{2}{3}a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{5}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{5c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (-5*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (2*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)
*x^3)/3 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (5*c^3*ArcTanh[Sqrt[1 -
 1/(a^2*x^2)]])/(8*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(\frac{8}{a}-\frac{5x}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{4}\left(5ac^3\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx,\right.$$

$$= \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{8}\left(5ac^3\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx,\right.$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{\left(5c^3\right)\text{Su}}{}$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{8}\left(5ac^3\right)$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{5c^3\tanh}{}$$

Mathematica [A] time = 0.17, size = 73, normalized size = 0.70

$$\frac{c^3\left(15\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-ax\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3-16a^2x^2+9ax+16\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3)) +
15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

fricas [A] time = 0.50, size = 115, normalized size = 1.10

$$\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^3x^4-10\,a^3c^3x^3-7\,a^2c^3x^2+25\,ac^3x+16\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^3*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (6*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 7*a^2*c^3*x^2 + 25*a*c
^3*x + 16*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [B] time = 0.17, size = 185, normalized size = 1.76

$$\frac{1}{24}\,ac^3\left(\frac{15\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{55\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{73\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{15\,(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}-15\sqrt{\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/24*a*c^3*(15*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*log(abs(sqrt((a*
x - 1)/(a*x + 1)) - 1))/a^2 + 2*(55*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*
x + 1) - 73*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 15*(a*x - 1
)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 - 15*sqrt((a*x - 1)/(a*x + 1)))/(
a^2*((a*x - 1)/(a*x + 1) - 1)^4))
```

maple [A] time = 0.05, size = 141, normalized size = 1.34

$$\frac{(ax-1)\,c^3\left(-6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;xa+16\left((ax-1)\,(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\;-15\sqrt{a^2x^2-1}\;\sqrt{a^2}\;xa+15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\;\sqrt{a^2}}{\sqrt{a^2}}\right)\right.}{24a\sqrt{\frac{ax-1}{ax+1}}\;\sqrt{(ax-1)\,(ax+1)}\;\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x)
```

```
[Out] 1/24*(a*x-1)*c^3/a*(-6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+16*((a*x-1)*(a*x+1
))^(3/2)*(a^2)^(1/2)-15*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+15*ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)
*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 221, normalized size = 2.10

$$\frac{1}{24}\left(\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(15\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+73\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c^3\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 + 2*(15*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 73*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - 55*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^3*sq
rt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*
x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^
2))*a
```

mupad [B] time = 0.08, size = 177, normalized size = 1.69

$$\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{55\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{73\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}+\frac{5\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{6\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{4\,a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}+\frac{a\,{(a\,x-1)}^4}{{(a\,x+1)}^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a) - ((5*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/4 - (55*c^3*((a*x - 1)/(a*x + 1))^(3/2))/12 + (73*c^3*((a*x -
1)/(a*x + 1))^(5/2))/12 + (5*c^3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a - (4*a*
(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2 - (4*a*(a*x - 1)^3)/(a
*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\frac{3ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a
**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3/sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(a*x +
1)), x))
```

3.160 $\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=78

$$-\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[Out] 1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3+1/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a-
1/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6175, 6178, 807, 266, 47, 63, 208}

$$\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

```
[Out] -(a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x^3
)/3 + (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx &= \left(a^2c^2\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx\\
&= -\left(\left(a^2c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\left(ac^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)\\
&= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{c^2\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a}\\
&= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\right.\\
&= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 64, normalized size = 0.82

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-3ax-2\right)+3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - 3*a*x + 2*a^2*x^2) + 3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

fricas [A] time = 0.51, size = 103, normalized size = 1.32

$$\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3c^2x^3-a^2c^2x^2-5\,ac^2x-2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) + (2*a^3*c^2*x^3 - a^2*c^2*x^2 - 5*a*c^2*x - 2*c^2)*sqrt((a*x
 - 1)/(a*x + 1)))/a
```

giac [B] time = 0.15, size = 154, normalized size = 1.97

$$\frac{1}{6}\,ac^2\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{8\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{3\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-3\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*a*c^2*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(abs(sqrt((a*x -
 1)/(a*x + 1)) - 1))/a^2 - 2*(8*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x +
1) + 3*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 3*sqrt((a*x - 1)
/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [A] time = 0.04, size = 121, normalized size = 1.55

$$\frac{(ax-1)\,c^2\left(2\left((ax-1)\,(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)\,(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x)
```

```
[Out] 1/6*(a*x-1)*c^2*(2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-3*(a^2*x^2-1)^(1/2)*
(a^2)^(1/2)*x*a+3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/
((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 181, normalized size = 2.32

$$\frac{1}{6}\,a\left(\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 8*c^2*((a*
x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2
/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 -
a^2))
```

mupad [B] time = 1.21, size = 138, normalized size = 1.77

$$\frac{\frac{8\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - c^2*((a*x - 1)/(a*x + 1))^(1/2) +
c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x
 - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (c^2*atanh(((a*x - 1)
/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\left(-\frac{2ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^{2}x^{2}}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2
*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1)), x))
```

3.161 $\int e^{\coth^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=47

$$\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a} - \frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] 1/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a-1/2*a*c*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6175, 6178, 266, 47, 63, 208}

$$\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a} - \frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x),x]
```

```
[Out] -(a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*
a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}(c-acx)\,dx &= -\left((ac)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)\\
&= (ac)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}(ac)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}\\
&= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{2}(ac)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 51, normalized size = 1.09

$$
\frac{c\left(\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x),x]
```

```
[Out] (c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]
))/(2*a)
```

fricas [A] time = 0.54, size = 77, normalized size = 1.64

$$
\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2cx^2+acx\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - c*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) - (a^2*c*x^2 + a*c*x)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [B] time = 0.20, size = 154, normalized size = 3.28

$$
\frac{1}{4}ac\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}+2\right)}{a^2}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}-2\right|\right)}{a^2}-\frac{4\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{\left(\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^2-4\right)a^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] 1/4*a*c*(log(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)) + 2)/a
^2 - log(abs(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)) - 2))/
a^2 - 4*(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)))/(((sqrt((
a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)))^2 - 4)*a^2))
```

maple [B] time = 0.04, size = 93, normalized size = 1.98

$$-\frac{(ax-1)\,c\left(x\sqrt{a^2x^2-1}\,\sqrt{a^2}-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x)
```

```
[Out] -1/2*(a*x-1)*c*(x*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)
*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)
/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 132, normalized size = 2.81

$$\frac{1}{2}\,a\left(\frac{2\left(c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(c*((a*x - 1)/(a*x + 1))^(3/2) + c*sqrt((a*x - 1)/(a*x + 1)))/(2*(
a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + c*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 1.20, size = 94, normalized size = 2.00

$$\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,(a\,x-1)}{a\,x+1}+\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2) +
 c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2*a*(a*x - 1))/(a*x + 1) + (a*(a*x -
1)^2)/(a*x + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\frac{ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a
*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.162 $\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=51

$$\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6175, 6178, 852, 1805, 266, 63, 208}

$$\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x),x]
```

```
[Out] (2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx &= -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}\\
&= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 60, normalized size = 1.18

$$
\frac{2ax\sqrt{1-\frac{1}{a^2x^2}}+(1-ax)\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac(ax-1)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*x + (1 - a*x)*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*
x])/(a*c*(-1 + a*x))
```

fricas [A] time = 0.47, size = 87, normalized size = 1.71

$$-\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - 2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x - a*c)
```

giac [A] time = 0.18, size = 79, normalized size = 1.55

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{2}{a^2c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(abs(sqrt((a*x - 1)/(a*
x + 1)) - 1))/(a^2*c) - 2/(a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

maple [B] time = 0.05, size = 249, normalized size = 4.88

$$\frac{-\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2-\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}+2\sqrt{(ax-1)(ax+1)}\ \cdots}{a\sqrt{a^2}\,(ax-1)\,c\sqrt{(ax-1)(ax\ \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x)
```

```
[Out] 1/a*(-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-ln((a^2*x+((a*x-1)*(a*x+1
))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1
/2)+2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+2*ln((a^2*x+((a*x-1)*(a*x+1))
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-
a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/(a^2)^(1/2)/
(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.30, size = 78, normalized size = 1.53

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2}{a^2c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c) - 2/(a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

mupad [B] time = 0.07, size = 48, normalized size = 0.94

$$\frac{2}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 2/(a*c*((a*x - 1)/(a*x + 1))^(1/2)) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
)/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))), x)/c
```

3.163 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

[Out] -1/3*a^2*(1-1/a^2/x^2)^(3/2)/c^2/(a-1/x)^3

Rubi [A] time = 0.10, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6175, 6178, 651}

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^2,x]

[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2))/(3*c^2*(a - x^(-1))^3)

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.03

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax+1)}{3c^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x))/(c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.46, size = 57, normalized size = 1.73

$$-\frac{\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*c
^2*x + a*c^2)
```

giac [A] time = 0.13, size = 35, normalized size = 1.06

$$-\frac{ax+1}{3\,(ax-1)ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -1/3*(a*x + 1)/((a*x - 1)*a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 36, normalized size = 1.09

$$-\frac{ax+1}{3\,(ax-1)\,c^2\sqrt{\frac{ax-1}{ax+1}}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x)
```

```
[Out] -1/3*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)/a
```

maxima [A] time = 0.31, size = 23, normalized size = 0.70

$$-\frac{1}{3\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/3/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

mupad [B] time = 1.18, size = 23, normalized size = 0.70

$$-\frac{1}{3\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] -1/(3*a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
[Out] Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

3.164 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=67

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

```
[Out] 1/5*a^3*(1-1/a^2/x^2)^(3/2)/c^3/(a-1/x)^4-4/15*a^2*(1-1/a^2/x^2)^(3/2)/c^3/
(a-1/x)^3
```

Rubi [A] time = 0.12, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6175, 6178, 793, 651}

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2))/(5*c^3*(a - x^(-1))^4) - (4*a^2*(1 - 1/(a^2*x
^2))^(3/2))/(15*c^3*(a - x^(-1))^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{a^3c^3}$$
$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4} - \frac{4\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{5a^2c^3}$$
$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4} - \frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 0.63

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(a^2x^2-3ax-4\right)}{15c^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(-4 - 3*a*x + a^2*x^2))/(c^3*(-1 + a*x)^3)
```

fricas [A] time = 0.48, size = 77, normalized size = 1.15

$$-\frac{\left(a^3x^3-2\,a^2x^2-7\,ax-4\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/15*(a^3*x^3 - 2*a^2*x^2 - 7*a*x - 4)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^3*
x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.13, size = 53, normalized size = 0.79

$$-\frac{(ax+1)^2\left(\frac{5\,(ax-1)}{ax+1}-3\right)}{30\,(ax-1)^2ac^3\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/30*(a*x + 1)^2*(5*(a*x - 1)/(a*x + 1) - 3)/((a*x - 1)^2*a*c^3*sqrt((a*x
- 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 41, normalized size = 0.61

$$-\frac{(ax-4)(ax+1)}{15(ax-1)^2 c^3 \sqrt{\frac{ax-1}{ax+1}}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/15*(a*x-4)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(1/2)/a
```

maxima [A] time = 0.31, size = 39, normalized size = 0.58

$$-\frac{\frac{5(ax-1)}{ax+1}-3}{30\,ac^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/30*(5*(a*x - 1)/(a*x + 1) - 3)/(a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

mupad [B] time = 1.17, size = 39, normalized size = 0.58

$$-\frac{\frac{a\,x-1}{3\,(a\,x+1)}-\frac{1}{5}}{2\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] -((a*x - 1)/(3*(a*x + 1)) - 1/5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x**2*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

3.165 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=100

$$-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}$$

```
[Out] -1/7*a^4*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^5+12/35*a^3*(1-1/a^2/x^2)^(3/2)/c^
4/(a-1/x)^4-23/105*a^2*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^3
```

Rubi [A] time = 0.23, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6175, 6178, 1639, 793, 659, 651}

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(3/2))/(7*c^4*(a - x^(-1))^5) + (12*a^3*(1 - 1/(a^2
*x^2))^(3/2))/(35*c^4*(a - x^(-1))^4) - (23*a^2*(1 - 1/(a^2*x^2))^(3/2))/(1
05*c^4*(a - x^(-1))^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}\\
&= -\frac{\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{a^4c^4}\\
&= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^4\left(a-\frac{1}{x}\right)^4} - \frac{\text{Subst}\left(\int \frac{\left(\frac{4}{a^2}-\frac{3x}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{c^4}\\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^4\left(a-\frac{1}{x}\right)^4} - \frac{23\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{7a^2c^4}\\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4} - \frac{23\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{35a^2c^4}\\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4} - \frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 51, normalized size = 0.51

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-8a^2x^2+13ax+23\right)}{105c^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -1/105*(Sqrt[1 - 1/(a^2*x^2)]*x*(23 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^4
*(-1 + a*x)^4)
```

fricas [A] time = 0.68, size = 96, normalized size = 0.96

$$-\frac{\left(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+36\,ax+23\right)\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + 36*a*x + 23)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.14, size = 69, normalized size = 0.69

$$\frac{(ax+1)^3\left(\frac{42\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}-15\right)}{420\,(ax-1)^3ac^4\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/420*(a*x + 1)^3*(42*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 - 15
)/((a*x - 1)^3*a*c^4*sqrt((a*x - 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 50, normalized size = 0.50

$$-\frac{\left(2a^2x^2-10ax+23\right)(ax+1)}{105\,(ax-1)^3\,c^4\sqrt{\frac{ax-1}{ax+1}}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)/
a
```

maxima [A] time = 0.30, size = 55, normalized size = 0.55

$$\frac{\frac{42\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}-15}{420\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/420*(42*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 - 15)/(a*c^4*((a
*x - 1)/(a*x + 1))^(7/2))
```

mupad [B] time = 0.04, size = 56, normalized size = 0.56

$$-\frac{\frac{(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{2\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] -((a*x - 1)^2/(3*(a*x + 1)^2) - (2*(a*x - 1))/(5*(a*x + 1)) + 1/7)/(4*a*c^4
*((a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x**3*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))), x)/c**4
```

3.166 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=133

$$-\frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}+\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6}-\frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5}+\frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4}$$

```
[Out] 1/9*a^5*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^6-8/21*a^4*(1-1/a^2/x^2)^(3/2)/c^5/
(a-1/x)^5+47/105*a^3*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^4-58/315*a^2*(1-1/a^2/
x^2)^(3/2)/c^5/(a-1/x)^3
```

Rubi [A] time = 0.34, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6175, 6178, 1639, 793, 659, 651}

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6}-\frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5}+\frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4}-\frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(3/2))/(9*c^5*(a - x^(-1))^6) - (8*a^4*(1 - 1/(a^2*x
^2))^(3/2))/(21*c^5*(a - x^(-1))^5) + (47*a^3*(1 - 1/(a^2*x^2))^(3/2))/(105
*c^5*(a - x^(-1))^4) - (58*a^2*(1 - 1/(a^2*x^2))^(3/2))/(315*c^5*(a - x^(-1
))^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{\text{Subst}\left(\int \frac{\left(\frac{4}{a^2}-\frac{7x}{a^3}+\frac{2x^2}{a^4}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{a^4\,\text{Subst}\left(\int \frac{\left(\frac{18}{a^6}-\frac{20x}{a^7}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{29\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{3a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{58\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{21a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{105a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.44

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^4x^4-10a^3x^3+21a^2x^2-25ax-58\right)}{315c^5(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^5,x]

[Out] -1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(-58 - 25*a*x + 21*a^2*x^2 - 10*a^3*x^3 + 2
*a^4*x^4))/(c^5*(-1 + a*x)^5)
```

fricas [A] time = 0.42, size = 116, normalized size = 0.87

$$-\frac{\left(2\,a^{5}x^{5}-8\,a^{4}x^{4}+11\,a^{3}x^{3}-4\,a^{2}x^{2}-83\,ax-58\right)\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^{6}c^{5}x^{5}-5\,a^{5}c^{5}x^{4}+10\,a^{4}c^{5}x^{3}-10\,a^{3}c^{5}x^{2}+5\,a^{2}c^{5}x-ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 - 4*a^2*x^2 - 83*a*x - 58)*sqrt(
(a*x - 1)/(a*x + 1))/(a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*a^3
*c^5*x^2 + 5*a^2*c^5*x - a*c^5)
```

giac [A] time = 0.16, size = 85, normalized size = 0.64

$$-\frac{(ax+1)^{4}\left(\frac{135\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^{2}}{(ax+1)^{2}}+\frac{105\,(ax-1)^{3}}{(ax+1)^{3}}-35\right)}{2520\,(ax-1)^{4}ac^{5}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/2520*(a*x + 1)^4*(135*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2
+ 105*(a*x - 1)^3/(a*x + 1)^3 - 35)/((a*x - 1)^4*a*c^5*sqrt((a*x - 1)/(a*x
+ 1)))
```

maple [A] time = 0.04, size = 58, normalized size = 0.44

$$-\frac{\left(2x^{3}a^{3}-12a^{2}x^{2}+33ax-58\right)(ax+1)}{315\,(ax-1)^{4}\,c^{5}\sqrt{\frac{ax-1}{ax+1}}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/315*(2*a^3*x^3-12*a^2*x^2+33*a*x-58)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a*x
+1))^(1/2)/a
```

maxima [A] time = 0.30, size = 71, normalized size = 0.53

$$-\frac{\frac{135\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^{2}}{(ax+1)^{2}}+\frac{105\,(ax-1)^{3}}{(ax+1)^{3}}-35}{2520\,ac^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/2520*(135*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 105*(a*x -
 1)^3/(a*x + 1)^3 - 35)/(a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

mupad [B] time = 1.17, size = 72, normalized size = 0.54

$$\frac{\frac{3\,(a\,x-1)^{2}}{5\,(a\,x+1)^{2}}-\frac{(a\,x-1)^{3}}{3\,(a\,x+1)^{3}}-\frac{3\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{9}}{8\,a\,c^{5}\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((3*(a*x - 1)^2)/(5*(a*x + 1)^2) - (a*x - 1)^3/(3*(a*x + 1)^3) - (3*(a*x -
1))/(7*(a*x + 1)) + 1/9)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 5*a**4*x**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 10*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 5*a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**5
```

3.167 $\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=42

$$\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

[Out] 2*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(1+p)/a/c/(1+p)

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6130, 21, 43}

$$\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]

[Out] (2*(c - a*c*x)^p)/(a*p) - (c - a*c*x)^(1 + p)/(a*c*(1 + p))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^p\,dx\\
&= -\int \frac{(1+ax)(c-acx)^p}{1-ax}\,dx\\
&= -\left(c\int (1+ax)(c-acx)^{-1+p}\,dx\right)\\
&= -\left(c\int \left(2(c-acx)^{-1+p}-\frac{(c-acx)^p}{c}\right)dx\right)\\
&= \frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.67

$$\frac{(apx+p+2)(c-acx)^p}{ap(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
[Out] ((c - a*c*x)^p*(2 + p + a*p*x))/(a*p*(1 + p))
```

fricas [A] time = 0.53, size = 28, normalized size = 0.67

$$\frac{\left(apx+p+2\right)(-acx+c)^p}{ap^2+ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="fricas")
[Out] (a*p*x + p + 2)*(-a*c*x + c)^p/(a*p^2 + a*p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)(-acx+c)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

maple [A] time = 0.03, size = 29, normalized size = 0.69

$$\frac{\left(apx+p+2\right)(-acx+c)^p}{ap\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^p,x)
[Out] (a*p*x+p+2)*(-a*c*x+c)^p/a/p/(1+p)
```

maxima [A] time = 0.32, size = 49, normalized size = 1.17

$$\frac{\left(ac^ppx+c^p\right)(-ax+1)^p}{\left(p^2+p\right)a}+\frac{(-ax+1)^pc^p}{ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] (a*c^p*p*x + c^p)*(-a*x + 1)^p/((p^2 + p)*a) + (-a*x + 1)^p*c^p/(a*p)
```

mupad [B] time = 1.21, size = 28, normalized size = 0.67

$$\frac{(c-a\,c\,x)^p\,\left(p+a\,p\,x+2\right)}{a\,p\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x + 1))/(a*x - 1),x)
```

```
[Out] ((c - a*c*x)^p*(p + a*p*x + 2))/(a*p*(p + 1))
```

sympy [A] time = 0.65, size = 124, normalized size = 2.95

$$\begin{cases} -c^p x & \text{for } a = 0 \\ -\frac{ax\log\left(x-\frac{1}{a}\right)}{a^2cx-ac} + \frac{\log\left(x-\frac{1}{a}\right)}{a^2cx-ac} + \frac{2}{a^2cx-ac} & \text{for } p = -1 \\ x + \frac{2\log\left(x-\frac{1}{a}\right)}{a} & \text{for } p = 0 \\ \frac{apx(-acx+c)^p}{ap^2+ap} + \frac{p(-acx+c)^p}{ap^2+ap} + \frac{2(-acx+c)^p}{ap^2+ap} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((-c**p*x, Eq(a, 0)), (-a*x*log(x - 1/a)/(a**2*c*x - a*c) + log(x
- 1/a)/(a**2*c*x - a*c) + 2/(a**2*c*x - a*c), Eq(p, -1)), (x + 2*log(x - 1/
a)/a, Eq(p, 0)), (a*p*x*(-a*c*x + c)**p/(a*p**2 + a*p) + p*(-a*c*x + c)**p/
(a*p**2 + a*p) + 2*(-a*c*x + c)**p/(a*p**2 + a*p), True))
```

3.168 $\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=37

$$\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

```
[Out] 2/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] (2*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx&=-\int e^{2\tanh^{-1}(ax)}(c-acx)^5\,dx\\&=-\left(c^5\int(1-ax)^4(1+ax)\,dx\right)\\&=-\left(c^5\int\left(2(1-ax)^4-(1-ax)^5\right)\,dx\right)\\&=\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.62

$$-\frac{c^5(ax-1)^5(5ax+7)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -1/30*(c^5*(-1 + a*x)^5*(7 + 5*a*x))/a
```

fricas [A] time = 0.51, size = 60, normalized size = 1.62

$$-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

giac [A] time = 0.12, size = 60, normalized size = 1.62

$$-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

maple [A] time = 0.03, size = 47, normalized size = 1.27

$$c^5\left(-\frac{1}{6}x^6a^5+\frac{3}{5}a^4x^5-\frac{1}{2}x^4a^3-\frac{2}{3}x^3a^2+\frac{3}{2}a\,x^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^5,x)
```

```
[Out] c^5*(-1/6*x^6*a^5+3/5*a^4*x^5-1/2*x^4*a^3-2/3*x^3*a^2+3/2*a*x^2-x)
```

maxima [A] time = 0.31, size = 60, normalized size = 1.62

$$-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

mupad [B] time = 0.03, size = 60, normalized size = 1.62

$$-\frac{a^5\,c^5\,x^6}{6}+\frac{3\,a^4\,c^5\,x^5}{5}-\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}+\frac{3\,a\,c^5\,x^2}{2}-c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (3*a*c^5*x^2)/2 - c^5*x - (2*a^2*c^5*x^3)/3 - (a^3*c^5*x^4)/2 + (3*a^4*c^5*
x^5)/5 - (a^5*c^5*x^6)/6
```

sympy [B] time = 0.08, size = 66, normalized size = 1.78

$$-\frac{a^5c^5x^6}{6}+\frac{3a^4c^5x^5}{5}-\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}+\frac{3ac^5x^2}{2}-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + 3*a**4*c**5*x**5/5 - a**3*c**5*x**4/2 - 2*a**2*c**5*x**
3/3 + 3*a*c**5*x**2/2 - c**5*x
```

3.169 $\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=37

$$\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

```
[Out] 1/2*c^4*(-a*x+1)^4/a-1/5*c^4*(-a*x+1)^5/a
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(1 - a*x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx&=-\int e^{2\tanh^{-1}(ax)}(c-acx)^4\,dx\\&=-\left(c^4\int(1-ax)^3(1+ax)\,dx\right)\\&=-\left(c^4\int\left(2(1-ax)^3-(1-ax)^4\right)dx\right)\\&=\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.81

$$\frac{1}{10}c^4x\left(2a^4x^4-5a^3x^3+10ax-10\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
[Out] (c^4*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

fricas [A] time = 0.63, size = 37, normalized size = 1.00

$$\frac{1}{5}\,a^4c^4x^5 - \frac{1}{2}\,a^3c^4x^4 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="fricas")
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

giac [A] time = 0.14, size = 37, normalized size = 1.00

$$\frac{1}{5}\,a^4c^4x^5 - \frac{1}{2}\,a^3c^4x^4 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="giac")
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

maple [A] time = 0.03, size = 30, normalized size = 0.81

$$c^4\left(\frac{1}{5}a^4x^5 - \frac{1}{2}x^4a^3 + a\,x^2 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^4,x)
[Out] c^4*(1/5*a^4*x^5-1/2*x^4*a^3+a*x^2-x)
```

maxima [A] time = 0.30, size = 37, normalized size = 1.00

$$\frac{1}{5}\,a^4c^4x^5 - \frac{1}{2}\,a^3c^4x^4 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="maxima")
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

mupad [B] time = 0.05, size = 37, normalized size = 1.00

$$\frac{a^4\,c^4\,x^5}{5} - \frac{a^3\,c^4\,x^4}{2} + a\,c^4\,x^2 - c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(a*x - 1),x)
[Out] a*c^4*x^2 - c^4*x - (a^3*c^4*x^4)/2 + (a^4*c^4*x^5)/5
```

sympy [A] time = 0.07, size = 36, normalized size = 0.97

$$\frac{a^4c^4x^5}{5} - \frac{a^3c^4x^4}{2} + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**4,x)
[Out] a**4*c**4*x**5/5 - a**3*c**4*x**4/2 + a*c**4*x**2 - c**4*x
```

3.170 $\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=37

$$\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

```
[Out] 2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x)^4)/(4*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^3\,dx\\ &= -\left(c^3\int (1-ax)^2(1+ax)\,dx\right)\\ &= -\left(c^3\int \left(2(1-ax)^2-(1-ax)^3\right)\,dx\right)\\ &= \frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.81

$$-\frac{1}{12}c^3x\left(3a^3x^3-4a^2x^2-6ax+12\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
[Out] -1/12*(c^3*x*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))
```

fricas [A] time = 0.92, size = 38, normalized size = 1.03

$$-\frac{1}{4}a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + \frac{1}{2}ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

giac [A] time = 0.14, size = 38, normalized size = 1.03

$$-\frac{1}{4}a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + \frac{1}{2}ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

maple [A] time = 0.03, size = 31, normalized size = 0.84

$$c^3\left(-\frac{1}{4}x^4a^3 + \frac{1}{3}x^3a^2 + \frac{1}{2}a\,x^2 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^3,x)
[Out] c^3*(-1/4*x^4*a^3+1/3*x^3*a^2+1/2*a*x^2-x)
```

maxima [A] time = 0.30, size = 38, normalized size = 1.03

$$-\frac{1}{4}a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + \frac{1}{2}ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

mupad [B] time = 0.05, size = 38, normalized size = 1.03

$$-\frac{a^3\,c^3\,x^4}{4} + \frac{a^2\,c^3\,x^3}{3} + \frac{a\,c^3\,x^2}{2} - c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(a*x - 1),x)
[Out] (a*c^3*x^2)/2 - c^3*x + (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

sympy [A] time = 0.07, size = 37, normalized size = 1.00

$$-\frac{a^3c^3x^4}{4} + \frac{a^2c^3x^3}{3} + \frac{ac^3x^2}{2} - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**3,x)
[Out] -a**3*c**3*x**4/4 + a**2*c**3*x**3/3 + a*c**3*x**2/2 - c**3*x
```

3.171 $\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}a^2c^2x^3-c^2x$$

```
[Out] -c^2*x+1/3*a^2*c^2*x^3
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 41}

$$\frac{1}{3}a^2c^2x^3-c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*x) + (a^2*c^2*x^3)/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx&=-\int e^{2\tanh^{-1}(ax)}(c-acx)^2\,dx\\&=-\left(c^2\int(1-ax)(1+ax)\,dx\right)\\&=-\left(c^2\int\left(1-a^2x^2\right)\,dx\right)\\&=-c^2x+\frac{1}{3}a^2c^2x^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.85

$$-c^2\left(x-\frac{a^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*(x - (a^2*x^3)/3))
```

fricas [A] time = 0.54, size = 18, normalized size = 0.90

$$\frac{1}{3}a^2c^2x^3 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="fricas")
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

giac [A] time = 0.13, size = 18, normalized size = 0.90

$$\frac{1}{3}a^2c^2x^3 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="giac")
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

maple [A] time = 0.03, size = 17, normalized size = 0.85

$$c^2\left(\frac{1}{3}x^3a^2 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^2,x)
[Out] c^2*(1/3*x^3*a^2-x)
```

maxima [A] time = 0.31, size = 18, normalized size = 0.90

$$\frac{1}{3}a^2c^2x^3 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="maxima")
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

mupad [B] time = 0.03, size = 15, normalized size = 0.75

$$\frac{c^2\,x\,\left(a^2\,x^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(a*x - 1),x)
[Out] (c^2*x*(a^2*x^2 - 3))/3
```

sympy [A] time = 0.06, size = 15, normalized size = 0.75

$$\frac{a^2c^2x^3}{3} - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**2,x)
[Out] a**2*c**2*x**3/3 - c**2*x
```

3.172 $\int e^{2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}acx^2-cx$$

[Out] -c*x-1/2*a*c*x^2

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.86, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2288}

$$\frac{c\left(1-a^2x^2\right)e^{2\coth^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x),x]

[Out] (c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=\frac{ce^{2\coth^{-1}(ax)}\left(1-a^2x^2\right)}{2a}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 1.86

$$\frac{c\left(1-a^2x^2\right)e^{2\coth^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x),x]

[Out] (c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)

fricas [A] time = 0.54, size = 12, normalized size = 0.86

$$-\frac{1}{2}\,acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="fricas")

[Out] -1/2*a*c*x^2 - c*x

giac [A] time = 0.12, size = 12, normalized size = 0.86

$$-\frac{1}{2}\,acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*a*c*x^2 - c*x
```

maple [A] time = 0.03, size = 13, normalized size = 0.93

$$c\left(-\frac{1}{2}a\,x^{2}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c),x)
```

```
[Out] c*(-1/2*a*x^2-x)
```

maxima [A] time = 0.32, size = 12, normalized size = 0.86

$$-\frac{1}{2}\,acx^{2}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 - c*x
```

mupad [B] time = 0.02, size = 9, normalized size = 0.64

$$-\frac{c\,x\,\left(a\,x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] -(c*x*(a*x + 2))/2
```

sympy [A] time = 0.05, size = 12, normalized size = 0.86

$$-\frac{acx^{2}}{2}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x)
```

```
[Out] -a*c*x**2/2 - c*x
```

3.173 $\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=32

$$-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

```
[Out] -2/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -2/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-acx}\,dx\\
&= -\frac{\int \frac{1+ax}{(1-ax)^2}\,dx}{c}\\
&= -\frac{\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{c}\\
&= -\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.94

$$-\frac{\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
[Out] -((2/(a*(1 - a*x)) + Log[1 - a*x]/a)/c)
```

fricas [A] time = 0.47, size = 29, normalized size = 0.91

$$-\frac{(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
[Out] -((a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

giac [A] time = 0.13, size = 31, normalized size = 0.97

$$-\frac{\log(|ax-1|)}{ac}+\frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="giac")
[Out] -log(abs(a*x - 1))/(a*c) + 2/((a*x - 1)*a*c)
```

maple [A] time = 0.04, size = 31, normalized size = 0.97

$$-\frac{\ln(ax-1)}{ca}+\frac{2}{ca(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c),x)
[Out] -1/c/a*ln(a*x-1)+2/c/a/(a*x-1)
```

maxima [A] time = 0.31, size = 30, normalized size = 0.94

$$\frac{2}{a^2cx-ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
[Out] 2/(a^2*c*x - a*c) - log(a*x - 1)/(a*c)
```

mupad [B] time = 1.20, size = 29, normalized size = 0.91

$$-\frac{2}{a\,(c-a\,c\,x)}-\frac{\ln(a\,x-1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)*(a*x - 1)),x)
[Out] - 2/(a*(c - a*c*x)) - log(a*x - 1)/(a*c)
```

sympy [A] time = 0.13, size = 20, normalized size = 0.62

$$\frac{2}{a^2cx-ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x)
[Out] 2/(a**2*c*x - a*c) - log(a*x - 1)/(a*c)
```

3.174 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{x}{c^2(1-ax)^2}$$

```
[Out] -x/c^2/(-a*x+1)^2
```

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6167, 6129, 34}

$$-\frac{x}{c^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(x/(c^2*(1 - a*x)^2))
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx\\ &= -\frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c^2}\\ &= -\frac{x}{c^2(1-ax)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.79

$$-\frac{(ax+1)^2}{4ac^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -1/4*(1 + a*x)^2/(a*c^2*(1 - a*x)^2)
```

fricas [A] time = 0.65, size = 26, normalized size = 1.86

$$-\frac{x}{a^2c^2x^2 - 2\,ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

giac [B] time = 0.14, size = 34, normalized size = 2.43

$$-\frac{1}{(acx - c)^2a} - \frac{1}{(acx - c)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -1/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c)
```

maple [B] time = 0.04, size = 30, normalized size = 2.14

$$\frac{-\frac{1}{a(ax-1)^2} - \frac{1}{a(ax-1)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^2,x)
[Out] 1/c^2*(-1/a/(a*x-1)^2-1/a/(a*x-1))
```

maxima [A] time = 0.31, size = 26, normalized size = 1.86

$$-\frac{x}{a^2c^2x^2 - 2\,ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

mupad [B] time = 1.19, size = 13, normalized size = 0.93

$$-\frac{x}{c^2\,(a\,x - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^2*(a*x - 1)),x)
[Out] -x/(c^2*(a*x - 1)^2)
```

sympy [A] time = 0.16, size = 24, normalized size = 1.71

$$-\frac{x}{a^2c^2x^2 - 2ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**2,x)
[Out] -x/(a**2*c**2*x**2 - 2*a*c**2*x + c**2)
```

3.175 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}$$

```
[Out] -2/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{1}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -2/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx\\
&= -\frac{\int \frac{1+ax}{(1-ax)^4}\,dx}{c^3}\\
&= -\frac{\int \left(\frac{2}{(-1+ax)^4}+\frac{1}{(-1+ax)^3}\right)dx}{c^3}\\
&= -\frac{2}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.62

$$\frac{3ax+1}{6ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 + 3*a*x)/(6*a*c^3*(-1 + a*x)^3)
```

fricas [A] time = 0.45, size = 47, normalized size = 1.27

$$\frac{3\,ax+1}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.14, size = 21, normalized size = 0.57

$$\frac{3\,ax+1}{6\,(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*a*x + 1)/((a*x - 1)^3*a*c^3)
```

maple [A] time = 0.04, size = 30, normalized size = 0.81

$$\frac{\frac{1}{2a(ax-1)^2}+\frac{2}{3a(ax-1)^3}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3*(1/2/a/(a*x-1)^2+2/3/a/(a*x-1)^3)
```

maxima [A] time = 0.30, size = 47, normalized size = 1.27

$$\frac{3\,ax+1}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

mupad [B] time = 1.20, size = 46, normalized size = 1.24

$$-\frac{\frac{x}{2}+\frac{1}{6\,a}}{-a^3\,c^3\,x^3+3\,a^2\,c^3\,x^2-3\,a\,c^3\,x+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^3*(a*x - 1)),x)
```

```
[Out] -(x/2 + 1/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c^3*x)
```

sympy [A] time = 0.22, size = 49, normalized size = 1.32

$$-\frac{-3ax-1}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**3,x)
```

```
[Out] -(-3*a*x - 1)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**2*c**3*x - 6*a*
c**3)
```

3.176 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}$$

```
[Out] -1/2/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/(2*a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx\\
&= -\frac{\int \frac{1+ax}{(1-ax)^5}\,dx}{c^4}\\
&= -\frac{\int \left(-\frac{2}{(-1+ax)^5}-\frac{1}{(-1+ax)^4}\right)dx}{c^4}\\
&= -\frac{1}{2ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.62

$$-\frac{2ax+1}{6ac^4(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/6*(1 + 2*a*x)/(a*c^4*(-1 + a*x)^4)
```

fricas [A] time = 0.46, size = 57, normalized size = 1.54

$$-\frac{2\,ax+1}{6\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

giac [A] time = 0.14, size = 21, normalized size = 0.57

$$-\frac{2\,ax+1}{6\,(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*a*x + 1)/((a*x - 1)^4*a*c^4)
```

maple [A] time = 0.04, size = 30, normalized size = 0.81

$$\frac{-\frac{1}{3a(ax-1)^3}-\frac{1}{2a(ax-1)^4}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-1/3/a/(a*x-1)^3-1/2/a/(a*x-1)^4)
```

maxima [A] time = 0.32, size = 57, normalized size = 1.54

$$-\frac{2\,ax+1}{6\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

mupad [B] time = 0.09, size = 56, normalized size = 1.51

$$-\frac{\frac{x}{3}+\frac{1}{6\,a}}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^4*(a*x - 1)),x)
```

```
[Out] -(x/3 + 1/(6*a))/(c^4 + 6*a^2*c^4*x^2 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*c
^4*x)
```

<u>**sympy**</u> [**B**] time = 0.28, size = 60, normalized size = 1.62

$$\frac{-2ax-1}{6a^5c^4x^4-24a^4c^4x^3+36a^3c^4x^2-24a^2c^4x+6ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**4,x)
```

```
[Out] (-2*a*x - 1)/(6*a**5*c**4*x**4 - 24*a**4*c**4*x**3 + 36*a**3*c**4*x**2 - 24
*a**2*c**4*x + 6*a*c**4)
```

3.177 $\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=202

$$-\frac{3\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}(c-acx)^p\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a^2p\left(1-p^2\right)x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^p}{(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{3\sqrt{\frac{1}{ax}+1}\,(c-acx)^p}{ap(p+1)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (1+1/a/x)^(3/2)*x*(-a*c*x+c)^p/(1+p)/(1-1/a/x)^(1/2)-3*((a-1/x)/(a+1/x))^(3
/2-p)*(-a*c*x+c)^p*hypergeom([1-p, 3/2-p],[2-p],2/(a+1/x)/x)*(1+1/a/x)^(1/2
)/a^2/p/(-p^2+1)/(1-1/a/x)^(3/2)/x+3*(-a*c*x+c)^p*(1+1/a/x)^(1/2)/a/p/(1+p)
/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6176, 6181, 94, 132}

$$-\frac{3\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}(c-acx)^p\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a^2p\left(1-p^2\right)x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^p}{(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{3\sqrt{\frac{1}{ax}+1}\,(c-acx)^p}{ap(p+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p)/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]) + ((1 +
 1/(a*x))^(3/2)*x*(c - a*c*x)^p)/((1 + p)*Sqrt[1 - 1/(a*x)]) - (3*((a - x^(
-1))/(a + x^(-1)))^(3/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric
2F1[1 - p, 3/2 - p, 2 - p, 2/((a + x^(-1))*x)])/(a^2*p*(1 - p^2)*(1 - 1/(a*
x))^(3/2)*x)
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{3/2}dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-1-p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\right.}{a(1+p)}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subs}}{a^2p(1+p)}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\sqrt{1+\frac{1}{ax}}(c-acx)^p\,{}_2F_1\left(\right.}{a^2p\left(1-p^2\right)\left(1-\right.}$$

Mathematica [A] time = 0.15, size = 155, normalized size = 0.77

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{ax-1}{ax+1}\right)^{-p}(c-acx)^p\left(3\sqrt{\frac{ax-1}{ax+1}}\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;\frac{2}{ax+1}\right)+(p-1)(ax+1)(apx+p+3)\left(\frac{ax-1}{ax+1}\right)^p\right)}{a(p-1)p(p+1)\sqrt{1-\frac{1}{ax}}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*((-1 + p)*((-1 + a*x)/(1 + a*x))^p*(1 + a*
x)*(3 + p + a*p*x) + 3*Sqrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1 - p,
3/2 - p, 2 - p, 2/(1 + a*x)]))/(a*(-1 + p)*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1
 + a*x)/(1 + a*x))^p*(1 + a*x))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x^2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c-a\,c\,x)^p}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.178 $\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=105

$$\frac{3}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}-\frac{1}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[Out] -1/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1/5*a^4*c^4*(1-1/a^2/x^2)^(5/2)*x^5-3/
8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+3/8*a*c^4*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6175, 6178, 807, 266, 47, 63, 208}

$$\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}-\frac{1}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 + (a^4*c^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/5 - (3*c^4*ArcTanh[Sqrt[1 - 1/
(a^2*x^2)]])/(8*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx=\left(a^4c^4\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^4x^4\,dx$$

$$=-\left(\left(a^4c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5+\left(a^3c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^5}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5+\frac{1}{2}\left(a^3c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a^2}\right)^{3/2}}{x^3}\,dx,x,\frac{1}{x^2}\right)$$

$$=-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{1}{8}\left(3ac^4\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\right.$$

$$=\frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5+\frac{\left(3c^4\right)\text{Su}}{}$$

$$=\frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{1}{8}\left(3ac^4\right)$$

$$=\frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{3c^4\tanh}{}$$

Mathematica [A] time = 0.21, size = 80, normalized size = 0.76

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)-15\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a ^4*x^4) - 15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)

fricas [A] time = 0.52, size = 126, normalized size = 1.20

$$-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(8\,a^5c^4x^5-2\,a^4c^4x^4-26\,a^3c^4x^3+9\,a^2c^4x^2+33\,ac^4x+8\,c^4\right)\sqrt{\ldots}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="fricas")

[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (8*a^5*c^4*x^5 - 2*a^4*c^4*x^4 - 26*a^3*c^4*x^3 + 9*a^2 *c^4*x^2 + 33*a*c^4*x + 8*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a

giac [B] time = 0.18, size = 216, normalized size = 2.06

$$-\frac{1}{40}\,ac^4\left(\frac{15\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{70\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{128\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{70\,(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}+\frac{15\ldots}{\ldots}\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="giac")

[Out] -1/40*a*c^4*(15*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*log(abs(sqrt((a *x - 1)/(a*x + 1)) - 1))/a^2 - 2*(70*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a *x + 1) - 128*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 70*(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 15*(a*x - 1)^4*sqrt((a*x - 1) /(a*x + 1))/(a*x + 1)^4 - 15*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a* x + 1) - 1)^5))

maple [B] time = 0.05, size = 192, normalized size = 1.83

$$\frac{(ax-1)^2\,c^4\left(24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^2a^2-30\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;xa+16\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;+45\sqrt{a^2x^2-1}\;\sqrt{a^2}\;xa-40\ldots\right.}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\;\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x)

[Out] 1/120*(a*x-1)^2*c^4/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-30*(a^2*x^2 -1)^(3/2)*(a^2)^(1/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+45*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-40*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-45*ln((a^2*x+(a ^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)

maxima [B] time = 0.33, size = 259, normalized size = 2.47

$$-\frac{1}{40}\left(\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\,c^4\ldots\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4a^2}{(ax+1)^4}\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^4*
((a*x - 1)/(a*x + 1))^(7/2) - 128*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^4*
((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x - 1
)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*x
+ 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^2)
)*a
```

mupad [B] time = 0.09, size = 214, normalized size = 2.04

$$\frac{\frac{3\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}+\frac{32\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{5}+\frac{7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{2}-\frac{3\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}-\frac{3\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((3*c^4*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^4*((a*x - 1)/(a*x + 1))^(3/2)
)/2 + (32*c^4*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^4*((a*x - 1)/(a*x + 1))
^(7/2))/2 - (3*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^4*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2*x**2/(a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1)), x) + Integral(-4*a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Inte
gral(a**4*x**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x))
```

3.179 $\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=78

$$\frac{3}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[Out] -1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4-3/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
+3/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 266, 47, 63, 208}

$$-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (3*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 - (3*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx &= -\left(\left(a^3c^3\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)\\
&= \left(a^3c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^5}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{2}\left(a^3c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a^2}\right)^{3/2}}{x^3}\,dx,x,\frac{1}{x^2}\right)\\
&= -\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{1}{8}\left(3ac^3\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx,x,\frac{1}{x^2}\right)\\
&= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{\left(3c^3\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{16a}\\
&= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{1}{8}\left(3ac^3\right)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x\right.\\
&= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 64, normalized size = 0.82

$$\frac{c^3\left(a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(5-2a^2x^2\right)-3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{8a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*(5 - 2*a^2*x^2) - 3*Log[a*(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(8*a)
```

fricas [A] time = 0.55, size = 109, normalized size = 1.40

$$-\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4+2\,a^3c^3x^3-5\,a^2c^3x^2-5\,ac^3x\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) + (2*a^4*c^3*x^4 + 2*a^3*c^3*x^3 - 5*a^2*c^3*x^2 - 5*a*c^3*x
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [B] time = 0.18, size = 196, normalized size = 2.51

$$-\frac{1}{16}ac^3\left(\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}+2\right)}{a^2}-\frac{3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}-2\right|\right)}{a^2}-\frac{4\left(3\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^3-20\sqrt{\frac{ax-1}{ax+1}}-\cdots\right.}{\left(\left(\sqrt{\frac{ax-1}{ax+1}}+\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^2-4\right)^2a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*a*c^3*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1))
+ 2)/a^2 - 3*log(abs(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)
) - 2))/a^2 - 4*(3*(sqrt((a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1))
)^3 - 20*sqrt((a*x - 1)/(a*x + 1)) - 20/sqrt((a*x - 1)/(a*x + 1)))/(((sqrt(
(a*x - 1)/(a*x + 1)) + 1/sqrt((a*x - 1)/(a*x + 1)))^2 - 4)^2*a^2))
```

maple [A] time = 0.04, size = 124, normalized size = 1.59

$$-\frac{(ax-1)^2c^3\left(2x\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-3x\sqrt{a^2x^2-1}\,\sqrt{a^2}+3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x)
```

```
[Out] -1/8*(a*x-1)^2*c^3*(2*x*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*x*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)+3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x
-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [B] time = 0.32, size = 221, normalized size = 2.83

$$-\frac{1}{8}\left(\frac{3c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(3c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-11c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-11c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3c^3\sqrt{\frac{ax-1}{ax+1}}\right.}{\frac{4(ax-1)a^2}{ax+1}-\frac{6(ax-1)^2a^2}{(ax+1)^2}+\frac{4(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1)/a^2 + 2*(3*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 11*c^3*((a*
x - 1)/(a*x + 1))^(5/2) - 11*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((
a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x +
1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*
a
```

mupad [B] time = 1.20, size = 176, normalized size = 2.26

$$\frac{\frac{3c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{11c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4}-\frac{11c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{4}+\frac{3c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a-\frac{4a\,(a\,x-1)}{a\,x+1}+\frac{6a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}-\frac{3c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((3*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(3/2
))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 + (3*c^3*((a*x - 1)/(a*x + 1)
)^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2
- (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) - (3*c^3*ata
nh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int \frac{3ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int \frac{a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2*x**2/(a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x) + Integral(a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integr
al(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1)), x))
```

3.180 $\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=78

$$\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

```
[Out] 1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3-1/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+
1/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6175, 6178, 850, 807, 266, 47, 63, 208}

$$\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)
/3 - (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx &= \left(a^2c^2\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx\\
&= -\left(\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^4\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\left(ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}\\
&= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x,\right.\\
&= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 64, normalized size = 0.82

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+3ax-2\right)-3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*a*x + 2*a^2*x^2) - 3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

fricas [A] time = 0.40, size = 103, normalized size = 1.32

$$-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3+5\,a^2c^2x^2+ac^2x-2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) - (2*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + a*c^2*x - 2*c^2)*sqrt((a*
x - 1)/(a*x + 1)))/a
```

giac [B] time = 0.18, size = 154, normalized size = 1.97

$$-\frac{1}{6}\,ac^2\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{8\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{3\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+3\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/6*a*c^2*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(abs(sqrt((a*x
- 1)/(a*x + 1)) - 1))/a^2 + 2*(8*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x +
 1) - 3*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 3*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [A] time = 0.04, size = 130, normalized size = 1.67

$$\frac{(ax-1)^2\,c^2\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+2\left((ax-1)\,(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\,-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,\sqrt{(ax-1)\,(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x)
```

```
[Out] 1/6*(a*x-1)^2*c^2*(3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+2*((a*x-1)*(a*x+1))^
(3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a
)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

maxima [B] time = 0.32, size = 181, normalized size = 2.32

$$-\frac{1}{6}\,a\left(\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 8*c^2*((a
*x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^
2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 -
 a^2))
```

mupad [B] time = 1.20, size = 139, normalized size = 1.78

$$\frac{c^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}}-\frac{c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (c^2*((a*x - 1)/(a*x + 1))^(1/2) + (8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 -
c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x
 - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^2*atanh(((a*x - 1)
/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{2ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2*x**2/(a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

3.181 $\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}-2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -3/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a-2*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2
*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6175, 6178, 852, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}-2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\coth^{-1}(ax)}(c-acx)\,dx &= -\left((ac)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)\\
&= (ac)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^3\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= (ac)\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{2}(ac)\,\text{Subst}\left(\int \frac{-\frac{4}{a}-\frac{3x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}\\
&= -2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{2}(3ac)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 53, normalized size = 0.82

$$-\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax+4)+3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^2
*x^2)])*x]))/a
```

fricas [A] time = 0.51, size = 81, normalized size = 1.25

$$-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2+5\,acx+4\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (a^2*c*x^2 + 5*a*c*x + 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [B] time = 0.17, size = 121, normalized size = 1.86

$$-\frac{1}{2}\,ac\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{3\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-5\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*a*c*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(abs(sqrt((a*x -
1)/(a*x + 1)) - 1))/a^2 - 2*(3*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1
) - 5*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^2))
```

maple [B] time = 0.05, size = 162, normalized size = 2.49

$$-\frac{(ax-1)^2\,c\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+4\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}+4a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right.\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,\sqrt{(ax-1)\,(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x)
```

```
[Out] -1/2*(a*x-1)^2*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-ln((a^2*x+(a^2*x^2-1)^(
1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+4*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+4*a*
ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+
1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 135, normalized size = 2.08

$$-\frac{1}{2}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(3/2) - 5*c*sqrt((a*x - 1)/(a*x + 1)))
/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 3*c*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/a^2)
```

mupad [B] time = 1.20, size = 97, normalized size = 1.49

$$-\frac{5\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (5*c*((a*x - 1)/(a*x + 1))^(1/2) - 3*c*((a*x - 1)/(a*x + 1))^(3/2))/(a -
(2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a*x
 - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\frac{ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x))
```

3.182 $\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=80

$$\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{4}{3a^2cx\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] 8/3*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(3/2)-arctanh((1-1/a^2/x^2)^(1/2))/a/c+4/3/
a^2/c/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6175, 6178, 852, 1805, 12, 266, 63, 208}

$$\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{4}{3a^2cx\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) + 4/(3*a^2*c*Sqrt[1 - 1/
(a^2*x^2)]*x) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^4}{x\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3-\frac{4x}{a}+\frac{3x^2}{a^2}}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{3}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 0.79

$$\frac{\frac{4x\sqrt{1-\frac{1}{a^2x^2}}\,(2ax-1)}{(ax-1)^2} - \frac{3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] ((4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x))/(-1 + a*x)^2 - (3*Log[a*(1 + Sqrt
[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

fricas [A] time = 0.47, size = 120, normalized size = 1.50

$$-\frac{3\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-4\left(2\,a^{2}x^{2}+ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - 4*(2*a^2*x^2 + a*x - 1
)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

giac [A] time = 0.15, size = 108, normalized size = 1.35

$$-\frac{1}{3}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^{2}c}-\frac{2\,(ax+1)\left(\frac{3\,(ax-1)}{ax+1}+1\right)}{(ax-1)a^{2}c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/3*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 3*log(abs(sqrt((a*x
- 1)/(a*x + 1)) - 1))/(a^2*c) - 2*(a*x + 1)*(3*(a*x - 1)/(a*x + 1) + 1)/((a
*x - 1)*a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

maple [B] time = 0.05, size = 345, normalized size = 4.31

$$-\frac{3\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{3}a^{4}+3\sqrt{a^{2}}\,\sqrt{(ax-1)\,(ax+1)}\,x^{3}a^{3}-9\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{2}a^{3}-3\sqrt{a^{2}}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x)
```

```
[Out] -1/3/a*(3*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a
^4+3*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-9*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-3*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(3/2)*x*a-9*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+9*ln((a^2*x+((a*x-
1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(
a^2)^(1/2)+9*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-3*a*ln((a^2*x+((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1
))^(3/2)
```

maxima [A] time = 0.31, size = 95, normalized size = 1.19

$$-\frac{1}{3}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c}-\frac{2\left(\frac{3\,(ax-1)}{ax+1}+1\right)}{a^{2}c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 3*log(sqrt((a*x - 1)
/(a*x + 1)) - 1)/(a^2*c) - 2*(3*(a*x - 1)/(a*x + 1) + 1)/(a^2*c*((a*x - 1)/
(a*x + 1))^(3/2)))
```

mupad [B] time = 0.07, size = 63, normalized size = 0.79

$$\frac{\frac{2\,(a\,x-1)}{a\,x+1}+\frac{2}{3}}{a\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((2*(a*x - 1))/(a*x + 1) + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2)) - (2*atan
h(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1)), x)/c
```

3.183 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

```
[Out] -1/5*a^4*(1-1/a^2/x^2)^(5/2)/c^2/(a-1/x)^5
```

Rubi [A] time = 0.11, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6178, 651}

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2))/(5*c^2*(a - x^(-1))^5)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2x^2}\,dx}{a^2c^2}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 1.09

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax+1)^2}{5c^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] -1/5*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2)/(c^2*(-1 + a*x)^3)
```

fricas [B] time = 0.52, size = 77, normalized size = 2.33

$$-\frac{\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/5*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*x
^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [A] time = 0.16, size = 37, normalized size = 1.12

$$-\frac{(ax+1)^2}{5\,(ax-1)^2ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -1/5*(a*x + 1)^2/((a*x - 1)^2*a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 36, normalized size = 1.09

$$-\frac{ax+1}{5\,(ax-1)\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x)
```

```
[Out] -1/5*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

maxima [A] time = 0.31, size = 23, normalized size = 0.70

$$-\frac{1}{5\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/5/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

mupad [B] time = 0.04, size = 23, normalized size = 0.70

$$-\frac{1}{5\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -1/(5*a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**2*
x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x
)/c**2
```

3.184 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=67

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

```
[Out] 1/7*a^5*(1-1/a^2/x^2)^(5/2)/c^3/(a-1/x)^6-6/35*a^4*(1-1/a^2/x^2)^(5/2)/c^3/
(a-1/x)^5
```

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6175, 6178, 793, 651}

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2))/(7*c^3*(a - x^(-1))^6) - (6*a^4*(1 - 1/(a^2*x
^2))^(5/2))/(35*c^3*(a - x^(-1))^5)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{a^3c^3}$$
$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6} - \frac{6\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{7a^2c^3}$$
$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6} - \frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.61

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-6)(ax+1)^2}{35c^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-6 + a*x)*(1 + a*x)^2)/(c^3*(-1 + a*x)^4)
```

fricas [A] time = 0.69, size = 95, normalized size = 1.42

$$-\frac{\left(a^4x^4-3\,a^3x^3-15\,a^2x^2-17\,ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(a^4*x^4 - 3*a^3*x^3 - 15*a^2*x^2 - 17*a*x - 6)*sqrt((a*x - 1)/(a*x +
 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.16, size = 53, normalized size = 0.79

$$-\frac{(ax+1)^3\left(\frac{7\,(ax-1)}{ax+1}-5\right)}{70\,(ax-1)^3ac^3\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/70*(a*x + 1)^3*(7*(a*x - 1)/(a*x + 1) - 5)/((a*x - 1)^3*a*c^3*sqrt((a*x
- 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 41, normalized size = 0.61

$$-\frac{(ax-6)(ax+1)}{35(ax-1)^{2} c^{3} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/35*(a*x-6)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(3/2)/a
```

maxima [A] time = 0.32, size = 39, normalized size = 0.58

$$-\frac{\frac{7(ax-1)}{ax+1}-5}{70\,ac^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/70*(7*(a*x - 1)/(a*x + 1) - 5)/(a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

mupad [B] time = 1.18, size = 39, normalized size = 0.58

$$-\frac{\frac{a\,x-1}{5\,(a\,x+1)}-\frac{1}{7}}{2\,a\,c^{3}\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -((a*x - 1)/(5*(a*x + 1)) - 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^{4}x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**3
*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

3.185 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=94

$$-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] -47/315*(a+1/x)^5/a^6/c^4/(1-1/a^2/x^2)^(5/2)+16/63*(a+1/x)^6/a^7/c^4/(1-1/
a^2/x^2)^(7/2)-1/9*(a+1/x)^7/a^8/c^4/(1-1/a^2/x^2)^(9/2)
```

Rubi [A] time = 0.28, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 852, 1635, 789, 651}

$$-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] (-47*(a + x^(-1))^5)/(315*a^6*c^4*(1 - 1/(a^2*x^2))^(5/2)) + (16*(a + x^(-1
))^6)/(63*a^7*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (a + x^(-1))^7/(9*a^8*c^4*(1 -
 1/(a^2*x^2))^(9/2))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^7}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^7}{\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= -\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^6\left(7a^2+9ax\right)}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{9a^4c^4}$$

$$= \frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{47\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^5}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{63a^2c^4}$$

$$= -\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.53

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax+1)^2\left(2a^2x^2-14ax+47\right)}{315c^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(47 - 14*a*x + 2*a^2*x^2))/(c^4
*(-1 + a*x)^5)
```

fricas [A] time = 0.70, size = 116, normalized size = 1.23

$$-\frac{\left(2\,a^{5}x^{5}-8\,a^{4}x^{4}+11\,a^{3}x^{3}+101\,a^{2}x^{2}+127\,ax+47\right)\sqrt{\frac{ax-1}{ax+1}}}{315\,\left(a^{6}c^{4}x^{5}-5\,a^{5}c^{4}x^{4}+10\,a^{4}c^{4}x^{3}-10\,a^{3}c^{4}x^{2}+5\,a^{2}c^{4}x-ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 + 101*a^2*x^2 + 127*a*x + 47)*sq
rt((a*x - 1)/(a*x + 1))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*
a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.14, size = 69, normalized size = 0.73

$$\frac{(ax+1)^{4}\left(\frac{90\,(ax-1)}{ax+1}-\frac{63\,(ax-1)^{2}}{(ax+1)^{2}}-35\right)}{1260\,(ax-1)^{4}ac^{4}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/1260*(a*x + 1)^4*(90*(a*x - 1)/(a*x + 1) - 63*(a*x - 1)^2/(a*x + 1)^2 - 3
5)/((a*x - 1)^4*a*c^4*sqrt((a*x - 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 50, normalized size = 0.53

$$-\frac{\left(2a^{2}x^{2}-14ax+47\right)(ax+1)}{315\,(ax-1)^{3}\,c^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/315*(2*a^2*x^2-14*a*x+47)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)/
a
```

maxima [A] time = 0.30, size = 55, normalized size = 0.59

$$\frac{\frac{90\,(ax-1)}{ax+1}-\frac{63\,(ax-1)^{2}}{(ax+1)^{2}}-35}{1260\,ac^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/1260*(90*(a*x - 1)/(a*x + 1) - 63*(a*x - 1)^2/(a*x + 1)^2 - 35)/(a*c^4*((
a*x - 1)/(a*x + 1))^(9/2))
```

mupad [B] time = 1.19, size = 56, normalized size = 0.60

$$-\frac{\frac{(a\,x-1)^{2}}{5\,(a\,x+1)^{2}}-\frac{2\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{9}}{4\,a\,c^{4}\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -((a*x - 1)^2/(5*(a*x + 1)^2) - (2*(a*x - 1))/(7*(a*x + 1)) + 1/9)/(4*a*c^4
*((a*x - 1)/(a*x + 1))^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}}{c^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 5*a**4*
x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

3.186 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=125

$$\frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] -152/1155*(a+1/x)^5/a^6/c^5/(1-1/a^2/x^2)^(5/2)+79/231*(a+1/x)^6/a^7/c^5/(1
-1/a^2/x^2)^(7/2)-10/33*(a+1/x)^7/a^8/c^5/(1-1/a^2/x^2)^(9/2)+1/11*(a+1/x)^
8/a^9/c^5/(1-1/a^2/x^2)^(11/2)
```

Rubi [A] time = 0.39, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 852, 1635, 789, 651}

$$\frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

```
[Out] (-152*(a + x^(-1))^5)/(1155*a^6*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (79*(a + x^(
-1))^6)/(231*a^7*c^5*(1 - 1/(a^2*x^2))^(7/2)) - (10*(a + x^(-1))^7)/(33*a^8
*c^5*(1 - 1/(a^2*x^2))^(9/2)) + (a + x^(-1))^8/(11*a^9*c^5*(1 - 1/(a^2*x^2)
)^(11/2))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
```

```
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^8}dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^8}{\left(1-\frac{x^2}{a^2}\right)^{13/2}}dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^7(8a^3+11a^2x+11ax^2)}{\left(1-\frac{x^2}{a^2}\right)^{11/2}}dx, x, \frac{1}{x}\right)}{11a^5c^5}$$

$$= -\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^6(138a^3+99a^2x)}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}dx, x, \frac{1}{x}\right)}{99a^5c^5}$$

$$= \frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} - \frac{152\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^5}{\left(1-\frac{x^2}{a^2}\right)^{7}}\right.}{231a^2c^5}$$

$$= -\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)}$$

Mathematica [A] time = 0.08, size = 58, normalized size = 0.46

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}(ax+1)^2\left(2a^3x^3-16a^2x^2+61ax-152\right)}{1155c^5(ax-1)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

```
[Out] -1/1155*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(-152 + 61*a*x - 16*a^2*x^2 + 
2*a^3*x^3))/(c^5*(-1 + a*x)^6)
```

fricas [A] time = 0.89, size = 134, normalized size = 1.07

$$-\frac{\left(2\,a^6x^6-10\,a^5x^5+19\,a^4x^4-15\,a^3x^3-289\,a^2x^2-395\,ax-152\right)\sqrt{\frac{ax-1}{ax+1}}}{1155\left(a^7c^5x^6-6\,a^6c^5x^5+15\,a^5c^5x^4-20\,a^4c^5x^3+15\,a^3c^5x^2-6\,a^2c^5x+ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/1155*(2*a^6*x^6 - 10*a^5*x^5 + 19*a^4*x^4 - 15*a^3*x^3 - 289*a^2*x^2 - 3
95*a*x - 152)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 15*a
^5*c^5*x^4 - 20*a^4*c^5*x^3 + 15*a^3*c^5*x^2 - 6*a^2*c^5*x + a*c^5)
```

giac [A] time = 0.16, size = 85, normalized size = 0.68

$$-\frac{(ax+1)^5\left(\frac{385\,(ax-1)}{ax+1}-\frac{495\,(ax-1)^2}{(ax+1)^2}+\frac{231\,(ax-1)^3}{(ax+1)^3}-105\right)}{9240\,(ax-1)^5ac^5\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/9240*(a*x + 1)^5*(385*(a*x - 1)/(a*x + 1) - 495*(a*x - 1)^2/(a*x + 1)^2
+ 231*(a*x - 1)^3/(a*x + 1)^3 - 105)/((a*x - 1)^5*a*c^5*sqrt((a*x - 1)/(a*x
 + 1)))
```

maple [A] time = 0.04, size = 58, normalized size = 0.46

$$-\frac{\left(2x^3a^3-16a^2x^2+61ax-152\right)(ax+1)}{1155\,(ax-1)^4\,c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x)
```

```
[Out] -1/1155*(2*a^3*x^3-16*a^2*x^2+61*a*x-152)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a
*x+1))^(3/2)/a
```

maxima [A] time = 0.31, size = 71, normalized size = 0.57

$$-\frac{\frac{385\,(ax-1)}{ax+1}-\frac{495\,(ax-1)^2}{(ax+1)^2}+\frac{231\,(ax-1)^3}{(ax+1)^3}-105}{9240\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/9240*(385*(a*x - 1)/(a*x + 1) - 495*(a*x - 1)^2/(a*x + 1)^2 + 231*(a*x -
 1)^3/(a*x + 1)^3 - 105)/(a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

mupad [B] time = 0.04, size = 72, normalized size = 0.58

$$\frac{\frac{3\,(a\,x-1)^2}{7\,(a\,x+1)^2}-\frac{(a\,x-1)^3}{5\,(a\,x+1)^3}-\frac{a\,x-1}{3\,(a\,x+1)}+\frac{1}{11}}{8\,a\,c^5\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((3*(a*x - 1)^2)/(7*(a*x + 1)^2) - (a*x - 1)^3/(5*(a*x + 1)^3) - (a*x - 1)/
(3*(a*x + 1)) + 1/11)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^{6}x^{6}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6a^{5}x^{5}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^{4}x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{20a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}}{c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 6*a**5
*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 15*a**4*x**4*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 20*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1) + 15*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - 6*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1)), x)/c**5
```

3.187 $\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=66

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

```
[Out] 4*c*(-a*c*x+c)^(-1+p)/a/(1-p)+4*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(1+p)/a/c/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6130, 21, 43}

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*c*(c - a*c*x)^(-1 + p))/(a*(1 - p)) + (4*(c - a*c*x)^p)/(a*p) - (c - a*c
*x)^(1 + p)/(a*c*(1 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx = \int e^{4\tanh^{-1}(ax)}(c-acx)^p\,dx$$
$$= \int \frac{(1+ax)^2(c-acx)^p}{(1-ax)^2}\,dx$$
$$= c^2\int (1+ax)^2(c-acx)^{-2+p}\,dx$$
$$= c^2\int \left(4(c-acx)^{-2+p} - \frac{4(c-acx)^{-1+p}}{c} + \frac{(c-acx)^p}{c^2}\right)dx$$
$$= \frac{4c(c-acx)^{-1+p}}{a(1-p)} + \frac{4(c-acx)^p}{ap} - \frac{(c-acx)^{1+p}}{ac(1+p)}$$

Mathematica [A] time = 0.10, size = 50, normalized size = 0.76

$$\frac{\left(\frac{ax}{p+1}+\frac{4}{(p-1)(ax-1)}+\frac{3p+4}{p(p+1)}\right)(c-acx)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((c - a*c*x)^p*((4 + 3*p)/(p*(1 + p)) + (a*x)/(1 + p) + 4/((-1 + p)*(-1 + a
*x))))/a
```

fricas [A] time = 0.66, size = 81, normalized size = 1.23

$$-\frac{\left(\left(a^2p^2-a^2p\right)x^2+p^2+2\left(ap^2+ap-2\,a\right)x+3\,p+4\right)(-acx+c)^p}{ap^3-ap-\left(a^2p^3-a^2p\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] -((a^2*p^2 - a^2*p)*x^2 + p^2 + 2*(a*p^2 + a*p - 2*a)*x + 3*p + 4)*(-a*c*x
+ c)^p/(a*p^3 - a*p - (a^2*p^3 - a^2*p)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2(-acx+c)^p}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)^2*(-a*c*x + c)^p/(a*x - 1)^2, x)
```

maple [A] time = 0.04, size = 74, normalized size = 1.12

$$\frac{(-acx+c)^p\left(a^2p^2x^2-a^2x^2p+2a\,p^2x+2apx-4ax+p^2+3p+4\right)}{\left(p^2-1\right)ap\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x)
```

```
[Out] (-a*c*x+c)^p*(a^2*p^2*x^2-a^2*p*x^2+2*a*p^2*x+2*a*p*x-4*a*x+p^2+3*p+4)/(p^2
-1)/a/p/(a*x-1)
```

maxima [B] time = 0.33, size = 153, normalized size = 2.32

$$\frac{\left(\left(p^2-p\right)a^2c^px^2+2\,ac^p\left(p-1\right)x+2\,c^p\right)(-ax+1)^pa^2}{\left(p^3-p\right)a^4x-\left(p^3-p\right)a^3}+\frac{2\left(ac^p\left(p-1\right)x+c^p\right)(-ax+1)^pa}{\left(p^2-p\right)a^3x-\left(p^2-p\right)a^2}+\frac{(-ax+1)^pc^p}{a^2\left(p-1\right)x-a\left(p-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 - p)*a^2*c^p*x^2 + 2*a*c^p*(p - 1)*x + 2*c^p)*(-a*x + 1)^p*a^2/((p^3
- p)*a^4*x - (p^3 - p)*a^3) + 2*(a*c^p*(p - 1)*x + c^p)*(-a*x + 1)^p*a/((p^
2 - p)*a^3*x - (p^2 - p)*a^2) + (-a*x + 1)^p*c^p/(a^2*(p - 1)*x - a*(p - 1)
)
```

mupad [B] time = 1.35, size = 57, normalized size = 0.86

$$\frac{4\,{\left(c-a\,c\,x\right)}^p}{a\,\left(a\,x-1\right)\,\left(p-1\right)}+\frac{{\left(c-a\,c\,x\right)}^p\,\left(3\,p+a\,p\,x+4\right)}{a\,p\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (4*(c - a*c*x)^p)/(a*(a*x - 1)*(p - 1)) + ((c - a*c*x)^p*(3*p + a*p*x + 4))
/(a*p*(p + 1))
```

sympy [A] time = 1.18, size = 530, normalized size = 8.03

$$\begin{cases} c^px \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{2ax\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{4ax}{a^3cx^2-2a^2cx+ac}-\frac{\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}-\frac{2}{a^3cx^2-2a^2cx+ac} \\ \frac{a^2x^2}{a^2x-a}+\frac{4ax\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{4\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{5}{a^2x-a} \\ -\frac{acx^2}{2}-3cx-\frac{4c\log\left(x-\frac{1}{a}\right)}{a} \\ \frac{a^2p^2x^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{a^2px^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2ap^2x(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2apx(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{4ax(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{p^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((c**p*x, Eq(a, 0)), (-a**2*x**2*log(x - 1/a)/(a**3*c*x**2 - 2*a**
2*c*x + a*c) + 2*a*x*log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 4*a*x/
(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x +
 a*c) - 2/(a**3*c*x**2 - 2*a**2*c*x + a*c), Eq(p, -1)), (a**2*x**2/(a**2*x
- a) + 4*a*x*log(x - 1/a)/(a**2*x - a) - 4*log(x - 1/a)/(a**2*x - a) - 5/(a
**2*x - a), Eq(p, 0)), (-a*c*x**2/2 - 3*c*x - 4*c*log(x - 1/a)/a, Eq(p, 1))
, (a**2*p**2*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) -
 a**2*p*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*
p**2*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*p*x*(-
a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) - 4*a*x*(-a*c*x + c)*
*p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + p**2*(-a*c*x + c)**p/(a**2*p**
3*x - a**2*p*x - a*p**3 + a*p) + 3*p*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*
x - a*p**3 + a*p) + 4*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*
p), True))
```

3.188 $\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=53

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

```
[Out] -c^5*(-a*x+1)^4/a+4/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -((c^5*(1 - a*x)^4)/a) + (4*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^5\,dx\\
&= c^5\int (1-ax)^3(1+ax)^2\,dx\\
&= c^5\int \left(4(1-ax)^3-4(1-ax)^4+(1-ax)^5\right)dx\\
&= -\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.58

$$-\frac{c^5(ax-1)^4\left(5a^2x^2+14ax+11\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -1/30*(c^5*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2))/a
```

fricas [A] time = 0.42, size = 59, normalized size = 1.11

$$-\frac{1}{6}a^5c^5x^6 + \frac{1}{5}a^4c^5x^5 + \frac{1}{2}a^3c^5x^4 - \frac{2}{3}a^2c^5x^3 - \frac{1}{2}ac^5x^2 + c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

giac [A] time = 0.13, size = 42, normalized size = 0.79

$$-\frac{\left(5\,c^5 + \frac{24\,c^5}{ax-1} + \frac{30\,c^5}{(ax-1)^2}\right)(ax-1)^6}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/30*(5*c^5 + 24*c^5/(a*x - 1) + 30*c^5/(a*x - 1)^2)*(a*x - 1)^6/a
```

maple [A] time = 0.03, size = 45, normalized size = 0.85

$$c^5\left(-\frac{1}{6}x^6a^5 + \frac{1}{5}a^4x^5 + \frac{1}{2}x^4a^3 - \frac{2}{3}x^3a^2 - \frac{1}{2}a\,x^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x)
```

```
[Out] c^5*(-1/6*x^6*a^5+1/5*a^4*x^5+1/2*x^4*a^3-2/3*x^3*a^2-1/2*a*x^2+x)
```

maxima [A] time = 0.30, size = 59, normalized size = 1.11

$$-\frac{1}{6}a^5c^5x^6 + \frac{1}{5}a^4c^5x^5 + \frac{1}{2}a^3c^5x^4 - \frac{2}{3}a^2c^5x^3 - \frac{1}{2}ac^5x^2 + c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

mupad [B] time = 1.19, size = 59, normalized size = 1.11

$$-\frac{a^5\,c^5\,x^6}{6} + \frac{a^4\,c^5\,x^5}{5} + \frac{a^3\,c^5\,x^4}{2} - \frac{2\,a^2\,c^5\,x^3}{3} - \frac{a\,c^5\,x^2}{2} + c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^5*x - (a*c^5*x^2)/2 - (2*a^2*c^5*x^3)/3 + (a^3*c^5*x^4)/2 + (a^4*c^5*x^5)
/5 - (a^5*c^5*x^6)/6
```

sympy [**A**] time = 0.09, size = 63, normalized size = 1.19

$$-\frac{a^5c^5x^6}{6}+\frac{a^4c^5x^5}{5}+\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}-\frac{ac^5x^2}{2}+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + a**4*c**5*x**5/5 + a**3*c**5*x**4/2 - 2*a**2*c**5*x**3/
3 - a*c**5*x**2/2 + c**5*x
```

3.189 $\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=32

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

```
[Out] c^4*x-2/3*a^2*c^4*x^3+1/5*a^4*c^4*x^5
```

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 41, 194}

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^4\,dx\\
&= c^4\int (1-ax)^2(1+ax)^2\,dx\\
&= c^4\int \left(1-a^2x^2\right)^2\,dx\\
&= c^4\int \left(1-2a^2x^2+a^4x^4\right)\,dx\\
&= c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.81

$$c^4\left(\frac{a^4x^5}{5}-\frac{2a^2x^3}{3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
[Out] c^4*(x - (2*a^2*x^3)/3 + (a^4*x^5)/5)
```

fricas [A] time = 0.48, size = 28, normalized size = 0.88

$$\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="fricas")
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

giac [A] time = 0.13, size = 42, normalized size = 1.31

$$\frac{\left(3\,c^4+\frac{15\,c^4}{ax-1}+\frac{20\,c^4}{(ax-1)^2}\right)(ax-1)^5}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="giac")
[Out] 1/15*(3*c^4 + 15*c^4/(a*x - 1) + 20*c^4/(a*x - 1)^2)*(a*x - 1)^5/a
```

maple [A] time = 0.03, size = 23, normalized size = 0.72

$$c^4\left(\frac{1}{5}a^4x^5-\frac{2}{3}x^3a^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x)
[Out] c^4*(1/5*a^4*x^5-2/3*x^3*a^2+x)
```

maxima [A] time = 0.30, size = 28, normalized size = 0.88

$$\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="maxima")
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

mupad [B] time = 0.04, size = 24, normalized size = 0.75

$$\frac{c^4\,x\,\left(3\,a^4\,x^4-10\,a^2\,x^2+15\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^4*x*(3*a^4*x^4 - 10*a^2*x^2 + 15))/15
```

sympy [A] time = 0.08, size = 29, normalized size = 0.91

$$\frac{a^4c^4x^5}{5} - \frac{2a^2c^4x^3}{3} + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**4,x)
```

```
[Out] a**4*c**4*x**5/5 - 2*a**2*c**4*x**3/3 + c**4*x
```

3.190 $\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=35

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

```
[Out] 2/3*c^3*(a*x+1)^3/a-1/4*c^3*(a*x+1)^4/a
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 + a*x)^3)/(3*a) - (c^3*(1 + a*x)^4)/(4*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^3\,dx\\
&= c^3\int (1-ax)(1+ax)^2\,dx\\
&= c^3\int \left(2(1+ax)^2-(1+ax)^3\right)dx\\
&= \frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.86

$$-\frac{1}{12}c^3x\left(3a^3x^3+4a^2x^2-6ax-12\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -1/12*(c^3*x*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))
```

fricas [A] time = 0.47, size = 37, normalized size = 1.06

$$-\frac{1}{4}a^3c^3x^4 - \frac{1}{3}a^2c^3x^3 + \frac{1}{2}ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

giac [A] time = 0.14, size = 42, normalized size = 1.20

$$-\frac{\left(3\,c^3 + \frac{16\,c^3}{ax-1} + \frac{24\,c^3}{(ax-1)^2}\right)(ax-1)^4}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/12*(3*c^3 + 16*c^3/(a*x - 1) + 24*c^3/(a*x - 1)^2)*(a*x - 1)^4/a
```

maple [A] time = 0.03, size = 29, normalized size = 0.83

$$c^3\left(-\frac{1}{4}x^4a^3 - \frac{1}{3}x^3a^2 + \frac{1}{2}a\,x^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x)
[Out] c^3*(-1/4*x^4*a^3-1/3*x^3*a^2+1/2*a*x^2+x)
```

maxima [A] time = 0.30, size = 37, normalized size = 1.06

$$-\frac{1}{4}a^3c^3x^4 - \frac{1}{3}a^2c^3x^3 + \frac{1}{2}ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

mupad [B] time = 0.05, size = 37, normalized size = 1.06

$$-\frac{a^3\,c^3\,x^4}{4} - \frac{a^2\,c^3\,x^3}{3} + \frac{a\,c^3\,x^2}{2} + c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] c^3*x + (a*c^3*x^2)/2 - (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

sympy [A] time = 0.08, size = 37, normalized size = 1.06

$$-\frac{a^3c^3x^4}{4} - \frac{a^2c^3x^3}{3} + \frac{ac^3x^2}{2} + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**3,x)
[Out] -a**3*c**3*x**4/4 - a**2*c**3*x**3/3 + a*c**3*x**2/2 + c**3*x
```

3.191 $\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(ax+1)^3}{3a}$$

```
[Out] 1/3*c^2*(a*x+1)^3/a
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 32}

$$\frac{c^2(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
[Out] (c^2*(1 + a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^2\,dx\\ &= c^2\int (1+ax)^2\,dx\\ &= \frac{c^2(1+ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.24

$$c^2\left(\frac{a^2x^3}{3}+ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
[Out] c^2*(x + a*x^2 + (a^2*x^3)/3)
```

fricas [A] time = 0.59, size = 25, normalized size = 1.47

$$\frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="fricas")
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

giac [B] time = 0.14, size = 40, normalized size = 2.35

$$\frac{\left(c^2 + \frac{6\,c^2}{ax-1} + \frac{12\,c^2}{(ax-1)^2}\right)(ax-1)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="giac")
[Out] 1/3*(c^2 + 6*c^2/(a*x - 1) + 12*c^2/(a*x - 1)^2)*(a*x - 1)^3/a
```

maple [A] time = 0.03, size = 16, normalized size = 0.94

$$\frac{c^2\,(ax+1)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x)
[Out] 1/3*c^2*(a*x+1)^3/a
```

maxima [A] time = 0.31, size = 25, normalized size = 1.47

$$\frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="maxima")
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

mupad [B] time = 0.03, size = 19, normalized size = 1.12

$$\frac{c^2\,x\,\left(a^2\,x^2 + 3\,a\,x + 3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (c^2*x*(3*a*x + a^2*x^2 + 3))/3
```

sympy [A] time = 0.07, size = 24, normalized size = 1.41

$$\frac{a^2c^2x^3}{3} + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**2,x)
[Out] a**2*c**2*x**3/3 + a*c**2*x**2 + c**2*x
```

3.192 $\int e^{4\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}-3cx$$

```
[Out] -3*c*x-1/2*a*c*x^2-4*c*ln(-a*x+1)/a
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6167, 6129, 43}

$$-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}-3cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -3*c*x - (a*c*x^2)/2 - (4*c*Log[1 - a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)\,dx\\
&= c\int \frac{(1+ax)^2}{1-ax}\,dx\\
&= c\int \left(-3-ax+\frac{4}{1-ax}\right)dx\\
&= -3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.96

$$c\left(-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
[Out] c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

fricas [A] time = 0.57, size = 28, normalized size = 1.04

$$-\frac{a^2cx^2 + 6\,acx + 8\,c\log\left(ax - 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="fricas")
[Out] -1/2*(a^2*c*x^2 + 6*a*c*x + 8*c*log(a*x - 1))/a
```

giac [A] time = 0.14, size = 50, normalized size = 1.85

$$-\frac{(ax-1)^2\left(c + \frac{8\,c}{ax-1}\right)}{2\,a} + \frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="giac")
[Out] -1/2*(a*x - 1)^2*(c + 8*c/(a*x - 1))/a + 4*c*log(abs(a*x - 1)/((a*x - 1)^2*
abs(a)))/a
```

maple [A] time = 0.03, size = 25, normalized size = 0.93

$$-\frac{ac\,x^2}{2} - 3cx - \frac{4c\ln\left(ax - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x)
[Out] -1/2*a*c*x^2-3*c*x-4*c/a*ln(a*x-1)
```

maxima [A] time = 0.31, size = 24, normalized size = 0.89

$$-\frac{1}{2}\,acx^2 - 3\,cx - \frac{4\,c\log\left(ax - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="maxima")
[Out] -1/2*a*c*x^2 - 3*c*x - 4*c*log(a*x - 1)/a
```

mupad [B] time = 1.18, size = 26, normalized size = 0.96

$$-\frac{c\,\left(8\,\ln\left(a\,x - 1\right) + 6\,a\,x + a^2\,x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] -(c*(8*log(a*x - 1) + 6*a*x + a^2*x^2))/(2*a)
```

sympy [A] time = 0.11, size = 26, normalized size = 0.96

$$-\frac{acx^2}{2} - 3cx - \frac{4c\log\left(ax - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c),x)
[Out] -a*c*x**2/2 - 3*c*x - 4*c*log(a*x - 1)/a
```

3.193 $\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=48

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

```
[Out] 2/a/c/(-a*x+1)^2-4/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] 2/(a*c*(1 - a*x)^2) - 4/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{c-acx}\,dx\\
&= \frac{\int \frac{(1+ax)^2}{(1-ax)^3}\,dx}{c}\\
&= \frac{\int \left(\frac{1}{1-ax}-\frac{4}{(-1+ax)^3}-\frac{4}{(-1+ax)^2}\right)dx}{c}\\
&= \frac{2}{ac(1-ax)^2}-\frac{4}{ac(1-ax)}-\frac{\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.75

$$\frac{4ax+(ax-1)^2(-\log(1-ax))-2}{ac(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (-2 + 4*a*x - (-1 + a*x)^2*Log[1 - a*x])/(a*c*(-1 + a*x)^2)
```

fricas [A] time = 0.43, size = 49, normalized size = 1.02

$$\frac{4\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-2}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (4*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x - 1) - 2)/(a^3*c*x^2 - 2*a^2*c*x + a
*c)
```

giac [A] time = 0.14, size = 57, normalized size = 1.19

$$\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}+\frac{2\left(\frac{2\,ac}{ax-1}+\frac{ac}{(ax-1)^2}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) + 2*(2*a*c/(a*x - 1) + a*c/(a*
x - 1)^2)/(a^2*c^2)
```

maple [A] time = 0.04, size = 46, normalized size = 0.96

$$\frac{2}{ca\left(ax-1\right)^2}-\frac{\ln\left(ax-1\right)}{ca}+\frac{4}{ca\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x)
```

```
[Out] 2/c/a/(a*x-1)^2-1/c/a*ln(a*x-1)+4/c/a/(a*x-1)
```

maxima [A] time = 0.31, size = 44, normalized size = 0.92

$$\frac{2\left(2\,ax-1\right)}{a^3cx^2-2\,a^2cx+ac}-\frac{\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 2*(2*a*x - 1)/(a^3*c*x^2 - 2*a^2*c*x + a*c) - log(a*x - 1)/(a*c)
```

mupad [B] time = 0.06, size = 42, normalized size = 0.88

$$\frac{4\,x-\frac{2}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}-\frac{\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a*c*x)*(a*x - 1)^2),x)
```

```
[Out] (4*x - 2/a)/(c + a^2*c*x^2 - 2*a*c*x) - log(a*x - 1)/(a*c)
```

sympy [A] time = 0.21, size = 37, normalized size = 0.77

$$-\frac{-4ax+2}{a^3cx^2-2a^2cx+ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c),x)
```

```
[Out] -(-4*a*x + 2)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(a*x - 1)/(a*c)
```

3.194 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=25

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

```
[Out] 1/6*(a*x+1)^3/a/c^2/(-a*x+1)^3
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6167, 6129, 37}

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^2}\,dx\\
&= \frac{\int \frac{(1+ax)^2}{(1-ax)^4}\,dx}{c^2}\\
&= \frac{(1+ax)^3}{6ac^2(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

fricas [B] time = 0.55, size = 51, normalized size = 2.04

$$-\frac{3\,a^2x^2+1}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [B] time = 0.14, size = 50, normalized size = 2.00

$$-\frac{2}{(acx-c)^2a}-\frac{1}{(acx-c)ac}-\frac{4\,c}{3\,(acx-c)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -2/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c) - 4/3*c/((a*c*x - c)^3*a)
```

maple [A] time = 0.04, size = 42, normalized size = 1.68

$$\frac{-\frac{2}{a(ax-1)^2}-\frac{4}{3a(ax-1)^3}-\frac{1}{a(ax-1)}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x)
[Out] 1/c^2*(-2/a/(a*x-1)^2-4/3/a/(a*x-1)^3-1/a/(a*x-1))
```

maxima [B] time = 0.31, size = 51, normalized size = 2.04

$$-\frac{3\,a^2x^2+1}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

mupad [B] time = 1.19, size = 25, normalized size = 1.00

$$-\frac{3\,a^2\,x^2+1}{3\,a\,c^2\,(a\,x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a*c*x)^2*(a*x - 1)^2),x)
[Out] -(3*a^2*x^2 + 1)/(3*a*c^2*(a*x - 1)^3)
```

sympy [B] time = 0.24, size = 51, normalized size = 2.04

$$\frac{-3a^2x^2-1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**2,x)
```

```
[Out] (-3*a**2*x**2 - 1)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3
*a*c**2)
```

3.195 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=52

$$\frac{1}{2ac^3(1-ax)^2}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{ac^3(1-ax)^4}$$

```
[Out] 1/a/c^3/(-a*x+1)^4-4/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{1}{2ac^3(1-ax)^2}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] 1/(a*c^3*(1 - a*x)^4) - 4/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^3}\,dx\\
&= \frac{\int \frac{(1+ax)^2}{(1-ax)^5}\,dx}{c^3}\\
&= \frac{\int \left(-\frac{4}{(-1+ax)^5}-\frac{4}{(-1+ax)^4}-\frac{1}{(-1+ax)^3}\right)dx}{c^3}\\
&= \frac{1}{ac^3(1-ax)^4}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.60

$$\frac{3a^2x^2+2ax+1}{6ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 + 2*a*x + 3*a^2*x^2)/(6*a*c^3*(-1 + a*x)^4)
```

fricas [A] time = 0.63, size = 65, normalized size = 1.25

$$\frac{3\,a^2x^2+2\,ax+1}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.14, size = 42, normalized size = 0.81

$$\frac{\frac{3}{(ax-1)^2a}+\frac{8}{(ax-1)^3a}+\frac{6}{(ax-1)^4a}}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*(3/((a*x - 1)^2*a) + 8/((a*x - 1)^3*a) + 6/((a*x - 1)^4*a))/c^3
```

maple [A] time = 0.04, size = 41, normalized size = 0.79

$$\frac{\frac{1}{2a(ax-1)^2}+\frac{4}{3a(ax-1)^3}+\frac{1}{a(ax-1)^4}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x)
```

```
[Out] 1/c^3*(1/2/a/(a*x-1)^2+4/3/a/(a*x-1)^3+1/a/(a*x-1)^4)
```

maxima [A] time = 0.30, size = 65, normalized size = 1.25

$$\frac{3\,a^2x^2+2\,ax+1}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

mupad [B] time = 1.23, size = 29, normalized size = 0.56

$$\frac{3\,a^2\,x^2+2\,a\,x+1}{6\,a\,c^3\,{\left(a\,x-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a*c*x)^3*(a*x - 1)^2),x)
```

```
[Out] (2*a*x + 3*a^2*x^2 + 1)/(6*a*c^3*(a*x - 1)^4)
```

sympy [**A**] time = 0.31, size = 70, normalized size = 1.35

$$-\frac{-3a^2x^2-2ax-1}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**3,x)
```

```
[Out] -(-3*a**2*x**2 - 2*a*x - 1)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**3
*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3)
```

3.196 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

```
[Out] 4/5/a/c^4/(-a*x+1)^5-1/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] 4/(5*a*c^4*(1 - a*x)^5) - 1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^4}\,dx\\
&= \frac{\int \frac{(1+ax)^2}{(1-ax)^6}\,dx}{c^4}\\
&= \frac{\int \left(\frac{4}{(-1+ax)^6}+\frac{4}{(-1+ax)^5}+\frac{1}{(-1+ax)^4}\right)dx}{c^4}\\
&= \frac{4}{5ac^4(1-ax)^5}-\frac{1}{ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.58

$$-\frac{5a^2x^2+5ax+2}{15ac^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/15*(2 + 5*a*x + 5*a^2*x^2)/(a*c^4*(-1 + a*x)^5)
```

fricas [A] time = 0.69, size = 77, normalized size = 1.45

$$-\frac{5\,a^{2}x^{2}+5\,ax+2}{15\,\left(a^{6}c^{4}x^{5}-5\,a^{5}c^{4}x^{4}+10\,a^{4}c^{4}x^{3}-10\,a^{3}c^{4}x^{2}+5\,a^{2}c^{4}x-ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.14, size = 42, normalized size = 0.79

$$-\frac{\frac{5}{(ax-1)^{3}a}+\frac{15}{(ax-1)^{4}a}+\frac{12}{(ax-1)^{5}a}}{15\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/15*(5/((a*x - 1)^3*a) + 15/((a*x - 1)^4*a) + 12/((a*x - 1)^5*a))/c^4
```

maple [A] time = 0.04, size = 42, normalized size = 0.79

$$\frac{-\frac{4}{5a(ax-1)^{5}}-\frac{1}{3a(ax-1)^{3}}-\frac{1}{a(ax-1)^{4}}}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x)
```

```
[Out] 1/c^4*(-4/5/a/(a*x-1)^5-1/3/a/(a*x-1)^3-1/a/(a*x-1)^4)
```

maxima [A] time = 0.31, size = 77, normalized size = 1.45

$$-\frac{5\,a^{2}x^{2}+5\,ax+2}{15\,\left(a^{6}c^{4}x^{5}-5\,a^{5}c^{4}x^{4}+10\,a^{4}c^{4}x^{3}-10\,a^{3}c^{4}x^{2}+5\,a^{2}c^{4}x-ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

mupad [B] time = 1.24, size = 29, normalized size = 0.55

$$-\frac{5\,a^{2}\,x^{2}+5\,a\,x+2}{15\,a\,c^{4}\,{\left(a\,x-1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a*c*x)^4*(a*x - 1)^2),x)
```

```
[Out] -(5*a*x + 5*a^2*x^2 + 2)/(15*a*c^4*(a*x - 1)^5)
```

sympy [A] time = 0.35, size = 80, normalized size = 1.51

$$\frac{-5a^2x^2 - 5ax - 2}{15a^6c^4x^5 - 75a^5c^4x^4 + 150a^4c^4x^3 - 150a^3c^4x^2 + 75a^2c^4x - 15ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**4,x)
```

```
[Out] (-5*a**2*x**2 - 5*a*x - 2)/(15*a**6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**
4*c**4*x**3 - 150*a**3*c**4*x**2 + 75*a**2*c**4*x - 15*a*c**4)
```

3.197 $\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=94

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}(c-acx)^p\,{}_2F_1\left(-p-1,-p-\frac{1}{2};-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

```
[Out] ((a-1/x)/(a+1/x))^(-1/2-p)*x*(-a*c*x+c)^p*hypergeom([-1-p, -1/2-p],[-p],2/(
a+1/x)/x)*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/(1+p)
```

Rubi [A] time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6176, 6181, 132}

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}(c-acx)^p\,{}_2F_1\left(-p-1,-p-\frac{1}{2};-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^p*Hypergeometric2F1[-1 - p, -1/2 - p, -p, 2/((a + x^(-1))*x)
])/(1 + p)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-2-p}\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}}{\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)\right)$$
$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p\,{}_2F_1\left(-1-p,-\frac{1}{2}-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 0.81

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{-p-\frac{1}{2}}(c-acx)^p\,{}_2F_1\left(-p-1,-p-\frac{1}{2};-p;\frac{2}{ax+1}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^ArcCoth[a*x],x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(c - a*c*x)^p*Hy
pergeometric2F1[-1 - p, -1/2 - p, -p, 2/(1 + a*x)])/(1 + p)
```

fricas [F] time = 2.26, size = 0, normalized size = 0.00

$$\text{integral}\left((-acx+c)^p\sqrt{\frac{ax-1}{ax+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^p \sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c-a\,c\,x)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**p, x)
```

3.198 $\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=127

$$-\frac{27}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{20}{3}c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{35c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{4}{3}a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] -35/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+20/3*c^3*x*(1-1/a^2/x^2)^(1/2)-27/
8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)+4/3*a^2*c^3*x^3*(1-1/a^2/x^2)^(1/2)-1/4*a^3
*c^3*x^4*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6175, 6178, 1807, 807, 266, 63, 208}

$$-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{4}{3}a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{27}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{20}{3}c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{35c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

```
[Out] (20*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (27*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8
 + (4*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]
*x^4)/4 - (35*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{16}{a}-\frac{27x}{a^2}+\frac{16x^2}{a^3}-\frac{4x^3}{a^4}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{12}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{81}{a^2}-\frac{80x}{a^3}+\frac{12x^2}{a^4}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= -\frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{24}\left(a^3c^3\right)\text{S}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2}}$$

Mathematica [A] time = 0.21, size = 72, normalized size = 0.57

$$\frac{c^3\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-6a^3x^3+32a^2x^2-81ax+160\right)-105\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c - a*c*x)^3/E^ArcCoth[a*x],x]

[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(160 - 81*a*x + 32*a^2*x^2 - 6*a^3*x^3) - 1
05*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)

fricas [A] time = 1.84, size = 114, normalized size = 0.90

$$-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^3x^4-26\,a^3c^3x^3+49\,a^2c^3x^2-79\,ac^3x-160\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")

[Out] -1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (6*a^4*c^3*x^4 - 26*a^3*c^3*x^3 + 49*a^2*c^3*x^2 - 79
*a*c^3*x - 160*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a

giac [A] time = 0.16, size = 109, normalized size = 0.86

$$\frac{35\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{8\,|a|}+\frac{1}{24}\sqrt{a^2x^2-1}\left(\frac{160\,c^3\mathrm{sgn}\left(ax+1\right)}{a}-\left(81\,c^3\mathrm{sgn}\left(ax+1\right)+2\left(3\,a^2c^3x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")

[Out] 35/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/24
*sqrt(a^2*x^2 - 1)*(160*c^3*sgn(a*x + 1)/a - (81*c^3*sgn(a*x + 1) + 2*(3*a^
2*c^3*x*sgn(a*x + 1) - 16*a*c^3*sgn(a*x + 1))*x)*x)

maple [A] time = 0.05, size = 196, normalized size = 1.54

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^3\left(6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,xa+87\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa-32\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}-87\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\ldots}{\sqrt{\ldots}}\right.\right.}{24a\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x)

[Out] -1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3/a*(6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
)*x*a+87*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-32*((a*x-1)*(a*x+1))^(3/2)*(a^2)
^(1/2)-87*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-192*((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+192*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)

maxima [B] time = 0.32, size = 221, normalized size = 1.74

$$-\frac{1}{24}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(279\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-511\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+385\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-\ldots\right.}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")

```
[Out] -1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(279*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 511*
c^3*((a*x - 1)/(a*x + 1))^(5/2) + 385*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105
*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*
a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)
^4 - a^2))*a
```

mupad [B] time = 1.23, size = 176, normalized size = 1.39

$$\frac{\frac{35\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{385\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{511\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{35\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((35*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (385*c^3*((a*x - 1)/(a*x + 1))^(3
/2))/12 + (511*c^3*((a*x - 1)/(a*x + 1))^(5/2))/12 - (93*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x +
 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) - (35*
c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^{3}\left(\int 3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-3a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a
**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
), x))
```

3.199 $\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=100

$$-\frac{3}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{11}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] -5/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+11/3*c^2*x*(1-1/a^2/x^2)^(1/2)-3/2*
a*c^2*x^2*(1-1/a^2/x^2)^(1/2)+1/3*a^2*c^2*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6175, 6178, 1807, 807, 266, 63, 208}

$$\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{3}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{11}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

```
[Out] (11*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (3*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2
+ (a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (5*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = \left(a^2c^2\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{1}{3}\left(a^2c^2\right)\text{Subst}\left(\int \frac{\frac{9}{a}-\frac{11x}{a^2}+\frac{3x^2}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{6}\left(a^2c^2\right)\text{Subst}\left(\int \frac{\frac{22}{a^2}-\frac{15x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x\right.$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{\left(5c^2\right)\text{Subst}\left(\int\right.}{2}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{\left(5c^2\right)\text{Subst}\left(\int\right.}{4}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{2}\left(5ac^2\right)\text{Subst}\left(\right.$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{5c^2\tanh^{-1}\left(\sqrt{1\right.}{2a}$$

Mathematica [A] time = 0.13, size = 64, normalized size = 0.64

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-9ax+22\right)-15\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 - 9*a*x + 2*a^2*x^2) - 15*Log[a*(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

fricas [A] time = 0.49, size = 104, normalized size = 1.04

$$-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3-7\,a^2c^2x^2+13\,ac^2x+22\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 7*a^2*c^2*x^2 + 13*a*c^2*x + 22*c^2)*sq
rt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.18, size = 90, normalized size = 0.90

$$\frac{5\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}+\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(2\,ac^2x\operatorname{sgn}(ax+1)-9\,c^2\operatorname{sgn}(ax+1)\right)x+\frac{22\,c^2\operatorname{sgn}(a}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 5/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/6*s
qrt(a^2*x^2 - 1)*((2*a*c^2*x*sgn(a*x + 1) - 9*c^2*sgn(a*x + 1))*x + 22*c^2*
sgn(a*x + 1)/a)
```

maple [B] time = 0.05, size = 176, normalized size = 1.76

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^2\left(2\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+24\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}+9\ln\left(\frac{a^2x+\sqrt{a^2x}}{\sqrt{a}}\right.\right.}{6\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1
/2)-9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+24*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1
/2)+9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-24*a*ln((a^2*
x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))^(1/2
)/a/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 181, normalized size = 1.81

$$-\frac{1}{6}\,a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(33\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 + 2*(33*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 40*c^2
*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x -
```

```
1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1
)^3 - a^2))
```

mupad [B] time = 1.21, size = 140, normalized size = 1.40

$$\frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+11\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{5\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (40*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + 11*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*
a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c^2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2
*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1) -
 1/(a*x + 1)), x))
```

3.200 $\int e^{-\coth^{-1}(ax)}(c - acx)\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -3/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+2*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2
*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6175, 6178, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^ArcCoth[a*x],x]
```

```
[Out] 2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}(c-acx)\,dx &= -\left((ac)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)\\
&= (ac)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{2}(ac)\,\text{Subst}\left(\int \frac{\frac{4}{a}-\frac{3x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= 2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= 2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}\\
&= 2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{2}(3ac)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= 2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 53, normalized size = 0.82

$$-\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-4)+3\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^
2*x^2)])*x]))/a
```

fricas [A] time = 0.60, size = 81, normalized size = 1.25

$$-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-3\,acx-4\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (a^2*c*x^2 - 3*a*c*x - 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.15, size = 68, normalized size = 1.05

$$\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{2\,|a|}-\frac{1}{2}\,\sqrt{a^2x^2-1}\left(cx\mathrm{sgn}\left(ax+1\right)-\frac{4\,c\mathrm{sgn}\left(ax+1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/2*sqr
t(a^2*x^2 - 1)*(c*x*sgn(a*x + 1) - 4*c*sgn(a*x + 1)/a)
```

maple [B] time = 0.04, size = 153, normalized size = 2.35

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\ (ax+1)\,c\left(-\sqrt{a^2x^2-1}\ \sqrt{a^2}\ xa+4\sqrt{(ax-1)\,(ax+1)}\ \sqrt{a^2}\ +\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\ \sqrt{a^2}}{\sqrt{a^2}}\right)a-4a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax}{\sqrt{a^2}}\right.\right.}{2\sqrt{(ax-1)\,(ax+1)}\ a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+4
*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*a-4*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2)))/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 135, normalized size = 2.08

$$\frac{1}{2}\,a\left(\frac{2\left(5\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(5*c*((a*x - 1)/(a*x + 1))^(3/2) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/
(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 3*c*log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2)
```

mupad [B] time = 0.06, size = 96, normalized size = 1.48

$$\frac{3\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-5\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*c*((a*x - 1)/(a*x + 1))^(1/2) - 5*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2
*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a*x -
 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c*(Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x
/(a*x + 1) - 1/(a*x + 1)), x))
```

3.201 $\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c
```

Rubi [A] time = 0.10, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6175, 6178, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
```

```
[Out] -(ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$
$$= -\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.48

$$-\frac{\log\left(ax\left(\sqrt{\frac{a^2x^2-1}{a^2x^2}}+1\right)\right)}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
[Out] -(Log[a*x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]/(a*c))
```

fricas [B] time = 0.49, size = 47, normalized size = 2.04

$$-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
[Out] -(log(sqrt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/
(a*c)
```

giac [A] time = 0.13, size = 33, normalized size = 1.43

$$\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a))
```

maple [B] time = 0.05, size = 76, normalized size = 3.30

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)}{\sqrt{(ax-1)(ax+1)}\,c\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1
/2))/(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c/(a^2)^(1/2)
```

maxima [B] time = 0.31, size = 55, normalized size = 2.39

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c))
```

mupad [B] time = 0.06, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x),x)
```

```
[Out] -(2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

3.202 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

```
[Out] -(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)
```

Rubi [A] time = 0.10, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6178, 651}

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]/(c^2*(a - x^(-1))))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 0.96

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(-1 + a*x)))
```

fricas [A] time = 0.57, size = 39, normalized size = 1.39

$$-\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] undef
```

maple [A] time = 0.04, size = 36, normalized size = 1.29

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x)
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/(a*x-1)/a/c^2
```

maxima [A] time = 0.30, size = 23, normalized size = 0.82

$$-\frac{1}{ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/(a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

mupad [B] time = 0.03, size = 23, normalized size = 0.82

$$-\frac{1}{a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^2,x)
```

```
[Out] -1/(a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

3.203 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=62

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

[Out] 1/3*a*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)^2-2/3*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)

Rubi [A] time = 0.13, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6175, 6178, 793, 651}

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]

[Out] (a*Sqrt[1 - 1/(a^2*x^2)])/(3*c^3*(a - x^(-1))^2) - (2*Sqrt[1 - 1/(a^2*x^2)])/(3*c^3*(a - x^(-1)))

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3c^3}$$
$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3a^2c^3}$$
$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.55

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-2)}{3c^3(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

```
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(c^3*(-1 + a*x)^2)
```

fricas [A] time = 0.60, size = 57, normalized size = 0.92

$$-\frac{\left(a^2x^2-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^3*x^2 - 2*a^2*c^3
*x + a*c^3)
```

giac [A] time = 0.20, size = 45, normalized size = 0.73

$$\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^3*a*
c^3)
```

maple [A] time = 0.04, size = 41, normalized size = 0.66

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax-2)\,(ax+1)}{3\,(ax-1)^{2}\,c^{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x)
```

```
[Out] -1/3*((a*x-1)/(a*x+1))^(1/2)*(a*x-2)*(a*x+1)/(a*x-1)^2/c^3/a
```

maxima [A] time = 0.31, size = 39, normalized size = 0.63

$$-\frac{\frac{3\,(ax-1)}{ax+1}-1}{6\,ac^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a*x - 1)/(a*x + 1) - 1)/(a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

mupad [B] time = 0.03, size = 38, normalized size = 0.61

$$-\frac{\frac{a\,x-1}{a\,x+1}-\frac{1}{3}}{2\,a\,c^{3}\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^3,x)
```

```
[Out] -((a*x - 1)/(a*x + 1) - 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{3}x^{3}-3a^{2}x^{2}+3ax-1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x**2 + 3*a*
x - 1), x)/c**3
```

3.204 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal. Leaf size=95

$$-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

```
[Out] -1/5*a^2*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)^3+8/15*a*(1-1/a^2/x^2)^(1/2)/c^4/(
a-1/x)^2-7/15*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)
```

Rubi [A] time = 0.23, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 1639, 793, 659, 651}

$$-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

```
[Out] -(a^2*Sqrt[1 - 1/(a^2*x^2)])/(5*c^4*(a - x^(-1))^3) + (8*a*Sqrt[1 - 1/(a^2*
x^2)])/(15*c^4*(a - x^(-1))^2) - (7*Sqrt[1 - 1/(a^2*x^2)])/(15*c^4*(a - x^(
-1)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
```

```
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}\\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}\\
&= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2}{a^2}-\frac{x}{a^3}}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^4}\\
&= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5a^2c^4}\\
&= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15a^2c^4}\\
&= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 43, normalized size = 0.45

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-6ax+7\right)}{15c^4(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

```
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^4*(-1 + a*x)^3)
```

fricas [A] time = 0.58, size = 77, normalized size = 0.81

$$-\frac{\left(2\,a^{3}x^{3}-4\,a^{2}x^{2}+ax+7\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{4}c^{4}x^{3}-3\,a^{3}c^{4}x^{2}+3\,a^{2}c^{4}x-ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^4*
x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.21, size = 65, normalized size = 0.68

$$-\frac{4\left(10\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)^{2}x^{2}-5\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)x-1\right)^{5}ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^4)
```

maple [A] time = 0.04, size = 50, normalized size = 0.53

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(2a^{2}x^{2}-6ax+7\right)(ax+1)}{15\,(ax-1)^{3}\,c^{4}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x)
```

```
[Out] -1/15*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2-6*a*x+7)*(a*x+1)/(a*x-1)^3/c^4/a
```

maxima [A] time = 0.31, size = 55, normalized size = 0.58

$$\frac{\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^{2}}{(ax+1)^{2}}-3}{60\,ac^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^4*((a*x
 - 1)/(a*x + 1))^(5/2))
```

mupad [B] time = 0.04, size = 55, normalized size = 0.58

$$-\frac{\frac{(a\,x-1)^{2}}{(a\,x+1)^{2}}-\frac{2\,(a\,x-1)}{3\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^{4}\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^4,x)
```

```
[Out] -((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^4*((a
*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}{a^4x^4 - 4a^3x^3 + 6a^2x^2 - 4ax + 1}\, dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x**3 + 6*a**
2*x**2 - 4*a*x + 1), x)/c**4
```

3.205 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=128

$$-\frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3}+\frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2}-\frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}+\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4}$$

```
[Out] 1/7*a^3*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^4-18/35*a^2*(1-1/a^2/x^2)^(1/2)/c^5
/(a-1/x)^3+23/35*a*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^2-12/35*(1-1/a^2/x^2)^(1
/2)/c^5/(a-1/x)
```

Rubi [A] time = 0.25, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 1639, 793, 659, 651}

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4}-\frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3}+\frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2}-\frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)])/(7*c^5*(a - x^(-1))^4) - (18*a^2*Sqrt[1 - 1/(a^
2*x^2)])/(35*c^5*(a - x^(-1))^3) + (23*a*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a
- x^(-1))^2) - (12*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a - x^(-1)))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{\left(1-\frac{x}{a}\right)^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2}{a^2}-\frac{3x}{a^3}}{\left(1-\frac{x}{a}\right)^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{18\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{7a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{36\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{35a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{35a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

Mathematica [A] time = 0.07, size = 51, normalized size = 0.40

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-8a^2x^2+13ax-12\right)}{35c^5(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-12 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^5
*(-1 + a*x)^4)
```

fricas [A] time = 0.64, size = 95, normalized size = 0.74

$$-\frac{\left(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+ax-12\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^5x^4-4\,a^4c^5x^3+6\,a^3c^5x^2-4\,a^2c^5x+ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

[Out] -1/35*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + a*x - 12)*sqrt((a*x - 1)/(a*x + 1))/(a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 6*a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5)

giac [A] time = 0.30, size = 85, normalized size = 0.66

$$\frac{4\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3-21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")

[Out] 4/35*(35*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 21*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 7*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^5)

maple [A] time = 0.04, size = 58, normalized size = 0.45

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(2x^3a^3-8a^2x^2+13ax-12\right)(ax+1)}{35\,(ax-1)^4\,c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x)

[Out] -1/35*((a*x-1)/(a*x+1))^(1/2)*(2*a^3*x^3-8*a^2*x^2+13*a*x-12)*(a*x+1)/(a*x-1)^4/c^5/a

maxima [A] time = 0.31, size = 71, normalized size = 0.55

$$-\frac{\frac{21\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}+\frac{35\,(ax-1)^3}{(ax+1)^3}-5}{280\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")

[Out] -1/280*(21*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 + 35*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a*c^5*((a*x - 1)/(a*x + 1))^(7/2))

mupad [B] time = 1.19, size = 71, normalized size = 0.55

$$\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{8\,a\,c^5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^5,x)

[Out] ((a*x - 1)^2/(a*x + 1)^2 - (a*x - 1)^3/(a*x + 1)^3 - (3*(a*x - 1))/(5*(a*x + 1)) + 1/7)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 5*a**4*x**4 + 10*a
**3*x**3 - 10*a**2*x**2 + 5*a*x - 1), x)/c**5
```

3.206 $\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=44

$$\frac{(c-acx)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

[Out] 1/2*(-a*c*x+c)^(2+p)*hypergeom([1, 2+p],[3+p],-1/2*a*x+1/2)/a/c^2/(2+p)

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6130, 21, 68}

$$\frac{(c-acx)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

Antiderivative was successfully verified.

[In] Int[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]

[Out] ((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a*c^2*(2 + p))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx=-\int e^{-2\tanh^{-1}(ax)}(c-acx)^p\,dx$$
$$=-\int\frac{(1-ax)(c-acx)^p}{1+ax}\,dx$$
$$=-\frac{\int\frac{(c-acx)^{1+p}}{1+ax}\,dx}{c}$$
$$=\frac{(c-acx)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$-\frac{(ax-1)(c-acx)^p\left({}_2F_1\left(1,p+1;p+2;\frac{1}{2}(1-ax)\right)-1\right)}{a(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
[Out] -(((-1 + a*x)*(c - a*c*x)^p*(-1 + Hypergeometric2F1[1, 1 + p, 2 + p, (1 - a
*x)/2]))/(a*(1 + p)))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(ax-1)(-acx+c)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int\frac{(-acx+c)^p\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p/(a*x+1)*(a*x-1),x)
[Out] int((-a*c*x+c)^p/(a*x+1)*(a*x-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax-1)(-acx+c)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c-a\,c\,x)^p\,(a\,x-1)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1))^p\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(a*x - 1))**p*(a*x - 1)/(a*x + 1), x)
```

3.207 $\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=91

$$-\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}-\frac{4c^4(1-ax)^3}{3a}-\frac{4c^4(1-ax)^2}{a}-\frac{32c^4\log(ax+1)}{a}+16c^4x$$

```
[Out] 16*c^4*x-4*c^4*(-a*x+1)^2/a-4/3*c^4*(-a*x+1)^3/a-1/2*c^4*(-a*x+1)^4/a-1/5*c
^4*(-a*x+1)^5/a-32*c^4*ln(a*x+1)/a
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}-\frac{4c^4(1-ax)^3}{3a}-\frac{4c^4(1-ax)^2}{a}-\frac{32c^4\log(ax+1)}{a}+16c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 16*c^4*x - (4*c^4*(1 - a*x)^2)/a - (4*c^4*(1 - a*x)^3)/(3*a) - (c^4*(1 - a*
x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a) - (32*c^4*Log[1 + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^4\,dx\\ &= -\left(c^4\int\frac{(1-ax)^5}{1+ax}\,dx\right)\\ &= -\left(c^4\int\left(-16-8(1-ax)-4(1-ax)^2-2(1-ax)^3-(1-ax)^4+\frac{32}{1+ax}\right)dx\right)\\ &= 16c^4x-\frac{4c^4(1-ax)^2}{a}-\frac{4c^4(1-ax)^3}{3a}-\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}-\frac{32c^4\log(1+}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.62

$$\frac{c^4\left(6a^5x^5-45a^4x^4+160a^3x^3-390a^2x^2+930ax-960\log(ax+1)-181\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(-181 + 930*a*x - 390*a^2*x^2 + 160*a^3*x^3 - 45*a^4*x^4 + 6*a^5*x^5 -
 960*Log[1 + a*x]))/(30*a)
```

fricas [A] time = 0.81, size = 68, normalized size = 0.75

$$\frac{6\,a^5c^4x^5-45\,a^4c^4x^4+160\,a^3c^4x^3-390\,a^2c^4x^2+930\,ac^4x-960\,c^4\log\left(ax+1\right)}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/30*(6*a^5*c^4*x^5 - 45*a^4*c^4*x^4 + 160*a^3*c^4*x^3 - 390*a^2*c^4*x^2 +
930*a*c^4*x - 960*c^4*log(a*x + 1))/a
```

giac [A] time = 0.14, size = 75, normalized size = 0.82

$$-\frac{32\,c^4\log\left(|ax+1|\right)}{a}+\frac{6\,a^9c^4x^5-45\,a^8c^4x^4+160\,a^7c^4x^3-390\,a^6c^4x^2+930\,a^5c^4x}{30\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -32*c^4*log(abs(a*x + 1))/a + 1/30*(6*a^9*c^4*x^5 - 45*a^8*c^4*x^4 + 160*a^
7*c^4*x^3 - 390*a^6*c^4*x^2 + 930*a^5*c^4*x)/a^5
```

maple [A] time = 0.04, size = 64, normalized size = 0.70

$$\frac{a^4c^4x^5}{5}-\frac{3c^4x^4a^3}{2}+\frac{16a^2c^4x^3}{3}-13c^4x^2a+31c^4x-\frac{32c^4\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^4/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/5*a^4*c^4*x^5-3/2*c^4*x^4*a^3+16/3*a^2*c^4*x^3-13*c^4*x^2*a+31*c^4*x-32*c
^4*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 63, normalized size = 0.69

$$\frac{1}{5}\,a^4c^4x^5-\frac{3}{2}\,a^3c^4x^4+\frac{16}{3}\,a^2c^4x^3-13\,ac^4x^2+31\,c^4x-\frac{32\,c^4\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 3/2*a^3*c^4*x^4 + 16/3*a^2*c^4*x^3 - 13*a*c^4*x^2 + 31*c^
4*x - 32*c^4*log(a*x + 1)/a
```

mupad [B] time = 1.17, size = 63, normalized size = 0.69

$$31\,c^4\,x-13\,a\,c^4\,x^2+\frac{16\,a^2\,c^4\,x^3}{3}-\frac{3\,a^3\,c^4\,x^4}{2}+\frac{a^4\,c^4\,x^5}{5}-\frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x - 1))/(a*x + 1),x)
```

[Out] 31*c^4*x - 13*a*c^4*x^2 + (16*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + (a^4*c^4
*x^5)/5 - (32*c^4*log(a*x + 1))/a

sympy [A] time = 0.16, size = 68, normalized size = 0.75

$$\frac{a^4c^4x^5}{5} - \frac{3a^3c^4x^4}{2} + \frac{16a^2c^4x^3}{3} - 13ac^4x^2 + 31c^4x - \frac{32c^4\log{(ax+1)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)**4*(a*x-1)/(a*x+1),x)

[Out] a**4*c**4*x**5/5 - 3*a**3*c**4*x**4/2 + 16*a**2*c**4*x**3/3 - 13*a*c**4*x**
2 + 31*c**4*x - 32*c**4*log(a*x + 1)/a

3.208 $\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=73

$$-\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}-\frac{2c^3(1-ax)^2}{a}-\frac{16c^3\log(ax+1)}{a}+8c^3x$$

```
[Out] 8*c^3*x-2*c^3*(-a*x+1)^2/a-2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a-16*c^3
*ln(a*x+1)/a
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}-\frac{2c^3(1-ax)^2}{a}-\frac{16c^3\log(ax+1)}{a}+8c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 8*c^3*x - (2*c^3*(1 - a*x)^2)/a - (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x
)^4)/(4*a) - (16*c^3*Log[1 + a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^3\,dx\\
&= -\left(c^3\int\frac{(1-ax)^4}{1+ax}\,dx\right)\\
&= -\left(c^3\int\left(-8-4(1-ax)-2(1-ax)^2-(1-ax)^3+\frac{16}{1+ax}\right)dx\right)\\
&= 8c^3x-\frac{2c^3(1-ax)^2}{a}-\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}-\frac{16c^3\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.66

$$-\frac{c^3\left(3a^4x^4-20a^3x^3+66a^2x^2-180ax+192\log(ax+1)+35\right)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/12*(c^3*(35 - 180*a*x + 66*a^2*x^2 - 20*a^3*x^3 + 3*a^4*x^4 + 192*Log[1
+ a*x]))/a
```

fricas [A] time = 0.55, size = 57, normalized size = 0.78

$$-\frac{3\,a^{4}c^{3}x^{4}-20\,a^{3}c^{3}x^{3}+66\,a^{2}c^{3}x^{2}-180\,ac^{3}x+192\,c^{3}\log\left(ax+1\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*c^3*x^4 - 20*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 180*a*c^3*x + 192*
c^3*log(a*x + 1))/a
```

giac [A] time = 0.14, size = 64, normalized size = 0.88

$$-\frac{16\,c^{3}\log\left(|ax+1|\right)}{a}-\frac{3\,a^{7}c^{3}x^{4}-20\,a^{6}c^{3}x^{3}+66\,a^{5}c^{3}x^{2}-180\,a^{4}c^{3}x}{12\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -16*c^3*log(abs(a*x + 1))/a - 1/12*(3*a^7*c^3*x^4 - 20*a^6*c^3*x^3 + 66*a^5
*c^3*x^2 - 180*a^4*c^3*x)/a^4
```

maple [A] time = 0.03, size = 53, normalized size = 0.73

$$-\frac{c^{3}x^{4}a^{3}}{4}+\frac{5a^{2}c^{3}x^{3}}{3}-\frac{11c^{3}x^{2}a}{2}+15c^{3}x-\frac{16c^{3}\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/4*c^3*x^4*a^3+5/3*a^2*c^3*x^3-11/2*c^3*x^2*a+15*c^3*x-16*c^3*ln(a*x+1)/a
```

maxima [A] time = 0.30, size = 52, normalized size = 0.71

$$-\frac{1}{4}\,a^{3}c^{3}x^{4}+\frac{5}{3}\,a^{2}c^{3}x^{3}-\frac{11}{2}\,ac^{3}x^{2}+15\,c^{3}x-\frac{16\,c^{3}\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*a^3*c^3*x^4 + 5/3*a^2*c^3*x^3 - 11/2*a*c^3*x^2 + 15*c^3*x - 16*c^3*log
(a*x + 1)/a
```

mupad [B] time = 0.04, size = 52, normalized size = 0.71

$$15\,c^{3}\,x-\frac{11\,a\,c^{3}\,x^{2}}{2}+\frac{5\,a^{2}\,c^{3}\,x^{3}}{3}-\frac{a^{3}\,c^{3}\,x^{4}}{4}-\frac{16\,c^{3}\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] 15*c^3*x - (11*a*c^3*x^2)/2 + (5*a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4 - (16*c^3
*log(a*x + 1))/a
```

sympy [A] time = 0.14, size = 56, normalized size = 0.77

$$-\frac{a^3c^3x^4}{4}+\frac{5a^2c^3x^3}{3}-\frac{11ac^3x^2}{2}+15c^3x-\frac{16c^3\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**3*c**3*x**4/4 + 5*a**2*c**3*x**3/3 - 11*a*c**3*x**2/2 + 15*c**3*x - 16*
c**3*log(a*x + 1)/a
```

3.209 $\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=55

$$-\frac{c^2(1-ax)^3}{3a}-\frac{c^2(1-ax)^2}{a}-\frac{8c^2\log(ax+1)}{a}+4c^2x$$

```
[Out] 4*c^2*x-c^2*(-a*x+1)^2/a-1/3*c^2*(-a*x+1)^3/a-8*c^2*ln(a*x+1)/a
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 43}

$$-\frac{c^2(1-ax)^3}{3a}-\frac{c^2(1-ax)^2}{a}-\frac{8c^2\log(ax+1)}{a}+4c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 4*c^2*x - (c^2*(1 - a*x)^2)/a - (c^2*(1 - a*x)^3)/(3*a) - (8*c^2*Log[1 + a*
x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx&=-\int e^{-2\tanh^{-1}(ax)}(c-acx)^2\,dx\\&=-\left(c^2\int\frac{(1-ax)^3}{1+ax}\,dx\right)\\&=-\left(c^2\int\left(-4-2(1-ax)-(1-ax)^2+\frac{8}{1+ax}\right)dx\right)\\&=4c^2x-\frac{c^2(1-ax)^2}{a}-\frac{c^2(1-ax)^3}{3a}-\frac{8c^2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.71

$$\frac{c^2\left(a^3x^3-6a^2x^2+21ax-24\log(ax+1)-4\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-4 + 21*a*x - 6*a^2*x^2 + a^3*x^3 - 24*Log[1 + a*x]))/(3*a)
```

fricas [A] time = 0.64, size = 45, normalized size = 0.82

$$\frac{a^3c^2x^3 - 6\,a^2c^2x^2 + 21\,ac^2x - 24\,c^2\log\left(ax+1\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*c^2*x^3 - 6*a^2*c^2*x^2 + 21*a*c^2*x - 24*c^2*log(a*x + 1))/a
```

giac [A] time = 0.13, size = 52, normalized size = 0.95

$$-\frac{8\,c^2\log\left(|ax+1|\right)}{a} + \frac{a^5c^2x^3 - 6\,a^4c^2x^2 + 21\,a^3c^2x}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -8*c^2*log(abs(a*x + 1))/a + 1/3*(a^5*c^2*x^3 - 6*a^4*c^2*x^2 + 21*a^3*c^2*
x)/a^3
```

maple [A] time = 0.04, size = 42, normalized size = 0.76

$$\frac{a^2c^2x^3}{3} - 2c^2x^2a + 7c^2x - \frac{8c^2\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/3*a^2*c^2*x^3-2*c^2*x^2*a+7*c^2*x-8*c^2*ln(a*x+1)/a
```

maxima [A] time = 0.30, size = 41, normalized size = 0.75

$$\frac{1}{3}\,a^2c^2x^3 - 2\,ac^2x^2 + 7\,c^2x - \frac{8\,c^2\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*x^3 - 2*a*c^2*x^2 + 7*c^2*x - 8*c^2*log(a*x + 1)/a
```

mupad [B] time = 0.04, size = 41, normalized size = 0.75

$$7\,c^2\,x - 2\,a\,c^2\,x^2 + \frac{a^2\,c^2\,x^3}{3} - \frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x - 1))/(a*x + 1),x)
```

```
[Out] 7*c^2*x - 2*a*c^2*x^2 + (a^2*c^2*x^3)/3 - (8*c^2*log(a*x + 1))/a
```

sympy [A] time = 0.12, size = 41, normalized size = 0.75

$$\frac{a^2c^2x^3}{3} - 2ac^2x^2 + 7c^2x - \frac{8c^2\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*(a*x-1)/(a*x+1),x)
```

```
[Out] a**2*c**2*x**3/3 - 2*a*c**2*x**2 + 7*c**2*x - 8*c**2*log(a*x + 1)/a
```

3.210 $\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2}acx^2-\frac{4c\log(ax+1)}{a}+3cx$$

[Out] 3*c*x-1/2*a*c*x^2-4*c*ln(a*x+1)/a

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6167, 6129, 43}

$$-\frac{1}{2}acx^2-\frac{4c\log(ax+1)}{a}+3cx$$

Antiderivative was successfully verified.

[In] Int[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]

[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)\,dx\\ &= -\left(c\int \frac{(1-ax)^2}{1+ax}\,dx\right)\\ &= -\left(c\int \left(-3+ax+\frac{4}{1+ax}\right)dx\right)\\ &= 3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{1}{2}acx^2-\frac{4c\log(ax+1)}{a}+3cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

fricas [A] time = 0.47, size = 28, normalized size = 1.08

$$-\frac{a^2cx^2 - 6\,acx + 8\,c\log\left(ax+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*x^2 - 6*a*c*x + 8*c*log(a*x + 1))/a
```

giac [A] time = 0.13, size = 35, normalized size = 1.35

$$-\frac{4\,c\log\left(|ax+1|\right)}{a} - \frac{a^3cx^2 - 6\,a^2cx}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*c*log(abs(a*x + 1))/a - 1/2*(a^3*c*x^2 - 6*a^2*c*x)/a^2
```

maple [A] time = 0.03, size = 25, normalized size = 0.96

$$3cx - \frac{ac\,x^2}{2} - \frac{4c\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)/(a*x+1)*(a*x-1),x)
```

```
[Out] 3*c*x-1/2*a*c*x^2-4*c*ln(a*x+1)/a
```

maxima [A] time = 0.30, size = 24, normalized size = 0.92

$$-\frac{1}{2}\,acx^2 + 3\,cx - \frac{4\,c\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 + 3*c*x - 4*c*log(a*x + 1)/a
```

mupad [B] time = 0.04, size = 26, normalized size = 1.00

$$-\frac{c\,\left(8\,\ln\left(a\,x+1\right) - 6\,a\,x + a^2\,x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] -(c*(8*log(a*x + 1) - 6*a*x + a^2*x^2))/(2*a)
```

sympy [A] time = 0.11, size = 24, normalized size = 0.92

$$-\frac{acx^2}{2} + 3cx - \frac{4c\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a*c*x**2/2 + 3*c*x - 4*c*log(a*x + 1)/a
```

3.211 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=14

$$-\frac{\log(ax+1)}{ac}$$

```
[Out] -ln(a*x+1)/a/c
```

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6129, 31}

$$-\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] -(Log[1 + a*x]/(a*c))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-acx}\,dx\\
&= -\frac{\int \frac{1}{1+ax}\,dx}{c}\\
&= -\frac{\log(1+ax)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] -(Log[1 + a*x]/(a*c))
```

fricas [A] time = 0.46, size = 14, normalized size = 1.00

$$-\frac{\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
[Out] -log(a*x + 1)/(a*c)
```

giac [A] time = 0.12, size = 15, normalized size = 1.07

$$-\frac{\log(|ax+1|)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="giac")
[Out] -log(abs(a*x + 1))/(a*c)
```

maple [A] time = 0.04, size = 15, normalized size = 1.07

$$-\frac{\ln(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c),x)
[Out] -ln(a*x+1)/a/c
```

maxima [A] time = 0.30, size = 14, normalized size = 1.00

$$-\frac{\log(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
[Out] -log(a*x + 1)/(a*c)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.00

$$-\frac{\ln(a\,x+1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)*(a*x + 1)),x)
[Out] -log(a*x + 1)/(a*c)
```

sympy [A] time = 0.06, size = 12, normalized size = 0.86

$$-\frac{\log(acx+c)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x)
[Out] -log(a*c*x + c)/(a*c)
```

3.212 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

```
[Out] -arctanh(a*x)/a/c^2
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 35, 206}

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

```
[Out] -(ArcTanh[a*x]/(a*c^2))
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)(1+ax)}\,dx}{c^2}\\
&= -\frac{\int \frac{1}{1-a^2x^2}\,dx}{c^2}\\
&= -\frac{\tanh^{-1}(ax)}{ac^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
[Out] -(ArcTanh[a*x]/(a*c^2))
```

fricas [A] time = 0.70, size = 23, normalized size = 1.92

$$-\frac{\log(ax+1)-\log(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/2*(log(a*x + 1) - log(a*x - 1))/(a*c^2)
```

giac [B] time = 0.14, size = 25, normalized size = 2.08

$$-\frac{\log\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -1/2*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^2)
```

maple [B] time = 0.04, size = 30, normalized size = 2.50

$$\frac{\ln(ax-1)}{2c^2a}-\frac{\ln(ax+1)}{2a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^2,x)
[Out] 1/2/c^2/a*ln(a*x-1)-1/2*ln(a*x+1)/a/c^2
```

maxima [B] time = 0.31, size = 29, normalized size = 2.42

$$-\frac{\log(ax+1)}{2\,ac^2}+\frac{\log(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.06, size = 12, normalized size = 1.00

$$-\frac{\operatorname{atanh}(a\,x)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^2*(a*x + 1)),x)
```

```
[Out] -atanh(a*x)/(a*c^2)
```

sympy [A] time = 0.14, size = 20, normalized size = 1.67

$$\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**2,x)
```

```
[Out] (log(x - 1/a)/2 - log(x + 1/a)/2)/(a*c**2)
```

3.213 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3} dx$

Optimal. Leaf size=33

$$-\frac{1}{2ac^3(1-ax)}-\frac{\tanh^{-1}(ax)}{2ac^3}$$

```
[Out] -1/2/a/c^3/(-a*x+1)-1/2*arctanh(a*x)/a/c^3
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 44, 207}

$$-\frac{1}{2ac^3(1-ax)}-\frac{\tanh^{-1}(ax)}{2ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] -1/(2*a*c^3*(1 - a*x)) - ArcTanh[a*x]/(2*a*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)^2(1+ax)}\,dx}{c^3}$$
$$= -\frac{\int \left(\frac{1}{2(-1+ax)^2} - \frac{1}{2(-1+a^2x^2)}\right)dx}{c^3}$$
$$= -\frac{1}{2ac^3(1-ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{2c^3}$$
$$= -\frac{1}{2ac^3(1-ax)} - \frac{\tanh^{-1}(ax)}{2ac^3}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.97

$$-\frac{\frac{1}{2a(1-ax)} + \frac{\tanh^{-1}(ax)}{2a}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] -((1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3)
```

fricas [A] time = 0.73, size = 46, normalized size = 1.39

$$-\frac{(ax-1)\log(ax+1) - (ax-1)\log(ax-1) - 2}{4\left(a^2c^3x - ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*c^3*x - a*c
^3)
```

giac [A] time = 0.15, size = 46, normalized size = 1.39

$$-\frac{\log(|ax+1|)}{4\,ac^3} + \frac{\log(|ax-1|)}{4\,ac^3} + \frac{1}{2\,(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) + 1/2/((a*x
- 1)*a*c^3)
```

maple [A] time = 0.04, size = 45, normalized size = 1.36

$$\frac{1}{2c^3a\,(ax-1)} + \frac{\ln(ax-1)}{4c^3a} - \frac{\ln(ax+1)}{4a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^3,x)
```

[Out] 1/2/c^3/a/(a*x-1)+1/4/c^3/a*ln(a*x-1)-1/4*ln(a*x+1)/a/c^3

maxima [A] time = 0.30, size = 48, normalized size = 1.45

$$\frac{1}{2\left(a^{2}c^{3}x-ac^{3}\right)}-\frac{\log\left(ax+1\right)}{4\,ac^{3}}+\frac{\log\left(ax-1\right)}{4\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")

[Out] 1/2/(a^2*c^3*x - a*c^3) - 1/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)/(a*c^3)

mupad [B] time = 0.07, size = 31, normalized size = 0.94

$$-\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}-\frac{\mathrm{atanh}\left(a\,x\right)}{2\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x - 1)/((c - a*c*x)^3*(a*x + 1)),x)

[Out] - 1/(2*a*(c^3 - a*c^3*x)) - atanh(a*x)/(2*a*c^3)

sympy [A] time = 0.22, size = 39, normalized size = 1.18

$$\frac{1}{2a^{2}c^{3}x-2ac^{3}}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4}+\frac{\log\left(x+\frac{1}{a}\right)}{4}}{ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**3,x)

[Out] 1/(2*a**2*c**3*x - 2*a*c**3) - (-log(x - 1/a)/4 + log(x + 1/a)/4)/(a*c**3)

3.214 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^4}$$

```
[Out] -1/4/a/c^4/(-a*x+1)^2-1/4/a/c^4/(-a*x+1)-1/4*arctanh(a*x)/a/c^4
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 44, 207}

$$-\frac{1}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] -1/(4*a*c^4*(1 - a*x)^2) - 1/(4*a*c^4*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^4}$$
$$= -\frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^4}$$
$$= -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^4}$$
$$= -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}-\frac{\tanh^{-1}(ax)}{4ac^4}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.69

$$\frac{ax+(ax-1)^2\left(-\tanh^{-1}(ax)\right)-2}{4ac^4(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
[Out] (-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^4*(-1 + a*x)^2)
```

fricas [A] time = 0.51, size = 76, normalized size = 1.49

$$\frac{2\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-4}{8\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.14, size = 51, normalized size = 1.00

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^4}+\frac{\log\left(|ax-1|\right)}{8\,ac^4}+\frac{ax-2}{4\,(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
[Out] -1/8*log(abs(a*x + 1))/(a*c^4) + 1/8*log(abs(a*x - 1))/(a*c^4) + 1/4*(a*x -
 2)/((a*x - 1)^2*a*c^4)
```

maple [A] time = 0.04, size = 60, normalized size = 1.18

$$-\frac{1}{4c^4a\,(ax-1)^2}+\frac{1}{4c^4a\,(ax-1)}+\frac{\ln\left(ax-1\right)}{8c^4a}-\frac{\ln\left(ax+1\right)}{8a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^4,x)
```

```
[Out] -1/4/c^4/a/(a*x-1)^2+1/4/c^4/a/(a*x-1)+1/8/c^4/a*ln(a*x-1)-1/8*ln(a*x+1)/a/
c^4
```

maxima [A] time = 0.31, size = 63, normalized size = 1.24

$$\frac{ax-2}{4\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}-\frac{\log\left(ax+1\right)}{8\,ac^4}+\frac{\log\left(ax-1\right)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x - 2)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) - 1/8*log(a*x + 1)/(a*c^4
) + 1/8*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.07, size = 46, normalized size = 0.90

$$\frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}-\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^4*(a*x + 1)),x)
```

```
[Out] (x/4 - 1/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) - atanh(a*x)/(4*a*c^4)
```

sympy [A] time = 0.28, size = 54, normalized size = 1.06

$$\frac{ax-2}{4a^3c^4x^2-8a^2c^4x+4ac^4}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**4,x)
```

```
[Out] (a*x - 2)/(4*a**3*c**4*x**2 - 8*a**2*c**4*x + 4*a*c**4) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**4)
```

3.215 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{8ac^5(1-ax)}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{6ac^5(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8ac^5}$$

```
[Out] -1/6/a/c^5/(-a*x+1)^3-1/8/a/c^5/(-a*x+1)^2-1/8/a/c^5/(-a*x+1)-1/8*arctanh(a
*x)/a/c^5
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6129, 44, 207}

$$-\frac{1}{8ac^5(1-ax)}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{6ac^5(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] -1/(6*a*c^5*(1 - a*x)^3) - 1/(8*a*c^5*(1 - a*x)^2) - 1/(8*a*c^5*(1 - a*x))
- ArcTanh[a*x]/(8*a*c^5)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^5}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)^4(1+ax)}\,dx}{c^5}$$
$$= -\frac{\int \left(\frac{1}{2(-1+ax)^4} - \frac{1}{4(-1+ax)^3} + \frac{1}{8(-1+ax)^2} - \frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{c^5}$$
$$= -\frac{1}{6ac^5(1-ax)^3} - \frac{1}{8ac^5(1-ax)^2} - \frac{1}{8ac^5(1-ax)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{8c^5}$$
$$= -\frac{1}{6ac^5(1-ax)^3} - \frac{1}{8ac^5(1-ax)^2} - \frac{1}{8ac^5(1-ax)} - \frac{\tanh^{-1}(ax)}{8ac^5}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.64

$$\frac{3a^2x^2 - 9ax - 3(ax-1)^3\tanh^{-1}(ax) + 10}{24ac^5(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (10 - 9*a*x + 3*a^2*x^2 - 3*(-1 + a*x)^3*ArcTanh[a*x])/(24*a*c^5*(-1 + a*x)
^3)
```

fricas [A] time = 0.46, size = 113, normalized size = 1.64

$$\frac{6\,a^2x^2 - 18\,ax - 3\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log\left(ax+1\right) + 3\left(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1\right)\log\left(ax-1\right) + 20}{48\left(a^4c^5x^3 - 3\,a^3c^5x^2 + 3\,a^2c^5x - ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^2*x^2 - 18*a*x - 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x + 1)
 + 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x - 1) + 20)/(a^4*c^5*x^3 - 3*
a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

giac [A] time = 0.14, size = 89, normalized size = 1.29

$$-\frac{\log\left(\left|-\frac{2\,c}{acx-c} - 1\right|\right)}{16\,ac^5} + \frac{\frac{3\,a^2c^2}{acx-c} - \frac{3\,a^2c^3}{(acx-c)^2} + \frac{4\,a^2c^4}{(acx-c)^3}}{24\,a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^5) + 1/24*(3*a^2*c^2/(a*c*x - c)
- 3*a^2*c^3/(a*c*x - c)^2 + 4*a^2*c^4/(a*c*x - c)^3)/(a^3*c^6)
```

maple [A] time = 0.04, size = 75, normalized size = 1.09

$$\frac{1}{6c^5a\,(ax-1)^3} - \frac{1}{8c^5a\,(ax-1)^2} + \frac{1}{8c^5a\,(ax-1)} + \frac{\ln\left(ax-1\right)}{16c^5a} - \frac{\ln\left(ax+1\right)}{16c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^5,x)
```

```
[Out] 1/6/c^5/a/(a*x-1)^3-1/8/c^5/a/(a*x-1)^2+1/8/c^5/a/(a*x-1)+1/16/c^5/a*ln(a*x
-1)-1/16/c^5/a*ln(a*x+1)
```

maxima [A] time = 0.31, size = 84, normalized size = 1.22

$$\frac{3\,a^2x^2-9\,ax+10}{24\,\left(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5\right)}-\frac{\log\left(ax+1\right)}{16\,ac^5}+\frac{\log\left(ax-1\right)}{16\,ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*x^2 - 9*a*x + 10)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x -
a*c^5) - 1/16*log(a*x + 1)/(a*c^5) + 1/16*log(a*x - 1)/(a*c^5)
```

mupad [B] time = 1.20, size = 65, normalized size = 0.94

$$-\frac{\frac{a\,x^2}{8}-\frac{3\,x}{8}+\frac{5}{12\,a}}{-a^3\,c^5\,x^3+3\,a^2\,c^5\,x^2-3\,a\,c^5\,x+c^5}-\frac{\mathrm{atanh}\left(a\,x\right)}{8\,a\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^5*(a*x + 1)),x)
```

```
[Out] - ((a*x^2)/8 - (3*x)/8 + 5/(12*a))/(c^5 + 3*a^2*c^5*x^2 - a^3*c^5*x^3 - 3*a
*c^5*x) - atanh(a*x)/(8*a*c^5)
```

sympy [A] time = 0.36, size = 78, normalized size = 1.13

$$-\frac{-3a^2x^2+9ax-10}{24a^4c^5x^3-72a^3c^5x^2+72a^2c^5x-24ac^5}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{16}+\frac{\log\left(x+\frac{1}{a}\right)}{16}}{ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**5,x)
```

```
[Out] -(-3*a**2*x**2 + 9*a*x - 10)/(24*a**4*c**5*x**3 - 72*a**3*c**5*x**2 + 72*a*
*2*c**5*x - 24*a*c**5) - (-log(x - 1/a)/16 + log(x + 1/a)/16)/(a*c**5)
```

3.216 $\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=94

$$\frac{x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}(c-acx)^p\,{}_2F_1\left(-p-\frac{3}{2},-p-1;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(p+1)\sqrt{\frac{1}{ax}+1}}$$

```
[Out] ((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2)*x*(-a*c*x+c)^p*hypergeom([-1-p,
-3/2-p],[-p],2/(a+1/x)/x)/(1+p)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6176, 6181, 132}

$$\frac{x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}(c-acx)^p\,{}_2F_1\left(-p-\frac{3}{2},-p-1;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(p+1)\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2)*x*(c - a*c*x)^p
*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/((a + x^(-1))*x)])/((1 + p)*Sqrt
[1 + 1/(a*x)])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p} x^{-p}(c-acx)^p\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-2-p}\left(1-\frac{x}{a}\right)^{\frac{3}{2}+p}}{\left(1+\frac{x}{a}\right)^{3/2}}dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{3/2} x(c-acx)^p\,{}_2F_1\left(-\frac{3}{2}-p,-1-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] time = 0.07, size = 96, normalized size = 1.02

$$\frac{\sqrt{1-\frac{1}{ax}}(ax+1)\left(\frac{ax-1}{ax+1}\right)^{-p-\frac{1}{2}}(c-acx)^p\,{}_2F_1\left(-p-\frac{3}{2},-p-1;-p;\frac{2}{ax+1}\right)}{a(p+1)\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(1 + a*x)*(c - a*c*x)^
p*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*Sqrt[1 +
 1/(a*x)])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax-1)(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}}{ax+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int(-acx+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c-a\,c\,x)^p \left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.217 $\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=152

$$-\frac{67}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+30c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{315c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+2a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{4}a^3c$$

```
[Out] -315/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+32*c^3*(a-1/x)/a^2/(1-1/a^2/x^2)^
(1/2)+30*c^3*x*(1-1/a^2/x^2)^(1/2)-67/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)+2*a^2
*c^3*x^3*(1-1/a^2/x^2)^(1/2)-1/4*a^3*c^3*x^4*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6175, 6178, 1805, 1807, 807, 266, 63, 208}

$$-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}+2a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{67}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+30c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{315c^3\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 30*c^3*Sqrt[1 - 1/(a^2*
x^2)]*x - (67*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + 2*a^2*c^3*Sqrt[1 - 1/(a^
2*x^2)]*x^3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 - (315*c^3*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^6}{x^5\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \left(a^3c^3\right)\text{Subst}\left(\int \frac{-1+\frac{6x}{a}-\frac{16x^2}{a^2}+\frac{26x^3}{a^3}-\frac{31x^4}{a^4}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{-\frac{24}{a}+\frac{67x}{a^2}-\frac{104x^2}{a^3}+\frac{12}{}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{12}\left(a^3c^3\right)\text{Subst}\left(\int\right.$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 -$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 -$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 -$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 -$$

Mathematica [A] time = 0.25, size = 86, normalized size = 0.57

$$\frac{1}{8}c^3\left(\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-2a^4x^4+14a^3x^3-51a^2x^2+173ax+496\right)}{ax+1}-\frac{315\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*((Sqrt[1 - 1/(a^2*x^2)]*x*(496 + 173*a*x - 51*a^2*x^2 + 14*a^3*x^3 - 2
*a^4*x^4))/(1 + a*x) - (315*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/8
```

fricas [A] time = 0.49, size = 114, normalized size = 0.75

$$-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4-14\,a^3c^3x^3+51\,a^2c^3x^2-173\,ac^3x-496\,c^3\right)\sqrt{}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (2*a^4*c^3*x^4 - 14*a^3*c^3*x^3 + 51*a^2*c^3*x^2 - 173
*a*c^3*x - 496*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 542, normalized size = 3.57

$$\frac{\left(-2\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^3a^3+16\sqrt{a^2}\,\left((ax-1)(ax+1)\right)^{\frac{3}{2}}x^2a^2-4\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-69\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/8*(-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+16*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x^2*a^2-4*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-69*(a^2*x^2-1)^(1
/2)*(a^2)^(1/2)*x^3*a^3+32*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+384*((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a
-138*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+69*ln((a^2*x+(a^2*x^2-1)^(1/2)*(
a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-384*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2))*x^2*a^3-112*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+768*(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-69*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a
+138*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-768*ln((a^
2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+384*((a*x-1)*(a
*x+1))^(1/2)*(a^2)^(1/2)+69*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*a-384*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))
/a*c^3*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.32, size = 244, normalized size = 1.61

$$-\frac{1}{8}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{256\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}-\frac{2\left(325\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-765\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+64\right.}{\frac{4(ax-1)a^2}{ax+1}-\frac{6(ax-1)^2a^2}{(ax+1)^2}+\frac{4(ax-1}{(ax+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 256*c^3*sqrt((a*x - 1)/(a*x + 1))/a^2 - 2*(325
*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 765*c^3*((a*x - 1)/(a*x + 1))^(5/2) + 64
3*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 187*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(
a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/
(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*a
```

mupad [B] time = 0.09, size = 199, normalized size = 1.31

$$\frac{\frac{187\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{643\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{765\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}-\frac{325\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{32\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{315\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((187*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (643*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/4 + (765*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 - (325*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x +
 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) + (32*
c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (315*c^3*atanh(((a*x - 1)/(a*x + 1))^(
1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
-4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(6*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**3*x**3*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x**4*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.218 $\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=129

$$-\frac{5}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{35}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{35c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] -35/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+16*c^2*(a-1/x)/a^2/(1-1/a^2/x^2)^(
1/2)+35/3*c^2*x*(1-1/a^2/x^2)^(1/2)-5/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)+1/3*a
^2*c^2*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6175, 6178, 1805, 1807, 807, 266, 63, 208}

$$\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{5}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{35}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{35c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (35*c^2*Sqrt[1 - 1/(a^2
*x^2)]*x)/3 - (5*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*Sqrt[1 - 1/(
a^2*x^2)]*x^3)/3 - (35*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx=\left(a^2c^2\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2x^2\,dx$$

$$=-\left(\left(a^2c^2\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^4\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\left(a^2c^2\right)\mathrm{Subst}\left(\int\frac{-1+\frac{5x}{a}-\frac{11x^2}{a^2}+\frac{15x^3}{a^3}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{1}{3}\left(a^2c^2\right)\mathrm{Subst}\left(\int\frac{-\frac{15}{a}+\frac{35x}{a^2}-\frac{45x^2}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right.$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{1}{6}\left(a^2c^2\right)\mathrm{Subst}\left(\int\frac{-\frac{70}{a^2}}{x^2\sqrt{}}\right.$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{(3}{}$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{(3}{}$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{1}{2}$$

$$=\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{35}{}$$

Mathematica [A] time = 0.17, size = 78, normalized size = 0.60

$$\frac{1}{6}c^2\left(\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-13a^2x^2+55ax+166\right)}{ax+1}-\frac{105\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*((Sqrt[1 - 1/(a^2*x^2)]*x*(166 + 55*a*x - 13*a^2*x^2 + 2*a^3*x^3))/(1
+ a*x) - (105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/6
```

fricas [A] time = 0.57, size = 104, normalized size = 0.81

$$-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3-13\,a^2c^2x^2+55\,ac^2x+166\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 13*a^2*c^2*x^2 + 55*a*c^2*x + 166*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 474, normalized size = 3.67

$$\frac{\left(2\sqrt{a^2}\ \left((ax-1)(ax+1)\right)^{\frac{3}{2}} x^2a^2 - 15\sqrt{a^2x^2-1}\ \sqrt{a^2}\ x^3a^3 + 4\sqrt{a^2}\ \left((ax-1)(ax+1)\right)^{\frac{3}{2}} xa + 120\sqrt{(ax-1)(ax-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/6*(2*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-15*(a^2*x^2-1)^(1/2)*(a^
2)^(1/2)*x^3*a^3+4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+120*((a*x-1)*(a*
x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-30*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+15
*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-120*ln((a^2*
x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-46*((a*x-1)*(a*
x+1))^(3/2)*(a^2)^(1/2)+240*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-15*(a^2
*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(
a^2)^(1/2))*x*a^2-240*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*x*a^2+120*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+15*ln((a^2*x+(a^2*x^2-
1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-120*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2
)*(a^2)^(1/2))/(a^2)^(1/2)))/a*c^2*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*
x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.31, size = 204, normalized size = 1.58

$$-\frac{1}{6}a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2} - \frac{96\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2} + \frac{2\left(87\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 136\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + \right.}{\frac{3\,(ax-1)a^2}{ax+1} - \frac{3\,(ax-1)^2a^2}{(ax+1)^2} + \frac{(ax-1)^3}{(ax+1)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 96*c^2*sqrt((a*x - 1)/(a*x + 1))/a^2 + 2*(87
*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 136*c^2*((a*x - 1)/(a*x + 1))^(3/2) + 57
*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*
a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2))
```

mupad [B] time = 1.19, size = 163, normalized size = 1.26

$$\frac{19\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{136\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3} + 29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a - \frac{3\,a\,(a\,x-1)}{a\,x+1} + \frac{3\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2} - \frac{a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}} + \frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a} - \frac{35\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (19*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (136*c^2*((a*x - 1)/(a*x + 1))^(3/2))
/3 + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (
3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (16*c^2*((a*x
 - 1)/(a*x + 1))^(1/2))/a - (35*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.219 $\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=92

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+4cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{15c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -15/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+8*c*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+
4*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6175, 6178, 1805, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+4cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{15c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 4*c*Sqrt[1 - 1/(a^2*x^2)]*
x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (15*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]
])/(2*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx=-\left((ac)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$=(ac)\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-(ac)\,\text{Subst}\left(\int\frac{-1+\frac{4x}{a}-\frac{7x^2}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{2}(ac)\,\text{Subst}\left(\int\frac{-\frac{8}{a}+\frac{15x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{(15c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,\right.}{2a}$$
$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{(15c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,\right.}{4a}$$
$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{2}(15ac)\,\text{Subst}\left(\int\frac{1}{a^2-a^2x\right.$$
$$=\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{15c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A] time = 0.15, size = 68, normalized size = 0.74

$$\frac{1}{2}c\left(\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-a^2x^2+7ax+24\right)}{ax+1}-\frac{15\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*((Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 7*a*x - a^2*x^2))/(1 + a*x) - (15*Log[a*
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/2
```

fricas [A] time = 0.53, size = 81, normalized size = 0.88

$$-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-7\,acx-24\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) + (a^2*c*x^2 - 7*a*c*x - 24*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.05, size = 422, normalized size = 4.59

$$-\frac{\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-16\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2+2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+16\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/2*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-16*((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2)*x^2*a^2+2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-ln((a^2*x+(a^2*x^2-
1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+16*ln((a^2*x+((a*x-1)*(a*x+1))^(
1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+8*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2
)-32*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
x*a-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+32*ln((a^
2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-16*((a*x-1)*(a*
x+1))^(1/2)*(a^2)^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2
))*a+16*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*c*
((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.31, size = 156, normalized size = 1.70

$$\frac{1}{2}a\left(\frac{2\left(9c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-7c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{15c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{16c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(9*c*((a*x - 1)/(a*x + 1))^(3/2) - 7*c*sqrt((a*x - 1)/(a*x + 1)))/
(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 15*c*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/a^2 + 16*c*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

mupad [B] time = 0.07, size = 117, normalized size = 1.27

$$\frac{7c\sqrt{\frac{a\,x-1}{a\,x+1}}-9c\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{2a\,(a\,x-1)}{a\,x+1}+\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}-\frac{15\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8c\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (7*c*((a*x - 1)/(a*x + 1))^(1/2) - 9*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2
*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (15*c*atanh(((a*x
- 1)/(a*x + 1))^(1/2)))/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int \left(-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int \frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2*
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x**2*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.220 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=53

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6178, 1805, 266, 63, 208}

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
```

```
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx &= -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 54, normalized size = 1.02

$$
\frac{\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{ax+1}-\frac{\log\left(ax\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{c}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] ((2*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*
x]/a)/c
```

fricas [A] time = 0.67, size = 63, normalized size = 1.19

$$
\frac{2\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (2*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) + log(sqr
t((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.05, size = 248, normalized size = 4.68

$$-\frac{\left(-\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2+\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+((ax-1)(ax+1))^{\frac{3}{2}}\sqrt{a^2}-2\sqrt{(ax-1)(ax+1}\right.}{a\sqrt{a^2}\,c\,(ax-1)\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x)
```

```
[Out] -(-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+ln((a^2*x+((a*x-1)*(a*x+1))^
(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+2*ln((a^2*x+((a*x-1)*(a*x+1))^(1
/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+a*l
n((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x
+1))^(3/2)/(a^2)^(1/2)/c/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)
```

maxima [A] time = 0.30, size = 78, normalized size = 1.47

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c) - 2*sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

mupad [B] time = 1.17, size = 48, normalized size = 0.91

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2
)))/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Integra
l(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

3.221 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=28

$$\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

[Out] (a-1/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6178, 637}

$$\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]

[Out] (a - x^(-1))/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)])

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx &= \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}\\
&= -\frac{\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^2c^2}\\
&= \frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 26, normalized size = 0.93

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(1 + a*x))
```

fricas [A] time = 0.61, size = 22, normalized size = 0.79

$$\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
[Out] undef
```

maple [A] time = 0.04, size = 35, normalized size = 1.25

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x)
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)/a/c^2
```

maxima [A] time = 0.31, size = 22, normalized size = 0.79

$$\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] sqrt((a*x - 1)/(a*x + 1))/(a*c^2)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.79

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^2,x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(a*c^2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x +
 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*
x**2 - a*x + 1), x))/c**2
```

3.222 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=21

$$\frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

[Out] 1/a/c^3/(1-1/a^2/x^2)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6178, 261}

$$\frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]

[Out] 1/(a*c^3*Sqrt[1 - 1/(a^2*x^2)])

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^3c^3}\\
&= \frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 1.57

$$\frac{ax^2\sqrt{1-\frac{1}{a^2x^2}}}{c^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(c^3*(-1 + a^2*x^2))
```

fricas [A] time = 0.45, size = 31, normalized size = 1.48

$$\frac{x\sqrt{\frac{ax-1}{ax+1}}}{ac^3x-c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] x*sqrt((a*x - 1)/(a*x + 1))/(a*c^3*x - c^3)
```

giac [A] time = 0.14, size = 22, normalized size = 1.05

$$\frac{x\mathrm{sgn}\left(ax+1\right)}{\sqrt{a^2x^2-1}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] x*sgn(a*x + 1)/(sqrt(a^2*x^2 - 1)*c^3)
```

maple [A] time = 0.04, size = 33, normalized size = 1.57

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)x}{\left(ax-1\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*x/(a*x-1)^2/c^3
```

maxima [B] time = 0.31, size = 48, normalized size = 2.29

$$\frac{1}{2}a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*(sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + 1/(a^2*c^3*sqrt((a*x - 1)/(a*x
 + 1))))
```

mupad [B] time = 1.17, size = 38, normalized size = 1.81

$$\frac{\frac{a\,x-1}{a\,x+1}+1}{2\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^3,x)
```

```
[Out] ((a*x - 1)/(a*x + 1) + 1)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*x**3 + 2*
a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 -
2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

3.223 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=61

$$\frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}$$

```
[Out] 2/3/a/c^4/(1-1/a^2/x^2)^(1/2)-1/3/a^2/c^4/(a-1/x)/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6175, 6178, 855, 12, 261}

$$\frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4x^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] 2/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) - 1/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a -
 x^(-1))*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 855

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^n*(a + c*x^2)^(p + 1))/(2*a*e*p*(d + e*
x)), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$
$$= -\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2} + \frac{\text{Subst}\left(\int \frac{2x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^3c^4}$$
$$= -\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2} + \frac{2\,\text{Subst}\left(\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^3c^4}$$
$$= \frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.82

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-2ax-1\right)}{3c^4(ax-1)^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^4*(-1 + a*x)^2*(1 +
 a*x))
```

fricas [A] time = 0.41, size = 58, normalized size = 0.95

$$\frac{\left(2\,a^2x^2-2\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^4*x^2 - 2*a^2*
c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^4, x)
```

maple [A] time = 0.04, size = 50, normalized size = 0.82

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(2a^2x^2-2ax-1\right)(ax+1)}{3\left(ax-1\right)^3c^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x)
```

```
[Out] 1/3*((a*x-1)/(a*x+1))^(3/2)*(2*a^2*x^2-2*a*x-1)*(a*x+1)/(a*x-1)^3/c^4/a
```

maxima [A] time = 0.31, size = 65, normalized size = 1.07

$$\frac{1}{12}a\left(\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{\frac{6(ax-1)}{ax+1}-1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4) + (6*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

mupad [B] time = 1.23, size = 50, normalized size = 0.82

$$\frac{-2\,a^2\,x^2+2\,a\,x+1}{\left(3\,a\,c^4-3\,a^3\,c^4\,x^2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^4,x)
```

```
[Out] (2*a*x - 2*a^2*x^2 + 1)/((3*a*c^4 - 3*a^3*c^4*x^2)*((a*x - 1)/(a*x + 1))^(1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a
**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a*x
+ 1), x))/c**4
```

3.224 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5} dx$

Optimal. Leaf size=94

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] -4/5*(a+1/x)/a^2/c^5/(1-1/a^2/x^2)^(3/2)+1/5*(a+1/x)^2/a^3/c^5/(1-1/a^2/x^2
)^(5/2)+1/5*(5*a+2/x)/a^2/c^5/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6175, 6178, 852, 1635, 637}

$$\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) + (a + x^(-1))^2/(5*a
^3*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (5*a + 2/x)/(5*a^2*c^5*Sqrt[1 - 1/(a^2*x^
2)])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx &= -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}\\
&= \frac{\text{Subst}\left(\int \frac{x^3}{\left(1-\frac{x}{a}\right)^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}\\
&= \frac{\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}\\
&= \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)\left(2a^3+5a^2x+5ax^2\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5a^5c^5}\\
&= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{6a^3+15a^2x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15a^5c^5}\\
&= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.61

$$
\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^3x^3-4a^2x^2+ax+2\right)}{5c^5(ax-1)^3(ax+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3))/(5*c^5*(-1 + a*
x)^3*(1 + a*x))
```

fricas [A] time = 0.50, size = 77, normalized size = 0.82

$$
\frac{\left(2\,a^3x^3-4\,a^2x^2+ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^5*x^
3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^5, x)
```

maple [A] time = 0.04, size = 57, normalized size = 0.61

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(2x^3a^3-4a^2x^2+ax+2\right)(ax+1)}{5\,(ax-1)^4\,c^5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x)
```

```
[Out] 1/5*((a*x-1)/(a*x+1))^(3/2)*(2*a^3*x^3-4*a^2*x^2+a*x+2)*(a*x+1)/(a*x-1)^4/c
^5/a
```

maxima [A] time = 0.31, size = 82, normalized size = 0.87

$$\frac{1}{40}\,a\left(\frac{5\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5}-\frac{\frac{5\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-1}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/40*a*(5*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^5) - (5*(a*x - 1)/(a*x + 1) - 15
*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(5/2)))
```

mupad [B] time = 1.21, size = 51, normalized size = 0.54

$$\frac{2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2}{5\,a\,c^5\,{\left(a\,x+1\right)}^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^5,x)
```

```
[Out] (a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2)/(5*a*c^5*(a*x + 1)^3*((a*x - 1)/(a*x + 1)
)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*
a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*a*x
 - 1), x))/c**5
```

3.225 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$

Optimal. Leaf size=125

$$-\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] -46/35*(a+1/x)/a^2/c^6/(1-1/a^2/x^2)^(3/2)+24/35*(a+1/x)^2/a^3/c^6/(1-1/a^2
/x^2)^(5/2)-1/7*(a+1/x)^3/a^4/c^6/(1-1/a^2/x^2)^(7/2)+1/35*(35*a+13/x)/a^2/
c^6/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6175, 6178, 852, 1635, 637}

$$-\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (-46*(a + x^(-1)))/(35*a^2*c^6*(1 - 1/(a^2*x^2))^(3/2)) + (24*(a + x^(-1))^
2)/(35*a^3*c^6*(1 - 1/(a^2*x^2))^(5/2)) - (a + x^(-1))^3/(7*a^4*c^6*(1 - 1/
(a^2*x^2))^(7/2)) + (35*a + 13/x)/(35*a^2*c^6*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^6 x^6}\,dx}{a^6c^6}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-\frac{x}{a}\right)^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^6c^6}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a^6c^6}$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2\left(3a^4+7a^3x+7a^2x^2+7ax^3\right)}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7a^6c^6}$$

$$= \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)\left(33a^4+70a^3x+35a^2x^2\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35a^6c^6}$$

$$= -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{39a^4+105a^3x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,\right.}{105a^6c^6}$$

$$= -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.08, size = 66, normalized size = 0.53

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13\right)}{35c^6(ax-1)^4(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4
))/(35*c^6*(-1 + a*x)^4*(1 + a*x))
```

fricas [A] time = 0.62, size = 96, normalized size = 0.77

$$\frac{\left(8\,a^{4}x^{4}-24\,a^{3}x^{3}+20\,a^{2}x^{2}+4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^{5}c^{6}x^{4}-4\,a^{4}c^{6}x^{3}+6\,a^{3}c^{6}x^{2}-4\,a^{2}c^{6}x+ac^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^6*x^4 - 4*a^4*c^6*x^3 + 6*a^3*c^6*x^2 - 4*a^2*c^6*x + a*c^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(acx-c)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^6, x)
```

maple [A] time = 0.04, size = 66, normalized size = 0.53

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(8x^{4}a^{4}-24x^{3}a^{3}+20a^{2}x^{2}+4ax-13\right)(ax+1)}{35\,(ax-1)^{5}\,c^{6}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x)
```

```
[Out] 1/35*((a*x-1)/(a*x+1))^(3/2)*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)*(a*
x+1)/(a*x-1)^5/c^6/a
```

maxima [A] time = 0.31, size = 97, normalized size = 0.78

$$\frac{1}{560}\,a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{6}}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^{2}}{(ax+1)^{2}}+\frac{140\,(ax-1)^{3}}{(ax+1)^{3}}-5}{a^{2}c^{6}\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^6) + (28*(a*x - 1)/(a*x + 1) -
 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^6*((a
*x - 1)/(a*x + 1))^(7/2)))
```

mupad [B] time = 0.07, size = 60, normalized size = 0.48

$$\frac{8\,a^{4}\,x^{4}-24\,a^{3}\,x^{3}+20\,a^{2}\,x^{2}+4\,a\,x-13}{35\,a\,c^{6}\,(a\,x+1)^{4}\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^6,x)
```

```
[Out] (4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^6*(a*x + 1)^4*((
a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\,dx}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**6,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*a
**5*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x) + Integ
ral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*a**5
*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x))/c**6
```

3.226 $\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=254

$$\frac{9088\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{3465a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{32\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^4\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{768\left(\frac{1}{ax}\right.}{385\ldots}$$

```
[Out] -32/99*(a-1/x)^3*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)+9088/
3465*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)/x^3-768/385*(1+1/
a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+128/231*(1+1/a/x)^(3/2)
*(-a*c*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^4*(1+1/a/x)^(3/2)*x*(-
a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)
```

Rubi [A] time = 0.22, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{768\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{385a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9088\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{3465a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{32\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^4\left(\ldots\right.}{11a^4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (-32*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1/(
a*x))^(9/2)) + (9088*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(3465*a^4*(1 -
1/(a*x))^(9/2)*x^3) - (768*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(385*a^3*
(1 - 1/(a*x))^(9/2)*x^2) + (128*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(231
*a^2*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^4*(1 + 1/(a*x))^(3/2)*x*(c -
a*c*x)^(9/2))/(11*a^4*(1 - 1/(a*x))^(9/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{(c-acx)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}{x^{13/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(16\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^{11/2}}\cdots\right.}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(64\cdots\right.}{\cdots}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^4\cdots}{11\cdots}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{128\left(1+\frac{1}{ax}\cdots\right.}{231a^2\cdots}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9088\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{3465a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3}-\frac{768\left(1+\cdots\right.}{385a^3\cdots}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.33

$$\frac{2c^4\sqrt{\frac{1}{ax}+1}\,(ax+1)\left(315a^4x^4-1820a^3x^3+4530a^2x^2-6396ax+5419\right)\sqrt{c-acx}}{3465a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(5419 - 6396*a*x + 4530*
a^2*x^2 - 1820*a^3*x^3 + 315*a^4*x^4))/(3465*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.58, size = 105, normalized size = 0.41

$$\frac{2\left(315\,a^6c^4x^6-1190\,a^5c^4x^5+1205\,a^4c^4x^4+844\,a^3c^4x^3-2843\,a^2c^4x^2+4442\,ac^4x+5419\,c^4\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-\cdots}{ax+\cdots}}}{3465\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

[Out] 2/3465*(315*a^6*c^4*x^6 - 1190*a^5*c^4*x^5 + 1205*a^4*c^4*x^4 + 844*a^3*c^4
*x^3 - 2843*a^2*c^4*x^2 + 4442*a*c^4*x + 5419*c^4)*sqrt(-a*c*x + c)*sqrt((a
*x - 1)/(a*x + 1))/(a^2*x - a)

giac [A] time = 0.21, size = 147, normalized size = 0.58

$$\frac{2\left(\frac{4096\sqrt{2}\sqrt{-c}c^4}{\mathrm{sgn}(c)} + \frac{315(acx+c)^5\sqrt{-acx-c}-3080(acx+c)^4\sqrt{-acx-c}\,c+11880(acx+c)^3\sqrt{-acx-c}\,c^2-22176(acx+c)^2\sqrt{-acx-c}\,c^3-18480(-acx-c)^{\frac{3}{2}}c^4}{c\,\mathrm{sgn}(-acx-c)}\right)}{3465a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")

[Out] 2/3465*(4096*sqrt(2)*sqrt(-c)*c^4/sgn(c) + (315*(a*c*x + c)^5*sqrt(-a*c*x -
 c) - 3080*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 11880*(a*c*x + c)^3*sqrt(-a*c
*x - c)*c^2 - 22176*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3 - 18480*(-a*c*x - c)
^(3/2)*c^4)/(c*sgn(-a*c*x - c)))/a

maple [A] time = 0.04, size = 72, normalized size = 0.28

$$\frac{2(ax+1)\left(315x^4a^4-1820x^3a^3+4530a^2x^2-6396ax+5419\right)(-acx+c)^{\frac{9}{2}}}{3465a(ax-1)^4\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x)

[Out] 2/3465*(a*x+1)*(315*a^4*x^4-1820*a^3*x^3+4530*a^2*x^2-6396*a*x+5419)*(-a*c*
x+c)^(9/2)/a/(a*x-1)^4/((a*x-1)/(a*x+1))^(1/2)

maxima [A] time = 0.33, size = 99, normalized size = 0.39

$$\frac{2\left(315a^5\sqrt{-c}c^4x^5-1505a^4\sqrt{-c}c^4x^4+2710a^3\sqrt{-c}c^4x^3-1866a^2\sqrt{-c}c^4x^2-977a\sqrt{-c}c^4x+5419\sqrt{-c}c^4\right)\sqrt{ax}}{3465a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima")

[Out] 2/3465*(315*a^5*sqrt(-c)*c^4*x^5 - 1505*a^4*sqrt(-c)*c^4*x^4 + 2710*a^3*sqr
t(-c)*c^4*x^3 - 1866*a^2*sqrt(-c)*c^4*x^2 - 977*a*sqrt(-c)*c^4*x + 5419*sqr
t(-c)*c^4)*sqrt(a*x + 1)/a

mupad [B] time = 1.46, size = 76, normalized size = 0.30

$$\frac{2c^4\sqrt{c-a\,c\,x}\,(a\,x+1)^2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(315\,a^4\,x^4-1820\,a^3\,x^3+4530\,a^2\,x^2-6396\,a\,x+5419\right)}{3465\,a\,(a\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] (2*c^4*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(4530*a^2*
x^2 - 6396*a*x - 1820*a^3*x^3 + 315*a^4*x^4 + 5419))/(3465*a*(a*x - 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(9/2),x)

[Out] Timed out

3.227 $\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=197

$$-\frac{568\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{315a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{48\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{35a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -8/21*(1+1/a/x)^(3/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)-568/315*(1+1/a/x)^
(3/2)*(-a*c*x+c)^(7/2)/a^3/(1-1/a/x)^(7/2)/x^2+48/35*(1+1/a/x)^(3/2)*(-a*c*
x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(a-1/x)^3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)
^(7/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A] time = 0.19, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{568\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{315a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{48\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{35a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (-8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(21*a*(1 - 1/(a*x))^(7/2)) - (56
8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(315*a^3*(1 - 1/(a*x))^(7/2)*x^2)
+ (48*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(35*a^2*(1 - 1/(a*x))^(7/2)*x)
 + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(7/2))/(9*a^3*(1 - 1
/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{(c-acx)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx\right.}{3a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(8\left(\frac{1}{x}\right)^{7/2}(c-acx\right.}{}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-\right.}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{568\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{315a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2} + \frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.38

$$-\frac{2c^3\sqrt{\frac{1}{ax}+1}(ax+1)\left(35a^3x^3-165a^2x^2+321ax-319\right)\sqrt{c-acx}}{315a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(-319 + 321*a*x - 165*a
^2*x^2 + 35*a^3*x^3))/(315*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.39, size = 94, normalized size = 0.48

$$-\frac{2\left(35\,a^5c^3x^5-95\,a^4c^3x^4+26\,a^3c^3x^3+158\,a^2c^3x^2-317\,ac^3x-319\,c^3\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 95*a^4*c^3*x^4 + 26*a^3*c^3*x^3 + 158*a^2*c^3*x^2
- 317*a*c^3*x - 319*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
- a)
```

giac [A] time = 0.17, size = 124, normalized size = 0.63

$$\frac{2\left(\frac{256\sqrt{2}\sqrt{-c}\,c^3}{\mathrm{sgn}(c)}-\frac{35\,(acx+c)^4\sqrt{-acx-c}-270\,(acx+c)^3\sqrt{-acx-c}\,c+756\,(acx+c)^2\sqrt{-acx-c}\,c^2+840\,(-acx-c)^{\frac{3}{2}}c^3}{c\,\mathrm{sgn}(-acx-c)}\right)}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/315*(256*sqrt(2)*sqrt(-c)*c^3/sgn(c) - (35*(a*c*x + c)^4*sqrt(-a*c*x - c)
 - 270*(a*c*x + c)^3*sqrt(-a*c*x - c)*c + 756*(a*c*x + c)^2*sqrt(-a*c*x - c
)*c^2 + 840*(-a*c*x - c)^(3/2)*c^3)/(c*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.04, size = 64, normalized size = 0.32

$$\frac{2\,(ax+1)\left(35x^3a^3-165a^2x^2+321ax-319\right)(-acx+c)^{\frac{7}{2}}}{315a\,(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/315*(a*x+1)*(35*a^3*x^3-165*a^2*x^2+321*a*x-319)*(-a*c*x+c)^(7/2)/a/(a*x-
1)^3/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.33, size = 83, normalized size = 0.42

$$-\frac{2\left(35\,a^4\sqrt{-c}\,c^3x^4-130\,a^3\sqrt{-c}\,c^3x^3+156\,a^2\sqrt{-c}\,c^3x^2+2\,a\sqrt{-c}\,c^3x-319\,\sqrt{-c}\,c^3\right)\sqrt{ax+1}}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 130*a^3*sqrt(-c)*c^3*x^3 + 156*a^2*sqrt(-
c)*c^3*x^2 + 2*a*sqrt(-c)*c^3*x - 319*sqrt(-c)*c^3)*sqrt(a*x + 1)/a
```

mupad [B] time = 1.43, size = 102, normalized size = 0.52

$$\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-35\,a^4\,x^4+60\,a^3\,x^3+34\,a^2\,x^2-124\,a\,x+193\right)}{315\,a}+\frac{1024\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(34*a^2*x^2 - 124*a*x
+ 60*a^3*x^3 - 35*a^4*x^4 + 193))/(315*a) + (1024*c^3*(c - a*c*x)^(1/2)*((a
*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.228 $\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=115

$$\frac{64a^2c^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105(c-acx)^{3/2}}+\frac{16a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-acx}}+\frac{2}{7}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\sqrt{c-acx}$$

```
[Out] 64/105*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+16/35*a^2*c^3*(1-1/
a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)+2/7*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3*(-a
*c*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6176, 6181, 89, 78, 37}

$$\frac{142\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{105a^2x\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{36\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (-36*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (1
42*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(5/2))/(105*a^2*(1 - 1/(a*x))^(5/2)*x) +
 (2*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{9}{a}+\frac{7x}{2a^2}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{36\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(71\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Su}}{35a^2\left(1-\right.}$$

$$= -\frac{36\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{142\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{105a^2\left(1-\frac{1}{ax}\right)^{5/2}x} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.58

$$\frac{2c^2\sqrt{\frac{1}{ax}+1}\,(ax+1)\left(15a^2x^2-54ax+71\right)\sqrt{c-acx}}{105a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(71 - 54*a*x + 15*a^2*x^
2))/(105*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.46, size = 83, normalized size = 0.72

$$\frac{2\left(15\,a^4c^2x^4-24\,a^3c^2x^3-22\,a^2c^2x^2+88\,ac^2x+71\,c^2\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^4*c^2*x^4 - 24*a^3*c^2*x^3 - 22*a^2*c^2*x^2 + 88*a*c^2*x + 71*c
^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

giac [A] time = 0.16, size = 99, normalized size = 0.86

$$\frac{2\left(\frac{64\sqrt{2}\sqrt{-c}c^2}{\mathrm{sgn}(c)} + \frac{15(acx+c)^3\sqrt{-acx-c} - 84(acx+c)^2\sqrt{-acx-c}\,c - 140(-acx-c)^{\frac{3}{2}}c^2}{\mathrm{csgn}(-acx-c)}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/105*(64*sqrt(2)*sqrt(-c)*c^2/sgn(c) + (15*(a*c*x + c)^3*sqrt(-a*c*x - c)
- 84*(a*c*x + c)^2*sqrt(-a*c*x - c)*c - 140*(-a*c*x - c)^(3/2)*c^2)/(c*sgn(
-a*c*x - c)))/a
```

maple [A] time = 0.04, size = 56, normalized size = 0.49

$$\frac{2(ax+1)\left(15a^2x^2 - 54ax + 71\right)(-acx+c)^{\frac{5}{2}}}{105a(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/105*(a*x+1)*(15*a^2*x^2-54*a*x+71)*(-a*c*x+c)^(5/2)/a/(a*x-1)^2/((a*x-1)/
(a*x+1))^(1/2)
```

maxima [A] time = 0.33, size = 67, normalized size = 0.58

$$\frac{2\left(15\,a^3\sqrt{-c}\,c^2x^3 - 39\,a^2\sqrt{-c}\,c^2x^2 + 17\,a\sqrt{-c}\,c^2x + 71\,\sqrt{-c}\,c^2\right)\sqrt{ax+1}}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*a^3*sqrt(-c)*c^2*x^3 - 39*a^2*sqrt(-c)*c^2*x^2 + 17*a*sqrt(-c)*c^
2*x + 71*sqrt(-c)*c^2)*sqrt(a*x + 1)/a
```

mupad [B] time = 1.41, size = 60, normalized size = 0.52

$$\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^2\,x^2-54\,a\,x+71\right)}{105\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x^
2 - 54*a*x + 71))/(105*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.229 $\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=77

$$\frac{8a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15(c-acx)^{3/2}}+\frac{2a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-acx}}$$

```
[Out] 8/15*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+2/5*a^2*c^2*(1-1/a^2/
x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 89, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-14*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(3/2))/(15*a*(1 - 1/(a*x))^(3/2)) + (2
*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{(c-acx)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\left(7\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}dx,x,\frac{1}{x}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=-\frac{14\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.74

$$-\frac{2c\sqrt{\frac{1}{ax}+1}\,(ax+1)(3ax-7)\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a*Sqrt[
1 - 1/(a*x)])
```

fricas [A] time = 0.53, size = 64, normalized size = 0.83

$$-\frac{2\left(3\,a^3cx^3-a^2cx^2-11\,acx-7\,c\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 - a^2*c*x^2 - 11*a*c*x - 7*c)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1))/(a^2*x - a)
```

giac [A] time = 0.14, size = 74, normalized size = 0.96

$$\frac{2\left(\frac{8\sqrt{2}\sqrt{-c}\,c}{\text{sgn}(c)}-\frac{3\,(acx+c)^2\sqrt{-acx-c}+10\,(-acx-c)^{\frac{3}{2}}c}{c\text{sgn}(-acx-c)}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(8*sqrt(2)*sqrt(-c)*c/sgn(c) - (3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 10*
(-a*c*x - c)^(3/2)*c)/(c*sgn(-a*c*x - c)))/a
```

maple [**A**] time = 0.04, size = 48, normalized size = 0.62

$$\frac{2\left(ax+1\right)\left(3ax-7\right)\left(-acx+c\right)^{\frac{3}{2}}}{15a\left(ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/15*(a*x+1)*(3*a*x-7)*(-a*c*x+c)^(3/2)/a/(a*x-1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [**A**] time = 0.33, size = 45, normalized size = 0.58

$$-\frac{2\left(3\,a^{2}\sqrt{-c}\,cx^{2}-4\,a\sqrt{-c}\,cx-7\,\sqrt{-c}\,c\right)\sqrt{ax+1}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*a^2*sqrt(-c)*c*x^2 - 4*a*sqrt(-c)*c*x - 7*sqrt(-c)*c)*sqrt(a*x + 1
)/a
```

mupad [**B**] time = 1.37, size = 50, normalized size = 0.65

$$-\frac{2\,c\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\left(3\,a\,x-7\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] -(2*c*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 7)*((a*x - 1)/(a*x + 1))^(1/2)
)/(15*a*(a*x - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.230 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=29

$$\frac{2(ax+1)\sqrt{c-acx}\,e^{\coth^{-1}(ax)}}{3a}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6174}

$$\frac{2(ax+1)\sqrt{c-acx}\,e^{\coth^{-1}(ax)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.48

$$\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.63, size = 50, normalized size = 1.72

$$\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

giac [A] time = 0.16, size = 48, normalized size = 1.66

$$\frac{2\left(\frac{2\sqrt{2}\sqrt{-c}}{\operatorname{sgn}(c)} - \frac{(-acx-c)^{\frac{3}{2}}}{c\operatorname{sgn}(-acx-c)}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(2*sqrt(2)*sqrt(-c)/sgn(c) - (-a*c*x - c)^(3/2)/(c*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.03, size = 35, normalized size = 1.21

$$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x)
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

maxima [A] time = 0.33, size = 26, normalized size = 0.90

$$\frac{2\left(a\sqrt{-c}\,x + \sqrt{-c}\right)\sqrt{ax+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

mupad [B] time = 1.31, size = 43, normalized size = 1.48

$$\frac{2\sqrt{c-a\,c\,x}\,(a\,x+1)^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,(a\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.231 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=118

$$\frac{2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-acx}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[Out] 2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)-2*arctanh(2^(1/2)*(1/x
)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a^(1/2)/(1/x)^(1/2
)/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-acx}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - a*c*x] - (2*Sqrt[2]*Sqrt
[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\ \text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-acx}} - \frac{\left(2\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-acx}} - \frac{\left(4\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-acx}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.84

$$\frac{2x\sqrt{1-\frac{1}{ax}}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}-\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{\sqrt{a}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*x*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[2]*Sqrt[x^(-1)]*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[c
 - a*c*x])
```

fricas [A] time = 0.52, size = 239, normalized size = 2.03

$$\left[\frac{\sqrt{2}\,(acx-c)\sqrt{-\frac{1}{c}}\,\log\left(-\frac{a^2x^2-2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)-2\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac},\ -\frac{2\left(\sqrt{-acx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt(2)*sqrt(-a*c*x + c)
*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a
*x + 1)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x
 - a*c), -2*(sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - sqrt(2)
*(a*c*x - c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*
x - 1)*sqrt(c)))/sqrt(c))/(a^2*c*x - a*c)]
```

giac [C] time = 0.15, size = 91, normalized size = 0.77

$$\frac{2\left(\frac{\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-\sqrt{-acx-c}}{\operatorname{sgn}(-acx-c)}-\frac{-i\,\sqrt{2}\,\sqrt{-c}\,\arctan(-i)+\sqrt{2}\,\sqrt{-c}}{\operatorname{sgn}(c)}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*((sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - sqrt(-a*
c*x - c))/sgn(-a*c*x - c) - (-I*sqrt(2)*sqrt(-c)*arctan(-I) + sqrt(2)*sqrt(
-c))/sgn(c))/(a*c)
```

maple [A] time = 0.06, size = 82, normalized size = 0.69

$$-\frac{2\sqrt{-c\,(ax-1)}\,\left(\sqrt{-c\,(ax+1)}-\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax+1)}\,ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] -2*(-c*(a*x-1))^(1/2)*((-c*(a*x+1))^(1/2)-c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2)/c
/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))), x)
```

3.232 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -1/2*(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a
^(1/2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)-a*(1-1/a/x)^(3/2)*x*(1+1/a/x)^(
1/2)/(a-1/x)/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - a*c*x)^(3/
2))) - (Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]
*Sqrt[1 + 1/(a*x)])])/(Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Mathematica [A] time = 0.12, size = 116, normalized size = 0.91

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{ax}+1}+\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{2\sqrt{a}\,c(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(-
1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*S
qrt[a]*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

fricas [A] time = 0.47, size = 281, normalized size = 2.20

$$\left[-\frac{\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}, -\frac{\sqrt{2}}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/2*(sqrt(2)*(a^2*x^2 - 2*a*
x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))/(a*c*x - c)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

giac [A] time = 0.18, size = 71, normalized size = 0.55

$$-\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{-acx-c}}{acx-c}}{2\,ac\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + 2*sqrt
(-a*c*x - c)/(a*c*x - c))/(a*c*sgn(-a*c*x - c))
```

maple [A] time = 0.06, size = 118, normalized size = 0.92

$$-\frac{\sqrt{-c\left(ax-1\right)}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c+2\sqrt{-c\left(ax+1\right)}\,\sqrt{c}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\sqrt{-c\left(ax+1\right)}\,c^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -1/2/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a*x-1))^(1/2)*(2^(1/2)*arctan(1/2
*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2^(1/2)*arctan(1/2*(-c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/c
^(5/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-acx+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a\,c\,x\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**(3/2)), x)
```

3.233 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

```
[Out] -1/4*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)+1/1
6*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/
2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+1/8*a^2*(1-1/a/x)^(5/2)*x^2*(1+1/a
/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A] time = 0.20, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(8*(a - x^(-1))*(c - a*c*x)
^(5/2)) - (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(4*(a - x^(-1))
^2*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[
x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(8*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x
)^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2} \operatorname{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2} (c-acx)^{5/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{\ldots}}\ldots\right)}{16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, \ldots\right)}{8\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] time = 0.16, size = 123, normalized size = 0.64

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-2\sqrt{a}\sqrt{\frac{1}{ax}+1}\,(ax+3)\right)}{16\sqrt{a}\,c^2(ax-1)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(3 + a*x) + Sqrt[2]*Sqrt
[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])]))/(16*Sqrt[a]*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

fricas [A] time = 0.55, size = 337, normalized size = 1.75

$$\left[-\frac{\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4\left(a^2x^2+4ax+3\right)\sqrt{-c}}{32\left(a^4c^3x^3-3a^3c^3x^2+3a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2
+ 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3), -1/16*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan
(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) -
2*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^
3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

giac [A] time = 0.18, size = 89, normalized size = 0.46

$$-\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left((-acx-c)^{\frac{3}{2}}-2\sqrt{-acx-c}\,c\right)}{(acx-c)^2c}}{16\,ac\,\mathrm{sgn}(-acx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) + 2*((-
a*c*x - c)^(3/2) - 2*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c))/(a*c*sgn(-a*c*x
 - c))
```

maple [A] time = 0.06, size = 167, normalized size = 0.87

$$\frac{\sqrt{-c(ax-1)}\left(-\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c+2\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)xac+2xa\sqrt{-c(ax+1)}\sqrt{c}-\sqrt{}\right.}{16\sqrt{\frac{ax-1}{ax+1}}(ax-1)^2c^{\frac{7}{2}}\sqrt{-c(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] 1/16*(-c*(a*x-1))^(1/2)*(-2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(
1/2))*x^2*a^2*c+2*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*
a*c+2*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)-2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*
2^(1/2)/c^(1/2))*c+6*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a
*x-1)^2/c^(7/2)/(-c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-acx+c)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a\,c\,x\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
[Out] Timed out
```

3.234 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}+\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\cdots}{32\left(a-\frac{1}{x}\right)(c\cdots}$$

```
[Out] -1/6*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(7/2)+1/1
6*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)-1/64*a
^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))
/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/32*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/a/x
)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A] time = 0.22, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\cdots\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] -(a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^2)/(6*(a - x^(-1))^3*(c - a
*c*x)^(7/2)) - (a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(32*(a - x^(
-1))*(c - a*c*x)^(7/2)) + (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^3)
/(16*(a - x^(-1))^2*(c - a*c*x)^(7/2)) - (a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTa
nh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(32*Sqrt[2]*(x^(-1)
)^(7/2)*(c - a*c*x)^(7/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{1\cdots}}{\sqrt{x}\,(1\cdots}\right.}{32\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7\cdots}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{(a\cdots}{\cdots}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{(a\cdots}{\cdots}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{a^{5\cdots}}{\cdots}$$

Mathematica [A] time = 0.17, size = 139, normalized size = 0.56

$$\frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(-3a^2x^2+10ax+25\right)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}(ax-1)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{192\sqrt{a}\,c^3\sqrt{\frac{1}{x}}(ax-1)^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(25 + 10*a*x - 3*a^2*x^2))
/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])]))/(192*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 + a*x)^3*Sqrt[c
 - a*c*x])
```

fricas [A] time = 1.15, size = 393, normalized size = 1.57

$$\left[-\frac{3\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4\left(3a^3x^3-7\right.}{384\left(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2-4a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*l
og(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 - 7*a^
2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x
^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/192*(3*sqrt(2
)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt
(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^3*x^3
- 7*a^2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5
*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.20, size = 116, normalized size = 0.46

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{2\left(3(acx+c)^2\sqrt{-acx-c}+16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}\,c^2\right)}{(acx-c)^3c^2}}{192\,ac\,\mathrm{sgn}(-acx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/192*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*
(3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*
x - c)*c^2)/((a*c*x - c)^3*c^2))/(a*c*sgn(-a*c*x - c))
```

maple [A] time = 0.06, size = 219, normalized size = 0.88

$$\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^3a^3c+9\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c+6x^2a^2\sqrt{-c(ax+1)}\sqrt{\right.}{192\sqrt{\frac{ax-1}{ax+1}}\,(ax-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] 1/192*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*x^3*a^3*c+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
*x^2*a^2*c+6*x^2*a^2*(-c*(a*x+1))^(1/2)*c^(1/2)-9*2^(1/2)*arctan(1/2*(-c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-20*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)+3*2^(
1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-50*(-c*(a*x+1))^(1/2)
*c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^3/c^(9/2)/(-c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((-a*c*x + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c-a\,c\,x)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
[Out] Timed out
```

3.235 $\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

[Out] 4/7*(-a*c*x+c)^(7/2)/a-2/9*(-a*c*x+c)^(9/2)/a/c

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]

[Out] (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx\\
&= -\int \frac{(1+ax)(c-acx)^{7/2}}{1-ax}\,dx\\
&= -\left(c\int (1+ax)(c-acx)^{5/2}\,dx\right)\\
&= -\left(c\int \left(2(c-acx)^{5/2}-\frac{(c-acx)^{7/2}}{c}\right)dx\right)\\
&= \frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}
\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 34, normalized size = 0.85

$$-\frac{2c^3(ax-1)^3(7ax+11)\sqrt{c-acx}}{63a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*(-1 + a*x)^3*(11 + 7*a*x)*Sqrt[c - a*c*x])/(63*a)
```

fricas [**A**] time = 0.46, size = 60, normalized size = 1.50

$$-\frac{2\left(7\,a^4c^3x^4-10\,a^3c^3x^3-12\,a^2c^3x^2+26\,ac^3x-11\,c^3\right)\sqrt{-acx+c}}{63\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 12*a^2*c^3*x^2 + 26*a*c^3*x - 11*c^
3)*sqrt(-a*c*x + c)/a
```

giac [**B**] time = 0.16, size = 205, normalized size = 5.12

$$\frac{2\left(90\,(acx-c)^3\sqrt{-acx+c}+378\,(acx-c)^2\sqrt{-acx+c}\,c-630\,(-acx+c)^{\frac{3}{2}}c^2+945\,\sqrt{-acx+c}\,c^3+210\left((-acx+c\right.\right.}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/315*(90*(a*c*x - c)^3*sqrt(-a*c*x + c) + 378*(a*c*x - c)^2*sqrt(-a*c*x +
c)*c - 630*(-a*c*x + c)^(3/2)*c^2 + 945*sqrt(-a*c*x + c)*c^3 + 210*((-a*c*x
 + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c^2 - (35*(a*c*x - c)^4*sqrt(-a*c*x + c
) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 378*(a*c*x - c)^2*sqrt(-a*c*x +
c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sqrt(-a*c*x + c)*c^4)/c)/a
```

maple [**A**] time = 0.04, size = 21, normalized size = 0.52

$$\frac{2\,(-acx+c)^{\frac{7}{2}}\,(7ax+11)}{63a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/63*(-a*c*x+c)^(7/2)*(7*a*x+11)/a
```

maxima [**A**] time = 0.31, size = 32, normalized size = 0.80

$$-\frac{2\left(7\,(-acx+c)^{\frac{9}{2}}-18\,(-acx+c)^{\frac{7}{2}}c\right)}{63\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/63*(7*(-a*c*x + c)^(9/2) - 18*(-a*c*x + c)^(7/2)*c)/(a*c)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.80

$$\frac{4\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(7/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

sympy [A] time = 11.84, size = 170, normalized size = 4.25

$$\begin{cases} \frac{-c^{3}\left(\begin{cases} 0 & \text{for}\: c = 0 \\ - \frac{2 \left(- a c x + c\right)^{\frac{3}{2}}}{3 c} & \text{otherwise} \end{cases}\right) - \frac{2 c^{2} \left(- a c x + c\right)^{\frac{3}{2}}}{3} + 2 c \left(- \frac{c \left(- a c x + c\right)^{\frac{3}{2}}}{3} + \frac{\left(- a c x + c\right)^{\frac{5}{2}}}{5}\right) + \frac{4 c \left(- a c x + c\right)^{\frac{5}{2}}}{5} - \frac{2 \left(- a c x + c\right)^{\frac{7}{2}}}{7} - \frac{2 \left(- \frac{c^{3} \left(- a c x + c\right)^{\frac{3}{2}}}{3} + \frac{3 c^{2} \left(- a c x + c\right)^{\frac{5}{2}}}{5} - 3 c(\cdots}{c}}{a} \\ - c^{\frac{7}{2}} x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Piecewise(((-c**3*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), T
rue)) - 2*c**2*(-a*c*x + c)**(3/2)/3 + 2*c*(-c*(-a*c*x + c)**(3/2)/3 + (-a*
c*x + c)**(5/2)/5) + 4*c*(-a*c*x + c)**(5/2)/5 - 2*(-a*c*x + c)**(7/2)/7 -
2*(-c**3*(-a*c*x + c)**(3/2)/3 + 3*c**2*(-a*c*x + c)**(5/2)/5 - 3*c*(-a*c*x
 + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/c)/a, Ne(a, 0)), (-c**(7/2)*x, True
))
```

3.236 $\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

```
[Out] 4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c
```

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx\\
&= -\int \frac{(1+ax)(c-acx)^{5/2}}{1-ax}\,dx\\
&= -\left(c\int (1+ax)(c-acx)^{3/2}\,dx\right)\\
&= -\left(c\int \left(2(c-acx)^{3/2}-\frac{(c-acx)^{5/2}}{c}\right)dx\right)\\
&= \frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.85

$$\frac{2c^2(ax-1)^2(5ax+9)\sqrt{c-acx}}{35a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*(-1 + a*x)^2*(9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a)
```

fricas [A] time = 0.69, size = 49, normalized size = 1.22

$$\frac{2\left(5a^3c^2x^3-a^2c^2x^2-13ac^2x+9c^2\right)\sqrt{-acx+c}}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^3*c^2*x^3 - a^2*c^2*x^2 - 13*a*c^2*x + 9*c^2)*sqrt(-a*c*x + c)/a
```

giac [B] time = 0.15, size = 141, normalized size = 3.52

$$-\frac{2\left(21(acx-c)^2\sqrt{-acx+c}-70(-acx+c)^{\frac{3}{2}}c-35\left((-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c\right)c-\frac{3\left(5(acx-c)^3\sqrt{-acx+c}+21(acx\cdot\right.}{}\right.}{105a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/105*(21*(a*c*x - c)^2*sqrt(-a*c*x + c) - 70*(-a*c*x + c)^(3/2)*c - 35*((
-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c - 3*(5*(a*c*x - c)^3*sqrt(-a*c*
x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 +
35*sqrt(-a*c*x + c)*c^3)/c)/a
```

maple [A] time = 0.04, size = 21, normalized size = 0.52

$$\frac{2(-acx+c)^{\frac{5}{2}}(5ax+9)}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/35*(-a*c*x+c)^(5/2)*(5*a*x+9)/a
```

maxima [A] time = 0.31, size = 32, normalized size = 0.80

$$-\frac{2\left(5(-acx+c)^{\frac{7}{2}}-14(-acx+c)^{\frac{5}{2}}c\right)}{35ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/35*(5*(-a*c*x + c)^(7/2) - 14*(-a*c*x + c)^(5/2)*c)/(a*c)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.80

$$\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

sympy [A] time = 7.83, size = 80, normalized size = 2.00

$$\begin{cases} \frac{-c^{2}\left(\begin{cases} 0 & \text{for}\: c = 0 \\ -\frac{2\left(-acx+c\right)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right) - \frac{2\left(\frac{c^{2}\left(-acx+c\right)^{\frac{3}{2}}}{3} - \frac{2c\left(-acx+c\right)^{\frac{5}{2}}}{5} + \frac{\left(-acx+c\right)^{\frac{7}{2}}}{7}\right)}{c}}{a} & \text{for}\: a \neq 0 \\ -c^{\frac{5}{2}}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Piecewise(((-c**2*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), T
rue)) - 2*(c**2*(-a*c*x + c)**(3/2)/3 - 2*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x
 + c)**(7/2)/7)/c)/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

3.237 $\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

```
[Out] 4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c
```

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx\\ &= -\int \frac{(1+ax)(c-acx)^{3/2}}{1-ax}\,dx\\ &= -\left(c\int (1+ax)\sqrt{c-acx}\,dx\right)\\ &= -\left(c\int \left(2\sqrt{c-acx}-\frac{(c-acx)^{3/2}}{c}\right)dx\right)\\ &= \frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}\end{aligned}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.75

$$-\frac{2c(ax-1)(3ax+7)\sqrt{c-acx}}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
[Out] (-2*c*(-1 + a*x)*(7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a)
```

fricas [A] time = 0.50, size = 32, normalized size = 0.80

$$-\frac{2\left(3\,a^2cx^2+4\,acx-7\,c\right)\sqrt{-acx+c}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] -2/15*(3*a^2*c*x^2 + 4*a*c*x - 7*c)*sqrt(-a*c*x + c)/a
```

giac [B] time = 0.13, size = 71, normalized size = 1.78

$$\frac{2\left(15\sqrt{-acx+c}\,c-\frac{3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}\,c^2}{c}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] 2/15*(15*sqrt(-a*c*x + c)*c - (3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*
x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/c)/a
```

maple [A] time = 0.03, size = 21, normalized size = 0.52

$$\frac{2\,(-acx+c)^{\frac{3}{2}}\,(3ax+7)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(3/2),x)
[Out] 2/15*(-a*c*x+c)^(3/2)*(3*a*x+7)/a
```

maxima [A] time = 0.30, size = 32, normalized size = 0.80

$$-\frac{2\left(3\,(-acx+c)^{\frac{5}{2}}-10\,(-acx+c)^{\frac{3}{2}}c\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -2/15*(3*(-a*c*x + c)^(5/2) - 10*(-a*c*x + c)^(3/2)*c)/(a*c)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.80

$$\frac{4\,(c-a\,c\,x)^{3/2}}{3\,a}-\frac{2\,(c-a\,c\,x)^{5/2}}{5\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

sympy [**A**] time = 5.43, size = 61, normalized size = 1.52

$$\begin{cases} \frac{-c\left(\begin{cases} 0 & \text{for}\: c = 0 \\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right) - \frac{2\left(-\frac{c(-acx+c)^{\frac{3}{2}}}{3} + \frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{c}}{a} & \text{for}\: a \neq 0 \\ -c^{\frac{3}{2}}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Piecewise(((-c*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), True
)) - 2*(-c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/c)/a, Ne(a, 0)),
(-c**(3/2)*x, True))
```

3.238 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx&=-\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\&=-\int\frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\&=-\left(c\int\frac{1+ax}{\sqrt{c-acx}}\,dx\right)\\&=-\left(c\int\left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\right)\\&=\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.61

$$\frac{2(ax+5)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
[Out] (2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

fricas [A] time = 0.58, size = 19, normalized size = 0.50

$$\frac{2\sqrt{-acx+c}\,(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

giac [A] time = 0.15, size = 44, normalized size = 1.16

$$\frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c}{c}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

maple [A] time = 0.03, size = 20, normalized size = 0.53

$$\frac{2\sqrt{-acx+c}\ \ (ax+5)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2),x)
[Out] 2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

maxima [A] time = 0.31, size = 30, normalized size = 0.79

$$-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.84

$$\frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] (4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

sympy [A] time = 2.89, size = 31, normalized size = 0.82

$$-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
[Out] -2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

3.239 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=36

$$-\frac{2\sqrt{c-acx}}{ac}-\frac{4}{a\sqrt{c-acx}}$$

[Out] -4/a/(-a*c*x+c)^(1/2)-2*(-a*c*x+c)^(1/2)/a/c

Rubi [A] time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$-\frac{2\sqrt{c-acx}}{ac}-\frac{4}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]

[Out] -4/(a*Sqrt[c - a*c*x]) - (2*Sqrt[c - a*c*x])/(a*c)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx \\
&= -\int \frac{1+ax}{(1-ax)\sqrt{c-acx}}\,dx \\
&= -\left(c\int \frac{1+ax}{(c-acx)^{3/2}}\,dx\right) \\
&= -\left(c\int \left(\frac{2}{(c-acx)^{3/2}}-\frac{1}{c\sqrt{c-acx}}\right)dx\right) \\
&= -\frac{4}{a\sqrt{c-acx}}-\frac{2\sqrt{c-acx}}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 21, normalized size = 0.58

$$\frac{2ax-6}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
[Out] (-6 + 2*a*x)/(a*Sqrt[c - a*c*x])
```

fricas [A] time = 0.62, size = 29, normalized size = 0.81

$$-\frac{2\sqrt{-acx+c}\,(ax-3)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*(a*x - 3)/(a^2*c*x - a*c)
```

giac [A] time = 0.17, size = 32, normalized size = 0.89

$$-\frac{4}{\sqrt{-acx+c}\,a}-\frac{2\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] -4/(sqrt(-a*c*x + c)*a) - 2*sqrt(-a*c*x + c)/(a*c)
```

maple [A] time = 0.04, size = 20, normalized size = 0.56

$$\frac{2ax-6}{a\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^(1/2),x)
[Out] 2*(a*x-3)/a/(-a*c*x+c)^(1/2)
```

maxima [A] time = 0.31, size = 30, normalized size = 0.83

$$-\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(-a*c*x + c) + 2*c/sqrt(-a*c*x + c))/(a*c)
```

mupad [B] time = 1.21, size = 19, normalized size = 0.53

$$\frac{2\,a\,x-6}{a\,\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^(1/2)*(a*x - 1)),x)
```

```
[Out] (2*a*x - 6)/(a*(c - a*c*x)^(1/2))
```

sympy [A] time = 14.07, size = 49, normalized size = 1.36

$$\begin{cases} \frac{-\frac{2}{\sqrt{-acx+c}}+\frac{2\left(-\frac{c}{\sqrt{-acx+c}}-\sqrt{-acx+c}\right)}{c}}{a} & \text{for}\ a\neq 0 \\ -\frac{x}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2/sqrt(-a*c*x + c) + 2*(-c/sqrt(-a*c*x + c) - sqrt(-a*c*x + c)
)/c)/a, Ne(a, 0)), (-x/sqrt(c), True))
```

3.240 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}} dx$

Optimal. Leaf size=38

$$\frac{2}{ac\sqrt{c-acx}}-\frac{4}{3a(c-acx)^{3/2}}$$

```
[Out] -4/3/a/(-a*c*x+c)^(3/2)+2/a/c/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$\frac{2}{ac\sqrt{c-acx}}-\frac{4}{3a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] -4/(3*a*(c - a*c*x)^(3/2)) + 2/(a*c*Sqrt[c - a*c*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx\\ &= -\int \frac{1+ax}{(1-ax)(c-acx)^{3/2}}\,dx\\ &= -\left(c\int \frac{1+ax}{(c-acx)^{5/2}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{(c-acx)^{5/2}}-\frac{1}{c(c-acx)^{3/2}}\right)dx\right)\\ &= -\frac{4}{3a(c-acx)^{3/2}}+\frac{2}{ac\sqrt{c-acx}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 0.89

$$-\frac{2(3ax-1)\sqrt{c-acx}}{3ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*(-1 + 3*a*x)*Sqrt[c - a*c*x])/(3*a*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.52, size = 44, normalized size = 1.16

$$-\frac{2\sqrt{-acx+c}\,(3\,ax-1)}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-a*c*x + c)*(3*a*x - 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.13, size = 36, normalized size = 0.95

$$\frac{2\,(3\,acx-c)}{3\,(acx-c)\sqrt{-acx+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*a*c*x - c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c)
```

maple [A] time = 0.04, size = 21, normalized size = 0.55

$$-\frac{2\,(3ax-1)}{3a\,(-acx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -2/3*(3*a*x-1)/a/(-a*c*x+c)^(3/2)
```

maxima [A] time = 0.30, size = 26, normalized size = 0.68

$$-\frac{2\,(3\,acx-c)}{3\,(-acx+c)^{\frac{3}{2}}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a*c*x - c)/((-a*c*x + c)^(3/2)*a*c)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.53

$$-\frac{6\,a\,x-2}{3\,a\,{\left(c-a\,c\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^(3/2)*(a*x - 1)),x)
```

```
[Out] -(6*a*x - 2)/(3*a*(c - a*c*x)^(3/2))
```

sympy [A] time = 29.05, size = 29, normalized size = 0.76

$$-\frac{4}{3a\left(-acx+c\right)^{\frac{3}{2}}}+\frac{2}{ac\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(3/2),x)
```

```
[Out] -4/(3*a*(-a*c*x + c)**(3/2)) + 2/(a*c*sqrt(-a*c*x + c))
```

3.241 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{3ac(c-acx)^{3/2}}-\frac{4}{5a(c-acx)^{5/2}}$$

```
[Out] -4/5/a/(-a*c*x+c)^(5/2)+2/3/a/c/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$\frac{2}{3ac(c-acx)^{3/2}}-\frac{4}{5a(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -4/(5*a*(c - a*c*x)^(5/2)) + 2/(3*a*c*(c - a*c*x)^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx \\ &= -\int \frac{1+ax}{(1-ax)(c-acx)^{5/2}}\,dx \\ &= -\left(c\int \frac{1+ax}{(c-acx)^{7/2}}\,dx\right) \\ &= -\left(c\int \left(\frac{2}{(c-acx)^{7/2}}-\frac{1}{c(c-acx)^{5/2}}\right)dx\right) \\ &= -\frac{4}{5a(c-acx)^{5/2}}+\frac{2}{3ac(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.85

$$\frac{2(5ax+1)\sqrt{c-acx}}{15ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (2*(1 + 5*a*x)*Sqrt[c - a*c*x])/(15*a*c^3*(-1 + a*x)^3)
```

fricas [A] time = 0.51, size = 56, normalized size = 1.40

$$\frac{2\sqrt{-acx+c}\,(5\,ax+1)}{15\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(-a*c*x + c)*(5*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3)
```

giac [A] time = 0.14, size = 34, normalized size = 0.85

$$-\frac{2\,(5\,acx+c)}{15\,(acx-c)^2\sqrt{-acx+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a*c*x + c)/((a*c*x - c)^2*sqrt(-a*c*x + c)*a*c)
```

maple [A] time = 0.04, size = 21, normalized size = 0.52

$$-\frac{2\,(5ax+1)}{15a\,(-acx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -2/15*(5*a*x+1)/a/(-a*c*x+c)^(5/2)
```

maxima [A] time = 0.30, size = 24, normalized size = 0.60

$$-\frac{2\,(5\,acx+c)}{15\,(-acx+c)^{\frac{5}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] -2/15*(5*a*c*x + c)/((-a*c*x + c)^(5/2)*a*c)
```

mupad [B] time = 1.21, size = 20, normalized size = 0.50

$$-\frac{10\,a\,x+2}{15\,a\,{(c-a\,c\,x)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^(5/2)*(a*x - 1)),x)
[Out] -(10*a*x + 2)/(15*a*(c - a*c*x)^(5/2))
```

sympy [A] time = 21.49, size = 31, normalized size = 0.78

$$-\frac{4}{5a\,(-acx+c)^{\frac{5}{2}}}+\frac{2}{3ac\,(-acx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(5/2),x)
[Out] -4/(5*a*(-a*c*x + c)**(5/2)) + 2/(3*a*c*(-a*c*x + c)**(3/2))
```

3.242 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{5ac(c-acx)^{5/2}}-\frac{4}{7a(c-acx)^{7/2}}$$

```
[Out] -4/7/a/(-a*c*x+c)^(7/2)+2/5/a/c/(-a*c*x+c)^(5/2)
```

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$\frac{2}{5ac(c-acx)^{5/2}}-\frac{4}{7a(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -4/(7*a*(c - a*c*x)^(7/2)) + 2/(5*a*c*(c - a*c*x)^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx \\
&= -\int \frac{1+ax}{(1-ax)(c-acx)^{7/2}}\,dx \\
&= -\left(c\int \frac{1+ax}{(c-acx)^{9/2}}\,dx\right) \\
&= -\left(c\int \left(\frac{2}{(c-acx)^{9/2}}-\frac{1}{c(c-acx)^{7/2}}\right)dx\right) \\
&= -\frac{4}{7a(c-acx)^{7/2}}+\frac{2}{5ac(c-acx)^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.85

$$
-\frac{2(7ax+3)\sqrt{c-acx}}{35ac^4(ax-1)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*(3 + 7*a*x)*Sqrt[c - a*c*x])/(35*a*c^4*(-1 + a*x)^4)
```

fricas [B] time = 0.68, size = 66, normalized size = 1.65

$$
-\frac{2\sqrt{-acx+c}\,(7\,ax+3)}{35\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/35*sqrt(-a*c*x + c)*(7*a*x + 3)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4
*x^2 - 4*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.13, size = 36, normalized size = 0.90

$$
\frac{2\,(7\,acx+3\,c)}{35\,(acx-c)^3\sqrt{-acx+c}\,ac}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(7*a*c*x + 3*c)/((a*c*x - c)^3*sqrt(-a*c*x + c)*a*c)
```

maple [A] time = 0.04, size = 21, normalized size = 0.52

$$
-\frac{2\,(7ax+3)}{35a\,(-acx+c)^{\frac{7}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -2/35*(7*a*x+3)/a/(-a*c*x+c)^(7/2)
```

maxima [A] time = 0.32, size = 26, normalized size = 0.65

$$-\frac{2\,(7\,acx+3\,c)}{35\,(-acx+c)^{\frac{7}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/35*(7*a*c*x + 3*c)/((-a*c*x + c)^(7/2)*a*c)
```

mupad [B] time = 1.19, size = 20, normalized size = 0.50

$$-\frac{14\,a\,x+6}{35\,a\,(c-a\,c\,x)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^(7/2)*(a*x - 1)),x)
```

```
[Out] -(14*a*x + 6)/(35*a*(c - a*c*x)^(7/2))
```

sympy [A] time = 40.69, size = 31, normalized size = 0.78

$$-\frac{4}{7a\,(-acx+c)^{\frac{7}{2}}}+\frac{2}{5ac\,(-acx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] -4/(7*a*(-a*c*x + c)**(7/2)) + 2/(5*a*c*(-a*c*x + c)**(5/2))
```

3.243 $\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=197

$$-\frac{856\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{1155a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{16\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{21a^2x\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[Out] -8/33*(1+1/a/x)^(5/2)*(-a*c*x+c)^(9/2)/a/(1-1/a/x)^(9/2)-856/1155*(1+1/a/x)
^(5/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+16/21*(1+1/a/x)^(5/2)*(-a*c
*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^3*(1+1/a/x)^(5/2)*x*(-a*c*x+
c)^(9/2)/a^3/(1-1/a/x)^(9/2)
```

Rubi [A] time = 0.19, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{856\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{1155a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{16\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{21a^2x\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

```
[Out] (-8*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(33*a*(1 - 1/(a*x))^(9/2)) - (85
6*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(1155*a^3*(1 - 1/(a*x))^(9/2)*x^2)
 + (16*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(21*a^2*(1 - 1/(a*x))^(9/2)*x
) + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(9/2))/(11*a^3*(1 -
 1/(a*x))^(9/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{(c-acx)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\left(1+\frac{x}{a}\right)^{3/2}}{x^{13/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(12\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}}\cdots\right.}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(8\left(\frac{1}{x}\right)^{9/2}(c-ac\cdots\right.}{\cdots}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c\cdots}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{856\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{1155a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9\cdots}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.39

$$\frac{2c^4\sqrt{\frac{1}{ax}+1}\,(ax+1)^2\left(105a^3x^3-455a^2x^2+755ax-533\right)\sqrt{c-acx}}{1155a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(-533 + 755*a*x - 455*
a^2*x^2 + 105*a^3*x^3))/(1155*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.50, size = 105, normalized size = 0.53

$$\frac{2\left(105\,a^6c^4x^6-140\,a^5c^4x^5-295\,a^4c^4x^4+472\,a^3c^4x^3+211\,a^2c^4x^2-844\,ac^4x-533\,c^4\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{1155\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*a^6*c^4*x^6 - 140*a^5*c^4*x^5 - 295*a^4*c^4*x^4 + 472*a^3*c^4*x
^3 + 211*a^2*c^4*x^2 - 844*a*c^4*x - 533*c^4)*sqrt(-a*c*x + c)*sqrt((a*x -
1)/(a*x + 1))/(a^2*x - a)
```

giac [A] time = 0.20, size = 132, normalized size = 0.67

$$-\frac{2\left(\frac{512\,\sqrt{2}\,\sqrt{-c}\,c^4}{\mathrm{sgn}(c)}-\frac{105\,(acx+c)^5\,\sqrt{-acx-c}-770\,(acx+c)^4\,\sqrt{-acx-c}\,c+1980\,(acx+c)^3\,\sqrt{-acx-c}\,c^2-1848\,(acx+c)^2\,\sqrt{-acx-c}\,c^3}{c\mathrm{sgn}(-acx-c)}\right)}{1155\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -2/1155*(512*sqrt(2)*sqrt(-c)*c^4/sgn(c) - (105*(a*c*x + c)^5*sqrt(-a*c*x -
 c) - 770*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 1980*(a*c*x + c)^3*sqrt(-a*c*x
 - c)*c^2 - 1848*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3)/(c*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.04, size = 64, normalized size = 0.32

$$\frac{2\,(ax+1)\left(105x^3a^3-455a^2x^2+755ax-533\right)(-acx+c)^{\frac{9}{2}}}{1155a\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x)
```

```
[Out] 2/1155*(a*x+1)*(105*a^3*x^3-455*a^2*x^2+755*a*x-533)*(-a*c*x+c)^(9/2)/a/(a*
x-1)^3/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.35, size = 106, normalized size = 0.54

$$\frac{2\left(105\,a^5\sqrt{-c}\,c^4x^5-455\,a^4\sqrt{-c}\,c^4x^4+650\,a^3\sqrt{-c}\,c^4x^3-78\,a^2\sqrt{-c}\,c^4x^2-755\,a\sqrt{-c}\,c^4x+533\,\sqrt{-c}\,c^4\right)(ax+1)}{1155\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*a^5*sqrt(-c)*c^4*x^5 - 455*a^4*sqrt(-c)*c^4*x^4 + 650*a^3*sqrt(
-c)*c^4*x^3 - 78*a^2*sqrt(-c)*c^4*x^2 - 755*a*sqrt(-c)*c^4*x + 533*sqrt(-c)
*c^4)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

mupad [B] time = 1.45, size = 110, normalized size = 0.56

$$\frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(105\,a^5\,x^5-35\,a^4\,x^4-330\,a^3\,x^3+142\,a^2\,x^2+353\,a\,x-491\right)}{1155\,a}-\frac{2048\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{1155\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(353*a*x + 142*a^2*x^2
 - 330*a^3*x^3 - 35*a^4*x^4 + 105*a^5*x^5 - 491))/(1155*a) - (2048*c^4*(c -
 a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(1155*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.244 $\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{214\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{315a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{44\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -44/63*(1+1/a/x)^(5/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)+214/315*(1+1/a/x)
^(5/2)*(-a*c*x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(1+1/a/x)^(5/2)*x*(-a*c*x
+c)^(7/2)/(1-1/a/x)^(7/2)
```

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 89, 78, 37}

$$\frac{214\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{315a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{44\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-44*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(63*a*(1 - 1/(a*x))^(7/2)) + (2
14*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(315*a^2*(1 - 1/(a*x))^(7/2)*x) +
 (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(7/2))/(9*(1 - 1/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{(c-acx)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{11}{a}+\frac{9x}{2a^2}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}}\,dx,x,\right.}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(107\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)}{63a^2}$$

$$= -\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{214\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{315a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.50

$$-\frac{2c^3\sqrt{\frac{1}{ax}+1}\,(ax+1)^2\left(35a^2x^2-110ax+107\right)\sqrt{c-acx}}{315a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(107 - 110*a*x + 35*a
^2*x^2))/(315*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.60, size = 94, normalized size = 0.69

$$-\frac{2\left(35\,a^5c^3x^5-5\,a^4c^3x^4-118\,a^3c^3x^3+26\,a^2c^3x^2+211\,ac^3x+107\,c^3\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 5*a^4*c^3*x^4 - 118*a^3*c^3*x^3 + 26*a^2*c^3*x^2 +
 211*a*c^3*x + 107*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x -
 a)
```

giac [A] time = 0.21, size = 107, normalized size = 0.78

$$-\frac{2\left(\frac{128\sqrt{2}\sqrt{-c}c^3}{\mathrm{sgn}(c)}+\frac{35(acx+c)^4\sqrt{-acx-c}-180(acx+c)^3\sqrt{-acx-c}c+252(acx+c)^2\sqrt{-acx-c}c^2}{c\mathrm{sgn}(-acx-c)}\right)}{315a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/315*(128*sqrt(2)*sqrt(-c)*c^3/sgn(c) + (35*(a*c*x + c)^4*sqrt(-a*c*x - c
) - 180*(a*c*x + c)^3*sqrt(-a*c*x - c)*c + 252*(a*c*x + c)^2*sqrt(-a*c*x -
c)*c^2)/(c*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.04, size = 56, normalized size = 0.41

$$\frac{2(ax+1)\left(35a^2x^2-110ax+107\right)(-acx+c)^{\frac{7}{2}}}{315a(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x)
```

```
[Out] 2/315*(a*x+1)*(35*a^2*x^2-110*a*x+107)*(-a*c*x+c)^(7/2)/a/(a*x-1)^2/((a*x-1
)/(a*x+1))^(3/2)
```

maxima [A] time = 0.34, size = 90, normalized size = 0.66

$$-\frac{2\left(35a^4\sqrt{-c}c^3x^4-110a^3\sqrt{-c}c^3x^3+72a^2\sqrt{-c}c^3x^2+110a\sqrt{-c}c^3x-107\sqrt{-c}c^3\right)(ax+1)^{\frac{3}{2}}}{315(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 110*a^3*sqrt(-c)*c^3*x^3 + 72*a^2*sqrt(-c
)*c^3*x^2 + 110*a*sqrt(-c)*c^3*x - 107*sqrt(-c)*c^3)*(a*x + 1)^(3/2)/((a*x
- 1)*a)
```

mupad [B] time = 1.44, size = 102, normalized size = 0.74

$$-\frac{2c^3\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(35\,a^4\,x^4+30\,a^3\,x^3-88\,a^2\,x^2-62\,a\,x+149\right)}{315\,a}-\frac{512\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(30*a^3*x^3 - 88*a^2
*x^2 - 62*a*x + 35*a^4*x^4 + 149))/(315*a) - (512*c^3*(c - a*c*x)^(1/2)*((a
*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.245 $\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{18\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -18/35*(1+1/a/x)^(5/2)*(-a*c*x+c)^(5/2)/a/(1-1/a/x)^(5/2)+2/7*(1+1/a/x)^(5/
2)*x*(-a*c*x+c)^(5/2)/(1-1/a/x)^(5/2)
```

Rubi [A] time = 0.16, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{18\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-18*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (2
*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(9\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}}dx,x,\frac{1}{x}\right)}{7a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{18\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.66

$$\frac{2\sqrt{\frac{1}{ax}+1}\,(5ax-9)\sqrt{c-acx}\,(acx+c)^2}{35a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(-9 + 5*a*x)*Sqrt[c - a*c*x]*(c + a*c*x)^2)/(35*a*Sqrt
[1 - 1/(a*x)])
```

fricas [A] time = 0.52, size = 83, normalized size = 0.93

$$\frac{2\left(5\,a^4c^2x^4+6\,a^3c^2x^3-12\,a^2c^2x^2-22\,ac^2x-9\,c^2\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 + 6*a^3*c^2*x^3 - 12*a^2*c^2*x^2 - 22*a*c^2*x - 9*c^2)*
sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

giac [A] time = 0.19, size = 84, normalized size = 0.94

$$-\frac{2\left(\frac{16\sqrt{2}\sqrt{-c}\,c^2}{\mathrm{sgn}(c)}-\frac{5\,(acx+c)^3\sqrt{-acx-c}-14\,(acx+c)^2\sqrt{-acx-c}\,c}{c\,\mathrm{sgn}(-acx-c)}\right)}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/35*(16*sqrt(2)*sqrt(-c)*c^2/sgn(c) - (5*(a*c*x + c)^3*sqrt(-a*c*x - c) -
 14*(a*c*x + c)^2*sqrt(-a*c*x - c)*c)/(c*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.04, size = 48, normalized size = 0.54

$$\frac{2\left(ax+1\right)\left(5ax-9\right)\left(-acx+c\right)^{\frac{5}{2}}}{35a\left(ax-1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x)
```

```
[Out] 2/35*(a*x+1)*(5*a*x-9)*(-a*c*x+c)^(5/2)/a/(a*x-1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.34, size = 74, normalized size = 0.83

$$\frac{2\left(5\,a^{3}\sqrt{-c}\,c^{2}x^{3}-9\,a^{2}\sqrt{-c}\,c^{2}x^{2}-5\,a\sqrt{-c}\,c^{2}x+9\,\sqrt{-c}\,c^{2}\right)\left(ax+1\right)^{\frac{3}{2}}}{35\left(ax-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^3*sqrt(-c)*c^2*x^3 - 9*a^2*sqrt(-c)*c^2*x^2 - 5*a*sqrt(-c)*c^2*x
+ 9*sqrt(-c)*c^2)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

mupad [B] time = 1.39, size = 93, normalized size = 1.04

$$-\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-5\,a^3\,x^3-11\,a^2\,x^2+a\,x+23\right)}{35\,a}-\frac{64\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a*x - 11*a^2*x^2 -
5*a^3*x^3 + 23))/(35*a) - (64*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(
1/2))/(35*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.246 $\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2(ax+1)(c-acx)^{3/2}e^{3\coth^{-1}(ax)}}{5a}$$

```
[Out] 2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6174}

$$\frac{2(ax+1)(c-acx)^{3/2}e^{3\coth^{-1}(ax)}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
[Out] (2*E^(3*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(3/2))/(5*a)
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{2e^{3\coth^{-1}(ax)}(1+ax)(c-acx)^{3/2}}{5a}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 1.39

$$\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
[Out] (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

fricas [A] time = 0.58, size = 61, normalized size = 1.97

$$-\frac{2\left(a^{3}cx^{3}+3\,a^{2}cx^{2}+3\,acx+c\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] -2/5*(a^3*c*x^3 + 3*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1))/(a^2*x - a)
```

giac [A] time = 0.15, size = 56, normalized size = 1.81

$$-\frac{2\left(\frac{4\sqrt{2}\sqrt{-c}c}{\mathrm{sgn}(c)}+\frac{(acx+c)^2\sqrt{-acx-c}}{c\mathrm{sgn}(-acx-c)}\right)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/5*(4*sqrt(2)*sqrt(-c)*c/sgn(c) + (a*c*x + c)^2*sqrt(-a*c*x - c)/(c*sgn(-
a*c*x - c)))/a
```

maple [A] time = 0.04, size = 35, normalized size = 1.13

$$\frac{2(ax+1)(-acx+c)^{\frac{3}{2}}}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x)
```

```
[Out] 2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

maxima [A] time = 0.34, size = 41, normalized size = 1.32

$$-\frac{2\left(a^2\sqrt{-c}cx^2-\sqrt{-c}c\right)(ax+1)^{\frac{3}{2}}}{5(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^2*sqrt(-c)*c*x^2 - sqrt(-c)*c)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

mupad [B] time = 1.38, size = 81, normalized size = 2.61

$$-\frac{2c\sqrt{c-acx}\sqrt{\frac{ax-1}{ax+1}}\left(a^2x^2+4ax+7\right)}{5a}-\frac{16c\sqrt{c-acx}\sqrt{\frac{ax-1}{ax+1}}}{5a(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + a^2*x^2 + 7))
/(5*a) - (16*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x - 1
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.247 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=163

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1
/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(3/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{}{\sqrt{x}(1\ldots}\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}\ldots}\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{a}\sqrt{\ldots}}\right.}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.08, size = 105, normalized size = 0.64

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}\,(ax+7)-6\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x^
(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^(3
/2)*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.85, size = 250, normalized size = 1.53

$$\left[\frac{2\left(3\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(a^2x^2+8\,ax+7\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{3\left(a^2x-a\right)}, -\ 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2 + 8*a
*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

giac [C] time = 0.17, size = 105, normalized size = 0.64

$$-\frac{\frac{12i\,\sqrt{2}\,\sqrt{-c}\,\arctan(-i)-16\,\sqrt{2}\,\sqrt{-c}}{\mathrm{sgn}(c)}+\frac{2\left(6\,\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx-c}}{2\,\sqrt{c}}\right)+(-acx-c)^{\frac{3}{2}}-6\,\sqrt{-acx-c}\,c\right)}{c\mathrm{sgn}(-acx-c)}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*((12*I*sqrt(2)*sqrt(-c)*arctan(-I) - 16*sqrt(2)*sqrt(-c))/sgn(c) + 2*(
6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) + (-a*c*x -
c)^(3/2) - 6*sqrt(-a*c*x - c)*c)/(c*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.06, size = 107, normalized size = 0.66

$$-\frac{2\,(ax-1)\,\sqrt{-c\,(ax-1)}\,\left(6\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-xa\sqrt{-c\,(ax+1)}-7\sqrt{-c\,(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,\sqrt{-c\,(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-x*a*(-c*(a*x+1))^(1/
2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.248 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=177

$$\frac{2ax\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[Out] 2*a*(1+1/a/x)^(3/2)*x*(1-1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(1/2)-6*(1-1/a/x)^
(1/2)*(1+1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(1/2)-3*arctanh(2^(1/2)*(1/x)^(1/2
)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a^(1/2)/(1/x)^(1/2)/(-a*
c*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{2ax\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (-6*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - a*c*x]) + (
2*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x)/((a - x^(-1))*Sqrt[c - a*c*x])
 - (3*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqr
t[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{ax}}\,\sqrt{x}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\,\sqrt{x}}\,dx}{\sqrt{c-acx}}\\
&= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{3/2}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}\\
&= \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(6\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}\\
&= -\frac{6\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, \right.}{a\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}\\
&= -\frac{6\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(6\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right.}{a\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}\\
&= -\frac{6\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{3\sqrt{2}\,\sqrt{1-\frac{1}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 116, normalized size = 0.66

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\,\sqrt{\frac{1}{ax}+1}\,(ax-2)-3\sqrt{2}\,\sqrt{\frac{1}{x}}\,(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)\right)}{\sqrt{a}\,(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 3*Sqrt[2]*Sq
rt[x^(-1)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])]))/(Sqrt[a]*(-1 + a*x)*Sqrt[c - a*c*x])
```

fricas [A] time = 0.50, size = 288, normalized size = 1.63

$$\left[\frac{3\sqrt{2}\left(a^{2}cx^{2}-2\,acx+c\right)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^{2}x^{2}-2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^{2}x^{2}-2\,ax+1}\right)-4\left(a^{2}x^{2}-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-}{ax+}}}{2\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt
(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x
 - 3)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c), -(2*(a^2*x^2 - a*x - 2
)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)) - 3*sqrt(2)*(a^2*c*x^2 - 2*a*c
*x + c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1
)*sqrt(c)))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

giac [A] time = 0.17, size = 85, normalized size = 0.48

$$\frac{3\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-2\sqrt{-acx-c}+\frac{2\sqrt{-acx-c}\,c}{acx-c}}{ac\mathrm{sgn}\left(-acx-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] (3*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - 2*sqrt(-a
*c*x - c) + 2*sqrt(-a*c*x - c)*c/(a*c*x - c))/(a*c*sgn(-a*c*x - c))
```

maple [A] time = 0.07, size = 135, normalized size = 0.76

$$\frac{\sqrt{-c\,(ax-1)}\,\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac-2xa\sqrt{-c\,(ax+1)}\,\sqrt{c}\,-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c+4\sqrt{-c\,(ax}\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,c^{\frac{3}{2}}\sqrt{-c\,(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(-c*(a*x-1))^(1/2)*(3*2^(1/2)*arctan(1/2*
(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)-
3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+4*(-c*(a*x+1))^(
1/2)*c^(1/2))/c^(3/2)/(-c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-a\,c\,x}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
[Out] Timed out
```

3.249 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}}-\frac{3ax\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x/(a-1/x)^2/(-a*c*x+c)^(3/2)-3/8*(
1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a^(1/2)
/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)-3/4*a*(1-1/a/x)^(3/2)*x*(1+1/a/x)^(1/
2)/(a-1/x)/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}}-\frac{3ax\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-3*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(4*(a - x^(-1))*(c - a*c*x)^
(3/2)) - (a^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x)/(2*(a - x^(-1))^2*
(c - a*c*x)^(3/2)) - (3*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)
^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2} x^{3/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2} x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2} \operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2} x}{2\left(a-\frac{1}{x}\right)^2 (c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2} x}{2\left(a-\frac{1}{x}\right)^2 (c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1 \cdots}{\sqrt{x}\left(1-\frac{x}{a} \cdots\right.}\cdots\right.}{8\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3 \cdots}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2} x}{2\left(a-\frac{1}{x}\right)^2 (c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,d\cdots\right.}{4\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2} x}{2\left(a-\frac{1}{x}\right)^2 (c-acx)^{3/2}} - \frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{a}\sqrt{1+\cdots}}\right.}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Mathematica [A] time = 0.16, size = 125, normalized size = 0.67

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{ax}+1}\,(5ax-1)+3\sqrt{2}\sqrt{\frac{1}{x}}\,(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{8\sqrt{a}\,c(ax-1)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1 + 5*a*x) + 3*Sqrt[2]*
Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 +
1/(a*x)])]))/(8*Sqrt[a]*c*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

fricas [A] time = 0.67, size = 341, normalized size = 1.82

$$\left[-\frac{3\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+4\left(5a^2x^2+4ax-1\right)\sqrt{-}}{16\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*
c^2*x - a*c^2), -1/8*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*a
rctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) + 2*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

giac [A] time = 0.17, size = 89, normalized size = 0.48

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5(-acx-c)^{\frac{3}{2}}+6\sqrt{-acx-c}\,c\right)}{(acx-c)^2}}{8ac\operatorname{sgn}(-acx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - 2*(5
*(-a*c*x - c)^(3/2) + 6*sqrt(-a*c*x - c)*c)/(a*c*x - c)^2)/(a*c*sgn(-a*c*x
- c))
```

maple [A] time = 0.06, size = 174, normalized size = 0.93

$$-\frac{\sqrt{-c(ax-1)}\left(3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c-6\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)xac+10xa\sqrt{-c(ax+1)}\sqrt{c}+3\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)c^{\frac{5}{2}}\sqrt{-c(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -1/8/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(5/2)*(3*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-6*2^(1/2)*
arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+10*x*a*(-c*(a*x+1))^(1
/2)*c^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-2*(-
c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c - a\,c\,x)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
[Out] Timed out
```

3.250 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}+\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

```
[Out] 1/24*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)-1/6
*a^4*(1-1/a/x)^(5/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(5/2)+1/32*a^
(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))/
(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+1/16*a^2*(1-1/a/x)^(5/2)*x^2*(1+1/a/x)
^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A] time = 0.22, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}+\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(16*(a - x^(-1))*(c - a*c*x
)^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(24*(a - x^(-1
))^2*(c - a*c*x)^(5/2)) - (a^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^2)
/(6*(a - x^(-1))^3*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))
^(5/2)*(c - a*c*x)^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2} x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2} \operatorname{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{12\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{24\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1}}{\sqrt{x}(}\right.}{16\left(\frac{1}{x}\right)^{5/2}(c-acx)}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{24\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{5/2}} +$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{24\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{5/2}} +$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2} x^2}{24\left(a-\frac{1}{x}\right)^2 (c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2} x^2}{6\left(a-\frac{1}{x}\right)^3 (c-acx)^{5/2}} +$$

Mathematica [A] time = 0.15, size = 142, normalized size = 0.57

$$-\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\left(3a^2x^2+22ax+7\right)-\frac{3\sqrt{2}(ax-1)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{x}\right)}{96\sqrt{a}c^2\left(\frac{1}{x}\right)^{3/2}(ax-1)^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -1/96*(Sqrt[1 - 1/(a*x)]*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(7 + 22*
a*x + 3*a^2*x^2) - (3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)]])])/x))/(Sqrt[a]*c^2*(x^(-1))^(3/2)*(-1 + a*x)^3*S
qrt[c - a*c*x])
```

fricas [A] time = 0.61, size = 393, normalized size = 1.57

$$\left[-\frac{3\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4\left(3a^3x^3+25a\right.}{192\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*l
og(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 + 25*a
^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x
^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), -1/96*(3*sqrt(2)
*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^3*x^3 +
 25*a^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*
c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

giac [A] time = 0.22, size = 116, normalized size = 0.46

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{2\left(3(acx+c)^2\sqrt{-acx-c}-16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}c^2\right)}{(acx-c)^3c}}{96\,ac\,\mathrm{sgn}(-acx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/96*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - 2*(
3*(a*c*x + c)^2*sqrt(-a*c*x - c) - 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*x
 - c)*c^2)/((a*c*x - c)^3*c))/(a*c*sgn(-a*c*x - c))
```

maple [A] time = 0.07, size = 226, normalized size = 0.90

$$\frac{\sqrt{-c}(ax-1)\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^3a^3c+9\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c+6x^2a^2\sqrt{-c(ax+1)}\sqrt{c}-\right.}{96\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)^2(ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] 1/96*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c
^(1/2))*x^3*a^3*c+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*
x^2*a^2*c+6*x^2*a^2*(-c*(a*x+1))^(1/2)*c^(1/2)-9*2^(1/2)*arctan(1/2*(-c*(a*
x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+44*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)+3*2^(1
/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+14*(-c*(a*x+1))^(1/2)*
c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)/c^(7/2)/(-c*(a*x+1))^(1/
2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-acx+c)^{\frac{5}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c-a\,c\,x)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.251 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{256\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}+\frac{a^5x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^5x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}-\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\right.}{128\left(a-\frac{1}{x}\right)^2(c}$$

```
[Out] -1/8*a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^4/(-a*c*x+c)^(7/2)-1/1
28*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)+1/32*
a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^3/(a-1/x)^3/(-a*c*x+c)^(7/2)-3/512*a^
(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))/
(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-3/256*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/a/x
)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A] time = 0.24, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{a^5x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^5x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}-\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{3a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -(a^5*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^2)/(8*(a - x^(-1))^4*(c - a
*c*x)^(7/2)) - (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(256*(a -
x^(-1))*(c - a*c*x)^(7/2)) - (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x
^3)/(128*(a - x^(-1))^2*(c - a*c*x)^(7/2)) + (a^5*(1 - 1/(a*x))^(7/2)*(1 +
1/(a*x))^(5/2)*x^3)/(32*(a - x^(-1))^3*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1 -
 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]
)/(256*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^{5}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^{4}}\,dx, x, \frac{1}{x}\right)}{16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{(1+\frac{x}{a}\cdots}{\sqrt{x}(1\cdots}\right.}{64\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7\cdots}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \cdots$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \cdots$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \cdots$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \cdots$$

Mathematica [A] time = 0.21, size = 147, normalized size = 0.48

$$\frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(-3a^3x^3+13a^2x^2+79ax+39\right)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}\,(ax-1)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{512\sqrt{a}\,c^3\sqrt{\frac{1}{x}}\,(ax-1)^4\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(39 + 79*a*x + 13*a^2*x^2
- 3*a^3*x^3))/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^4*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(512*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 +
a*x)^4*Sqrt[c - a*c*x])
```

fricas [A] time = 0.50, size = 449, normalized size = 1.46

$$\left[-\frac{3\sqrt{2}\left(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4}{1024\left(a^6c^4x^5-5a^5c^4x^4+10a^4c^4x^3-10a^3c^4x^2+5a^2c^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/1024*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x
- 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x +
 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3
*a^4*x^4 - 10*a^3*x^3 - 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a
*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4), -1/512*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10
*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^4*x^4 - 10*a^3*x^3
- 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a
^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x
- a*c^4)]
```

giac [A] time = 0.22, size = 140, normalized size = 0.46

$$-\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{2\left(3(acx+c)^3\sqrt{-acx-c}-22(acx+c)^2\sqrt{-acx-c}\,c+44(-acx-c)^{\frac{3}{2}}c^2+24\sqrt{-acx-c}\,c^3\right)}{(acx-c)^4c^2}}{512\,ac\,\mathrm{sgn}(-acx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/512*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*
(3*(a*c*x + c)^3*sqrt(-a*c*x - c) - 22*(a*c*x + c)^2*sqrt(-a*c*x - c)*c + 4
4*(-a*c*x - c)^(3/2)*c^2 + 24*sqrt(-a*c*x - c)*c^3)/((a*c*x - c)^4*c^2))/(a
*c*sgn(-a*c*x - c))
```

maple [A] time = 0.07, size = 278, normalized size = 0.91

$$\frac{\sqrt{-c(ax-1)}\left(-3\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^4a^4c+12\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^3a^3c+6x^3a^3\sqrt{-c(ax+1)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] 1/512*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*x^4*a^4*c+12*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2)
)*x^3*a^3*c+6*x^3*a^3*(-c*(a*x+1))^(1/2)*c^(1/2)-18*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c-26*x^2*a^2*(-c*(a*x+1))^(1/2)*c^(
1/2)+12*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-158*x*
a*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2
)/c^(1/2))*c-78*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)
^3/(a*x+1)/c^(9/2)/(-c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-acx+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((-a*c*x + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-a\,c\,x\right)}^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
[Out] Timed out
```

3.252 $\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=194

$$\frac{16384c^5x\sqrt{1-\frac{1}{a^2x^2}}}{693\sqrt{c-acx}}+\frac{4096}{693}c^4x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{512}{231}c^3x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}+\frac{640}{693}c^2x\sqrt{1-\frac{1}{a^2x^2}}(c-acx$$

```
[Out] 512/231*c^3*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+640/693*c^2*x*(-a*c*x+c)
^(5/2)*(1-1/a^2/x^2)^(1/2)+40/99*c*x*(-a*c*x+c)^(7/2)*(1-1/a^2/x^2)^(1/2)+2
/11*x*(-a*c*x+c)^(9/2)*(1-1/a^2/x^2)^(1/2)+16384/693*c^5*x*(1-1/a^2/x^2)^(1
/2)/(-a*c*x+c)^(1/2)+4096/693*c^4*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 311, normalized size of antiderivative = 1.60, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{512\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{231a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{1024\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{99a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{22016\sqrt{\frac{1}{ax}+1}(c-acx)^{9/2}}{693a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^5(c-ac}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-40*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(99*a^5*(1 - 1/(a*
x))^(9/2)) - (22016*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(693*a^5*(1 - 1/(a
*x))^(9/2)*x^4) + (1024*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1
/(a*x))^(9/2)*x^3) - (512*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(231*a^3*(1
- 1/(a*x))^(9/2)*x^2) + (640*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(
9/2))/(693*a^5*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^5*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^(9/2))/(11*a^5*(1 - 1/(a*x))^(9/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{(c-acx)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^{13/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(20\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{11/2}\sqrt{1+\frac{x}{a}}}\,d\right.}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(320\left(\frac{1}{x}\right.\right.}{}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\right.}{}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{512\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1}}{693a^5\left(1\right.}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{1024\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3} - \frac{512\sqrt{1+\frac{1}{ax}}(c}{231a^3\left(1-\frac{1}{a}\right.}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{22016\sqrt{1+\frac{1}{ax}}(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x^4} + \frac{1024\sqrt{1+\frac{1}{ax}}}{99a^4\left(1-\right.}$$

Mathematica [A] time = 0.05, size = 86, normalized size = 0.44

$$\frac{2c^4\sqrt{\frac{1}{ax}+1}\left(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531\right)\sqrt{c-acx}}{693a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-11531 + 5419*a*x - 3198*a^2*x^2
+ 1510*a^3*x^3 - 455*a^4*x^4 + 63*a^5*x^5))/(693*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.53, size = 105, normalized size = 0.54

$$\frac{2\left(63a^6c^4x^6-392a^5c^4x^5+1055a^4c^4x^4-1688a^3c^4x^3+2221a^2c^4x^2-6112ac^4x-11531c^4\right)\sqrt{-acx+c}\sqrt{\frac{ax-}{ax+}}}{693\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^6*c^4*x^6 - 392*a^5*c^4*x^5 + 1055*a^4*c^4*x^4 - 1688*a^3*c^4*x
^3 + 2221*a^2*c^4*x^2 - 6112*a*c^4*x - 11531*c^4)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 80, normalized size = 0.41

$$\frac{2\left(ax+1\right)\left(63x^{5}a^{5}-455x^{4}a^{4}+1510x^{3}a^{3}-3198a^{2}x^{2}+5419ax-11531\right)\left(-acx+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}}{693a\left(ax-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/693*(a*x+1)*(63*a^5*x^5-455*a^4*x^4+1510*a^3*x^3-3198*a^2*x^2+5419*a*x-11
531)*(-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2)/a/(a*x-1)^5
```

maxima [A] time = 0.33, size = 128, normalized size = 0.66

$$\frac{2\left(63\,a^{6}\sqrt{-c}\,c^{4}x^{6}-392\,a^{5}\sqrt{-c}\,c^{4}x^{5}+1055\,a^{4}\sqrt{-c}\,c^{4}x^{4}-1688\,a^{3}\sqrt{-c}\,c^{4}x^{3}+2221\,a^{2}\sqrt{-c}\,c^{4}x^{2}-6112\,a\sqrt{-c}\,c^{4}x-}{693\left(a^{2}x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*a^6*sqrt(-c)*c^4*x^6 - 392*a^5*sqrt(-c)*c^4*x^5 + 1055*a^4*sqrt(-
c)*c^4*x^4 - 1688*a^3*sqrt(-c)*c^4*x^3 + 2221*a^2*sqrt(-c)*c^4*x^2 - 6112*a
*sqrt(-c)*c^4*x - 11531*sqrt(-c)*c^4)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

mupad [B] time = 1.41, size = 110, normalized size = 0.57

$$\frac{2\,c^{4}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(63\,a^{5}\,x^{5}-329\,a^{4}\,x^{4}+726\,a^{3}\,x^{3}-962\,a^{2}\,x^{2}+1259\,a\,x-4853\right)}{693\,a}-\frac{32768\,c^{4}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x}{a\,x}}}{693\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(1259*a*x - 962*a^2*x^
2 + 726*a^3*x^3 - 329*a^4*x^4 + 63*a^5*x^5 - 4853))/(693*a) - (32768*c^4*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(693*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.253 $\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=161

$$\frac{4096c^4x\sqrt{1-\frac{1}{a^2x^2}}}{315\sqrt{c-acx}}+\frac{1024}{315}c^3x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{128}{105}c^2x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}+\frac{32}{63}cx\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{5/2}+\frac{2}{9}$$

```
[Out] 128/105*c^2*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+32/63*c*x*(-a*c*x+c)^(5/
2)*(1-1/a^2/x^2)^(1/2)+2/9*x*(-a*c*x+c)^(7/2)*(1-1/a^2/x^2)^(1/2)+4096/315*
c^4*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+1024/315*c^3*x*(1-1/a^2/x^2)^(1/
2)*(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 254, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{256\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}{45a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{5504\sqrt{\frac{1}{ax}+1}(c-acx)^{7/2}}{315a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{32\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3(c-}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-32*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*
x))^(7/2)) + (5504*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(315*a^4*(1 - 1/(a*
x))^(7/2)*x^3) - (256*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(45*a^3*(1 - 1/(
a*x))^(7/2)*x^2) + (128*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(105*a^2*(1 -
1/(a*x))^(7/2)*x) + (2*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^(7/2)
)/(9*a^4*(1 - 1/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{(c-acx)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{11/2}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{\left(16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{9/2}\sqrt{1+\frac{x}{a}}}dx,x,\right.}{9a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(64\left(\frac{1}{x}\right)^{7/2}\right.}{}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{128\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{105a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}}{9a^4\left(1-\frac{1}{a}\right.}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{256\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{45a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2} + \frac{128\sqrt{1+\frac{1}{ax}}(c-ac}{105a^2\left(1-\frac{1}{ax}\right)^{7/}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{5504\sqrt{1+\frac{1}{ax}}(c-acx)^{7/2}}{315a^4\left(1-\frac{1}{ax}\right)^{7/2}x^3} - \frac{256\sqrt{1+\frac{1}{ax}}(c-}{45a^3\left(1-\frac{1}{ax}\right)^{7/}}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.48

$$-\frac{2c^3\sqrt{\frac{1}{ax}+1}\left(35a^4x^4-220a^3x^3+642a^2x^2-1276ax+2867\right)\sqrt{c-acx}}{315a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(2867 - 1276*a*x + 642*a^2*x^2 -
220*a^3*x^3 + 35*a^4*x^4))/(315*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.59, size = 94, normalized size = 0.58

$$-\frac{2\left(35\,a^5c^3x^5-185\,a^4c^3x^4+422\,a^3c^3x^3-634\,a^2c^3x^2+1591\,ac^3x+2867\,c^3\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 185*a^4*c^3*x^4 + 422*a^3*c^3*x^3 - 634*a^2*c^3*x^
2 + 1591*a*c^3*x + 2867*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 72, normalized size = 0.45

$$\frac{2\left(ax+1\right)\left(35x^{4}a^{4}-220x^{3}a^{3}+642a^{2}x^{2}-1276ax+2867\right)\left(-acx+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}{315a\left(ax-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/315*(a*x+1)*(35*a^4*x^4-220*a^3*x^3+642*a^2*x^2-1276*a*x+2867)*(-a*c*x+c)
^(7/2)*((a*x-1)/(a*x+1))^(1/2)/a/(a*x-1)^4
```

maxima [A] time = 0.34, size = 112, normalized size = 0.70

$$-\frac{2\left(35\,a^{5}\sqrt{-c}\,c^{3}x^{5}-185\,a^{4}\sqrt{-c}\,c^{3}x^{4}+422\,a^{3}\sqrt{-c}\,c^{3}x^{3}-634\,a^{2}\sqrt{-c}\,c^{3}x^{2}+1591\,a\sqrt{-c}\,c^{3}x+2867\,\sqrt{-c}\,c^{3}\right)(ax}{315\left(a^{2}x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^5*sqrt(-c)*c^3*x^5 - 185*a^4*sqrt(-c)*c^3*x^4 + 422*a^3*sqrt(-
c)*c^3*x^3 - 634*a^2*sqrt(-c)*c^3*x^2 + 1591*a*sqrt(-c)*c^3*x + 2867*sqrt(-
c)*c^3)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

mupad [B] time = 1.36, size = 102, normalized size = 0.63

$$-\frac{2\,c^{3}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(35\,a^{4}\,x^{4}-150\,a^{3}\,x^{3}+272\,a^{2}\,x^{2}-362\,a\,x+1229\right)}{315\,a}-\frac{8192\,c^{3}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(272*a^2*x^2 - 362*a
*x - 150*a^3*x^3 + 35*a^4*x^4 + 1229))/(315*a) - (8192*c^3*(c - a*c*x)^(1/2
)*((a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.254 $\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=128

$$\frac{256c^3x\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-acx}}+\frac{64}{35}c^2x\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-acx}+\frac{24}{35}cx\sqrt{1-\frac{1}{a^2x^2}}\,(c-acx)^{3/2}+\frac{2}{7}x\sqrt{1-\frac{1}{a^2x^2}}\,(c-acx)^{5/2}$$

```
[Out] 24/35*c*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+2/7*x*(-a*c*x+c)^(5/2)*(1-1/
a^2/x^2)^(1/2)+256/35*c^3*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+64/35*c^2*
x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 197, normalized size of antiderivative = 1.54, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{344\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}{35a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\,\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{16\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}{5a^2x\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-24*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) - (344
*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(35*a^3*(1 - 1/(a*x))^(5/2)*x^2) + (1
6*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(5*a^2*(1 - 1/(a*x))^(5/2)*x) + (2*(
a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(
5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(12\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx \cdots\right.}{7a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{24\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(24\left(\frac{1}{x}\right)^{5/2}(c-ac \cdots\right.}{\cdots}$$

$$= -\frac{24\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{16\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{5a^2\left(1-\frac{1}{ax}\right)^{5/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,x(c-a \cdots}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{24\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{344\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{35a^3\left(1-\frac{1}{ax}\right)^{5/2}x^2} + \frac{16\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{5a^2\left(1-\frac{1}{ax}\right)^{5/2}x} + \cdots$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.55

$$\frac{2c^2\sqrt{\frac{1}{ax}+1}\left(5a^3x^3-27a^2x^2+71ax-177\right)\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-177 + 71*a*x - 27*a^2*x^2 + 5*a^
3*x^3))/(35*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.56, size = 83, normalized size = 0.65

$$\frac{2\left(5\,a^4c^2x^4-22\,a^3c^2x^3+44\,a^2c^2x^2-106\,ac^2x-177\,c^2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 - 22*a^3*c^2*x^3 + 44*a^2*c^2*x^2 - 106*a*c^2*x - 177*c
^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 64, normalized size = 0.50

$$\frac{2\left(ax+1\right)\left(5x^3a^3-27a^2x^2+71ax-177\right)\left(-acx+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{35a\left(ax-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/35*(a*x+1)*(5*a^3*x^3-27*a^2*x^2+71*a*x-177)*(-a*c*x+c)^(5/2)*((a*x-1)/(a
*x+1))^(1/2)/a/(a*x-1)^3
```

maxima [A] time = 0.34, size = 96, normalized size = 0.75

$$\frac{2\left(5\,a^4\sqrt{-c}\,c^2x^4-22\,a^3\sqrt{-c}\,c^2x^3+44\,a^2\sqrt{-c}\,c^2x^2-106\,a\sqrt{-c}\,c^2x-177\,\sqrt{-c}\,c^2\right)(ax-1)}{35\left(a^2x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^4*sqrt(-c)*c^2*x^4 - 22*a^3*sqrt(-c)*c^2*x^3 + 44*a^2*sqrt(-c)*c^
2*x^2 - 106*a*sqrt(-c)*c^2*x - 177*sqrt(-c)*c^2)*(a*x - 1)/((a^2*x - a)*sqr
t(a*x + 1))
```

mupad [B] time = 1.37, size = 94, normalized size = 0.73

$$\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-17\,a^2\,x^2+27\,a\,x-79\right)}{35\,a}-\frac{512\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(27*a*x - 17*a^2*x^2 +
 5*a^3*x^3 - 79))/(35*a) - (512*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))
^(1/2))/(35*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.255 $\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{64c^2x\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-acx}}+\frac{16}{15}cx\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-acx}+\frac{2}{5}x\sqrt{1-\frac{1}{a^2x^2}}(c-acx)^{3/2}$$

```
[Out] 2/5*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+64/15*c^2*x*(1-1/a^2/x^2)^(1/2)/
(-a*c*x+c)^(1/2)+16/15*c*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 89, 78, 37}

$$\frac{86\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{15a^2x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{28\sqrt{\frac{1}{ax}+1}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-28*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))/(15*a*(1 - 1/(a*x))^(3/2)) + (86*
Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))/(15*a^2*(1 - 1/(a*x))^(3/2)*x) + (2*Sq
rt[1 + 1/(a*x)]*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{(c-acx)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{-\frac{7}{a}+\frac{5x}{2a^2}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{28\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(43\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Su}}{15a^2\left(1-\right.}$$

$$= -\frac{28\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{86\sqrt{1+\frac{1}{ax}}(c-acx)^{3/2}}{15a^2\left(1-\frac{1}{ax}\right)^{3/2}x} + \frac{2\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.63

$$-\frac{2c\sqrt{\frac{1}{ax}+1}\left(3a^2x^2-14ax+43\right)\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*c*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(43 - 14*a*x + 3*a^2*x^2))/(15*a*Sq
rt[1 - 1/(a*x)])
```

fricas [A] time = 0.60, size = 64, normalized size = 0.67

$$-\frac{2\left(3\,a^3cx^3-11\,a^2cx^2+29\,acx+43\,c\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 - 11*a^2*c*x^2 + 29*a*c*x + 43*c)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1))/(a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 56, normalized size = 0.59

$$\frac{2\left(ax+1\right)\left(3a^{2}x^{2}-14ax+43\right)\left(-acx+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}{15a\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/15*(a*x+1)*(3*a^2*x^2-14*a*x+43)*(-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2)
/a/(a*x-1)^2
```

maxima [A] time = 0.33, size = 72, normalized size = 0.76

$$-\frac{2\left(3\,a^{3}\sqrt{-c}\,cx^{3}-11\,a^{2}\sqrt{-c}\,cx^{2}+29\,a\sqrt{-c}\,cx+43\,\sqrt{-c}\,c\right)(ax-1)}{15\left(a^{2}x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*a^3*sqrt(-c)*c*x^3 - 11*a^2*sqrt(-c)*c*x^2 + 29*a*sqrt(-c)*c*x + 4
3*sqrt(-c)*c)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

mupad [B] time = 1.34, size = 82, normalized size = 0.86

$$-\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(3\,a^{2}\,x^{2}-8\,a\,x+21\right)}{15\,a}-\frac{128\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 8*a*x + 2
1))/(15*a) - (128*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(15*a*(a
*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.256 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=62

$$\frac{2}{3}x\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-acx}+\frac{8cx\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-acx}}$$

```
[Out] 8/3*c*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+2/3*x*(1-1/a^2/x^2)^(1/2)*(-a*
c*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 89, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{10\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (-10*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1
 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{10\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.81

$$\frac{2\sqrt{\frac{1}{ax}+1}\,(ax-5)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.58, size = 50, normalized size = 0.81

$$\frac{2\left(a^2x^2-4\,ax-5\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

giac [A] time = 0.14, size = 43, normalized size = 0.69

$$\frac{2\,(-acx-c)^{\frac{3}{2}}|c|}{3\,ac^2} + \frac{4\,\sqrt{-acx-c}\,|c|}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

maple [A] time = 0.04, size = 47, normalized size = 0.76

$$\frac{2\,(ax+1)\,(ax-5)\,\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\,(ax-1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/3*(a*x+1)*(a*x-5)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/a
```

maxima [A] time = 0.33, size = 54, normalized size = 0.87

$$\frac{2\left(a^{2}\sqrt{-c}\,x^{2}-4\,a\sqrt{-c}\,x-5\,\sqrt{-c}\right)(ax-1)}{3\left(a^{2}x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a)
*sqrt(a*x + 1))
```

mupad [B] time = 1.28, size = 71, normalized size = 1.15

$$\frac{2\,\sqrt{c-a\,c\,x}\,\left(a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

sympy [C] time = 42.84, size = 66, normalized size = 1.06

$$\frac{4icx\sqrt{\frac{1}{acx+c}}}{3}+\frac{4ic\sqrt{\frac{1}{acx+c}}}{a}-\frac{2i\left(-acx+c\right)^{2}\sqrt{\frac{1}{acx+c}}}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] 4*I*c*x*sqrt(1/(a*c*x + c))/3 + 4*I*c*sqrt(1/(a*c*x + c))/a - 2*I*(-a*c*x +
 c)**2*sqrt(1/(a*c*x + c))/(3*a*c)
```

3.257 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=29

$$\frac{2(ax+1)e^{-\coth^{-1}(ax)}}{a\sqrt{c-acx}}$$

```
[Out] 2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6174}

$$\frac{2(ax+1)e^{-\coth^{-1}(ax)}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
[Out] (2*(1 + a*x))/(a*E^ArcCoth[a*x]*Sqrt[c - a*c*x])
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2e^{-\coth^{-1}(ax)}(1+ax)}{a\sqrt{c-acx}}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.97

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - a*c*x]
```

fricas [A] time = 0.72, size = 44, normalized size = 1.52

$$-\frac{2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 35, normalized size = 1.21

$$\frac{2\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

maxima [A] time = 0.33, size = 29, normalized size = 1.00

$$-\frac{2\left(a\sqrt{-c}\,x+\sqrt{-c}\right)}{\sqrt{ax+1}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(a*sqrt(-c)*x + sqrt(-c))/(sqrt(a*x + 1)*a*c)
```

mupad [B] time = 1.25, size = 34, normalized size = 1.17

$$\frac{\left(2\,x+\frac{2}{a}\right)\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(1/2),x)
```

```
[Out] ((2*x + 2/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(ax-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)), x)
```

3.258 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a^(1/
2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6176, 6181, 93, 206}

$$-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$
$$= -\frac{\left(2\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$
$$= -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Mathematica [**A**] time = 0.06, size = 76, normalized size = 1.00

$$-\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

fricas [**A**] time = 0.59, size = 141, normalized size = 1.86

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\log\left(-\frac{a^2x^2+2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)}{2\,ac}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{ac^{\frac{3}{2}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/c)*log(-(a^2*x^2 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1
)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a*x + 1))/
(a*c), -sqrt(2)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(
(a*x - 1)*sqrt(c)))/(a*c^(3/2))]
```

giac [**A**] time = 0.14, size = 64, normalized size = 0.84

$$\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{a\sqrt{c}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)|c|\text{sgn}\,(ax+1)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*sqrt(c)) - sqrt(2)
*arctan(sqrt(-c)/sqrt(c))/(a*sqrt(c)))*abs(c)*sgn(a*x + 1)/c^2
```

maple [A] time = 0.06, size = 78, normalized size = 1.03

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{(ax-1)\sqrt{-c\,(ax+1)}\,c^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)/c^(3/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a\,c\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-c\,(ax-1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1))**(3/2), x)
```

3.259 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=136

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

```
[Out] 1/4*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(
1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)-1/2*a^2*(1-1/a/x)^(5/2)*x^2*(1+1
/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A] time = 0.19, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(2*(a - x^(-1))*(c - a*c*x
)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqr
t[a]*Sqrt[1 + 1/(a*x)])])/(2*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] time = 0.14, size = 116, normalized size = 0.85

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(\sqrt{2}\sqrt{\frac{1}{x}}(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)-2\sqrt{a}\sqrt{\frac{1}{ax}+1}\right)}{4\sqrt{a}\,c^2(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(
-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(4*
Sqrt[a]*c^2*(-1 + a*x)*Sqrt[c - a*c*x])
```

fricas [A] time = 0.48, size = 281, normalized size = 2.07

$$\left[-\frac{\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{8\left(a^3c^3x^2-2\,a^2c^3x+ac^3\right)}, -\frac{\sqrt{2}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) - 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), -1/4*(sqrt(2)*(a^2*x^2 - 2*a*
x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.06, size = 123, normalized size = 0.90

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)c-2\sqrt{-c\,(ax+1)}\right)}{4c^{\frac{7}{2}}\,(ax-1)^{2}\,\sqrt{-c\,(ax+1)}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)/c^(7/2)*(2^(1/2)*ar
ctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c-2^(1/2)*arctan(1/2*(-c*(
a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-2*(-c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)^2/(-
c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{(c-a\,c\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.260 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}+\frac{3a^3x^3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^3x^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}$$

```
[Out] -3/32*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)
^(1/2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/4*a^3*(1-1/a/x)^(7/2)*x^2*(1
+1/a/x)^(1/2)/(a-1/x)^2/(-a*c*x+c)^(7/2)+3/16*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/
a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A] time = 0.20, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{3a^3x^3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^3x^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] -(a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^2)/(4*(a - x^(-1))^2*(c - a*c
*x)^(7/2)) + (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(16*(a - x^(
-1))*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*S
qrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))^(7/2)*(c -
a*c*x)^(7/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2} \text{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x}{a}\right)^3 \sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right.}\right.}{32\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,d\right.}{16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right.}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Mathematica [A] time = 0.18, size = 125, normalized size = 0.65

$$\frac{x\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\sqrt{\frac{1}{ax}+1}\,(7-3ax)+3\sqrt{2}\sqrt{\frac{1}{x}}\,(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{32\sqrt{a}\,c^3(ax-1)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 - 3*a*x) + 3*Sqrt[2]*S
qrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1
/(a*x)])]))/(32*Sqrt[a]*c^3*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

fricas [A] time = 0.53, size = 341, normalized size = 1.77

$$\left[-\frac{3\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4\left(3a^2x^2-4ax-7\right)\cdots}{64\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*
c^4*x - a*c^4), -1/32*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*
arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x -
c)) - 2*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))
/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

giac [A] time = 0.23, size = 88, normalized size = 0.46

$$\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3(-acx-c)^{\frac{3}{2}}+10\sqrt{-acx-c}\,c\right)}{(acx-c)^2c^2}\right)|c|\mathrm{sgn}\,(ax+1)}{32\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/32*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) + 2*(3
*(-a*c*x - c)^(3/2) + 10*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c^2))*abs(c)*sg
n(a*x + 1)/(a*c^2)
```

maple [A] time = 0.06, size = 172, normalized size = 0.89

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c+6\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac+6xa\sqrt{-\cdots}\right.}{32c^{\frac{9}{2}}\,(ax-1)^3\,\sqrt{-c\,(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] 1/32*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(
1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x^2*a^2*c+6*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+6*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^
(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-14*(-c*(a*x+1))^(1/2
)*c^(1/2))/c^(9/2)/(a*x-1)^3/(-c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a\,c\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
[Out] Timed out
```

3.261 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{32c^3\sqrt{c-acx}}{a}-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}-\frac{8c(c-acx)^{5/2}}{5a}$$

```
[Out] -16/3*c^2*(-a*c*x+c)^(3/2)/a-8/5*c*(-a*c*x+c)^(5/2)/a-4/7*(-a*c*x+c)^(7/2)/
a-2/9*(-a*c*x+c)^(9/2)/a/c+32*c^(7/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/
c^(1/2))*2^(1/2)/a-32*c^3*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 50, 63, 206}

$$-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{32c^3\sqrt{c-acx}}{a}+\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}-\frac{8c(c-acx)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-32*c^3*Sqrt[c - a*c*x])/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c -
a*c*x)^(5/2))/(5*a) - (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(
9*a*c) + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$$
$$= -\int \frac{(1-ax)(c-acx)^{7/2}}{1+ax}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{9/2}}{1+ax}\,dx}{c}$$
$$= -\frac{2(c-acx)^{9/2}}{9ac} - 2\int \frac{(c-acx)^{7/2}}{1+ax}\,dx$$
$$= -\frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (4c)\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$
$$= -\frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - \left(8c^2\right)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$
$$= -\frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - \left(16c^3\right)\int \frac{\sqrt{c}}{1}$$
$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/}}{9ac}$$

$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/}}{9ac}$$

$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/}}{9ac}$$

Mathematica [A] time = 0.09, size = 88, normalized size = 0.64

$$\frac{2c^3\left(\left(-35a^4x^4+230a^3x^3-732a^2x^2+1754ax-6257\right)\sqrt{c-acx}+5040\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*c^3*(Sqrt[c - a*c*x]*(-6257 + 1754*a*x - 732*a^2*x^2 + 230*a^3*x^3 - 35*
a^4*x^4) + 5040*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])
)/(315*a)
```

fricas [A] time = 0.51, size = 204, normalized size = 1.49

$$\left[\frac{2\left(2520\sqrt{2}\,c^{\frac{7}{2}}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\left(35a^4c^3x^4-230a^3c^3x^3+732a^2c^3x^2-1754ac^3x+6257c^3\right)\sqrt{-acx}\right.}{315a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/315*(2520*sqrt(2)*c^(7/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) - (35*a^4*c^3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2 -
 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c))/a, -2/315*(5040*sqrt(2)*sqrt(-c
)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*c^3*x^4 - 2
30*a^3*c^3*x^3 + 732*a^2*c^3*x^2 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c
))/a]
```

giac [A] time = 0.16, size = 161, normalized size = 1.18

$$-\frac{32\,\sqrt{2}\,c^{4}\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(35\,(acx-c)^{4}\sqrt{-acx+c}\,a^{8}c^{8}-90\,(acx-c)^{3}\sqrt{-acx+c}\,a^{8}c^{9}+252\,(acx-c)^{2}\cdots\right.}{315\,a^{9}c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -32*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
- 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^8*c^8 - 90*(a*c*x - c)^3*sqrt(
-a*c*x + c)*a^8*c^9 + 252*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^10 + 840*(-a
*c*x + c)^(3/2)*a^8*c^11 + 5040*sqrt(-a*c*x + c)*a^8*c^12)/(a^9*c^9)
```

maple [A] time = 0.04, size = 101, normalized size = 0.74

$$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}+\frac{2(-acx+c)^{\frac{7}{2}}c}{7}+\frac{4(-acx+c)^{\frac{5}{2}}c^{2}}{5}+\frac{8c^{3}(-acx+c)^{\frac{3}{2}}}{3}+16\sqrt{-acx+c}\;c^{4}-16c^{\frac{9}{2}}\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\;\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/9*(-a*c*x+c)^(9/2)+2/7*(-a*c*x+c)^(7/2)*c+4/5*(-a*c*x+c)^(5/2)*c^
2+8/3*c^3*(-a*c*x+c)^(3/2)+16*(-a*c*x+c)^(1/2)*c^4-16*c^(9/2)*2^(1/2)*arcta
nh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.40, size = 123, normalized size = 0.90

$$-\frac{2\left(2520\,\sqrt{2}\,c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+35\,(-acx+c)^{\frac{9}{2}}+90\,(-acx+c)^{\frac{7}{2}}c+252\,(-acx+c)^{\frac{5}{2}}c^{2}+840\,(-acx+\cdots\right.}{315\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/315*(2520*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqr
t(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) + 90*(-a*c*x + c)
^(7/2)*c + 252*(-a*c*x + c)^(5/2)*c^2 + 840*(-a*c*x + c)^(3/2)*c^3 + 5040*s
qrt(-a*c*x + c)*c^4)/(a*c)
```

mupad [B] time = 0.08, size = 112, normalized size = 0.82

$$-\frac{4\,(c-a\,c\,x)^{7/2}}{7\,a}-\frac{8\,c\,(c-a\,c\,x)^{5/2}}{5\,a}-\frac{32\,c^{3}\,\sqrt{c-a\,c\,x}}{a}-\frac{16\,c^{2}\,(c-a\,c\,x)^{3/2}}{3\,a}-\frac{2\,(c-a\,c\,x)^{9/2}}{9\,a\,c}-\frac{\sqrt{2}\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,\cdots}}{2\,\sqrt{c}}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(7/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(7/2))/(7*a) - (8*c*(c - a*c*x)^(5/2))/(5*a) - (32*c^3*(c
- a*c*x)^(1/2))/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(9/2)
```

```
)/(9*a*c) - (2^(1/2)*c^(7/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)
))*32i)/a
```

sympy [A] time = 76.39, size = 129, normalized size = 0.94

$$-\frac{32\sqrt{2}\,c^4\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{32c^3\sqrt{-acx+c}}{a}-\frac{16c^2\left(-acx+c\right)^{\frac{3}{2}}}{3a}-\frac{8c\left(-acx+c\right)^{\frac{5}{2}}}{5a}-\frac{4\left(-acx+c\right)^{\frac{7}{2}}}{7a}-\frac{2\left(-acx+c\right)^{\frac{9}{2}}}{9ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -32*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) -
 32*c**3*sqrt(-a*c*x + c)/a - 16*c**2*(-a*c*x + c)**(3/2)/(3*a) - 8*c*(-a*c
*x + c)**(5/2)/(5*a) - 4*(-a*c*x + c)**(7/2)/(7*a) - 2*(-a*c*x + c)**(9/2)/
(9*a*c)
```

3.262 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{16c^2\sqrt{c-acx}}{a}-\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}-\frac{8c(c-acx)^{3/2}}{3a}$$

```
[Out] -8/3*c*(-a*c*x+c)^(3/2)/a-4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c+1
6*c^(5/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-16*c^2*(-
a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 50, 63, 206}

$$-\frac{16c^2\sqrt{c-acx}}{a}+\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}-\frac{8c(c-acx)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-16*c^2*Sqrt[c - a*c*x])/a - (8*c*(c - a*c*x)^(3/2))/(3*a) - (4*(c - a*c*x
)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c) + (16*Sqrt[2]*c^(5/2)*ArcTan
h[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx \\
&= -\int \frac{(1-ax)(c-acx)^{5/2}}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{7/2}}{1+ax}\,dx}{c} \\
&= -\frac{2(c-acx)^{7/2}}{7ac} - 2\int \frac{(c-acx)^{5/2}}{1+ax}\,dx \\
&= -\frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - (4c)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx \\
&= -\frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - \left(8c^2\right)\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - \left(16c^3\right)\int \frac{}{(1+a} \\
&= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} + \frac{\left(32c^2\right)\text{Subst}\Big(}{} \\
&= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} + \frac{16\sqrt{2}\,c^{5/2}\tanh}{a}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 80, normalized size = 0.69

$$\frac{2c^2\left(\left(15a^3x^3-87a^2x^2+269ax-1037\right)\sqrt{c-acx}+840\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*c^2*(Sqrt[c - a*c*x]*(-1037 + 269*a*x - 87*a^2*x^2 + 15*a^3*x^3) + 840*S
qrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(105*a)
```

fricas [A] time = 0.52, size = 182, normalized size = 1.57

$$\left[\frac{2\left(420\sqrt{2}\,c^{\frac{5}{2}}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(15a^3c^2x^3-87a^2c^2x^2+269ac^2x-1037c^2\right)\sqrt{-acx+c}\right)}{105a}, -\frac{2\left(840\sqrt{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/105*(420*sqrt(2)*c^(5/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c^2*x - 1037*
```

```
c^2)*sqrt(-a*c*x + c))/a, -2/105*(840*sqrt(2)*sqrt(-c)*c^2*arctan(1/2*sqrt(
2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*
c^2*x - 1037*c^2)*sqrt(-a*c*x + c))/a]
```

giac [A] time = 0.16, size = 134, normalized size = 1.16

$$-\frac{16\sqrt{2}c^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left(15\,(acx-c)^3\sqrt{-acx+c}\,a^6c^6-42\,(acx-c)^2\sqrt{-acx+c}\,a^6c^7-140\,(-acx+c\right.}{105\,a^7c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -16*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
+ 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^6*c^6 - 42*(a*c*x - c)^2*sqrt(
-a*c*x + c)*a^6*c^7 - 140*(-a*c*x + c)^(3/2)*a^6*c^8 - 840*sqrt(-a*c*x + c)
*a^6*c^9)/(a^7*c^7)
```

maple [A] time = 0.04, size = 87, normalized size = 0.75

$$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{4(-acx+c)^{\frac{3}{2}}c^2}{3}+8\sqrt{-acx+c}\,c^3-8c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/7*(-a*c*x+c)^(7/2)+2/5*c*(-a*c*x+c)^(5/2)+4/3*(-a*c*x+c)^(3/2)*c^
2+8*(-a*c*x+c)^(1/2)*c^3-8*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(
1/2)/c^(1/2)))
```

maxima [A] time = 0.41, size = 109, normalized size = 0.94

$$-\frac{2\left(420\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+15\,(-acx+c)^{\frac{7}{2}}+42\,(-acx+c)^{\frac{5}{2}}c+140\,(-acx+c)^{\frac{3}{2}}c^2+840\sqrt{-acx+\right.}}{105\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/105*(420*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt
(2)*sqrt(c) + sqrt(-a*c*x + c))) + 15*(-a*c*x + c)^(7/2) + 42*(-a*c*x + c)^
(5/2)*c + 140*(-a*c*x + c)^(3/2)*c^2 + 840*sqrt(-a*c*x + c)*c^3)/(a*c)
```

mupad [B] time = 0.07, size = 95, normalized size = 0.82

$$-\frac{4\,(c-a\,c\,x)^{5/2}}{5\,a}-\frac{8\,c\,(c-a\,c\,x)^{3/2}}{3\,a}-\frac{16\,c^2\sqrt{c-a\,c\,x}}{a}-\frac{2\,(c-a\,c\,x)^{7/2}}{7\,a\,c}-\frac{\sqrt{2}\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\sqrt{c}}\right)16\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(5/2))/(5*a) - (8*c*(c - a*c*x)^(3/2))/(3*a) - (16*c^2*(c
- a*c*x)^(1/2))/a - (2*(c - a*c*x)^(7/2))/(7*a*c) - (2^(1/2)*c^(5/2)*atan((
2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*16i)/a
```

sympy [A] time = 51.78, size = 110, normalized size = 0.95

$$-\frac{16\sqrt{2}c^3\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{16c^2\sqrt{-acx+c}}{a}-\frac{8c\,(-acx+c)^{\frac{3}{2}}}{3a}-\frac{4\,(-acx+c)^{\frac{5}{2}}}{5a}-\frac{2\,(-acx+c)^{\frac{7}{2}}}{7ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -16*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) -
 16*c**2*sqrt(-a*c*x + c)/a - 8*c*(-a*c*x + c)**(3/2)/(3*a) - 4*(-a*c*x + c
)**(5/2)/(5*a) - 2*(-a*c*x + c)**(7/2)/(7*a*c)
```

3.263 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}-\frac{8c\sqrt{c-acx}}{a}$$

```
[Out] -4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c+8*c^(3/2)*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-8*c*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 50, 63, 206}

$$\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}-\frac{8c\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-8*c*Sqrt[c - a*c*x])/a - (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/
2))/(5*a*c) + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
)/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx \\
&= -\int \frac{(1-ax)(c-acx)^{3/2}}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{5/2}}{1+ax}\,dx}{c} \\
&= -\frac{2(c-acx)^{5/2}}{5ac}-2\int \frac{(c-acx)^{3/2}}{1+ax}\,dx \\
&= -\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}-(4c)\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{8c\sqrt{c-acx}}{a}-\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}-\left(8c^2\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= -\frac{8c\sqrt{c-acx}}{a}-\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}+\frac{(16c)\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-ac
\end{aligned}$$

$$= -\frac{8c\sqrt{c-acx}}{a}-\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}+\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.75

$$\frac{120\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-2c\left(3a^2x^2-16ax+73\right)\sqrt{c-acx}}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*c*Sqrt[c - a*c*x]*(73 - 16*a*x + 3*a^2*x^2) + 120*Sqrt[2]*c^(3/2)*ArcTa
nh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a)
```

fricas [A] time = 0.60, size = 146, normalized size = 1.54

$$\left[\frac{2\left(30\sqrt{2}\,c^{\frac{3}{2}}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)-\left(3a^2cx^2-16acx+73c\right)\sqrt{-acx+c}\right)}{15a},-\frac{2\left(60\sqrt{2}\sqrt{-c}\,c\arctan\left(\frac{\sqrt{2}\sqrt{-a
\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*c^(3/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) - (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a, -2/
15*(60*sqrt(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) +
 (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a]
```

giac [A] time = 0.14, size = 107, normalized size = 1.13

$$-\frac{8\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}\,a^4c^4+10\,(-acx+c)^{\frac{3}{2}}a^4c^5+60\,\sqrt{-acx+c}\,a^4c^6\right)}{15\,a^5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -8*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^4 + 10*(-a*c*x + c)^(3/2)*a^4
*c^5 + 60*sqrt(-a*c*x + c)*a^4*c^6)/(a^5*c^5)
```

maple [A] time = 0.04, size = 73, normalized size = 0.77

$$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c^2-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/5*(-a*c*x+c)^(5/2)+2/3*c*(-a*c*x+c)^(3/2)+4*(-a*c*x+c)^(1/2)*c^2-
4*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.41, size = 95, normalized size = 1.00

$$-\frac{2\left(30\,\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+3\,(-acx+c)^{\frac{5}{2}}+10\,(-acx+c)^{\frac{3}{2}}c+60\,\sqrt{-acx+c}\,c^2\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/15*(30*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 10*(-a*c*x + c)^(3/
2)*c + 60*sqrt(-a*c*x + c)*c^2)/(a*c)
```

mupad [B] time = 1.25, size = 78, normalized size = 0.82

$$-\frac{4\,(c-a\,c\,x)^{3/2}}{3\,a}-\frac{8\,c\,\sqrt{c-a\,c\,x}}{a}-\frac{2\,(c-a\,c\,x)^{5/2}}{5\,a\,c}-\frac{\sqrt{2}\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,8\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^
(5/2))/(5*a*c) - (2^(1/2)*c^(3/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^
(1/2)))*8i)/a
```

sympy [A] time = 32.54, size = 92, normalized size = 0.97

$$-\frac{8\sqrt{2}\,c^2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{8c\sqrt{-acx+c}}{a}-\frac{4\,(-acx+c)^{\frac{3}{2}}}{3a}-\frac{2\,(-acx+c)^{\frac{5}{2}}}{5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -8*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) -
8*c*sqrt(-a*c*x + c)/a - 4*(-a*c*x + c)**(3/2)/(3*a) - 2*(-a*c*x + c)**(5/2
)/(5*a*c)
```

3.264 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)/a-4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 50, 63, 206}

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{2(c-acx)^{3/2}}{3ac}-2\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}-(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{8\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.80

$$\frac{2(ax-7)\sqrt{c-acx}+12\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/
(Sqrt[2]*Sqrt[c])])/(3*a)
```

fricas [A] time = 0.85, size = 119, normalized size = 1.57

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}\,(ax-7)\right)}{3a},\ -\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx\right.}{3a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*ar
ctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))
/a]
```

giac [A] time = 0.15, size = 77, normalized size = 1.01

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left((-acx+c)^{\frac{3}{2}}a^{2}c^{2}+6\sqrt{-acx+c}\,a^{2}c^{3}\right)}{3\,a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2
/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

maple [A] time = 0.04, size = 59, normalized size = 0.78

$$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh
(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.41, size = 79, normalized size = 1.04

$$-\frac{2\left(3\sqrt{2}c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*
sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)/(
a*c)
```

mupad [B] time = 1.24, size = 61, normalized size = 0.80

$$\frac{4\sqrt{2}\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a}-\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}-\frac{4\sqrt{c-a\,c\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*(
c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

sympy [A] time = 3.92, size = 73, normalized size = 0.96

$$-\frac{2\left(\frac{2\sqrt{2}c^{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

3.265 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-acx}}{ac}$$

```
[Out] 2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)-2*(-a*c*x
+c)^(1/2)/a/c
```

Rubi [A] time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 50, 63, 206}

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-acx}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx\\ &= -\int \frac{1-ax}{(1+ax)\sqrt{c-acx}}\,dx\\ &= -\frac{\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{c}\\ &= -\frac{2\sqrt{c-acx}}{ac}-2\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\ &= -\frac{2\sqrt{c-acx}}{ac}+\frac{4\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{ac}\\ &= -\frac{2\sqrt{c-acx}}{ac}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.00

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-acx}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

fricas [A] time = 0.56, size = 118, normalized size = 2.03

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)-2\sqrt{-acx+c}}{ac},\ \frac{2\left(\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)-\sqrt{-acx+c}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*sqrt(c)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x +
 1)) - 2*sqrt(-a*c*x + c))/(a*c), 2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(2)*sq
rt(-a*c*x + c)*sqrt(-1/c)/(a*x - 1)) - sqrt(-a*c*x + c))/(a*c)]
```

giac [A] time = 0.16, size = 51, normalized size = 0.88

$$-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2*s
qrt(-a*c*x + c)/(a*c)
```

maple [A] time = 0.04, size = 45, normalized size = 0.78

$$-\frac{2\left(\sqrt{-acx+c}-\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/c/a*((-a*c*x+c)^(1/2)-arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1
/2)*c^(1/2))
```

maxima [A] time = 0.41, size = 68, normalized size = 1.17

$$-\frac{\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+2\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c))) + 2*sqrt(-a*c*x + c))/(a*c)
```

mupad [B] time = 0.07, size = 47, normalized size = 0.81

$$\frac{2\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-a\,c\,x}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(1/2)*(a*x + 1)),x)
```

```
[Out] (2*2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(1/2)) - (2
*(c - a*c*x)^(1/2))/(a*c)
```

sympy [A] time = 18.12, size = 60, normalized size = 1.03

$$-\frac{2\sqrt{-acx+c}}{ac}-\frac{2\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}}{\sqrt{-\frac{1}{c}}\sqrt{-acx+c}}\right)}{ac\sqrt{-\frac{1}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*sqrt(-a*c*x + c)/(a*c) - 2*sqrt(2)*atan(sqrt(2)/(sqrt(-1/c)*sqrt(-a*c*x
+ c)))/(a*c*sqrt(-1/c))
```

3.266 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

[Out] arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6130, 21, 63, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]

[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx\\ &= -\int \frac{1-ax}{(1+ax)(c-acx)^{3/2}}\,dx\\ &= -\frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{c}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^2}\\ &= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

fricas [A] time = 0.88, size = 88, normalized size = 2.38

$$\left[\frac{\sqrt{2}\log\left(\frac{ax-\frac{2\sqrt{2}\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)}{2\,ac^{\frac{3}{2}}}, \frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-\frac{1}{c}}}{ax-1}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x + 1))/
(a*c^(3/2)), sqrt(2)*sqrt(-1/c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-1/c)/
(a*x - 1))/(a*c)]
```

giac [A] time = 0.16, size = 36, normalized size = 0.97

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

maple [A] time = 0.04, size = 29, normalized size = 0.78

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(3/2),x)
```

```
[Out] arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

maxima [A] time = 0.41, size = 52, normalized size = 1.41

$$-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{2\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + s
qrt(-a*c*x + c)))/(a*c^(3/2))
```

mupad [B] time = 0.08, size = 28, normalized size = 0.76

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(3/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(3/2))
```

sympy [A] time = 15.75, size = 41, normalized size = 1.11

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(3/2),x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*c*sqrt(-c))
```

3.267 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}-\frac{1}{ac^2\sqrt{c-acx}}$$

```
[Out] 1/2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)-1/a/c^2
/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}-\frac{1}{ac^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(1/(a*c^2*Sqrt[c - a*c*x])) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(
Sqrt[2]*a*c^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx \\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{5/2}}\,dx \\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{c} \\
&= -\frac{1}{ac^2\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{2c^2} \\
&= -\frac{1}{ac^2\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^3} \\
&= -\frac{1}{ac^2\sqrt{c-acx}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 37, normalized size = 0.65

$$
-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1-ax)\right)}{ac^2\sqrt{c-acx}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]/(a*c^2*Sqrt[c - a*c*x]))
```

fricas [A] time = 0.71, size = 146, normalized size = 2.56

$$
\left[\frac{\sqrt{2}(ax-1)\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+4\sqrt{-acx+c}}{4\left(a^2c^3x-ac^3\right)}, -\frac{\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-2\sqrt{-}}{2\left(a^2c^3x-ac^3\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3), -1/2*(sqr
t(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2
*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3)]
```

giac [A] time = 0.14, size = 54, normalized size = 0.95

$$
-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2a\sqrt{-c}\,c^2} - \frac{1}{\sqrt{-acx+c}\,ac^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^2)
 - 1/(sqrt(-a*c*x + c)*a*c^2)
```

maple [A] time = 0.04, size = 50, normalized size = 0.88

$$-\frac{2\left(\frac{1}{2c\sqrt{-acx+c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -2/c/a*(1/2/c/(-a*c*x+c)^(1/2)-1/4/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(
1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.42, size = 71, normalized size = 1.25

$$-\frac{\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}+\frac{4}{\sqrt{-acx+c}\,c}}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) +
sqrt(-a*c*x + c)))/c^(3/2) + 4/(sqrt(-a*c*x + c)*c))/(a*c)
```

mupad [B] time = 1.23, size = 47, normalized size = 0.82

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{2\,a\,c^{5/2}}-\frac{1}{a\,c^2\,\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(5/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(2*a*c^(5/2)) - 1/
(a*c^2*(c - a*c*x)^(1/2))
```

sympy [A] time = 14.83, size = 61, normalized size = 1.07

$$-\frac{1}{ac^2\sqrt{-acx+c}}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2ac^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] -1/(a*c**2*sqrt(-a*c*x + c)) - sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqr
t(-c)))/(2*a*c**2*sqrt(-c))
```

3.268 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}-\frac{1}{2ac^3\sqrt{c-acx}}-\frac{1}{3ac^2(c-acx)^{3/2}}$$

```
[Out] -1/3/a/c^2/(-a*c*x+c)^(3/2)+1/4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2
))/a/c^(7/2)*2^(1/2)-1/2/a/c^3/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 51, 63, 206}

$$-\frac{1}{2ac^3\sqrt{c-acx}}-\frac{1}{3ac^2(c-acx)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/(3*a*c^2*(c - a*c*x)^(3/2)) - 1/(2*a*c^3*Sqrt[c - a*c*x]) + ArcTanh[Sqrt
[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx\\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{7/2}}\,dx\\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{c}\\
&= -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{2c^2}\\
&= -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{1}{2ac^3\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{4c^3}\\
&= -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{1}{2ac^3\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{2ac^4}\\
&= -\frac{1}{3ac^2(c-acx)^{3/2}} - \frac{1}{2ac^3\sqrt{c-acx}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.47

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(1-ax)\right)}{3ac^2(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(3/2)
)
```

fricas [A] time = 0.64, size = 196, normalized size = 2.36

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(\frac{acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{ax+1}\right)+4\,\sqrt{-acx+c}\,(3\,ax-5)}{24\,\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}, -\frac{3\,\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\text{arc}}{12\,\left(a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 5))/(a^
3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sq
rt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c)
*(3*a*x - 5))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.15, size = 73, normalized size = 0.88

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4\,a\sqrt{-c}\,c^3}-\frac{3\,acx-5\,c}{6\,(acx-c)\sqrt{-acx+c}\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^3)
 - 1/6*(3*a*c*x - 5*c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c^3)
```

maple [A] time = 0.05, size = 64, normalized size = 0.77

$$-\frac{2\left(\frac{1}{4c^2\sqrt{-acx+c}}+\frac{1}{6c(-acx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -2/c/a*(1/4/c^2/(-a*c*x+c)^(1/2)+1/6/c/(-a*c*x+c)^(3/2)-1/8/c^(5/2)*2^(1/2)
*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.41, size = 81, normalized size = 0.98

$$-\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}-\frac{4\,(3\,acx-5\,c)}{(-acx+c)^{\frac{3}{2}}c^2}}{24\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c)
 + sqrt(-a*c*x + c)))/c^(5/2) - 4*(3*a*c*x - 5*c)/((-a*c*x + c)^(3/2)*c^2))
/(a*c)
```

mupad [B] time = 0.10, size = 65, normalized size = 0.78

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{4\,a\,c^{7/2}}-\frac{\frac{c-a\,c\,x}{2\,c^2}+\frac{1}{3\,c}}{a\,c\,{(c-a\,c\,x)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(7/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(4*a*c^(7/2)) - ((
c - a*c*x)/(2*c^2) + 1/(3*c))/(a*c*(c - a*c*x)^(3/2))
```

sympy [A] time = 28.54, size = 82, normalized size = 0.99

$$-\frac{1}{3ac^2\,(-acx+c)^{\frac{3}{2}}}-\frac{1}{2ac^3\sqrt{-acx+c}}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4ac^3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] -1/(3*a*c**2*(-a*c*x + c)**(3/2)) - 1/(2*a*c**3*sqrt(-a*c*x + c)) - sqrt(2)
*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(4*a*c**3*sqrt(-c))
```

3.269 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}-\frac{1}{4ac^4\sqrt{c-acx}}-\frac{1}{6ac^3(c-acx)^{3/2}}-\frac{1}{5ac^2(c-acx)^{5/2}}$$

```
[Out] -1/5/a/c^2/(-a*c*x+c)^(5/2)-1/6/a/c^3/(-a*c*x+c)^(3/2)+1/8*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)-1/4/a/c^4/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 51, 63, 206}

$$-\frac{1}{4ac^4\sqrt{c-acx}}-\frac{1}{6ac^3(c-acx)^{3/2}}-\frac{1}{5ac^2(c-acx)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] -1/(5*a*c^2*(c - a*c*x)^(5/2)) - 1/(6*a*c^3*(c - a*c*x)^(3/2)) - 1/(4*a*c^4
*Sqrt[c - a*c*x]) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(4*Sqrt[2]*a
*c^(9/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx \\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{9/2}}\,dx \\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{7/2}}\,dx}{c} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{2c^2} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{4c^3} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{8c^4} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{4ac^5} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 39, normalized size = 0.38

$$-\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(1-ax)\right)}{5ac^2(c-acx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(5/2)
)
```

fricas [A] time = 0.48, size = 252, normalized size = 2.42

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{c}\,\log\left(\frac{acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{ax+1}\right)+4\left(15\,a^2x^2-40\,ax+37\right)\sqrt{-acx+c}}{240\left(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5\right)}, -\frac{15\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*(15*a^2*x^2 - 40*a*
x + 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^
5), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*(15*a^2*x^2 - 40*a*x + 37)*sqrt(
-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

giac [A] time = 0.13, size = 93, normalized size = 0.89

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8\,a\sqrt{-c}\,c^4}-\frac{15\,(acx-c)^2-10\,(acx-c)c+12\,c^2}{60\,(acx-c)^2\sqrt{-acx+c}\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^4)
 - 1/60*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 12*c^2)/((a*c*x - c)^2*sqrt(
-a*c*x + c)*a*c^4)
```

maple [A] time = 0.05, size = 78, normalized size = 0.75

$$-\frac{2\left(\frac{1}{8c^3\sqrt{-acx+c}}+\frac{1}{12c^2(-acx+c)^{\frac{3}{2}}}+\frac{1}{10c(-acx+c)^{\frac{5}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a*c*x+c)^(9/2),x)
```

```
[Out] -2/c/a*(1/8/c^3/(-a*c*x+c)^(1/2)+1/12/c^2/(-a*c*x+c)^(3/2)+1/10/c/(-a*c*x+c
)^(5/2)-1/16/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.41, size = 101, normalized size = 0.97

$$-\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{7}{2}}}+\frac{4\left(15\,(acx-c)^2-10\,(acx-c)c+12\,c^2\right)}{(-acx+c)^{\frac{5}{2}}c^3}}{240\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c)))/c^(7/2) + 4*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c +
12*c^2)/((-a*c*x + c)^(5/2)*c^3))/(a*c)
```

mupad [B] time = 0.09, size = 79, normalized size = 0.76

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{8\,a\,c^{9/2}}-\frac{\frac{c-a\,c\,x}{6\,c^2}+\frac{1}{5\,c}+\frac{(c-a\,c\,x)^2}{4\,c^3}}{a\,c\,(c-a\,c\,x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(9/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(8*a*c^(9/2)) - ((
c - a*c*x)/(6*c^2) + 1/(5*c) + (c - a*c*x)^2/(4*c^3))/(a*c*(c - a*c*x)^(5/2
))
```

sympy [A] time = 21.75, size = 100, normalized size = 0.96

$$-\frac{1}{5ac^2\left(-acx+c\right)^{\frac{5}{2}}}-\frac{1}{6ac^3\left(-acx+c\right)^{\frac{3}{2}}}-\frac{1}{4ac^4\sqrt{-acx+c}}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8ac^4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(9/2),x)
```

```
[Out] -1/(5*a*c**2*(-a*c*x + c)**(5/2)) - 1/(6*a*c**3*(-a*c*x + c)**(3/2)) - 1/(4
*a*c**4*sqrt(-a*c*x + c)) - sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-
c)))/(8*a*c**4*sqrt(-c))
```

3.270 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=368

$$-\frac{94208(c-acx)^{9/2}}{231a^6x^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6x^2\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^6(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] -16/33*(a-1/x)^5*(-a*c*x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)-94208
/231*(-a*c*x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/x^5/(1+1/a/x)^(1/2)-40960/231*(-a
*c*x+c)^(9/2)/a^5/(1-1/a/x)^(9/2)/x^4/(1+1/a/x)^(1/2)+4096/231*(-a*c*x+c)^(
9/2)/a^4/(1-1/a/x)^(9/2)/x^3/(1+1/a/x)^(1/2)-1024/231*(a-1/x)^3*(-a*c*x+c)^
(9/2)/a^6/(1-1/a/x)^(9/2)/x^2/(1+1/a/x)^(1/2)+320/231*(a-1/x)^4*(-a*c*x+c)^
(9/2)/a^6/(1-1/a/x)^(9/2)/x/(1+1/a/x)^(1/2)+2/11*(a-1/x)^6*x*(-a*c*x+c)^(9/
2)/a^6/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6x^2\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{4096(c-acx)^{9/2}}{231a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{40960(c-acx)^{9/2}}{231a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{94208(c-acx)^{9/2}}{231a^6x^5\left(1-\frac{1}{ax}\right)^{9/2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-16*(a - x^(-1))^5*(c - a*c*x)^(9/2))/(33*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 +
 1/(a*x)]) - (94208*(c - a*c*x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1
+ 1/(a*x)]*x^5) - (40960*(c - a*c*x)^(9/2))/(231*a^5*(1 - 1/(a*x))^(9/2)*Sq
rt[1 + 1/(a*x)]*x^4) + (4096*(c - a*c*x)^(9/2))/(231*a^4*(1 - 1/(a*x))^(9/2
)*Sqrt[1 + 1/(a*x)]*x^3) - (1024*(a - x^(-1))^3*(c - a*c*x)^(9/2))/(231*a^6
*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x^2) + (320*(a - x^(-1))^4*(c - a*c*
x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1
))^6*x*(c - a*c*x)^(9/2))/(11*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{(c-acx)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^6}{x^{13/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(24\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^5}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(160\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\cdots}{33a^2\cdots}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2\left(a-\frac{1}{x}\right)^6 x(c-acx\cdots}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\cdots}}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^2} + \frac{320\left(a-\frac{1}{x}\right)^4(c-a\cdots}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1\cdots}}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} + \frac{4096(c-acx)^{9/2}}{231a^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{1024\left(a-\frac{1}{x}\right)^3(c-\cdots}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1\cdots}}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{40960(c-acx)^{9/2}}{231a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^4} + \frac{4096(c-acx)^{\cdots}}{231a^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1\cdots}}$$

$$= -\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}} - \frac{94208(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^5} - \frac{40960(c-acx\cdots}{231a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1\cdots}}$$

Mathematica [A] time = 0.06, size = 84, normalized size = 0.23

$$\frac{2c^4\left(21a^6x^6-182a^5x^5+755a^4x^4-2132a^3x^3+5419a^2x^2-23062ax-46355\right)\sqrt{c-acx}}{231a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*c^4*Sqrt[c - a*c*x]*(-46355 - 23062*a*x + 5419*a^2*x^2 - 2132*a^3*x^3 +
755*a^4*x^4 - 182*a^5*x^5 + 21*a^6*x^6))/(231*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.48, size = 105, normalized size = 0.29

$$\frac{2\left(21\,a^{6}c^{4}x^{6}-182\,a^{5}c^{4}x^{5}+755\,a^{4}c^{4}x^{4}-2132\,a^{3}c^{4}x^{3}+5419\,a^{2}c^{4}x^{2}-23062\,ac^{4}x-46355\,c^{4}\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{231\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*a^6*c^4*x^6 - 182*a^5*c^4*x^5 + 755*a^4*c^4*x^4 - 2132*a^3*c^4*x^
3 + 5419*a^2*c^4*x^2 - 23062*a*c^4*x - 46355*c^4)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 88, normalized size = 0.24

$$\frac{2\,(ax+1)\left(21x^{6}a^{6}-182x^{5}a^{5}+755x^{4}a^{4}-2132x^{3}a^{3}+5419a^{2}x^{2}-23062ax-46355\right)(-acx+c)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{231a\,(ax-1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/231*(a*x+1)*(21*a^6*x^6-182*a^5*x^5+755*a^4*x^4-2132*a^3*x^3+5419*a^2*x^2
-23062*a*x-46355)*(-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^6
```

maxima [A] time = 0.34, size = 152, normalized size = 0.41

$$\frac{2\left(21\,a^{7}\sqrt{-c}\,c^{4}x^{7}-161\,a^{6}\sqrt{-c}\,c^{4}x^{6}+573\,a^{5}\sqrt{-c}\,c^{4}x^{5}-1377\,a^{4}\sqrt{-c}\,c^{4}x^{4}+3287\,a^{3}\sqrt{-c}\,c^{4}x^{3}-17643\,a^{2}\sqrt{-c}\,c^{4}\cdots}{231\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*a^7*sqrt(-c)*c^4*x^7 - 161*a^6*sqrt(-c)*c^4*x^6 + 573*a^5*sqrt(-c
)*c^4*x^5 - 1377*a^4*sqrt(-c)*c^4*x^4 + 3287*a^3*sqrt(-c)*c^4*x^3 - 17643*a
^2*sqrt(-c)*c^4*x^2 - 69417*a*sqrt(-c)*c^4*x - 46355*sqrt(-c)*c^4)*(a*x - 1
)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

mupad [B] time = 1.45, size = 110, normalized size = 0.30

$$\frac{2\,c^{4}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(21\,a^{5}\,x^{5}-161\,a^{4}\,x^{4}+594\,a^{3}\,x^{3}-1538\,a^{2}\,x^{2}+3881\,a\,x-19181\right)}{231\,a}-\frac{131072\,c^{4}\,\sqrt{c-a\,c\,x}\cdots}{231\,a\,\left(a\,x-1\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3881*a*x - 1538*a^2*x
^2 + 594*a^3*x^3 - 161*a^4*x^4 + 21*a^5*x^5 - 19181))/(231*a) - (131072*c^4
*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(231*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.271 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=311

$$\frac{11776(c-acx)^{7/2}}{63a^5x^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^5(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5x\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+$$

```
[Out] -40/63*(a-1/x)^4*(-a*c*x+c)^(7/2)/a^5/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+11776
/63*(-a*c*x+c)^(7/2)/a^5/(1-1/a/x)^(7/2)/x^4/(1+1/a/x)^(1/2)+5120/63*(-a*c*
x+c)^(7/2)/a^4/(1-1/a/x)^(7/2)/x^3/(1+1/a/x)^(1/2)-512/63*(-a*c*x+c)^(7/2)/
a^3/(1-1/a/x)^(7/2)/x^2/(1+1/a/x)^(1/2)+128/63*(a-1/x)^3*(-a*c*x+c)^(7/2)/a
^5/(1-1/a/x)^(7/2)/x/(1+1/a/x)^(1/2)+2/9*(a-1/x)^5*x*(-a*c*x+c)^(7/2)/a^5/(
1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{512(c-acx)^{7/2}}{63a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{5120(c-acx)^{7/2}}{63a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{11776(c-acx)^{7/2}}{63a^5x^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^5(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-40*(a - x^(-1))^4*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]) + (11776*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]*x^4) + (5120*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*x))^(7/2)*Sqrt[
1 + 1/(a*x)]*x^3) - (512*(c - a*c*x)^(7/2))/(63*a^3*(1 - 1/(a*x))^(7/2)*Sqr
t[1 + 1/(a*x)]*x^2) + (128*(a - x^(-1))^3*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1
/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1))^5*x*(c - a*c*x)^(7/2))
/(9*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
```

```
+ 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{(c-acx)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(20\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right.}{9a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(320\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/}\right.}{63}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2\left(a-\frac{1}{x}\right)^5x(c-ac}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2}+\frac{128\left(a-\frac{1}{x}\right)^3(c-}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}-\frac{512(c-acx)}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{}}$$

$$=-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{11776(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^4}+\frac{5120(c-acx}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{}}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 0.24

$$-\frac{2c^3\left(7a^5x^5-55a^4x^4+214a^3x^3-638a^2x^2+2867ax+5797\right)\sqrt{c-acx}}{63a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*c^3*Sqrt[c - a*c*x]*(5797 + 2867*a*x - 638*a^2*x^2 + 214*a^3*x^3 - 55*a
^4*x^4 + 7*a^5*x^5))/(63*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.71, size = 94, normalized size = 0.30

$$-\frac{2\left(7\,a^5c^3x^5-55\,a^4c^3x^4+214\,a^3c^3x^3-638\,a^2c^3x^2+2867\,ac^3x+5797\,c^3\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^5*c^3*x^5 - 55*a^4*c^3*x^4 + 214*a^3*c^3*x^3 - 638*a^2*c^3*x^2 +
 2867*a*c^3*x + 5797*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 80, normalized size = 0.26

$$\frac{2\left(ax+1\right)\left(7x^{5}a^{5}-55x^{4}a^{4}+214x^{3}a^{3}-638a^{2}x^{2}+2867ax+5797\right)\left(-acx+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{63a\left(ax-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/63*(a*x+1)*(7*a^5*x^5-55*a^4*x^4+214*a^3*x^3-638*a^2*x^2+2867*a*x+5797)*(
-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^5
```

maxima [A] time = 0.34, size = 136, normalized size = 0.44

$$-\frac{2\left(7\,a^{6}\sqrt{-c}\,c^{3}x^{6}-48\,a^{5}\sqrt{-c}\,c^{3}x^{5}+159\,a^{4}\sqrt{-c}\,c^{3}x^{4}-424\,a^{3}\sqrt{-c}\,c^{3}x^{3}+2229\,a^{2}\sqrt{-c}\,c^{3}x^{2}+8664\,a\sqrt{-c}\,c^{3}x+57}{63\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(7*a^6*sqrt(-c)*c^3*x^6 - 48*a^5*sqrt(-c)*c^3*x^5 + 159*a^4*sqrt(-c)*
c^3*x^4 - 424*a^3*sqrt(-c)*c^3*x^3 + 2229*a^2*sqrt(-c)*c^3*x^2 + 8664*a*sqr
t(-c)*c^3*x + 5797*sqrt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x
+ 1)^(3/2))
```

mupad [B] time = 1.41, size = 102, normalized size = 0.33

$$-\frac{2\,c^{3}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(7\,a^{4}\,x^{4}-48\,a^{3}\,x^{3}+166\,a^{2}\,x^{2}-472\,a\,x+2395\right)}{63\,a}-\frac{16384\,c^{3}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{63\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(166*a^2*x^2 - 472*a
*x - 48*a^3*x^3 + 7*a^4*x^4 + 2395))/(63*a) - (16384*c^3*(c - a*c*x)^(1/2)*
((a*x - 1)/(a*x + 1))^(1/2))/(63*a*(a*x - 1))
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.272 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=254

$$-\frac{2944(c-acx)^{5/2}}{35a^4x^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^4(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{256(c-acx)^{5/2}}{7a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+-\ 3$$

```
[Out] -32/35*(a-1/x)^3*(-a*c*x+c)^(5/2)/a^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)-2944/
35*(-a*c*x+c)^(5/2)/a^4/(1-1/a/x)^(5/2)/x^3/(1+1/a/x)^(1/2)-256/7*(-a*c*x+c
)^(5/2)/a^3/(1-1/a/x)^(5/2)/x^2/(1+1/a/x)^(1/2)+128/35*(-a*c*x+c)^(5/2)/a^2
/(1-1/a/x)^(5/2)/x/(1+1/a/x)^(1/2)+2/7*(a-1/x)^4*x*(-a*c*x+c)^(5/2)/a^4/(1-
1/a/x)^(5/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6176, 6181, 94, 89, 78, 37}

$$-\frac{256(c-acx)^{5/2}}{7a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{2944(c-acx)^{5/2}}{35a^4x^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^4(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+-\ 3$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-32*(a - x^(-1))^3*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]) - (2944*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
1/(a*x)]*x^3) - (256*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]*x^2) + (128*(c - a*c*x)^(5/2))/(35*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1
 + 1/(a*x)]*x) + (2*(a - x^(-1))^4*x*(c - a*c*x)^(5/2))/(7*a^4*(1 - 1/(a*x)
)^(5/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{(c-acx)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{7a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(192\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{S}}{35a^2\left(\right.}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{128(c-acx)^{5/2}}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+}}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{256(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}+\frac{128(c-acx)^{5/2}}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+}}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{2944(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3}-\frac{256(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+}}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.27

$$\frac{2c^2\left(5a^4x^4-36a^3x^3+142a^2x^2-708ax-1451\right)\sqrt{c-acx}}{35a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*c^2*Sqrt[c - a*c*x]*(-1451 - 708*a*x + 142*a^2*x^2 - 36*a^3*x^3 + 5*a^4*
x^4))/(35*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.69, size = 83, normalized size = 0.33

$$\frac{2\left(5\,a^4c^2x^4-36\,a^3c^2x^3+142\,a^2c^2x^2-708\,ac^2x-1451\,c^2\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 - 36*a^3*c^2*x^3 + 142*a^2*c^2*x^2 - 708*a*c^2*x - 1451
*c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [**A**] time = 0.04, size = 72, normalized size = 0.28

$$\frac{2\,(ax+1)\left(5x^4a^4-36x^3a^3+142a^2x^2-708ax-1451\right)(-acx+c)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{35a\,(ax-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/35*(a*x+1)*(5*a^4*x^4-36*a^3*x^3+142*a^2*x^2-708*a*x-1451)*(-a*c*x+c)^(5/
2)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^4
```

maxima [**A**] time = 0.35, size = 120, normalized size = 0.47

$$\frac{2\left(5\,a^5\sqrt{-c}\,c^2x^5-31\,a^4\sqrt{-c}\,c^2x^4+106\,a^3\sqrt{-c}\,c^2x^3-566\,a^2\sqrt{-c}\,c^2x^2-2159\,a\sqrt{-c}\,c^2x-1451\,\sqrt{-c}\,c^2\right)(ax-1)^2}{35\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^5*sqrt(-c)*c^2*x^5 - 31*a^4*sqrt(-c)*c^2*x^4 + 106*a^3*sqrt(-c)*c
^2*x^3 - 566*a^2*sqrt(-c)*c^2*x^2 - 2159*a*sqrt(-c)*c^2*x - 1451*sqrt(-c)*c
^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

mupad [**B**] time = 1.38, size = 94, normalized size = 0.37

$$\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-31\,a^2\,x^2+111\,a\,x-597\right)}{35\,a}-\frac{4096\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(111*a*x - 31*a^2*x^2
+ 5*a^3*x^3 - 597))/(35*a) - (4096*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x +
1))^(1/2))/(35*a*(a*x - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.273 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=195

$$\frac{184(c-acx)^{3/2}}{5a^3x^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^3(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{16(c-acx)^{3/2}}{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] -8/5*(-a*c*x+c)^(3/2)/a/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+184/5*(-a*c*x+c)^(3
/2)/a^3/(1-1/a/x)^(3/2)/x^2/(1+1/a/x)^(1/2)+16*(-a*c*x+c)^(3/2)/a^2/(1-1/a/
x)^(3/2)/x/(1+1/a/x)^(1/2)+2/5*(a-1/x)^3*x*(-a*c*x+c)^(3/2)/a^3/(1-1/a/x)^(
3/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6176, 6181, 94, 89, 78, 37}

$$\frac{184(c-acx)^{3/2}}{5a^3x^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^3(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{16(c-acx)^{3/2}}{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-8*(c - a*c*x)^(3/2))/(5*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + (184*(
c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x^2) + (16*(
c - a*c*x)^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x
^(-1))^3*x*(c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{(c-acx)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3 x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(12\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right.}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^3 x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(8\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Su}}{5a\,(1}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2\left(a-\frac{1}{x}\right)^3 x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

$$= -\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{184(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2} + \frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.29

$$-\frac{2c\left(a^3x^3-7a^2x^2+43ax+91\right)\sqrt{c-acx}}{5a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*c*Sqrt[c - a*c*x]*(91 + 43*a*x - 7*a^2*x^2 + a^3*x^3))/(5*a^2*Sqrt[1 -
1/(a^2*x^2)]*x)
```

fricas [A] time = 0.72, size = 63, normalized size = 0.32

$$-\frac{2\left(a^{3}cx^{3}-7\,a^{2}cx^{2}+43\,acx+91\,c\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^3*c*x^3 - 7*a^2*c*x^2 + 43*a*c*x + 91*c)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1))/(a^2*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 63, normalized size = 0.32

$$\frac{2\left(ax+1\right)\left(x^{3}a^{3}-7a^{2}x^{2}+43ax+91\right)\left(-acx+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{5a\left(ax-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/5*(a*x+1)*(a^3*x^3-7*a^2*x^2+43*a*x+91)*(-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1)
)^(3/2)/a/(a*x-1)^3
```

maxima [A] time = 0.34, size = 93, normalized size = 0.48

$$-\frac{2\left(a^{4}\sqrt{-c}\,cx^{4}-6\,a^{3}\sqrt{-c}\,cx^{3}+36\,a^{2}\sqrt{-c}\,cx^{2}+134\,a\sqrt{-c}\,cx+91\,\sqrt{-c}\,c\right)(ax-1)^{2}}{5\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^4*sqrt(-c)*c*x^4 - 6*a^3*sqrt(-c)*c*x^3 + 36*a^2*sqrt(-c)*c*x^2 + 1
34*a*sqrt(-c)*c*x + 91*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*
x + 1)^(3/2))
```

mupad [B] time = 1.36, size = 81, normalized size = 0.42

$$-\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2-6\,a\,x+37\right)}{5\,a}-\frac{256\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a^2*x^2 - 6*a*x + 37)
)/(5*a) - (256*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x -
 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.274 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=137

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] -20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(1
/2)/a^2/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6176, 6181, 89, 78, 37}

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(23\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}(1+\cdots}\right.}{3a^2\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.35

$$\frac{2\left(a^2x^2-10ax-23\right)\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

fricas [A] time = 0.77, size = 50, normalized size = 0.36

$$\frac{2\left(a^2x^2-10\,ax-23\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [**A**] time = 0.04, size = 55, normalized size = 0.40

$$\frac{2\left(ax+1\right)\left(a^{2}x^{2}-10ax-23\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a/
(a*x-1)^2
```

maxima [**A**] time = 0.33, size = 75, normalized size = 0.55

$$\frac{2\left(a^{3}\sqrt{-c}\,x^{3}-9\,a^{2}\sqrt{-c}\,x^{2}-33\,a\sqrt{-c}\,x-23\,\sqrt{-c}\right)(ax-1)^{2}}{3\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c))
*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

mupad [**B**] time = 1.30, size = 71, normalized size = 0.52

$$\frac{2\sqrt{c-a\,c\,x}\,\left(a\,x-9\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{64\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.275 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=85

$$\frac{2x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}+\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}$$

```
[Out] 6*(1-1/a/x)^(1/2)/a/(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)+2*x*(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}+\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]) + (2*Sqrt[1 - 1
/(a*x)]*x)/(Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\,x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}} + \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}} + \frac{2\sqrt{1-\frac{1}{ax}}\,x}{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.56

$$\frac{2\sqrt{1-\frac{1}{ax}}\,(ax+3)}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
[Out] (2*Sqrt[1 - 1/(a*x)]*(3 + a*x))/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.61, size = 44, normalized size = 0.52

$$-\frac{2\sqrt{-acx+c}\,(ax+3)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*(a*x + 3)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

giac [A] time = 0.16, size = 42, normalized size = 0.49

$$2\left(\frac{\sqrt{-acx-c}}{ac^2} - \frac{2}{\sqrt{-acx-c}\,ac}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2*(sqrt(-a*c*x - c)/(a*c^2) - 2/(sqrt(-a*c*x - c)*a*c))*abs(c)
```

maple [A] time = 0.04, size = 47, normalized size = 0.55

$$\frac{2\,(ax+1)\,(ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a\,(ax-1)\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x)
```

```
[Out] 2*(a*x+1)*(a*x+3)*((a*x-1)/(a*x+1))^(3/2)/a/(a*x-1)/(-a*c*x+c)^(1/2)
```

maxima [A] time = 0.34, size = 48, normalized size = 0.56

$$\frac{2\left(a^2x^2+4\,ax+3\right)(ax-1)}{\left(a^2\sqrt{-c}\,x-a\sqrt{-c}\,\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(a^2*x^2 + 4*a*x + 3)*(a*x - 1)/((a^2*sqrt(-c)*x - a*sqrt(-c))*(a*x + 1)^
(3/2))
```

mupad [B] time = 1.40, size = 34, normalized size = 0.40

$$\frac{\left(2\,x+\frac{6}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(1/2),x)
```

```
[Out] ((2*x + 6/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.276 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2(ax+1)e^{-3\coth^{-1}(ax)}}{a(c-acx)^{3/2}}$$

```
[Out] -2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6174}

$$-\frac{2(ax+1)e^{-3\coth^{-1}(ax)}}{a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*(1 + a*x))/(a*E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2))
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2e^{-3\coth^{-1}(ax)}(1+ax)}{a(c-acx)^{3/2}}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.41

$$-\frac{2x\left(1-\frac{1}{ax}\right)^{3/2}}{\sqrt{\frac{1}{ax}+1}\,(c-acx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(3/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))
```

fricas [A] time = 0.49, size = 43, normalized size = 1.48

$$-\frac{2\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

giac [A] time = 0.16, size = 41, normalized size = 1.41

$$\frac{\left(\frac{\sqrt{2}}{a\sqrt{-c}} - \frac{2}{\sqrt{-acx-c}\,a}\right)|c|\mathrm{sgn}\left(ax+1\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] (sqrt(2)/(a*sqrt(-c)) - 2/(sqrt(-a*c*x - c)*a))*abs(c)*sgn(a*x + 1)/c^2
```

maple [A] time = 0.03, size = 35, normalized size = 1.21

$$-\frac{2\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a\left(-acx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x)
[Out] -2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

maxima [A] time = 0.34, size = 45, normalized size = 1.55

$$-\frac{2\left(a\sqrt{-c}\,x+\sqrt{-c}\right)\left(ax-1\right)}{\left(a^2c^2x-ac^2\right)\left(ax+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -2*(a*sqrt(-c)*x + sqrt(-c))*(a*x - 1)/((a^2*c^2*x - a*c^2)*(a*x + 1)^(3/2)
)
```

mupad [B] time = 1.35, size = 32, normalized size = 1.10

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c\,\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(3/2),x)
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c*(c - a*c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
[Out] Timed out
```

3.277 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{ax^2\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}-\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] -1/2*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^
(1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+a*(1-1/a/x)^(5/2)*x^2/(-a*c*x+c
)^(5/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{ax^2\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}-\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] (a*(1 - 1/(a*x))^(5/2)*x^2)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2)) - (a^(3/2
)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])])/(Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}} - \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 1.02

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}-\sqrt{2}\sqrt{a}\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{2ac^2\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)] - Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*ArcT
anh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*a*c^2*Sqrt[1 +
 1/(a*x)]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

fricas [A] time = 0.58, size = 235, normalized size = 1.96

$$\left[-\frac{\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{4\left(a^2c^3x-ac^3\right)}, \frac{\sqrt{2}\,(ax-1)\sqrt{c}\,\text{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
- 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3*x -
a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*sqrt((a*x -
 1)/(a*x + 1)))/(a^2*c^3*x - a*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.06, size = 85, normalized size = 0.71

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1)\sqrt{-c\,(ax-1)}\,\left(\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c\,(ax+1)}+2\sqrt{c}\right)}{2\,(ax-1)^{2}\,c^{\frac{7}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)/c^(7/2)*(
arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^(1/2)+2
*c^(1/2))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{{\left(c-a\,c\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
[Out] Timed out
```

3.278 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=184

$$\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}-\frac{3a^2x^3\left(1-\frac{1}{ax}\right)^{7/2}}{4\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}$$

```
[Out] 3/8*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(
1/2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/2*a^2*(1-1/a/x)^(7/2)*x^2/(a-1
/x)/(-a*c*x+c)^(7/2)/(1+1/a/x)^(1/2)-3/4*a^2*(1-1/a/x)^(7/2)*x^3/(-a*c*x+c)
^(7/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{3a^2x^3\left(1-\frac{1}{ax}\right)^{7/2}}{4\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}+\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -(a^2*(1 - 1/(a*x))^(7/2)*x^2)/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - a*c*x
)^(7/2)) - (3*a^2*(1 - 1/(a*x))^(7/2)*x^3)/(4*Sqrt[1 + 1/(a*x)]*(c - a*c*x)
^(7/2)) + (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} + \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{}{\sqrt{x}\,(\cdots}\right.}{8\left(\frac{1}{x}\right)^{7/2}(c-acx\cdots}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} + \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2\cdots}{}}\right.}{4\left(\frac{1}{x}\right)^{7/2}(c-acx)\cdots}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} + \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\cdots}{\sqrt{a}\,\sqrt{1\cdots}}\right.}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

Mathematica [A] time = 0.16, size = 140, normalized size = 0.76

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}\,(3ax-1)-3\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{1}{ax}+1}\,(ax-1)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)\right)}{8ac^3\sqrt{\frac{1}{x}}\,\sqrt{\frac{1}{ax}+1}\,(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)]*(-1 + 3*a*x) - 3*Sqrt[2]*Sqrt[a]*Sqrt[1
+ 1/(a*x)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
```

```
*x)])]))/(8*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(-1 + a*x)*Sqrt[c - a*c*x]
)
```

fricas [A] time = 0.64, size = 285, normalized size = 1.55

$$\left[-\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-acx+c}\,(3\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{16\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)},\ -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
+ 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x - 1
)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/8*(3*sqrt(2)*(a^2*x^2
- 2*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.22, size = 90, normalized size = 0.49

$$-\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{5}{2}}}-\frac{2\,(3\,acx-c)}{\left((-acx-c)^{\frac{3}{2}}+2\,\sqrt{-acx-c}\,c\right)ac^2}\right)|c|\mathrm{sgn}\,(ax+1)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(5/2)) -
2*(3*a*c*x - c)/(((-a*c*x - c)^(3/2) + 2*sqrt(-a*c*x - c)*c)*a*c^2))*abs(c)
*sgn(a*x + 1)/c^2
```

maple [A] time = 0.07, size = 129, normalized size = 0.70

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xa\sqrt{-c\,(ax+1)}-3\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c\,(\right.}{8\,(ax-1)^3\,c^{\frac{9}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x)
```

```
[Out] -1/8*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^3*(-c*(a*x-1))^(1/2)/c^(9/2)*(
3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*(-c*(a*x+1))^(
1/2)-3*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^
(1/2)+6*x*a*c^(1/2)-2*c^(1/2))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-a\,c\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.279 $\int e^{\coth^{-1}(x)}x(1+x)\,dx$

Optimal. Leaf size=99

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{1}{3}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

```
[Out] arctanh((1/x+1)^(1/2)*((-1+x)/x)^(1/2))+1/3*(1/x+1)^(3/2)*x^2*((-1+x)/x)^(1
/2)+1/3*(1/x+1)^(5/2)*x^3*((-1+x)/x)^(1/2)+x*(1/x+1)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6175, 6180, 96, 94, 92, 206}

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{1}{3}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x),x]
```

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x + ((1 + x^(-1))^(3/2)*Sqrt[(-1 + x)/x]*
x^2)/3 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + ArcTanh[Sqrt[1 + x^(
-1)]*Sqrt[(-1 + x)/x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(x)}x(1+x)\,dx &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)x^2\,dx\\
&= -\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3-\frac{2}{3}\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3-\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,x^2}\,dx, x, \frac{1}{x}\right.\\
&= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3-\text{Subst}\\
&= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3+\text{Subst}\\
&= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3+\tanh^{-}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 41, normalized size = 0.41

$$\frac{1}{3}\sqrt{1-\frac{1}{x^2}}\,x\left(x^2+3x+5\right)+\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x),x]
[Out] (Sqrt[1 - x^(-2)]*x*(5 + 3*x + x^2))/3 + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

fricas [A] time = 0.80, size = 57, normalized size = 0.58

$$\frac{1}{3}\left(x^3+4x^2+8x+5\right)\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="fricas")
[Out] 1/3*(x^3 + 4*x^2 + 8*x + 5)*sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1
)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [A] time = 0.16, size = 105, normalized size = 1.06

$$\frac{2\left(\frac{8(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-\frac{3(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2}-9\sqrt{\frac{x-1}{x+1}}\right)}{3\left(\frac{x-1}{x+1}-1\right)^3}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="giac")
```

```
[Out] 2/3*(8*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 3*(x - 1)^2*sqrt((x - 1)/(x
+ 1))/(x + 1)^2 - 9*sqrt((x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^3 + log(sq
rt((x - 1)/(x + 1)) + 1) - log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

maple [A] time = 0.04, size = 67, normalized size = 0.68

$$\frac{(-1+x)\left(((1+x)(-1+x))^{\frac{3}{2}}+3x\sqrt{x^2-1}+6\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{3\sqrt{\frac{-1+x}{1+x}}\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x)
```

```
[Out] 1/3*(-1+x)*(((1+x)*(-1+x))^(3/2)+3*x*(x^2-1)^(1/2)+6*(x^2-1)^(1/2)+3*ln(x+(
x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

maxima [A] time = 0.31, size = 110, normalized size = 1.11

$$-\frac{2\left(3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{x-1}{x+1}}\right)}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="maxima")
```

```
[Out] -2/3*(3*((x - 1)/(x + 1))^(5/2) - 8*((x - 1)/(x + 1))^(3/2) + 9*sqrt((x - 1
)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^
3 - 1) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

mupad [B] time = 1.23, size = 94, normalized size = 0.95

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{6\sqrt{\frac{x-1}{x+1}}-\frac{16\left(\frac{x-1}{x+1}\right)^{3/2}}{3}+2\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - (6*((x - 1)/(x + 1))^(1/2) - (16*((x - 1
)/(x + 1))^(3/2))/3 + 2*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3*
(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(x+1)}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x),x)
```

```
[Out] Integral(x*(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

3.280 $\int e^{\coth^{-1}(x)}(1+x)\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{3}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\frac{3}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

```
[Out] 3/2*arctanh((1/x+1)^(1/2)*((-1+x)/x)^(1/2))+1/2*(1/x+1)^(3/2)*x^2*((-1+x)/x
)^(1/2)+3/2*x*(1/x+1)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6175, 6180, 94, 92, 206}

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{3}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\frac{3}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x),x]
```

```
[Out] (3*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + ((1 + x^(-1))^(3/2)*Sqrt[(-1 +
x)/x]*x^2)/2 + (3*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
```

m]

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(x)}(1+x)\,dx &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)x\,dx\\ &= -\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^3}\,dx,x,\frac{1}{x}\right)\\ &= \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2-\frac{3}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,x^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx,x\right.\\ &= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2+\frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1+\frac{1}{x}}\right.\\ &= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2+\frac{3}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.51

$$\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x(x+4)+\frac{3}{2}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 + x),x]
[Out] (Sqrt[1 - x^(-2)]*x*(4 + x))/2 + (3*Log[(1 + Sqrt[1 - x^(-2)])*x])/2
```

fricas [A] time = 0.52, size = 54, normalized size = 0.68

$$\frac{1}{2}\left(x^2+5x+4\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 5*x + 4)*sqrt((x - 1)/(x + 1)) + 3/2*log(sqrt((x - 1)/(x + 1)) +
 1) - 3/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [A] time = 0.15, size = 84, normalized size = 1.06

$$-\frac{\frac{3(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-5\sqrt{\frac{x-1}{x+1}}}{\left(\frac{x-1}{x+1}-1\right)^2}+\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="giac")
[Out] -(3*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 5*sqrt((x - 1)/(x + 1)))/((x -
1)/(x + 1) - 1)^2 + 3/2*log(sqrt((x - 1)/(x + 1)) + 1) - 3/2*log(abs(sqrt((
x - 1)/(x + 1)) - 1))
```

maple [A] time = 0.04, size = 57, normalized size = 0.72

$$\frac{(-1+x)\left(x\sqrt{x^2-1}+4\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{2\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x),x)
```

```
[Out] 1/2*(-1+x)*(x*(x^2-1)^(1/2)+4*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/((-1+x)/
(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

maxima [A] time = 0.31, size = 87, normalized size = 1.10

$$\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1}+\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="maxima")
```

```
[Out] (3*((x - 1)/(x + 1))^(3/2) - 5*sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) -
(x - 1)^2/(x + 1)^2 - 1) + 3/2*log(sqrt((x - 1)/(x + 1)) + 1) - 3/2*log(sqr
t((x - 1)/(x + 1)) - 1)
```

mupad [B] time = 0.04, size = 68, normalized size = 0.86

$$3\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)+\frac{5\sqrt{\frac{x-1}{x+1}}-3\left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] 3*atanh(((x - 1)/(x + 1))^(1/2)) + (5*((x - 1)/(x + 1))^(1/2) - 3*((x - 1)/
(x + 1))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x),x)
```

```
[Out] Integral((x + 1)/sqrt((x - 1)/(x + 1)), x)
```

3.281 $\int e^{\coth^{-1}(x)}(1-x)x\,dx$

Optimal. Leaf size=18

$$-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[Out] -1/3*(1-1/x^2)^(3/2)*x^3
```

Rubi [A] time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6175, 6178, 264}

$$-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)*x,x]
```

```
[Out] -((1 - x^(-2))^(3/2)*x^3)/3
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(x)}(1-x)x\,dx &= -\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)x^2\,dx\\ &= \text{Subst}\left(\int\frac{\sqrt{1-x^2}}{x^4}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.17

$$-\frac{1}{3}\sqrt{1-\frac{1}{x^2}}\,x\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)*x,x]
[Out] -1/3*(Sqrt[1 - x^(-2)]*x*(-1 + x^2))
```

fricas [A] time = 0.55, size = 24, normalized size = 1.33

$$-\frac{1}{3}\left(x^{3}+x^{2}-x-1\right)\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="fricas")
[Out] -1/3*(x^3 + x^2 - x - 1)*sqrt((x - 1)/(x + 1))
```

giac [B] time = 0.15, size = 29, normalized size = 1.61

$$\frac{8}{3\left(\sqrt{\frac{x-1}{x+1}}-\frac{1}{\sqrt{\frac{x-1}{x+1}}}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="giac")
[Out] 8/3/(sqrt((x - 1)/(x + 1)) - 1/sqrt((x - 1)/(x + 1)))^3
```

maple [A] time = 0.04, size = 22, normalized size = 1.22

$$-\frac{(1+x)(-1+x)^{2}}{3\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x)
[Out] -1/3*(1+x)*(-1+x)^2/((-1+x)/(1+x))^(1/2)
```

maxima [B] time = 0.31, size = 50, normalized size = 2.78

$$\frac{8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^{2}}{(x+1)^{2}}+\frac{(x-1)^{3}}{(x+1)^{3}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="maxima")
[Out] 8/3*((x - 1)/(x + 1))^(3/2)/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x
 - 1)^3/(x + 1)^3 - 1)
```

mupad [B] time = 1.21, size = 18, normalized size = 1.00

$$-\frac{\left(\frac{x-1}{x+1}\right)^{3/2}(x+1)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x - 1))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] -(((x - 1)/(x + 1))^(3/2)*(x + 1)^3)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right) dx - \int \frac{x^2}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)*x,x)
```

```
[Out] -Integral(-x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(x**2/sqrt(x/(x + 1)
 - 1/(x + 1)), x)
```

3.282 $\int e^{\coth^{-1}(x)}(1-x)\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2$$

```
[Out] 1/2*arctanh((1-1/x^2)^(1/2))-1/2*x^2*(1-1/x^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6175, 6178, 266, 47, 63, 206}

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x),x]
```

```
[Out] -(Sqrt[1 - x^(-2)]*x^2)/2 + ArcTanh[Sqrt[1 - x^(-2)]]/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)\,dx &= -\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)x\,dx\\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2-\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.11

$$\frac{1}{2}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x),x]
[Out] -1/2*(Sqrt[1 - x^(-2)]*x^2) + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

fricas [A] time = 0.45, size = 51, normalized size = 1.46

$$-\frac{1}{2}\left(x^2+x\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="fricas")
[Out] -1/2*(x^2 + x)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) -
 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [B] time = 0.14, size = 110, normalized size = 3.14

$$-\frac{\sqrt{\frac{x-1}{x+1}}+\frac{1}{\sqrt{\frac{x-1}{x+1}}}}{\left(\sqrt{\frac{x-1}{x+1}}+\frac{1}{\sqrt{\frac{x-1}{x+1}}}\right)^2-4}+\frac{1}{4}\log\left(\sqrt{\frac{x-1}{x+1}}+\frac{1}{\sqrt{\frac{x-1}{x+1}}}+2\right)-\frac{1}{4}\log\left(\left|\sqrt{\frac{x-1}{x+1}}+\frac{1}{\sqrt{\frac{x-1}{x+1}}}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="giac")
```

```
[Out] -(sqrt((x - 1)/(x + 1)) + 1/sqrt((x - 1)/(x + 1)))/((sqrt((x - 1)/(x + 1))
+ 1/sqrt((x - 1)/(x + 1)))^2 - 4) + 1/4*log(sqrt((x - 1)/(x + 1)) + 1/sqrt(
(x - 1)/(x + 1)) + 2) - 1/4*log(abs(sqrt((x - 1)/(x + 1)) + 1/sqrt((x - 1)/
(x + 1)) - 2))
```

maple [A] time = 0.04, size = 48, normalized size = 1.37

$$-\frac{(-1+x)\left(x\sqrt{x^2-1}-\ln\left(x+\sqrt{x^2-1}\right)\right)}{2\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x),x)
```

```
[Out] -1/2*(-1+x)*(x*(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+
x)*(-1+x))^(1/2)
```

maxima [B] time = 0.32, size = 83, normalized size = 2.37

$$\frac{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="maxima")
```

```
[Out] (((x - 1)/(x + 1))^(3/2) + sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) - (x -
 1)^2/(x + 1)^2 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x
 - 1)/(x + 1)) - 1)
```

mupad [B] time = 0.03, size = 63, normalized size = 1.80

$$\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\sqrt{\frac{x-1}{x+1}}+\left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] atanh(((x - 1)/(x + 1))^(1/2)) - (((x - 1)/(x + 1))^(1/2) + ((x - 1)/(x + 1
))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx-\int\left(-\frac{1}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x),x)
```

```
[Out] -Integral(x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(-1/sqrt(x/(x + 1) -
1/(x + 1)), x)
```

3.283 $\int e^{\coth^{-1}(x)}x(1+x)^2\,dx$

Optimal. Leaf size=133

$$\frac{1}{4}\left(\frac{1}{x}+1\right)^{7/2}\sqrt{\frac{x-1}{x}}\,x^4+\frac{1}{4}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{5}{8}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{15}{8}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\frac{15}{8}\tanh^{-1}\Bigg($$

```
[Out] 15/8*arctanh((1/x+1)^(1/2)*((-1+x)/x)^(1/2))+5/8*(1/x+1)^(3/2)*x^2*((-1+x)/
x)^(1/2)+1/4*(1/x+1)^(5/2)*x^3*((-1+x)/x)^(1/2)+1/4*(1/x+1)^(7/2)*x^4*((-1+
x)/x)^(1/2)+15/8*x*(1/x+1)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6175, 6180, 96, 94, 92, 206}

$$\frac{1}{4}\left(\frac{1}{x}+1\right)^{7/2}\sqrt{\frac{x-1}{x}}\,x^4+\frac{1}{4}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{5}{8}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{15}{8}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\frac{15}{8}\tanh^{-1}\Bigg($$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x)^2,x]
```

```
[Out] (15*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/8 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
 + x)/x]*x^2)/8 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/4 + ((1 + x^(-1
))^(7/2)*Sqrt[(-1 + x)/x]*x^4)/4 + (15*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]])/8
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx=\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^2x^3\,dx$$
$$=-\text{Subst}\left(\int\frac{(1+x)^{5/2}}{\sqrt{1-x}\,x^5}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}\,x^4-\frac{3}{4}\text{Subst}\left(\int\frac{(1+x)^{5/2}}{\sqrt{1-x}\,x^4}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}\,x^3+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}\,x^4-\frac{5}{4}\text{Subst}\left(\int\frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^3}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}\,x^3+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}\,x^4-\frac{15}{8}$$
$$=\frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}\,x^3+\frac{1}{4}\left(1-\right.$$
$$=\frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}\,x^3+\frac{1}{4}\left(1-\right.$$
$$=\frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}\,x^3+\frac{1}{4}\left(1-\right.$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.39

$$\frac{15}{8}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)+\frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x\left(2x^3+8x^2+15x+24\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(24 + 15*x + 8*x^2 + 2*x^3))/8 + (15*Log[(1 + Sqrt[1 -
x^(-2)])*x])/8
```

fricas [A] time = 0.46, size = 66, normalized size = 0.50

$$\frac{1}{8}\left(2\,x^4+10\,x^3+23\,x^2+39\,x+24\right)\sqrt{\frac{x-1}{x+1}}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^4 + 10*x^3 + 23*x^2 + 39*x + 24)*sqrt((x - 1)/(x + 1)) + 15/8*log(
sqrt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [A] time = 0.14, size = 130, normalized size = 0.98

$$-\frac{\frac{73\,(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-\frac{55\,(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2}+\frac{15\,(x-1)^3\sqrt{\frac{x-1}{x+1}}}{(x+1)^3}-49\sqrt{\frac{x-1}{x+1}}}{4\left(\frac{x-1}{x+1}-1\right)^4}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(73*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 55*(x - 1)^2*sqrt((x - 1)/
(x + 1))/(x + 1)^2 + 15*(x - 1)^3*sqrt((x - 1)/(x + 1))/(x + 1)^3 - 49*sqrt
((x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^4 + 15/8*log(sqrt((x - 1)/(x + 1))
 + 1) - 15/8*log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

maple [A] time = 0.04, size = 79, normalized size = 0.59

$$\frac{(-1+x)\left(2x\left(x^2-1\right)^{\frac{3}{2}}+8\left((1+x)(-1+x)\right)^{\frac{3}{2}}+17x\sqrt{x^2-1}+32\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)\right)}{8\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x)
```

```
[Out] 1/8*(-1+x)*(2*x*(x^2-1)^(3/2)+8*((1+x)*(-1+x))^(3/2)+17*x*(x^2-1)^(1/2)+32*
(x^2-1)^(1/2)+15*ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(
1/2)
```

maxima [A] time = 0.31, size = 138, normalized size = 1.04

$$\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-55\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+73\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-49\sqrt{\frac{x-1}{x+1}}}{4\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(15*((x - 1)/(x + 1))^(7/2) - 55*((x - 1)/(x + 1))^(5/2) + 73*((x - 1)/
(x + 1))^(3/2) - 49*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2
/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 15/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

mupad [B] time = 0.05, size = 118, normalized size = 0.89

$$\frac{15\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}+\frac{\frac{49\sqrt{\frac{x-1}{x+1}}}{4}-\frac{73\left(\frac{x-1}{x+1}\right)^{3/2}}{4}+\frac{55\left(\frac{x-1}{x+1}\right)^{5/2}}{4}-\frac{15\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6(x-1)^2}{(x+1)^2}-\frac{4(x-1)}{x+1}-\frac{4(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (15*atanh(((x - 1)/(x + 1))^(1/2)))/4 + ((49*((x - 1)/(x + 1))^(1/2))/4 - (
73*((x - 1)/(x + 1))^(3/2))/4 + (55*((x - 1)/(x + 1))^(5/2))/4 - (15*((x -
1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (4*(
x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(x+1\right)^{2}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**2,x)
```

```
[Out] Integral(x*(x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

3.284 $\int e^{\coth^{-1}(x)}(1+x)^2\,dx$

Optimal. Leaf size=106

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{5}{6}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{5}{2}\sqrt{\frac{1}{x}+1}\,\sqrt{\frac{x-1}{x}}\,x+\frac{5}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\,\sqrt{\frac{x-1}{x}}\right)$$

```
[Out] 5/2*arctanh((1/x+1)^(1/2)*((-1+x)/x)^(1/2))+5/6*(1/x+1)^(3/2)*x^2*((-1+x)/x
)^(1/2)+1/3*(1/x+1)^(5/2)*x^3*((-1+x)/x)^(1/2)+5/2*x*(1/x+1)^(1/2)*((-1+x)/
x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6175, 6180, 94, 92, 206}

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{5}{6}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{5}{2}\sqrt{\frac{1}{x}+1}\,\sqrt{\frac{x-1}{x}}\,x+\frac{5}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\,\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x)^2,x]
```

```
[Out] (5*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
+ x)/x]*x^2)/6 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + (5*ArcTanh[S
qrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
```

```
- a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1+x)^2\,dx = \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^2 x^2\,dx$$
$$= -\text{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}\,x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3 - \frac{5}{3}\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3 - \frac{5}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3 - \frac{5}{2}\text{Subst}$$
$$= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3 + \frac{5}{2}\text{Subst}$$
$$= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3 + \frac{5}{2}\tanh^{-}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.44

$$\frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x\left(2x^2+9x+22\right)+\frac{5}{2}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 + x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(22 + 9*x + 2*x^2))/6 + (5*Log[(1 + Sqrt[1 - x^(-2)])*x
])/2
```

fricas [A] time = 0.45, size = 61, normalized size = 0.58

$$\frac{1}{6}\left(2\,x^3+11\,x^2+31\,x+22\right)\sqrt{\frac{x-1}{x+1}}+\frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3 + 11*x^2 + 31*x + 22)*sqrt((x - 1)/(x + 1)) + 5/2*log(sqrt((x -
1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [A] time = 0.13, size = 107, normalized size = 1.01

$$\frac{\frac{40\,(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-\frac{15\,(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2}-33\,\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{x-1}{x+1}-1\right)^3}+\frac{5}{2}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\,\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/3*(40*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 15*(x - 1)^2*sqrt((x - 1)/(
x + 1))/(x + 1)^2 - 33*sqrt((x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^3 + 5/2
*log(sqrt((x - 1)/(x + 1)) + 1) - 5/2*log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

maple [A] time = 0.04, size = 69, normalized size = 0.65

$$\frac{(-1+x)\left(2\left((1+x)(-1+x)\right)^{\frac{3}{2}}+9x\sqrt{x^2-1}+24\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)\right)}{6\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x)
```

```
[Out] 1/6*(-1+x)*(2*((1+x)*(-1+x))^(3/2)+9*x*(x^2-1)^(1/2)+24*(x^2-1)^(1/2)+15*ln
(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

maxima [A] time = 0.32, size = 112, normalized size = 1.06

$$-\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-40\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+33\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*((x - 1)/(x + 1))^(5/2) - 40*((x - 1)/(x + 1))^(3/2) + 33*sqrt((x
 - 1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x +
1)^3 - 1) + 5/2*log(sqrt((x - 1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x +
1)) - 1)
```

mupad [B] time = 0.05, size = 94, normalized size = 0.89

$$5\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{11\sqrt{\frac{x-1}{x+1}}-\frac{40\left(\frac{x-1}{x+1}\right)^{3/2}}{3}+5\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] 5*atanh(((x - 1)/(x + 1))^(1/2)) - (11*((x - 1)/(x + 1))^(1/2) - (40*((x -
1)/(x + 1))^(3/2))/3 + 5*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3
*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**2,x)
```

```
[Out] Integral((x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

3.285 $\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx$

Optimal. Leaf size=71

$$\frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2-\frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[Out] -1/3*(1-1/x^2)^(3/2)*x^3+1/4*(1-1/x^2)^(3/2)*x^4-1/8*arctanh((1-1/x^2)^(1/2
))+1/8*x^2*(1-1/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6175, 6178, 835, 807, 266, 47, 63, 206}

$$\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2-\frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^2*x,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x^2)/8 - ((1 - x^(-2))^(3/2)*x^3)/3 + ((1 - x^(-2))^(3/2)
*x^4)/4 - ArcTanh[Sqrt[1 - x^(-2)]]/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx &= \int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^2 x^3\,dx \\
&= -\text{Subst}\left(\int \frac{(1-x)\sqrt{1-x^2}}{x^5}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{(4-x)\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 - \frac{1}{8}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 + \frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x}\right. \\
&= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\right. \\
&= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2} x^4 - \frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.73

$$
\frac{1}{24}\sqrt{1-\frac{1}{x^2}}\,x\left(6x^3-8x^2-3x+8\right)-\frac{1}{8}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^2*x,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(8 - 3*x - 8*x^2 + 6*x^3))/24 - Log[(1 + Sqrt[1 - x^(-2
)])*x]/8
```

fricas [A] time = 0.48, size = 66, normalized size = 0.93

$$\frac{1}{24}\left(6x^4 - 2x^3 - 11x^2 + 5x + 8\right)\sqrt{\frac{x-1}{x+1}} - \frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) + \frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^4 - 2*x^3 - 11*x^2 + 5*x + 8)*sqrt((x - 1)/(x + 1)) - 1/8*log(sqr
t((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [B] time = 0.13, size = 130, normalized size = 1.83

$$-\frac{\frac{11(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1} - \frac{53(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2} - \frac{3(x-1)^3\sqrt{\frac{x-1}{x+1}}}{(x+1)^3} - 3\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{x-1}{x+1} - 1\right)^4} - \frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) + \frac{1}{8}\log\left(\left|\sqrt{\frac{x-1}{x+1}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="giac")
```

```
[Out] -1/12*(11*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - 53*(x - 1)^2*sqrt((x - 1)
/(x + 1))/(x + 1)^2 - 3*(x - 1)^3*sqrt((x - 1)/(x + 1))/(x + 1)^3 - 3*sqrt(
(x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^4 - 1/8*log(sqrt((x - 1)/(x + 1)) +
 1) + 1/8*log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

maple [A] time = 0.04, size = 70, normalized size = 0.99

$$-\frac{(-1+x)\left(-6x\left(x^2-1\right)^{\frac{3}{2}} + 8\left((1+x)(-1+x)\right)^{\frac{3}{2}} - 3x\sqrt{x^2-1} + 3\ln\left(x + \sqrt{x^2-1}\right)\right)}{24\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x)
```

```
[Out] -1/24*(-1+x)*(-6*x*(x^2-1)^(3/2)+8*((1+x)*(-1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3
*ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

maxima [B] time = 0.32, size = 138, normalized size = 1.94

$$-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}} + 53\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} - 11\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} + 3\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1} - \frac{6(x-1)^2}{(x+1)^2} + \frac{4(x-1)^3}{(x+1)^3} - \frac{(x-1)^4}{(x+1)^4} - 1\right)} - \frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) + \frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="maxima")
```

```
[Out] -1/12*(3*((x - 1)/(x + 1))^(7/2) + 53*((x - 1)/(x + 1))^(5/2) - 11*((x - 1)
/(x + 1))^(3/2) + 3*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2
```

```
/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) - 1/8*log(sqr
t((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

mupad [B] time = 1.19, size = 118, normalized size = 1.66

$$\frac{\frac{\sqrt{\frac{x-1}{x+1}}}{4}-\frac{11\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{53\left(\frac{x-1}{x+1}\right)^{5/2}}{12}+\frac{\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6(x-1)^2}{(x+1)^2}-\frac{4(x-1)}{x+1}-\frac{4(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}-\frac{\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x - 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (((x - 1)/(x + 1))^(1/2)/4 - (11*((x - 1)/(x + 1))^(3/2))/12 + (53*((x - 1)
/(x + 1))^(5/2))/12 + ((x - 1)/(x + 1))^(7/2)/4)/((6*(x - 1)^2)/(x + 1)^2 -
 (4*(x - 1))/(x + 1) - (4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1) -
 atanh(((x - 1)/(x + 1))^(1/2))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2*x,x)
```

```
[Out] Integral(x*(x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

3.286 $\int e^{\coth^{-1}(x)}(1-x)^2\,dx$

Optimal. Leaf size=53

$$-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[Out] 1/3*(1-1/x^2)^(3/2)*x^3+1/2*arctanh((1-1/x^2)^(1/2))-1/2*x^2*(1-1/x^2)^(1/2
)
```

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6175, 6178, 807, 266, 47, 63, 206}

$$\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^2,x]
```

```
[Out] -(Sqrt[1 - x^(-2)]*x^2)/2 + ((1 - x^(-2))^(3/2)*x^3)/3 + ArcTanh[Sqrt[1 - x
^(-2)]]/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)^2\,dx &= \int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^2 x^2\,dx\\
&= -\text{Subst}\left(\int \frac{(1-x)\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2 + \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} x^3 + \frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.89

$$
\frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x\left(2x^2-3x-2\right)+\frac{1}{2}\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*x*(-2 - 3*x + 2*x^2))/6 + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

fricas [A] time = 0.55, size = 61, normalized size = 1.15

$$
\frac{1}{6}\left(2x^3-x^2-5x-2\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3 - x^2 - 5*x - 2)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x
 + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [B] time = 0.15, size = 107, normalized size = 2.02

$$-\frac{\frac{8\,(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}+\frac{3\,(x-1)^2\sqrt{\frac{x-1}{x+1}}}{(x+1)^2}-3\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{x-1}{x+1}-1\right)^3}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="giac")
```

```
[Out] -1/3*(8*(x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) + 3*(x - 1)^2*sqrt((x - 1)/(x
 + 1))/(x + 1)^2 - 3*sqrt((x - 1)/(x + 1)))/((x - 1)/(x + 1) - 1)^3 + 1/2*l
og(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

maple [A] time = 0.04, size = 60, normalized size = 1.13

$$\frac{(-1+x)\left(2\left((1+x)(-1+x)\right)^{\frac{3}{2}}-3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{6\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x)
```

```
[Out] 1/6*(-1+x)*(2*((1+x)*(-1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))
/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

maxima [B] time = 0.31, size = 112, normalized size = 2.11

$$-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3\,(x-1)}{x+1}-\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*((x - 1)/(x + 1))^(5/2) + 8*((x - 1)/(x + 1))^(3/2) - 3*sqrt((x - 1
)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^
3 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1))
 - 1)
```

mupad [B] time = 1.18, size = 90, normalized size = 1.70

$$\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{3/2}}{3}-\sqrt{\frac{x-1}{x+1}}+\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3\,(x-1)}{x+1}-\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] atanh(((x - 1)/(x + 1))^(1/2)) - ((8*((x - 1)/(x + 1))^(3/2))/3 - ((x - 1)/
(x + 1))^(1/2) + ((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3*(x - 1)
^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2,x)
[Out] Integral((x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

3.287 $\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx$

Optimal. Leaf size=22

$$\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x$$

```
[Out] x*(1/x+1)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6175, 6179, 95}

$$\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x),x]
```

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{1+\frac{1}{x}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x\end{aligned}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 0.68

$$x\sqrt{\frac{x^2-1}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x),x]
[Out] x*Sqrt[(-1 + x^2)/x^2]
```

fricas [A] time = 0.44, size = 15, normalized size = 0.68

$$(x+1)\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="fricas")
[Out] (x + 1)*sqrt((x - 1)/(x + 1))
```

giac [A] time = 0.14, size = 29, normalized size = 1.32

$$-\frac{2}{\sqrt{\frac{x-1}{x+1}}-\frac{1}{\sqrt{\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="giac")
[Out] -2/(sqrt((x - 1)/(x + 1)) - 1/sqrt((x - 1)/(x + 1)))
```

maple [A] time = 0.04, size = 16, normalized size = 0.73

$$\frac{-1+x}{\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x)
[Out] (-1+x)/((-1+x)/(1+x))^(1/2)
```

maxima [A] time = 0.31, size = 26, normalized size = 1.18

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="maxima")
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

mupad [B] time = 0.05, size = 26, normalized size = 1.18

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] -(2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x + 1) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)), x)
```

3.288 $\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx$

Optimal. Leaf size=22

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

```
[Out] arctanh((1/x+1)^(1/2)*((-1+x)/x)^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6175, 6180, 92, 206}

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 + x),x]
```

```
[Out] ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)x}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right) \\
&= \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 18, normalized size = 0.82

$$\log\left(x\left(\sqrt{\frac{x^2-1}{x^2}}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x),x]
[Out] Log[x*(1 + Sqrt[(-1 + x^2)/x^2])]
```

fricas [**A**] time = 0.61, size = 31, normalized size = 1.41

$$\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="fricas")
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

giac [**A**] time = 0.13, size = 32, normalized size = 1.45

$$\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="giac")
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

maple [**A**] time = 0.05, size = 35, normalized size = 1.59

$$\frac{(-1+x)\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{\frac{-1+x}{1+x}}\sqrt{(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x),x)
[Out] 1/((-1+x)/(1+x))^(1/2)*(-1+x)/((1+x)*(-1+x))^(1/2)*ln(x+(x^2-1)^(1/2))
```

maxima [A] time = 0.32, size = 31, normalized size = 1.41

$$\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="maxima")
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.64

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2))
```

sympy [A] time = 4.03, size = 29, normalized size = 1.32

$$-\log\left(-1+\frac{1}{\sqrt{1-\frac{2}{x+1}}}\right)+\log\left(1+\frac{1}{\sqrt{1-\frac{2}{x+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x),x)
[Out] -log(-1 + 1/sqrt(1 - 2/(x + 1))) + log(1 + 1/sqrt(1 - 2/(x + 1)))
```

3.289 $\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx$

Optimal. Leaf size=47

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\sqrt{1-\frac{1}{x^2}}\,x-2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -2*arctanh((1-1/x^2)^(1/2))+2*(1/x+1)/(1-1/x^2)^(1/2)-x*(1-1/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6175, 6177, 852, 1805, 807, 266, 63, 206}

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\sqrt{1-\frac{1}{x^2}}\,x-2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x),x]
```

```
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - Sqrt[1 - x^(-2)]*x - 2*ArcTanh[Sqrt[1 -
 x^(-2)]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
```

```
&&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx &= -\int \frac{e^{\coth^{-1}(x)}}{1-\frac{1}{x}}\,dx\\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^2x^2}\,dx, x, \frac{1}{x}\right)\\
&= \text{Subst}\left(\int \frac{(1+x)^2}{x^2\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{-1-2x}{x^2\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}\,x + 2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}\,x + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}\,x - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}\,x - 2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 41, normalized size = 0.87

$$-\frac{\sqrt{1-\frac{1}{x^2}}(x-3)x}{x-1}-2\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x),x]
[Out] -((Sqrt[1 - x^(-2)]*(-3 + x)*x)/(-1 + x)) - 2*Log[(1 + Sqrt[1 - x^(-2)])*x]
```

fricas [A] time = 0.58, size = 66, normalized size = 1.40

$$-\frac{2(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-2(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)+\left(x^2-2x-3\right)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="fricas")
[Out] -(2*(x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 2*(x - 1)*log(sqrt((x - 1)/(x
+ 1)) - 1) + (x^2 - 2*x - 3)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

giac [B] time = 0.13, size = 84, normalized size = 1.79

$$\frac{2\left(\frac{2(x-1)}{x+1}-1\right)}{\frac{(x-1)\sqrt{\frac{x-1}{x+1}}}{x+1}-\sqrt{\frac{x-1}{x+1}}}-2\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+2\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="giac")
[Out] 2*(2*(x - 1)/(x + 1) - 1)/((x - 1)*sqrt((x - 1)/(x + 1))/(x + 1) - sqrt((x
- 1)/(x + 1))) - 2*log(sqrt((x - 1)/(x + 1)) + 1) + 2*log(abs(sqrt((x - 1)/
(x + 1)) - 1))
```

maple [B] time = 0.05, size = 106, normalized size = 2.26

$$\frac{\left(x^2-1\right)^{\frac{3}{2}}-2x^2\sqrt{x^2-1}-2\ln\left(x+\sqrt{x^2-1}\right)x^2+4x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-2\sqrt{x^2-1}-2\ln\left(x+\sqrt{x^2}\right.}{(-1+x)\sqrt{(1+x)(-1+x)}\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x)
[Out] ((x^2-1)^(3/2)-2*x^2*(x^2-1)^(1/2)-2*ln(x+(x^2-1)^(1/2))*x^2+4*x*(x^2-1)^(1
/2)+4*ln(x+(x^2-1)^(1/2))*x-2*(x^2-1)^(1/2)-2*ln(x+(x^2-1)^(1/2)))/(-1+x)/(
(1+x)*(-1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

maxima [A] time = 0.31, size = 74, normalized size = 1.57

$$\frac{2\left(\frac{2(x-1)}{x+1}-1\right)}{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-\sqrt{\frac{x-1}{x+1}}}-2\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+2\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="maxima")
```

```
[Out] 2*(2*(x - 1)/(x + 1) - 1)/(((x - 1)/(x + 1))^(3/2) - sqrt((x - 1)/(x + 1)))
 - 2*log(sqrt((x - 1)/(x + 1)) + 1) + 2*log(sqrt((x - 1)/(x + 1)) - 1)
```

mupad [B] time = 1.19, size = 43, normalized size = 0.91

$$-\frac{2x+8\,\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)\sqrt{\frac{x-1}{x+1}}-6}{2\sqrt{\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
```

```
[Out] -(2*x + 8*atanh(((x - 1)/(x + 1))^(1/2))*((x - 1)/(x + 1))^(1/2) - 6)/(2*((
x - 1)/(x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x),x)
```

```
[Out] -Integral(x/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
x)
```

3.290 $\int \frac{e^{\coth^{-1}(x)}}{1-x} dx$

Optimal. Leaf size=33

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -arctanh((1-1/x^2)^(1/2))+2*(1/x+1)/(1-1/x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6175, 6178, 852, 1805, 266, 63, 206}

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 - x),x]
```

```
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx &= -\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)x}\,dx\\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^2 x}\,dx, x, \frac{1}{x}\right)\\
&= \text{Subst}\left(\int \frac{(1+x)^2}{x\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.15

$$
\frac{2\sqrt{1-\frac{1}{x^2}}\,x}{x-1}-\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 - x),x]
[Out] (2*Sqrt[1 - x^(-2)]*x)/(-1 + x) - Log[(1 + Sqrt[1 - x^(-2)])*x]
```

fricas [**B**] time = 0.54, size = 61, normalized size = 1.85

$$-\frac{(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-2\,(x+1)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="fricas")
```

```
[Out] -((x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - (x - 1)*log(sqrt((x - 1)/(x + 1)
) - 1) - 2*(x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

giac [**A**] time = 0.14, size = 45, normalized size = 1.36

$$\frac{2}{\sqrt{\frac{x-1}{x+1}}}-\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="giac")
```

```
[Out] 2/sqrt((x - 1)/(x + 1)) - log(sqrt((x - 1)/(x + 1)) + 1) + log(abs(sqrt((x
- 1)/(x + 1)) - 1))
```

maple [**B**] time = 0.04, size = 106, normalized size = 3.21

$$\frac{\left(x^{2}-1\right)^{\frac{3}{2}}-x^{2}\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)x^{2}+2x\sqrt{x^{2}-1}+2\ln\left(x+\sqrt{x^{2}-1}\right)x-\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)}{(-1+x)\sqrt{(1+x)(-1+x)}\,\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x),x)
```

```
[Out] ((x^2-1)^(3/2)-x^2*(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2))*x^2+2*x*(x^2-1)^(1/2)+
2*ln(x+(x^2-1)^(1/2))*x-(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/(-1+x)/((1+x)*(-
1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

maxima [**A**] time = 0.31, size = 44, normalized size = 1.33

$$\frac{2}{\sqrt{\frac{x-1}{x+1}}}-\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="maxima")
```

```
[Out] 2/sqrt((x - 1)/(x + 1)) - log(sqrt((x - 1)/(x + 1)) + 1) + log(sqrt((x - 1)
/(x + 1)) - 1)
```

mupad [**B**] time = 1.19, size = 28, normalized size = 0.85

$$\frac{2}{\sqrt{\frac{x-1}{x+1}}}-2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
```

```
[Out] 2/((x - 1)/(x + 1))^(1/2) - 2*atanh(((x - 1)/(x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x\sqrt{\frac{x}{x+1} - \frac{1}{x+1}} - \sqrt{\frac{x}{x+1} - \frac{1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x),x)
```

```
[Out] -Integral(1/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
x)
```

3.291 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2} dx$

Optimal. Leaf size=45

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)-\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

[Out] arctanh((1/x+1)^(1/2)*((-1+x)/x)^(1/2))-((-1+x)/x)^(1/2)/(1/x+1)^(1/2)

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6175, 6180, 96, 92, 206}

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)-\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcCoth[x]*x)/(1 + x)^2,x]

[Out] -(Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]]

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
```

```
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^2 x}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{3/2}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} - \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)\\ &= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} + \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 36, normalized size = 0.80

$$\log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)-\frac{\sqrt{1-\frac{1}{x^2}}\,x}{x+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x)^2,x]
[Out] -((Sqrt[1 - x^(-2)]*x)/(1 + x)) + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

fricas [A] time = 0.43, size = 44, normalized size = 0.98

$$-\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="fricas")
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/
(x + 1)) - 1)
```

giac [A] time = 0.13, size = 45, normalized size = 1.00

$$-\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="giac")
```

```
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(abs(sqrt((x -
 1)/(x + 1)) - 1))
```

maple [B] time = 0.05, size = 110, normalized size = 2.44

$$\frac{(-1+x)\left(\left(x^2-1\right)^{\frac{3}{2}}-x^2\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)x^2-2x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-\sqrt{x^2-1}+2\ln\left(x\right.\right.}{2\sqrt{\frac{-1+x}{1+x}}\sqrt{(1+x)(-1+x)}\,(1+x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x)
```

```
[Out] 1/2*(-1+x)*((x^2-1)^(3/2)-x^2*(x^2-1)^(1/2)+2*ln(x+(x^2-1)^(1/2))*x^2-2*x*(
x^2-1)^(1/2)+4*ln(x+(x^2-1)^(1/2))*x-(x^2-1)^(1/2)+2*ln(x+(x^2-1)^(1/2)))/(
(-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)/(1+x)^2
```

maxima [A] time = 0.31, size = 44, normalized size = 0.98

$$-\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/
(x + 1)) - 1)
```

mupad [B] time = 0.03, size = 28, normalized size = 0.62

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
```

```
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - ((x - 1)/(x + 1))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**2,x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**2), x)
```

3.292 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

[Out] ((-1+x)/x)^(1/2)/(1/x+1)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6175, 6180, 37}

$$\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[x]/(1 + x)^2,x]

[Out] Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^2 x^2} dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{\sqrt{1-\frac{1}{x^2}}\,x}{x+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x)/(1 + x)
```

fricas [A] time = 0.64, size = 11, normalized size = 0.52

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] sqrt((x - 1)/(x + 1))
```

giac [A] time = 0.13, size = 11, normalized size = 0.52

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] sqrt((x - 1)/(x + 1))
```

maple [A] time = 0.04, size = 21, normalized size = 1.00

$$\frac{-1+x}{(1+x)\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x)
```

```
[Out] 1/(1+x)*(-1+x)/((-1+x)/(1+x))^(1/2)
```

maxima [A] time = 0.31, size = 11, normalized size = 0.52

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] sqrt((x - 1)/(x + 1))
```

mupad [B] time = 0.18, size = 11, normalized size = 0.52

$$\sqrt{1-\frac{2}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
```

```
[Out] (1 - 2/(x + 1))^(1/2)
```

sympy [**A**] time = 5.38, size = 8, normalized size = 0.38

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**2,x)
```

```
[Out] sqrt((x - 1)/(x + 1))
```

3.293 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{\frac{5}{x}+3}{3\sqrt{1-\frac{1}{x^2}}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -4/3*(1/x+1)/(1-1/x^2)^(3/2)+arctanh((1-1/x^2)^(1/2))+1/3*(-3-5/x)/(1-1/x^2
)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6175, 6178, 852, 1805, 823, 12, 266, 63, 206}

$$-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{\frac{5}{x}+3}{3\sqrt{1-\frac{1}{x^2}}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

```
[Out] (-4*(1 + x^(-1)))/(3*(1 - x^(-2))^(3/2)) - (3 + 5/x)/(3*Sqrt[1 - x^(-2)]) +
 ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx = \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^2 x}\,dx$$
$$= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^3 x}\,dx, x, \frac{1}{x}\right)$$
$$= -\text{Subst}\left(\int \frac{(1+x)^3}{x\left(1-x^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{1}{3}\text{Subst}\left(\int \frac{-3-5x}{x\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \frac{1}{3}\text{Subst}\left(\int -\frac{3}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$
$$= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}} + \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [A] time = 0.07, size = 43, normalized size = 0.78

$$\frac{\sqrt{1-\frac{1}{x^2}}(5-7x)x}{3(x-1)^2} + \log\left(\left(\sqrt{1-\frac{1}{x^2}}+1\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*(5 - 7*x)*x)/(3*(-1 + x)^2) + Log[(1 + Sqrt[1 - x^(-2)])*
x]
```

fricas [A] time = 0.61, size = 84, normalized size = 1.53

$$\frac{3\left(x^2-2x+1\right)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-3\left(x^2-2x+1\right)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-\left(7x^2+2x-5\right)\sqrt{\frac{x-1}{x+1}}}{3\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(x^2 - 2*x + 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 3*(x^2 - 2*x + 1)*l
og(sqrt((x - 1)/(x + 1)) - 1) - (7*x^2 + 2*x - 5)*sqrt((x - 1)/(x + 1)))/(x
^2 - 2*x + 1)
```

giac [A] time = 0.15, size = 65, normalized size = 1.18

$$-\frac{(x+1)\left(\frac{6(x-1)}{x+1}+1\right)}{3(x-1)\sqrt{\frac{x-1}{x+1}}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\left|\sqrt{\frac{x-1}{x+1}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="giac")
```

```
[Out] -1/3*(x + 1)*(6*(x - 1)/(x + 1) + 1)/((x - 1)*sqrt((x - 1)/(x + 1))) + log(
sqrt((x - 1)/(x + 1)) + 1) - log(abs(sqrt((x - 1)/(x + 1)) - 1))
```

maple [B] time = 0.05, size = 146, normalized size = 2.65

$$-\frac{3x\left(x^2-1\right)^{\frac{3}{2}}-3x^3\sqrt{x^2-1}-3\ln\left(x+\sqrt{x^2-1}\right)x^3-2\left(x^2-1\right)^{\frac{3}{2}}+9x^2\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)x^2-9x}{3(-1+x)^2\sqrt{(1+x)(-1+x)}\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x)
```

```
[Out] -1/3*(3*x*(x^2-1)^(3/2)-3*x^3*(x^2-1)^(1/2)-3*ln(x+(x^2-1)^(1/2))*x^3-2*(x^
2-1)^(3/2)+9*x^2*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2))*x^2-9*x*(x^2-1)^(1/2)-
9*ln(x+(x^2-1)^(1/2))*x+3*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/(-1+x)^2/((1
+x)*(-1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

maxima [A] time = 0.32, size = 56, normalized size = 1.02

$$-\frac{\frac{6(x-1)}{x+1}+1}{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*(x - 1)/(x + 1) + 1)/((x - 1)/(x + 1))^(3/2) + log(sqrt((x - 1)/(x
+ 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

mupad [B] time = 0.04, size = 40, normalized size = 0.73

$$2\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\frac{2(x-1)}{x+1}+\frac{1}{3}}{\left(\frac{x-1}{x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
```

```
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - ((2*(x - 1))/(x + 1) + 1/3)/((x - 1)/(x
+ 1))^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**2,x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

3.294 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2} dx$

Optimal. Leaf size=24

$$-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

[Out] -1/3*(1-1/x^2)^(3/2)/(1-1/x)^3

Rubi [A] time = 0.07, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6175, 6178, 651}

$$-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[x]/(1 - x)^2,x]

[Out] -(1 - x^(-2))^(3/2)/(3*(1 - x^(-1))^3)

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^2 x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$-\frac{\sqrt{1-\frac{1}{x^2}}\,x(x+1)}{3(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/(1 - x)^2,x]
[Out] -1/3*(Sqrt[1 - x^(-2)]*x*(1 + x))/(-1 + x)^2
```

fricas [A] time = 0.51, size = 31, normalized size = 1.29

$$-\frac{\left(x^2+2\,x+1\right)\sqrt{\frac{x-1}{x+1}}}{3\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="fricas")
[Out] -1/3*(x^2 + 2*x + 1)*sqrt((x - 1)/(x + 1))/(x^2 - 2*x + 1)
```

giac [A] time = 0.14, size = 21, normalized size = 0.88

$$-\frac{x+1}{3\,(x-1)\sqrt{\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="giac")
[Out] -1/3*(x + 1)/((x - 1)*sqrt((x - 1)/(x + 1)))
```

maple [A] time = 0.03, size = 22, normalized size = 0.92

$$-\frac{1+x}{3\,(-1+x)\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x)
[Out] -1/3*(1+x)/(-1+x)/((-1+x)/(1+x))^(1/2)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.54

$$-\frac{1}{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="maxima")
[Out] -1/3/((x - 1)/(x + 1))^(3/2)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.54

$$-\frac{1}{3\left(\frac{x-1}{x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
[Out] -1/(3*((x - 1)/(x + 1))^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**2,x)
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

3.295 $\int e^{\coth^{-1}(ax)} x^m \sqrt{c - acx}\, dx$

Optimal. Leaf size=65

$$\frac{2x^{m+1}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{3}{2};-m-\frac{1}{2};-\frac{1}{ax}\right)}{(2m+3)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*x^(1+m)*hypergeom([-1/2, -3/2-m],[-1/2-m],-1/a/x)*(-a*c*x+c)^(1/2)/(3+2*m
)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6176, 6181, 64}

$$\frac{2x^{m+1}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{3}{2};-m-\frac{1}{2};-\frac{1}{ax}\right)}{(2m+3)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{\frac{1}{2}+m}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$
$$=-\frac{\left(\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\text{Subst}\left(\int x^{-\frac{5}{2}-m}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$=\frac{2x^{1+m}\sqrt{c-acx}\;{}_2F_1\left(-\frac{1}{2},-\frac{3}{2}-m;-\frac{1}{2}-m;-\frac{1}{ax}\right)}{(3+2m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.03

$$-\frac{x^{m+1}\sqrt{c-acx}\;{}_2F_1\left(-\frac{1}{2},-m-\frac{3}{2};-m-\frac{1}{2};-\frac{1}{ax}\right)}{\left(-m-\frac{3}{2}\right)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] -((x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((-3/2 - m)*Sqrt[1 - 1/(a*x)]))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-acx+c}\,(ax+1)x^m\sqrt{\frac{ax-1}{ax+1}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-a*c*x + c)*(a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}\,x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int\frac{x^m\sqrt{-acx+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\,x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\sqrt{c-a\,c\,x}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.296 $\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=140

$$\frac{16x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{8x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 16/105*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)-8/35*(1+1/a/x
)^(3/2)*x^2*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*(1+1/a/x)^(3/2)*x^3*(-a*
c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6176, 6181, 45, 37}

$$\frac{16x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{8x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (16*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (8
*(1 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*(1
+ 1/(a*x))^(3/2)*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
```

```
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$
$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7a\sqrt{1-\frac{1}{ax}}}$$
$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\right.}{35a^2\sqrt{1-\frac{1}{a}}}$$
$$= \frac{16\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.46

$$\frac{2\sqrt{\frac{1}{ax}+1}\,(ax+1)\left(15a^2x^2-12ax+8\right)\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(8 - 12*a*x + 15*a^2*x^2))/(
105*a^3*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.51, size = 69, normalized size = 0.49

$$\frac{2\left(15\,a^4x^4+18\,a^3x^3-a^2x^2+4\,ax+8\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/105*(15*a^4*x^4 + 18*a^3*x^3 - a^2*x^2 + 4*a*x + 8)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

giac [A] time = 0.17, size = 102, normalized size = 0.73

$$\frac{2\left(\frac{22\sqrt{2}\sqrt{-c}}{a^2\text{sgn}(c)}+\frac{15\,(acx+c)^3\sqrt{-acx-c}-42\,(acx+c)^2\sqrt{-acx-c}\,c-35\,(-acx-c)^{\frac{3}{2}}c^2}{a^2c^3\text{sgn}(-acx-c)}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] 2/105*(22*sqrt(2)*sqrt(-c)/(a^2*sgn(c)) + (15*(a*c*x + c)^3*sqrt(-a*c*x - c
) - 42*(a*c*x + c)^2*sqrt(-a*c*x - c)*c - 35*(-a*c*x - c)^(3/2)*c^2)/(a^2*c
^3*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.04, size = 49, normalized size = 0.35

$$\frac{2\left(ax+1\right)\left(15a^{2}x^{2}-12ax+8\right)\sqrt{-acx+c}}{105a^{3}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/105*(a*x+1)*(15*a^2*x^2-12*a*x+8)*(-a*c*x+c)^(1/2)/a^3/((a*x-1)/(a*x+1))^
(1/2)
```

maxima [A] time = 0.32, size = 55, normalized size = 0.39

$$\frac{2\left(15\,a^{3}\sqrt{-c}\,x^{3}+3\,a^{2}\sqrt{-c}\,x^{2}-4\,a\sqrt{-c}\,x+8\,\sqrt{-c}\right)\sqrt{ax+1}}{105\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2/105*(15*a^3*sqrt(-c)*x^3 + 3*a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x + 8*sqrt(-
c))*sqrt(a*x + 1)/a^3
```

mupad [B] time = 1.38, size = 57, normalized size = 0.41

$$\frac{2\,\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^{2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^{2}\,x^{2}-12\,a\,x+8\right)}{105\,a^{3}\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x^2 -
12*a*x + 8))/(105*a^3*(a*x - 1))
```

sympy [C] time = 50.08, size = 105, normalized size = 0.75

$$-\frac{94ix}{105a^{2}\sqrt{\frac{1}{acx+c}}}+\frac{10i}{21a^{3}\sqrt{\frac{1}{acx+c}}}-\frac{32i\left(-acx+c\right)^{2}}{35a^{3}c^{2}\sqrt{\frac{1}{acx+c}}}+\frac{2i\left(-acx+c\right)^{3}}{7a^{3}c^{3}\sqrt{\frac{1}{acx+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] -94*I*x/(105*a**2*sqrt(1/(a*c*x + c))) + 10*I/(21*a**3*sqrt(1/(a*c*x + c)))
 - 32*I*(-a*c*x + c)**2/(35*a**3*c**2*sqrt(1/(a*c*x + c))) + 2*I*(-a*c*x +
c)**3/(7*a**3*c**3*sqrt(1/(a*c*x + c)))
```

3.297 $\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=92

$$\frac{2x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -4/15*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(3
/2)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6176, 6181, 45, 37}

$$\frac{2x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]) + (2*(1
 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$
$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{7/2}}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}dx,x,\frac{1}{x}\right)}{5a\sqrt{1-\frac{1}{ax}}}$$
$$= -\frac{4\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.61

$$\frac{2\sqrt{\frac{1}{ax}+1}\,(ax+1)(3ax-2)\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-2 + 3*a*x)*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1
 - 1/(a*x)])
```

fricas [A] time = 0.74, size = 61, normalized size = 0.66

$$\frac{2\left(3a^3x^3+4a^2x^2-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(3*a^3*x^3 + 4*a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1))/(a^3*x - a^2)
```

giac [A] time = 0.15, size = 78, normalized size = 0.85

$$\frac{2\left(\frac{2\sqrt{2}\sqrt{-c}}{a\,\text{sgn}(c)} + \frac{3(acx+c)^2\sqrt{-acx-c}+5(-acx-c)^{\frac{3}{2}}c}{ac^2\,\text{sgn}(-acx-c)}\right)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(2*sqrt(2)*sqrt(-c)/(a*sgn(c)) + (3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 5
*(-a*c*x - c)^(3/2)*c)/(a*c^2*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.04, size = 41, normalized size = 0.45

$$\frac{2(ax+1)(3ax-2)\sqrt{-acx+c}}{15a^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/15*(a*x+1)*(3*a*x-2)*(-a*c*x+c)^(1/2)/a^2/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.34, size = 41, normalized size = 0.45

$$\frac{2\left(3a^2\sqrt{-c}x^2+a\sqrt{-c}x-2\sqrt{-c}\right)\sqrt{ax+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/15*(3*a^2*sqrt(-c)*x^2 + a*sqrt(-c)*x - 2*sqrt(-c))*sqrt(a*x + 1)/a^2
```

mupad [B] time = 1.37, size = 49, normalized size = 0.53

$$\frac{2\sqrt{c-acx}\,(ax+1)^2\,(3ax-2)\,\sqrt{\frac{ax-1}{ax+1}}}{15a^2\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/(
15*a^2*(a*x - 1))
```

sympy [C] time = 28.09, size = 71, normalized size = 0.77

$$-\frac{14ix}{15a\sqrt{\frac{1}{acx+c}}}+\frac{2i}{3a^2\sqrt{\frac{1}{acx+c}}}-\frac{2i(-acx+c)^2}{5a^2c^2\sqrt{\frac{1}{acx+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] -14*I*x/(15*a*sqrt(1/(a*c*x + c))) + 2*I/(3*a**2*sqrt(1/(a*c*x + c))) - 2*I
*(-a*c*x + c)**2/(5*a**2*c**2*sqrt(1/(a*c*x + c)))
```

3.298 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=29

$$\frac{2(ax+1)\sqrt{c-acx}\,e^{\coth^{-1}(ax)}}{3a}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6174}

$$\frac{2(ax+1)\sqrt{c-acx}\,e^{\coth^{-1}(ax)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.48

$$\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 1.17, size = 50, normalized size = 1.72

$$\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

giac [A] time = 0.16, size = 48, normalized size = 1.66

$$\frac{2\left(\frac{2\sqrt{2}\sqrt{-c}}{\operatorname{sgn}(c)}-\frac{(-acx-c)^{\frac{3}{2}}}{c\operatorname{sgn}(-acx-c)}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(2*sqrt(2)*sqrt(-c)/sgn(c) - (-a*c*x - c)^(3/2)/(c*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.03, size = 35, normalized size = 1.21

$$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x)
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

maxima [A] time = 0.34, size = 26, normalized size = 0.90

$$\frac{2\left(a\sqrt{-c}\,x+\sqrt{-c}\right)\sqrt{ax+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

mupad [B] time = 0.00, size = 43, normalized size = 1.48

$$\frac{2\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.299 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6176, 6181, 47, 54, 215}

$$\frac{2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] - (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.80

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{a+\frac{1}{x}}{a}}-\sqrt{\frac{1}{x}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[(a + x^(-1))/a] - Sqrt[x^(-1)]*ArcSinh[Sqr
t[x^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.53, size = 207, normalized size = 2.20

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, -\frac{2\left((ax-1)\sqrt{c}\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(
-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

giac [C] time = 0.19, size = 86, normalized size = 0.91

$$-2\,c\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}\,\mathrm{sgn}\left(-acx-c\right)}+\frac{\sqrt{2}-i\,\arctan\left(-i\,\sqrt{2}\right)}{\sqrt{-c}\,\mathrm{sgn}(c)}-\frac{\sqrt{-acx-c}}{c\mathrm{sgn}\left(-acx-c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*c*(arctan(sqrt(-a*c*x - c)/sqrt(c))/(sqrt(c)*sgn(-a*c*x - c)) + (sqrt(2)
 - I*arctan(-I*sqrt(2)))/(sqrt(-c)*sgn(c)) - sqrt(-a*c*x - c)/(c*sgn(-a*c*x
 - c)))
```

maple [A] time = 0.05, size = 70, normalized size = 0.74

$$-\frac{2\sqrt{-c\left(ax-1\right)}\,\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)-\sqrt{-c\left(ax+1\right)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(ax+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] -2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(ax-1\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.300 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)-arcsinh((1/x)^(1/2)/a^(
1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6176, 6181, 50, 54, 215}

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x)) - (Sqrt[a]*Sqr
t[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.78

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}+\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.44, size = 229, normalized size = 2.36

$$\left[\frac{\left(a^2x^2-ax\right)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)-2\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)}, -\frac{\left(a^2x^2-ax\right)\sqrt{c}\,\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)
*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -((a^2*x^2 -
a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c
*x - c)) + sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x
)]
```

giac [C] time = 0.20, size = 100, normalized size = 1.03

$$-\frac{\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}+\frac{a^2\sqrt{c}\arctan\left(-i\sqrt{2}\right)+\sqrt{2}a^2\sqrt{-c}}{\mathrm{sgn}(c)}+\frac{\sqrt{-acx-c}\,a}{x\mathrm{sgn}(-acx-c)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] -(a^2*sqrt(c)*arctan(sqrt(-a*c*x - c)/sqrt(c))/sgn(-a*c*x - c) + (a^2*sqrt(
c)*arctan(-I*sqrt(2)) + sqrt(2)*a^2*sqrt(-c))/sgn(c) + sqrt(-a*c*x - c)*a/(
x*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.06, size = 78, normalized size = 0.80

$$-\frac{\left(\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)xac+\sqrt{-c\,(ax+1)}\,\sqrt{c}\right)\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax+1)}\,x\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] -(arctan((-c*(a*x+1))^(1/2)/c^(1/2))*x*a*c+(-c*(a*x+1))^(1/2)*c^(1/2))*(-c*
(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2)/x/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a\,c\,x}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.301 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=101

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}$$

```
[Out] -14/3*(-a*c*x+c)^(3/2)/a^4/c+18/5*(-a*c*x+c)^(5/2)/a^4/c^2-10/7*(-a*c*x+c)^
(7/2)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4+4*(-a*c*x+c)^(1/2)/a^4
```

Rubi [A] time = 0.23, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6167, 6130, 21, 77}

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*c*x
)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^
(9/2))/(9*a^4*c^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx\\ &= -\int \frac{x^3(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= -\left(c\int \frac{x^3(1+ax)}{\sqrt{c-acx}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{a^3\sqrt{c-acx}}-\frac{7\sqrt{c-acx}}{a^3c}+\frac{9(c-acx)^{3/2}}{a^3c^2}-\frac{5(c-acx)^{5/2}}{a^3c^3}+\frac{(c-acx)^{7/2}}{a^3c^4}\right)\right)\\ &= \frac{4\sqrt{c-acx}}{a^4}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}\end{aligned}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.48

$$\frac{2\left(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272\right)\sqrt{c-acx}}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(272 + 136*a*x + 102*a^2*x^2 + 85*a^3*x^3 + 35*a^4*x^4))
/(315*a^4)
```

fricas [A] time = 0.45, size = 44, normalized size = 0.44

$$\frac{2\left(35\,a^4x^4+85\,a^3x^3+102\,a^2x^2+136\,ax+272\right)\sqrt{-acx+c}}{315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^4*x^4 + 85*a^3*x^3 + 102*a^2*x^2 + 136*a*x + 272)*sqrt(-a*c*x +
 c)/a^4
```

giac [B] time = 0.14, size = 189, normalized size = 1.87

$$\frac{2\left(\frac{9\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}\,c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{-acx+c}\,c^3\right)}{a^3c^3}+\frac{35\,(acx-c)^4\sqrt{-acx+c}+180\,(acx-c)^3\sqrt{-acx+c}\,c+378\,(acx-c)^2}{a^3c^4}\right.}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(9*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x +
 c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*x + c)*c^3)/(a^3*c^3) + (3
5*(a*c*x - c)^4*sqrt(-a*c*x + c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 3
78*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sq
rt(-a*c*x + c)*c^4)/(a^3*c^4))/a
```

maple [A] time = 0.04, size = 45, normalized size = 0.45

$$\frac{2\sqrt{-acx+c}\,\left(35x^4a^4+85x^3a^3+102a^2x^2+136ax+272\right)}{315a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/315*(-a*c*x+c)^(1/2)*(35*a^4*x^4+85*a^3*x^3+102*a^2*x^2+136*a*x+272)/a^4
```

maxima [A] time = 0.33, size = 74, normalized size = 0.73

$$\frac{2\left(35\left(-acx+c\right)^{\frac{9}{2}}-225\left(-acx+c\right)^{\frac{7}{2}}c+567\left(-acx+c\right)^{\frac{5}{2}}c^{2}-735\left(-acx+c\right)^{\frac{3}{2}}c^{3}+630\sqrt{-acx+c}\,c^{4}\right)}{315\,a^{4}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(-a*c*x + c)^(9/2) - 225*(-a*c*x + c)^(7/2)*c + 567*(-a*c*x + c)^
(5/2)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c^4
)
```

mupad [B] time = 0.04, size = 83, normalized size = 0.82

$$\frac{4\sqrt{c-a\,c\,x}}{a^4}-\frac{14\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^4\,c}+\frac{18\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^4\,c^2}-\frac{10\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^4\,c^3}+\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a^4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*c
*x)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x
)^(9/2))/(9*a^4*c^4)
```

sympy [A] time = 3.53, size = 83, normalized size = 0.82

$$\frac{2\left(2c^{4}\sqrt{-acx+c}-\frac{7c^{3}\left(-acx+c\right)^{\frac{3}{2}}}{3}+\frac{9c^{2}\left(-acx+c\right)^{\frac{5}{2}}}{5}-\frac{5c\left(-acx+c\right)^{\frac{7}{2}}}{7}+\frac{\left(-acx+c\right)^{\frac{9}{2}}}{9}\right)}{a^{4}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(2*c**4*sqrt(-a*c*x + c) - 7*c**3*(-a*c*x + c)**(3/2)/3 + 9*c**2*(-a*c*x
+ c)**(5/2)/5 - 5*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4*c*
*4)
```

3.302 $\int e^{2\coth^{-1}(ax)} x^2 \sqrt{c-acx}\, dx$

Optimal. Leaf size=80

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}$$

```
[Out] -10/3*(-a*c*x+c)^(3/2)/a^3/c+8/5*(-a*c*x+c)^(5/2)/a^3/c^2-2/7*(-a*c*x+c)^(7
/2)/a^3/c^3+4*(-a*c*x+c)^(1/2)/a^3
```

Rubi [A] time = 0.22, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6167, 6130, 21, 77}

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c*x)
^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = -\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$= -\int \frac{x^2(1+ax)\sqrt{c-acx}}{1-ax}\,dx$$
$$= -\left(c\int \frac{x^2(1+ax)}{\sqrt{c-acx}}\,dx\right)$$
$$= -\left(c\int \left(\frac{2}{a^2\sqrt{c-acx}} - \frac{5\sqrt{c-acx}}{a^2c} + \frac{4(c-acx)^{3/2}}{a^2c^2} - \frac{(c-acx)^{5/2}}{a^2c^3}\right)dx\right)$$
$$= \frac{4\sqrt{c-acx}}{a^3} - \frac{10(c-acx)^{3/2}}{3a^3c} + \frac{8(c-acx)^{5/2}}{5a^3c^2} - \frac{2(c-acx)^{7/2}}{7a^3c^3}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.50

$$\frac{2\left(15a^3x^3+39a^2x^2+52ax+104\right)\sqrt{c-acx}}{105a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(104 + 52*a*x + 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3)
```

fricas [A] time = 0.49, size = 36, normalized size = 0.45

$$\frac{2\left(15\,a^3x^3+39\,a^2x^2+52\,ax+104\right)\sqrt{-acx+c}}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*a^3*x^3 + 39*a^2*x^2 + 52*a*x + 104)*sqrt(-a*c*x + c)/a^3
```

giac [B] time = 0.14, size = 142, normalized size = 1.78

$$\frac{2\left(\frac{7\left(3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}\,c^2\right)}{a^2c^2}+\frac{3\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}\,c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{-acx+c}\,c^3\right)}{a^2c^3}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/105*(7*(3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*s
qrt(-a*c*x + c)*c^2)/(a^2*c^2) + 3*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(
a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*
x + c)*c^3)/(a^2*c^3))/a
```

maple [A] time = 0.04, size = 37, normalized size = 0.46

$$\frac{2\sqrt{-acx+c}\,\left(15x^3a^3+39a^2x^2+52ax+104\right)}{105a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] 2/105*(-a*c*x+c)^(1/2)*(15*a^3*x^3+39*a^2*x^2+52*a*x+104)/a^3
```

maxima [A] time = 0.31, size = 60, normalized size = 0.75

$$-\frac{2\left(15\left(-acx+c\right)^{\frac{7}{2}}-84\left(-acx+c\right)^{\frac{5}{2}}c+175\left(-acx+c\right)^{\frac{3}{2}}c^2-210\sqrt{-acx+c}\,c^3\right)}{105\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/105*(15*(-a*c*x + c)^(7/2) - 84*(-a*c*x + c)^(5/2)*c + 175*(-a*c*x + c)^
(3/2)*c^2 - 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

mupad [B] time = 0.05, size = 66, normalized size = 0.82

$$\frac{4\sqrt{c-a\,c\,x}}{a^3}-\frac{10\left(c-a\,c\,x\right)^{3/2}}{3\,a^3\,c}+\frac{8\left(c-a\,c\,x\right)^{5/2}}{5\,a^3\,c^2}-\frac{2\left(c-a\,c\,x\right)^{7/2}}{7\,a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c*
x)^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

sympy [A] time = 3.31, size = 68, normalized size = 0.85

$$-\frac{2\left(-2c^3\sqrt{-acx+c}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(-2*c**3*sqrt(-a*c*x + c) + 5*c**2*(-a*c*x + c)**(3/2)/3 - 4*c*(-a*c*x +
 c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a**3*c**3)
```

3.303 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=57

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{4\sqrt{c-acx}}{a^2}$$

```
[Out] -2*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c^2+4*(-a*c*x+c)^(1/2)/a
^2
```

Rubi [A] time = 0.15, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6167, 6130, 21, 77}

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{4\sqrt{c-acx}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^2 - (2*(c - a*c*x)^(3/2))/(a^2*c) + (2*(c - a*c*x)^(5
/2))/(5*a^2*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx\\
&= -\int \frac{x(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\
&= -\left(c\int \frac{x(1+ax)}{\sqrt{c-acx}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{a\sqrt{c-acx}}-\frac{3\sqrt{c-acx}}{ac}+\frac{(c-acx)^{3/2}}{ac^2}\right)dx\right)\\
&= \frac{4\sqrt{c-acx}}{a^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 31, normalized size = 0.54

$$\frac{2\left(a^2x^2+3ax+6\right)\sqrt{c-acx}}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(6 + 3*a*x + a^2*x^2))/(5*a^2)
```

fricas [A] time = 0.50, size = 27, normalized size = 0.47

$$\frac{2\left(a^2x^2+3\,ax+6\right)\sqrt{-acx+c}}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/5*(a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)/a^2
```

giac [A] time = 0.13, size = 92, normalized size = 1.61

$$-\frac{2\left(\frac{5\left((-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c\right)}{ac}-\frac{3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\sqrt{-acx+c}\,c^2}{ac^2}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] -2/15*(5*((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/(a*c) - (3*(a*c*x - c)
^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/(a
*c^2))/a
```

maple [A] time = 0.03, size = 28, normalized size = 0.49

$$\frac{2\sqrt{-acx+c}\,\left(a^2x^2+3ax+6\right)}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x*(-a*c*x+c)^(1/2),x)
[Out] 2/5*(-a*c*x+c)^(1/2)*(a^2*x^2+3*a*x+6)/a^2
```

maxima [A] time = 0.31, size = 44, normalized size = 0.77

$$\frac{2\left((-acx+c)^{\frac{5}{2}}-5\,(-acx+c)^{\frac{3}{2}}c+10\,\sqrt{-acx+c}\,c^2\right)}{5\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*((-a*c*x + c)^(5/2) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c*x + c)*c^2)
/(a^2*c^2)
```

mupad [B] time = 0.05, size = 46, normalized size = 0.81

$$\frac{2\,(c-a\,c\,x)^{5/2}-10\,c\,(c-a\,c\,x)^{3/2}+20\,c^2\,\sqrt{c-a\,c\,x}}{5\,a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*(c - a*c*x)^(5/2) - 10*c*(c - a*c*x)^(3/2) + 20*c^2*(c - a*c*x)^(1/2))/(
5*a^2*c^2)
```

sympy [A] time = 3.89, size = 48, normalized size = 0.84

$$\frac{2\left(2c^2\sqrt{-acx+c}-c\,(-acx+c)^{\frac{3}{2}}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(2*c**2*sqrt(-a*c*x + c) - c*(-a*c*x + c)**(3/2) + (-a*c*x + c)**(5/2)/5)
/(a**2*c**2)
```

3.304 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6130, 21, 43}

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\ &= -\int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= -\left(c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{\sqrt{c-acx}}-\frac{\sqrt{c-acx}}{c}\right)dx\right)\\ &= \frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.61

$$\frac{2(ax+5)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
[Out] (2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

fricas [A] time = 0.46, size = 19, normalized size = 0.50

$$\frac{2\sqrt{-acx+c}\,(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

giac [A] time = 0.14, size = 44, normalized size = 1.16

$$\frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c}{c}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

maple [A] time = 0.03, size = 20, normalized size = 0.53

$$\frac{2\sqrt{-acx+c}\;(ax+5)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2),x)
[Out] 2/3*(-a*c*x+c)^(1/2)*(a*x+5)/a
```

maxima [A] time = 0.32, size = 30, normalized size = 0.79

$$-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

mupad [B] time = 0.00, size = 32, normalized size = 0.84

$$\frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] (4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

sympy [A] time = 2.90, size = 31, normalized size = 0.82

$$-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
[Out] -2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

3.305 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=39

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6167, 6130, 21, 80, 63, 208}

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x(1-ax)}\,dx \\
&= -\left(c\int \frac{1+ax}{x\sqrt{c-acx}}\,dx\right) \\
&= 2\sqrt{c-acx} - c\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= 2\sqrt{c-acx} + \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right)}{a} \\
&= 2\sqrt{c-acx} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.00

$$2\sqrt{c-acx} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

fricas [A] time = 0.46, size = 82, normalized size = 2.10

$$\left[\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c}, -2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)+2\sqrt{-acx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x
+ c), -2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x + c)]
```

giac [A] time = 0.14, size = 40, normalized size = 1.03

$$-2c\left(\frac{\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*c*(arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/c)
```

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}\,+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

maxima [A] time = 0.40, size = 49, normalized size = 1.26

$$-\sqrt{c}\,\log\left(\frac{\sqrt{-acx+c}\,-\sqrt{c}}{\sqrt{-acx+c}\,+\sqrt{c}}\right)+2\,\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c))) + 2
*sqrt(-a*c*x + c)
```

mupad [B] time = 1.19, size = 31, normalized size = 0.79

$$2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+2\,\sqrt{c-a\,c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

```
[Out] 2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2)
```

sympy [A] time = 4.09, size = 39, normalized size = 1.00

$$-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] -2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

3.306 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 3*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.20, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6167, 6130, 21, 78, 63, 208}

$$\frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^2(1-ax)}\,dx\\
&= -\left(c\int \frac{1+ax}{x^2\sqrt{c-acx}}\,dx\right)\\
&= \frac{\sqrt{c-acx}}{x}-\frac{1}{2}(3ac)\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{x}+3\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)\\
&= \frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.00

$$\frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

fricas [A] time = 0.58, size = 97, normalized size = 2.31

$$\left[\frac{3\,a\sqrt{c}\,x\log\left(\frac{acx-2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}}{2\,x},-\frac{3\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fricas")
[Out] [1/2*(3*a*sqrt(c)*x*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*s
qrt(-a*c*x + c))/x, -(3*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) -
sqrt(-a*c*x + c))/x]
```

giac [A] time = 0.14, size = 48, normalized size = 1.14

$$-\frac{\frac{3\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}\,a}{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")
[Out] -(3*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)*a/x
)/a
```

maple [A] time = 0.04, size = 45, normalized size = 1.07

$$-2ac\left(-\frac{\sqrt{-acx+c}}{2xac}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] -2*a*c*(-1/2*(-a*c*x+c)^(1/2)/x/a/c-3/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^
(1/2)))
```

maxima [A] time = 0.42, size = 62, normalized size = 1.48

$$-\frac{1}{2}\,ac\left(\frac{3\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\,\sqrt{-acx+c}}{acx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*c*(3*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/
sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

mupad [B] time = 0.06, size = 34, normalized size = 0.81

$$\frac{\sqrt{c-a\,c\,x}}{x}+3\,a\,\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

```
[Out] (c - a*c*x)^(1/2)/x + 3*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2))
```

sympy [B] time = 8.45, size = 119, normalized size = 2.83

$$-\frac{ac^2\sqrt{\frac{1}{c^3}}\,\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{ac^2\sqrt{\frac{1}{c^3}}\,\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{2ac\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] -a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + a*c**
2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - 2*a*c*atan(s
qrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

3.307 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=68

$$\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}$$

```
[Out] 7/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a*c*x+c)^(1/2)/x^2+
7/4*a*(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.21, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6167, 6130, 21, 78, 51, 63, 208}

$$\frac{7}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] Sqrt[c - a*c*x]/(2*x^2) + (7*a*Sqrt[c - a*c*x])/(4*x) + (7*a^2*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^3(1-ax)}\,dx \\
&= -\left(c\int \frac{1+ax}{x^3\sqrt{c-acx}}\,dx\right) \\
&= \frac{\sqrt{c-acx}}{2x^2}-\frac{1}{4}(7ac)\int \frac{1}{x^2\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}-\frac{1}{8}\left(7a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}+\frac{1}{4}(7a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right) \\
&= \frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}+\frac{7}{4}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.81

$$\frac{7}{4}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{(7ax+2)\sqrt{c-acx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] ((2 + 7*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (7*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c
*x]/Sqrt[c]])/4
```

fricas [A] time = 0.56, size = 117, normalized size = 1.72

$$\left[\frac{7\,a^2\sqrt{c}\,x^2\log\left(\frac{acx-2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}\,(7\,ax+2)}{8\,x^2},-\frac{7\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-\sqrt{-acx+c}\,(7\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(7*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) +
 2*sqrt(-a*c*x + c)*(7*a*x + 2))/x^2, -1/4*(7*a^2*sqrt(-c)*x^2*arctan(sqrt(
-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(7*a*x + 2))/x^2]
```

giac [A] time = 0.14, size = 76, normalized size = 1.12

$$-\frac{\frac{7\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{7\,(-acx+c)^{\frac{3}{2}}a^3c-9\,\sqrt{-acx+c}\,a^3c^2}{a^2c^2x^2}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(7*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + (7*(-a*c*x + c)^
(3/2)*a^3*c - 9*sqrt(-a*c*x + c)*a^3*c^2)/(a^2*c^2*x^2))/a
```

maple [A] time = 0.05, size = 65, normalized size = 0.96

$$2a^2c^2\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{x^2a^2c^2}+\frac{7\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x^3,x)
```

```
[Out] 2*a^2*c^2*((-7/8/c*(-a*c*x+c)^(3/2)+9/8*(-a*c*x+c)^(1/2))/x^2/a^2/c^2+7/8/c
^(3/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.62, size = 103, normalized size = 1.51

$$-\frac{1}{8}\,a^2c^2\left(\frac{2\left(7\,(-acx+c)^{\frac{3}{2}}-9\,\sqrt{-acx+c}\,c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{7\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*c^2*(2*(7*(-a*c*x + c)^(3/2) - 9*sqrt(-a*c*x + c)*c)/((a*c*x - c)^
2*c + 2*(a*c*x - c)*c^2 + c^3) + 7*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-
a*c*x + c) + sqrt(c)))/c^(3/2))
```

mupad [B] time = 1.23, size = 54, normalized size = 0.79

$$\frac{9\,\sqrt{c-a\,c\,x}}{4\,x^2}+\frac{7\,a^2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)}{4}-\frac{7\,{\left(c-a\,c\,x\right)}^{3/2}}{4\,c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] (9*(c - a*c*x)^(1/2))/(4*x^2) + (7*a^2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1
/2)))/4 - (7*(c - a*c*x)^(3/2))/(4*c*x^2)
```

sympy [B] time = 18.12, size = 270, normalized size = 3.97

$$\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2\,(-acx+c)^2}-\frac{6a^2c^3\,(-acx+c)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2\,(-acx+c)^2}-\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\,\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}+\frac{3a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] 10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**
2) - 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x
 + c)**2) - 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x
 + c))/8 + 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x +
 c))/8 - a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)
)/2 + a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2
+ a*sqrt(-a*c*x + c)/x
```

3.308 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=89

$$\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}$$

```
[Out] 11/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a*c*x+c)^(1/2)/x^3
+11/12*a*(-a*c*x+c)^(1/2)/x^2+11/8*a^2*(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.22, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6167, 6130, 21, 78, 51, 63, 208}

$$\frac{11a^2\sqrt{c-acx}}{8x}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{\sqrt{c-acx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] Sqrt[c - a*c*x]/(3*x^3) + (11*a*Sqrt[c - a*c*x])/(12*x^2) + (11*a^2*Sqrt[c
- a*c*x])/(8*x) + (11*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^4(1-ax)}\,dx\\
&= -\left(c\int \frac{1+ax}{x^4\sqrt{c-acx}}\,dx\right)\\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{1}{6}(11ac)\int \frac{1}{x^3\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}-\frac{1}{8}\left(11a^2c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}-\frac{1}{16}\left(11a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{1}{8}\left(11a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-}\right.\\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.71

$$\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{\left(33a^2x^2+22ax+8\right)\sqrt{c-acx}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (Sqrt[c - a*c*x]*(8 + 22*a*x + 33*a^2*x^2))/(24*x^3) + (11*a^3*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

fricas [A] time = 0.64, size = 133, normalized size = 1.49

$$\left[\frac{33\,a^3\sqrt{c}\,x^3\log\left(\frac{acx-2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)+2\left(33\,a^2x^2+22\,ax+8\right)\sqrt{-acx+c}}{48\,x^3},-\frac{33\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-}{24\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(33*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x)
 + 2*(33*a^2*x^2 + 22*a*x + 8)*sqrt(-a*c*x + c))/x^3, -1/24*(33*a^3*sqrt(-c
)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (33*a^2*x^2 + 22*a*x + 8)*sqrt(
-a*c*x + c))/x^3]
```

giac [A] time = 0.15, size = 104, normalized size = 1.17

$$-\frac{\frac{33\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{33\,(acx-c)^2\sqrt{-acx+c}\,a^4c-88\,(-acx+c)^{\frac{3}{2}}a^4c^2+63\,\sqrt{-acx+c}\,a^4c^3}{a^3c^3x^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*(33*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (33*(a*c*x - c
)^2*sqrt(-a*c*x + c)*a^4*c - 88*(-a*c*x + c)^(3/2)*a^4*c^2 + 63*sqrt(-a*c*x
 + c)*a^4*c^3)/(a^3*c^3*x^3))/a
```

maple [A] time = 0.05, size = 80, normalized size = 0.90

$$-2c^3a^3\left(-\frac{\frac{11(-acx+c)^{\frac{5}{2}}}{16c^2}-\frac{11(-acx+c)^{\frac{3}{2}}}{6c}+\frac{21\sqrt{-acx+c}}{16}}{x^3a^3c^3}-\frac{11\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x^4,x)
```

```
[Out] -2*c^3*a^3*(-(11/16/c^2*(-a*c*x+c)^(5/2)-11/6/c*(-a*c*x+c)^(3/2)+21/16*(-a*
c*x+c)^(1/2))/x^3/a^3/c^3-11/16/c^(5/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 1.08, size = 134, normalized size = 1.51

$$\frac{1}{48}\,a^3c^3\left(\frac{2\left(33\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+63\,\sqrt{-acx+c}\,c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{33\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*a^3*c^3*(2*(33*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 63*sqrt(
-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)*c
^4 + c^5) - 33*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)
))/c^(5/2))
```

mupad [B] time = 1.20, size = 74, normalized size = 0.83

$$\frac{21\,\sqrt{c-a\,c\,x}}{8\,x^3}-\frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3}+\frac{11\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3}-\frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,11{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

```
[Out] (21*(c - a*c*x)^(1/2))/(8*x^3) - (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c
^(1/2))*11i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (11*(c - a*c*x)^(5/2))/
(8*c^2*x^3)
```

sympy [B] time = 17.81, size = 439, normalized size = 4.93

$$-\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}+\frac{80a^3c^5\left(-acx+c\right)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] -66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x
+ c)**2 + 48*c**3*(-a*c*x + c)**3) + 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144
*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) -
 30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c
*x + c)**2 + 48*c**3*(-a*c*x + c)**3) + 10*a**3*c**4*sqrt(-a*c*x + c)/(16*a
*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 5*a**3*c**4*sqrt(c**(-7))*log(
-c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 5*a**3*c**4*sqrt(c**(-7))*log(
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 6*a**3*c**3*(-a*c*x + c)**(3/2)
/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 3*a**3*c**3*sqrt(c**(-5)
)*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 + 3*a**3*c**3*sqrt(c**(-5))
*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8
```

3.309 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=110

$$\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}$$

```
[Out] 75/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a*c*x+c)^(1/2)/x^
4+5/8*a*(-a*c*x+c)^(1/2)/x^3+25/32*a^2*(-a*c*x+c)^(1/2)/x^2+75/64*a^3*(-a*c
*x+c)^(1/2)/x
```

Rubi [A] time = 0.23, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6167, 6130, 21, 78, 51, 63, 208}

$$\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{\sqrt{c-acx}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] Sqrt[c - a*c*x]/(4*x^4) + (5*a*Sqrt[c - a*c*x])/(8*x^3) + (25*a^2*Sqrt[c -
a*c*x])/(32*x^2) + (75*a^3*Sqrt[c - a*c*x])/(64*x) + (75*a^4*Sqrt[c]*ArcTan
h[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^5(1-ax)}\,dx\\
&= -\left(c\int \frac{1+ax}{x^5\sqrt{c-acx}}\,dx\right)\\
&= \frac{\sqrt{c-acx}}{4x^4}-\frac{1}{8}(15ac)\int \frac{1}{x^4\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}-\frac{1}{16}\left(25a^2c\right)\int \frac{1}{x^3\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}-\frac{1}{64}\left(75a^3c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}-\frac{1}{128}\left(75a^4c\right)\int \frac{1}{x\sqrt{c-}}\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{1}{64}\left(75a^3\right)\text{Subst}\left(\int\right.\\
&= \frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\sqrt{c}\right.
\end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.65

$$\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{\left(75a^3x^3+50a^2x^2+40ax+16\right)\sqrt{c-acx}}{64x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (Sqrt[c - a*c*x]*(16 + 40*a*x + 50*a^2*x^2 + 75*a^3*x^3))/(64*x^4) + (75*a^
4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

fricas [A] time = 0.53, size = 149, normalized size = 1.35

$$\left[\frac{75\,a^4\sqrt{c}\,x^4\log\left(\frac{acx-2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)+2\left(75\,a^3x^3+50\,a^2x^2+40\,ax+16\right)\sqrt{-acx+c}}{128\,x^4},-\frac{75\,a^4\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{-a}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(75*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x
) + 2*(75*a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4, -1/64*
(75*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (75*a^3*x^3 + 50
*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4]
```

giac [A] time = 0.13, size = 131, normalized size = 1.19

$$-\frac{\frac{75\,a^5c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{75\,(acx-c)^3\sqrt{-acx+c}\,a^5c+275\,(acx-c)^2\sqrt{-acx+c}\,a^5c^2-365\,(-acx+c)^{\frac{3}{2}}a^5c^3+181\sqrt{-acx+c}\,a^5c^4}{a^4c^4x^4}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/64*(75*a^5*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (75*(a*c*x - c
)^3*sqrt(-a*c*x + c)*a^5*c + 275*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^5*c^2 - 3
65*(-a*c*x + c)^(3/2)*a^5*c^3 + 181*sqrt(-a*c*x + c)*a^5*c^4)/(a^4*c^4*x^4)
)/a
```

maple [A] time = 0.05, size = 93, normalized size = 0.85

$$2c^4a^4\left(\frac{-\frac{75(-acx+c)^{\frac{7}{2}}}{128c^3}+\frac{275(-acx+c)^{\frac{5}{2}}}{128c^2}-\frac{365(-acx+c)^{\frac{3}{2}}}{128c}+\frac{181\sqrt{-acx+c}}{128}}{x^4a^4c^4}+\frac{75\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{128c^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a*c*x+c)^(1/2)/x^5,x)
```

```
[Out] 2*c^4*a^4*((-75/128/c^3*(-a*c*x+c)^(7/2)+275/128/c^2*(-a*c*x+c)^(5/2)-365/1
28/c*(-a*c*x+c)^(3/2)+181/128*(-a*c*x+c)^(1/2))/x^4/a^4/c^4+75/128/c^(7/2)*
arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.45, size = 163, normalized size = 1.48

$$-\frac{1}{128}\,a^4c^4\left(\frac{2\left(75\,(-acx+c)^{\frac{7}{2}}-275\,(-acx+c)^{\frac{5}{2}}c+365\,(-acx+c)^{\frac{3}{2}}c^2-181\sqrt{-acx+c}\,c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{75\log\left(\frac{\sqrt{-acx+c}-}{\sqrt{-acx+c}+}\right.}{c^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/128*a^4*c^4*(2*(75*(-a*c*x + c)^(7/2) - 275*(-a*c*x + c)^(5/2)*c + 365*(
-a*c*x + c)^(3/2)*c^2 - 181*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*(a
*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 75*log((
sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(7/2))
```

mupad [B] time = 0.07, size = 91, normalized size = 0.83

$$\frac{181\sqrt{c-a\,c\,x}}{64\,x^4}-\frac{365\,{\left(c-a\,c\,x\right)}^{3/2}}{64\,c\,x^4}+\frac{275\,{\left(c-a\,c\,x\right)}^{5/2}}{64\,c^2\,x^4}-\frac{75\,{\left(c-a\,c\,x\right)}^{7/2}}{64\,c^3\,x^4}-\frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,75{}\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

```
[Out] (181*(c - a*c*x)^(1/2))/(64*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)
/c^(1/2))*75i)/64 - (365*(c - a*c*x)^(3/2))/(64*c*x^4) + (275*(c - a*c*x)^(
5/2))/(64*c^2*x^4) - (75*(c - a*c*x)^(7/2))/(64*c^3*x^4)
```

sympy [B] time = 28.19, size = 639, normalized size = 5.81

$$\frac{558a^4c^8\sqrt{-acx+c}}{1536ac^8x-1152c^8+2304c^6(-acx+c)^2-1536c^5(-acx+c)^3+384c^4(-acx+c)^4}-\frac{}{1536ac^8x-1152c^8+2304c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] 558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c
*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 1022*a
**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x
 + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 770*a**4
*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 66*a**4*c**
6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48
*c**3*(-a*c*x + c)**3) - 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x -
 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c*
*4*(-a*c*x + c)**4) + 80*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x + 96*
c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 35*a**4*c**5*s
qrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 35*a**4*c**5
*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 - 30*a**4*c**
4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 +
 48*c**3*(-a*c*x + c)**3) - 5*a**4*c**4*sqrt(c**(-7))*log(-c**4*sqrt(c**(-7
)) + sqrt(-a*c*x + c))/16 + 5*a**4*c**4*sqrt(c**(-7))*log(c**4*sqrt(c**(-7)
) + sqrt(-a*c*x + c))/16
```

3.310 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=309

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{9/2}\sqrt{1-\frac{1}{ax}}}+\frac{1576\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-}}{105a^2\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 1576/315*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(1-1/a/x)^(1/2)+472/315*x*(1+
1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+92/105*x^2*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+38/63*x^3*(1+1/a/x)^(1/2)*(-a*c*x+c)
^(1/2)/a/(1-1/a/x)^(1/2)+2/9*x^4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)
^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)
^(1/2)*(-a*c*x+c)^(1/2)/a^(9/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 98, 152, 12, 93, 206}

$$\frac{92x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{472x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{1576\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (1576*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(315*a^4*Sqrt[1 - 1/(a*x)]) + (472
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(315*a^3*Sqrt[1 - 1/(a*x)]) + (92*Sqr
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) + (38*Sqrt[
1 + 1/(a*x)]*x^3*Sqrt[c - a*c*x])/(63*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/
(a*x)]*x^4*Sqrt[c - a*c*x])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]
*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])
])/(a^(9/2)*Sqrt[1 - 1/(a*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{7/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{19}{2a}-\frac{17x}{2a^2}}{x^{9/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{9\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{38\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \ldots\right.}{63\sqrt{1-\ldots}}$$

$$= \frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}} + \ldots$$

$$= \frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}} + \ldots$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \ldots$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \ldots$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \ldots$$

$$= \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}} + \frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} + \ldots$$

Mathematica [A] time = 0.13, size = 130, normalized size = 0.42

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(35a^4x^4+95a^3x^3+138a^2x^2+236ax+788\right)-630\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{315a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(788 + 236*a*x + 138*a^2*x^2
+ 95*a^3*x^3 + 35*a^4*x^4) - 630*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt
[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(315*a^(9/2)*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.72, size = 304, normalized size = 0.98

$$\left[\frac{2\left(315\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+\left(35a^5x^5+130a^4x^4+233a^3x^3+374\right.\right.}{315\left(a^5x-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [2/315*(315*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2
)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) + (35*a^5*x^5 + 130*a^4*x^4 + 233*a^3*x^3 + 374*a^2*x^2
+ 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*x - a^4)
, -2/315*(630*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (35*a^5*x^5 + 130*a^4*x^4 + 2
33*a^3*x^3 + 374*a^2*x^2 + 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/(a^5*x - a^4)]
```

giac [C] time = 0.20, size = 185, normalized size = 0.60

$$-\frac{\frac{1260i\sqrt{2}\sqrt{-c}\arctan(-i)-2584\sqrt{2}\sqrt{-c}}{a^3\mathrm{sgn}(c)}+\frac{2\left(630\sqrt{2}c^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-35(acx+c)^4\sqrt{-acx-c}+45(acx+c)^3\sqrt{-acx-c}\,c-63(acx+c)^2\sqrt{-acx-c}\right.}{a^3c^4\mathrm{sgn}(-acx-c)}}{315a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/315*((1260*I*sqrt(2)*sqrt(-c)*arctan(-I) - 2584*sqrt(2)*sqrt(-c))/(a^3*s
gn(c)) + 2*(630*sqrt(2)*c^(9/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)
) - 35*(a*c*x + c)^4*sqrt(-a*c*x - c) + 45*(a*c*x + c)^3*sqrt(-a*c*x - c)*c
 - 63*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^2 + 105*(-a*c*x - c)^(3/2)*c^3 - 630
*sqrt(-a*c*x - c)*c^4)/(a^3*c^4*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.06, size = 161, normalized size = 0.52

$$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-35x^4a^4\sqrt{-c(ax+1)}-95x^3a^3\sqrt{-c(ax+1)}-138x^2a^2\sqrt{-c(ax+1)}+630\sqrt{c}\sqrt{2}\,\mathrm{arc}\right.}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/315/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-35*x^4*
a^4*(-c*(a*x+1))^(1/2)-95*x^3*a^3*(-c*(a*x+1))^(1/2)-138*x^2*a^2*(-c*(a*x+1
))^(1/2)+630*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
-236*x*a*(-c*(a*x+1))^(1/2)-788*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}\,x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{- c \left(a x - 1\right)}}{\left(\frac{a x - 1}{a x + 1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.311 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=261

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{7/2}\sqrt{1-\frac{1}{ax}}}+\frac{104\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}+$$

```
[Out] 104/21*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+32/21*x*(1+1/a/
x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+6/7*x^2*(1+1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*x^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1
/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*
(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(7/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 98, 152, 12, 93, 206}

$$\frac{32x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{104\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{7/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}+6$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (104*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(21*a^3*Sqrt[1 - 1/(a*x)]) + (32*Sq
rt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(21*a^2*Sqrt[1 - 1/(a*x)]) + (6*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(7*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*x
)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(
a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}+\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{15}{2a}-\frac{13x}{2a^2}}{x^{7/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{2(}{x^{5/2}(1}\right.}{35\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}+\frac{\left(8\sqrt{\frac{1}{x}}\right.}{}$$

$$=\frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+}}{7}$$

$$=\frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+}}{7}$$

$$=\frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+}}{7}$$

$$=\frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+}}{7}$$

Mathematica [A] time = 0.09, size = 122, normalized size = 0.47

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(3a^3x^3+9a^2x^2+16ax+52\right)-42\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{21a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(52 + 16*a*x + 9*a^2*x^2 + 3*
a^3*x^3) - 42*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*
Sqrt[1 + 1/(a*x)])]))/(21*a^(7/2)*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.57, size = 288, normalized size = 1.10

$$\left[\frac{2\left(21\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+\left(3a^4x^4+12a^3x^3+25a^2x^2+68ax\right.\right.}{21\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [2/21*(21*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*sqrt
(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3), -2/21*(42*sqrt(2)*(a
*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
 + 1))/(a*c*x - c)) - (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*s
qrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)]
```

giac [C] time = 0.20, size = 139, normalized size = 0.53

$$-\frac{\frac{84i\sqrt{2}\sqrt{-c}\arctan(-i)-160\sqrt{2}\sqrt{-c}}{a^2\mathrm{sgn}(c)}+\frac{2\left(42\sqrt{2}c^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-3(acx+c)^3\sqrt{-acx-c}+7(-acx-c)^{\frac{3}{2}}c^2-42\sqrt{-acx-c}\,c^3\right)}{a^2c^3\mathrm{sgn}(-acx-c)}}{21a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/21*((84*I*sqrt(2)*sqrt(-c)*arctan(-I) - 160*sqrt(2)*sqrt(-c))/(a^2*sgn(c
)) + 2*(42*sqrt(2)*c^(7/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - 3
*(a*c*x + c)^3*sqrt(-a*c*x - c) + 7*(-a*c*x - c)^(3/2)*c^2 - 42*sqrt(-a*c*x
 - c)*c^3)/(a^2*c^3*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.06, size = 143, normalized size = 0.55

$$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-3x^3a^3\sqrt{-c(ax+1)}-9x^2a^2\sqrt{-c(ax+1)}+42\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-16x\right.}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/21/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*x^3*a^
3*(-c*(a*x+1))^(1/2)-9*x^2*a^2*(-c*(a*x+1))^(1/2)+42*c^(1/2)*2^(1/2)*arctan
(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-16*x*a*(-c*(a*x+1))^(1/2)-52*(-c*(
a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}\,x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{-c\left(ax-1\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.312 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=211

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{5/2}\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-a}}{3a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(5/2
)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a
^2/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2
^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(5/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6176, 6181, 96, 94, 93, 206}

$$\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{5/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-ac}}{3a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(
a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*x))^(
5/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])
/(a^(5/2)*Sqrt[1 - 1/(a*x)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\cdots\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{\frac{1}{x}}\cdots\right.}{\cdots}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{\frac{1}{x}}\cdots\right.}{\cdots}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\cdots}{\cdots}$$

Mathematica [A] time = 0.08, size = 114, normalized size = 0.54

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}\left(3a^2x^2+11ax+38\right)-30\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{15a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(38 + 11*a*x + 3*a^2*x^2) - 3
0*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(
a*x)])]))/(15*a^(5/2)*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.53, size = 272, normalized size = 1.29

$$\left[\frac{2\left(15\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+\left(3a^3x^3+14a^2x^2+49ax+38\right)\sqrt{-ac}\right.}{15\left(a^3x-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] [2/15*(15*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*
sqrt((a*x - 1)/(a*x + 1)))/(a^3*x - a^2), -2/15*(30*sqrt(2)*(a*x - 1)*sqrt(
c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1)))/(a^3*x - a^2)]
```

giac [C] time = 0.18, size = 137, normalized size = 0.65

$$-\frac{\frac{60i\sqrt{2}\sqrt{-c}\arctan(-i)-104\sqrt{2}\sqrt{-c}}{a\operatorname{sgn}(c)}+\frac{2\left(30\sqrt{2}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-3(acx+c)^2\sqrt{-acx-c}+5(-acx-c)^{\frac{3}{2}}c-30\sqrt{-acx-c}\,c^2\right)}{ac^2\operatorname{sgn}(-acx-c)}}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*((60*I*sqrt(2)*sqrt(-c)*arctan(-I) - 104*sqrt(2)*sqrt(-c))/(a*sgn(c))
 + 2*(30*sqrt(2)*c^(5/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - 3*(
a*c*x + c)^2*sqrt(-a*c*x - c) + 5*(-a*c*x - c)^(3/2)*c - 30*sqrt(-a*c*x - c
)*c^2)/(a*c^2*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.06, size = 125, normalized size = 0.59

$$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(-3x^2a^2\sqrt{-c(ax+1)}+30\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-11xa\sqrt{-c(ax+1)}-38\sqrt{\right.}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*x^2*a^
2*(-c*(a*x+1))^(1/2)+30*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/
2)/c^(1/2))-11*x*a*(-c*(a*x+1))^(1/2)-38*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(
1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx + c}\,x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(ax-1\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.313 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=163

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1
/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(3/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\Bigg(\int\frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)}}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\Bigg(\int\frac{1}{1-\frac{2x^2}{a}}\,dx}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\Bigg(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\cdots}}}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.64

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}\,(ax+7)-6\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x^
(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^(3
/2)*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.57, size = 250, normalized size = 1.53

$$\left[\frac{2\left(3\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+\left(a^2x^2+8ax+7\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{3\left(a^2x-a\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2 + 8*a
*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

giac [C] time = 0.18, size = 105, normalized size = 0.64

$$-\frac{\frac{12i\sqrt{2}\sqrt{-c}\arctan(-i)-16\sqrt{2}\sqrt{-c}}{\mathrm{sgn}(c)}+\frac{2\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)+(-acx-c)^{\frac{3}{2}}-6\sqrt{-acx-c}\,c\right)}{c\,\mathrm{sgn}(-acx-c)}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*((12*I*sqrt(2)*sqrt(-c)*arctan(-I) - 16*sqrt(2)*sqrt(-c))/sgn(c) + 2*(
6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) + (-a*c*x -
c)^(3/2) - 6*sqrt(-a*c*x - c)*c)/(c*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.05, size = 107, normalized size = 0.66

$$-\frac{2(ax-1)\sqrt{-c(ax-1)}\left(6\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)-xa\sqrt{-c(ax+1)}-7\sqrt{-c(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x)
```

```
[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-x*a*(-c*(a*x+1))^(1/
2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.314 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=170

$$\frac{2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(2^(1/
2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2
)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6176, 6181, 98, 157, 54, 215, 93, 206}

$$\frac{2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)]) -
 (4*Sqrt[2]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{x}{2a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-\right.}{}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c\right.}{}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\right.}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.07, size = 120, normalized size = 0.71

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}+\sqrt{\frac{1}{x}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-2\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]] - 2*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 1.23, size = 352, normalized size = 2.07

$$\left[\frac{2\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax-}{ax^2-x}\right.}{ax-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [(2*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2
*a*x + 1)) + (a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt
(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*(2*sqrt(2)*
(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a
*x + 1))/(a*c*x - c)) - (a*x - 1)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*s
qrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*
x - 1)/(a*x + 1)))/(a*x - 1)]
```

giac [C] time = 0.21, size = 136, normalized size = 0.80

$$-c\left(\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}\,\mathrm{sgn}\left(-acx-c\right)}-\frac{2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}\,\mathrm{sgn}\left(-acx-c\right)}+\frac{-4i\sqrt{2}\arctan\left(-i\right)+2\sqrt{2}+2i\arctan\left(-i\sqrt{2}\right)}{\sqrt{-c}\,\mathrm{sgn}(c)}-\frac{2\sqrt{-}}{c\mathrm{sgn}\,(-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -c*(4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(sqrt(c)*sgn(-a*
c*x - c)) - 2*arctan(sqrt(-a*c*x - c)/sqrt(c))/(sqrt(c)*sgn(-a*c*x - c)) +
(-4*I*sqrt(2)*arctan(-I) + 2*sqrt(2) + 2*I*arctan(-I*sqrt(2)))/(sqrt(-c)*sg
n(c)) - 2*sqrt(-a*c*x - c)/(c*sgn(-a*c*x - c)))
```

maple [A] time = 0.06, size = 107, normalized size = 0.63

$$\frac{2\left(ax-1\right)\sqrt{-c\left(ax-1\right)}\left(-2\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)+\sqrt{c}\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c\left(ax+1\right)}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)\sqrt{-c\left(ax+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x)
```

```
[Out] 2*(a*x-1)*(-c*(a*x-1))^(1/2)*(-2*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1
/2)*2^(1/2)/c^(1/2))+c^(1/2)*arctan((-c*(a*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1)
)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a\,c\,x}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.315 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=172

$$\frac{\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+5*arcsinh((1/x)^(1/2)/a^
(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(2^(1/
2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*a^(1/2)*(1/x)^(1/2)*(-a*c*x
+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6176, 6181, 102, 157, 54, 215, 93, 206}

$$\frac{\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (5*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)] -
 (4*Sqrt[2]*Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{5x}{2a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right.}{}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-ac\right.}{}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{5\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.08, size = 120, normalized size = 0.70

$$\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}+5\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-4\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] (Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + 5*Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]] - 4*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/Sqrt[1 - 1/(a*x)]
```

fricas [A] time = 0.58, size = 390, normalized size = 2.27

$$\left[\frac{4\sqrt{2}\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+5\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-ac}}{}\right.}{2\left(ax^2-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt
(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) + 5*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x -
 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*
x^2 - x)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2
- x), -(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 5*(a^2*x^2 - a*x)*sqrt(c)*a
rctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)]
```

giac [C] time = 0.23, size = 164, normalized size = 0.95

$$-\frac{\frac{4\sqrt{2}a^2\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}-\frac{5a^2\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}-\frac{-4i\sqrt{2}a^2\sqrt{-c}\arctan(-i)+5ia^2\sqrt{-c}\arctan\left(-i\sqrt{2}\right)+\sqrt{2}a^2\sqrt{-c}}{\mathrm{sgn}(c)}-\frac{\sqrt{-acx-c}}{x\,\mathrm{sgn}(-acx-c)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] -(4*sqrt(2)*a^2*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sgn(-a
*c*x - c) - 5*a^2*sqrt(c)*arctan(sqrt(-a*c*x - c)/sqrt(c))/sgn(-a*c*x - c)
- (-4*I*sqrt(2)*a^2*sqrt(-c)*arctan(-I) + 5*I*a^2*sqrt(-c)*arctan(-I*sqrt(2
)) + sqrt(2)*a^2*sqrt(-c))/sgn(c) - sqrt(-a*c*x - c)*a/(x*sgn(-a*c*x - c)))
/a
```

maple [A] time = 0.07, size = 117, normalized size = 0.68

$$\frac{(ax-1)\sqrt{-c\,(ax-1)}\,\left(-4\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)xac+5\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)xac+\sqrt{-c\,(ax+1)}\,\sqrt{c}\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,x\sqrt{c}\,\sqrt{-c\,(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-4*2^(1/2)*ar
ctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*x*a*c+5*arctan((-c*(a*x+1))^(1
/2)/c^(1/2))*x*a*c+(-c*(a*x+1))^(1/2)*c^(1/2))/x/c^(1/2)/(-c*(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a\,c\,x}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

3.316 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=224

$$\frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{2x\sqrt{1-\frac{1}{ax}}}+\frac{7a\sqrt{\frac{1}{ax}}}{4x}$$

```
[Out] 1/2*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+7/4*a*(1+1/a/x)^(1
/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+23/4*a^(3/2)*arcsinh((1/x)^(1/2)/a^(
1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(3/2)*arctanh(2^(1/2
)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)
/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6176, 6181, 101, 154, 157, 54, 215, 93, 206}

$$\frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{2x\sqrt{1-\frac{1}{ax}}}+\frac{7a\sqrt{\frac{1}{ax}}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] (7*a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (a*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x) + (23*a^(3/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(3/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{5/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}\left(\frac{1}{2}+\frac{7x}{2a}\right)}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{}{\sqrt{x}\,(}\right.}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(23a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \right.}{8\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(23a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \right.}{4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} + \frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right.}{4\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.16, size = 132, normalized size = 0.59

$$\frac{\sqrt{c-acx}\left(\frac{23a^{3/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}} - \frac{16\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}} + \sqrt{\frac{1}{ax}+1}\,(9ax+2)\right)}{4x^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(2 + 9*a*x) + (23*a^(3/2)*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]])/(x^(-1))^(3/2) - (16*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqr
t[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(3/2)))/(4*Sqrt[1 - 1/(a*
x)]*x^2)
```

fricas [A] time = 0.48, size = 428, normalized size = 1.91

$$\left[\frac{16\sqrt{2}\left(a^3x^3-a^2x^2\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+23\left(a^3x^3-a^2x^2\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx\cdots}{\cdots}\right.\cdots}{8\left(ax^3-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] [1/8*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) + 23*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2
 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
- 2*c)/(a*x^2 - x)) + 2*(9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1)))/(a*x^3 - x^2), -1/4*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(
c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - 23*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqr
t((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x
+ c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

giac [C] time = 0.21, size = 194, normalized size = 0.87

$$-\frac{\frac{16\,\sqrt{2}\,a^3\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx-c}}{2\,\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}-\frac{23\,a^3\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}+\frac{16i\,\sqrt{2}\,a^3\sqrt{-c}\,\arctan(-i)-23i\,a^3\sqrt{-c}\,\arctan\left(-i\,\sqrt{2}\right)-11\,\sqrt{2}\,a^3\sqrt{-c}}{\mathrm{sgn}(c)}+\frac{9\,(-acx\cdots}{a\cdots}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] -1/4*(16*sqrt(2)*a^3*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/s
gn(-a*c*x - c) - 23*a^3*sqrt(c)*arctan(sqrt(-a*c*x - c)/sqrt(c))/sgn(-a*c*x
 - c) + (16*I*sqrt(2)*a^3*sqrt(-c)*arctan(-I) - 23*I*a^3*sqrt(-c)*arctan(-I
*sqrt(2)) - 11*sqrt(2)*a^3*sqrt(-c))/sgn(c) + (9*(-a*c*x - c)^(3/2)*a^3*c +
 7*sqrt(-a*c*x - c)*a^3*c^2)/(a^2*c^2*x^2*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.07, size = 144, normalized size = 0.64

$$\frac{(ax-1)\sqrt{-c\,(ax-1)}\,\left(-16\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^2a^2c+23c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)x^2a^2+9xa\sqrt{-c\,(ax+1)}\,\sqrt{c}\cdots\right.}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\,\sqrt{-c\,(ax+1)}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x)
```

```
[Out] 1/4*(a*x-1)*(-c*(a*x-1))^(1/2)*(-16*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2
^(1/2)/c^(1/2))*x^2*a^2*c+23*c*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*x^2*a^2+9
*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)+2*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x
+1))^(3/2)/(a*x+1)/c^(1/2)/(-c*(a*x+1))^(1/2)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

3.317 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=274

$$\frac{45a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{3a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}+\frac{13a^2\sqrt{\cdots}}{8x\cdots}$$

```
[Out] 1/3*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+3/4*a^2*(1+1/a/x
)^(3/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+13/8*a^2*(1+1/a/x)^(1/2)*(-a*c*x
+c)^(1/2)/x/(1-1/a/x)^(1/2)+45/8*a^(5/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)
^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(5/2)*arctanh(2^(1/2)*(1/x)^(1/
2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6176, 6181, 101, 154, 157, 54, 215, 93, 206}

$$\frac{3a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}+\frac{13a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}+\frac{45a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh\cdots}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^2) + (13*a^2
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (3*a^2*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (45*a^(5/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(5/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
```

```
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{7/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}\left(\frac{3}{2}+\frac{9x}{2a}\right)}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{(-\cdots}{\cdots}\right.}{6\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(a^4\cdots\right.}{\cdots}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(45a\cdots\right.}{\cdots}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(45a\cdots\right.}{\cdots}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{45a^{5}\cdots}{\cdots}$$

Mathematica [A] time = 0.23, size = 140, normalized size = 0.51

$$\frac{\sqrt{c-acx}\left(\frac{135a^{5/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}} - \frac{96\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{5/2}} + \sqrt{\frac{1}{ax}+1}\left(57a^2x^2+26ax+8\right)\right)}{24x^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(8 + 26*a*x + 57*a^2*x^2) + (135*a^(5/2
)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2) - (96*Sqrt[2]*a^(5/2)*ArcTa
nh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(5/2)))/(2
4*Sqrt[1 - 1/(a*x)]*x^3)
```

fricas [A] time = 0.72, size = 444, normalized size = 1.62

$$\left[\frac{96\sqrt{2}\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+135\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{a^2cx^2}{}\right.}{48\left(ax^4-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] [1/48*(96*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x +
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3
*c)/(a^2*x^2 - 2*a*x + 1)) + 135*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x
^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)
) - 2*c)/(a*x^2 - x)) + 2*(57*a^3*x^3 + 83*a^2*x^2 + 34*a*x + 8)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^3), -1/24*(96*sqrt(2)*(a^4*x^4
 - a^3*x^3)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/
(a*x + 1))/(a*c*x - c)) - 135*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(sqrt(-a*c*
x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (57*a^3*x^3 + 83*a^
2*x^2 + 34*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^
3)]
```

giac [C] time = 0.22, size = 222, normalized size = 0.81

$$-\frac{\frac{96\sqrt{2}a^4\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}-\frac{135\,a^4\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}+\frac{96i\sqrt{2}a^4\sqrt{-c}\arctan(-i)-135i\,a^4\sqrt{-c}\arctan\left(-i\sqrt{2}\right)-91\sqrt{2}a^4\sqrt{-c}}{\mathrm{sgn}(c)}-\frac{5}{}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] -1/24*(96*sqrt(2)*a^4*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/
sgn(-a*c*x - c) - 135*a^4*sqrt(c)*arctan(sqrt(-a*c*x - c)/sqrt(c))/sgn(-a*c
*x - c) + (96*I*sqrt(2)*a^4*sqrt(-c)*arctan(-I) - 135*I*a^4*sqrt(-c)*arctan
(-I*sqrt(2)) - 91*sqrt(2)*a^4*sqrt(-c))/sgn(c) - (57*(a*c*x + c)^2*sqrt(-a*
c*x - c)*a^4*c + 88*(-a*c*x - c)^(3/2)*a^4*c^2 + 39*sqrt(-a*c*x - c)*a^4*c^
3)/(a^3*c^3*x^3*sgn(-a*c*x - c)))/a
```

maple [A] time = 0.07, size = 165, normalized size = 0.60

$$\frac{(ax-1)\sqrt{-c\,(ax-1)}\,\left(-96\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)x^3a^3c+135c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)x^3a^3+57x^2a^2\sqrt{-c\,(ax}\right.}{24\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\,\sqrt{-c\,(ax+1)}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x)
```

```
[Out] 1/24*(a*x-1)*(-c*(a*x-1))^(1/2)*(-96*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*
2^(1/2)/c^(1/2))*x^3*a^3*c+135*c*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*x^3*a^3
+57*x^2*a^2*(-c*(a*x+1))^(1/2)*c^(1/2)+26*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)+8*
(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/c^(1/2)/(-c*(a*
x+1))^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.318 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=322

$$\frac{363a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{21a^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{32x\sqrt{1-\frac{1}{ax}}}+10$$

```
[Out] 1/4*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)+11/24*a^2*(1+1/a
/x)^(3/2)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+21/32*a^3*(1+1/a/x)^(3/2)*(-
a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+107/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2
)/x/(1-1/a/x)^(1/2)+363/64*a^(7/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)
*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(
1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6176, 6181, 101, 154, 157, 54, 215, 93, 206}

$$\frac{11a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{24x^2\sqrt{1-\frac{1}{ax}}}+\frac{21a^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{32x\sqrt{1-\frac{1}{ax}}}+\frac{107a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}+\frac{363a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}}{64\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^3) + (11*a^2
*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^2) + (107*a^3
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) + (21*a^3*(1 +
 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(32*Sqrt[1 - 1/(a*x)]*x) + (363*a^(7/2)*Sq
rt[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a
*x)]) - (4*Sqrt[2]*a^(7/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
```

```
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{9/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\sqrt{1+\frac{x}{a}}\left(\frac{5}{2}+\frac{11x}{2a}\right)}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{\left(a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\Big(}{12\sqrt{\cdots}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{32\sqrt{1-\frac{1}{ax}}\,x}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}\,x} +$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}\,x} +$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}\,x} +$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}\,x} +$$

Mathematica [A] time = 0.27, size = 148, normalized size = 0.46

$$\frac{\sqrt{c-acx}\left(\frac{1089a^{7/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}} - \frac{768\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{7/2}} + \sqrt{\frac{1}{ax}+1}\left(447a^3x^3+214a^2x^2+136ax+48\right)\right)}{192x^4\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(48 + 136*a*x + 214*a^2*x^2 + 447*a^3*x
^3) + (1089*a^(7/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2) - (768*Sq
rt[2]*a^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(x^(-1))^(7/2)))/(192*Sqrt[1 - 1/(a*x)]*x^4)
```

fricas [A] time = 0.66, size = 460, normalized size = 1.43

$$\left[\frac{768\sqrt{2}\left(a^5x^5-a^4x^4\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+1089\left(a^5x^5-a^4x^4\right)\sqrt{-c}\log\left(-\frac{a^2cx^2\cdots}{\cdots}\right.}{384\left(ax^5-x\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fric
as")
```

```
[Out] [1/384*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
+ 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 1089*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*
c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x +
 1)) - 2*c)/(a*x^2 - x)) + 2*(447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184
*a*x + 48)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/19
2*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 1089*(a^5*x^5 - a^4*x^4)*s
qrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) - (447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184*a*x + 48)*sqrt(-a*c*x +
 c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

giac [C] time = 0.22, size = 249, normalized size = 0.77

$$-\frac{\frac{768\sqrt{2}a^5\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}-\frac{1089a^5\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\mathrm{sgn}(-acx-c)}+\frac{768i\sqrt{2}a^5\sqrt{-c}\arctan(-i)-1089ia^5\sqrt{-c}\arctan\left(-i\sqrt{2}\right)-845\sqrt{2}a^5\sqrt{-c}}{\mathrm{sgn}(c)}-\cdots}{192a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac
")
```

```
[Out] -1/192*(768*sqrt(2)*a^5*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)
)/sgn(-a*c*x - c) - 1089*a^5*sqrt(c)*arctan(sqrt(-a*c*x - c)/sqrt(c))/sgn(-
a*c*x - c) + (768*I*sqrt(2)*a^5*sqrt(-c)*arctan(-I) - 1089*I*a^5*sqrt(-c)*a
rctan(-I*sqrt(2)) - 845*sqrt(2)*a^5*sqrt(-c))/sgn(c) - (447*(a*c*x + c)^3*s
qrt(-a*c*x - c)*a^5*c - 1127*(a*c*x + c)^2*sqrt(-a*c*x - c)*a^5*c^2 - 1049*
(-a*c*x - c)^(3/2)*a^5*c^3 - 321*sqrt(-a*c*x - c)*a^5*c^4)/(a^4*c^4*x^4*sgn
(-a*c*x - c)))/a
```

maple [A] time = 0.07, size = 186, normalized size = 0.58

$$\frac{(ax-1)\sqrt{-c\,(ax-1)}\left(-768\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)x^4a^4c+1089\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)cx^4a^4+447x^3a^3\sqrt{-c\,(ax\cdots}\right.}{192\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\sqrt{-c\,(ax\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x)
```

```
[Out] 1/192*(a*x-1)*(-c*(a*x-1))^(1/2)*(-768*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2
)*2^(1/2)/c^(1/2))*x^4*a^4*c+1089*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*c*x^4*
```

```
a^4+447*x^3*a^3*(-c*(a*x+1))^(1/2)*c^(1/2)+214*x^2*a^2*(-c*(a*x+1))^(1/2)*c
^(1/2)+136*x*a*(-c*(a*x+1))^(1/2)*c^(1/2)+48*(-c*(a*x+1))^(1/2)*c^(1/2))/((
a*x-1)/(a*x+1))^(3/2)/(a*x+1)/c^(1/2)/(-c*(a*x+1))^(1/2)/x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}}{x^5 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

3.319 $\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{2\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}x^2}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{8\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}x}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{46\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}}+\frac{92\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}x}$$

```
[Out] 46/21*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1/x+1)^(3/2)+92/21*(1+x)^(3/2)*((-1+x)/
x)^(1/2)/(1/x+1)^(3/2)/x+8/7*x*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1/x+1)^(3/2)+2
/7*x^2*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1/x+1)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6176, 6181, 89, 78, 45, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}x^2}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{8\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}x}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{46\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}}+\frac{92\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

```
[Out] (46*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)) + (92*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)*x) + (8*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(7*(1 + x^(-1))^(3/2)) + (2*Sqrt[-((1 - x)/x)]*x^2*(1 +
x)^(3/2))/(7*(1 + x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx &= \frac{(1+x)^{3/2}\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^{3/2}x^{5/2}\,dx}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\operatorname{Subst}\left(\int\frac{(1+x)^2}{\sqrt{1-x}\,x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{3/2}}\\
&= \frac{2\sqrt{-\frac{1-x}{x}}\,x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}-\frac{\left(2\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\operatorname{Subst}\left(\int\frac{10+\frac{7x}{2}}{\sqrt{1-x}\,x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7\left(1+\frac{1}{x}\right)^{3/2}}\\
&= \frac{8\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}+\frac{2\sqrt{-\frac{1-x}{x}}\,x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}-\frac{\left(23\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x}\ldots}\right.}{7\left(1+\frac{1}{x}\right)^{3/2}}\\
&= \frac{46\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}}+\frac{8\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}+\frac{2\sqrt{-\frac{1-x}{x}}\,x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}-\frac{\left(46\left(\frac{1}{x}\right)^{3/2}(1\ldots\right.}{}\\
&= \frac{46\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}}+\frac{92\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}x}+\frac{8\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}+\frac{2\sqrt{-\frac{1-x}{x}}\,x^2(1\ldots}{7\left(1+\frac{1}{x}\right)^{3\ldots}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.32

$$
\frac{2\sqrt{\frac{x-1}{x}}\sqrt{x+1}\left(3x^3+12x^2+23x+46\right)}{21\sqrt{\frac{1}{x}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(46 + 23*x + 12*x^2 + 3*x^3))/(21*Sqrt[1 +
x^(-1)])
```

fricas [A] time = 0.63, size = 33, normalized size = 0.23

$$\frac{2}{21}\left(3\,x^{3}+12\,x^{2}+23\,x+46\right)\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*x^3 + 12*x^2 + 23*x + 46)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.04, size = 37, normalized size = 0.26

$$\frac{2\left(-1+x\right)\left(3x^{3}+12x^{2}+23x+46\right)}{21\sqrt{1+x}\,\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x)
```

```
[Out] 2/21*(-1+x)*(3*x^3+12*x^2+23*x+46)/(1+x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

maxima [A] time = 0.32, size = 27, normalized size = 0.19

$$\frac{2\left(3\,x^{4}+9\,x^{3}+11\,x^{2}+23\,x-46\right)}{21\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^4 + 9*x^3 + 11*x^2 + 23*x - 46)/sqrt(x - 1)
```

mupad [B] time = 1.33, size = 48, normalized size = 0.33

$$\sqrt{\frac{x-1}{x+1}}\,\left(\frac{46\,x\,\sqrt{x+1}}{21}+\frac{92\,\sqrt{x+1}}{21}+\frac{8\,x^{2}\,\sqrt{x+1}}{7}+\frac{2\,x^{3}\,\sqrt{x+1}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] ((x - 1)/(x + 1))^(1/2)*((46*x*(x + 1)^(1/2))/21 + (92*(x + 1)^(1/2))/21 +
(8*x^2*(x + 1)^(1/2))/7 + (2*x^3*(x + 1)^(1/2))/7)
```

sympy [C] time = 91.98, size = 197, normalized size = 1.37

$$-2\left(\begin{cases}\frac{8x\sqrt{x-1}}{15}+\frac{\sqrt{x-1}\,(x+1)^2}{5}+\frac{8\sqrt{x-1}}{3} & \text{for } \frac{|x+1|}{2}>1\\ \frac{8ix\sqrt{1-x}}{15}+\frac{i\sqrt{1-x}\,(x+1)^2}{5}+\frac{8i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}\right)+2\left(\begin{cases}\frac{32x\sqrt{x-1}}{35}+\frac{\sqrt{x-1}\,(x+1)^3}{7}+\frac{12\sqrt{x-1}\,(x+1)^2}{35}+\frac{32\sqrt{x-1}}{7} & \text{fo}\\ \frac{32ix\sqrt{1-x}}{35}+\frac{i\sqrt{1-x}\,(x+1)^3}{7}+\frac{12i\sqrt{1-x}\,(x+1)^2}{35}+\frac{32i\sqrt{1-x}}{7} & \text{o}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(3/2),x)
```

```
[Out] -2*Piecewise((8*x*sqrt(x - 1)/15 + sqrt(x - 1)*(x + 1)**2/5 + 8*sqrt(x - 1)
/3, Abs(x + 1)/2 > 1), (8*I*x*sqrt(1 - x)/15 + I*sqrt(1 - x)*(x + 1)**2/5 +
 8*I*sqrt(1 - x)/3, True)) + 2*Piecewise((32*x*sqrt(x - 1)/35 + sqrt(x - 1)
*(x + 1)**3/7 + 12*sqrt(x - 1)*(x + 1)**2/35 + 32*sqrt(x - 1)/7, Abs(x + 1)
/2 > 1), (32*I*x*sqrt(1 - x)/35 + I*sqrt(1 - x)*(x + 1)**3/7 + 12*I*sqrt(1
- x)*(x + 1)**2/35 + 32*I*sqrt(1 - x)/7, True))
```

3.320 $\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt{-\frac{1-x}{x}}\,x(x+1)^{3/2}}{5\left(\frac{1}{x}+1\right)^{3/2}}+\frac{28\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}x}$$

```
[Out] 28/15*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1/x+1)^(3/2)+86/15*(1+x)^(3/2)*((-1+x)/
x)^(1/2)/(1/x+1)^(3/2)/x+2/5*x*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1/x+1)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6176, 6181, 89, 78, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\,x(x+1)^{3/2}}{5\left(\frac{1}{x}+1\right)^{3/2}}+\frac{28\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

```
[Out] (28*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)) + (86*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)*x) + (2*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(5*(1 + x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \frac{(1+x)^{3/2}\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\,dx}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}$$
$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{(1+x)^2}{\sqrt{1-x}\,x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{3/2}}$$
$$= \frac{2\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{7+\frac{5x}{2}}{\sqrt{1-x}\,x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\left(1+\frac{1}{x}\right)^{3/2}}$$
$$= \frac{28\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(43\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x^{3/2}}\right.}{15\left(1+\frac{1}{x}\right)^{3/2}}$$
$$= \frac{28\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}} + \frac{86\sqrt{-\frac{1-x}{x}}(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}x} + \frac{2\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.38

$$\frac{2\sqrt{\frac{x-1}{x}}\sqrt{x+1}\left(3x^2+14x+43\right)}{15\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 + x)^(3/2),x]
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(43 + 14*x + 3*x^2))/(15*Sqrt[1 + x^(-1)])
```

fricas [A] time = 0.54, size = 28, normalized size = 0.26

$$\frac{2}{15}\left(3x^2+14x+43\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + 14*x + 43)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.04, size = 32, normalized size = 0.30

$$\frac{2\left(-1+x\right)\left(3x^{2}+14x+43\right)}{15\sqrt{1+x}\,\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x)
```

```
[Out] 2/15*(-1+x)*(3*x^2+14*x+43)/(1+x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

maxima [A] time = 1.03, size = 22, normalized size = 0.21

$$\frac{2\left(3\,x^{3}+11\,x^{2}+29\,x-43\right)}{15\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^3 + 11*x^2 + 29*x - 43)/sqrt(x - 1)
```

mupad [B] time = 1.26, size = 38, normalized size = 0.36

$$\sqrt{\frac{x-1}{x+1}}\left(\frac{28\,x\,\sqrt{x+1}}{15}+\frac{86\,\sqrt{x+1}}{15}+\frac{2\,x^{2}\,\sqrt{x+1}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] ((x - 1)/(x + 1))^(1/2)*((28*x*(x + 1)^(1/2))/15 + (86*(x + 1)^(1/2))/15 +
(2*x^2*(x + 1)^(1/2))/5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x+1\right)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(3/2),x)
```

```
[Out] Integral((x + 1)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

3.321 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx$

Optimal. Leaf size=104

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{44\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[Out] 44/105*(1/x+1)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)-22/35*(1/x+1)^(3/2)*(1-x)^(3
/2)*x/(1-1/x)^(3/2)+2/7*(1/x+1)^(3/2)*(1-x)^(3/2)*x^2/(1-1/x)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 78, 45, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{44\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (44*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(105*(1 - x^(-1))^(3/2)) - (22*(1 + x
^(-1))^(3/2)*(1 - x)^(3/2)*x)/(35*(1 - x^(-1))^(3/2)) + (2*(1 + x^(-1))^(3/
2)*(1 - x)^(3/2)*x^2)/(7*(1 - x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx = \frac{(1-x)^{3/2}\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^{3/2}x^{5/2}\,dx}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left((1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{(1-x)\sqrt{1+x}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}} + \frac{\left(11(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}} - \frac{\left(22(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\right.}{35\left(1-\frac{1}{x}\right)^{3/2}}$$

$$= \frac{44\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}} - \frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.44

$$-\frac{2\sqrt{\frac{1}{x}+1}\sqrt{1-x}\,(x+1)\left(15x^2-33x+22\right)}{105\sqrt{\frac{x-1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(22 - 33*x + 15*x^2))/(105*Sqrt[(-
1 + x)/x])
```

fricas [A] time = 0.50, size = 45, normalized size = 0.43

$$-\frac{2\left(15\,x^4-3\,x^3-29\,x^2+11\,x+22\right)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="fricas")
```

[Out] -2/105*(15*x^4 - 3*x^3 - 29*x^2 + 11*x + 22)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)

giac [C] time = 0.17, size = 58, normalized size = 0.56

$$\frac{1}{105}\left(16i\sqrt{2} + \frac{2\left(15\,(x+1)^3\sqrt{-x-1} - 63\,(x+1)^2\sqrt{-x-1} - 70\,(-x-1)^{\frac{3}{2}}\right)}{\operatorname{sgn}(-x-1)}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="giac")

[Out] 1/105*(16*I*sqrt(2) + 2*(15*(x + 1)^3*sqrt(-x - 1) - 63*(x + 1)^2*sqrt(-x - 1) - 70*(-x - 1)^(3/2))/sgn(-x - 1))*sgn(x)

maple [A] time = 0.04, size = 34, normalized size = 0.33

$$-\frac{2\,(1+x)\left(15x^2-33x+22\right)\sqrt{1-x}}{105\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x)

[Out] -2/105*(1+x)*(15*x^2-33*x+22)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)

maxima [C] time = 0.99, size = 22, normalized size = 0.21

$$-\frac{1}{105}\left(30i\,x^3 - 36i\,x^2 - 22i\,x + 44i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="maxima")

[Out] -1/105*(30*I*x^3 - 36*I*x^2 - 22*I*x + 44*I)*sqrt(x + 1)

mupad [B] time = 1.31, size = 35, normalized size = 0.34

$$\frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2\,\left(15\,x^2-33\,x+22\right)}{105\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(1 - x)^(3/2))/((x - 1)/(x + 1))^(1/2),x)

[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2*(15*x^2 - 33*x + 22))/(105*(1 - x)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2)*x,x)

[Out] Timed out

3.322 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx$

Optimal. Leaf size=68

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}-\frac{14\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[Out] -14/15*(1/x+1)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)+2/5*(1/x+1)^(3/2)*(1-x)^(3/2
)*x/(1-1/x)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6176, 6181, 78, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}-\frac{14\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

```
[Out] (-14*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(15*(1 - x^(-1))^(3/2)) + (2*(1 + x^
(-1))^(3/2)*(1 - x)^(3/2)*x)/(5*(1 - x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx = \frac{(1-x)^{3/2}\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}$$
$$= -\frac{\left((1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{(1-x)\sqrt{1+x}}{x^{7/2}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}$$
$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}} + \frac{\left(7(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^{5/2}}\,dx, x, \frac{1}{x}\right)}{5\left(1-\frac{1}{x}\right)^{3/2}}$$
$$= -\frac{14\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.60

$$-\frac{2\sqrt{\frac{1}{x}+1}\,\sqrt{1-x}\,(x+1)(3x-7)}{15\sqrt{\frac{x-1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^(3/2),x]
[Out] (-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(-7 + 3*x))/(15*Sqrt[(-1 + x)/x])
```

fricas [A] time = 0.41, size = 40, normalized size = 0.59

$$-\frac{2\left(3\,x^3-x^2-11\,x-7\right)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="fricas")
[Out] -2/15*(3*x^3 - x^2 - 11*x - 7)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

giac [C] time = 0.16, size = 44, normalized size = 0.65

$$\frac{1}{15}\left(-16i\,\sqrt{2}+\frac{2\left(3\,(x+1)^2\sqrt{-x-1}\,+10\,(-x-1)^{\frac{3}{2}}\right)}{\text{sgn}\,(-x-1)}\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="giac")
[Out] 1/15*(-16*I*sqrt(2) + 2*(3*(x + 1)^2*sqrt(-x - 1) + 10*(-x - 1)^(3/2))/sgn(
-x - 1))*sgn(x)
```

maple [A] time = 0.04, size = 29, normalized size = 0.43

$$-\frac{2\,(1+x)\,(3x-7)\,\sqrt{1-x}}{15\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x)
[Out] -2/15*(1+x)*(3*x-7)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

maxima [C] time = 0.46, size = 17, normalized size = 0.25

$$-\frac{1}{15}\left(6i\,x^2-8i\,x-14i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="maxima")
[Out] -1/15*(6*I*x^2 - 8*I*x - 14*I)*sqrt(x + 1)
```

mupad [B] time = 1.25, size = 30, normalized size = 0.44

$$\frac{2\,(3\,x-7)\,\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{15\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
[Out] (2*(3*x - 7)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(1-x)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2),x)
[Out] Integral((1 - x)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

3.323 $\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt{-\frac{1-x}{x}}\,\sqrt{x+1}\,x^2}{5\sqrt{\frac{1}{x}+1}}+\frac{6\sqrt{-\frac{1-x}{x}}\,\sqrt{x+1}\,x}{5\sqrt{\frac{1}{x}+1}}+\frac{12\sqrt{-\frac{1-x}{x}}\,\sqrt{x+1}}{5\sqrt{\frac{1}{x}+1}}$$

```
[Out] 12/5*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1/x+1)^(1/2)+6/5*x*((-1+x)/x)^(1/2)*(1+x
)^(1/2)/(1/x+1)^(1/2)+2/5*x^2*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1/x+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6176, 6181, 78, 45, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\,\sqrt{x+1}\,x^2}{5\sqrt{\frac{1}{x}+1}}+\frac{6\sqrt{-\frac{1-x}{x}}\,\sqrt{x+1}\,x}{5\sqrt{\frac{1}{x}+1}}+\frac{12\sqrt{-\frac{1-x}{x}}\,\sqrt{x+1}}{5\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

```
[Out] (12*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (6*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x)/x)]*x^2*Sqrt[
1 + x])/(5*Sqrt[1 + x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}x\sqrt{1+x}\,dx=\frac{\sqrt{1+x}\int e^{\coth^{-1}(x)}\sqrt{1+\frac{1}{x}}\,x^{3/2}\,dx}{\sqrt{1+\frac{1}{x}}\sqrt{x}}$$
$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1+x}{\sqrt{1-x}\,x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1+\frac{1}{x}}}$$
$$=\frac{2\sqrt{-\frac{1-x}{x}}\,x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}-\frac{\left(9\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1+\frac{1}{x}}}$$
$$=\frac{6\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}\,x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}-\frac{\left(6\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1+\frac{1}{x}}}$$
$$=\frac{12\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{6\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}\,x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.36

$$\frac{2\sqrt{\frac{x-1}{x}}\sqrt{x+1}\left(x^2+3x+6\right)}{5\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*x*Sqrt[1 + x],x]
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(6 + 3*x + x^2))/(5*Sqrt[1 + x^(-1)])
```

fricas [A] time = 0.41, size = 26, normalized size = 0.24

$$\frac{2}{5}\left(x^2+3x+6\right)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(x^2 + 3*x + 6)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.04, size = 30, normalized size = 0.28

$$\frac{2\left(-1+x\right)\left(x^{2}+3x+6\right)}{5\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x)
```

```
[Out] 2/5*(-1+x)*(x^2+3*x+6)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

maxima [A] time = 0.46, size = 20, normalized size = 0.19

$$\frac{2\left(x^{3}+2\,x^{2}+3\,x-6\right)}{5\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x^3 + 2*x^2 + 3*x - 6)/sqrt(x - 1)
```

mupad [B] time = 1.25, size = 38, normalized size = 0.36

$$\sqrt{\frac{x-1}{x+1}}\,\left(\frac{6\,x\,\sqrt{x+1}}{5}+\frac{12\,\sqrt{x+1}}{5}+\frac{2\,x^2\,\sqrt{x+1}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] ((x - 1)/(x + 1))^(1/2)*((6*x*(x + 1)^(1/2))/5 + (12*(x + 1)^(1/2))/5 + (2*
x^2*(x + 1)^(1/2))/5)
```

sympy [C] time = 7.93, size = 133, normalized size = 1.24

$$-2\left(\begin{cases}\frac{x\sqrt{x-1}}{3}+\frac{5\sqrt{x-1}}{3} & \text{for}\ \frac{|x+1|}{2}>1\\ \frac{ix\sqrt{1-x}}{3}+\frac{5i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}\right)+2\left(\begin{cases}\frac{8x\sqrt{x-1}}{15}+\frac{\sqrt{x-1}\,(x+1)^2}{5}+\frac{8\sqrt{x-1}}{3} & \text{for}\ \frac{|x+1|}{2}>1\\ \frac{8ix\sqrt{1-x}}{15}+\frac{i\sqrt{1-x}\,(x+1)^2}{5}+\frac{8i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(1/2),x)
```

```
[Out] -2*Piecewise((x*sqrt(x - 1)/3 + 5*sqrt(x - 1)/3, Abs(x + 1)/2 > 1), (I*x*sq
rt(1 - x)/3 + 5*I*sqrt(1 - x)/3, True)) + 2*Piecewise((8*x*sqrt(x - 1)/15 +
 sqrt(x - 1)*(x + 1)**2/5 + 8*sqrt(x - 1)/3, Abs(x + 1)/2 > 1), (8*I*x*sqrt
(1 - x)/15 + I*sqrt(1 - x)*(x + 1)**2/5 + 8*I*sqrt(1 - x)/3, True))
```

3.324 $\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx$

Optimal. Leaf size=70

$$\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}\,x}{3\sqrt{\frac{1}{x}+1}}+\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{3\sqrt{\frac{1}{x}+1}}$$

```
[Out] 10/3*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1/x+1)^(1/2)+2/3*x*((-1+x)/x)^(1/2)*(1+x
)^(1/2)/(1/x+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 78, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}\,x}{3\sqrt{\frac{1}{x}+1}}+\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{3\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 + x],x]
```

```
[Out] (10*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{\sqrt{1+x}\int e^{\coth^{-1}(x)}\sqrt{1+\frac{1}{x}}\sqrt{x}\,dx}{\sqrt{1+\frac{1}{x}}\sqrt{x}}$$
$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1+x}{\sqrt{1-x}\,x^{5/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1+\frac{1}{x}}}$$
$$= \frac{2\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}} - \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1+\frac{1}{x}}}$$
$$= \frac{10\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.49

$$\frac{2\sqrt{\frac{x-1}{x}}\sqrt{x+1}\,(x+5)}{3\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 + x],x]
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(5 + x))/(3*Sqrt[1 + x^(-1)])
```

fricas [A] time = 0.52, size = 21, normalized size = 0.30

$$\frac{2}{3}\,(x+5)\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x + 5)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.04, size = 25, normalized size = 0.36

$$\frac{2\,(-1+x)\,(x+5)}{3\sqrt{\frac{-1+x}{1+x}}\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x)
```

```
[Out] 2/3*(-1+x)*(x+5)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

maxima [A] time = 0.89, size = 15, normalized size = 0.21

$$\frac{2\left(x^{2}+4x-5\right)}{3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 + 4*x - 5)/sqrt(x - 1)
```

mupad [B] time = 1.24, size = 21, normalized size = 0.30

$$\frac{2\sqrt{\frac{x-1}{x+1}}\sqrt{x+1}\,(x+5)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 5))/3
```

sympy [A] time = 4.83, size = 39, normalized size = 0.56

$$2\left(\begin{cases}2\sqrt{2}\left(\frac{\sqrt{2}(x-1)^{\frac{3}{2}}}{12}+\frac{\sqrt{2}\sqrt{x-1}}{2}\right) & \text{for}\ x\geq -1\wedge x<1\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(1/2),x)
```

```
[Out] 2*Piecewise((2*sqrt(2)*(sqrt(2)*(x - 1)**(3/2)/12 + sqrt(2)*sqrt(x - 1)/2),
 (x >= -1) & (x < 1)))
```

3.325 $\int e^{\coth^{-1}(x)}\sqrt{1-x}\,x\,dx$

Optimal. Leaf size=71

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}\,x^2}{5\sqrt{1-\frac{1}{x}}}-\frac{4\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}\,x}{15\sqrt{1-\frac{1}{x}}}$$

```
[Out] -4/15*(1/x+1)^(3/2)*x*(1-x)^(1/2)/(1-1/x)^(1/2)+2/5*(1/x+1)^(3/2)*x^2*(1-x)
^(1/2)/(1-1/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6176, 6181, 45, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}\,x^2}{5\sqrt{1-\frac{1}{x}}}-\frac{4\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}\,x}{15\sqrt{1-\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

```
[Out] (-4*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(15*Sqrt[1 - x^(-1)]) + (2*(1 + x^(-1
))^(3/2)*Sqrt[1 - x]*x^2)/(5*Sqrt[1 - x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,x\,dx = \frac{\sqrt{1-x}\int e^{\coth^{-1}(x)}\sqrt{1-\frac{1}{x}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{x}}\sqrt{x}}$$
$$= -\frac{\left(\sqrt{1-x}\sqrt{\frac{1}{x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^{7/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{x}}}$$
$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x^2}{5\sqrt{1-\frac{1}{x}}} + \frac{\left(2\sqrt{1-x}\sqrt{\frac{1}{x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^{5/2}}\,dx, x, \frac{1}{x}\right)}{5\sqrt{1-\frac{1}{x}}}$$
$$= -\frac{4\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x}{15\sqrt{1-\frac{1}{x}}} + \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x^2}{5\sqrt{1-\frac{1}{x}}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.58

$$\frac{2\sqrt{\frac{1}{x}+1}\,\sqrt{1-x}\,(x+1)(3x-2)}{15\sqrt{\frac{x-1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(-2 + 3*x))/(15*Sqrt[(-1 + x)/x])
```

fricas [A] time = 0.63, size = 40, normalized size = 0.56

$$\frac{2\left(3\,x^3+4\,x^2-x-2\right)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^3 + 4*x^2 - x - 2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

giac [C] time = 0.18, size = 44, normalized size = 0.62

$$\frac{1}{15}\left(-4i\sqrt{2} - \frac{2\left(3\,(x+1)^2\sqrt{-x-1} + 5\,(-x-1)^{\frac{3}{2}}\right)}{\text{sgn}\,(-x-1)}\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="giac")
[Out] 1/15*(-4*I*sqrt(2) - 2*(3*(x + 1)^2*sqrt(-x - 1) + 5*(-x - 1)^(3/2))/sgn(-x
 - 1))*sgn(x)
```

maple [A] time = 0.03, size = 29, normalized size = 0.41

$$\frac{2\,(1+x)\,(3x-2)\,\sqrt{1-x}}{15\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x)
```

```
[Out] 2/15*(1+x)*(3*x-2)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

maxima [C] time = 1.28, size = 17, normalized size = 0.24

$$\frac{1}{15}\left(6i\,x^2 + 2i\,x - 4i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(6*I*x^2 + 2*I*x - 4*I)*sqrt(x + 1)
```

mupad [B] time = 1.27, size = 30, normalized size = 0.42

$$-\frac{2\,(3\,x-2)\,\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{15\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] -(2*(3*x - 2)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

sympy [C] time = 16.15, size = 46, normalized size = 0.65

$$-\frac{14ix}{15\sqrt{\frac{1}{x+1}}} - \frac{2i\,(1-x)^2}{5\sqrt{\frac{1}{x+1}}} + \frac{2i}{3\sqrt{\frac{1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1-x)**(1/2),x)
```

```
[Out] -14*I*x/(15*sqrt(1/(x + 1))) - 2*I*(1 - x)**2/(5*sqrt(1/(x + 1))) + 2*I/(3*
sqrt(1/(x + 1)))
```

3.326 $\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx$

Optimal. Leaf size=20

$$\frac{2}{3}\sqrt{1-x}\,(x+1)e^{\coth^{-1}(x)}$$

```
[Out] 2/3/((-1+x)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6174}

$$\frac{2}{3}\sqrt{1-x}\,(x+1)e^{\coth^{-1}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 - x],x]
```

```
[Out] (2*E^ArcCoth[x]*Sqrt[1 - x]*(1 + x))/3
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx=\frac{2}{3}e^{\coth^{-1}(x)}\sqrt{1-x}\,(1+x)$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.70

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}\,x}{3\sqrt{1-\frac{1}{x}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 - x],x]
```

```
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(3*Sqrt[1 - x^(-1)])
```

fricas [A] time = 0.60, size = 33, normalized size = 1.65

$$\frac{2\left(x^2+2x+1\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + 2*x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

giac [C] time = 0.15, size = 27, normalized size = 1.35

$$\frac{1}{3}\left(-4i\sqrt{2}+\frac{2(-x-1)^{\frac{3}{2}}}{\mathrm{sgn}(-x-1)}\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="giac")
[Out] 1/3*(-4*I*sqrt(2) + 2*(-x - 1)^(3/2)/sgn(-x - 1))*sgn(x)
```

maple [A] time = 0.03, size = 24, normalized size = 1.20

$$\frac{2\left(1+x\right)\sqrt{1-x}}{3\sqrt{\frac{-1+x}{1+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x)
[Out] 2/3/((-1+x)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

maxima [C] time = 0.50, size = 12, normalized size = 0.60

$$\frac{1}{3}\sqrt{x+1}\,(2i\,x+2i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x + 1)*(2*I*x + 2*I)
```

mupad [B] time = 1.27, size = 25, normalized size = 1.25

$$-\frac{2\sqrt{\frac{x-1}{x+1}}\,\left(x+1\right)^2}{3\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
[Out] -(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(3*(1 - x)^(1/2))
```

sympy [C] time = 6.80, size = 29, normalized size = 1.45

$$-\frac{2ix}{3\sqrt{\frac{1}{x+1}}}-\frac{2i}{3\sqrt{\frac{1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(1/2),x)
[Out] -2*I*x/(3*sqrt(1/(x + 1))) - 2*I/(3*sqrt(1/(x + 1)))
```

3.327 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{x+1}}+\frac{4\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x}{3\sqrt{x+1}}$$

```
[Out] 4/3*x*(1/x+1)^(1/2)*((-1+x)/x)^(1/2)/(1+x)^(1/2)+2/3*x^2*(1/x+1)^(1/2)*((-1
+x)/x)^(1/2)/(1+x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6176, 6181, 45, 37}

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{x+1}}+\frac{4\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

```
[Out] (4*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/(3*Sqrt[1 + x]) + (2*Sqrt[1 + x^(
-1)]*Sqrt[-((1 - x)/x)]*x^2)/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx = \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}\sqrt{x}}{\sqrt{1+\frac{1}{x}}}\,dx}{\sqrt{1+x}}$$
$$= -\frac{\sqrt{1+\frac{1}{x}}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{5/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{1+x}}$$
$$= \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{1+x}} - \frac{\left(2\sqrt{1+\frac{1}{x}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{\frac{1}{x}}\sqrt{1+x}}$$
$$= \frac{4\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x}{3\sqrt{1+x}} + \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{1+x}}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.36

$$\frac{2\sqrt{1-\frac{1}{x^2}}\,x(x+2)}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
[Out] (2*Sqrt[1 - x^(-2)]*x*(2 + x))/(3*Sqrt[1 + x])
```

fricas [A] time = 0.63, size = 21, normalized size = 0.29

$$\frac{2}{3}\,(x+2)\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x + 2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.04, size = 25, normalized size = 0.34

$$\frac{2\,(-1+x)\,(x+2)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x)
[Out] 2/3*(-1+x)*(x+2)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

maxima [A] time = 1.00, size = 13, normalized size = 0.18

$$\frac{2\left(x^2+x-2\right)}{3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x^2 + x - 2)/sqrt(x - 1)
```

mupad [B] time = 1.24, size = 21, normalized size = 0.29

$$\frac{2\sqrt{\frac{x-1}{x+1}}\sqrt{x+1}\,(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 2))/3
```

sympy [A] time = 7.82, size = 48, normalized size = 0.66

$$\begin{cases} \frac{2x\sqrt{x-1}}{3} + \frac{4\sqrt{x-1}}{3} & \text{for } |x| > 1 \\ \frac{2ix\sqrt{1-x}}{3} + \frac{4i\sqrt{1-x}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(1/2),x)
[Out] Piecewise((2*x*sqrt(x - 1)/3 + 4*sqrt(x - 1)/3, Abs(x) > 1), (2*I*x*sqrt(1
- x)/3 + 4*I*sqrt(1 - x)/3, True))
```

3.328 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x}{\sqrt{x+1}}$$

[Out] 2*x*(1/x+1)^(1/2)*((-1+x)/x)^(1/2)/(1+x)^(1/2)

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 37}

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[x]/Sqrt[1 + x],x]

[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/Sqrt[1 + x]

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+\frac{1}{x}}\sqrt{x}}\,dx}{\sqrt{1+x}}\\
&= -\frac{\sqrt{1+\frac{1}{x}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{1+x}}\\
&= \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x}{\sqrt{1+x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.64

$$\frac{2\sqrt{1-\frac{1}{x^2}}\,x}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/Sqrt[1 + x],x]
[Out] (2*Sqrt[1 - x^(-2)]*x)/Sqrt[1 + x]
```

fricas [A] time = 0.59, size = 18, normalized size = 0.55

$$2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

giac [C] time = 0.14, size = 13, normalized size = 0.39

$$-2i\sqrt{2}+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*I*sqrt(2) + 2*sqrt(x - 1)
```

maple [A] time = 0.03, size = 22, normalized size = 0.67

$$\frac{-2+2x}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x)
[Out] 2*(-1+x)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

maxima [A] time = 0.37, size = 7, normalized size = 0.21

$$2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x - 1)
```

mupad [B] time = 1.22, size = 18, normalized size = 0.55

$$2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] 2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)
```

sympy [A] time = 7.49, size = 19, normalized size = 0.58

$$\begin{cases} 2\sqrt{x-1} & \text{for } |x| > 1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(1/2),x)
[Out] Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x), True))
```

3.329 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=126

$$\frac{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{3\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\,x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

```
[Out] 2/3*(1/x+1)^(3/2)*x^2*(1-1/x)^(1/2)/(1-x)^(1/2)+2*x*(1-1/x)^(1/2)*(1/x+1)^(
1/2)/(1-x)^(1/2)-2*arctanh(2^(1/2)*(1/x)^(1/2)/(1/x+1)^(1/2))*2^(1/2)*(1-1/
x)^(1/2)/(1-x)^(1/2)/(1/x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6176, 6181, 96, 94, 93, 206}

$$\frac{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{3\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\,x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] + (2*Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2)/(3*Sqrt[1 - x]) - (2*Sqrt[2]*Sqrt[1 - x^(-1)]*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[x^(-1)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}\sqrt{x}}{\sqrt{1-\frac{1}{x}}}\,dx}{\sqrt{1-x}}\\
&= -\frac{\sqrt{1-\frac{1}{x}}\,\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{5/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{\sqrt{1-\frac{1}{x}}\,\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{\left(2\sqrt{1-\frac{1}{x}}\right)\mathrm{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{\left(4\sqrt{1-\frac{1}{x}}\right)\mathrm{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}\\
&= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.55

$$
\frac{2\sqrt{\frac{x-1}{x}}\,x\left(\sqrt{\frac{1}{x}+1}\,(x+4)-3\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{x+1}}\right)\right)}{3\sqrt{1-x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)]*(4 + x) - 3*Sqrt[2]*Sqrt[x^(-1)]*Ar
cTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(3*Sqrt[1 - x])
```

fricas [A] time = 0.52, size = 72, normalized size = 0.57

$$\frac{2\left(3\sqrt{2}(x-1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-\left(x^2+5x+4\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x
 - 1)) - (x^2 + 5*x + 4)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*sig
n(x)*((-3*sqrt(2)*atan(i)-(-5*i)*sqrt(2))/3+(-sqrt(2)*atan(sqrt(-x-1)/sqrt(
2))-1/3*sqrt(-x-1)*(-x-1)+sqrt(-x-1))/sign(-x-1))
```

maple [A] time = 0.05, size = 65, normalized size = 0.52

$$-\frac{2\sqrt{1-x}\,\left(\sqrt{-1-x}\,x-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)+4\sqrt{-1-x}\right)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{-1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x)
```

```
[Out] -2/3*(1-x)^(1/2)*((-1-x)^(1/2)*x-3*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))
+4*(-1-x)^(1/2))/((-1+x)/(1+x))^(1/2)/(-1-x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

```
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*sqrt(1 - x)), x)
```

3.330 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=90

$$\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\,x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

```
[Out] 2*x*(1-1/x)^(1/2)*(1/x+1)^(1/2)/(1-x)^(1/2)-2*arctanh(2^(1/2)*(1/x)^(1/2)/(
1/x+1)^(1/2))*2^(1/2)*(1-1/x)^(1/2)/(1-x)^(1/2)/(1/x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6176, 6181, 94, 93, 206}

$$\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\,x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] - (2*Sqrt[2]*Sqrt[1 - x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[
x^(-1)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{\left(\sqrt{1-\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-\frac{1}{x}}\sqrt{x}}\,dx}{\sqrt{1-x}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} - \frac{\left(2\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} - \frac{\left(4\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 0.70

$$\frac{2\sqrt{\frac{x-1}{x}}\,x\left(\sqrt{\frac{1}{x}+1}-\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{x+1}}\right)\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)] - Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[Sqrt
[2]*Sqrt[(1 + x)^(-1)]]))/Sqrt[1 - x]
```

fricas [A] time = 0.48, size = 66, normalized size = 0.73

$$\frac{2\left(\sqrt{2}\,(x-1)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1
)) - (x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*sig
n(x)*(-sqrt(2)*atan(i)-(-i)*sqrt(2)+(sqrt(-x-1)-sqrt(2)*atan(sqrt(-x-1)/sqr
t(2)))/sign(-x-1))
```

maple [A] time = 0.04, size = 54, normalized size = 0.60

$$-\frac{2\sqrt{1-x}\,\left(-\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)+\sqrt{-1-x}\right)}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{-1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x)
```

```
[Out] -2*(1-x)^(1/2)*(-2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))+(-1-x)^(1/2))/((-
1+x)/(1+x))^(1/2)/(-1-x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

```
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*sqrt(1 - x)), x)
```

3.331 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(x+1)^{3/2}}+\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

```
[Out] (1/x+1)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/((-1+x)/x)^(1/2))*2^(1/2)/(1/x)^(3
/2)/(1+x)^(3/2)+2*(1/x+1)^(3/2)*x^2*((-1+x)/x)^(1/2)/(1+x)^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6176, 6181, 96, 93, 203}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(x+1)^{3/2}}+\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

```
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[-((1 - x)/x)]*x^2)/(1 + x)^(3/2) + (Sqrt[2]*(1 +
 x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/x)]])/((x^(-1))
^(3/2)*(1 + x)^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx &= \frac{\left(\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^{3/2}\sqrt{x}}\,dx}{(1+x)^{3/2}}\\
&= -\frac{\left(1+\frac{1}{x}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{3/2}(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}}+\frac{\left(1+\frac{1}{x}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{x}(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}}+\frac{\left(2\left(1+\frac{1}{x}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{-1+x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}}+\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.70

$$
\frac{\sqrt{\frac{1}{x}+1}\,x\left(2\sqrt{\frac{x-1}{x}}-\sqrt{2}\sqrt{\frac{1}{x}}\tan^{-1}\left(\frac{\sqrt{\frac{x-1}{x^2}}\,x}{\sqrt{2}}\right)\right)}{\sqrt{x+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 + x^(-1)]*x*(2*Sqrt[(-1 + x)/x] - Sqrt[2]*Sqrt[x^(-1)]*ArcTan[(Sqrt
[(-1 + x)/x^2]*x)/Sqrt[2]]))/Sqrt[1 + x]
```

fricas [A] time = 0.87, size = 46, normalized size = 0.49

$$
-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)+2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))) + 2*sqrt(x +
 1)*sqrt((x - 1)/(x + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(sq
rt(x-1)-atan(sqrt(x-1)/sqrt(2))/sqrt(2)+(atan(i)-2*i)/sqrt(2))
```

maple [A] time = 0.05, size = 47, normalized size = 0.51

$$\frac{\sqrt{-1+x}\,\left(2\sqrt{-1+x}-\sqrt{2}\,\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\right)}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x)
```

```
[Out] 1/((-1+x)/(1+x))^(1/2)*(-1+x)^(1/2)/(1+x)^(1/2)*(2*(-1+x)^(1/2)-2^(1/2)*arc
tan(1/2*(-1+x)^(1/2)*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
```

```
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(3/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**(3/2)), x)
```

3.332 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}} \, dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

```
[Out] -(1/x+1)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/((-1+x)/x)^(1/2))*2^(1/2)/(1/x)^(
3/2)/(1+x)^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6176, 6181, 93, 203}

$$-\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

```
[Out] -((Sqrt[2]*(1 + x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/
x)]])/((x^(-1))^(3/2)*(1 + x)^(3/2)))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx = \frac{\left(\left(1+\frac{1}{x}\right)^{3/2} x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^{3/2} x^{3/2}}\,dx}{(1+x)^{3/2}}$$
$$= -\frac{\left(1+\frac{1}{x}\right)^{3/2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,(1+x)}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2} (1+x)^{3/2}}$$
$$= -\frac{\left(2\left(1+\frac{1}{x}\right)^{3/2}\right) \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{-1+x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2} (1+x)^{3/2}}$$
$$= -\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2} (1+x)^{3/2}}$$

Mathematica [A]   time = 0.02, size = 41, normalized size = 0.71

$$\sqrt{2}\sqrt{\frac{1}{x+1}}\sqrt{x+1}\,\tan^{-1}\left(\frac{\sqrt{\frac{x-1}{x^2}}\,x}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

```
[Out] Sqrt[2]*Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcTan[(Sqrt[(-1 + x)/x^2]*x)/Sqrt[2
]]
```

fricas [A]   time = 0.64, size = 26, normalized size = 0.45

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1)))
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(-s
qrt(2)*atan(i)/2+1/2*sqrt(2)*atan(sqrt(x-1)/sqrt(2)))
```

maple [A]   time = 0.05, size = 37, normalized size = 0.64

$$\frac{\sqrt{-1+x}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x)
```

```
[Out] 1/((-1+x)/(1+x))^(1/2)*(-1+x)^(1/2)/(1+x)^(1/2)*2^(1/2)*arctan(1/2*(-1+x)^(
1/2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
```

```
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*(x + 1)**(3/2)), x)
```

3.333 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{2(1-x)^{3/2}}+\frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{x}+1}\,x^2}{2(1-x)^{3/2}}-\frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] -5/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1/x+1)^(1/2))/(1-x)^(3/2)/(
1/x)^(3/2)*2^(1/2)-1/2*(1/x+1)^(3/2)*x^2*(1-1/x)^(1/2)/(1-x)^(3/2)+5/2*(1-1
/x)^(3/2)*x^2*(1/x+1)^(1/2)/(1-x)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6176, 6181, 96, 94, 93, 206}

$$-\frac{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{2(1-x)^{3/2}}+\frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{x}+1}\,x^2}{2(1-x)^{3/2}}-\frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] (5*(1 - x^(-1))^(3/2)*Sqrt[1 + x^(-1)]*x^2)/(2*(1 - x)^(3/2)) - (Sqrt[1 - x
^(-1)]*(1 + x^(-1))^(3/2)*x^2)/(2*(1 - x)^(3/2)) - (5*(1 - x^(-1))^(3/2)*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*(x^(
-1))^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^{3/2}\sqrt{x}}\,dx}{(1-x)^{3/2}}\\
&= -\frac{\left(1-\frac{1}{x}\right)^{3/2}\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)^2x^{3/2}}\,dx,x,\frac{1}{x}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}\\
&= -\frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}}-\frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx,x,\frac{1}{x}\right)}{4(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}\\
&= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}\,x^2}{2(1-x)^{3/2}}-\frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}}-\frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)}{2(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}\\
&= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}\,x^2}{2(1-x)^{3/2}}-\frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}}-\frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}\\
&= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}\,x^2}{2(1-x)^{3/2}}-\frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}}-\frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 75, normalized size = 0.58

$$-\frac{\sqrt{\frac{x-1}{x}}\,x\left(2\sqrt{\frac{1}{x}+1}\,(3-2x)+5\sqrt{2}\,(x-1)\sqrt{\frac{1}{x}}\,\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{x+1}}\right)\right)}{2(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)]*(3 - 2*x) + 5*Sqrt[2]*(-1 + x)
*Sqrt[x^(-1)]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(1 - x)^(3/2)
```

fricas [A] time = 0.51, size = 84, normalized size = 0.65

$$-\frac{5\sqrt{2}\left(x^2-2x+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-2\left(2x^2-x-3\right)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(5*sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x
 + 1))/(x - 1)) - 2*(2*x^2 - x - 3)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^
2 - 2*x + 1)
```

giac [A] time = 0.13, size = 54, normalized size = 0.42

$$\frac{\left(5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right)-4\sqrt{-x-1}+\frac{2\sqrt{-x-1}}{x-1}\right)\operatorname{sgn}(x)}{2\operatorname{sgn}(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(5*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) - 4*sqrt(-x - 1) + 2*sqrt(-
x - 1)/(x - 1))*sgn(x)/sgn(-x - 1)
```

maple [A] time = 0.06, size = 90, normalized size = 0.69

$$-\frac{\sqrt{1-x}\left(5\sqrt{2}\arctan\left(\frac{\sqrt{-1-x}\sqrt{2}}{2}\right)x-5\sqrt{2}\arctan\left(\frac{\sqrt{-1-x}\sqrt{2}}{2}\right)-4\sqrt{-1-x}\,x+6\sqrt{-1-x}\right)}{2\sqrt{\frac{-1+x}{1+x}}\,(-1+x)\sqrt{-1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x)
```

```
[Out] -1/2/((-1+x)/(1+x))^(1/2)/(-1+x)*(1-x)^(1/2)*(5*2^(1/2)*arctan(1/2*(-1-x)^(
1/2)*2^(1/2))*x-5*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))-4*(-1-x)^(1/2)*x
+6*(-1-x)^(1/2))/(-1-x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

```
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(3/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(1 - x)**(3/2)), x)
```

3.334 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{\sqrt{\frac{1}{x}+1}\,x\sqrt{1-\frac{1}{x}}}{(1-x)^{3/2}}-\frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] -1/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1/x+1)^(1/2))/(1-x)^(3/2)/(
1/x)^(3/2)*2^(1/2)-x*(1-1/x)^(1/2)*(1/x+1)^(1/2)/(1-x)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6176, 6181, 94, 93, 206}

$$-\frac{\sqrt{\frac{1}{x}+1}\,x\sqrt{1-\frac{1}{x}}}{(1-x)^{3/2}}-\frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

```
[Out] -((Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/(1 - x)^(3/2)) - ((1 - x^(-1))^(3/2
)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*
(x^(-1))^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}\,dx}{(1-x)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)^2\sqrt{x}}\,dx, x, \frac{1}{x}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)}{2(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.10, size = 58, normalized size = 0.64

$$\frac{\frac{2}{\sqrt{\frac{1}{x+1}}}+\sqrt{2}(x-1)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{x+1}}\right)}{2\sqrt{-\frac{(x-1)^2}{x^2}}\,x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

```
[Out] (2/Sqrt[(1 + x)^(-1)] + Sqrt[2]*(-1 + x)*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]
])/(2*Sqrt[-((-1 + x)^2/x^2)]*x)
```

fricas [A] time = 0.51, size = 76, normalized size = 0.84

$$-\frac{\sqrt{2}\left(x^2-2x+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{x-1}\right)+2(x+1)\sqrt{-x+1}\sqrt{\frac{x-1}{x+1}}}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x +
 1))/(x - 1)) + 2*(x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^2 - 2*x +
1)
```

giac [A] time = 0.14, size = 44, normalized size = 0.49

$$\frac{\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right)+\frac{2\sqrt{-x-1}}{x-1}\right)\operatorname{sgn}(x)}{2\operatorname{sgn}(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) + 2*sqrt(-x - 1)/(x - 1))*sgn
(x)/sgn(-x - 1)
```

maple [A] time = 0.05, size = 79, normalized size = 0.88

$$-\frac{\sqrt{1-x}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)x-\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)+2\sqrt{-1-x}\right)}{2\sqrt{\frac{-1+x}{1+x}}\,\left(-1+x\right)\sqrt{-1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x)
```

```
[Out] -1/2/((-1+x)/(1+x))^(1/2)/(-1+x)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/
2)*2^(1/2))*x-2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))+2*(-1-x)^(1/2))/(-1-
x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-x+1)^{\frac{3}{2}}\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

```
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*(1 - x)**(3/2)), x)
```

3.335 $\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-acx}\, dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{\frac{1}{ax}+1}\,x^{m+1}\sqrt{c-acx}}{(2m+3)\sqrt{1-\frac{1}{ax}}}-\frac{2(4m+5)x^m\sqrt{c-acx}\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{1}{2}-m;-\frac{1}{ax}\right)}{a(2m+1)(2m+3)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -2*(5+4*m)*x^m*hypergeom([1/2, -1/2-m],[1/2-m],-1/a/x)*(-a*c*x+c)^(1/2)/a/(
4*m^2+8*m+3)/(1-1/a/x)^(1/2)+2*x^(1+m)*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(3+
2*m)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6176, 6181, 79, 64}

$$\frac{2\sqrt{\frac{1}{ax}+1}\,x^{m+1}\sqrt{c-acx}}{(2m+3)\sqrt{1-\frac{1}{ax}}}-\frac{2(4m+5)x^m\sqrt{c-acx}\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{1}{2}-m;-\frac{1}{ax}\right)}{a(2m+1)(2m+3)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*x^(1 + m)*Sqrt[c - a*c*x])/((3 + 2*m)*Sqrt[1 - 1/(a*x)
]) - (2*(5 + 4*m)*x^m*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2, -1/2 - m, 1/2
- m, -(1/(a*x))])/(a*(1 + 2*m)*(3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{\frac{1}{2}+m}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\mathrm{Subst}\left(\int\frac{x^{-\frac{5}{2}-m}\left(1-\frac{x}{a}\right)}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} + \frac{\left((5+4m)\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\mathrm{Subst}\left(\int\frac{x^{-\frac{3}{2}-m}}{\sqrt{1+\frac{x}{a}}}\,dx\right.}{a(3+2m)\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} - \frac{2(5+4m)x^m\sqrt{c-acx}\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2}-m;-\frac{1}{ax}\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.08, size = 102, normalized size = 0.78

$$\frac{2x^m\sqrt{c-acx}\left(a(2m+1)x\sqrt{\frac{1}{ax}+1}-(4m+5)\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{1}{2}-m;-\frac{1}{ax}\right)\right)}{a(2m+1)(2m+3)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*x^m*Sqrt[c - a*c*x]*(a*(1 + 2*m)*Sqrt[1 + 1/(a*x)]*x - (5 + 4*m)*Hyperge
ometric2F1[1/2, -1/2 - m, 1/2 - m, -(1/(a*x))]))/(a*(1 + 2*m)*(3 + 2*m)*Sqr
t[1 - 1/(a*x)])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-acx+c}\,x^m\sqrt{\frac{ax-1}{ax+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-acx+c}\,x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int x^{m}\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-acx+c}\,x^{m}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

3.336 $\int e^{-\coth^{-1}(ax)} x^2 \sqrt{c - acx}\, dx$

Optimal. Leaf size=142

$$-\frac{2x^2\sqrt{1-\frac{1}{a^2x^2}}\,(c-acx)^{3/2}}{7ac}+\frac{6x\sqrt{1-\frac{1}{a^2x^2}}\,(c-acx)^{3/2}}{35a^2c}+\frac{38x\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-acx}}{105a^2}+\frac{152cx\sqrt{1-\frac{1}{a^2x^2}}}{105a^2\sqrt{c-acx}}$$

```
[Out] 6/35*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)/a^2/c-2/7*x^2*(-a*c*x+c)^(3/2)*
(1-1/a^2/x^2)^(1/2)/a/c+152/105*c*x*(1-1/a^2/x^2)^(1/2)/a^2/(-a*c*x+c)^(1/2
)+38/105*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)/a^2
```

Rubi [A] time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6176, 6181, 78, 45, 37}

$$\frac{104x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{208\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{26x^2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (-208*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) + (104
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (26*Sqr
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 +
1/(a*x)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(13\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{7a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{26\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(52\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \cdots\right.}{35a^2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{26\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{(104\cdots}{\cdots}$$

$$= -\frac{208\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}} + \frac{104\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{26\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\cdots}{\cdots}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.47

$$\frac{2\sqrt{\frac{1}{ax}+1}\left(15a^3x^3-39a^2x^2+52ax-104\right)\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-104 + 52*a*x - 39*a^2*x^2 + 15*a^3*x
^3))/(105*a^3*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.58, size = 69, normalized size = 0.49

$$\frac{2\left(15\,a^4x^4-24\,a^3x^3+13\,a^2x^2-52\,ax-104\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(15*a^4*x^4 - 24*a^3*x^3 + 13*a^2*x^2 - 52*a*x - 104)*sqrt(-a*c*x + c
)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 64, normalized size = 0.45

$$\frac{2\left(ax+1\right)\left(15x^{3}a^{3}-39a^{2}x^{2}+52ax-104\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105a^{3}\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/105*(a*x+1)*(15*a^3*x^3-39*a^2*x^2+52*a*x-104)*(-a*c*x+c)^(1/2)*((a*x-1)/
(a*x+1))^(1/2)/a^3/(a*x-1)
```

maxima [A] time = 0.41, size = 83, normalized size = 0.58

$$\frac{2\left(15\,a^{4}\sqrt{-c}\,x^{4}-24\,a^{3}\sqrt{-c}\,x^{3}+13\,a^{2}\sqrt{-c}\,x^{2}-52\,a\sqrt{-c}\,x-104\,\sqrt{-c}\right)(ax-1)}{105\left(a^{4}x-a^{3}\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/105*(15*a^4*sqrt(-c)*x^4 - 24*a^3*sqrt(-c)*x^3 + 13*a^2*sqrt(-c)*x^2 - 52
*a*sqrt(-c)*x - 104*sqrt(-c))*(a*x - 1)/((a^4*x - a^3)*sqrt(a*x + 1))
```

mupad [B] time = 1.29, size = 88, normalized size = 0.62

$$\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^{3}\,x^{3}-9\,a^{2}\,x^{2}+4\,a\,x-48\right)}{105\,a^{3}}-\frac{304\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^{3}\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x - 9*a^2*x^2 + 15*a^
3*x^3 - 48))/(105*a^3) - (304*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)
)/(105*a^3*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.337 $\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=104

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\,(c-acx)^{3/2}}{5ac}-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-acx}}{5a}-\frac{8cx\sqrt{1-\frac{1}{a^2x^2}}}{5a\sqrt{c-acx}}$$

```
[Out] -2/5*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)/a/c-8/5*c*x*(1-1/a^2/x^2)^(1/2)
/a/(-a*c*x+c)^(1/2)-2/5*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6176, 6181, 78, 45, 37}

$$\frac{12\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{6x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (12*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(5*a^2*Sqrt[1 - 1/(a*x)]) - (6*Sqrt[
1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(5*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*
x)]*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}+\frac{\left(9\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{5a\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{6\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(6\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1}}\right.}{5a^2\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{12\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}}-\frac{6\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.56

$$\frac{2\sqrt{\frac{1}{ax}+1}\left(a^2x^2-3ax+6\right)\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(6 - 3*a*x + a^2*x^2))/(5*a^2*Sqrt[1 -
 1/(a*x)])
```

fricas [A] time = 0.66, size = 60, normalized size = 0.58

$$\frac{2\left(a^3x^3-2\,a^2x^2+3\,ax+6\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5\left(a^3x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^3*x^3 - 2*a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
+ 1))/(a^3*x - a^2)
```

giac [A] time = 0.14, size = 69, normalized size = 0.66

$$-\frac{4\sqrt{-acx-c}\,|c|}{a^2c}-\frac{2\left((acx+c)^2\sqrt{-acx-c}\,|c|+5\,(-acx-c)^{\frac{3}{2}}c|c|\right)}{5\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-a*c*x - c)*abs(c)/(a^2*c) - 2/5*((a*c*x + c)^2*sqrt(-a*c*x - c)*ab
s(c) + 5*(-a*c*x - c)^(3/2)*c*abs(c))/(a^2*c^3)
```

maple [A] time = 0.04, size = 55, normalized size = 0.53

$$\frac{2\,(ax+1)\left(a^2x^2-3ax+6\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5a^2\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/5*(a*x+1)*(a^2*x^2-3*a*x+6)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/a^2/
(a*x-1)
```

maxima [A] time = 0.34, size = 69, normalized size = 0.66

$$\frac{2\left(a^3\sqrt{-c}\,x^3-2\,a^2\sqrt{-c}\,x^2+3\,a\sqrt{-c}\,x+6\,\sqrt{-c}\,\right)(ax-1)}{5\left(a^3x-a^2\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a^3*sqrt(-c)*x^3 - 2*a^2*sqrt(-c)*x^2 + 3*a*sqrt(-c)*x + 6*sqrt(-c))*(
a*x - 1)/((a^3*x - a^2)*sqrt(a*x + 1))
```

mupad [B] time = 1.29, size = 57, normalized size = 0.55

$$\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^3\,x^3-2\,a^2\,x^2+3\,a\,x+6\right)}{5\,a^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a*x - 2*a^2*x^2 + a^3*x
^3 + 6))/(5*a^2*(a*x - 1))
```

sympy [C] time = 90.92, size = 105, normalized size = 1.01

$$-\frac{4icx\sqrt{\frac{1}{acx+c}}}{5a}-\frac{12ic\sqrt{\frac{1}{acx+c}}}{5a^2}-\frac{2i\,(-acx+c)^2\,\sqrt{\frac{1}{acx+c}}}{5a^2c}+\frac{2i\,(-acx+c)^3\,\sqrt{\frac{1}{acx+c}}}{5a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -4*I*c*x*sqrt(1/(a*c*x + c))/(5*a) - 12*I*c*sqrt(1/(a*c*x + c))/(5*a**2) -
2*I*(-a*c*x + c)**2*sqrt(1/(a*c*x + c))/(5*a**2*c) + 2*I*(-a*c*x + c)**3*sq
rt(1/(a*c*x + c))/(5*a**2*c**2)
```

3.338 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=62

$$\frac{2}{3}x\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-acx}+\frac{8cx\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-acx}}$$

```
[Out] 8/3*c*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+2/3*x*(1-1/a^2/x^2)^(1/2)*(-a*
c*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 89, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6176, 6181, 78, 37}

$$\frac{2x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{10\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (-10*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1
 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{10\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [**A**] time = 0.03, size = 50, normalized size = 0.81

$$\frac{2\sqrt{\frac{1}{ax}+1}\,(ax-5)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

fricas [**A**] time = 0.53, size = 50, normalized size = 0.81

$$\frac{2\left(a^2x^2-4\,ax-5\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

giac [**A**] time = 0.13, size = 43, normalized size = 0.69

$$\frac{2\,(-acx-c)^{\frac{3}{2}}|c|}{3\,ac^2} + \frac{4\,\sqrt{-acx-c}\,|c|}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] 2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

maple [**A**] time = 0.03, size = 47, normalized size = 0.76

$$\frac{2\,(ax+1)\,(ax-5)\,\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\,(ax-1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 2/3*(a*x+1)*(a*x-5)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/a
```

maxima [A] time = 0.34, size = 54, normalized size = 0.87

$$\frac{2\left(a^2\sqrt{-c}\,x^2 - 4\,a\sqrt{-c}\,x - 5\,\sqrt{-c}\right)(ax-1)}{3\left(a^2x - a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a)
*sqrt(a*x + 1))
```

mupad [B] time = 0.00, size = 71, normalized size = 1.15

$$\frac{2\sqrt{c-a\,c\,x}\ \ (a\,x-3)\ \sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a} - \frac{16\,\sqrt{c-a\,c\,x}\ \sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\ (a\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

sympy [C] time = 43.14, size = 66, normalized size = 1.06

$$\frac{4icx\sqrt{\frac{1}{acx+c}}}{3} + \frac{4ic\sqrt{\frac{1}{acx+c}}}{a} - \frac{2i\left(-acx+c\right)^2\sqrt{\frac{1}{acx+c}}}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] 4*I*c*x*sqrt(1/(a*c*x + c))/3 + 4*I*c*sqrt(1/(a*c*x + c))/a - 2*I*(-a*c*x +
c)**2*sqrt(1/(a*c*x + c))/(3*a*c)
```

3.339 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6176, 6181, 78, 54, 215}

$$\frac{2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,\sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.79

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{\frac{1}{ax}+1}+\sqrt{\frac{1}{x}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.45, size = 206, normalized size = 2.19

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, \frac{2\left((ax-1)\sqrt{c}\arctan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
```

```
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), 2*((a*x - 1)*sqrt(c)*arct
an(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) + sqrt(-
a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

giac [A] time = 0.14, size = 40, normalized size = 0.43

$$-2\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{-acx-c}}{c}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] -2*(arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + sqrt(-a*c*x - c)/c)*abs(c)
```

maple [A] time = 0.06, size = 80, normalized size = 0.85

$$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c\,(ax+1)}\right)}{(ax-1)\sqrt{-c\,(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
[Out] 2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a
*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1))^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x, x)
```

3.340 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=96

$$\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)-3*arcsinh((1/x)^(1/2)/a^
(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6176, 6181, 80, 54, 215}

$$\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) - (3*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
```

```
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.81

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + 3*Sqrt
[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.70, size = 232, normalized size = 2.42

$$\left[\frac{3\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)}, -\frac{3\left(a^2x^2-ax\right)}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(3*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x +
c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqr
```

```
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -(3*(a^2*x^
2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/
(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2
 - x)]
```

giac [A] time = 0.16, size = 48, normalized size = 0.50

$$a\left(\frac{3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-acx-c}}{acx}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] a*(3*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - sqrt(-a*c*x - c)/(a*c*x))*a
bs(c)
```

maple [A] time = 0.06, size = 92, normalized size = 0.96

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\left(3\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)xac-\sqrt{-c\,(ax+1)}\,\sqrt{c}\right)}{(ax-1)\sqrt{-c\,(ax+1)}\,x\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(3*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))*x*a*c-(-c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
/x/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

```
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x**2, x)
```

3.341 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=139

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a^4/c+2/5*(-a*c*x+c)^(5/2)/a^4/c^2-2/7*(-a*c*x+c)^(7/2
)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4-4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/
2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4+4*(-a*c*x+c)^(1/2)/a^4
```

Rubi [A] time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6167, 6130, 21, 88, 50, 63, 206}

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*x)^
(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^(9/
2))/(9*a^4*c^4) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[
c])])/a^4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= -\int \frac{x^3(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$= -\frac{\int \frac{x^3(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$= -\frac{\int \left(\frac{(c-acx)^{3/2}}{a^3} - \frac{(c-acx)^{3/2}}{a^3(1+ax)} - \frac{(c-acx)^{5/2}}{a^3c} + \frac{(c-acx)^{7/2}}{a^3c^2}\right)dx}{c}$$
$$= \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^3c}$$
$$= \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^3}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{(4c)}{}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{8\,\mathrm{Su}}{}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} - \frac{4\sqrt{2}}{}$$

Mathematica [A] time = 0.14, size = 85, normalized size = 0.61

$$\frac{2\left(\left(35a^4x^4 - 95a^3x^3 + 138a^2x^2 - 236ax + 788\right)\sqrt{c-acx} - 630\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]

[Out] (2*(Sqrt[c - a*c*x]*(788 - 236*a*x + 138*a^2*x^2 - 95*a^3*x^3 + 35*a^4*x^4)
 - 630*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(315*a^
4)
```

fricas [A] time = 0.48, size = 168, normalized size = 1.21

$$\left[\frac{2\left(315\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788\right)\sqrt{-acx+c}\right)}{315a^4}, \frac{2\left(63\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*x + 788
)*sqrt(-a*c*x + c))/a^4, 2/315*(630*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqr
t(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*
x + 788)*sqrt(-a*c*x + c))/a^4]
```

giac [A] time = 0.15, size = 159, normalized size = 1.14

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^4\sqrt{-c}}+\frac{2\left(35(acx-c)^4\sqrt{-acx+c}\,a^{32}c^{32}+45(acx-c)^3\sqrt{-acx+c}\,a^{32}c^{33}+63(acx-c)^2\right.}{315a^{36}c^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^4*sqrt(-c)) +
2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^32*c^32 + 45*(a*c*x - c)^3*sqrt(
-a*c*x + c)*a^32*c^33 + 63*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^32*c^34 + 105*(
-a*c*x + c)^(3/2)*a^32*c^35 + 630*sqrt(-a*c*x + c)*a^32*c^36)/(a^36*c^36)
```

maple [A] time = 0.04, size = 101, normalized size = 0.73

$$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2(-acx+c)^{\frac{7}{2}}c}{7}+\frac{2(-acx+c)^{\frac{5}{2}}c^2}{5}+\frac{2c^3(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\,c^4-4c^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 2/c^4/a^4*(1/9*(-a*c*x+c)^(9/2)-1/7*(-a*c*x+c)^(7/2)*c+1/5*(-a*c*x+c)^(5/2)
*c^2+1/3*c^3*(-a*c*x+c)^(3/2)+2*(-a*c*x+c)^(1/2)*c^4-2*c^(9/2)*2^(1/2)*arct
anh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.41, size = 123, normalized size = 0.88

$$\frac{2\left(315\sqrt{2}c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+35(-acx+c)^{\frac{9}{2}}-45(-acx+c)^{\frac{7}{2}}c+63(-acx+c)^{\frac{5}{2}}c^2+105(-acx+c)^{\frac{3}{2}}c\right.}{315a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(
2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) - 45*(-a*c*x + c)^(
7/2)*c + 63*(-a*c*x + c)^(5/2)*c^2 + 105*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(
-a*c*x + c)*c^4)/(a^4*c^4)
```

mupad [B] time = 0.08, size = 114, normalized size = 0.82

$$\frac{4\sqrt{c-a\,c\,x}}{a^4}+\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^4\,c}+\frac{2\,(c-a\,c\,x)^{5/2}}{5\,a^4\,c^2}-\frac{2\,(c-a\,c\,x)^{7/2}}{7\,a^4\,c^3}+\frac{2\,(c-a\,c\,x)^{9/2}}{9\,a^4\,c^4}+\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*x
)^(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^(
9/2))/(9*a^4*c^4) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2
*c^(1/2)))*4i)/a^4
```

sympy [A] time = 7.53, size = 126, normalized size = 0.91

$$\frac{2\left(\frac{2\sqrt{2}c^5\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^4\sqrt{-acx+c}+\frac{c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] 2*(2*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*
c**4*sqrt(-a*c*x + c) + c**3*(-a*c*x + c)**(3/2)/3 + c**2*(-a*c*x + c)**(5/
2)/5 - c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4*c**4)
```

3.342 $\int e^{-2\coth^{-1}(ax)} x^2 \sqrt{c-acx}\, dx$

Optimal. Leaf size=97

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a^3/c-2/7*(-a*c*x+c)^(7/2)/a^3/c^3+4*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3-4*(-a*c*x+c)^(1/2)/a^3
```

Rubi [A] time = 0.25, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6167, 6130, 21, 88, 50, 63, 206}

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*c*x)
^(7/2))/(7*a^3*c^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/a^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx\\
&= -\int \frac{x^2(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{\int \frac{x^2(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{\int \left(\frac{(c-acx)^{3/2}}{a^2(1+ax)}-\frac{(c-acx)^{5/2}}{a^2c}\right)dx}{c}\\
&= -\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^2c}\\
&= -\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a^2}\\
&= -\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a^2}\\
&= -\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8\,\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a^3}\\
&= -\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.80

$$\frac{2\left(3a^3x^3-9a^2x^2+16ax-52\right)\sqrt{c-acx}+84\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{21a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-52 + 16*a*x - 9*a^2*x^2 + 3*a^3*x^3) + 84*Sqrt[2]*Sqrt
[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(21*a^3)
```

fricas [A] time = 0.56, size = 153, normalized size = 1.58

$$\left[\frac{2\left(21\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^3x^3-9a^2x^2+16ax-52\right)\sqrt{-acx+c}\right)}{21a^3}, -\frac{2\left(42\sqrt{2}\sqrt{-c}\arctan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/
a^3, -2/21*(42*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c
)/c) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/a^3]
```

giac [A] time = 0.14, size = 105, normalized size = 1.08

$$-\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^3\sqrt{-c}}+\frac{2\left(3(acx-c)^3\sqrt{-acx+c}\,a^{18}c^{18}-7(-acx+c)^{\frac{3}{2}}a^{18}c^{20}-42\sqrt{-acx+c}\,a^{18}c^{21}\right)}{21\,a^{21}c^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^3*sqrt(-c)) +
 2/21*(3*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^18*c^18 - 7*(-a*c*x + c)^(3/2)*a^
18*c^20 - 42*sqrt(-a*c*x + c)*a^18*c^21)/(a^21*c^21)
```

maple [A] time = 0.04, size = 75, normalized size = 0.77

$$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{3}{2}}c^2}{3}+2\sqrt{-acx+c}\,c^3-2c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{c^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c^3/a^3*(1/7*(-a*c*x+c)^(7/2)+1/3*(-a*c*x+c)^(3/2)*c^2+2*(-a*c*x+c)^(1/2
)*c^3-2*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.41, size = 97, normalized size = 1.00

$$-\frac{2\left(21\sqrt{2}c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+3(-acx+c)^{\frac{7}{2}}+7(-acx+c)^{\frac{3}{2}}c^2+42\sqrt{-acx+c}\,c^3\right)}{21\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/21*(21*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(7/2) + 7*(-a*c*x + c)^(3/2
)*c^2 + 42*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

mupad [B] time = 1.26, size = 80, normalized size = 0.82

$$-\frac{4\sqrt{c-a\,c\,x}}{a^3}-\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^3\,c}-\frac{2\,(c-a\,c\,x)^{7/2}}{7\,a^3\,c^3}-\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(1/2))/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*c
*x)^(7/2))/(7*a^3*c^3) - (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1
i)/(2*c^(1/2)))*4i)/a^3
```

sympy [A] time = 6.25, size = 95, normalized size = 0.98

$$-\frac{2\left(\frac{2\sqrt{2}\,c^4\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^3\sqrt{-acx+c}+\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c**3*sqrt(-a*c*x + c) + c**2*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(7/2)/7
)/(a**3*c**3)
```

3.343 $\int e^{-2\coth^{-1}(ax)} x\sqrt{c-acx}\,dx$

Optimal. Leaf size=97

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{4\sqrt{c-acx}}{a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c^2-4*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2+4*(-a*c*x+c)^(1/2)/a^2
```

Rubi [A] time = 0.17, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6130, 21, 80, 50, 63, 206}

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{4\sqrt{c-acx}}{a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*x)^
(5/2))/(5*a^2*c^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/a^2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx &= -\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx\\
&= -\int \frac{x(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{\int \frac{x(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= \frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{ac}\\
&= \frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a}\\
&= \frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a}\\
&= \frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a^2}\\
&= \frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 70, normalized size = 0.72

$$
\frac{2\left(3a^2x^2-11ax+38\right)\sqrt{c-acx}-60\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(38 - 11*a*x + 3*a^2*x^2) - 60*Sqrt[2]*Sqrt[c]*ArcTanh[S
qrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a^2)
```

fricas [A] time = 0.63, size = 136, normalized size = 1.40

$$
\left[\frac{2\left(15\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\left(3a^2x^2-11ax+38\right)\sqrt{-acx+c}\right)}{15a^2}, \frac{2\left(30\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2c}\right.\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2, 2/15*(3
0*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (3*a^2
*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2]
```

giac [A] time = 0.12, size = 105, normalized size = 1.08

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}}+\frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}\,a^8c^8+5\,(-acx+c)^{\frac{3}{2}}a^8c^9+30\,\sqrt{-acx+c}\,a^8c^{10}\right)}{15\,a^{10}c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^2*sqrt(-c)) +
2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^8 + 5*(-a*c*x + c)^(3/2)*a^8*c
^9 + 30*sqrt(-a*c*x + c)*a^8*c^10)/(a^10*c^10)
```

maple [A] time = 0.04, size = 73, normalized size = 0.75

$$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4\sqrt{-acx+c}\;c^2-4c^{\frac{5}{2}}\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\;\sqrt{2}}{2\sqrt{c}}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 2/c^2/a^2*(1/5*(-a*c*x+c)^(5/2)+1/3*c*(-a*c*x+c)^(3/2)+2*(-a*c*x+c)^(1/2)*c
^2-2*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.42, size = 95, normalized size = 0.98

$$\frac{2\left(15\,\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+3\,(-acx+c)^{\frac{5}{2}}+5\,(-acx+c)^{\frac{3}{2}}c+30\,\sqrt{-acx+c}\,c^2\right)}{15\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 5*(-a*c*x + c)^(3/2)
*c + 30*sqrt(-a*c*x + c)*c^2)/(a^2*c^2)
```

mupad [B] time = 0.09, size = 80, normalized size = 0.82

$$\frac{4\,\sqrt{c-a\,c\,x}}{a^2}+\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^2\,c}+\frac{2\,(c-a\,c\,x)^{5/2}}{5\,a^2\,c^2}+\frac{\sqrt{2}\,\sqrt{c}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*x
)^(5/2))/(5*a^2*c^2) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)
/(2*c^(1/2)))*4i)/a^2
```

sympy [A] time = 5.29, size = 92, normalized size = 0.95

$$\frac{2\left(\frac{2\sqrt{2}\,c^3\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^2\sqrt{-acx+c}+\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] 2*(2*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*
c**2*sqrt(-a*c*x + c) + c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/(a
**2*c**2)
```

3.344 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)/a-4*(-a*c*x+c)^(1/2)/a
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6167, 6130, 21, 50, 63, 206}

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
```

```
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{2(c-acx)^{3/2}}{3ac}-2\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}-(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.80

$$\frac{2(ax-7)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/
(Sqrt[2]*Sqrt[c])])/(3*a)
```

fricas [A] time = 0.50, size = 119, normalized size = 1.57

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}(ax-7)\right)}{3a}, -\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx}\right.}{3a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*ar
ctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))
/a]
```

giac [A] time = 0.14, size = 77, normalized size = 1.01

$$-\frac{4\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\,\sqrt{-acx+c}\,a^2c^3\right)}{3\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2
/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

maple [A] time = 0.04, size = 59, normalized size = 0.78

$$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\ \sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh
(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

maxima [A] time = 0.42, size = 79, normalized size = 1.04

$$-\frac{2\left(3\,\sqrt{2}\,c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\,\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*
sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)/(
a*c)
```

mupad [B] time = 0.00, size = 61, normalized size = 0.80

$$\frac{4\,\sqrt{2}\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}-\frac{4\,\sqrt{c-a\,c\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*(
c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

sympy [A] time = 3.85, size = 73, normalized size = 0.96

$$-\frac{2\left(\frac{2\sqrt{2}\,c^2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

3.345 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=74

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6167, 6130, 21, 84, 156, 63, 208, 206}

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x(1+ax)}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x(1+ax)}\,dx}{c}\\
&= 2\sqrt{c-acx}-\frac{\int \frac{ac^2-3a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{ac}\\
&= 2\sqrt{c-acx}-c\int \frac{1}{x\sqrt{c-acx}}\,dx+(4ac)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= 2\sqrt{c-acx}-8\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)+\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-\right.}{a}\\
&= 2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.00

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.51, size = 157, normalized size = 2.12

$$\left[2\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c},4\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(
a*x + 1)) + sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*s
qrt(-a*c*x + c), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(-c)/c) - 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x +
 c)]
```

giac [A] time = 0.13, size = 67, normalized size = 0.91

$$\frac{4\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*c*ar
ctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

maple [A] time = 0.05, size = 58, normalized size = 0.78

$$2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}-4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

maxima [A] time = 0.41, size = 98, normalized size = 1.32

$$2\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)-\sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c))) - sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*
c*x + c) + sqrt(c))) + 2*sqrt(-a*c*x + c)
```

mupad [B] time = 0.08, size = 57, normalized size = 0.77

$$2\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+2\sqrt{c-a\,c\,x}-4\sqrt{2}\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

```
[Out] 2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2) - 4*2^(1/2
)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

sympy [A] time = 5.49, size = 80, normalized size = 1.08

$$-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}c\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] -2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*atan(sqrt(2)*sq
rt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

3.346 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -5*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*arctanh(1/2*(-a*c*x+c)^(
1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.23, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6167, 6130, 21, 98, 156, 63, 208, 206}

$$\frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^2(1+ax)}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^2(1+ax)}\,dx}{c}\\
&= \frac{\sqrt{c-acx}}{x}+\frac{\int \frac{\frac{5ac^2}{2}-\frac{3}{2}a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{c}\\
&= \frac{\sqrt{c-acx}}{x}+\frac{1}{2}(5ac)\int \frac{1}{x\sqrt{c-acx}}\,dx-\left(4a^2c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= \frac{\sqrt{c-acx}}{x}-5\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)+(8a)\,\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)\\
&= \frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 1.00

$$\frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.60, size = 176, normalized size = 2.26

$$\left[\frac{4\sqrt{2}\,a\sqrt{c}\,x\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+5\,a\sqrt{c}\,x\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c}}{2x},-\frac{4\sqrt{2}\,a\sqrt{-c}\,x\,\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*sqrt(2)*a*sqrt(c)*x*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + 5*a*sqrt(c)*x*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c)
- 2*c)/x) + 2*sqrt(-a*c*x + c))/x, -(4*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt
(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 5*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*s
qrt(-c)/c) - sqrt(-a*c*x + c))/x]
```

giac [A] time = 0.14, size = 71, normalized size = 0.91

$$-\frac{4\sqrt{2}\,ac\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{5\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 5*a
*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

maple [A] time = 0.05, size = 71, normalized size = 0.91

$$-2ac\left(-\frac{2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\ \sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-acx+c}}{2xac}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] -2*a*c*(-2*2^(1/2)/c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))-1/
2*(-a*c*x+c)^(1/2)/x/a/c+5/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.42, size = 111, normalized size = 1.42

$$-\frac{1}{2}\,ac\left(\frac{4\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)}{\sqrt{c}}-\frac{5\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\sqrt{-acx+c}}{acx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*c*(4*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt
(c) + sqrt(-a*c*x + c)))/sqrt(c) - 5*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt
(-a*c*x + c) + sqrt(c)))/sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

mupad [B] time = 1.24, size = 61, normalized size = 0.78

$$\frac{\sqrt{c-a\,c\,x}}{x}-5\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+4\,\sqrt{2}\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

```
[Out] (c - a*c*x)^(1/2)/x - 5*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 4*2^(1
/2)*a*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

sympy [B] time = 8.10, size = 162, normalized size = 2.08

$$-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{6ac\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{4\sqrt{2}\,ac\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] -a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + a*c**
2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + 6*a*c*atan(s
qrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*a*c*atan(sqrt(2)*sqrt(-a*c*x
 + c)/(2*sqrt(-c)))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

3.347 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=106

$$\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}$$

```
[Out] 23/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^2*arctanh(1/2*(-a*c*
x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/2*(-a*c*x+c)^(1/2)/x^2-9/4*a*
(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.26, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6167, 6130, 21, 98, 151, 156, 63, 208, 206}

$$\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] Sqrt[c - a*c*x]/(2*x^2) - (9*a*Sqrt[c - a*c*x])/(4*x) + (23*a^2*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= -\int \frac{(1-ax)\sqrt{c-acx}}{x^3(1+ax)}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{3/2}}{x^3(1+ax)}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{2x^2} + \frac{\int \frac{\frac{9ac^2}{2}-\frac{7}{2}a^2c^2x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{2c}$$
$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} - \frac{\int \frac{\frac{23a^2c^3}{4}-\frac{9}{4}a^3c^3x}{x(1+ax)\sqrt{c-acx}}\,dx}{2c^2}$$
$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} - \frac{1}{8}\left(23a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx + \left(4a^3c\right)\int \frac{1}{(1+ax)\sqrt{c-a}}$$
$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} + \frac{1}{4}(23a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) - \left(8a^2\right)\text{Subs}$$
$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} + \frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{2}}\right.$$

Mathematica [**A**] time = 0.08, size = 93, normalized size = 0.88

$$\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) + \frac{(2-9ax)\sqrt{c-acx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] ((2 - 9*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (23*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])]
```

fricas [**A**] time = 0.59, size = 204, normalized size = 1.92

$$\left[\frac{16\sqrt{2}a^2\sqrt{c}x^2\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right) + 23a^2\sqrt{c}x^2\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right) - 2\sqrt{-acx+c}(9ax-2)}{8x^2}, \frac{16\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(c) - 3*c)/(a*x + 1)) + 23*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c
)*sqrt(c) - 2*c)/x) - 2*sqrt(-a*c*x + c)*(9*a*x - 2))/x^2, 1/4*(16*sqrt(2)*
a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 23*a^2*s
qrt(-c)*x^2*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(9*a*x -
 2))/x^2]
```

giac [**A**] time = 0.16, size = 106, normalized size = 1.00

$$\frac{4\sqrt{2}a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{23a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{9(-acx+c)^{\frac{3}{2}}a^2c - 7\sqrt{-acx+c}\,a^2c^2}{4a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^2*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 23
/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/4*(9*(-a*c*x + c)^(
3/2)*a^2*c - 7*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

maple [A] time = 0.05, size = 95, normalized size = 0.90

$$2a^2c^2\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\frac{-\frac{9(-acx+c)^{\frac{3}{2}}}{8}+\frac{7c\sqrt{-acx+c}}{8}}{x^2a^2c^2}-\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] 2*a^2*c^2*(-2/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
-1/c*((-9/8*(-a*c*x+c)^(3/2)+7/8*c*(-a*c*x+c)^(1/2))/x^2/a^2/c^2-23/8/c^(1/
2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

maxima [A] time = 0.42, size = 152, normalized size = 1.43

$$\frac{1}{8}\,a^2c^2\left(\frac{2\left(9\,(-acx+c)^{\frac{3}{2}}-7\,\sqrt{-acx+c}\,c\right)}{(acx-c)^2c+2\,(acx-c)c^2+c^3}+\frac{16\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}-\frac{23\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*c^2*(2*(9*(-a*c*x + c)^(3/2) - 7*sqrt(-a*c*x + c)*c)/((a*c*x - c)^2
*c + 2*(a*c*x - c)*c^2 + c^3) + 16*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*
c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*c*x + c)))/c^(3/2) - 23*log((sqrt(-a*c
*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(3/2))
```

mupad [B] time = 1.24, size = 88, normalized size = 0.83

$$\frac{9\,(c-a\,c\,x)^{3/2}}{4\,c\,x^2}-\frac{a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,23\mathrm{i}}{4}-\frac{7\,\sqrt{c-a\,c\,x}}{4\,x^2}+\sqrt{2}\,a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] (9*(c - a*c*x)^(3/2))/(4*c*x^2) - (a^2*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/
c^(1/2))*23i)/4 - (7*(c - a*c*x)^(1/2))/(4*x^2) + 2^(1/2)*a^2*c^(1/2)*atan(
(2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

sympy [B] time = 14.86, size = 352, normalized size = 3.32

$$\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2\,(-acx+c)^2}-\frac{6a^2c^3\,(-acx+c)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2\,(-acx+c)^2}-\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\,\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}+\frac{3a^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] 10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**
2) - 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x
 + c)**2) - 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x
 + c))/8 + 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x +
 c))/8 + 3*a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x +
c))/2 - 3*a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)
)/2 - 8*a**2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*a**2*c*
atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 3*a*sqrt(-a*c*x + c)
/x
```

3.348 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=127

$$-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{19a^2\sqrt{c-acx}}{8x}+\frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}$$

```
[Out] -45/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a^3*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/3*(-a*c*x+c)^(1/2)/x^3-13/12
*a*(-a*c*x+c)^(1/2)/x^2+19/8*a^2*(-a*c*x+c)^(1/2)/x
```

Rubi [A] time = 0.28, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6167, 6130, 21, 98, 151, 156, 63, 208, 206}

$$\frac{19a^2\sqrt{c-acx}}{8x}-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{\sqrt{c-acx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] Sqrt[c - a*c*x]/(3*x^3) - (13*a*Sqrt[c - a*c*x])/(12*x^2) + (19*a^2*Sqrt[c
- a*c*x])/(8*x) - (45*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$= -\int \frac{(1-ax)\sqrt{c-acx}}{x^4(1+ax)}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{3/2}}{x^4(1+ax)}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{3x^3} + \frac{\int \frac{\frac{13ac^2}{2}-\frac{11}{2}a^2c^2x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{3c}$$
$$= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} - \frac{\int \frac{\frac{57a^2c^3}{4}-\frac{39}{4}a^3c^3x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{6c^2}$$
$$= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} + \frac{\int \frac{\frac{135a^3c^4}{8}-\frac{57}{8}a^4c^4x}{x(1+ax)\sqrt{c-acx}}\,dx}{6c^3}$$
$$= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} + \frac{1}{16}\left(45a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx - (4$$
$$= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} - \frac{1}{8}\left(45a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \right.$$
$$= \frac{\sqrt{c-acx}}{3x^3} - \frac{13a\sqrt{c-acx}}{12x^2} + \frac{19a^2\sqrt{c-acx}}{8x} - \frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.80

$$-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{\left(57a^2x^2-26ax+8\right)\sqrt{c-acx}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a*c*x]*(8 - 26*a*x + 57*a^2*x^2))/(24*x^3) - (45*a^3*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.57, size = 220, normalized size = 1.73

$$\left[\frac{96\sqrt{2}\,a^3\sqrt{c}\,x^3\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+135\,a^3\sqrt{c}\,x^3\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\left(57\,a^2x^2-26\,ax+8\right)\sqrt{}}{48\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c) - 3*c)/(a*x + 1)) + 135*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(-a*c*x +
 c)*sqrt(c) - 2*c)/x) + 2*(57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3,
-1/24*(96*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt
(-c)/c) - 135*a^3*sqrt(-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (57*a^
2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3]
```

giac [A] time = 0.13, size = 133, normalized size = 1.05

$$-\frac{4\sqrt{2}a^3c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{45a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{57(acx-c)^2\sqrt{-acx+c}\,a^3c-88(-acx+c)^{\frac{3}{2}}a^3c^2+39}{24a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4
5/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/24*(57*(a*c*x - c)
^2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 39*sqrt(-a*c*x
+ c)*a^3*c^3)/(a^3*c^3*x^3)
```

maple [A] time = 0.05, size = 110, normalized size = 0.87

$$-2c^3a^3\left(-\frac{2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\ \sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{-\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11c(-acx+c)^{\frac{3}{2}}}{6}-\frac{13\sqrt{-acx+c}\,c^2}{16}}{x^3a^3c^3}-\frac{45\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16\sqrt{c}}}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] -2*c^3*a^3*(-2/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)-1/c^2*(-(-19/16*(-a*c*x+c)^(5/2)+11/6*c*(-a*c*x+c)^(3/2)-13/16*(-a*c*x+c)
^(1/2)*c^2)/x^3/a^3/c^3-45/16/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

maxima [A] time = 0.43, size = 183, normalized size = 1.44

$$\frac{1}{48}a^3c^3\left(\frac{2\left(57(-acx+c)^{\frac{5}{2}}-88(-acx+c)^{\frac{3}{2}}c+39\sqrt{-acx+c}\,c^2\right)}{(acx-c)^3c^2+3(acx-c)^2c^3+3(acx-c)c^4+c^5}-\frac{96\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}+\frac{135\log\left(\frac{\sqrt{-a}}{\sqrt{-a}}\right.}{c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*a^3*c^3*(2*(57*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 39*sqrt(
-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)*c
^4 + c^5) - 96*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*s
qrt(c) + sqrt(-a*c*x + c)))/c^(5/2) + 135*log((sqrt(-a*c*x + c) - sqrt(c))/
(sqrt(-a*c*x + c) + sqrt(c)))/c^(5/2))
```

mupad [B] time = 0.13, size = 105, normalized size = 0.83

$$\frac{13\sqrt{c-a\,c\,x}}{8\,x^3}+\frac{a^3\sqrt{c}\ \mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\ 1\mathrm{i}}{\sqrt{c}}\right)\,45\mathrm{i}}{8}-\frac{11\,(c-a\,c\,x)^{3/2}}{3\,c\,x^3}+\frac{19\,(c-a\,c\,x)^{5/2}}{8\,c^2\,x^3}-\sqrt{2}\,a^3\sqrt{c}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \sqrt{c-a\,c\,x}\ 1\mathrm{i}}{2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

```
[Out] (13*(c - a*c*x)^(1/2))/(8*x^3) + (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c
^(1/2))*45i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (19*(c - a*c*x)^(5/2))/
(8*c^2*x^3) - 2^(1/2)*a^3*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^
(1/2)))*4i
```

sympy [B] time = 18.38, size = 614, normalized size = 4.83

$$-\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}+\frac{80a^3c^5\left(-acx+c\right)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] -66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x
+ c)**2 + 48*c**3*(-a*c*x + c)**3) + 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144
*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) -
 30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c
*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 30*a**3*c**4*sqrt(-a*c*x + c)/(16*a
*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 5*a**3*c**4*sqrt(c**(-7))*log(
-c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 5*a**3*c**4*sqrt(c**(-7))*log(
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 18*a**3*c**3*(-a*c*x + c)**(3/2
)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 9*a**3*c**3*sqrt(c**(-5
))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 - 9*a**3*c**3*sqrt(c**(-5)
)*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 - 2*a**3*c**2*sqrt(c**(-3))*
log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) + 2*a**3*c**2*sqrt(c**(-3))*log
(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) + 8*a**3*c*atan(sqrt(-a*c*x + c)/sq
rt(-c))/sqrt(-c) - 4*sqrt(2)*a**3*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-
c)))/sqrt(-c) + 4*a**2*sqrt(-a*c*x + c)/x
```

3.349 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=148

$$\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{107a^2\sqrt{c-acx}}{96x^2}+\frac{\sqrt{c-acx}}{4x^4}-\frac{17a}{}$$

```
[Out] 363/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^4*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/4*(-a*c*x+c)^(1/2)/x^4-17/2
4*a*(-a*c*x+c)^(1/2)/x^3+107/96*a^2*(-a*c*x+c)^(1/2)/x^2-149/64*a^3*(-a*c*x
+c)^(1/2)/x
```

Rubi [A] time = 0.31, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6167, 6130, 21, 98, 151, 156, 63, 208, 206}

$$\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{17a\sqrt{c-acx}}{24x^3}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] Sqrt[c - a*c*x]/(4*x^4) - (17*a*Sqrt[c - a*c*x])/(24*x^3) + (107*a^2*Sqrt[c
 - a*c*x])/(96*x^2) - (149*a^3*Sqrt[c - a*c*x])/(64*x) + (363*a^4*Sqrt[c]*A
rcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c
- a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6130

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[(u*(c + d*x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= -\int \frac{(1-ax)\sqrt{c-acx}}{x^5(1+ax)}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{3/2}}{x^5(1+ax)}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{4x^4} + \frac{\int \frac{\frac{17ac^2}{2}-\frac{15}{2}a^2c^2x}{x^4(1+ax)\sqrt{c-acx}}\,dx}{4c}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} - \frac{\int \frac{\frac{107a^2c^3}{4}-\frac{85}{4}a^3c^3x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{12c^2}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{\int \frac{\frac{447a^3c^4}{8}-\frac{321}{8}a^4c^4x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{24c^3}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} - \frac{\int \frac{\frac{1089a^4c^5}{16}-\frac{447}{16}a^5c^5x}{x(1+ax)\sqrt{c-acx}}}{24c^4}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} - \frac{1}{128}\left(363a^4c\right)\int$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{1}{64}\left(363a^3\right)\text{Subs}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{363}{64}a^4\sqrt{c}\tanh^{-1}$$

Mathematica [A] time = 0.13, size = 109, normalized size = 0.74

$$\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right) + \frac{\left(-447a^3x^3+214a^2x^2-136ax+48\right)\sqrt{c-acx}}{192x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a*c*x]*(48 - 136*a*x + 214*a^2*x^2 - 447*a^3*x^3))/(192*x^4) + (3
63*a^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.41, size = 236, normalized size = 1.59

$$\left[\frac{768\sqrt{2}a^4\sqrt{c}x^4\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right) + 1089a^4\sqrt{c}x^4\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right) - 2\left(447a^3x^3 - 214a^2x^2 + \right.}{384x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(768*sqrt(2)*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)
*sqrt(c) - 3*c)/(a*x + 1)) + 1089*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*
x + c)*sqrt(c) - 2*c)/x) - 2*(447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqr
```

```
t(-a*c*x + c))/x^4, 1/192*(768*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(1/2*sqrt(2)*
sqrt(-a*c*x + c)*sqrt(-c)/c) - 1089*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c
)*sqrt(-c)/c) - (447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqrt(-a*c*x + c)
)/x^4]
```

giac [A] time = 0.15, size = 160, normalized size = 1.08

$$\frac{4\sqrt{2}a^4c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{363a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\sqrt{-c}}-\frac{447(acx-c)^3\sqrt{-acx+c}\,a^4c+1127(acx-c)^2\sqrt{-ac}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 36
3/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/192*(447*(a*c*x -
 c)^3*sqrt(-a*c*x + c)*a^4*c + 1127*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^2
- 1049*(-a*c*x + c)^(3/2)*a^4*c^3 + 321*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*c^4*
x^4)
```

maple [A] time = 0.05, size = 123, normalized size = 0.83

$$2c^4a^4\left(-\frac{2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\ \sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{\frac{-\frac{149(-acx+c)^{\frac{7}{2}}}{128}+\frac{1127c(-acx+c)^{\frac{5}{2}}}{384}-\frac{1049(-acx+c)^{\frac{3}{2}}c^2}{384}+\frac{107\sqrt{-acx+c}\ c^3}{128}}{x^4a^4c^4}-\frac{363\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{128\sqrt{c}}}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/(a*x+1)*(a*x-1)/x^5,x)
```

```
[Out] 2*c^4*a^4*(-2/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
-1/c^3*((-149/128*(-a*c*x+c)^(7/2)+1127/384*c*(-a*c*x+c)^(5/2)-1049/384*(-a
*c*x+c)^(3/2)*c^2+107/128*(-a*c*x+c)^(1/2)*c^3)/x^4/a^4/c^4-363/128/c^(1/2)
*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

maxima [A] time = 0.42, size = 212, normalized size = 1.43

$$\frac{1}{384}a^4c^4\left(\frac{2\left(447(-acx+c)^{\frac{7}{2}}-1127(-acx+c)^{\frac{5}{2}}c+1049(-acx+c)^{\frac{3}{2}}c^2-321\sqrt{-acx+c}\,c^3\right)}{(acx-c)^4c^3+4(acx-c)^3c^4+6(acx-c)^2c^5+4(acx-c)c^6+c^7}+\frac{768\sqrt{2}\log\left(-\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] 1/384*a^4*c^4*(2*(447*(-a*c*x + c)^(7/2) - 1127*(-a*c*x + c)^(5/2)*c + 1049
*(-a*c*x + c)^(3/2)*c^2 - 321*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*
(a*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 768*sq
rt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*
c*x + c)))/c^(7/2) - 1089*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c
) + sqrt(c)))/c^(7/2))
```

mupad [B] time = 1.26, size = 122, normalized size = 0.82

$$\frac{1049(c-a\,c\,x)^{3/2}}{192\,c\,x^4}-\frac{a^4\sqrt{c}\ \mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\ 1\mathrm{i}}{\sqrt{c}}\right)363\mathrm{i}}{64}-\frac{107\sqrt{c-a\,c\,x}}{64\,x^4}-\frac{1127(c-a\,c\,x)^{5/2}}{192\,c^2\,x^4}+\frac{149(c-a\,c\,x)^{7/2}}{64\,c^3\,x^4}+\sqrt{2}\ a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

```
[Out] (1049*(c - a*c*x)^(3/2))/(192*c*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)
*1i)/c^(1/2))*363i)/64 - (107*(c - a*c*x)^(1/2))/(64*x^4) - (1127*(c - a*c*
x)^(5/2))/(192*c^2*x^4) + (149*(c - a*c*x)^(7/2))/(64*c^3*x^4) + 2^(1/2)*a^
4*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

sympy [B] time = 32.76, size = 991, normalized size = 6.70

$$\frac{558a^4c^8\sqrt{-acx+c}}{1536ac^8x-1152c^8+2304c^6(-acx+c)^2-1536c^5(-acx+c)^3+384c^4(-acx+c)^4}-\frac{}{1536ac^8x-1152c^8+2304c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] 558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c
*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 1022*a
**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x
 + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 770*a**4
*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 198*a**4*c*
*6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 4
8*c**3*(-a*c*x + c)**3) - 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x
- 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c
**4*(-a*c*x + c)**4) - 240*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x + 9
6*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 35*a**4*c**5
*sqrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 35*a**4*c*
*5*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 90*a**4*c
**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2
 + 48*c**3*(-a*c*x + c)**3) + 40*a**4*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x -
8*c**4 + 8*c**2*(-a*c*x + c)**2) + 15*a**4*c**4*sqrt(c**(-7))*log(-c**4*sqr
t(c**(-7)) + sqrt(-a*c*x + c))/16 - 15*a**4*c**4*sqrt(c**(-7))*log(c**4*sqr
t(c**(-7)) + sqrt(-a*c*x + c))/16 - 24*a**4*c**3*(-a*c*x + c)**(3/2)/(16*a*
c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 3*a**4*c**3*sqrt(c**(-5))*log(-
c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/2 + 3*a**4*c**3*sqrt(c**(-5))*log(c*
*3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/2 + 2*a**4*c**2*sqrt(c**(-3))*log(-c**
2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) - 2*a**4*c**2*sqrt(c**(-3))*log(c**2*sq
rt(c**(-3)) + sqrt(-a*c*x + c)) - 8*a**4*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/
sqrt(-c) + 4*sqrt(2)*a**4*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqr
t(-c) - 4*a**3*sqrt(-a*c*x + c)/x
```

3.350 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=281

$$\frac{1312\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}-\frac{656x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}+\frac{164x^2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}+\frac{2x^4\sqrt{c-}}{9\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -82/9*x^2*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-8/9*x^3*(-a*
c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/9*x^4*(-a*c*x+c)^(1/2)/(1-
1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1312/45*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(
1-1/a/x)^(1/2)-656/45*x*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2
)+164/15*x^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6176, 6181, 89, 78, 45, 37}

$$\frac{164x^2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}-\frac{656x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}+\frac{1312\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}+\frac{2x^4\sqrt{c-}}{9\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (1312*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(45*a^4*Sqrt[1 - 1/(a*x)]) - (656*
Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(45*a^3*Sqrt[1 - 1/(a*x)]) - (82*x^2*S
qrt[c - a*c*x])/(9*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (164*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (8*x^3*Sqrt[c -
 a*c*x])/(9*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^4*Sqrt[c - a*c*x]
)/(9*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{7/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{14}{a}+\frac{9x}{2a^2}}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{9\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(41\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{}{x^{7/2}}\right.}{9a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(82\sqrt{\frac{1}{x}}\right.}{}$$

$$= -\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{164\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{}{9}$$

$$= -\frac{656\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}} - \frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{164\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} -$$

$$= \frac{1312\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}} - \frac{656\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}} - \frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1}{}$$

Mathematica [**A**] time = 0.05, size = 73, normalized size = 0.26

$$\frac{2\left(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656\right)\sqrt{c-acx}}{45a^5x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(656 + 328*a*x - 82*a^2*x^2 + 41*a^3*x^3 - 20*a^4*x^4 +
5*a^5*x^5))/(45*a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [**A**] time = 0.52, size = 77, normalized size = 0.27

$$\frac{2\left(5\,a^5x^5-20\,a^4x^4+41\,a^3x^3-82\,a^2x^2+328\,ax+656\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{45\left(a^5x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(5*a^5*x^5 - 20*a^4*x^4 + 41*a^3*x^3 - 82*a^2*x^2 + 328*a*x + 656)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^5*x - a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 80, normalized size = 0.28

$$\frac{2\left(ax+1\right)\left(5x^{5}a^{5}-20x^{4}a^{4}+41x^{3}a^{3}-82a^{2}x^{2}+328ax+656\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45a^{4}\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/45*(a*x+1)*(5*a^5*x^5-20*a^4*x^4+41*a^3*x^3-82*a^2*x^2+328*a*x+656)*(-a*c
*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

maxima [A] time = 0.35, size = 117, normalized size = 0.42

$$\frac{2\left(5\,a^{6}\sqrt{-c}\,x^{6}-15\,a^{5}\sqrt{-c}\,x^{5}+21\,a^{4}\sqrt{-c}\,x^{4}-41\,a^{3}\sqrt{-c}\,x^{3}+246\,a^{2}\sqrt{-c}\,x^{2}+984\,a\sqrt{-c}\,x+656\,\sqrt{-c}\,\right)\left(ax-1\right)^{2}}{45\left(a^{6}x^{2}-2\,a^{5}x+a^{4}\right)\left(ax+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(5*a^6*sqrt(-c)*x^6 - 15*a^5*sqrt(-c)*x^5 + 21*a^4*sqrt(-c)*x^4 - 41*a
^3*sqrt(-c)*x^3 + 246*a^2*sqrt(-c)*x^2 + 984*a*sqrt(-c)*x + 656*sqrt(-c))*(
a*x - 1)^2/((a^6*x^2 - 2*a^5*x + a^4)*(a*x + 1)^(3/2))
```

mupad [B] time = 1.41, size = 74, normalized size = 0.26

$$\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^{5}\,x^{5}-20\,a^{4}\,x^{4}+41\,a^{3}\,x^{3}-82\,a^{2}\,x^{2}+328\,a\,x+656\right)}{45\,a^{4}\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(328*a*x - 82*a^2*x^2 + 41
*a^3*x^3 - 20*a^4*x^4 + 5*a^5*x^5 + 656))/(45*a^4*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.351 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=231

$$-\frac{2672\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{1336x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] -334/35*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-44/35*x^2*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/7*x^3*(-a*c*x+c)^(1/2)/(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2672/105*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^
3/(1-1/a/x)^(1/2)+1336/105*x*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)
^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6176, 6181, 89, 78, 45, 37}

$$\frac{1336x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{2672\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2672*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) - (33
4*x*Sqrt[c - a*c*x])/(35*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1336*S
qrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (44*x^2*S
qrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^3*Sqrt[c
- a*c*x])/(7*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{11}{a}+\frac{7x}{2a^2}}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{7\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(167\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\ldots\right.}{35a^2\sqrt{1-\ldots}}$$

$$=-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{(668\ldots}{\ldots}$$

$$=-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1336\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\ldots$$

$$=-\frac{2672\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1336\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}\ldots$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.28

$$\frac{2\left(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336\right)\sqrt{c-acx}}{105a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-1336 - 668*a*x + 167*a^2*x^2 - 66*a^3*x^3 + 15*a^4*x^4
))/(105*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.65, size = 69, normalized size = 0.30

$$\frac{2\left(15\,a^4x^4-66\,a^3x^3+167\,a^2x^2-668\,ax-1336\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(15*a^4*x^4 - 66*a^3*x^3 + 167*a^2*x^2 - 668*a*x - 1336)*sqrt(-a*c*x
+ c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 72, normalized size = 0.31

$$\frac{2\left(ax+1\right)\left(15x^4a^4-66x^3a^3+167a^2x^2-668ax-1336\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105a^3\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/105*(a*x+1)*(15*a^4*x^4-66*a^3*x^3+167*a^2*x^2-668*a*x-1336)*(-a*c*x+c)^(
1/2)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

maxima [A] time = 0.35, size = 104, normalized size = 0.45

$$\frac{2\left(15\,a^5\sqrt{-c}\,x^5-51\,a^4\sqrt{-c}\,x^4+101\,a^3\sqrt{-c}\,x^3-501\,a^2\sqrt{-c}\,x^2-2004\,a\sqrt{-c}\,x-1336\,\sqrt{-c}\right)(ax-1)^2}{105\left(a^5x^2-2\,a^4x+a^3\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 2/105*(15*a^5*sqrt(-c)*x^5 - 51*a^4*sqrt(-c)*x^4 + 101*a^3*sqrt(-c)*x^3 - 5
01*a^2*sqrt(-c)*x^2 - 2004*a*sqrt(-c)*x - 1336*sqrt(-c))*(a*x - 1)^2/((a^5*
x^2 - 2*a^4*x + a^3)*(a*x + 1)^(3/2))
```

mupad [B] time = 1.34, size = 88, normalized size = 0.38

$$\frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^3\,x^3-51\,a^2\,x^2+116\,a\,x-552\right)}{105\,a^3}-\frac{3776\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(116*a*x - 51*a^2*x^2 + 15
*a^3*x^3 - 552))/(105*a^3) - (3776*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^
(1/2))/(105*a^3*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.352 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=182

$$\frac{316\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] -158/15*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-32/15*x*(-a*c*
x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/5*x^2*(-a*c*x+c)^(1/2)/(1-1/
a/x)^(1/2)/(1+1/a/x)^(1/2)+316/15*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1
/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6176, 6181, 89, 78, 45, 37}

$$\frac{316\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-158*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (316*
Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (32*x*Sqrt[
c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^2*Sqrt[c - a*
c*x])/(5*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{8}{a}+\frac{5x}{2a^2}}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(79\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{}{x^{3/2}}\right.}{15a^2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(158\right.}{}$$

$$=-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{316\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{}{5}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.31

$$\frac{2\left(3a^3x^3-16a^2x^2+79ax+158\right)\sqrt{c-acx}}{15a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(158 + 79*a*x - 16*a^2*x^2 + 3*a^3*x^3))/(15*a^3*Sqrt[1
- 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.72, size = 61, normalized size = 0.34

$$\frac{2\left(3\,a^{3}x^{3}-16\,a^{2}x^{2}+79\,ax+158\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{3}x-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^3*x^3 - 16*a^2*x^2 + 79*a*x + 158)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1))/(a^3*x - a^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 64, normalized size = 0.35

$$\frac{2\left(ax+1\right)\left(3x^{3}a^{3}-16a^{2}x^{2}+79ax+158\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15a^{2}\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/15*(a*x+1)*(3*a^3*x^3-16*a^2*x^2+79*a*x+158)*(-a*c*x+c)^(1/2)*((a*x-1)/(a
*x+1))^(3/2)/a^2/(a*x-1)^2
```

maxima [A] time = 0.34, size = 91, normalized size = 0.50

$$\frac{2\left(3\,a^{4}\sqrt{-c}\,x^{4}-13\,a^{3}\sqrt{-c}\,x^{3}+63\,a^{2}\sqrt{-c}\,x^{2}+237\,a\sqrt{-c}\,x+158\,\sqrt{-c}\right)\left(ax-1\right)^{2}}{15\left(a^{4}x^{2}-2\,a^{3}x+a^{2}\right)\left(ax+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*a^4*sqrt(-c)*x^4 - 13*a^3*sqrt(-c)*x^3 + 63*a^2*sqrt(-c)*x^2 + 237*
a*sqrt(-c)*x + 158*sqrt(-c))*(a*x - 1)^2/((a^4*x^2 - 2*a^3*x + a^2)*(a*x +
1)^(3/2))
```

mupad [B] time = 1.33, size = 58, normalized size = 0.32

$$\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(3\,a^{3}\,x^{3}-16\,a^{2}\,x^{2}+79\,a\,x+158\right)}{15\,a^{2}\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a*x - 16*a^2*x^2 + 3*a
^3*x^3 + 158))/(15*a^2*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.353 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=137

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] -20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(1
/2)/a^2/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6176, 6181, 89, 78, 37}

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
&& !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(23\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+ \cdots\right.}\right.}{3a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.35

$$\frac{2\left(a^2x^2-10ax-23\right)\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

fricas [A] time = 0.53, size = 50, normalized size = 0.36

$$\frac{2\left(a^2x^2-10\,ax-23\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.04, size = 55, normalized size = 0.40

$$\frac{2\left(ax+1\right)\left(a^{2}x^{2}-10ax-23\right)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 2/3*(a*x+1)*(a^2*x^2-10*a*x-23)*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/a/
(a*x-1)^2
```

maxima [A] time = 0.34, size = 75, normalized size = 0.55

$$\frac{2\left(a^{3}\sqrt{-c}\,x^{3}-9\,a^{2}\sqrt{-c}\,x^{2}-33\,a\sqrt{-c}\,x-23\,\sqrt{-c}\right)(ax-1)^{2}}{3\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c))
*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

mupad [B] time = 0.00, size = 71, normalized size = 0.52

$$\frac{2\sqrt{c-a\,c\,x}\,\left(a\,x-9\right)\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{64\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.354 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=140

$$\frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{10\sqrt{c-acx}}{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+10*(-a*c*x+c)^(1/2)/a/x/
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*
(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6176, 6181, 89, 78, 54, 215}

$$\frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{10\sqrt{c-acx}}{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (10*Sqrt[c - a*
c*x])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (2*Sqrt[x^(-1)]*Sqrt[c -
a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{-\frac{2}{a}+\frac{x}{2a^2}}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x\ldots\right.}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x\ldots\right.}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.12, size = 78, normalized size = 0.56

$$\frac{2\sqrt{c-acx}\left(-\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{\frac{1}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)+a+\frac{5}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(a + 5/x - Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*ArcSin
h[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.64, size = 207, normalized size = 1.48

$$\left[\frac{(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}\,(ax+5)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}, -\frac{2\left((ax-1)\sqrt{c}\,\mathrm{arct}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(
-a*c*x + c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.06, size = 80, normalized size = 0.57

$$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)\sqrt{-c\,(ax+1)}+acx+5c\right)}{(ax-1)^2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] 2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(c^(1/2)*arc
tan((-c*(a*x+1))^(1/2)/c^(1/2))*(-c*(a*x+1))^(1/2)+a*c*x+5*c)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

```
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

3.355 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-(1+1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+7*arcsinh((1/x)^(1/2)/a^(1/2))*a^(1/2)*(1/x)^
(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6176, 6181, 89, 80, 54, 215}

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (Sqrt[1 + 1/
(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (7*Sqrt[a]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\frac{3}{2a^2}-\frac{x}{2a^3}}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(7\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(7\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.56

$$\frac{\sqrt{c-acx}\left(\frac{7a^{3/2}\sqrt{\frac{1}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}-9ax-1\right)}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-1 - 9*a*x + (7*a^(3/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^
(-1)]/Sqrt[a]])/(x^(-1))^(3/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

fricas [A] time = 0.43, size = 233, normalized size = 1.66

$$\left[\frac{7\left(a^2x^2-ax\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\sqrt{-acx+c}(9ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^2-x\right)}, \frac{7\left(a^2x^2-ax\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(7*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x +
c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqr
t(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), (7*(a^2*x
^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))
/(a*c*x - c)) - sqrt(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*
x^2 - x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.07, size = 86, normalized size = 0.61

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(7\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)xa\sqrt{-c(ax+1)}+9xa\sqrt{c}+\sqrt{c}\right)\sqrt{-c(ax-1)}}{(ax-1)^2\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(7*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*x*a*
(-c*(a*x+1))^(1/2)+9*x*a*c^(1/2)+c^(1/2))*(-c*(a*x-1))^(1/2)/(a*x-1)^2/c^(1
/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
[Out] Timed out
```

3.356 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=190

$$-\frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{2x^2\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/2*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+47/4*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/
2)/x/(1-1/a/x)^(1/2)-47/4*a^(3/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*
(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 89, 80, 50, 54, 215}

$$-\frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{2x^2\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^2) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x^2) + (47*a*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) - (47*a^(3/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{5/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(\frac{11}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{\left(47\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{1+\frac{x}{a}}}\right.}{4\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(47a\sqrt{\frac{1}{x}}\right.}{}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(47a\sqrt{\frac{1}{x}}\right.}{}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} - \frac{47a^{3/2}\sqrt{}}{}$$

Mathematica [A] time = 0.09, size = 90, normalized size = 0.47

$$-\frac{\sqrt{c-acx}\left(\frac{47a^{5/2}\sqrt{\frac{1}{ax}+1}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}} - 47a^2x^2 - 13ax + 2\right)}{4ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -1/4*(Sqrt[c - a*c*x]*(2 - 13*a*x - 47*a^2*x^2 + (47*a^(5/2)*Sqrt[1 + 1/(a*
x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2)))/(a*Sqrt[1 - 1/(a^2*x^2)
]*x^3)
```

fricas [A] time = 0.63, size = 262, normalized size = 1.38

$$\left[\frac{47\left(a^3x^3-a^2x^2\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\left(47a^2x^2+13ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{8\left(ax^3-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas
")
```

```
[Out] [1/8*(47*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c
*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) +
2*(47*a^2*x^2 + 13*a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*
x^3 - x^2), -1/4*(47*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (47*a^2*x^2 + 13*a*x - 2)*sq
rt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.07, size = 103, normalized size = 0.54

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1)\sqrt{-c\,(ax-1)}\,\left(47\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)x^{2}a^{2}\sqrt{-c\,(ax+1)}\,+47x^{2}a^{2}\sqrt{c}\,+13xa\sqrt{c}\,-2\sqrt{c}\right)}{4\,(ax-1)^{2}\sqrt{c}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(47*arctan
((-c*(a*x+1))^(1/2)/c^(1/2))*x^2*a^2*(-c*(a*x+1))^(1/2)+47*x^2*a^2*c^(1/2)+
13*x*a*c^(1/2)-2*c^(1/2))/c^(1/2)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

```
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

3.357 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=238

$$\frac{119a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}-\frac{119a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3x^3\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{119a\sqrt{\frac{1}{ax}}}{12x^2}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/3*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)+119/12*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/x^2/(1-1/a/x)^(1/2)-119/8*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/
a/x)^(1/2)+119/8*a^(5/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*(-a*c*x+c
)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6176, 6181, 89, 80, 50, 54, 215}

$$-\frac{119a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}+\frac{119a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}+\frac{119a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{12x^2\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3x^3\sqrt{1-\frac{1}{ax}}}-\frac{}{x^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^3) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^3) + (119*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(12*Sqrt[1 - 1/(a*x)]*x^2) - (119*a^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (119*a^(5/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{7/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(\frac{19}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{\left(119\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}}{\sqrt{1+\frac{x}{a}}}\,d\right.}{6\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}\,x^2} - \frac{\left(119a\sqrt{\frac{1}{x}}\right.}{}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}\,x^2} - \frac{119a^2\sqrt{1+}}{8\sqrt{1}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}\,x^2} - \frac{119a^2\sqrt{1+}}{8\sqrt{1}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}\,x^2} - \frac{119a^2\sqrt{1+}}{8\sqrt{1}}$$

Mathematica [A] time = 0.10, size = 98, normalized size = 0.41

$$\frac{\sqrt{c-acx}\left(\frac{357a^{7/2}\sqrt{\frac{1}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}} - 357a^3x^3 - 119a^2x^2 + 38ax - 8\right)}{24ax^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-8 + 38*a*x - 119*a^2*x^2 - 357*a^3*x^3 + (357*a^(7/2)*Sq
rt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2)))/(24*a*Sqrt[
1 - 1/(a^2*x^2)]*x^4)
```

fricas [A] time = 0.54, size = 278, normalized size = 1.17

$$\left[\frac{357\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)-2\left(357a^3x^3+119a^2x^2-38ax+8\right)\sqrt{-a}}{48\left(ax^4-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas
")
```

```
[Out] [1/48*(357*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a
*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x))
- 2*(357*a^3*x^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/(a*x^4 - x^3), 1/24*(357*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(s
qrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (357*a^3*x
^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a*x^4 - x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.07, size = 114, normalized size = 0.48

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(357\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)x^3a^3\sqrt{-c(ax+1)}+357x^3a^3\sqrt{c}+119x^2a^2\sqrt{c}-38xa\right.}{24(ax-1)^2\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] -1/24*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(357*arc
tan((-c*(a*x+1))^(1/2)/c^(1/2))*x^3*a^3*(-c*(a*x+1))^(1/2)+357*x^3*a^3*c^(1
/2)+119*x^2*a^2*c^(1/2)-38*x*a*c^(1/2)+8*c^(1/2))/c^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
[Out] Timed out
```

3.358 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=286

$$-\frac{1115a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}+\frac{1115a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}-\frac{1115a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{96x^2\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x^4\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^4/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/4*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^4/(1-1/a/x)^(1/2)+223/24*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/x^3/(1-1/a/x)^(1/2)-1115/96*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x^2/(
1-1/a/x)^(1/2)+1115/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/
2)-1115/64*a^(7/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2
)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6176, 6181, 89, 80, 50, 54, 215}

$$-\frac{1115a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{96x^2\sqrt{1-\frac{1}{ax}}}+\frac{1115a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}-\frac{1115a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}+\frac{223a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{24x^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^4) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^4) + (223*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^3) - (1115*a^2*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(96*Sqrt[1 - 1/(a*x)]*x^2) + (1115*a^3*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) - (1115*a^(7/2)*Sqrt[x^(-1)]*Sq
rt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a*x)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{9/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(\frac{27}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{\left(223\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}}{\sqrt{1+ \cdots}}\right.}{8\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^3} - \frac{\left(1115a \cdots\right.}{\cdots}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^3} - \frac{1115a^2 \cdots}{96 \cdots}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^3} - \frac{1115a^2 \cdots}{96 \cdots}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^3} - \frac{1115a^2 \cdots}{96 \cdots}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^3} - \frac{1115a^2 \cdots}{96 \cdots}$$

Mathematica [A] time = 0.10, size = 106, normalized size = 0.37

$$-\frac{\sqrt{c-acx}\left(\frac{3345a^{9/2}\sqrt{\frac{1}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{9/2}} - 3345a^4x^4 - 1115a^3x^3 + 446a^2x^2 - 200ax + 48\right)}{192ax^5\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/192*(Sqrt[c - a*c*x]*(48 - 200*a*x + 446*a^2*x^2 - 1115*a^3*x^3 - 3345*a
^4*x^4 + (3345*a^(9/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^
(-1))^(9/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5)
```

fricas [A] time = 0.60, size = 294, normalized size = 1.03

$$\left[\frac{3345\left(a^5x^5-a^4x^4\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\left(3345a^4x^4+1115a^3x^3-446a^2x^2+20\right.}{384\left(ax^5-x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas
")
```

```
[Out] [1/384*(3345*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)
) + 2*(3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/192*(3345*(a^5*x^5 - a^
4*x^4)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a
*c*x - c)) - (3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.07, size = 125, normalized size = 0.44

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,\left(3345\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)x^4a^4\sqrt{-c\,(ax+1)}\,+3345x^4a^4\sqrt{c}\,+1115x^3a^3\sqrt{c}\,-446x^2\right.}{192\,(ax-1)^2\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] 1/192*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(-c*(a*x-1))^(1/2)*(3345*ar
ctan((-c*(a*x+1))^(1/2)/c^(1/2))*x^4*a^4*(-c*(a*x+1))^(1/2)+3345*x^4*a^4*c^
(1/2)+1115*x^3*a^3*c^(1/2)-446*x^2*a^2*c^(1/2)+200*x*a*c^(1/2)-48*c^(1/2))/
c^(1/2)/x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

```
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.359 $\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$

Optimal. Leaf size=278

$$\frac{2\left(n^2+14n+56\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a^2(n+6)\left(n^2+6n+8\right)x}-\frac{\left(n^2+14n+56\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a(n+4)(n+6)}+$$

```
[Out] -(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(2+1/2*n
)/a/(4+n)/(6+n)+2*(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-
a*c*x+c)^(2+1/2*n)/a^2/(6+n)/(n^2+6*n+8)/x+(8+n)*(1-1/a/x)^(-2-1/2*n)*(1+1/
a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/(6+n)-(a-1/x)*(1-1/a/x)^(-2-1/2*n)*(1
+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/a
```

Rubi [A] time = 0.27, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6176, 6181, 90, 79, 45, 37}

$$\frac{2\left(n^2+14n+56\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a^2(n+6)\left(n^2+6n+8\right)x}-\frac{\left(n^2+14n+56\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a(n+4)(n+6)}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

```
[Out] -(((56 + 14*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c
- a*c*x)^((4 + n)/2))/(a*(4 + n)*(6 + n))) + (2*(56 + 14*n + n^2)*(1 - 1/(a
*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^((4 + n)/2))/(a^2*(6
+ n)*(8 + 6*n + n^2)*x) + ((8 + n)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^(
(2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/(6 + n) - ((a - x^(-1))*(1 - 1/(a*x))
^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/a
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}} x^{-2-\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{2+\frac{n}{2}} x^{2+\frac{n}{2}}\,dx$$

$$= -\left(\left(\left(\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-4-\frac{n}{2}}\left(1-\frac{x}{a}\right)^{2}\left(1+\frac{x}{a}\right)^{n/2}\,dx\right.\right.\right.$$

$$= -\frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x(c-acx)^{\frac{4+n}{2}}}{a} + \left(a\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-\right.$$

$$= \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x(c-acx)^{\frac{4+n}{2}}}{6+n} - \frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)}{a}$$

$$= -\frac{\left(56+14n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}}{}$$

$$= -\frac{\left(56+14n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{2\left(56+14n+n^2\right)\left(\right.}{a^2(2}$$

Mathematica [A] time = 0.08, size = 116, normalized size = 0.42

$$\frac{2c^2(ax+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(2n\left(3a^2x^2-10ax+7\right)+8\left(a^2x^2-4ax+7\right)+n^2(ax-1)^2\right)(c-acx)^{n/2}}{a(n+2)(n+4)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

```
[Out] (2*c^2*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(n^2*(-1 + a*x)^2 +
8*(7 - 4*a*x + a^2*x^2) + 2*n*(7 - 10*a*x + 3*a^2*x^2)))/(a*(2 + n)*(4 + n)
*(6 + n)*(1 - 1/(a*x))^(n/2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left((-acx+c)^{\frac{1}{2}n+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="fricas")
[Out] integral((-a*c*x + c)^(1/2*n + 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-acx+c)^{\frac{1}{2}n+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^(1/2*n + 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [A] time = 0.04, size = 104, normalized size = 0.37

$$\frac{2\,(ax+1)\left(a^2n^2x^2+6a^2n\,x^2+8a^2x^2-2a\,n^2x-20anx-32ax+n^2+14n+56\right)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}\,(-acx+c)^{2+\frac{n}{2}}}{(ax-1)^2\,a\left(n^3+12n^2+44n+48\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x)
[Out] 2*(a*x+1)*(a^2*n^2*x^2+6*a^2*n*x^2+8*a^2*x^2-2*a*n^2*x-20*a*n*x-32*a*x+n^2+
14*n+56)*exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n)/(a*x-1)^2/a/(n^3+12*n^2+4
4*n+48)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-acx+c)^{\frac{1}{2}n+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^(1/2*n + 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [B] time = 1.79, size = 223, normalized size = 0.80

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{x^3\,(c-a\,c\,x)^{\frac{n}{2}+2}\,\left(2\,n^2+12\,n+16\right)}{n^3+12\,n^2+44\,n+48}+\frac{(c-a\,c\,x)^{\frac{n}{2}+2}\,\left(2\,n^2+28\,n+112\right)}{a^3\,\left(n^3+12\,n^2+44\,n+48\right)}-\frac{2\,x\,(c-a\,c\,x)^{\frac{n}{2}+2}\,\left(n^2+6\,n-24\right)}{a^2\,\left(n^3+12\,n^2+44\,n+48\right)}-\frac{x^2\,(c-a\,c\,x)^{\frac{n}{2}+2}\,\left(2\,n^2+28\,n+48\right)}{a\,\left(n^3+12\,n^2+44\,n+48\right)}\right.}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 2),x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((x^3*(c - a*c*x)^(n/2 + 2)*(12*n + 2*n^2 + 16))/(
44*n + 12*n^2 + n^3 + 48) + ((c - a*c*x)^(n/2 + 2)*(28*n + 2*n^2 + 112))/(a
^3*(44*n + 12*n^2 + n^3 + 48)) - (2*x*(c - a*c*x)^(n/2 + 2)*(6*n + n^2 - 24
))/(a^2*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(c - a*c*x)^(n/2 + 2)*(28*n + 2*
n^2 + 48))/(a*(44*n + 12*n^2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*(1/a^2
 - (2*x)/a + x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(2+1/2*n),x)
```

```
[Out] Timed out
```

3.360 $\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$

Optimal. Leaf size=127

$$\frac{2x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{n+4}-\frac{2(n+6)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{a(n+2)(n+4)}$$

```
[Out] -2*(6+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(1+1/2*n)/a/(n
^2+6*n+8)+2*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(1+1/2*n)
/(4+n)
```

Rubi [A] time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6176, 6181, 79, 37}

$$\frac{2x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{n+4}-\frac{2(n+6)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{a(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

```
[Out] (-2*(6 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^
((2 + n)/2))/(a*(2 + n)*(4 + n)) + (2*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x)
)^((2 + n)/2)*x*(c - a*c*x)^((2 + n)/2))/(4 + n)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx&=\left(\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}x^{-1-\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{1+\frac{n}{2}}x^{1+\frac{n}{2}}\,dx\\&=-\left(\left(\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(\frac{1}{x}\right)^{1+\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-3-\frac{n}{2}}\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{n/2}dx\right.\right.\\&=\frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n}+\frac{\left((6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(\frac{1}{x}\right)^{1+\frac{n}{2}}(c-a\right.}{}\\&=-\frac{2(6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{2+n}{2}}}{a(2+n)(4+n)}+\frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{4+n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.61

$$-\frac{2c(ax+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}(n(ax-1)+2ax-6)(c-acx)^{n/2}}{a(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

```
[Out] (-2*c*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(-6 + 2*a*x + n*(-1 +
 a*x)))/(a*(2 + n)*(4 + n)*(1 - 1/(a*x))^(n/2))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left((-acx+c)^{\frac{1}{2}n+1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n + 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-acx+c)^{\frac{1}{2}n+1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n + 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [A] time = 0.04, size = 61, normalized size = 0.48

$$\frac{2(-acx+c)^{1+\frac{n}{2}}e^{n\,\text{arccoth}(ax)}(anx+2ax-n-6)(ax+1)}{(ax-1)a\left(n^2+6n+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x)
```

```
[Out] 2*(-a*c*x+c)^(1+1/2*n)*exp(n*arccoth(a*x))*(a*n*x+2*a*x-n-6)*(a*x+1)/(a*x-1
)/a/(n^2+6*n+8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^{\frac{1}{2}n+1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n + 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [B] time = 1.34, size = 140, normalized size = 1.10

$$-\frac{\left(\frac{(2\,n+12)\,(c-a\,c\,x)^{\frac{n}{2}+1}}{a^2\,\left(n^2+6\,n+8\right)}-\frac{x^2\,(2\,n+4)\,(c-a\,c\,x)^{\frac{n}{2}+1}}{n^2+6\,n+8}+\frac{8\,x\,(c-a\,c\,x)^{\frac{n}{2}+1}}{a\,\left(n^2+6\,n+8\right)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\left(x-\frac{1}{a}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 1),x)
```

```
[Out] -((((2*n + 12)*(c - a*c*x)^(n/2 + 1))/(a^2*(6*n + n^2 + 8)) - (x^2*(2*n + 4
)*(c - a*c*x)^(n/2 + 1))/(6*n + n^2 + 8) + (8*x*(c - a*c*x)^(n/2 + 1))/(a*(
6*n + n^2 + 8)))*((a*x + 1)/(a*x))^(n/2))/((x - 1/a)*((a*x - 1)/(a*x))^(n/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} c^{\frac{n}{2}+1} x e^{\frac{i\pi n}{2}} \\ -\frac{\int \frac{1}{axe^{4\operatorname{acoth}(ax)}-e^{4\operatorname{acoth}(ax)}}\,dx}{c} \\ \int e^{-2\operatorname{acoth}(ax)}\,dx \\ -\frac{2a^2cnx^2(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a}-\frac{4a^2cx^2(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a}+\frac{8acx(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a}+\frac{2cn(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a}+\frac{12c(-acx+c)^{\frac{n}{2}}e^{n\operatorname{ac}}}{an^2+6an+8a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1+1/2*n),x)
```

```
[Out] Piecewise((c**(n/2 + 1)*x*exp(I*pi*n/2), Eq(a, 0)), (-Integral(1/(a*x*exp(4
*acoth(a*x)) - exp(4*acoth(a*x))), x)/c, Eq(n, -4)), (Integral(exp(-2*acoth
(a*x)), x), Eq(n, -2)), (-2*a**2*c*n*x**2*(-a*c*x + c)**(n/2)*exp(n*acoth(a
*x))/(a*n**2 + 6*a*n + 8*a) - 4*a**2*c*x**2*(-a*c*x + c)**(n/2)*exp(n*acoth
(a*x))/(a*n**2 + 6*a*n + 8*a) + 8*a*c*x*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x
))/(a*n**2 + 6*a*n + 8*a) + 2*c*n*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*
n**2 + 6*a*n + 8*a) + 12*c*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**2 +
6*a*n + 8*a), True))
```

3.361 $\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx$

Optimal. Leaf size=36

$$\frac{2(ax+1)(c-acx)^{n/2}e^{n\coth^{-1}(ax)}}{a(n+2)}$$

```
[Out] 2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6174}

$$\frac{2(ax+1)(c-acx)^{n/2}e^{n\coth^{-1}(ax)}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

```
[Out] (2*E^(n*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(n/2))/(a*(2 + n))
```

Rule 6174

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[((1 + a*x)*(c + d*x)^p*E^(n*ArcCoth[a*x]))/(a*(p + 1)), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] && !IntegerQ[n/2]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2e^{n\coth^{-1}(ax)}(1+ax)(c-acx)^{n/2}}{a(2+n)}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.61

$$-\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+1}(c-acx)^{n/2}}{-\frac{n}{2}-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

```
[Out] -(((1 + 1/(a*x))^(1 + n/2)*x*(c - a*c*x)^(n/2))/((-1 - n/2)*(1 - 1/(a*x))^(
n/2)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((-acx+c)^{\frac{1}{2}n}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^{\frac{1}{2}n}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [A] time = 0.04, size = 34, normalized size = 0.94

$$\frac{2\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}\,(ax+1)\,(-acx+c)^{\frac{n}{2}}}{a\,(2+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x)
```

```
[Out] 2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^{\frac{1}{2}n}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [B] time = 1.26, size = 55, normalized size = 1.53

$$\frac{2\left(\frac{1}{a\,x}+1\right)^{n/2}\,(c-a\,c\,x)^{n/2}\,(a\,x+1)}{a\left(1-\frac{1}{a\,x}\right)^{n/2}\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2),x)
```

```
[Out] (2*(1/(a*x) + 1)^(n/2)*(c - a*c*x)^(n/2)*(a*x + 1))/(a*(1 - 1/(a*x))^(n/2)*
(n + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{x}{c} & \text{for}\: a = 0 \wedge n = -2 \\ c^{\frac{n}{2}} x e^{\frac{i\pi n}{2}} & \text{for}\: a = 0 \\ -\frac{\int \frac{1}{axe^{2\,\mathrm{acoth}\,(ax)} - e^{2\,\mathrm{acoth}\,(ax)}}\, dx}{c} & \text{for}\: n = -2 \\ \frac{2ax(-acx+c)^{\frac{n}{2}} e^{n\,\mathrm{acoth}\,(ax)}}{an+2a} + \frac{2(-acx+c)^{\frac{n}{2}} e^{n\,\mathrm{acoth}\,(ax)}}{an+2a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2*n),x)
```

```
[Out] Piecewise((-x/c, Eq(a, 0) & Eq(n, -2)), (c**(n/2)*x*exp(I*pi*n/2), Eq(a, 0)
), (-Integral(1/(a*x*exp(2*acoth(a*x)) - exp(2*acoth(a*x))), x)/c, Eq(n, -2
)), (2*a*x*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a) + 2*(-a*c*x +
c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a), True))
```

3.362 $\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx$

Optimal. Leaf size=80

$$\frac{2x\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{\frac{n-2}{2}}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

```
[Out] 2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x*(-a*c*x+c)^(-1+1/2*n)*hypergeom([
1, -1/2*n],[1-1/2*n],2/(a+1/x)/x)/n
```

Rubi [A] time = 0.15, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6176, 6181, 131}

$$\frac{2x\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{\frac{n-2}{2}}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

```
[Out] (2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x*(c - a*c*x)^((-2 + n)/2)*H
ypergeometric2F1[1, -n/2, 1 - n/2, 2/((a + x^(-1))*x)])/n
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}x^{1-\frac{n}{2}}(c-acx)^{-1+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{-1+\frac{n}{2}}x^{-1+\frac{n}{2}}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{x}\right)^{-1+\frac{n}{2}}(c-acx)^{-1+\frac{n}{2}}\right)\text{Subst}\left(\int\frac{x^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{1-\frac{x}{a}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x(c-acx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.98

$$-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{ax+1}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

```
[Out] (-2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, 2/(1 + a*x)])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((-acx+c)^{\frac{1}{2}n-1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n - 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-acx+c)^{\frac{1}{2}n-1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n - 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{-1+\frac{n}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^{\frac{1}{2}n-1}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^(1/2*n - 1)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{\frac{n}{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-c(ax-1))^{\frac{n}{2}-1}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-1+1/2*n),x)
[Out] Integral((-c*(a*x - 1))**(n/2 - 1)*exp(n*acoth(a*x)), x)
```

3.363 $\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx$

Optimal. Leaf size=88

$$-\frac{2x\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}(c-acx)^{\frac{n-4}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

```
[Out] -2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-1+1/2*n)*x*(-a*c*x+c)^(-2+1/2*n)*hyperge
om([2, 1-1/2*n],[2-1/2*n],2/(a+1/x)/x)/(2-n)
```

Rubi [A] time = 0.15, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6176, 6181, 131}

$$-\frac{2x\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}(c-acx)^{\frac{n-4}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x*(c - a*c*x)^((-4 +
 n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, 2/((a + x^(-1))*x)])/(2 - n)
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}x^{2-\frac{n}{2}}(c-acx)^{-2+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{-2+\frac{n}{2}}x^{-2+\frac{n}{2}}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{x}\right)^{-2+\frac{n}{2}}(c-acx)^{-2+\frac{n}{2}}\right)\text{Subst}\left(\int\frac{x^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{\left(1-\frac{x}{a}\right)^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x(c-acx)^{\frac{1}{2}(-4+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 1.01

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{n/2}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{ax+1}\right)}{ac^2(n-2)(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, 2/(1 + a*x)])/(a*c^2*(-2 + n)*(1 - 1/(a*x))^(n/2)*(1 + a*x))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left((-acx+c)^{\frac{1}{2}n-2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^(1/2*n - 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-acx+c)^{\frac{1}{2}n-2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n - 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{-2+\frac{n}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx + c)^{\frac{1}{2}n-2} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^(1/2*n - 2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)}\,(c - a\,c\,x)^{\frac{n}{2}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-2+1/2*n),x)
[Out] Timed out
```

3.364 $\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=104

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}{}_2F_1\left(\frac{1}{2}(n-2p),-p-1;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2*n-p)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^p*hypergeom([-
1-p, 1/2*n-p],[-p],2/(a+1/x)/x)/(1+p)/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6176, 6181, 132}

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}{}_2F_1\left(\frac{1}{2}(n-2p),-p-1;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c -
 a*c*x)^p*Hypergeometric2F1[(n - 2*p)/2, -1 - p, -p, 2/((a + x^(-1))*x)])/(
(1 + p)*(1 - 1/(a*x))^(n/2))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)\right)$$
$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^p\,{}_2F_1\left(\frac{1}{2}(n-2p),-1-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)}\right)}{1+p}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 1.00

$$\frac{(ax+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}(n-2p)}{}_2F_1\left(-p-1,\frac{n}{2}-p;-p;\frac{2}{ax+1}\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((n - 2*p)/2)*(1 + a*x)*(c - a*
c*x)^p*Hypergeometric2F1[-1 - p, n/2 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*(1 -
 1/(a*x))^(n/2))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left((-acx+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-acx+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx + c)^p \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c - a\,c\,x)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^p,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-c\,(ax - 1))^p\, e^{n\,\mathrm{acoth}\,(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1))**p*exp(n*acoth(a*x)), x)
```

3.365 $\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=81

$$-\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}{}_2F_1\left(5,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[Out] -32*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([5, 4-1/2*n],[5-
1/2*n],(a-1/x)/(a+1/x))/a/(8-n)
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6180, 131}

$$-\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}{}_2F_1\left(5,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (-32*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric2
F1[5, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$
$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^5}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\,{}_2F_1\left(5,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Mathematica [B] time = 1.98, size = 190, normalized size = 2.35

$$-\frac{c^3e^{n\coth^{-1}(ax)}\left((n+2)\left(n^2\left(a^2x^2-12ax-1\right)+2n\left(a^3x^3-6a^2x^2+21ax+6\right)+6\left(a^4x^4-4a^3x^3+6a^2x^2-4ax-\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -1/24*(c^3*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(-48 + 44*n - 12*n^2 +
n^3)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(
a*n^3*x + n^2*(-1 - 12*a*x + a^2*x^2) + 2*n*(6 + 21*a*x - 6*a^2*x^2 + a^3*x
^3) + 6*(-7 - 4*a*x + 6*a^2*x^2 - 4*a^3*x^3 + a^4*x^4) + (-48 + 44*n - 12*n
^2 + n^3)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*(2 +
 n))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^3c^3x^3-3\,a^2c^3x^2+3\,ac^3x-c^3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*((a*x - 1)/(a*x +
 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -(acx-c)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)^3*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}\left(-acx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (acx-c)^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*c*x - c)^3*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^3,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int 3axe^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-3a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\right)dx+\int a^3x^3e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-e^{n\,\mathrm{acoth}\,(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x*exp(n*acoth(a*x)), x) + Integral(-3*a**2*x**2*exp(n*a
coth(a*x)), x) + Integral(a**3*x**3*exp(n*acoth(a*x)), x) + Integral(-exp(n
*acoth(a*x)), x))
```

3.366 $\int e^{n \coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=81

$$\frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}{}_2F_1\left(4,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[Out] 16*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([4, 3-1/2*n],[4-1
/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{6175, 6180, 131\}$

$$\frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}{}_2F_1\left(4,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (16*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[4, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx = \left(a^2c^2\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$
$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^4}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\,{}_2F_1\left(4,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Mathematica [A] time = 1.23, size = 144, normalized size = 1.78

$$\frac{c^2e^{n\coth^{-1}(ax)}\left((n+2)\left(2a^3x^3+n\left(a^2x^2-6ax-1\right)-6a^2x^2+\left(n^2-6n+8\right)\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+an^2x\right.\right.}{6a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(8 - 6*n + n^2)*Hypergeometri
c2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(6 + 6*a*x + a*n^2*
x - 6*a^2*x^2 + 2*a^3*x^3 + n*(-1 - 6*a*x + a^2*x^2) + (8 - 6*n + n^2)*Hype
rgeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(6*a*(2 + n))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^2x^2-2\,ac^2x+c^2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (acx-c)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*c*x - c)^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}\left(-acx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (acx-c)^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*c*x - c)^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^2,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \left(-2axe^{n\,\mathrm{acoth}\,(ax)}\right)dx+\int a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx+\int e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x*exp(n*acoth(a*x)), x) + Integral(a**2*x**2*exp(n*acot
h(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

3.367 $\int e^{n\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=79

$$-\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\,{}_2F_1\left(3,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[Out] -8*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([3, 2-1/2*n],[3-1/2
*n],(a-1/x)/(a+1/x))/a/(4-n)
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6175, 6180, 131}

$$-\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\,{}_2F_1\left(3,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] (-8*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1[
3, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)\,dx = -\left((ac)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$= (ac)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\,{}_2F_1\left(3,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Mathematica [A] time = 0.55, size = 104, normalized size = 1.32

$$-\frac{ce^{n\coth^{-1}(ax)}\left((n+2)\left(a^2x^2+(n-2)\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+a(n-2)x-1\right)+(n-2)ne^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}\right.}{2a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -1/2*(c*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1
[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*(-2 + n)*x + a^
2*x^2 + (-2 + n)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/
(a*(2 + n))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\text{arccoth}(ax)}\,(-acx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (acx - c)\left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="maxima")
[Out] -integrate((a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a x)} (c - a c x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int axe^{n \operatorname{acoth}(ax)} dx + \int \left(-e^{n \operatorname{acoth}(ax)}\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c),x)
[Out] -c*(Integral(a*x*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)), x))
```

3.368 $\int \frac{e^{n \coth^{-1}(ax)}}{c-acx} dx$

Optimal. Leaf size=71

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[Out] 2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n/
((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6180, 131}

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
+ x^(-1))])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int \frac{e^{n \coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{ac}$$
$$= \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Mathematica [A] time = 0.21, size = 87, normalized size = 1.23

$$-\frac{e^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(ax)}\right)+(n+2)\left({}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)-1\right)\right)}{acn(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + Hypergeometric2F1[1, n/2, 1 + n
/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n)))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{-acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="maxima")
[Out] -integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c),x)
[Out] -Integral(exp(n*acoth(a*x))/(a*x - 1), x)/c
```

3.369 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^2} dx$

Optimal. Leaf size=48

$$-\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

```
[Out] -(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)
```

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6175, 6180, 37}

$$-\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 + n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= -\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}$$

Mathematica [A] time = 0.18, size = 33, normalized size = 0.69

$$-\frac{(ax+1)e^{n\coth^{-1}(ax)}}{ac^2(n+2)(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] -((E^(n*ArcCoth[a*x])*(1 + a*x))/(a*c^2*(2 + n)*(-1 + a*x)))
```

fricas [A] time = 0.44, size = 59, normalized size = 1.23

$$-\frac{(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ac^2n-2\,ac^2-\left(a^2c^2n-2\,a^2c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^2*n - 2*a*c^2 - (a^2*c^2*n -
2*a^2*c^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^2, x)
```

maple [A] time = 0.04, size = 33, normalized size = 0.69

$$-\frac{e^{n\,\mathrm{arccoth}(ax)}\,(ax+1)}{(ax-1)\,c^2\,(2+n)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x)
[Out] -exp(n*arccoth(a*x))*(a*x+1)/(a*x-1)/c^2/(2+n)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(acx-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^2, x)
```

mupad [B] time = 1.53, size = 32, normalized size = 0.67

$$-\frac{e^{n\,\mathrm{acoth}(a\,x)}\,(a\,x+1)}{a\,c^2\,(a\,x-1)\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^2,x)
[Out] -(exp(n*acoth(a*x))*(a*x + 1))/(a*c^2*(a*x - 1)*(n + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \text{NaN} & \text{for}\ a = \frac{1}{x} \wedge c = 0 \wedge n = -2 \\ \tilde{\infty} x e^{\infty n} & \text{for}\ a = \frac{1}{x} \\ \tilde{\infty} \int e^{n \operatorname{acoth}(ax)}\,dx & \text{for}\ c = 0 \\ -\frac{ax \operatorname{acoth}(ax)}{a^2c^2xe^{2\operatorname{acoth}(ax)} - ac^2e^{2\operatorname{acoth}(ax)}} - \frac{\operatorname{acoth}(ax)}{a^2c^2xe^{2\operatorname{acoth}(ax)} - ac^2e^{2\operatorname{acoth}(ax)}} & \text{for}\ n = -2 \\ -\frac{axe^{n\operatorname{acoth}(ax)}}{a^2c^2nx + 2a^2c^2x - ac^2n - 2ac^2} - \frac{e^{n\operatorname{acoth}(ax)}}{a^2c^2nx + 2a^2c^2x - ac^2n - 2ac^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**2,x)
[Out] Piecewise((nan, Eq(c, 0) & Eq(n, -2) & Eq(a, 1/x)), (zoo*x*exp(oo*n), Eq(a,
 1/x)), (zoo*Integral(exp(n*acoth(a*x)), x), Eq(c, 0)), (-a*x*acoth(a*x)/(a
**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*x))) - acoth(a*x)/(a**2
*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*x))), Eq(n, -2)), (-a*x*ex
p(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a*c**2*n - 2*a*c**2) - exp
(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a*c**2*n - 2*a*c**2), True)
)
```

3.370 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=104

$$\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+4)}-\frac{(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+2)(n+4)}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(n^2+6*n+8)+(1-1/a/x)
^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(4+n)
```

Rubi [A] time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6175, 6180, 79, 37}

$$\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+4)}-\frac{(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] ((1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(4 + n)) - ((3
+ n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(2 + n)*(4
+ n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}$$

$$= \frac{\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{a^3c^3}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\,\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{a^2c^3(4+n)}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)} - \frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}$$

Mathematica [A] time = 0.25, size = 64, normalized size = 0.62

$$\frac{(-ax+n+3)e^{n\coth^{-1}(ax)}\left(\cosh\left(3\coth^{-1}(ax)\right)+\sinh\left(3\coth^{-1}(ax)\right)\right)}{a^2c^3(n+2)(n+4)x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(3 + n - a*x)*(Cosh[3*ArcCoth[a*x]] + Sinh[3*ArcCoth[a*
x]]))/(a^2*c^3*(2 + n)*(4 + n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.54, size = 125, normalized size = 1.20

$$-\frac{\left(a^2x^2+(an-2\,a)x+n-3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{ac^3n^2-6\,ac^3n+8\,ac^3+\left(a^3c^3n^2-6\,a^3c^3n+8\,a^3c^3\right)x^2-2\left(a^2c^3n^2-6\,a^2c^3n+8\,a^2c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 + (a*n - 2*a)*x + n - 3)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^3*n^2
 - 6*a*c^3*n + 8*a*c^3 + (a^3*c^3*n^2 - 6*a^3*c^3*n + 8*a^3*c^3)*x^2 - 2*(a
^2*c^3*n^2 - 6*a^2*c^3*n + 8*a^2*c^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{(acx-c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^3, x)
```

maple [A] time = 0.04, size = 46, normalized size = 0.44

$$-\frac{e^{n\,\text{arccoth}(ax)}\,(ax-n-3)\,(ax+1)}{(ax-1)^2\,c^3\left(n^2+6n+8\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x)
```

```
[Out] -exp(n*arccoth(a*x))*(a*x-n-3)*(a*x+1)/(a*x-1)^2/c^3/(n^2+6*n+8)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(acx-c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^3, x)
```

mupad [B] time = 1.65, size = 113, normalized size = 1.09

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{n+3}{a^3\,c^3\,\left(n^2+6\,n+8\right)}-\frac{x^2}{a\,c^3\,\left(n^2+6\,n+8\right)}+\frac{x\,\left(n+2\right)}{a^2\,c^3\,\left(n^2+6\,n+8\right)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^3,x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((n + 3)/(a^3*c^3*(6*n + n^2 + 8)) - x^2/(a*c^3*(6
*n + n^2 + 8)) + (x*(n + 2))/(a^2*c^3*(6*n + n^2 + 8))))/(((a*x - 1)/(a*x))
^(n/2)*(1/a^2 - (2*x)/a + x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**3,x)
```

```
[Out] Piecewise((zoo*Integral(exp(n*acoth(a*x)), x), Eq(c, 0)), (a**2*x**2*acoth(
a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x))
+ 2*a*c**3*exp(4*acoth(a*x))) + 2*a*x*acoth(a*x)/(2*a**3*c**3*x**2*exp(4*ac
oth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) -
 a*x/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x))
+ 2*a*c**3*exp(4*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*
x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - 1/(2*
a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c*
*3*exp(4*acoth(a*x))), Eq(n, -4)), (-a**2*x**2*acoth(a*x)/(2*a**3*c**3*x**2
*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth
(a*x))) + a*x/(2*a**3*c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*aco
th(a*x)) + 2*a*c**3*exp(2*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(2
*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x))
) + 1/(2*a**3*c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x))
 + 2*a*c**3*exp(2*acoth(a*x))), Eq(n, -2)), (-a**2*x**2*exp(n*acoth(a*x))/(
a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n
**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*
c**3) + a*n*x*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 +
 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x
+ a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + 2*a*x*exp(n*acoth(a*x))/(a**3*c**3
*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 1
```

```
2*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + n
*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*
x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2
 + 6*a*c**3*n + 8*a*c**3) + 3*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a*
*3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x -
 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3), True))
```

3.371 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal. Leaf size=224

$$-\frac{\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{a^2c^4x}-\frac{\left(n^2+8n+14\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}-\frac{\left(n^2+8n+14\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+ \cdots$$

```
[Out] -(n^2+8*n+14)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(n^2+10*n+24)-
(n^2+8*n+14)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(n^3+12*n^2+44*
n+48)+(5+n)*(1-1/a/x)^(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(6+n)-(1-1/a/x)^
(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a^2/c^4/x
```

Rubi [A] time = 0.26, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6175, 6180, 90, 79, 45, 37}

$$-\frac{\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{a^2c^4x}-\frac{\left(n^2+8n+14\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}-\frac{\left(n^2+8n+14\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+ \cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((5 + n)*(1 - 1/(a*x))^(-3 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^4*(6 + n)
) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(
a*c^4*(4 + n)*(6 + n)) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/
(a*x))^((2 + n)/2))/(a*c^4*(6 + n)*(8 + 6*n + n^2)) - ((1 - 1/(a*x))^(-3 -
n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a^2*c^4*x)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6175

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x} - \frac{\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\left(-1-\frac{(4+n)x}{a}\right)dx, x, \frac{1}{x}\right)}{a^2c^4}$$

$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x} - \frac{\left(14+8n+n^2\right)\text{Subst}\left(\int (1\right.}{a^2c^4}$$

$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{\left(14+8n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} - \frac{\left(1-\frac{1}{ax}\right)^{-3-}}{a^2}$$

$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{\left(14+8n+n^2\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} - \frac{\left(14+8n+\right.}{ac^4(}$$

Mathematica [A] time = 0.31, size = 83, normalized size = 0.37

$$-\frac{e^{n\coth^{-1}(ax)}\left(\cosh\left(4\coth^{-1}(ax)\right)+\sinh\left(4\coth^{-1}(ax)\right)\right)\left((n+4)^2\cosh\left(2\coth^{-1}(ax)\right)-2(n+4)\sinh\left(2\coth\right.\right.}{2ac^4(n+2)(n+4)(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/2*(E^(n*ArcCoth[a*x])*(-12 - 8*n - n^2 + (4 + n)^2*Cosh[2*ArcCoth[a*x]]
- 2*(4 + n)*Sinh[2*ArcCoth[a*x]])*(Cosh[4*ArcCoth[a*x]] + Sinh[4*ArcCoth[a*
x]]))/(a*c^4*(2 + n)*(4 + n)*(6 + n))
```

fricas [A] time = 0.91, size = 229, normalized size = 1.02

$$-\frac{\left(2\,a^{3}x^{3}+2\left(a^{2}n-3\,a^{2}\right)x^{2}+n^{2}+\left(an^{2}-6\,an+6\,a\right)x-8\,n}{ac^{4}n^{3}-12\,ac^{4}n^{2}+44\,ac^{4}n-48\,ac^{4}-\left(a^{4}c^{4}n^{3}-12\,a^{4}c^{4}n^{2}+44\,a^{4}c^{4}n-48\,a^{4}c^{4}\right)x^{3}+3\left(a^{3}c^{4}n^{3}-12\,a^{3}c^{4}n^{2}+44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -(2*a^3*x^3 + 2*(a^2*n - 3*a^2)*x^2 + n^2 + (a*n^2 - 6*a*n + 6*a)*x - 8*n +
 14)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^4*n^3 - 12*a*c^4*n^2 + 44*a*c^4*n -
 48*a*c^4 - (a^4*c^4*n^3 - 12*a^4*c^4*n^2 + 44*a^4*c^4*n - 48*a^4*c^4)*x^3
+ 3*(a^3*c^4*n^3 - 12*a^3*c^4*n^2 + 44*a^3*c^4*n - 48*a^3*c^4)*x^2 - 3*(a^2
*c^4*n^3 - 12*a^2*c^4*n^2 + 44*a^2*c^4*n - 48*a^2*c^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{(acx-c)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^4, x)
```

maple [A] time = 0.04, size = 68, normalized size = 0.30

$$-\frac{(ax+1)\left(2a^{2}x^{2}-2anx-8ax+n^{2}+8n+14\right)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{(ax-1)^{3}\,c^{4}a\left(n^{2}+8n+12\right)(4+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x)
```

```
[Out] -(a*x+1)*(2*a^2*x^2-2*a*n*x-8*a*x+n^2+8*n+14)*exp(n*arccoth(a*x))/(a*x-1)^3
/c^4/a/(n^2+8*n+12)/(4+n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{(acx-c)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c)^4, x)
```

mupad [B] time = 1.81, size = 180, normalized size = 0.80

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,x^{3}}{a\,c^{4}\left(n^{3}+12\,n^{2}+44\,n+48\right)}+\frac{n^{2}+8\,n+14}{a^{4}\,c^{4}\left(n^{3}+12\,n^{2}+44\,n+48\right)}-\frac{x^{2}\,(2\,n+6)}{a^{2}\,c^{4}\left(n^{3}+12\,n^{2}+44\,n+48\right)}+\frac{x\left(n^{2}+6\,n+6\right)}{a^{3}\,c^{4}\left(n^{3}+12\,n^{2}+44\,n+48\right)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\left(\frac{3\,x}{a^{2}}-\frac{1}{a^{3}}+x^{3}-\frac{3\,x^{2}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^4,x)
```

```
[Out] -(((a*x + 1)/(a*x))^(n/2)*((2*x^3)/(a*c^4*(44*n + 12*n^2 + n^3 + 48)) + (8*
n + n^2 + 14)/(a^4*c^4*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(2*n + 6))/(a^2*c
^4*(44*n + 12*n^2 + n^3 + 48)) + (x*(6*n + n^2 + 6))/(a^3*c^4*(44*n + 12*n^
2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*((3*x)/a^2 - 1/a^3 + x^3 - (3*x^2
)/a))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**4,x)
```

```
[Out] Timed out
```

3.372 $\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{2}{7}x(c-acx)^{5/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-5}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{7}{2},\frac{n-5}{2};-\frac{5}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/7*((a-1/x)/(a+1/x))^(-5/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(5/2)*h
ypergeom([-7/2, -5/2+1/2*n],[-5/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6176, 6181, 132}

$$\frac{2}{7}x(c-acx)^{5/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-5}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{7}{2},\frac{n-5}{2};-\frac{5}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-5 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(5/2)*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/((a + x^(-1))*x)
])/(7*(1 - 1/(a*x))^(n/2))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx=\frac{(c-acx)^{5/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{9/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{5/2}\,{}_2F_1\left(-\frac{7}{2},\frac{1}{2}(-5+n);-\frac{5}{2}\right.$$

Mathematica [A] time = 0.10, size = 103, normalized size = 1.05

$$\frac{2c^2(ax+1)^3\sqrt{c-acx}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n-1}{2}}{}_2F_1\left(-\frac{7}{2},\frac{n-5}{2};-\frac{5}{2};\frac{2}{ax+1}\right)}{7a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^3*
Sqrt[c - a*c*x]*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/(1 + a*x)])/(7*
a*(1 - 1/(a*x))^(n/2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^2x^2-2\,ac^2x+c^2\right)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*sqrt(-a*c*x + c)*((a*x - 1)/(a*x +
 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-1,[0,6,1,0,0]%%%}+%%%{3,[0,4,1,1,0]%%%}+%%%{-3,[0,2,1,2,0]%%%}+%%%{
1,[0,0,1,3,0]%%%} / %%%{1,[0,0,0,3,3]%%%} Error: Bad Argument Value
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^{\frac{5}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a x)} (c - a c x)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(5/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.373 $\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{2}{5}x(c-acx)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{5}{2},\frac{n-3}{2};-\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/5*((a-1/x)/(a+1/x))^(-3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(3/2)*h
ypergeom([-5/2, -3/2+1/2*n],[-3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6176, 6181, 132}

$$\frac{2}{5}x(c-acx)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{5}{2},\frac{n-3}{2};-\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(3/2)*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/((a + x^(-1))*x)
])/(5*(1 - 1/(a*x))^(n/2))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{(c-acx)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{7/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=\frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{3/2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-3+n);-\frac{3}{2};-\right.$$

Mathematica [A] time = 0.08, size = 101, normalized size = 1.03

$$-\frac{2c(ax+1)^2\sqrt{c-acx}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n-1}{2}}{}_2F_1\left(-\frac{5}{2},\frac{n-3}{2};-\frac{3}{2};\frac{2}{ax+1}\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^2*S
qrt[c - a*c*x]*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/(1 + a*x)])/(5*a
*(1 - 1/(a*x))^(n/2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-(acx-c)\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,4,1,0,0]%%%}+%%%{-2,[0,2,1,1,0]%%%}+%%%{1,[0,0,1,2,0]%%%} / %%%
{1,[0,0,0,2,2]%%%} Error: Bad Argument Value
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-acx+c)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(3/2),x)
[Out] Timed out
```

3.374 $\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=98

$$\frac{2}{3}x\sqrt{c-acx}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{3}{2},\frac{n-1}{2};-\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/3*((a-1/x)/(a+1/x))^(-1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-3/2, -
1/2+1/2*n],[-1/2],2/(a+1/x)/x)*(-a*c*x+c)^(1/2)/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.18, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6176, 6181, 132}

$$\frac{2}{3}x\sqrt{c-acx}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{3}{2},\frac{n-1}{2};-\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Sqr
t[c - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/((a + x^(-1))*x)])
/(3*(1 - 1/(a*x))^(n/2))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{5/2}}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\sqrt{c-acx}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-1+n);-\frac{1}{2};\frac{}{(a}\right.$$

Mathematica [A] time = 0.06, size = 98, normalized size = 1.00

$$\frac{2(ax+1)\sqrt{c-acx}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n-1}{2}}{}_2F_1\left(-\frac{3}{2},\frac{n-1}{2};-\frac{1}{2};\frac{2}{ax+1}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)*Sqrt[c
 - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/(1 + a*x)])/(3*a*(1 -
 1/(a*x))^(n/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\sqrt{-acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2),x)
[Out] Timed out
```

3.375 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=96

$$\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

```
[Out] 2*((a-1/x)/(a+1/x))^(1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-1/2, 1/2+
1/2*n],[1/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6176, 6181, 132}

$$\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hype
rgeometric2F1[-1/2, (1 + n)/2, 1/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*Sqrt[c - a*c*x])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\ \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 1.00

$$\frac{2(ax+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n+1}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{1}{2};\frac{2}{ax+1}\right)}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*(1 + a*x)*Hyperge
ometric2F1[-1/2, (1 + n)/2, 1/2, 2/(1 + a*x)])/(a*(1 - 1/(a*x))^(n/2)*Sqrt[
c - a*c*x])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a\,c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\,(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)), x)
```

3.376 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

```
[Out] -2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([1/2, 3/2+
1/2*n],[3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(3/2)
```

Rubi [A] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6176, 6181, 132}

$$-\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hyp
ergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*(c - a*c*x)^(3/2))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

Mathematica [A] time = 0.04, size = 94, normalized size = 0.98

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(\frac{ax-1}{ax+1}\right)^{\frac{n+1}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{ax+1}\right)}{ac\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1
[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)])/(a*c*(1 - 1/(a*x))^(n/2)*Sqrt[c - a*c*x
])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*
c^2*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\text{arccoth}(ax)}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{(c-a\,c\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2),x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(3/2), x)
```

3.377 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-acx)^{5/2}} dx$

Optimal. Leaf size=167

$$\frac{ax^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(n+3)(c-acx)^{5/2}}-\frac{ax^2\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+3)(c-acx)^{5/2}}$$

```
[Out] -a*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/(3+n)/(-a*c*x+c)^(5/2)+a*((a
-1/x)/(a+1/x))^(3/2+1/2*n)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2*hype
rgeom([1/2, 3/2+1/2*n],[3/2],2/(a+1/x)/x)/(3+n)/(-a*c*x+c)^(5/2)
```

Rubi [A] time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6176, 6181, 94, 132}

$$\frac{ax^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(n+3)(c-acx)^{5/2}}-\frac{ax^2\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+3)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((3 + n)*(c -
 a*c*x)^(5/2))) + (a*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^
((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2*Hypergeometric2F1[1/2, (3 + n)/2,
 3/2, 2/((a + x^(-1))*x)])/((3 + n)*(c - a*c*x)^(5/2))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
```

```
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}dx,x,\frac{1}{x}\right)}{2(3+n)\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(3+n)(c-acx)^{5/2}}$$

Mathematica [A] time = 0.12, size = 117, normalized size = 0.70

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left((ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{n+1}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{ax+1}\right)-ax-1\right)}{ac^2(n+3)(ax-1)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*(-1 - a*x + (-1 + a*x)*((-1 + a*x)/(1 + a*x))^((1 + n)
/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)]))/(a*c^2*(3 + n)*(1
 - 1/(a*x))^(n/2)*(-1 + a*x)*Sqrt[c - a*c*x])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^3c^3x^3-3a^2c^3x^2+3ac^3x-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^3*c^3*x^3 - 3*a
^2*c^3*x^2 + 3*a*c^3*x - c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{e^{n \,\mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \,\mathrm{acoth}(a\,x)}}{{(c-a\,c\,x)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(5/2),x)
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(5/2), x)
```

3.378 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=245

$$-\frac{3a^2x^3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}+\frac{3a^2x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}-\frac{ax^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\cdots}{(n+5)(c-\cdots}$$

```
[Out] -a*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/(5+n)/(-a*c*x+c)^(7/2)+3/2*a
^2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^3/(n^2+8*n+15)/(-a*c*x+c)^(7/2
)-3/2*a^2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1+1/
2*n)*x^3*hypergeom([1/2, 3/2+1/2*n],[3/2],2/(a+1/x)/x)/(n^2+8*n+15)/(-a*c*x
+c)^(7/2)
```

Rubi [A] time = 0.27, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6176, 6181, 94, 132}

$$-\frac{3a^2x^3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}+\frac{3a^2x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}-\frac{ax^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\cdots}{(n+5)(c-\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((5 + n)*(c -
 a*c*x)^(7/2))) + (3*a^2*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2
)*x^3)/(2*(15 + 8*n + n^2)*(c - a*c*x)^(7/2)) - (3*a^2*((a - x^(-1))/(a + x
^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3
*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(2*(15 + 8*n +
 n^2)*(c - a*c*x)^(7/2))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6176

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
```

```
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int x^{3/2}\left(1-\frac{x}{a}\right)^{-\frac{7}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}dx,\cdots\right.}{2(5+n)\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\cdots}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\cdots}{4(3+n)(5+n\cdots}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} - \frac{3a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n\cdots}{2}}\cdots}{2\left(15+\cdots\right.}$$

Mathematica [A] time = 0.17, size = 138, normalized size = 0.56

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(3(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{n+1}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{ax+1}\right)+(ax+1)(-3ax+2n+9)\right)}{2ac^3(n+3)(n+5)(ax-1)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*((9 + 2*n - 3*a*x)*(1 + a*x) + 3*(-1 + a*x)^2*((-1 + a
*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*
x)]))/(2*a*c^3*(3 + n)*(5 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x)^2*Sqrt[c - a*
c*x])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-acx+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^4x^4-4a^3c^4x^3+6a^2c^4x^2-4ac^4x+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^4*x^4 - 4*a^
3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(-acx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{(c-a\,c\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2),x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.379 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=114

$$-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c^4\csc^{-1}(ax)}{2a}$$

```
[Out] -1/3*c^4*(1-1/a^2/x^2)^(3/2)/a+c^4*(1-1/a^2/x^2)^(3/2)*x-1/2*c^4*arccsc(a*x
)/a-3*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+1/2*c^4*(6*a-1/x)*(1-1/a^2/x^2)^(1
/2)/a^2
```

Rubi [A] time = 0.27, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6177, 1807, 1809, 815, 844, 216, 266, 63, 208}

$$-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] -(c^4*(1 - 1/(a^2*x^2))^(3/2))/(3*a) + (c^4*Sqrt[1 - 1/(a^2*x^2)]*(6*a - x^
(-1)))/(2*a^2) + c^4*(1 - 1/(a^2*x^2))^(3/2)*x - (c^4*ArcCsc[a*x])/(2*a) -
(3*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
```

```
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\left(c\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(\frac{3c^3}{a}-\frac{c^3x}{a^2}+\frac{c^3x^2}{a^3}\right)}{x}dx, x, \frac{1}{x}\right)$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{1}{3}\left(a^2c\right)\text{Subst}\left(\int \frac{\left(-\frac{9c^3}{a^3}+\frac{3c^3x}{a^4}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}\right.$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{1}{6}\left(a^4c\right)\text{Subst}\left(\int\right.$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\,\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.}{2a^2}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} + -$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} - (3$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{c^4\csc^{-1}(ax)}{2a} - \frac{3c}{}$$

Mathematica [A] time = 0.20, size = 175, normalized size = 1.54

$$\frac{c^4\left(6a^5x^5+16a^4x^4-15a^3x^3-14a^2x^2+24a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+9a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-18a^4x\right.}{6a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*(-2 + 9*a*x - 14*a^2*x^2 - 15*a^3*x^3 + 16*a^4*x^4 + 6*a^5*x^5 + 24*a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 9*a^4*Sqrt[
1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 18*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

fricas [A] time = 0.44, size = 156, normalized size = 1.37

$$\frac{6\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^4x^4+22\,a^3c^4x^3+\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^4*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 7*a^2*c^4*x^2 - 7*a*c^4*x + 2*c^
4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

giac [B] time = 0.18, size = 211, normalized size = 1.85

$$\frac{1}{3}\,ac^4\left(\frac{3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{6\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{28\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{27\,(ax-1)}{(ax-}}{a^2\left(\frac{ax-1}{ax+1}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/3*a*c^4*(3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*log(sqrt((a*x - 1)/(
a*x + 1)) + 1)/a^2 + 9*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 - 6*sqrt
((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (28*(a*x - 1)*sqrt(
(a*x - 1)/(a*x + 1))/(a*x + 1) + 27*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(
a*x + 1)^2 + 9*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^3)
)
```

maple [B] time = 0.05, size = 224, normalized size = 1.96

$$-\frac{(ax-1)\,c^4\left(-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+18\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)\,(ax+1)}\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x)
```

```
[Out] -1/6*(a*x-1)*c^4*(-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+18*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2)*x^2*a^2+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+3*a^3*x^3
*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^
2)^(1/2))/(a^2)^(1/2))*x^3*a^4-9*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+2*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^
4/x^3/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 224, normalized size = 1.96

$$\frac{1}{3}\left(\frac{3\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-37\,c^4\left(\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-}{(a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^4*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^2 + 9*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21*c
^4*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 37*c^
4*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x +
1)^4 + a^2))*a
```

mupad [B] time = 1.32, size = 183, normalized size = 1.61

$$\frac{5\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{17\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (17*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^4*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^4
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^3}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**4,x)
```

```
[Out] c**4*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a/(x**3*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x) + Integral(6*a**2/(x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))), x) + Integral(-4*a**3/(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)
)/a**4
```

3.380 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=88

$$\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{2c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^3\csc^{-1}(ax)}{2a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x+1/2*c^3*arccsc(a*x)/a-2*c^3*arctanh((1-1/a^2/x^2)
^(1/2))/a+1/2*c^3*(4*a+1/x)*(1-1/a^2/x^2)^(1/2)/a^2
```

Rubi [A] time = 0.19, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6177, 1807, 815, 844, 216, 266, 63, 208}

$$\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{2c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*Sqrt[1 - 1/(a^2*x^2)]*(4*a + x^(-1)))/(2*a^2) + c^3*(1 - 1/(a^2*x^2))^
(3/2)*x + (c^3*ArcCsc[a*x])/(2*a) - (2*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/
a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
```

```
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx=-\left(c\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+c\,\text{Subst}\left(\int\frac{\left(\frac{2c^2}{a}+\frac{c^2x}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x-\frac{1}{2}\left(a^2c\right)\text{Subst}\left(\int\frac{-\frac{4c^2}{a^3}-\frac{c^2x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right.$$

$$=\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{c^3\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a^2}+\frac{\left(2c^3\right)\cdots}{\cdots}$$

$$=\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{c^3\csc^{-1}(ax)}{2a}+\frac{c^3\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\cdots\right.}{a}$$

$$=\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{c^3\csc^{-1}(ax)}{2a}-\left(2ac^3\right)\text{Subst}\left(\int\frac{\cdots}{a^2-\cdots}\right.$$

$$=\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{c^3\csc^{-1}(ax)}{2a}-\frac{2c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x\cdots}}\right.}{a}$$

Mathematica [A] time = 0.16, size = 167, normalized size = 1.90

$$\frac{c^3\left(2a^4x^4+4a^3x^3-3a^2x^2+2a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+2a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-4a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\tanh\right.}{2a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(1 - 4*a*x - 3*a^2*x^2 + 4*a^3*x^3 + 2*a^4*x^4 + 2*a^3*Sqrt[1 - 1/(a^2
*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 2*a^3*Sqrt[1 - 1/(a^2*x^2)]*
x^3*ArcSin[1/(a*x)] - 4*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

fricas [A] time = 0.47, size = 146, normalized size = 1.66

$$-\frac{2a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2a^3c^3x^3+6a^2c^3x^2+3ac^3\right.}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*a^2*c^3*x^2*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 4*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) - (2*a^3*c^3*x^3 + 6*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*x^2)
```

giac [B] time = 0.16, size = 180, normalized size = 2.05

$$-ac^3\left(\frac{\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}-\frac{\frac{5(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+3\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}+1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -a*c^3*(arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^2 - 2*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 2*sqrt((a*x
 - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) - (5*(a*x - 1)*sqrt((a*x -
 1)/(a*x + 1))/(a*x + 1) + 3*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*
x + 1) + 1)^2))
```

maple [B] time = 0.06, size = 200, normalized size = 2.27

$$-\frac{(ax-1)c^3\left(-4\sqrt{a^2x^2-1}\sqrt{a^2}\,x^3a^3+4\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,xa-\sqrt{a^2x^2-1}\sqrt{a^2}\,x^2a^2-a^2x^2\sqrt{a^2}\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right.}{2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax-1)(ax+1)}\,a^3x^2\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x)
```

```
[Out] -1/2*(a*x-1)*c^3*(-4*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+4*(a^2*x^2-1)^(3
/2)*(a^2)^(1/2)*x*a-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-a^2*x^2*(a^2)^(1/
2)*arctan(1/(a^2*x^2-1)^(1/2))+4*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(
```

```
a^2)^(1/2))*x^2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/
((a*x-1)*(a*x+1))^(1/2)/a^3/x^2/(a^2)^(1/2)
```

maxima [B] time = 0.42, size = 201, normalized size = 2.28

$$-\left(\frac{c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c^3*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1)/a^2 - 2*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (3*c^3*((a
*x - 1)/(a*x + 1))^(5/2) - 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqrt((
a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2
 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

mupad [B] time = 0.11, size = 163, normalized size = 1.85

$$\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-3\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,(a\,x-1)}{a\,x+1}-\frac{a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}}-\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 3*
c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x - 1
)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^3*atan(((a*x - 1)/(a*x
 + 1))^(1/2)))/a - (4*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{a^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**3,x)
```

```
[Out] c**3*(Integral(a**3/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a/(x**2*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**2/(x*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))), x))/a**3
```

3.381 $\int e^{\coth^{-1}(ax)} \left(c - \frac{c}{ax}\right)^2 dx$

Optimal. Leaf size=62

$$\frac{c^2 x \sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{a} - \frac{c^2 \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a} + \frac{c^2 \csc^{-1}(ax)}{a}$$

```
[Out] c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+c^2*(a+1/x)*x*(1-1/a^2
/x^2)^(1/2)/a
```

Rubi [A] time = 0.10, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6177, 813, 844, 216, 266, 63, 208}

$$\frac{c^2 x \sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{a} - \frac{c^2 \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a} + \frac{c^2 \csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a + x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a - (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\left(c\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{\frac{2c}{a}+\frac{2cx}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x\right.}{a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - \left(ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2}}\right.$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - \frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [B] time = 0.17, size = 158, normalized size = 2.55

$$\frac{c^2\left(2a^3x^3+2a^2x^2-2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\tanh^{-1}\left(\sqrt{1\right.\right.}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*(-2 - 2*a*x + 2*a^2*x^2 + 2*a^3*x^3 - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*
ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/
(a*x)] - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(
2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

fricas [B] time = 0.42, size = 119, normalized size = 1.92

$$-\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2c^2x^2+2\,ac^2x+c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c^2*x*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x^2
 + 2*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

giac [A] time = 0.16, size = 113, normalized size = 1.82

$$-ac^2\left(\frac{2\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{(ax-1)^2}{(ax+1)^2}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -a*c^2*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^2 - log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 4*sqrt((a*x -
 1)/(a*x + 1))/(a^2*((a*x - 1)^2/(a*x + 1)^2 - 1)))
```

maple [B] time = 0.05, size = 168, normalized size = 2.71

$$-\frac{(ax-1)\,c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa-ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x\cdots}{}\right.\right.}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)\,(ax+1)}\,a^2x\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x)
```

```
[Out] -(a*x-1)*c^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2
-1)^(1/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/((a
*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

maxima [B] time = 0.40, size = 125, normalized size = 2.02

$$-\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 - c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

mupad [B] time = 1.22, size = 90, normalized size = 1.45

$$\frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int \frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \left(-\frac{2a}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a/(x*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))), x))/a**2
```

3.382 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=27

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{c\csc^{-1}(ax)}{a}$$

[Out] c*arccsc(a*x)/a+c*x*(1-1/a^2/x^2)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6177, 277, 216}

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]*(c - c/(a*x)),x]

[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x + (c*ArcCsc[a*x])/a

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx&=-\left(c\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\&=c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}\\&=c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{c\csc^{-1}(ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.15

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}+\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x + ArcSin[1/(a*x)]))/a
```

fricas [A] time = 0.56, size = 48, normalized size = 1.78

$$-\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*c*x + c)*sqrt((a*x - 1)/(a*x +
 1)))/a
```

giac [B] time = 0.13, size = 86, normalized size = 3.19

$$-\frac{1}{2}\,ac\left(\frac{\pi+2\,\arctan\left(\frac{\frac{ax-1}{ax+1}-1}{2\,\sqrt{\frac{ax-1}{ax+1}}}\right)}{a^2}+\frac{4}{a^2\left(\sqrt{\frac{ax-1}{ax+1}}-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] -1/2*a*c*((pi + 2*arctan(1/2*((a*x - 1)/(a*x + 1) - 1)/sqrt((a*x - 1)/(a*x
+ 1))))/a^2 + 4/(a^2*(sqrt((a*x - 1)/(a*x + 1)) - 1/sqrt((a*x - 1)/(a*x + 1
)))))
```

maple [B] time = 0.04, size = 63, normalized size = 2.33

$$\frac{(ax-1)\,c\left(\sqrt{a^2x^2-1}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)\,(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/((a*x-1)*(a*x+1))^(1/2)*c/a*((a^2*x^2-1)^
(1/2)+arctan(1/(a^2*x^2-1)^(1/2)))
```

maxima [B] time = 0.40, size = 66, normalized size = 2.44

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2)
```

mupad [B] time = 0.06, size = 60, normalized size = 2.22

$$\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*atan
(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{a}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x),x)
```

```
[Out] c*(Integral(a/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))), x))/a
```

3.383 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=70

$$-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] 2*arctanh((1-1/a^2/x^2)^(1/2))/a/c-2*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(1
-1/a^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.20, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

```
[Out] (-2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)
/c + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^3}\\
&= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int \frac{-c^2-\frac{2c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c^3}\\
&= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}-\frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{ac}\\
&= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{ac}\\
&= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 63, normalized size = 0.90

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}(ax-3)+2(ax-1)\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac(ax-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3 + a*x) + 2*(-1 + a*x)*Log[(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/(a*c*(-1 + a*x))
```

fricas [A] time = 0.54, size = 94, normalized size = 1.34

$$\frac{2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-2\,ax-3\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (2*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2*(a*x - 1)*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 2*a*x - 3)*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*c*x - a*c)
```

giac [A] time = 0.15, size = 128, normalized size = 1.83

$$2\,a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{\frac{2\,(ax-1)}{ax+1}-1}{a^2c\left(\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\sqrt{\frac{ax-1}{ax+1}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2*a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(abs(sqrt((a*x - 1)/(a
*x + 1)) - 1))/(a^2*c) - (2*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)*sqrt
((a*x - 1)/(a*x + 1))/(a*x + 1) - sqrt((a*x - 1)/(a*x + 1)))))
```

maple [B] time = 0.06, size = 250, normalized size = 3.57

$$\frac{2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+2\sqrt{(ax-1)\,(ax+1)}\ \sqrt{a^2}\ x^2a^2-4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}}{\sqrt{a^2}}\right)x\,a^2-((ax-1)(}{a\sqrt{a^2}\ \ (ax-1)\,c\sqrt{(ax-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x)
```

```
[Out] (2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+2*((
a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-4*ln((a^2*x+((a*x-1)*(a*x+1))^(1/
2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-4*((
a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+2*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)
*(a^2)^(1/2))/(a^2)^(1/2))+2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/a/(a^2)^(
1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.30, size = 116, normalized size = 1.66

$$-2\,a\left(\frac{\frac{2\,(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*((2*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*
c*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

mupad [B] time = 1.22, size = 62, normalized size = 0.89

$$\frac{2\,a\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-6}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a*x + 8*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 6)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int\frac{x}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c
```

3.384 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=105

$$-\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -4/3*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(3/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c
^2+1/3*(-9*a-11/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A] time = 0.29, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (-4*(a + x^(-1)))/(3*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (9*a + 11/x)/(3*a^2
*c^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (3*ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(a*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^3}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^3}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^3-\frac{9c^3x}{a}-\frac{8c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^3+\frac{9c^3x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.90

$$\frac{3a^3x^3-16a^2x^2+9ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-5ax+14}{3a^2c^2x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (14 - 5*a*x - 16*a^2*x^2 + 3*a^3*x^3 + 9*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*
x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

fricas [A] time = 0.73, size = 134, normalized size = 1.28

$$\frac{9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-16\,a^2x^2-5\,ax+14\right)\sqrt{}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x^2
 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^
2)
```

giac [A] time = 0.17, size = 148, normalized size = 1.41

$$\frac{1}{3}\,a\left(\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{9\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^2}-\frac{(ax+1)\left(\frac{12\,(ax-1)}{ax+1}+1\right)}{(ax-1)a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{6\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 1/3*a*(9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 9*log(abs(sqrt((a*x
 - 1)/(a*x + 1)) - 1))/(a^2*c^2) - (a*x + 1)*(12*(a*x - 1)/(a*x + 1) + 1)/(
(a*x - 1)*a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) - 6*sqrt((a*x - 1)/(a*x + 1))/
(a^2*c^2*((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.06, size = 339, normalized size = 3.23

$$\frac{9\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^3a^4+9\sqrt{a^2}\,\sqrt{(ax-1)\,(ax+1)}\,x^3a^3-27\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3-6\sqrt{a^2}\,\left((ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x)
```

```
[Out] 1/3*(9*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+
9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-27*ln((a^2*x+((a*x-1)*(a*x+1)
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
3/2)*x*a-27*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+27*ln((a^2*x+((a*x-
1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+5*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)+27*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-9*a*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-9*((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2))/a/(a*x-1)^2/(a^2)^(1/2)/c^2/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+
1))^(1/2)
```

maxima [A] time = 0.31, size = 137, normalized size = 1.30

$$\frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*(
(a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 9*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c^2))
```

mupad [B] time = 0.10, size = 104, normalized size = 0.99

$$\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}-\frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)}-\frac{6\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{a\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((11*(a*x - 1))/(3*(a*x +
1)) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2)
 - a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**
2
```

3.385 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] -8/5*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(5/2)-4/15*(5*a+8/x)/a^2/c^3/(1-1/a^2/x^
2)^(3/2)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c^3+1/15*(-60*a-79/x)/a^2/c^3/(1-
1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^3
```

Rubi [A] time = 0.39, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-8*(a + x^(-1)))/(5*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 8/x))/(15
*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (60*a + 79/x)/(15*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]])/(a*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^4-\frac{20c^4x}{a}-\frac{27c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^4+\frac{60c^4x}{a}+\frac{64c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^4-\frac{60c^4x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{4\,\text{Subst}\left(\int \frac{\ldots}{x\sqrt{\ldots}}\right.}{ac\ldots}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{2\,\text{Subst}\left(\int \frac{\ldots}{x\sqrt{\ldots}}\right.}{ac\ldots}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{(4a)\,\text{Subst}\left(\int\right.}{\ldots}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{4\tanh^{-1}\left(\sqrt{1\ldots}\right.}{ac^3}$$

Mathematica [A] time = 0.08, size = 104, normalized size = 0.75

$$\frac{15a^4x^4-134a^3x^3+73a^2x^2+60ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+128ax-94}{15a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-94 + 128*a*x + 73*a^2*x^2 - 134*a^3*x^3 + 15*a^4*x^4 + 60*a*Sqrt[1 - 1/(a
^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^3*Sqrt[1
- 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

fricas [A] time = 0.58, size = 170, normalized size = 1.23

$$\frac{60\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-60\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(15\,a^{4}x^{4}-13\right.}{15\left(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 134*a^3*x^3 + 73*a^2*x^2 + 128*a*x - 94)*sqrt((a*x - 1)/(
a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.18, size = 166, normalized size = 1.20

$$\frac{1}{30}\,a\left(\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c^{3}}-\frac{120\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^{2}c^{3}}-\frac{(ax+1)^{2}\left(\frac{25\,(ax-1)}{ax+1}+\frac{180\,(ax-1)^{2}}{(ax+1)^{2}}+3\right)}{(ax-1)^{2}a^{2}c^{3}\sqrt{\frac{ax-1}{ax+1}}}-\frac{60\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{3}\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] 1/30*a*(120*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 120*log(abs(sqrt
((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^3) - (a*x + 1)^2*(25*(a*x - 1)/(a*x + 1)
 + 180*(a*x - 1)^2/(a*x + 1)^2 + 3)/((a*x - 1)^2*a^2*c^3*sqrt((a*x - 1)/(a*
x + 1))) - 60*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*((a*x - 1)/(a*x + 1) - 1))
)
```

maple [B] time = 0.06, size = 431, normalized size = 3.12

$$60\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{4}a^{5}+60\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,x^{4}a^{4}-240\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{3}a^{4}-45\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x)
```

```
[Out] 1/15*(60*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^
5+60*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-240*ln((a^2*x+((a*x-1)*(a*
x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-45*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x^2*a^2-240*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+360*ln((a
^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+76*(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(3/2)*x*a+360*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*
a^2-240*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-3
4*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-240*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/
2)*x*a+60*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))+60*
((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/a/(a*x-1)^3/(a^2)^(1/2)/c^3/((a*x-1)*(
a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.31, size = 153, normalized size = 1.11

$$\frac{1}{30}\,a\left(\frac{\frac{22\,(ax-1)}{ax+1}+\frac{155\,(ax-1)^2}{(ax+1)^2}-\frac{240\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*a*((22*(a*x - 1)/(a*x + 1) + 155*(a*x - 1)^2/(a*x + 1)^2 - 240*(a*x -
1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*x
- 1)/(a*x + 1))^(5/2)) + 120*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) -
 120*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

mupad [B] time = 1.22, size = 121, normalized size = 0.88

$$\frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{31\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{16\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{22\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{2\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-2\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((31*(a*x - 1)^2)/(3*(a*x
+ 1)^2) - (16*(a*x - 1)^3)/(a*x + 1)^3 + (22*(a*x - 1))/(15*(a*x + 1)) + 1/
5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^(7/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\int\frac{x^3}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

3.386 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=171

$$-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{5\tanh^{-1}\left(\right.}{a}$$

```
[Out] -16/7*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-4/35*(7*a+17/x)/a^2/c^4/(1-1/a^2/
x^2)^(5/2)+1/105*(-175*a-307/x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)+5*arctanh((1-1/
a^2/x^2)^(1/2))/a/c^4+1/105*(-525*a-719/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+x*(1
-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A] time = 0.50, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{5\tanh^{-1}\left(\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (-16*(a + x^(-1)))/(7*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (4*(7*a + 17/x))/(
35*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (175*a + 307/x)/(105*a^2*c^4*(1 - 1/(
a^2*x^2))^(3/2)) - (525*a + 719/x)/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (S
qrt[1 - 1/(a^2*x^2)]*x)/c^4 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^5}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-7c^5-\frac{35c^5x}{a}-\frac{61c^5x^2}{a^2}+\frac{7c^5x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{35c^5+\frac{175c^5x}{a}+\frac{272c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-105c^5-\frac{525c^5x}{a}-\cdots}{x^2\left(1-\frac{x^2}{a^2}\right)\cdots}\cdots\right)}{105c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} - \cdots$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \cdots$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \cdots$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \cdots$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \cdots$$

Mathematica [A] time = 0.09, size = 112, normalized size = 0.65

$$\frac{105a^5x^5-1339a^4x^4+1812a^3x^3+485a^2x^2+525ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-1947ax+824}{105a^2c^4x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (824 - 1947*a*x + 485*a^2*x^2 + 1812*a^3*x^3 - 1339*a^4*x^4 + 105*a^5*x^5 +
 525*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

fricas [A] time = 0.57, size = 204, normalized size = 1.19

$$\frac{525\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-525\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-\right.}{105\left(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2-4a^2c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) - 525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sq
rt((a*x - 1)/(a*x + 1)) - 1) + (105*a^5*x^5 - 1339*a^4*x^4 + 1812*a^3*x^3 +
 485*a^2*x^2 - 1947*a*x + 824)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x^4 - 4*
a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.18, size = 182, normalized size = 1.06

$$\frac{1}{420}a\left(\frac{2100\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{2100\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^4}-\frac{(ax+1)^3\left(\frac{126(ax-1)}{ax+1}+\frac{595(ax-1)^2}{(ax+1)^2}+\frac{3360(ax-1)^3}{(ax+1)^3}+15\right)}{(ax-1)^3a^2c^4\sqrt{\frac{ax-1}{ax+1}}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/420*a*(2100*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 2100*log(abs(s
qrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^4) - (a*x + 1)^3*(126*(a*x - 1)/(a*x
+ 1) + 595*(a*x - 1)^2/(a*x + 1)^2 + 3360*(a*x - 1)^3/(a*x + 1)^3 + 15)/((a
*x - 1)^3*a^2*c^4*sqrt((a*x - 1)/(a*x + 1))) - 840*sqrt((a*x - 1)/(a*x + 1)
)/(a^2*c^4*((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.06, size = 523, normalized size = 3.06

$$-\frac{-525\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^5a^5-525\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^5a^6+420\sqrt{a^2}\,\left((ax-1)(ax+1)\right)^{\frac{3}{2}}x^3a^3+26}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x)
```

```
[Out] -1/105*(-525*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-525*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+420*(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(3/2)*x^3*a^3+2625*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4
+2625*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-1
076*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-5250*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(1/2)*x^3*a^3-5250*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(
a^2)^(1/2))*x^3*a^4+970*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+5250*((a*x-
1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+5250*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2
)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-299*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-2625*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-2625*ln((a^2*x+((a*x-1)*(a*x+
```

```
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+525*((a*x-1)*(a*x+1))^(1/2)*(a^2)
^(1/2)+525*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a
/(a*x-1)^4/(a^2)^(1/2)/c^4/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.30, size = 169, normalized size = 0.99

$$\frac{1}{420}\,a\left(\frac{\frac{111\,(ax-1)}{ax+1}+\frac{469\,(ax-1)^2}{(ax+1)^2}+\frac{2765\,(ax-1)^3}{(ax+1)^3}-\frac{4200\,(ax-1)^4}{(ax+1)^4}+15}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{2100\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{2100\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/420*a*((111*(a*x - 1)/(a*x + 1) + 469*(a*x - 1)^2/(a*x + 1)^2 + 2765*(a*x
 - 1)^3/(a*x + 1)^3 - 4200*(a*x - 1)^4/(a*x + 1)^4 + 15)/(a^2*c^4*((a*x - 1
)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 2100*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 2100*log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/(a^2*c^4))
```

mupad [B] time = 0.11, size = 137, normalized size = 0.80

$$\frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}-\frac{\frac{67\,(a\,x-1)^2}{15\,(a\,x+1)^2}+\frac{79\,(a\,x-1)^3}{3\,(a\,x+1)^3}-\frac{40\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{37\,(a\,x-1)}{35\,(a\,x+1)}+\frac{1}{7}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((67*(a*x - 1)^2)/(15*(a*
x + 1)^2) + (79*(a*x - 1)^3)/(3*(a*x + 1)^3) - (40*(a*x - 1)^4)/(a*x + 1)^4
 + (37*(a*x - 1))/(35*(a*x + 1)) + 1/7)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(7/2
) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c**4
```

3.387 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=61

$$-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}-\frac{3c^5\log(x)}{a}+c^5x$$

```
[Out] -1/4*c^5/a^5/x^4+c^5/a^4/x^3-c^5/a^3/x^2-2*c^5/a^2/x+c^5*x-3*c^5*ln(x)/a
```

Rubi [A] time = 0.14, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 75}

$$-\frac{c^5}{a^3x^2}+\frac{c^5}{a^4x^3}-\frac{c^5}{4a^5x^4}-\frac{2c^5}{a^2x}-\frac{3c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] -c^5/(4*a^5*x^4) + c^5/(a^4*x^3) - c^5/(a^3*x^2) - (2*c^5)/(a^2*x) + c^5*x
- (3*c^5*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx$$
$$= \frac{c^5\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$
$$= \frac{c^5\int \frac{(1-ax)^4(1+ax)}{x^5}dx}{a^5}$$
$$= \frac{c^5\int \left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5}$$
$$= -\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(x)}{a}$$

Mathematica [A] time = 0.24, size = 63, normalized size = 1.03

$$-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}-\frac{3c^5\log(ax)}{a}+c^5x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] -1/4*c^5/(a^5*x^4) + c^5/(a^4*x^3) - c^5/(a^3*x^2) - (2*c^5)/(a^2*x) + c^5*
x - (3*c^5*Log[a*x])/a
```

fricas [A] time = 0.53, size = 67, normalized size = 1.10

$$\frac{4\,a^5c^5x^5-12\,a^4c^5x^4\log(x)-8\,a^3c^5x^3-4\,a^2c^5x^2+4\,ac^5x-c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) - 8*a^3*c^5*x^3 - 4*a^2*c^5*x^2
+ 4*a*c^5*x - c^5)/(a^5*x^4)
```

giac [A] time = 0.12, size = 58, normalized size = 0.95

$$c^5x-\frac{3\,c^5\log\left(|x|\right)}{a}-\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*x - 3*c^5*log(abs(x))/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*
x + c^5)/(a^5*x^4)
```

maple [A] time = 0.04, size = 60, normalized size = 0.98

$$-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{x^2a^3}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^5,x)
```

```
[Out] -1/4*c^5/a^5/x^4+c^5/a^4/x^3-c^5/x^2/a^3-2*c^5/a^2/x+c^5*x-3*c^5*ln(x)/a
```

maxima [A] time = 0.30, size = 57, normalized size = 0.93

$$c^5x - \frac{3\,c^5\log(x)}{a} - \frac{8\,a^3c^5x^3 + 4\,a^2c^5x^2 - 4\,ac^5x + c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - 3*c^5*log(x)/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*x + c
^5)/(a^5*x^4)
```

mupad [B] time = 0.07, size = 51, normalized size = 0.84

$$-\frac{c^5\,\left(4\,a^2\,x^2-4\,a\,x+8\,a^3\,x^3-4\,a^5\,x^5+12\,a^4\,x^4\,\ln(x)+1\right)}{4\,a^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] -(c^5*(4*a^2*x^2 - 4*a*x + 8*a^3*x^3 - 4*a^5*x^5 + 12*a^4*x^4*log(x) + 1))/
(4*a^5*x^4)
```

sympy [A] time = 0.27, size = 63, normalized size = 1.03

$$\frac{a^5c^5x - 3a^4c^5\log{(x)} + \frac{-8a^3c^5x^3-4a^2c^5x^2+4ac^5x-c^5}{4x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**5,x)
```

```
[Out] (a**5*c**5*x - 3*a**4*c**5*log(x) + (-8*a**3*c**5*x**3 - 4*a**2*c**5*x**2 +
 4*a*c**5*x - c**5)/(4*x**4))/a**5
```

3.388 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=40

$$\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}-\frac{2c^4\log(x)}{a}+c^4x$$

```
[Out] 1/3*c^4/a^4/x^3-c^4/a^3/x^2+c^4*x-2*c^4*ln(x)/a
```

Rubi [A] time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 75}

$$-\frac{c^4}{a^3x^2}+\frac{c^4}{3a^4x^3}-\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] c^4/(3*a^4*x^3) - c^4/(a^3*x^2) + c^4*x - (2*c^4*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= -\frac{c^4\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= -\frac{c^4\int\frac{(1-ax)^3(1+ax)}{x^4}dx}{a^4}$$
$$= -\frac{c^4\int\left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}$$
$$= \frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(x)}{a}$$

Mathematica [A] time = 0.17, size = 42, normalized size = 1.05

$$\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}-\frac{2c^4\log(ax)}{a}+c^4x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
[Out] c^4/(3*a^4*x^3) - c^4/(a^3*x^2) + c^4*x - (2*c^4*Log[a*x])/a
```

fricas [A] time = 0.62, size = 43, normalized size = 1.08

$$\frac{3\,a^4c^4x^4-6\,a^3c^4x^3\log(x)-3\,ac^4x+c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="fricas")
[Out] 1/3*(3*a^4*c^4*x^4 - 6*a^3*c^4*x^3*log(x) - 3*a*c^4*x + c^4)/(a^4*x^3)
```

giac [A] time = 0.15, size = 38, normalized size = 0.95

$$c^4x-\frac{2\,c^4\log(|x|)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="giac")
[Out] c^4*x - 2*c^4*log(abs(x))/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

maple [A] time = 0.04, size = 39, normalized size = 0.98

$$\frac{c^4}{3a^4x^3}-\frac{c^4}{x^2a^3}+c^4x-\frac{2c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^4,x)
[Out] 1/3*c^4/a^4/x^3-c^4/x^2/a^3+c^4*x-2*c^4*ln(x)/a
```

maxima [A] time = 0.31, size = 37, normalized size = 0.92

$$c^4x-\frac{2\,c^4\log(x)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] c^4*x - 2*c^4*log(x)/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.88

$$-\frac{c^4\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln(x)-1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x + 1))/(a*x - 1),x)
```

```
[Out] -(c^4*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

sympy [A] time = 0.18, size = 39, normalized size = 0.98

$$\frac{a^4c^4x-2a^3c^4\log{(x)}+\frac{-3ac^4x+c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**4,x)
```

```
[Out] (a**4*c**4*x - 2*a**3*c**4*log(x) + (-3*a*c**4*x + c**4)/(3*x**3))/a**4
```

3.389 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=39

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}-\frac{c^3\log(x)}{a}+c^3x$$

```
[Out] -1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x-c^3*ln(x)/a
```

Rubi [A] time = 0.13, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6167, 6131, 6129, 75}

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}-\frac{c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] -c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x - (c^3*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$= \frac{c^3\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^3}{x^3} dx}{a^3}$$
$$= \frac{c^3\int \frac{(1-ax)^2(1+ax)}{x^3} dx}{a^3}$$
$$= \frac{c^3\int \left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right) dx}{a^3}$$
$$= -\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(x)}{a}$$

Mathematica [A] time = 0.13, size = 41, normalized size = 1.05

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}-\frac{c^3\log(ax)}{a}+c^3x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
[Out] -1/2*c^3/(a^3*x^2) + c^3/(a^2*x) + c^3*x - (c^3*Log[a*x])/a
```

fricas [A] time = 0.73, size = 45, normalized size = 1.15

$$\frac{2\,a^3c^3x^3-2\,a^2c^3x^2\log(x)+2\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="fricas")
[Out] 1/2*(2*a^3*c^3*x^3 - 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x - c^3)/(a^3*x^2)
```

giac [A] time = 0.12, size = 38, normalized size = 0.97

$$c^3x-\frac{c^3\log\left(|x|\right)}{a}+\frac{2\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="giac")
[Out] c^3*x - c^3*log(abs(x))/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

maple [A] time = 0.04, size = 38, normalized size = 0.97

$$-\frac{c^3}{2x^2a^3}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^3,x)
[Out] -1/2*c^3/x^2/a^3+c^3/a^2/x+c^3*x-c^3*ln(x)/a
```

maxima [A] time = 0.30, size = 37, normalized size = 0.95

$$c^3x-\frac{c^3\log(x)}{a}+\frac{2\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] c^3*x - c^3*log(x)/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

mupad [B] time = 1.18, size = 35, normalized size = 0.90

$$\frac{c^3\,\left(2\,a\,x+2\,a^3\,x^3-2\,a^2\,x^2\,\ln(x)-1\right)}{2\,a^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^3*(2*a*x + 2*a^3*x^3 - 2*a^2*x^2*log(x) - 1))/(2*a^3*x^2)
```

sympy [A] time = 0.16, size = 37, normalized size = 0.95

$$\frac{a^3c^3x - a^2c^3\log(x) + \frac{2ac^3x-c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**3,x)
```

```
[Out] (a**3*c**3*x - a**2*c**3*log(x) + (2*a*c**3*x - c**3)/(2*x**2))/a**3
```

3.390 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=16

$$\frac{c^2}{a^2x}+c^2x$$

```
[Out] c^2/a^2/x+c^2*x
```

Rubi [A] time = 0.12, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6167, 6131, 6129, 73, 14}

$$\frac{c^2}{a^2x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] c^2/(a^2*x) + c^2*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$= -\frac{c^2\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$
$$= -\frac{c^2\int \frac{(1-ax)(1+ax)}{x^2}dx}{a^2}$$
$$= -\frac{c^2\int \frac{1-a^2x^2}{x^2}dx}{a^2}$$
$$= -\frac{c^2\int \left(-a^2+\frac{1}{x^2}\right)dx}{a^2}$$
$$= \frac{c^2}{a^2x}+c^2x$$

Mathematica [A] time = 0.10, size = 16, normalized size = 1.00

$$\frac{c^2}{a^2x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
[Out] c^2/(a^2*x) + c^2*x
```

fricas [A] time = 0.49, size = 21, normalized size = 1.31

$$\frac{a^2c^2x^2+c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="fricas")
[Out] (a^2*c^2*x^2 + c^2)/(a^2*x)
```

giac [A] time = 0.13, size = 16, normalized size = 1.00

$$c^2x+\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="giac")
[Out] c^2*x + c^2/(a^2*x)
```

maple [A] time = 0.04, size = 17, normalized size = 1.06

$$\frac{c^2\left(a^2x+\frac{1}{x}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^2,x)
[Out] c^2/a^2*(a^2*x+1/x)
```

maxima [A] time = 0.30, size = 16, normalized size = 1.00

$$c^2x + \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="maxima")
[Out] c^2*x + c^2/(a^2*x)
```

mupad [B] time = 0.03, size = 19, normalized size = 1.19

$$\frac{c^2 \left(a^2 x^2 + 1\right)}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x + 1))/(a*x - 1),x)
[Out] (c^2*(a^2*x^2 + 1))/(a^2*x)
```

sympy [A] time = 0.09, size = 15, normalized size = 0.94

$$\frac{a^2c^2x + \frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**2,x)
[Out] (a**2*c**2*x + c**2/x)/a**2
```

3.391 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=11

$$\frac{c\log(x)}{a}+cx$$

[Out] c*x+c*ln(x)/a

Rubi [A] time = 0.07, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6131, 6129, 43}

$$\frac{c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]

[Out] c*x + (c*Log[x])/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$$
$$= \frac{c\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$= \frac{c\int \frac{1+ax}{x}\,dx}{a}$$
$$= \frac{c\int \left(a+\frac{1}{x}\right)dx}{a}$$
$$= cx+\frac{c\log(x)}{a}$$

Mathematica [A] time = 0.04, size = 11, normalized size = 1.00

$$\frac{c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
[Out] c*x + (c*Log[x])/a
```

fricas [A] time = 0.66, size = 13, normalized size = 1.18

$$\frac{acx+c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="fricas")
[Out] (a*c*x + c*log(x))/a
```

giac [A] time = 0.12, size = 12, normalized size = 1.09

$$cx+\frac{c\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="giac")
[Out] c*x + c*log(abs(x))/a
```

maple [A] time = 0.03, size = 12, normalized size = 1.09

$$cx+\frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x),x)
[Out] c*x+c*ln(x)/a
```

maxima [A] time = 0.30, size = 11, normalized size = 1.00

$$cx+\frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] c*x + c*log(x)/a
```

mupad [B] time = 1.17, size = 11, normalized size = 1.00

$$\frac{c\,\left(\ln(x)+a\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c*(log(x) + a*x))/a
```

sympy [A] time = 0.08, size = 10, normalized size = 0.91

$$\frac{acx+c\log{(x)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x)
```

```
[Out] (a*c*x + c*log(x))/a
```

3.392 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}+\frac{x}{c}$$

[Out] x/c+2/a/c/(-a*x+1)+3*ln(-a*x+1)/a/c

Rubi [A] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]

[Out] x/c + 2/(a*c*(1 - a*x)) + (3*Log[1 - a*x])/(a*c)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$= \frac{a\int \frac{e^{2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= \frac{a\int \frac{x(1+ax)}{(1-ax)^2}\,dx}{c}$$
$$= \frac{a\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{c}$$
$$= \frac{x}{c}+\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.81

$$\frac{ax+\frac{2}{1-ax}+3\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
[Out] (a*x + 2/(1 - a*x) + 3*Log[1 - a*x])/(a*c)
```

fricas [A] time = 0.56, size = 40, normalized size = 1.08

$$\frac{a^2x^2-ax+3\,(ax-1)\log\,(ax-1)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

giac [A] time = 0.14, size = 36, normalized size = 0.97

$$\frac{x}{c}+\frac{3\,\log\,(|ax-1|)}{ac}-\frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="giac")
[Out] x/c + 3*log(abs(a*x - 1))/(a*c) - 2/((a*x - 1)*a*c)
```

maple [A] time = 0.04, size = 36, normalized size = 0.97

$$\frac{x}{c}+\frac{3\ln\,(ax-1)}{ca}-\frac{2}{ca\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x),x)
[Out] x/c+3/c/a*ln(a*x-1)-2/c/a/(a*x-1)
```

maxima [A] time = 0.30, size = 35, normalized size = 0.95

$$\frac{x}{c} - \frac{2}{a^2cx - ac} + \frac{3 \log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="maxima")
[Out] x/c - 2/(a^2*c*x - a*c) + 3*log(a*x - 1)/(a*c)
```

mupad [B] time = 0.06, size = 34, normalized size = 0.92

$$\frac{x}{c} + \frac{2}{a\,(c - a\,c\,x)} + \frac{3\,\ln(a\,x-1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))*(a*x - 1)),x)
[Out] x/c + 2/(a*(c - a*c*x)) + (3*log(a*x - 1))/(a*c)
```

sympy [A] time = 0.15, size = 26, normalized size = 0.70

$$-\frac{2}{a^2cx - ac} + \frac{x}{c} + \frac{3\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x)
[Out] -2/(a**2*c*x - a*c) + x/c + 3*log(a*x - 1)/(a*c)
```

3.393 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=53

$$\frac{5}{ac^2(1-ax)}-\frac{1}{ac^2(1-ax)^2}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2-1/a/c^2/(-a*x+1)^2+5/a/c^2/(-a*x+1)+4*ln(-a*x+1)/a/c^2
```

Rubi [A] time = 0.15, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{5}{ac^2(1-ax)}-\frac{1}{ac^2(1-ax)^2}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 - 1/(a*c^2*(1 - a*x)^2) + 5/(a*c^2*(1 - a*x)) + (4*Log[1 - a*x])/(a*c
^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= -\frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c^2}$$
$$= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c^2}$$
$$= \frac{x}{c^2}-\frac{1}{ac^2(1-ax)^2}+\frac{5}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

Mathematica [A] time = 0.11, size = 51, normalized size = 0.96

$$-\frac{5}{ac^2(ax-1)}-\frac{1}{ac^2(ax-1)^2}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 - 1/(a*c^2*(-1 + a*x)^2) - 5/(a*c^2*(-1 + a*x)) + (4*Log[1 - a*x])/(a
*c^2)
```

fricas [A] time = 0.56, size = 70, normalized size = 1.32

$$\frac{a^3x^3-2\,a^2x^2-4\,ax+4\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+4}{a^3c^2x^2-2\,a^2c^2x+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.12, size = 42, normalized size = 0.79

$$\frac{x}{c^2}+\frac{4\,\log\left(|ax-1|\right)}{ac^2}-\frac{5\,ax-4}{(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] x/c^2 + 4*log(abs(a*x - 1))/(a*c^2) - (5*a*x - 4)/((a*x - 1)^2*a*c^2)
```

maple [A] time = 0.04, size = 51, normalized size = 0.96

$$\frac{x}{c^2}+\frac{4\ln\left(ax-1\right)}{a\,c^2}-\frac{5}{a\,c^2\left(ax-1\right)}-\frac{1}{a\,c^2\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^2,x)
```

```
[Out] x/c^2+4/a/c^2*ln(a*x-1)-5/a/c^2/(a*x-1)-1/a/c^2/(a*x-1)^2
```

maxima [A] time = 0.30, size = 55, normalized size = 1.04

$$-\frac{5\,ax-4}{a^3c^2x^2-2\,a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\,\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 + 4*log(a*x - 1)/(
a*c^2)
```

mupad [B] time = 1.21, size = 54, normalized size = 1.02

$$\frac{x}{c^2}-\frac{5\,x-\frac{4}{a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^2*(a*x - 1)),x)
```

```
[Out] x/c^2 - (5*x - 4/a)/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (4*log(a*x - 1))/(a*c
^2)
```

sympy [A] time = 0.23, size = 49, normalized size = 0.92

$$\frac{-5ax+4}{a^3c^2x^2-2a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**2,x)
```

```
[Out] (-5*a*x + 4)/(a**3*c**2*x**2 - 2*a**2*c**2*x + a*c**2) + x/c**2 + 4*log(a*x
 - 1)/(a*c**2)
```

3.394 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=73

$$\frac{9}{ac^3(1-ax)}-\frac{7}{2ac^3(1-ax)^2}+\frac{2}{3ac^3(1-ax)^3}+\frac{5\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3+2/3/a/c^3/(-a*x+1)^3-7/2/a/c^3/(-a*x+1)^2+9/a/c^3/(-a*x+1)+5*ln(-a*x+
1)/a/c^3
```

Rubi [A] time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{9}{ac^3(1-ax)}-\frac{7}{2ac^3(1-ax)^2}+\frac{2}{3ac^3(1-ax)^3}+\frac{5\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 + 2/(3*a*c^3*(1 - a*x)^3) - 7/(2*a*c^3*(1 - a*x)^2) + 9/(a*c^3*(1 - a
*x)) + (5*Log[1 - a*x])/(a*c^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= \frac{a^3\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= \frac{a^3\int \frac{x^3(1+ax)}{(1-ax)^4}\,dx}{c^3}$$
$$= \frac{a^3\int \left(\frac{1}{a^3}+\frac{2}{a^3(-1+ax)^4}+\frac{7}{a^3(-1+ax)^3}+\frac{9}{a^3(-1+ax)^2}+\frac{5}{a^3(-1+ax)}\right)dx}{c^3}$$
$$= \frac{x}{c^3}+\frac{2}{3ac^3(1-ax)^3}-\frac{7}{2ac^3(1-ax)^2}+\frac{9}{ac^3(1-ax)}+\frac{5\log(1-ax)}{ac^3}$$

Mathematica [A] time = 0.13, size = 63, normalized size = 0.86

$$\frac{6a^4x^4-18a^3x^3-36a^2x^2+81ax+30(ax-1)^3\log(1-ax)-37}{6ac^3(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (-37 + 81*a*x - 36*a^2*x^2 - 18*a^3*x^3 + 6*a^4*x^4 + 30*(-1 + a*x)^3*Log[1
 - a*x])/(6*a*c^3*(-1 + a*x)^3)
```

fricas [A] time = 0.49, size = 100, normalized size = 1.37

$$\frac{6\,a^4x^4-18\,a^3x^3-36\,a^2x^2+81\,ax+30\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-37}{6\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^4*x^4 - 18*a^3*x^3 - 36*a^2*x^2 + 81*a*x + 30*(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)*log(a*x - 1) - 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
 - a*c^3)
```

giac [A] time = 0.14, size = 50, normalized size = 0.68

$$\frac{x}{c^3}+\frac{5\,\log\left(|ax-1|\right)}{ac^3}-\frac{54\,a^2x^2-87\,ax+37}{6\left(ax-1\right)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 + 5*log(abs(a*x - 1))/(a*c^3) - 1/6*(54*a^2*x^2 - 87*a*x + 37)/((a*x
- 1)^3*a*c^3)
```

maple [A] time = 0.04, size = 66, normalized size = 0.90

$$\frac{x}{c^3}+\frac{5\ln\left(ax-1\right)}{a\,c^3}-\frac{9}{a\,c^3\left(ax-1\right)}-\frac{2}{3a\,c^3\left(ax-1\right)^3}-\frac{7}{2a\,c^3\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^3,x)
```

```
[Out] x/c^3+5/a/c^3*ln(a*x-1)-9/a/c^3/(a*x-1)-2/3/a/c^3/(a*x-1)^3-7/2/a/c^3/(a*x-
1)^2
```

maxima [A] time = 0.31, size = 75, normalized size = 1.03

$$-\frac{54\,a^2x^2 - 87\,ax + 37}{6\left(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3\right)} + \frac{x}{c^3} + \frac{5\,\log\left(ax - 1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(54*a^2*x^2 - 87*a*x + 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
- a*c^3) + x/c^3 + 5*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.08, size = 71, normalized size = 0.97

$$\frac{9\,a\,x^2 - \frac{29\,x}{2} + \frac{37}{6\,a}}{-a^3\,c^3\,x^3 + 3\,a^2\,c^3\,x^2 - 3\,a\,c^3\,x + c^3} + \frac{x}{c^3} + \frac{5\,\ln\left(a\,x - 1\right)}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^3*(a*x - 1)),x)
```

```
[Out] (9*a*x^2 - (29*x)/2 + 37/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c^
3*x) + x/c^3 + (5*log(a*x - 1))/(a*c^3)
```

sympy [A] time = 0.33, size = 73, normalized size = 1.00

$$\frac{-54a^2x^2 + 87ax - 37}{6a^4c^3x^3 - 18a^3c^3x^2 + 18a^2c^3x - 6ac^3} + \frac{x}{c^3} + \frac{5\log\left(ax - 1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**3,x)
```

```
[Out] (-54*a**2*x**2 + 87*a*x - 37)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a*
*2*c**3*x - 6*a*c**3) + x/c**3 + 5*log(a*x - 1)/(a*c**3)
```

3.395 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=87

$$\frac{14}{ac^4(1-ax)}-\frac{8}{ac^4(1-ax)^2}+\frac{3}{ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}+\frac{6\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

```
[Out] x/c^4-1/2/a/c^4/(-a*x+1)^4+3/a/c^4/(-a*x+1)^3-8/a/c^4/(-a*x+1)^2+14/a/c^4/(
-a*x+1)+6*ln(-a*x+1)/a/c^4
```

Rubi [A] time = 0.17, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6167, 6131, 6129, 77}

$$\frac{14}{ac^4(1-ax)}-\frac{8}{ac^4(1-ax)^2}+\frac{3}{ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}+\frac{6\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 - 1/(2*a*c^4*(1 - a*x)^4) + 3/(a*c^4*(1 - a*x)^3) - 8/(a*c^4*(1 - a*x
)^2) + 14/(a*c^4*(1 - a*x)) + (6*Log[1 - a*x])/(a*c^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx \\
&= -\frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4} \\
&= -\frac{a^4\int \frac{x^4(1+ax)}{(1-ax)^5}\,dx}{c^4} \\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{2}{a^4(-1+ax)^5}-\frac{9}{a^4(-1+ax)^4}-\frac{16}{a^4(-1+ax)^3}-\frac{14}{a^4(-1+ax)^2}-\frac{6}{a^4(-1+ax)}\right)dx}{c^4} \\
&= \frac{x}{c^4}-\frac{1}{2ac^4(1-ax)^4}+\frac{3}{ac^4(1-ax)^3}-\frac{8}{ac^4(1-ax)^2}+\frac{14}{ac^4(1-ax)}+\frac{6\log(1-ax)}{ac^4}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 71, normalized size = 0.82

$$
\frac{2a^5x^5-8a^4x^4-16a^3x^3+60a^2x^2-56ax+12(ax-1)^4\log(1-ax)+17}{2ac^4(ax-1)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (17 - 56*a*x + 60*a^2*x^2 - 16*a^3*x^3 - 8*a^4*x^4 + 2*a^5*x^5 + 12*(-1 + a
*x)^4*Log[1 - a*x])/(2*a*c^4*(-1 + a*x)^4)
```

fricas [A] time = 1.50, size = 126, normalized size = 1.45

$$
\frac{2\,a^5x^5-8\,a^4x^4-16\,a^3x^3+60\,a^2x^2-56\,ax+12\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax-1\right)+17}{2\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 60*a^2*x^2 - 56*a*x + 12*(a^4*x^4
 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 17)/(a^5*c^4*x^4 - 4*a
^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.14, size = 58, normalized size = 0.67

$$
\frac{x}{c^4}+\frac{6\,\log\left(|ax-1|\right)}{ac^4}-\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\left(ax-1\right)^4ac^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 + 6*log(abs(a*x - 1))/(a*c^4) - 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x
 - 17)/((a*x - 1)^4*a*c^4)
```

maple [A] time = 0.04, size = 81, normalized size = 0.93

$$
\frac{x}{c^4}+\frac{6\ln\left(ax-1\right)}{a\,c^4}-\frac{14}{a\,c^4\left(ax-1\right)}-\frac{1}{2a\,c^4\left(ax-1\right)^4}-\frac{3}{a\,c^4\left(ax-1\right)^3}-\frac{8}{a\,c^4\left(ax-1\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^4,x)
```

```
[Out] x/c^4+6/a/c^4*ln(a*x-1)-14/a/c^4/(a*x-1)-1/2/a/c^4/(a*x-1)^4-3/a/c^4/(a*x-1
)^3-8/a/c^4/(a*x-1)^2
```

maxima [A] time = 0.30, size = 93, normalized size = 1.07

$$-\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}+\frac{6\,\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x - 17)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4) + x/c^4 + 6*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.09, size = 90, normalized size = 1.03

$$\frac{x}{c^4}-\frac{29\,x-34\,a\,x^2-\frac{17}{2\,a}+14\,a^2\,x^3}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}+\frac{6\,\ln\left(a\,x-1\right)}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^4*(a*x - 1)),x)
```

```
[Out] x/c^4 - (29*x - 34*a*x^2 - 17/(2*a) + 14*a^2*x^3)/(c^4 + 6*a^2*c^4*x^2 - 4*
a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*c^4*x) + (6*log(a*x - 1))/(a*c^4)
```

sympy [A] time = 0.42, size = 94, normalized size = 1.08

$$\frac{-28a^3x^3+68a^2x^2-58ax+17}{2a^5c^4x^4-8a^4c^4x^3+12a^3c^4x^2-8a^2c^4x+2ac^4}+\frac{x}{c^4}+\frac{6\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**4,x)
```

```
[Out] (-28*a**3*x**3 + 68*a**2*x**2 - 58*a*x + 17)/(2*a**5*c**4*x**4 - 8*a**4*c**
4*x**3 + 12*a**3*c**4*x**2 - 8*a**2*c**4*x + 2*a*c**4) + x/c**4 + 6*log(a*x
 - 1)/(a*c**4)
```

3.396 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=103

$$\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)}{3a}-\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{3c^4\csc^{-1}(ax)}{2a}$$

```
[Out] 1/3*c^4*(1-1/a^2/x^2)^(3/2)*(3*a+1/x)*x/a+3/2*c^4*arccsc(a*x)/a-c^4*arctanh
((1-1/a^2/x^2)^(1/2))/a+1/2*c^4*(2*a+3/x)*(1-1/a^2/x^2)^(1/2)/a^2
```

Rubi [A] time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 813, 815, 844, 216, 266, 63, 208}

$$\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)}{3a}-\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{3c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*Sqrt[1 - 1/(a^2*x^2)]*(2*a + 3/x))/(2*a^2) + (c^4*(1 - 1/(a^2*x^2))^(3
/2)*(3*a + x^(-1))*x)/(3*a) + (3*c^4*ArcCsc[a*x])/(2*a) - (c^4*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
```

```
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{1}{2}c^3\,\text{Subst}\left(\int \frac{\left(\frac{2c}{a}+\frac{6cx}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} - \frac{1}{4}\left(a^2c^3\right)\text{Subst}\left(\int \frac{-\frac{4c}{a^3}-}{x\sqrt{1-}}\right.$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{\left(3c^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx,\right.}{2a^2}$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{3c^4\csc^{-1}(ax)}{2a} + \frac{c^4\,\text{Subst}\left(\right.}{}$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{3c^4\csc^{-1}(ax)}{2a} - \left(ac^4\right)\text{Subs}$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{3c^4\csc^{-1}(ax)}{2a} - \frac{c^4\tanh^{-1}}{}$$

Mathematica [**A**] time = 0.23, size = 175, normalized size = 1.70

$$-\frac{c^4\left(-24a^5x^5-32a^4x^4+12a^3x^3+40a^2x^2+42a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-15a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)+\right.}{24a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -1/24*(c^4*(-8 + 12*a*x + 40*a^2*x^2 + 12*a^3*x^3 - 32*a^4*x^4 - 24*a^5*x^5
 + 42*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 15*
a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] + 24*a^4*Sqrt[1 - 1/(a^2*x^2)
]*x^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

fricas [**A**] time = 0.75, size = 156, normalized size = 1.51

$$-\frac{18\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^4x^4+14\,a^3c^4x^3+\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^4*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^4*x^4 + 14*a^3*c^4*x^3 + 11*a^2*c^4*x^2 + a*c^4*x - 2*c^4
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

giac [B] time = 0.18, size = 212, normalized size = 2.06

$$-\frac{1}{3}\,ac^4\left(\frac{9\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{6\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}-\frac{\frac{20\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{3\,(ax-1}{(a}}{a^2\left(\frac{ax-1}{ax+1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -1/3*a*c^4*(9*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*log(sqrt((a*x - 1)/
(a*x + 1)) + 1)/a^2 - 3*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 6*sqr
t((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) - (20*(a*x - 1)*sqrt
((a*x - 1)/(a*x + 1))/(a*x + 1) + 3*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(
a*x + 1)^2 + 9*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^3)
)
```

maple [B] time = 0.05, size = 233, normalized size = 2.26

$$-\frac{(ax-1)^2\,c^4\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-9a^3x^3\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{a^2}}\right.\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x)
```

```
[Out] -1/6*(a*x-1)^2*c^4*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+6*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2)*x^2*a^2-9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-9*a^3*x^3
*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2))*x^3*a^4+3*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-2*(a^2*x^
2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(
1/2)/a^4/x^3/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 223, normalized size = 2.17

$$-\frac{1}{3}\left(\frac{9\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+29\,c^4\left(\frac{ax}{ax}\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-}{(ax}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(9*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^2 - 3*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3*c
^4*((a*x - 1)/(a*x + 1))^(7/2) + c^4*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^4*(
(a*x - 1)/(a*x + 1))^(3/2) + 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)
*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^
4 + a^2))*a
```

mupad [B] time = 0.13, size = 183, normalized size = 1.78

$$\frac{5\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+\frac{c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{3}+c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{a+\frac{2\,a\,(a\,x-1)}{a\,x+1}-\frac{2\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}-\frac{3\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^4*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^4*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\left(-\frac{4a}{\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^3}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**4,x)
```

```
[Out] c**4*(Integral(-4*a/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2/(a*x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**3/(a*x**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x) + Integral(a**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1)), x))/a**4
```

3.397 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=61

$$c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{3c^3\csc^{-1}(ax)}{2a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x+3/2*c^3*arccsc(a*x)/a+3/2*c^3*(1-1/a^2/x^2)^(1/2)
/a^2/x
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6177, 277, 195, 216}

$$c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{3c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (3*c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*(1 - 1/(a^2*x^2))^(3/2)*x + (
3*c^3*ArcCsc[a*x])/(2*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$
$$= c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{(3c^3)\,\text{Subst}\left(\int \sqrt{1-\frac{x^2}{a^2}}dx, x, \frac{1}{x}\right)}{a^2}$$
$$= \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{(3c^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2a^2}$$
$$= \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{3c^3\csc^{-1}(ax)}{2a}$$

Mathematica [A] time = 0.08, size = 51, normalized size = 0.84

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+1\right)+3ax\sin^{-1}\left(\frac{1}{ax}\right)\right)}{2a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a^2*x^2) + 3*a*x*ArcSin[1/(a*x)]))/(2*a^
2*x)
```

fricas [A] time = 0.49, size = 85, normalized size = 1.39

$$-\frac{6\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(2\,a^3c^3x^3+2\,a^2c^3x^2+ac^3x+c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^3*c^3*x^3 + 2*
a^2*c^3*x^2 + a*c^3*x + c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x^2)
```

giac [B] time = 0.16, size = 165, normalized size = 2.70

$$-\frac{1}{4}ac^3\left(\frac{3\left(\pi+2\arctan\left(\frac{\frac{ax-1}{ax+1}-1}{2\sqrt{\frac{ax-1}{ax+1}}}\right)\right)}{a^2}+\frac{4\left(3\left(\sqrt{\frac{ax-1}{ax+1}}-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^2+8\right)}{\left(\left(\sqrt{\frac{ax-1}{ax+1}}-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)^3+4\sqrt{\frac{ax-1}{ax+1}}-\frac{4}{\sqrt{\frac{ax-1}{ax+1}}}\right)a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*a*c^3*(3*(pi + 2*arctan(1/2*((a*x - 1)/(a*x + 1) - 1)/sqrt((a*x - 1)/(
a*x + 1))))/a^2 + 4*(3*(sqrt((a*x - 1)/(a*x + 1)) - 1/sqrt((a*x - 1)/(a*x +
 1)))^2 + 8)/(((sqrt((a*x - 1)/(a*x + 1)) - 1/sqrt((a*x - 1)/(a*x + 1)))^3
+ 4*sqrt((a*x - 1)/(a*x + 1)) - 4/sqrt((a*x - 1)/(a*x + 1)))*a^2))
```

maple [A] time = 0.05, size = 105, normalized size = 1.72

$$-\frac{(ax-1)^2 c^3 \left(-3a^2x^2\sqrt{a^2x^2-1}-3a^2x^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\left(a^2x^2-1\right)^{\frac{3}{2}}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x)
```

```
[Out] -1/2*(a*x-1)^2*c^3*(-3*a^2*x^2*(a^2*x^2-1)^(1/2)-3*a^2*x^2*arctan(1/(a^2*x^
2-1)^(1/2))+(a^2*x^2-1)^(3/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*
x+1))^(1/2)/a^3/x^2
```

maxima [B] time = 0.41, size = 151, normalized size = 2.48

$$-\left(\frac{3c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(3*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - (3*c^3*((a*x - 1)/(a*x + 1)
)^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((a*x - 1)/(a*x + 1
)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^
2/(a*x + 1)^3 + a^2))*a
```

mupad [B] time = 1.23, size = 119, normalized size = 1.95

$$\frac{c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+c^3\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,x}+\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/a + c^3*x*((a*x - 1)/(a*x + 1))^(1/2) + (c^3*((a*x - 1)/(a*x + 1))^(1/
2))/(2*a^2*x) + (c^3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a^3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{3a}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^2}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\Bigg(-\cdots}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**3,x)
```

```
[Out] c**3*(Integral(3*a/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2/(a*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1)), x) + Integral(a**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integr
al(-1/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**3*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**3
```

3.398 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

```
[Out] c^2*arccsc(a*x)/a+c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+c^2*(a-1/x)*x*(1-1/a^2
/x^2)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 850, 813, 844, 216, 266, 63, 208}

$$\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a + (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 850

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + (c*x)/e)*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(c^3\,\text{Subst}\left(\int \frac{\left(\frac{1}{c}+\frac{x}{ac}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{1}{2}c^3\,\text{Subst}\left(\int \frac{-\frac{2}{ac}+\frac{2x}{a^2c}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + \left(ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + \frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [B] time = 0.16, size = 154, normalized size = 2.44

$$-\frac{c^2\left(-a^3x^3+a^2x^2+4a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\tanh^{-1}\left(\sqrt{1\right.\right.}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -((c^2*(-1 + a*x + a^2*x^2 - a^3*x^3 + 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcS
in[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/(a*x
)] - a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^3*Sq
rt[1 - 1/(a^2*x^2)]*x^2))
```

fricas [A] time = 0.53, size = 114, normalized size = 1.81

$$-\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2c^2x^2-c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c^2*x*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) + a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x^2
 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

giac [B] time = 0.17, size = 125, normalized size = 1.98

$$-ac^2\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{4\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)a^2\left(\frac{(ax-1)^2}{(ax+1)^2}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -a*c^2*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^2 + log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 4*(a*x - 1)*s
qrt((a*x - 1)/(a*x + 1))/((a*x + 1)*a^2*((a*x - 1)^2/(a*x + 1)^2 - 1)))
```

maple [B] time = 0.05, size = 174, normalized size = 2.76

$$\frac{(ax-1)^2c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\right.\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x)
```

```
[Out] (a*x-1)^2*c^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^2*x^2-1)^(3/2)*(a^
2)^(1/2)+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+a*x*(a^2)^(1/2)*arctan(1/(a^2*x^
2-1)^(1/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/((
a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 125, normalized size = 1.98

$$-\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c^2*log(sqrt((a*x - 1)/(a*x +
1)) + 1)/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

mupad [B] time = 1.21, size = 90, normalized size = 1.43

$$\frac{4\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}-\frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{2a}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(-2*a/(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2/(a*x*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a*x + 1)), x) + Integral(1/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**2
```

3.399 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=49

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

```
[Out] -c*arccsc(a*x)/a+2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6177, 852, 1807, 844, 216, 266, 63, 208}

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a + (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\left(c^3\,\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{\text{Subst}\left(\int\frac{\left(c+\frac{cx}{a}\right)^2}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{\text{Subst}\left(\int\frac{-\frac{2c^2}{a}-\frac{c^2x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}-\frac{(2c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}-\frac{c\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{a}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}+(2ac)\,\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}+\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 73, normalized size = 1.49

$$
\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}+2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Arc
Sin[1/(a*x)] + 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

fricas [A] time = 0.60, size = 88, normalized size = 1.80

$$\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 2*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

giac [B] time = 0.16, size = 108, normalized size = 2.20

$$2\,ac\left(\frac{\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2*a*c*(arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 - log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 - sqrt((a*x - 1)/(
a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.05, size = 145, normalized size = 2.96

$$-\frac{(ax-1)^2\,c\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-2a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)-2\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,\sqrt{(ax-1)\,(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x)
```

```
[Out] -(a*x-1)^2*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^
2)^(1/2)-2*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-2*
((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-
1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 114, normalized size = 2.33

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2
 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 0.09, size = 82, normalized size = 1.67

$$\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (4*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{a}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x),x)
```

```
[Out] c*(Integral(a/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**2*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1)), x))/a
```

3.400 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=105

$$-\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -8/3*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(3/2)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c-4
/3*(3*a+4/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.30, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] (-8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) - (4*(3*a + 4/x))/(3*a^
2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c + (4*ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^4-\frac{12c^4x}{a}-\frac{13c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^4+\frac{12c^4x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{4\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A] time = 0.14, size = 70, normalized size = 0.67

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(3a^2x^2-26ax+19\right)}{(ax-1)^2}+12\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{3ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 26*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + 12*Lo
g[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(3*a*c)
```

fricas [A] time = 0.58, size = 128, normalized size = 1.22

$$\frac{12\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-23\,a^2x^2-7\,ax+19\right)}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] 1/3*(12*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*(a^2*
x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 23*a^2*x
^2 - 7*a*x + 19)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

giac [A] time = 0.18, size = 148, normalized size = 1.41

$$\frac{2}{3}\,a\left(\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{6\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{(ax+1)\left(\frac{9\,(ax-1)}{ax+1}+1\right)}{(ax-1)a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2/3*a*(6*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 6*log(abs(sqrt((a*x -
 1)/(a*x + 1)) - 1))/(a^2*c) - (a*x + 1)*(9*(a*x - 1)/(a*x + 1) + 1)/((a*x
- 1)*a^2*c*sqrt((a*x - 1)/(a*x + 1))) - 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*
((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.06, size = 346, normalized size = 3.30

$$\frac{12\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^3a^4+12\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^3a^3-36\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3-9\sqrt{a^2}\,((a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x)
```

```
[Out] 1/3*(12*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4
+12*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-36*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-9*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(3/2)*x*a-36*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+36*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+7*((a*x-1)*(a*x+1))^(3/
2)*(a^2)^(1/2)+36*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-12*a*ln((a^2*x+((
a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-12*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2))/a/(a*x-1)/(a^2)^(1/2)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-
1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.32, size = 133, normalized size = 1.27

$$\frac{2}{3}\,a\left(\frac{\frac{8\,(ax-1)}{ax+1}-\frac{12\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] 2/3*a*((8*(a*x - 1)/(a*x + 1) - 12*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a*
x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 6*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 6*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^
2*c))
```

mupad [B] time = 1.24, size = 100, normalized size = 0.95

$$\frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}-\frac{\frac{16\,(a\,x-1)}{3\,(a\,x+1)}-\frac{8\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{2}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((16*(a*x - 1))/(3*(a*x + 1)
) - (8*(a*x - 1)^2)/(a*x + 1)^2 + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) - a
*c*((a*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int\frac{x}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x)/c
```

3.401 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=138

$$-\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -16/5*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(5/2)-4/15*(5*a+11/x)/a^2/c^2/(1-1/a^2/
x^2)^(3/2)+5*arctanh((1-1/a^2/x^2)^(1/2))/a/c^2+1/15*(-75*a-103/x)/a^2/c^2/
(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A] time = 0.40, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-16*(a + x^(-1)))/(5*a^2*c^2*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 11/x))/(
15*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (75*a + 103/x)/(15*a^2*c^2*Sqrt[1 - 1
/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x
^2)]])/(a*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^5}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^5-\frac{25c^5x}{a}-\frac{39c^5x^2}{a^2}+\frac{5c^5x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{5c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^5+\frac{75c^5x}{a}+\frac{88c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{15c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^5-\frac{75c^5x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{15c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{5\,\text{Subst}\left(\int -\right.}{a}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{5\,\text{Subst}\left(\int -\right.}{2}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{(5a)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{5\tanh^{-1}\left(\sqrt{\right.}{ac^2}$$

Mathematica [A] time = 0.08, size = 104, normalized size = 0.75

$$\frac{15a^4x^4-173a^3x^3+91a^2x^2+75ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+161ax-118}{15a^2c^2x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-118 + 161*a*x + 91*a^2*x^2 - 173*a^3*x^3 + 15*a^4*x^4 + 75*a*Sqrt[1 - 1/(
a^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^2*Sqrt[1
 - 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

fricas [A] time = 0.47, size = 170, normalized size = 1.23

$$\frac{75\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-75\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(15\,a^{4}x^{4}-1\right.}{15\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 173*a^3*x^3 + 91*a^2*x^2 + 161*a*x - 118)*sqrt((a*x - 1)/
(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [A] time = 0.17, size = 166, normalized size = 1.20

$$\frac{1}{15}\,a\left(\frac{75\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c^{2}}-\frac{75\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^{2}c^{2}}-\frac{(ax+1)^{2}\left(\frac{20\,(ax-1)}{ax+1}+\frac{120\,(ax-1)^{2}}{(ax+1)^{2}}+3\right)}{(ax-1)^{2}a^{2}c^{2}\sqrt{\frac{ax-1}{ax+1}}}-\frac{30\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{2}\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] 1/15*a*(75*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 75*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/(a^2*c^2) - (a*x + 1)^2*(20*(a*x - 1)/(a*x + 1) +
 120*(a*x - 1)^2/(a*x + 1)^2 + 3)/((a*x - 1)^2*a^2*c^2*sqrt((a*x - 1)/(a*x
+ 1))) - 30*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.06, size = 438, normalized size = 3.17

$$\frac{75\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{4}a^{5}+75\sqrt{a^{2}}\,\sqrt{(ax-1)(ax+1)}\,x^{4}a^{4}-300\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{3}a^{4}-60\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x)
```

```
[Out] 1/15*(75*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^
5+75*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-300*ln((a^2*x+((a*x-1)*(a*
x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-60*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x^2*a^2-300*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+450*ln((a
^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+97*(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(3/2)*x*a+450*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*
a^2-300*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-4
3*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-300*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/
2)*x*a+75*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))+75*
((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/a/(a*x-1)^2/(a^2)^(1/2)/c^2/((a*x-1)*(
a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.32, size = 153, normalized size = 1.11

$$\frac{1}{15}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}+\frac{100\,(ax-1)^{2}}{(ax+1)^{2}}-\frac{150\,(ax-1)^{3}}{(ax+1)^{3}}+3}{a^{2}c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^{2}c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{75\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c^{2}}-\frac{75\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a*((17*(a*x - 1)/(a*x + 1) + 100*(a*x - 1)^2/(a*x + 1)^2 - 150*(a*x -
1)^3/(a*x + 1)^3 + 3)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x
- 1)/(a*x + 1))^(5/2)) + 75*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) -
75*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

mupad [B] time = 0.09, size = 120, normalized size = 0.87

$$\frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}-\frac{\frac{20\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{10\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{17\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((20*(a*x - 1)^2)/(3*(a*x
 + 1)^2) - (10*(a*x - 1)^3)/(a*x + 1)^3 + (17*(a*x - 1))/(15*(a*x + 1)) + 1
/5)/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - a*c^2*((a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1)), x)/c**2
```

3.402 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=165

$$-\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}-\frac{16}{7a^2c^3x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{6\tanh^{-1}\left(\sqrt{1-\right.}{ac^3}$$

```
[Out] -32/7*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(7/2)-2/7*(7*a+13/x)/a^2/c^3/(1-1/a^2/x
^2)^(3/2)-16/7/a^2/c^3/(1-1/a^2/x^2)^(5/2)/x+6*arctanh((1-1/a^2/x^2)^(1/2))
/a/c^3+1/7*(-42*a-59/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c
^3
```

Rubi [A] time = 0.52, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}-\frac{16}{7a^2c^3x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{6\tanh^{-1}\left(\sqrt{1-\right.}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (-32*(a + x^(-1)))/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(7/2)) - (2*(7*a + 13/x))/(
7*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (42*a + 59/x)/(7*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) - 16/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)*x) + (Sqrt[1 - 1/(a^2*x^2
)]*x)/c^3 + (6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^6}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx,x,\frac{1}{x}\right)}{c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-7c^6-\frac{42c^6x}{a}-\frac{80c^6x^2}{a^2}+\frac{42c^6x^3}{a^3}+\frac{7c^6x^4}{a^4}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{7c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} - \frac{\text{Subst}\left(\int \frac{35c^6+\frac{210c^6x}{a}+\frac{355c^6x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{35c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\text{Subst}\left(\int \frac{-105c^6-\frac{630c^6x}{a}-\cdots}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/\cdots}}\cdots\right.}{105c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} - \frac{\text{Sub}\cdots}{\cdots}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1\cdots}}{\cdots}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1\cdots}}{\cdots}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1\cdots}}{\cdots}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\sqrt{1\cdots}}{\cdots}$$

Mathematica [A] time = 0.09, size = 112, normalized size = 0.68

$$\frac{7a^5x^5-109a^4x^4+145a^3x^3+39a^2x^2+42ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-156ax+66}{7a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (66 - 156*a*x + 39*a^2*x^2 + 145*a^3*x^3 - 109*a^4*x^4 + 7*a^5*x^5 + 42*a*S
qrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(7*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

fricas [A] time = 0.60, size = 204, normalized size = 1.24

$$\frac{42\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{7\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*(42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/(a
*x + 1)) + 1) - 42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((
a*x - 1)/(a*x + 1)) - 1) + (7*a^5*x^5 - 109*a^4*x^4 + 145*a^3*x^3 + 39*a^2*
x^2 - 156*a*x + 66)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3
 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.18, size = 182, normalized size = 1.10

$$\frac{1}{14}a\left(\frac{84\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{84\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^3}-\frac{(ax+1)^3\left(\frac{7(ax-1)}{ax+1}+\frac{28(ax-1)^2}{(ax+1)^2}+\frac{140(ax-1)^3}{(ax+1)^3}+1\right)}{(ax-1)^3a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{28\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] 1/14*a*(84*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 84*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/(a^2*c^3) - (a*x + 1)^3*(7*(a*x - 1)/(a*x + 1) +
28*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 + 1)/((a*x - 1)^3*
a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - 28*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*
((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.06, size = 530, normalized size = 3.21

$$-\frac{-42\sqrt{a^2}\sqrt{(ax-1)(ax+1)}\,x^5a^5-42\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\sqrt{a^2}}{\sqrt{a^2}}\right)x^5a^6+35\sqrt{a^2}\left((ax-1)(ax+1)\right)^{\frac{3}{2}}x^3a^3+210\sqrt{a^2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x)
```

```
[Out] -1/7*(-42*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-42*ln((a^2*x+((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+35*(a^2)^(1/2)*((a*x-1)*(
a*x+1))^(3/2)*x^3*a^3+210*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+210*l
```

```
n((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-87*(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-420*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2)*x^3*a^3-420*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*x^3*a^4+78*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+420*((a*x-1)*(a*x+1))^(
1/2)*(a^2)^(1/2)*x^2*a^2+420*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))
/(a^2)^(1/2))*x^2*a^3-24*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-210*((a*x-1)*(
a*x+1))^(1/2)*(a^2)^(1/2)*x*a-210*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2))*x*a^2+42*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+42*a*ln((a^
2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a/(a*x-1)^3/(a^2)^(1
/2)/c^3/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.31, size = 169, normalized size = 1.02

$$\frac{1}{14}\,a\left(\frac{\frac{6\,(ax-1)}{ax+1}+\frac{21\,(ax-1)^2}{(ax+1)^2}+\frac{112\,(ax-1)^3}{(ax+1)^3}-\frac{168\,(ax-1)^4}{(ax+1)^4}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{84\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{84\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/14*a*((6*(a*x - 1)/(a*x + 1) + 21*(a*x - 1)^2/(a*x + 1)^2 + 112*(a*x - 1)
^3/(a*x + 1)^3 - 168*(a*x - 1)^4/(a*x + 1)^4 + 1)/(a^2*c^3*((a*x - 1)/(a*x
+ 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 84*log(sqrt((a*x - 1)/
(a*x + 1)) + 1)/(a^2*c^3) - 84*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3)
)
```

mupad [B] time = 0.11, size = 137, normalized size = 0.83

$$\frac{12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{3\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{16\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{24\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{6\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{7}}{2\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-2\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (12*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((3*(a*x - 1)^2)/(a*x + 1
)^2 + (16*(a*x - 1)^3)/(a*x + 1)^3 - (24*(a*x - 1)^4)/(a*x + 1)^4 + (6*(a*x
 - 1))/(7*(a*x + 1)) + 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 2*a*c^3*
((a*x - 1)/(a*x + 1))^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\int\frac{x^3}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 4*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

3.403 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=204

$$\frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\cdots}}{c\cdots}$$

```
[Out] 16/63*(9*a-5/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-64/9*(a+1/x)/a^2/c^4/(1-1/a^2/x
^2)^(9/2)-8/105*(21*a+41/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/315*(-735*a-1417/
x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)+7*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4+1/315*(
-2205*a-3149/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A] time = 0.64, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$\frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\cdots}}{c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (16*(9*a - 5/x))/(63*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (64*(a + x^(-1)))/(
9*a^2*c^4*(1 - 1/(a^2*x^2))^(9/2)) - (8*(21*a + 41/x))/(105*a^2*c^4*(1 - 1/
(a^2*x^2))^(5/2)) - (735*a + 1417/x)/(315*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2))
- (2205*a + 3149/x)/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*
x^2)]*x)/c^4 + (7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^7}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^7}{x^2\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^{11}}$$

$$= -\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{-9c^7-\frac{63c^7x}{a}-\frac{134c^7x^2}{a^2}+\frac{198c^7x^3}{a^3}+\frac{63c^7x^4}{a^4}+\frac{9c^7x^5}{a^5}}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{9c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{\text{Subst}\left(\int \frac{63c^7+\frac{441c^7x}{a}+\frac{921c^7x^2}{a^2}+\frac{63c^7x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{63c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-315c^7-\frac{2205c^7x}{a}-393\cdots}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\cdots\right)}{315c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \cdots$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\cdots}{31\cdots}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\cdots}{31\cdots}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\cdots}{31\cdots}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\cdots}{31\cdots}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\cdots}{31\cdots}$$

Mathematica [A] time = 0.10, size = 120, normalized size = 0.59

$$\frac{315a^6x^6 - 6224a^5x^5 + 13241a^4x^4 - 5567a^3x^3 - 10232a^2x^2 + 2205ax\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) +}{315a^2c^4x\sqrt{1-\frac{1}{a^2x^2}}(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-3464 + 11651*a*x - 10232*a^2*x^2 - 5567*a^3*x^3 + 13241*a^4*x^4 - 6224*a^
5*x^5 + 315*a^6*x^6 + 2205*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4)
```

fricas [A] time = 0.58, size = 240, normalized size = 1.18

$$\frac{2205\left(a^5x^5 - 5a^4x^4 + 10a^3x^3 - 10a^2x^2 + 5ax - 1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right) - 2205\left(a^5x^5 - 5a^4x^4 + 10a^3x^3 - 10a^2x\right.}{315\left(a^6c^4x^5 - 5a^5c^4x^4 + 1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 -
10*a^2*x^2 + 5*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^6*x^6 -
 6224*a^5*x^5 + 13241*a^4*x^4 - 5567*a^3*x^3 - 10232*a^2*x^2 + 11651*a*x -
3464)*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*
x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.18, size = 198, normalized size = 0.97

$$\frac{1}{1260}a\left(\frac{8820\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4} - \frac{8820\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^4} - \frac{(ax+1)^4\left(\frac{270(ax-1)}{ax+1} + \frac{1071(ax-1)^2}{(ax+1)^2} + \frac{3360(ax-1)^3}{(ax+1)^3} + \right.}{(ax-1)^4a^2c^4\sqrt{\frac{ax-1}{ax+1}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] 1/1260*a*(8820*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 8820*log(abs(
sqrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^4) - (a*x + 1)^4*(270*(a*x - 1)/(a*x
 + 1) + 1071*(a*x - 1)^2/(a*x + 1)^2 + 3360*(a*x - 1)^3/(a*x + 1)^3 + 15120
*(a*x - 1)^4/(a*x + 1)^4 + 35)/((a*x - 1)^4*a^2*c^4*sqrt((a*x - 1)/(a*x + 1
))) - 2520*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4*((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.07, size = 622, normalized size = 3.05

$$-\frac{-2205\sqrt{(ax-1)(ax+1)}\sqrt{a^2}\,x^6a^6 - 2205\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\sqrt{a^2}}{\sqrt{a^2}}\right)x^6a^7 + 1890\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x)
```

```
[Out] -1/315*(-2205*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^6*a^6-2205*ln((a^2*x+((
a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6*a^7+1890*((a*x-1)*(a*x+
```

```
1))^(3/2)*(a^2)^(1/2)*x^4*a^4+13230*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5
*a^5+13230*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*
a^6-6376*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3-33075*(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2)*x^4*a^4-33075*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2))*x^4*a^5+8646*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2
+44100*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+44100*ln((a^2*x+((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-5349*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(3/2)*x*a-33075*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-33075
*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+1259*(
(a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+13230*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2
)*x*a+13230*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a
^2-2205*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-2205*a*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a/(a*x-1)^4/(a^2)^(1/2)/c^4/((a*x-1)*(
a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.31, size = 185, normalized size = 0.91

$$\frac{1}{1260}a\left(\frac{\frac{235\,(ax-1)}{ax+1}+\frac{801\,(ax-1)^2}{(ax+1)^2}+\frac{2289\,(ax-1)^3}{(ax+1)^3}+\frac{11760\,(ax-1)^4}{(ax+1)^4}-\frac{17640\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{8820\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{8820\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/1260*a*((235*(a*x - 1)/(a*x + 1) + 801*(a*x - 1)^2/(a*x + 1)^2 + 2289*(a*
x - 1)^3/(a*x + 1)^3 + 11760*(a*x - 1)^4/(a*x + 1)^4 - 17640*(a*x - 1)^5/(a
*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x - 1)/
(a*x + 1))^(9/2)) + 8820*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 882
0*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

mupad [B] time = 1.25, size = 153, normalized size = 0.75

$$\frac{14\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}-\frac{\frac{89\,(a\,x-1)^2}{35\,(a\,x+1)^2}+\frac{109\,(a\,x-1)^3}{15\,(a\,x+1)^3}+\frac{112\,(a\,x-1)^4}{3\,(a\,x+1)^4}-\frac{56\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{47\,(a\,x-1)}{63\,(a\,x+1)}+\frac{1}{9}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}-4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (14*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((89*(a*x - 1)^2)/(35*(a*
x + 1)^2) + (109*(a*x - 1)^3)/(15*(a*x + 1)^3) + (112*(a*x - 1)^4)/(3*(a*x
+ 1)^4) - (56*(a*x - 1)^5)/(a*x + 1)^5 + (47*(a*x - 1))/(63*(a*x + 1)) + 1/
9)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(11
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 5*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*sqr
```

```
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

3.404 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=64

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

```
[Out] 1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/a^3/x^2+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

Rubi [A] time = 0.14, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$-\frac{c^5}{a^3x^2}-\frac{c^5}{3a^4x^3}+\frac{c^5}{4a^5x^4}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5dx=\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5dx$$
$$=-\frac{c^5\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)^5}{x^5}dx}{a^5}$$
$$=-\frac{c^5\int\frac{(1-ax)^3(1+ax)^2}{x^5}dx}{a^5}$$
$$=-\frac{c^5\int\left(-a^5+\frac{1}{x^5}-\frac{a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5}$$
$$=\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}$$

Mathematica [A] time = 0.30, size = 66, normalized size = 1.03

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}-\frac{c^5\log(ax)}{a}+c^5x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[a*x])/a
```

fricas [A] time = 0.46, size = 67, normalized size = 1.05

$$\frac{12\,a^5c^5x^5-12\,a^4c^5x^4\log(x)+24\,a^3c^5x^3-12\,a^2c^5x^2-4\,ac^5x+3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) + 24*a^3*c^5*x^3 - 12*a^2*c^5*
x^2 - 4*a*c^5*x + 3*c^5)/(a^5*x^4)
```

giac [B] time = 0.12, size = 123, normalized size = 1.92

$$\frac{c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(12\,c^5+\frac{37\,c^5}{ax-1}+\frac{52\,c^5}{(ax-1)^2}+\frac{42\,c^5}{(ax-1)^3}+\frac{12\,c^5}{(ax-1)^4}\right)(ax-1)}{12\,a\left(\frac{1}{ax-1}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c^5*log(abs(-1/(a*x - 1) - 1
))/a + 1/12*(12*c^5 + 37*c^5/(a*x - 1) + 52*c^5/(a*x - 1)^2 + 42*c^5/(a*x -
 1)^3 + 12*c^5/(a*x - 1)^4)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^4)
```

maple [A] time = 0.04, size = 61, normalized size = 0.95

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{x^2a^3}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x)
```

```
[Out] 1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/x^2/a^3+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

maxima [A] time = 0.31, size = 59, normalized size = 0.92

$$c^5x - \frac{c^5 \log(x)}{a} + \frac{24\, a^3c^5x^3 - 12\, a^2c^5x^2 - 4\, ac^5x + 3\, c^5}{12\, a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - c^5*log(x)/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x +
3*c^5)/(a^5*x^4)
```

mupad [B] time = 0.06, size = 51, normalized size = 0.80

$$-\frac{c^5 \left(4\, a\, x + 12\, a^2\, x^2 - 24\, a^3\, x^3 - 12\, a^5\, x^5 + 12\, a^4\, x^4\, \ln(x) - 3\right)}{12\, a^5\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] -(c^5*(4*a*x + 12*a^2*x^2 - 24*a^3*x^3 - 12*a^5*x^5 + 12*a^4*x^4*log(x) - 3
))/(12*a^5*x^4)
```

sympy [A] time = 0.28, size = 63, normalized size = 0.98

$$\frac{a^5c^5x - a^4c^5 \log{(x)} + \frac{24a^3c^5x^3 - 12a^2c^5x^2 - 4ac^5x + 3c^5}{12x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**5,x)
```

```
[Out] (a**5*c**5*x - a**4*c**5*log(x) + (24*a**3*c**5*x**3 - 12*a**2*c**5*x**2 -
4*a*c**5*x + 3*c**5)/(12*x**4))/a**5
```

3.405 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=30

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

```
[Out] -1/3*c^4/a^4/x^3+2*c^4/a^2/x+c^4*x
```

Rubi [A] time = 0.13, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6167, 6131, 6129, 73, 270}

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -c^4/(3*a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$$
$$=\frac{c^4\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}$$
$$=\frac{c^4\int\frac{(1-ax)^2(1+ax)^2}{x^4}\,dx}{a^4}$$
$$=\frac{c^4\int\frac{\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}$$
$$=\frac{c^4\int\left(a^4+\frac{1}{x^4}-\frac{2a^2}{x^2}\right)dx}{a^4}$$
$$=-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Mathematica [A] time = 0.20, size = 30, normalized size = 1.00

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -1/3*c^4/(a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

fricas [A] time = 0.45, size = 36, normalized size = 1.20

$$\frac{3\,a^4c^4x^4+6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 + 6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

giac [B] time = 0.12, size = 59, normalized size = 1.97

$$\frac{(ax-1)c^4}{a}-\frac{5\,c^4+\frac{9\,c^4}{ax-1}+\frac{3\,c^4}{(ax-1)^2}}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] (a*x - 1)*c^4/a - 1/3*(5*c^4 + 9*c^4/(a*x - 1) + 3*c^4/(a*x - 1)^2)/(a*(1/(
a*x - 1) + 1)^3)
```

maple [A] time = 0.04, size = 27, normalized size = 0.90

$$\frac{c^4\left(x\,a^4+\frac{2a^2}{x}-\frac{1}{3x^3}\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x)
```

```
[Out] c^4/a^4*(x*a^4+2*a^2/x-1/3/x^3)
```

maxima [A] time = 0.31, size = 31, normalized size = 1.03

$$c^4x + \frac{6\,a^2c^4x^2 - c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

mupad [B] time = 0.05, size = 27, normalized size = 0.90

$$\frac{c^4\,\left(a^4\,x^4 + 2\,a^2\,x^2 - \frac{1}{3}\right)}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^4*(2*a^2*x^2 + a^4*x^4 - 1/3))/(a^4*x^3)
```

sympy [A] time = 0.15, size = 31, normalized size = 1.03

$$\frac{a^4c^4x + \frac{6a^2c^4x^2 - c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**4,x)
```

```
[Out] (a**4*c**4*x + (6*a**2*c**4*x**2 - c**4)/(3*x**3))/a**4
```

3.406 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=38

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

```
[Out] 1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

Rubi [A] time = 0.13, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 75}

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3dx=\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3dx$$
$$=-\frac{c^3\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$
$$=-\frac{c^3\int\frac{(1-ax)(1+ax)^2}{x^3}dx}{a^3}$$
$$=-\frac{c^3\int\left(-a^3+\frac{1}{x^3}+\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3}$$
$$=\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}$$

Mathematica [A] time = 0.15, size = 40, normalized size = 1.05

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(ax)}{a}+c^3x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[a*x])/a
```

fricas [A] time = 0.47, size = 43, normalized size = 1.13

$$\frac{2\,a^3c^3x^3+2\,a^2c^3x^2\log(x)+2\,ac^3x+c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="fricas")
[Out] 1/2*(2*a^3*c^3*x^3 + 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x + c^3)/(a^3*x^2)
```

giac [B] time = 0.15, size = 98, normalized size = 2.58

$$-\frac{c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(2\,c^3+\frac{c^3}{ax-1}-\frac{2\,c^3}{(ax-1)^2}\right)(ax-1)}{2\,a\left(\frac{1}{ax-1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="giac")
[Out] -c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + c^3*log(abs(-1/(a*x - 1) -
1))/a + 1/2*(2*c^3 + c^3/(a*x - 1) - 2*c^3/(a*x - 1)^2)*(a*x - 1)/(a*(1/(a*
x - 1) + 1)^2)
```

maple [A] time = 0.04, size = 37, normalized size = 0.97

$$\frac{c^3}{2x^2a^3}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x)
```

```
[Out] 1/2*c^3/x^2/a^3+c^3/a^2/x+c^3*x+c^3*ln(x)/a
```

maxima [A] time = 0.31, size = 34, normalized size = 0.89

$$c^3x + \frac{c^3 \log(x)}{a} + \frac{2\,ac^3x + c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="maxima")
[Out] c^3*x + c^3*log(x)/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

mupad [B] time = 1.18, size = 31, normalized size = 0.82

$$\frac{c^3\,\left(a\,x + a^3\,x^3 + a^2\,x^2\,\ln(x) + \frac{1}{2}\right)}{a^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (c^3*(a*x + a^3*x^3 + a^2*x^2*log(x) + 1/2))/(a^3*x^2)
```

sympy [A] time = 0.16, size = 37, normalized size = 0.97

$$\frac{a^3c^3x + a^2c^3\log{(x)} + \frac{2ac^3x+c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**3,x)
[Out] (a**3*c**3*x + a**2*c**3*log(x) + (2*a*c**3*x + c**3)/(2*x**2))/a**3
```

3.407 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=27

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

```
[Out] -c^2/a^2/x+c^2*x+2*c^2*ln(x)/a
```

Rubi [A] time = 0.12, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6167, 6131, 6129, 43}

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$$
$$=\frac{c^2\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}$$
$$=\frac{c^2\int \frac{(1+ax)^2}{x^2}\,dx}{a^2}$$
$$=\frac{c^2\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx}{a^2}$$
$$=-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}$$

Mathematica [A] time = 0.11, size = 29, normalized size = 1.07

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(ax)}{a}+c^2x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[a*x])/a
```

fricas [A] time = 0.54, size = 32, normalized size = 1.19

$$\frac{a^2c^2x^2+2\,ac^2x\log(x)-c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="fricas")
[Out] (a^2*c^2*x^2 + 2*a*c^2*x*log(x) - c^2)/(a^2*x)
```

giac [B] time = 0.12, size = 94, normalized size = 3.48

$$-\frac{2\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{2\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{c^2+\frac{2\,c^2}{ax-1}}{a^2\left(\frac{1}{(ax-1)a}+\frac{1}{(ax-1)^2a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="giac")
[Out] -2*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 2*c^2*log(abs(-1/(a*x - 1
) - 1))/a + (c^2 + 2*c^2/(a*x - 1))/(a^2*(1/((a*x - 1)*a) + 1/((a*x - 1)^2*
a)))
```

maple [A] time = 0.04, size = 28, normalized size = 1.04

$$-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x)
```

```
[Out] -c^2/a^2/x+c^2*x+2*c^2*ln(x)/a
```

maxima [A] time = 0.31, size = 27, normalized size = 1.00

$$c^2x + \frac{2\,c^2\log(x)}{a} - \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 2*c^2*log(x)/a - c^2/(a^2*x)
```

mupad [B] time = 1.18, size = 25, normalized size = 0.93

$$\frac{c^2\,\left(a^2\,x^2+2\,a\,x\,\ln(x)-1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^2*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

sympy [A] time = 0.12, size = 26, normalized size = 0.96

$$\frac{a^2c^2x + 2ac^2\log{(x)} - \frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**2,x)
```

```
[Out] (a**2*c**2*x + 2*a*c**2*log(x) - c**2/x)/a**2
```

3.408 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=25

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

[Out] c*x-c*ln(x)/a+4*c*ln(-a*x+1)/a

Rubi [A] time = 0.08, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6131, 6129, 72}

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]

[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx\\
&= -\frac{c\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= -\frac{c\int \frac{(1+ax)^2}{x(1-ax)}\,dx}{a}\\
&= -\frac{c\int \left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)dx}{a}\\
&= cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 1.00

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

fricas [A] time = 0.66, size = 23, normalized size = 0.92

$$\frac{acx+4\,c\log\left(ax-1\right)-c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="fricas")
[Out] (a*c*x + 4*c*log(a*x - 1) - c*log(x))/a
```

giac [B] time = 0.12, size = 55, normalized size = 2.20

$$\frac{(ax-1)c}{a}-\frac{3\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="giac")
[Out] (a*x - 1)*c/a - 3*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c*log(abs(-1
/(a*x - 1) - 1))/a
```

maple [A] time = 0.04, size = 25, normalized size = 1.00

$$cx+\frac{4c\ln\left(ax-1\right)}{a}-\frac{c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x)
[Out] c*x+4*c/a*ln(a*x-1)-c*ln(x)/a
```

maxima [A] time = 0.31, size = 24, normalized size = 0.96

$$cx+\frac{4\,c\log\left(ax-1\right)}{a}-\frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="maxima")
[Out] c*x + 4*c*log(a*x - 1)/a - c*log(x)/a
```

mupad [B] time = 0.07, size = 24, normalized size = 0.96

$$c\,x-\frac{c\,\ln(x)}{a}+\frac{4\,c\,\ln\left(a\,x-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c*x - (c*log(x))/a + (4*c*log(a*x - 1))/a
```

sympy [A] time = 0.21, size = 17, normalized size = 0.68

$$cx + \frac{c\left(-\log\left(x\right) + 4\log\left(x - \frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x),x)
[Out] c*x + c*(-log(x) + 4*log(x - 1/a))/a
```

3.409 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=53

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

[Out] x/c-2/a/c/(-a*x+1)^2+8/a/c/(-a*x+1)+5*ln(-a*x+1)/a/c

Rubi [A] time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 77}

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]

[Out] x/c - 2/(a*c*(1 - a*x)^2) + 8/(a*c*(1 - a*x)) + (5*Log[1 - a*x])/(a*c)

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$= -\frac{a\int \frac{e^{4\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= -\frac{a\int \frac{x(1+ax)^2}{(1-ax)^3}\,dx}{c}$$
$$= -\frac{a\int \left(-\frac{1}{a}-\frac{4}{a(-1+ax)^3}-\frac{8}{a(-1+ax)^2}-\frac{5}{a(-1+ax)}\right)dx}{c}$$
$$= \frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.96

$$-\frac{a\left(-\frac{8}{a^2(1-ax)}+\frac{2}{a^2(1-ax)^2}-\frac{5\log(1-ax)}{a^2}-\frac{x}{a}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] -((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/a
^2))/c)
```

fricas [A] time = 0.54, size = 64, normalized size = 1.21

$$\frac{a^3x^3-2\,a^2x^2-7\,ax+5\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+6}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 7*a*x + 5*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 6)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

giac [A] time = 0.13, size = 74, normalized size = 1.40

$$\frac{ax-1}{ac}-\frac{5\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{2\left(\frac{4\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}\right)}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c) - 5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - 2*(4*a^3
*c/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

maple [A] time = 0.04, size = 51, normalized size = 0.96

$$\frac{x}{c}+\frac{5\ln\left(ax-1\right)}{ca}-\frac{8}{ca\left(ax-1\right)}-\frac{2}{ca\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x)
[Out] x/c+5/c/a*ln(a*x-1)-8/c/a/(a*x-1)-2/c/a/(a*x-1)^2
```

maxima [A] time = 0.30, size = 49, normalized size = 0.92

$$-\frac{2\,(4\,ax-3)}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{5\,\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="maxima")
[Out] -2*(4*a*x - 3)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

mupad [B] time = 0.06, size = 48, normalized size = 0.91

$$\frac{x}{c}-\frac{8\,x-\frac{6}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{5\,\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a*x))*(a*x - 1)^2),x)
[Out] x/c - (8*x - 6/a)/(c + a^2*c*x^2 - 2*a*c*x) + (5*log(a*x - 1))/(a*c)
```

sympy [A] time = 0.23, size = 41, normalized size = 0.77

$$\frac{-8ax+6}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{5\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x),x)
[Out] (-8*a*x + 6)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

3.410 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=71

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2+4/3/a/c^2/(-a*x+1)^3-6/a/c^2/(-a*x+1)^2+13/a/c^2/(-a*x+1)+6*ln(-a*x+1
)/a/c^2
```

Rubi [A] time = 0.16, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 + 4/(3*a*c^2*(1 - a*x)^3) - 6/(a*c^2*(1 - a*x)^2) + 13/(a*c^2*(1 - a*
x)) + (6*Log[1 - a*x])/(a*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= \frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= \frac{a^2\int \frac{x^2(1+ax)^2}{(1-ax)^4}\,dx}{c^2}$$
$$= \frac{a^2\int \left(\frac{1}{a^2}+\frac{4}{a^2(-1+ax)^4}+\frac{12}{a^2(-1+ax)^3}+\frac{13}{a^2(-1+ax)^2}+\frac{6}{a^2(-1+ax)}\right)dx}{c^2}$$
$$= \frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}$$

Mathematica [A] time = 0.12, size = 63, normalized size = 0.89

$$\frac{3a^4x^4-9a^3x^3-30a^2x^2+57ax+18(ax-1)^3\log(1-ax)-25}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-25 + 57*a*x - 30*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 18*(-1 + a*x)^3*Log[1
- a*x])/(3*a*c^2*(-1 + a*x)^3)
```

fricas [A] time = 0.52, size = 100, normalized size = 1.41

$$\frac{3\,a^4x^4-9\,a^3x^3-30\,a^2x^2+57\,ax+18\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-25}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 30*a^2*x^2 + 57*a*x + 18*(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)*log(a*x - 1) - 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2)
```

giac [A] time = 0.14, size = 94, normalized size = 1.32

$$\frac{ax-1}{ac^2}-\frac{6\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{39\,a^5c^4}{ax-1}+\frac{18\,a^5c^4}{(ax-1)^2}+\frac{4\,a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^2) - 6*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3*
(39*a^5*c^4/(a*x - 1) + 18*a^5*c^4/(a*x - 1)^2 + 4*a^5*c^4/(a*x - 1)^3)/(a^
6*c^6)
```

maple [A] time = 0.04, size = 66, normalized size = 0.93

$$\frac{x}{c^2}+\frac{6\ln\left(ax-1\right)}{a\,c^2}-\frac{13}{a\,c^2\left(ax-1\right)}-\frac{4}{3a\,c^2\left(ax-1\right)^3}-\frac{6}{a\,c^2\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x)
```

```
[Out] x/c^2+6/a/c^2*ln(a*x-1)-13/a/c^2/(a*x-1)-4/3/a/c^2/(a*x-1)^3-6/a/c^2/(a*x-1
)^2
```

maxima [A] time = 0.31, size = 75, normalized size = 1.06

$$-\frac{39\,a^2x^2 - 60\,ax + 25}{3\left(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2\right)} + \frac{x}{c^2} + \frac{6\,\log\left(ax - 1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(39*a^2*x^2 - 60*a*x + 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 6*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.07, size = 71, normalized size = 1.00

$$\frac{13\,a\,x^2-20\,x+\frac{25}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{6\,\ln\left(a\,x-1\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a*x))^2*(a*x - 1)^2),x)
```

```
[Out] (13*a*x^2 - 20*x + 25/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x
) + x/c^2 + (6*log(a*x - 1))/(a*c^2)
```

sympy [A] time = 0.33, size = 73, normalized size = 1.03

$$\frac{-39a^2x^2 + 60ax - 25}{3a^4c^2x^3 - 9a^3c^2x^2 + 9a^2c^2x - 3ac^2} + \frac{x}{c^2} + \frac{6\log\left(ax - 1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**2,x)
```

```
[Out] (-39*a**2*x**2 + 60*a*x - 25)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2
*c**2*x - 3*a*c**2) + x/c**2 + 6*log(a*x - 1)/(a*c**2)
```

3.411 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=89

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3-1/a/c^3/(-a*x+1)^4+16/3/a/c^3/(-a*x+1)^3-25/2/a/c^3/(-a*x+1)^2+19/a/c
^3/(-a*x+1)+7*ln(-a*x+1)/a/c^3
```

Rubi [A] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 - 1/(a*c^3*(1 - a*x)^4) + 16/(3*a*c^3*(1 - a*x)^3) - 25/(2*a*c^3*(1 -
 a*x)^2) + 19/(a*c^3*(1 - a*x)) + (7*Log[1 - a*x])/(a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= -\frac{a^3\int \frac{e^{4\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= -\frac{a^3\int \frac{x^3(1+ax)^2}{(1-ax)^5}\,dx}{c^3}$$
$$= -\frac{a^3\int \left(-\frac{1}{a^3}-\frac{4}{a^3(-1+ax)^5}-\frac{16}{a^3(-1+ax)^4}-\frac{25}{a^3(-1+ax)^3}-\frac{19}{a^3(-1+ax)^2}-\frac{7}{a^3(-1+ax)}\right)dx}{c^3}$$
$$= \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}$$

Mathematica [A] time = 0.16, size = 71, normalized size = 0.80

$$\frac{6a^5x^5-24a^4x^4-78a^3x^3+243a^2x^2-218ax+42(ax-1)^4\log(1-ax)+65}{6ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (65 - 218*a*x + 243*a^2*x^2 - 78*a^3*x^3 - 24*a^4*x^4 + 6*a^5*x^5 + 42*(-1
+ a*x)^4*Log[1 - a*x])/(6*a*c^3*(-1 + a*x)^4)
```

fricas [A] time = 0.46, size = 126, normalized size = 1.42

$$\frac{6\,a^5x^5-24\,a^4x^4-78\,a^3x^3+243\,a^2x^2-218\,ax+42\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax-1\right)+65}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^5*x^5 - 24*a^4*x^4 - 78*a^3*x^3 + 243*a^2*x^2 - 218*a*x + 42*(a^4*
x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 65)/(a^5*c^3*x^4 -
4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.12, size = 109, normalized size = 1.22

$$\frac{ax-1}{ac^3}-\frac{7\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3}-\frac{\frac{114\,a^7c^9}{ax-1}+\frac{75\,a^7c^9}{(ax-1)^2}+\frac{32\,a^7c^9}{(ax-1)^3}+\frac{6\,a^7c^9}{(ax-1)^4}}{6\,a^8c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^3) - 7*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/6*
(114*a^7*c^9/(a*x - 1) + 75*a^7*c^9/(a*x - 1)^2 + 32*a^7*c^9/(a*x - 1)^3 +
6*a^7*c^9/(a*x - 1)^4)/(a^8*c^12)
```

maple [A] time = 0.04, size = 81, normalized size = 0.91

$$\frac{x}{c^3}+\frac{7\ln\left(ax-1\right)}{a\,c^3}-\frac{19}{a\,c^3\left(ax-1\right)}-\frac{1}{a\,c^3\left(ax-1\right)^4}-\frac{16}{3a\,c^3\left(ax-1\right)^3}-\frac{25}{2a\,c^3\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x)
```

```
[Out] x/c^3+7/a/c^3*ln(a*x-1)-19/a/c^3/(a*x-1)-1/a/c^3/(a*x-1)^4-16/3/a/c^3/(a*x-
1)^3-25/2/a/c^3/(a*x-1)^2
```

maxima [A] time = 0.31, size = 93, normalized size = 1.04

$$-\frac{114\,a^3x^3-267\,a^2x^2+224\,ax-65}{6\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}+\frac{x}{c^3}+\frac{7\,\log\left(ax-1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*a*x - 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x^
3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 + 7*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 1.23, size = 90, normalized size = 1.01

$$\frac{x}{c^3}-\frac{\frac{112\,x}{3}-\frac{89\,a\,x^2}{2}-\frac{65}{6\,a}+19\,a^2\,x^3}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}+\frac{7\,\ln\left(a\,x-1\right)}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a*x))^3*(a*x - 1)^2),x)
```

```
[Out] x/c^3 - ((112*x)/3 - (89*a*x^2)/2 - 65/(6*a) + 19*a^2*x^3)/(c^3 + 6*a^2*c^3
*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (7*log(a*x - 1))/(a*c^3)
```

sympy [A] time = 0.42, size = 94, normalized size = 1.06

$$\frac{-114a^3x^3+267a^2x^2-224ax+65}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}+\frac{x}{c^3}+\frac{7\log\left(ax-1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**3,x)
```

```
[Out] (-114*a**3*x**3 + 267*a**2*x**2 - 224*a*x + 65)/(6*a**5*c**3*x**4 - 24*a**4
*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3) + x/c**3 + 7*lo
g(a*x - 1)/(a*c**3)
```

3.412 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=105

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

```
[Out] x/c^4+4/5/a/c^4/(-a*x+1)^5-5/a/c^4/(-a*x+1)^4+41/3/a/c^4/(-a*x+1)^3-22/a/c^
4/(-a*x+1)^2+26/a/c^4/(-a*x+1)+8*ln(-a*x+1)/a/c^4
```

Rubi [A] time = 0.18, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 + 4/(5*a*c^4*(1 - a*x)^5) - 5/(a*c^4*(1 - a*x)^4) + 41/(3*a*c^4*(1 -
a*x)^3) - 22/(a*c^4*(1 - a*x)^2) + 26/(a*c^4*(1 - a*x)) + (8*Log[1 - a*x])/
(a*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$$
$$= \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}$$
$$= \frac{a^4\int \frac{x^4(1+ax)^2}{(1-ax)^6}\,dx}{c^4}$$
$$= \frac{a^4\int \left(\frac{1}{a^4}+\frac{4}{a^4(-1+ax)^6}+\frac{20}{a^4(-1+ax)^5}+\frac{41}{a^4(-1+ax)^4}+\frac{44}{a^4(-1+ax)^3}+\frac{26}{a^4(-1+ax)^2}+\frac{8}{a^4(-1+ax)}\right)dx}{c^4}$$
$$= \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3}-\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1}{ac}$$

Mathematica [A] time = 0.19, size = 79, normalized size = 0.75

$$\frac{15a^6x^6-75a^5x^5-240a^4x^4+1080a^3x^3-1480a^2x^2+890ax+120(ax-1)^5\log(1-ax)-202}{15ac^4(ax-1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-202 + 890*a*x - 1480*a^2*x^2 + 1080*a^3*x^3 - 240*a^4*x^4 - 75*a^5*x^5 +
15*a^6*x^6 + 120*(-1 + a*x)^5*Log[1 - a*x])/(15*a*c^4*(-1 + a*x)^5)
```

fricas [A] time = 0.51, size = 154, normalized size = 1.47

$$\frac{15\,a^6x^6-75\,a^5x^5-240\,a^4x^4+1080\,a^3x^3-1480\,a^2x^2+890\,ax+120\left(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,a}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*x^6 - 75*a^5*x^5 - 240*a^4*x^4 + 1080*a^3*x^3 - 1480*a^2*x^2 +
 890*a*x + 120*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*
log(a*x - 1) - 202)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.14, size = 124, normalized size = 1.18

$$\frac{ax-1}{ac^4}-\frac{8\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4}-\frac{\frac{390\,a^9c^{16}}{ax-1}+\frac{330\,a^9c^{16}}{(ax-1)^2}+\frac{205\,a^9c^{16}}{(ax-1)^3}+\frac{75\,a^9c^{16}}{(ax-1)^4}+\frac{12\,a^9c^{16}}{(ax-1)^5}}{15\,a^{10}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^4) - 8*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 1/15
*(390*a^9*c^16/(a*x - 1) + 330*a^9*c^16/(a*x - 1)^2 + 205*a^9*c^16/(a*x - 1
)^3 + 75*a^9*c^16/(a*x - 1)^4 + 12*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

maple [A] time = 0.04, size = 96, normalized size = 0.91

$$\frac{x}{c^4}+\frac{8\ln(ax-1)}{a\,c^4}-\frac{26}{a\,c^4\,(ax-1)}-\frac{5}{a\,c^4\,(ax-1)^4}-\frac{4}{5a\,c^4\,(ax-1)^5}-\frac{41}{3a\,c^4\,(ax-1)^3}-\frac{22}{a\,c^4\,(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x)
```

```
[Out] x/c^4+8/a/c^4*ln(a*x-1)-26/a/c^4/(a*x-1)-5/a/c^4/(a*x-1)^4-4/5/a/c^4/(a*x-1
)^5-41/3/a/c^4/(a*x-1)^3-22/a/c^4/(a*x-1)^2
```

maxima [A] time = 0.31, size = 113, normalized size = 1.08

$$-\frac{390\,a^4x^4-1230\,a^3x^3+1555\,a^2x^2-905\,ax+202}{15\left(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4\right)}+\frac{x}{c^4}+\frac{8\,\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 1555*a^2*x^2 - 905*a*x + 202)/(a^6*c^4*
x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4
) + x/c^4 + 8*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 1.25, size = 109, normalized size = 1.04

$$\frac{x}{c^4}+\frac{\frac{311\,a\,x^2}{3}-\frac{181\,x}{3}+\frac{202}{15\,a}-82\,a^2\,x^3+26\,a^3\,x^4}{-a^5\,c^4\,x^5+5\,a^4\,c^4\,x^4-10\,a^3\,c^4\,x^3+10\,a^2\,c^4\,x^2-5\,a\,c^4\,x+c^4}+\frac{8\,\ln\left(a\,x-1\right)}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a*x))^4*(a*x - 1)^2),x)
```

```
[Out] x/c^4 + ((311*a*x^2)/3 - (181*x)/3 + 202/(15*a) - 82*a^2*x^3 + 26*a^3*x^4)/
(c^4 + 10*a^2*c^4*x^2 - 10*a^3*c^4*x^3 + 5*a^4*c^4*x^4 - a^5*c^4*x^5 - 5*a*
c^4*x) + (8*log(a*x - 1))/(a*c^4)
```

sympy [A] time = 0.53, size = 114, normalized size = 1.09

$$\frac{-390a^4x^4+1230a^3x^3-1555a^2x^2+905ax-202}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4}+\frac{x}{c^4}+\frac{8\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**4,x)
```

```
[Out] (-390*a**4*x**4 + 1230*a**3*x**3 - 1555*a**2*x**2 + 905*a*x - 202)/(15*a**6
*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**4*c**4*x**3 - 150*a**3*c**4*x**2 +
75*a**2*c**4*x - 15*a*c**4) + x/c**4 + 8*log(a*x - 1)/(a*c**4)
```

3.413 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=135

$$c^4x\sqrt{1-\frac{1}{a^2x^2}}-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{5c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{25c^4\csc^{-1}(ax)}{2a}$$

```
[Out] -25/2*c^4*arccsc(a*x)/a-5*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a-32/3*c^4*(1-1/
a^2/x^2)^(1/2)/a-1/3*c^4*(1-1/a^2/x^2)^(1/2)/a^3/x^2+5/2*c^4*(1-1/a^2/x^2)^
(1/2)/a^2/x+c^4*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 1807, 1809, 844, 216, 266, 63, 208}

$$c^4x\sqrt{1-\frac{1}{a^2x^2}}-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{5c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{25c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

```
[Out] (-32*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) - (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*
x^2) + (5*c^4*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*
x - (25*c^4*ArcCsc[a*x])/(2*a) - (5*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^5}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c^4\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{5c^5}{a}-\frac{10c^5x}{a^2}+\frac{10c^5x^2}{a^3}-\frac{5c^5x^3}{a^4}+\frac{c^5x^4}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{15c^5}{a^3}+\frac{30c^5x}{a^4}-\frac{32c^5x^2}{a^5}+\frac{15c^5x^3}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^4\,\text{Subst}\left(\int \frac{\frac{30c^5}{a^5}-\frac{75c^5x}{a^6}+\frac{64c^5x^2}{a^7}}{x\sqrt{1-\frac{x^2}{a^2}}}\right.}{6c}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^6\,\text{Subst}\left(\right.}{}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{(25c^4)\,\text{Sub}}{}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{25c^4\csc^{-1}}{2a}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{25c^4\csc^{-1}}{2a}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{25c^4\csc^{-1}}{2a}$$

Mathematica [A] time = 0.16, size = 175, normalized size = 1.30

$$\frac{c^4\left(6a^5x^5-64a^4x^4+9a^3x^3+62a^2x^2+90a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-30a^4x^4\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-30a^4\right.}{6a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(2 - 15*a*x + 62*a^2*x^2 + 9*a^3*x^3 - 64*a^4*x^4 + 6*a^5*x^5 + 90*a^4
*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 30*a^4*Sqrt[
1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 30*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

fricas [A] time = 0.58, size = 156, normalized size = 1.16

$$\frac{150\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^4x^4-58\,a^3c^4x^3\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(150*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 30*a^3*c^4*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) + 30*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^4*x^4 - 58*a^3*c^4*x^3 - 49*a^2*c^4*x^2 + 13*a*c^4*x -
2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

giac [B] time = 0.17, size = 265, normalized size = 1.96

$$\frac{25\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}\left(ax+1\right)}{a}+\frac{5\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^4\mathrm{sgn}\,(ax}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 25*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 5*c^4*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
4*sgn(a*x + 1)/a - 1/3*(15*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(
a*x + 1) + 60*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 132*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 15*(x*abs(a) - sqrt(a^2*
x^2 - 1))*c^4*abs(a)*sgn(a*x + 1) + 64*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sq
rt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

maple [B] time = 0.06, size = 290, normalized size = 2.15

$$\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^4\left(-66\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+96\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^3a^3+66\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-75\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4*(-66*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
x^4*a^4+96*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+66*(a^2*x^2-1)^(3/2)
*(a^2)^(1/2)*x^2*a^2-75*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-75*a^3*x^3*(a
^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+66*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^
(1/2))/(a^2)^(1/2))*x^3*a^4-96*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*x^3*a^4-15*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+2*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^4/x^3/(a^2)^(1/2)
```

maxima [A] time = 0.41, size = 223, normalized size = 1.65

$$\frac{1}{3}\left(\frac{75\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{87\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+61\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(75*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 15*c^4*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 + 15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (8
7*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 61*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 55
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 45*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
```

```
x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
 + 1)^4 + a^2))*a
```

mupad [B] time = 0.12, size = 185, normalized size = 1.37

$$\frac{25\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{15\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{55\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{61\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{10\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (25*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (15*c^4*((a*x - 1)/(a*x + 1)
)^(1/2) + (55*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3 - (61*c^4*((a*x - 1)/(a*x
+ 1))^(5/2))/3 - 29*c^4*((a*x - 1)/(a*x + 1))^(7/2))/(a + (2*a*(a*x - 1))/(
a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1)^4)/(a*x + 1)^4) - (
10*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right.\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-4*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x**3, x) + Integral(6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x*
*2, x) + Integral(-4*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x, x))/a**4
```

3.414 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=106

$$c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{4c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{13c^3\csc^{-1}(ax)}{2a}$$

```
[Out] -13/2*c^3*arccsc(a*x)/a-4*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a-4*c^3*(1-1/a^2
/x^2)^(1/2)/a+1/2*c^3*(1-1/a^2/x^2)^(1/2)/a^2/x+c^3*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 1807, 1809, 844, 216, 266, 63, 208}

$$c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{4c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{13c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

```
[Out] (-4*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) +
c^3*Sqrt[1 - 1/(a^2*x^2)]*x - (13*c^3*ArcCsc[a*x])/(2*a) - (4*c^3*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^4}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{4c^4}{a}-\frac{6c^4x}{a^2}+\frac{4c^4x^2}{a^3}-\frac{c^4x^3}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{8c^4}{a^3}+\frac{13c^4x}{a^4}-\frac{8c^4x^2}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^4\,\text{Subst}\left(\int \frac{\frac{8c^4}{a^5}-\frac{13c^4x}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{(13c^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{13c^3\csc^{-1}(ax)}{2a} + \frac{(2c^3)\,\text{Subst}(}{}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{13c^3\csc^{-1}(ax)}{2a} - (4ac^3)\,\text{Subst}($$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{13c^3\csc^{-1}(ax)}{2a} - \frac{4c^3\tanh^{-1}(\sqrt{}}{a}$$

Mathematica [A] time = 0.13, size = 167, normalized size = 1.58

$$\frac{c^3\left(2a^4x^4-8a^3x^3-a^2x^2+10a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\sin^{-1}\left(\frac{1}{ax}\right)-8a^3x^3\sqrt{1-\frac{1}{a^2x^2}}\tanh\right.}{2a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(-1 + 8*a*x - a^2*x^2 - 8*a^3*x^3 + 2*a^4*x^4 + 10*a^3*Sqrt[1 - 1/(a^2
*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*
x^3*ArcSin[1/(a*x)] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

fricas [A] time = 0.55, size = 143, normalized size = 1.35

$$\frac{26\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+8\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3c^3x^3-6\,a^2c^3x^2-7\,ac^3\right.}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(26*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 8*a^2*c^3*x^2*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 8*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) + (2*a^3*c^3*x^3 - 6*a^2*c^3*x^2 - 7*a*c^3*x + c^3)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*x^2)
```

giac [B] time = 0.18, size = 232, normalized size = 2.19

$$\frac{13\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}\left(ax+1\right)}{a}+\frac{4\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^3\operatorname{sgn}(}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 13*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 4*c^3*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
3*sgn(a*x + 1)/a - ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn(a*x + 1
) + 8*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - (x*abs(a) - sqr
t(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 8*a*c^3*sgn(a*x + 1))/(((x*abs(a)
 - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a))
```

maple [B] time = 0.06, size = 266, normalized size = 2.51

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)c^3\left(-8\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3+16\sqrt{\left(ax-1\right)\left(ax+1\right)}\,\sqrt{a^2}\,x^2a^2+8\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,xa-13\sqrt{a^2}}{2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3*(-8*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x
^3*a^3+16*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+8*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*x*a-13*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-13*a^2*x^2*(a^2)^(1
/2)*arctan(1/(a^2*x^2-1)^(1/2))+8*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/
(a^2)^(1/2))*x^2*a^3-16*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2
)^(1/2))*x^2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^3
/x^2/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 201, normalized size = 1.90

$$\left(\frac{13\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c^3\sqrt{}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] (13*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 4*c^3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^2 + 4*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (11*c^3*
((a*x - 1)/(a*x + 1))^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqr
t((a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1
)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

mupad [B] time = 1.25, size = 163, normalized size = 1.54

$$\frac{2\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,\left(a\,x-1\right)}{a\,x+1}-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{13\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{8\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 11
*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x -
1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (13*c^3*atan(((a*x - 1)/
(a*x + 1))^(1/2)))/a - (8*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{3}\left(\int a^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{3}}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{2}}\,dx+\int\left(-\frac{3a^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**3*(Integral(a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**3, x) + Integral(3*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x**2, x) + Integral(-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x
, x))/a**3
```

3.415 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=77

$$c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{3c^2\csc^{-1}(ax)}{a}$$

```
[Out] -3*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a-c^2*(1-1/a^2/x^2)
^(1/2)/a+c^2*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6177, 1807, 1809, 844, 216, 266, 63, 208}

$$c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{3c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

```
[Out] -((c^2*Sqrt[1 - 1/(a^2*x^2)])/a) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x - (3*c^2*Arc
Csc[a*x])/a - (3*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^3}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{3c^3}{a}-\frac{3c^3x}{a^2}+\frac{c^3x^2}{a^3}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{3c^3}{a^3}+\frac{3c^3x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{\left(3c^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right.}{2a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} - \left(3ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x\right.$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} - \frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A] time = 0.22, size = 55, normalized size = 0.71

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)-3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-3\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]]))/a
```

fricas [A] time = 0.54, size = 113, normalized size = 1.47

$$\frac{6\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2c^2x^2-c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c^2*x*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) + 3*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c^2*
x^2 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

giac [A] time = 0.17, size = 130, normalized size = 1.69

$$\frac{6\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^2\operatorname{sgn}(a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 6*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2
*sgn(a*x + 1)/a - 2*c^2*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1
)*abs(a))
```

maple [B] time = 0.05, size = 227, normalized size = 2.95

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,xa+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\right.}{\sqrt{(ax-1)(ax+1)}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2
+4*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*
(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-3*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1
/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-4*ln((a^2*
x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/((a*x-1)*(a*x+1)
)^(1/2)/a^2/x/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 126, normalized size = 1.64

$$-\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

mupad [B] time = 1.22, size = 90, normalized size = 1.17

$$\frac{4\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}+\frac{6\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) + (6*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**2, x) + Integral(-2*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x, x))/a**2
```

3.416 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=49

$$cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

```
[Out] -c*arccsc(a*x)/a-2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6177, 1807, 844, 216, 266, 63, 208}

$$cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a - (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^2}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{\text{Subst}\left(\int\frac{\frac{2c^2}{a}-\frac{c^2x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}+\frac{(2c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}+\frac{c\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{a}\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}-(2ac)\,\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}-\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 73, normalized size = 1.49

$$
\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}-2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-2\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Arc
Sin[1/(a*x)] - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

fricas [A] time = 0.52, size = 88, normalized size = 1.80

$$
\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - 2*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

giac [A] time = 0.15, size = 85, normalized size = 1.73

$$\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}\left(ax+1\right)}{a}+\frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c\operatorname{sgn}\left(ax\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 2*c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a
*x + 1)/a
```

maple [B] time = 0.05, size = 137, normalized size = 2.80

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)c\left(2\sqrt{\left(ax-1\right)\left(ax+1\right)}\,\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}-\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-2a\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+}}{\sqrt{a^2}}\right.\right.}{\sqrt{\left(ax-1\right)\left(ax+1\right)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-(a
^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-2*a*ln(
(a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)*(a*x+1))
^(1/2)/a/(a^2)^(1/2)
```

maxima [B] time = 0.41, size = 114, normalized size = 2.33

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2
 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 0.07, size = 82, normalized size = 1.67

$$\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (4*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c*(Integral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/x, x))/a
```

3.417 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=19

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6177, 264}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}} dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\, x}{c}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 19, normalized size = 1.00

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

fricas [A] time = 1.50, size = 27, normalized size = 1.42

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
[Out] (a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

giac [A] time = 0.14, size = 24, normalized size = 1.26

$$\frac{\sqrt{a^2x^2-1}\,\mathrm{sgn}\,(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
[Out] sqrt(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

maple [A] time = 0.04, size = 28, normalized size = 1.47

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x)
[Out] 1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c
```

maxima [B] time = 0.31, size = 44, normalized size = 2.32

$$-\frac{2\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
[Out] -2*a*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c)
```

mupad [B] time = 0.05, size = 39, normalized size = 2.05

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x)),x)
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

3.418 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] arctanh((1-1/a^2/x^2)^(1/2))/a/c^2+2*x*(1-1/a^2/x^2)^(1/2)/c^2-a*x*(1-1/a^2
/x^2)^(1/2)/c^2/(a-1/x)
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6177, 857, 807, 266, 63, 208}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - (a*Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(a - x^(
-1))) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
```

```
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)}+\frac{a^2\,\text{Subst}\left(\int \frac{-\frac{2c}{a^2}-\frac{cx}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)}+\frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.95

$$\frac{a^2x^2+ax\sqrt{1-\frac{1}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-ax-2}{a^2c^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] (-2 - a*x + a^2*x^2 + a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^2
)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.80, size = 97, normalized size = 1.33

$$\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^2
*x - a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 256, normalized size = 3.51

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2-4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x}{2a\sqrt{(ax-1)(ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2
*a^2-4*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-((
a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-6*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a
+2*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))+3*((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c^2/(a*x-1)^2/(a^2)^(1
/2)
```

maxima [A] time = 0.32, size = 120, normalized size = 1.64

$$-a\left(\frac{\frac{3(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*
c^2*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^
2) + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

mupad [B] time = 1.23, size = 62, normalized size = 0.85

$$\frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^2,x)
```

```
[Out] (2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 4)/(2*a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2 \sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}{a^2x^2 - 2ax + 1}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1
), x)/c**2
```

3.419 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=105

$$-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] -2/3*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c
^3+1/3*(-6*a-7/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^3
```

Rubi [A] time = 0.29, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] (-2*(a + x^(-1)))/(3*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (6*a + 7/x)/(3*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (2*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
```

```
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^2-\frac{6c^2x}{a}-\frac{4c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^2+\frac{6c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.90

$$\frac{3a^3x^3-11a^2x^2+6ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-4ax+10}{3a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] (10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*
x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

fricas [A] time = 0.56, size = 134, normalized size = 1.28

$$\frac{6\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^{3}x^{3}-11\,a^{2}x^{2}-4\,ax+10\right)\sqrt{}}{3\left(a^{3}c^{3}x^{2}-2\,a^{2}c^{3}x+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 11*a^2*x^2
 - 4*a*x + 10)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^
3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [B] time = 0.06, size = 344, normalized size = 3.28

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)\left(24\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{3}a^{4}+27\sqrt{a^{2}}\,\sqrt{(ax-1)\left(ax+1\right)}\,x^{3}a^{3}-72\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}}{\sqrt{a^{2}}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x)
```

```
[Out] 1/12*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(24*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2
)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+27*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*
x^3*a^3-72*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*
a^3-15*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-81*((a*x-1)*(a*x+1))^(1/2)*(
a^2)^(1/2)*x^2*a^2+72*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*x*a^2+13*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+81*((a*x-1)*(a*x+1))^(1
/2)*(a^2)^(1/2)*x*a-24*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^
2)^(1/2))-27*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/c
^3/(a^2)^(1/2)/(a*x-1)^3
```

maxima [A] time = 0.31, size = 137, normalized size = 1.30

$$\frac{1}{6}\,a\left(\frac{\frac{14\,(ax-1)}{ax+1}-\frac{27\,(ax-1)^{2}}{(ax+1)^{2}}+1}{a^{2}c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^{2}c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c^{3}}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*a*((14*(a*x - 1)/(a*x + 1) - 27*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log(
```

```
sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 12*log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^3))
```

mupad [B] time = 1.25, size = 105, normalized size = 1.00

$$\frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{14\,(a\,x-1)}{3\,(a\,x+1)}-\frac{9\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^3,x)
```

```
[Out] (4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((14*(a*x - 1))/(3*(a*x +
1)) - (9*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(3/
2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\int\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-3a^2x^2+3ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x*
*2 + 3*a*x - 1), x)/c**3
```

3.420 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=138

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] -4/5*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/5*(-5*a-7/x)/a^2/c^4/(1-1/a^2/x^
2)^(3/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4+1/5*(-15*a-19/x)/a^2/c^4/(1-1
/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A] time = 0.39, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (5*a + 7/x)/(5*a^2*
c^4*(1 - 1/(a^2*x^2))^(3/2)) - (15*a + 19/x)/(5*a^2*c^4*Sqrt[1 - 1/(a^2*x^2
)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^4 + (3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a
*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^3}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^3-\frac{15c^3x}{a}-\frac{16c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^3+\frac{45c^3x}{a}+\frac{42c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^3-\frac{45c^3x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \right.}{15c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{(3a)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{3\tanh^{-1}\left(\sqrt{\right.}}{ac^4}$$

Mathematica [A] time = 0.09, size = 104, normalized size = 0.75

$$\frac{5a^4x^4-34a^3x^3+18a^2x^2+15ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+33ax-24}{5a^2c^4x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (-24 + 33*a*x + 18*a^2*x^2 - 34*a^3*x^3 + 5*a^4*x^4 + 15*a*Sqrt[1 - 1/(a^2*
x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(5*a^2*c^4*Sqrt[1 - 1/
(a^2*x^2)]*x*(-1 + a*x)^2)
```

fricas [A] time = 0.60, size = 170, normalized size = 1.23

$$\frac{15\left(a^3x^3-3a^2x^2+3ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^3x^3-3a^2x^2+3ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(5a^4x^4-34a^3x\right.}{5\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
 + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.19, size = 59, normalized size = 0.43

$$-\frac{3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c^4|a|}+\frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^4*abs(a)) + sqrt
(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^4)
```

maple [B] time = 0.06, size = 436, normalized size = 3.16

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-125\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^4a^4-120\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^4a^5+85\sqrt{a^2}\,((ax-1)(ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x)
```

```
[Out] -1/40*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(-125*(a^2)^(1/2)*((a*x-1)*(a*x+1))
^(1/2)*x^4*a^4-120*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/
2))*x^4*a^5+85*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+500*(a^2)^(1/2)*
((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+480*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)
^(1/2))/(a^2)^(1/2))*x^3*a^4-148*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-75
0*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-720*ln((a^2*x+((a*x-1)*(a*x+1
))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+67*((a*x-1)*(a*x+1))^(3/2)*(a^2)
^(1/2)+500*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+480*ln((a^2*x+((a*x-1)*(
a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-125*((a*x-1)*(a*x+1))^(1/2)*(
a^2)^(1/2)-120*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)
))/((a*x-1)*(a*x+1))^(1/2)/c^4/(a^2)^(1/2)/(a*x-1)^4
```

maxima [A] time = 0.31, size = 153, normalized size = 1.11

$$\frac{1}{20}a\left(\frac{\frac{9(ax-1)}{ax+1}+\frac{75(ax-1)^2}{(ax+1)^2}-\frac{125(ax-1)^3}{(ax+1)^3}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1)
^3/(a*x + 1)^3 + 1)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*((a*x -
1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 60
*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

mupad [B] time = 1.24, size = 121, normalized size = 0.88

$$\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}-\frac{\frac{15\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{25\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{9\,\left(a\,x-1\right)}{5\,\left(a\,x+1\right)}+\frac{1}{5}}{4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^4,x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((15*(a*x - 1)^2)/(a*x + 1
)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(4*
a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{4}\int\frac{x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{4}x^{4}-4a^{3}x^{3}+6a^{2}x^{2}-4ax+1}\,dx}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x*
*3 + 6*a**2*x**2 - 4*a*x + 1), x)/c**4
```

3.421 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=65

$$\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(ax+1)}{a}+c^4x$$

```
[Out] 1/3*c^4/a^4/x^3-3*c^4/a^3/x^2+16*c^4/a^2/x+c^4*x+26*c^4*ln(x)/a-32*c^4*ln(a
*x+1)/a
```

Rubi [A] time = 0.15, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$-\frac{3c^4}{a^3x^2}+\frac{c^4}{3a^4x^3}+\frac{16c^4}{a^2x}+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(ax+1)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(3*a^4*x^3) - (3*c^4)/(a^3*x^2) + (16*c^4)/(a^2*x) + c^4*x + (26*c^4*Lo
g[x])/a - (32*c^4*Log[1 + a*x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx \\
&= -\frac{c^4\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4} \\
&= -\frac{c^4\int \frac{(1-ax)^5}{x^4(1+ax)}dx}{a^4} \\
&= -\frac{c^4\int \left(-a^4+\frac{1}{x^4}-\frac{6a}{x^3}+\frac{16a^2}{x^2}-\frac{26a^3}{x}+\frac{32a^4}{1+ax}\right)dx}{a^4} \\
&= \frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(1+ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 67, normalized size = 1.03

$$
\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+\frac{26c^4\log(ax)}{a}-\frac{32c^4\log(ax+1)}{a}+c^4x
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(3*a^4*x^3) - (3*c^4)/(a^3*x^2) + (16*c^4)/(a^2*x) + c^4*x + (26*c^4*Lo
g[a*x])/a - (32*c^4*Log[1 + a*x])/a
```

fricas [A] time = 0.47, size = 71, normalized size = 1.09

$$
\frac{3\,a^4c^4x^4-96\,a^3c^4x^3\log\left(ax+1\right)+78\,a^3c^4x^3\log\left(x\right)+48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 - 96*a^3*c^4*x^3*log(a*x + 1) + 78*a^3*c^4*x^3*log(x) +
48*a^2*c^4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

giac [A] time = 0.14, size = 62, normalized size = 0.95

$$
c^4x-\frac{32\,c^4\log\left(|ax+1|\right)}{a}+\frac{26\,c^4\log\left(|x|\right)}{a}+\frac{48\,a^2c^4x^2-9\,ac^4x+c^4}{3\,a^4x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^4*x - 32*c^4*log(abs(a*x + 1))/a + 26*c^4*log(abs(x))/a + 1/3*(48*a^2*c^4
*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

maple [A] time = 0.04, size = 64, normalized size = 0.98

$$
\frac{c^4}{3a^4x^3}-\frac{3c^4}{x^2a^3}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\ln(x)}{a}-\frac{32c^4\ln\left(ax+1\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/3*c^4/a^4/x^3-3*c^4/x^2/a^3+16*c^4/a^2/x+c^4*x+26*c^4*ln(x)/a-32*c^4*ln(a
*x+1)/a
```

maxima [A] time = 0.30, size = 60, normalized size = 0.92

$$c^4x - \frac{32\,c^4\log\left(ax+1\right)}{a} + \frac{26\,c^4\log(x)}{a} + \frac{48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^4*x - 32*c^4*log(a*x + 1)/a + 26*c^4*log(x)/a + 1/3*(48*a^2*c^4*x^2 - 9*a
*c^4*x + c^4)/(a^4*x^3)
```

mupad [B] time = 0.10, size = 61, normalized size = 0.94

$$c^4\,x + \frac{16\,a^2\,c^4\,x^2 - 3\,a\,c^4\,x + \frac{c^4}{3}}{a^4\,x^3} + \frac{26\,c^4\,\ln(x)}{a} - \frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] c^4*x + (c^4/3 + 16*a^2*c^4*x^2 - 3*a*c^4*x)/(a^4*x^3) + (26*c^4*log(x))/a
- (32*c^4*log(a*x + 1))/a
```

sympy [A] time = 0.42, size = 56, normalized size = 0.86

$$c^4x + \frac{2c^4\left(13\log\left(x\right) - 16\log\left(x + \frac{1}{a}\right)\right)}{a} + \frac{48a^2c^4x^2 - 9ac^4x + c^4}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] c**4*x + 2*c**4*(13*log(x) - 16*log(x + 1/a))/a + (48*a**2*c**4*x**2 - 9*a*
c**4*x + c**4)/(3*a**4*x**3)
```

3.422 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=54

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(ax+1)}{a}+c^3x$$

```
[Out] -1/2*c^3/a^3/x^2+5*c^3/a^2/x+c^3*x+11*c^3*ln(x)/a-16*c^3*ln(a*x+1)/a
```

Rubi [A] time = 0.14, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(ax+1)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -c^3/(2*a^3*x^2) + (5*c^3)/(a^2*x) + c^3*x + (11*c^3*Log[x])/a - (16*c^3*Lo
g[1 + a*x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$= \frac{c^3\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$
$$= \frac{c^3\int \frac{(1-ax)^4}{x^3(1+ax)}dx}{a^3}$$
$$= \frac{c^3\int \left(a^3+\frac{1}{x^3}-\frac{5a}{x^2}+\frac{11a^2}{x}-\frac{16a^3}{1+ax}\right)dx}{a^3}$$
$$= -\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(1+ax)}{a}$$

Mathematica [A] time = 0.15, size = 56, normalized size = 1.04

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+\frac{11c^3\log(ax)}{a}-\frac{16c^3\log(ax+1)}{a}+c^3x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/2*c^3/(a^3*x^2) + (5*c^3)/(a^2*x) + c^3*x + (11*c^3*Log[a*x])/a - (16*c^
3*Log[1 + a*x])/a
```

fricas [A] time = 0.49, size = 62, normalized size = 1.15

$$\frac{2\,a^3c^3x^3-32\,a^2c^3x^2\log\left(ax+1\right)+22\,a^2c^3x^2\log\left(x\right)+10\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 - 32*a^2*c^3*x^2*log(a*x + 1) + 22*a^2*c^3*x^2*log(x) +
10*a*c^3*x - c^3)/(a^3*x^2)
```

giac [A] time = 0.14, size = 53, normalized size = 0.98

$$c^3x-\frac{16\,c^3\log\left(|ax+1|\right)}{a}+\frac{11\,c^3\log\left(|x|\right)}{a}+\frac{10\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^3*x - 16*c^3*log(abs(a*x + 1))/a + 11*c^3*log(abs(x))/a + 1/2*(10*a*c^3*x
 - c^3)/(a^3*x^2)
```

maple [A] time = 0.04, size = 53, normalized size = 0.98

$$-\frac{c^3}{2x^2a^3}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\ln(x)}{a}-\frac{16c^3\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/2*c^3/x^2/a^3+5*c^3/a^2/x+c^3*x+11*c^3*ln(x)/a-16*c^3*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 51, normalized size = 0.94

$$c^3x - \frac{16\,c^3\log\left(ax+1\right)}{a} + \frac{11\,c^3\log(x)}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^3*x - 16*c^3*log(a*x + 1)/a + 11*c^3*log(x)/a + 1/2*(10*a*c^3*x - c^3)/(a
^3*x^2)
```

mupad [B] time = 0.08, size = 51, normalized size = 0.94

$$c^3\,x - \frac{\frac{c^3}{2} - 5\,a\,c^3\,x}{a^3\,x^2} + \frac{11\,c^3\,\ln(x)}{a} - \frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] c^3*x - (c^3/2 - 5*a*c^3*x)/(a^3*x^2) + (11*c^3*log(x))/a - (16*c^3*log(a*x
 + 1))/a
```

sympy [A] time = 0.35, size = 42, normalized size = 0.78

$$c^3x + \frac{c^3\left(11\log\left(x\right) - 16\log\left(x + \frac{1}{a}\right)\right)}{a} + \frac{10ac^3x - c^3}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] c**3*x + c**3*(11*log(x) - 16*log(x + 1/a))/a + (10*a*c**3*x - c**3)/(2*a**
3*x**2)
```

3.423 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=40

$$\frac{c^2}{a^2x}+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(ax+1)}{a}+c^2x$$

```
[Out] c^2/a^2/x+c^2*x+4*c^2*ln(x)/a-8*c^2*ln(a*x+1)/a
```

Rubi [A] time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{c^2}{a^2x}+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(ax+1)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(a^2*x) + c^2*x + (4*c^2*Log[x])/a - (8*c^2*Log[1 + a*x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$= -\frac{c^2\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}$$
$$= -\frac{c^2\int \frac{(1-ax)^3}{x^2(1+ax)}dx}{a^2}$$
$$= -\frac{c^2\int \left(-a^2+\frac{1}{x^2}-\frac{4a}{x}+\frac{8a^2}{1+ax}\right)dx}{a^2}$$
$$= \frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(1+ax)}{a}$$

Mathematica [A] time = 0.11, size = 42, normalized size = 1.05

$$\frac{c^2}{a^2x}+\frac{4c^2\log(ax)}{a}-\frac{8c^2\log(ax+1)}{a}+c^2x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
[Out] c^2/(a^2*x) + c^2*x + (4*c^2*Log[a*x])/a - (8*c^2*Log[1 + a*x])/a
```

fricas [A] time = 0.66, size = 43, normalized size = 1.08

$$\frac{a^2c^2x^2-8\,ac^2x\log(ax+1)+4\,ac^2x\log(x)+c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a^2*c^2*x^2 - 8*a*c^2*x*log(a*x + 1) + 4*a*c^2*x*log(x) + c^2)/(a^2*x)
```

giac [A] time = 0.14, size = 42, normalized size = 1.05

$$c^2x-\frac{8\,c^2\log(|ax+1|)}{a}+\frac{4\,c^2\log(|x|)}{a}+\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c^2*x - 8*c^2*log(abs(a*x + 1))/a + 4*c^2*log(abs(x))/a + c^2/(a^2*x)
```

maple [A] time = 0.04, size = 41, normalized size = 1.02

$$\frac{c^2}{a^2x}+c^2x+\frac{4c^2\ln(x)}{a}-\frac{8c^2\ln(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2/(a*x+1)*(a*x-1),x)
[Out] c^2/a^2/x+c^2*x+4*c^2*ln(x)/a-8*c^2*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 40, normalized size = 1.00

$$c^2x-\frac{8\,c^2\log(ax+1)}{a}+\frac{4\,c^2\log(x)}{a}+\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^2*x - 8*c^2*log(a*x + 1)/a + 4*c^2*log(x)/a + c^2/(a^2*x)
```

mupad [B] time = 1.22, size = 40, normalized size = 1.00

$$c^2\,x+\frac{c^2}{a^2\,x}+\frac{4\,c^2\,\ln(x)}{a}-\frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x - 1))/(a*x + 1),x)
```

```
[Out] c^2*x + c^2/(a^2*x) + (4*c^2*log(x))/a - (8*c^2*log(a*x + 1))/a
```

sympy [A] time = 0.27, size = 31, normalized size = 0.78

$$c^2x+\frac{4c^2\left(\log\left(x\right)-2\log\left(x+\frac{1}{a}\right)\right)}{a}+\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*(a*x-1)/(a*x+1),x)
```

```
[Out] c**2*x + 4*c**2*(log(x) - 2*log(x + 1/a))/a + c**2/(a**2*x)
```

3.424 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=23

$$\frac{c\log(x)}{a}-\frac{4c\log(ax+1)}{a}+cx$$

```
[Out] c*x+c*ln(x)/a-4*c*ln(a*x+1)/a
```

Rubi [A] time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6131, 6129, 72}

$$\frac{c\log(x)}{a}-\frac{4c\log(ax+1)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx\\
&= \frac{c\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}\\
&= \frac{c\int \frac{(1-ax)^2}{x(1+ax)}\,dx}{a}\\
&= \frac{c\int\left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a}\\
&= cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$\frac{c\log(x)}{a}-\frac{4c\log(ax+1)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

fricas [A] time = 0.50, size = 22, normalized size = 0.96

$$\frac{acx-4\,c\log\left(ax+1\right)+c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a*c*x - 4*c*log(a*x + 1) + c*log(x))/a
```

giac [A] time = 0.14, size = 25, normalized size = 1.09

$$cx-\frac{4\,c\log\left(|ax+1|\right)}{a}+\frac{c\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c*x - 4*c*log(abs(a*x + 1))/a + c*log(abs(x))/a
```

maple [A] time = 0.04, size = 24, normalized size = 1.04

$$cx+\frac{c\ln(x)}{a}-\frac{4c\ln\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)/(a*x+1)*(a*x-1),x)
[Out] c*x+c*ln(x)/a-4*c*ln(a*x+1)/a
```

maxima [A] time = 0.31, size = 23, normalized size = 1.00

$$cx-\frac{4\,c\log\left(ax+1\right)}{a}+\frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c*x - 4*c*log(a*x + 1)/a + c*log(x)/a
```

mupad [B] time = 0.06, size = 23, normalized size = 1.00

$$c\,x+\frac{c\,\ln(x)}{a}-\frac{4\,c\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x - 1))/(a*x + 1),x)
[Out] c*x + (c*log(x))/a - (4*c*log(a*x + 1))/a
```

sympy [A] time = 0.21, size = 17, normalized size = 0.74

$$cx + \frac{c\left(\log\left(x\right) - 4\log\left(x + \frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x)
```

```
[Out] c*x + c*(log(x) - 4*log(x + 1/a))/a
```

3.425 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=20

$$\frac{x}{c}-\frac{\log(ax+1)}{ac}$$

[Out] x/c-ln(a*x+1)/a/c

Rubi [A] time = 0.12, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 43}

$$\frac{x}{c}-\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]

[Out] x/c - Log[1 + a*x]/(a*c)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$= \frac{a\int \frac{e^{-2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= \frac{a\int \frac{x}{1+ax}\,dx}{c}$$
$$= \frac{a\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{c}$$
$$= \frac{x}{c}-\frac{\log(1+ax)}{ac}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.10

$$\frac{a\left(\frac{x}{a}-\frac{\log(ax+1)}{a^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
[Out] (a*(x/a - Log[1 + a*x]/a^2))/c
```

fricas [A] time = 0.93, size = 19, normalized size = 0.95

$$\frac{ax-\log\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="fricas")
[Out] (a*x - log(a*x + 1))/(a*c)
```

giac [A] time = 0.13, size = 21, normalized size = 1.05

$$\frac{x}{c}-\frac{\log\left(|ax+1|\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="giac")
[Out] x/c - log(abs(a*x + 1))/(a*c)
```

maple [A] time = 0.03, size = 21, normalized size = 1.05

$$\frac{x}{c}-\frac{\ln\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x),x)
[Out] x/c-ln(a*x+1)/a/c
```

maxima [A] time = 0.31, size = 20, normalized size = 1.00

$$\frac{x}{c}-\frac{\log\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] x/c - log(a*x + 1)/(a*c)
```

mupad [B] time = 1.20, size = 19, normalized size = 0.95

$$-\frac{\ln\left(a\,x+1\right)-a\,x}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))*(a*x + 1)),x)
```

```
[Out] -(log(a*x + 1) - a*x)/(a*c)
```

sympy [A] time = 0.10, size = 17, normalized size = 0.85

$$a\left(\frac{x}{ac}-\frac{\log\left(ax+1\right)}{a^{2}c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x)
```

```
[Out] a*(x/(a*c) - log(a*x + 1)/(a**2*c))
```

3.426 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}$$

```
[Out] x/c^2-arctanh(a*x)/a/c^2
```

Rubi [A] time = 0.14, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6167, 6131, 6129, 72, 207}

$$\frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] x/c^2 - ArcTanh[a*x]/(a*c^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$
$$= -\frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}$$
$$= -\frac{a^2\int \frac{x^2}{(1-ax)(1+ax)}\,dx}{c^2}$$
$$= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{1}{a^2(-1+a^2x^2)}\right)dx}{c^2}$$
$$= \frac{x}{c^2}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2}$$
$$= \frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}$$

Mathematica [B] time = 0.09, size = 39, normalized size = 2.17

$$\frac{\log(1-ax)}{2ac^2}-\frac{\log(ax+1)}{2ac^2}+\frac{x}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
[Out] x/c^2 + Log[1 - a*x]/(2*a*c^2) - Log[1 + a*x]/(2*a*c^2)
```

fricas [A] time = 0.57, size = 27, normalized size = 1.50

$$\frac{2\,ax-\log\left(ax+1\right)+\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
[Out] 1/2*(2*a*x - log(a*x + 1) + log(a*x - 1))/(a*c^2)
```

giac [A] time = 0.12, size = 36, normalized size = 2.00

$$\frac{x}{c^2}-\frac{\log\left(|ax+1|\right)}{2\,ac^2}+\frac{\log\left(|ax-1|\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
[Out] x/c^2 - 1/2*log(abs(a*x + 1))/(a*c^2) + 1/2*log(abs(a*x - 1))/(a*c^2)
```

maple [A] time = 0.04, size = 35, normalized size = 1.94

$$\frac{x}{c^2}+\frac{\ln\left(ax-1\right)}{2a\,c^2}-\frac{\ln\left(ax+1\right)}{2a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^2,x)
```

```
[Out] x/c^2+1/2/a/c^2*ln(a*x-1)-1/2*ln(a*x+1)/a/c^2
```

maxima [**A**] time = 0.31, size = 34, normalized size = 1.89

$$\frac{x}{c^2} - \frac{\log(ax+1)}{2\,ac^2} + \frac{\log(ax-1)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
[Out] x/c^2 - 1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

mupad [**B**] time = 1.24, size = 17, normalized size = 0.94

$$-\frac{\operatorname{atanh}(a\,x) - a\,x}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^2*(a*x + 1)),x)
[Out] -(atanh(a*x) - a*x)/(a*c^2)
```

sympy [**B**] time = 0.15, size = 34, normalized size = 1.89

$$a^2\left(\frac{x}{a^2c^2} + \frac{\frac{\log\left(x-\frac{1}{a}\right)}{2} - \frac{\log\left(x+\frac{1}{a}\right)}{2}}{a^3c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**2,x)
[Out] a**2*(x/(a**2*c**2) + (log(x - 1/a)/2 - log(x + 1/a)/2)/(a**3*c**2))
```

3.427 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} \, dx$

Optimal. Leaf size=57

$$\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3+1/2/a/c^3/(-a*x+1)+5/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

Rubi [A] time = 0.15, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] x/c^3 + 1/(2*a*c^3*(1 - a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/(
4*a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$$
$$= \frac{a^3\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}$$
$$= \frac{a^3\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{c^3}$$
$$= \frac{a^3\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c^3}$$
$$= \frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

Mathematica [A] time = 0.13, size = 56, normalized size = 0.98

$$-\frac{1}{2ac^3(ax-1)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] x/c^3 - 1/(2*a*c^3*(-1 + a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/
(4*a*c^3)
```

fricas [A] time = 0.75, size = 59, normalized size = 1.04

$$\frac{4\,a^2x^2-4\,ax-(ax-1)\log(ax+1)+5\,(ax-1)\log(ax-1)-2}{4\left(a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)/(a^2*c^3*x - a*c^3)
```

giac [A] time = 0.13, size = 51, normalized size = 0.89

$$\frac{x}{c^3}-\frac{\log(|ax+1|)}{4\,ac^3}+\frac{5\,\log(|ax-1|)}{4\,ac^3}-\frac{1}{2\,(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 5/4*log(abs(a*x - 1))/(a*c^3) - 1/2
/((a*x - 1)*a*c^3)
```

maple [A] time = 0.04, size = 50, normalized size = 0.88

$$\frac{x}{c^3}-\frac{1}{2a\,c^3\,(ax-1)}+\frac{5\ln(ax-1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^3,x)
```

```
[Out] x/c^3-1/2/a/c^3/(a*x-1)+5/4/a/c^3*ln(a*x-1)-1/4*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.30, size = 53, normalized size = 0.93

$$-\frac{1}{2\left(a^2c^3x - ac^3\right)} + \frac{x}{c^3} - \frac{\log\left(ax+1\right)}{4\,ac^3} + \frac{5\,\log\left(ax-1\right)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 5/4*log(a*x -
 1)/(a*c^3)
```

mupad [B] time = 0.09, size = 52, normalized size = 0.91

$$\frac{x}{c^3} + \frac{1}{2\,a\,\left(c^3 - a\,c^3\,x\right)} + \frac{5\,\ln\left(a\,x-1\right)}{4\,a\,c^3} - \frac{\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^3*(a*x + 1)),x)
```

```
[Out] x/c^3 + 1/(2*a*(c^3 - a*c^3*x)) + (5*log(a*x - 1))/(4*a*c^3) - log(a*x + 1)
/(4*a*c^3)
```

sympy [A] time = 0.30, size = 56, normalized size = 0.98

$$a^3\left(-\frac{1}{2a^5c^3x - 2a^4c^3} + \frac{x}{a^3c^3} + \frac{\frac{5\log\left(x-\frac{1}{a}\right)}{4} - \frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^4c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**3,x)
```

```
[Out] a**3*(-1/(2*a**5*c**3*x - 2*a**4*c**3) + x/(a**3*c**3) + (5*log(x - 1/a)/4
- log(x + 1/a)/4)/(a**4*c**3))
```

3.428 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4} dx$

Optimal. Leaf size=75

$$\frac{7}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(ax+1)}{8ac^4}+\frac{x}{c^4}$$

```
[Out] x/c^4-1/4/a/c^4/(-a*x+1)^2+7/4/a/c^4/(-a*x+1)+17/8*ln(-a*x+1)/a/c^4-1/8*ln(
a*x+1)/a/c^4
```

Rubi [A] time = 0.16, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6131, 6129, 88}

$$\frac{7}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(ax+1)}{8ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] x/c^4 - 1/(4*a*c^4*(1 - a*x)^2) + 7/(4*a*c^4*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^4) - Log[1 + a*x]/(8*a*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6131

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[(u*(1 + (c*x)/d)^p*E^(n*ArcTanh[a*x]))/x^p, x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx \\
&= -\frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4} \\
&= -\frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^4} \\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^4} \\
&= \frac{x}{c^4}-\frac{1}{4ac^4(1-ax)^2}+\frac{7}{4ac^4(1-ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}
\end{aligned}
$$

Mathematica [**A**] time = 0.17, size = 73, normalized size = 0.97

$$
-\frac{7}{4ac^4(ax-1)}-\frac{1}{4ac^4(ax-1)^2}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(ax+1)}{8ac^4}+\frac{x}{c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] x/c^4 - 1/(4*a*c^4*(-1 + a*x)^2) - 7/(4*a*c^4*(-1 + a*x)) + (17*Log[1 - a*x
])/(8*a*c^4) - Log[1 + a*x]/(8*a*c^4)
```

fricas [**A**] time = 0.55, size = 93, normalized size = 1.24

$$
\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+17\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+12}{8\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*
c^4)
```

giac [**A**] time = 0.14, size = 57, normalized size = 0.76

$$
\frac{x}{c^4}-\frac{\log\left(|ax+1|\right)}{8\,ac^4}+\frac{17\,\log\left(|ax-1|\right)}{8\,ac^4}-\frac{7\,ax-6}{4\,(ax-1)^2ac^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 1/8*log(abs(a*x + 1))/(a*c^4) + 17/8*log(abs(a*x - 1))/(a*c^4) - 1/
4*(7*a*x - 6)/((a*x - 1)^2*a*c^4)
```

maple [**A**] time = 0.04, size = 65, normalized size = 0.87

$$
\frac{x}{c^4}-\frac{1}{4a\,c^4\,(ax-1)^2}-\frac{7}{4a\,c^4\,(ax-1)}+\frac{17\ln\left(ax-1\right)}{8a\,c^4}-\frac{\ln\left(ax+1\right)}{8a\,c^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^4,x)
```

```
[Out] x/c^4-1/4/a/c^4/(a*x-1)^2-7/4/a/c^4/(a*x-1)+17/8/a/c^4*ln(a*x-1)-1/8*ln(a*x
+1)/a/c^4
```

maxima [A] time = 0.31, size = 69, normalized size = 0.92

$$-\frac{7\,ax-6}{4\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}-\frac{\log\left(ax+1\right)}{8\,ac^4}+\frac{17\,\log\left(ax-1\right)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x - 6)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 1/8*log(a*x
+ 1)/(a*c^4) + 17/8*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.10, size = 68, normalized size = 0.91

$$\frac{x}{c^4}-\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}+\frac{17\,\ln\left(a\,x-1\right)}{8\,a\,c^4}-\frac{\ln\left(a\,x+1\right)}{8\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^4*(a*x + 1)),x)
```

```
[Out] x/c^4 - ((7*x)/4 - 3/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) + (17*log(a*x -
 1))/(8*a*c^4) - log(a*x + 1)/(8*a*c^4)
```

sympy [A] time = 0.38, size = 73, normalized size = 0.97

$$a^4\left(\frac{-7ax+6}{4a^7c^4x^2-8a^6c^4x+4a^5c^4}+\frac{x}{a^4c^4}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**4,x)
```

```
[Out] a**4*((-7*a*x + 6)/(4*a**7*c**4*x**2 - 8*a**6*c**4*x + 4*a**5*c**4) + x/(a*
*4*c**4) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**4))
```

3.429 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$

Optimal. Leaf size=164

$$\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^4x\sqrt{1-\frac{1}{a^2x^2}}+\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}+\frac{91c^4\csc^{-}}{2a}$$

```
[Out] 91/2*c^4*arccsc(a*x)/a-7*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+64*c^4*(a-1/x)/
a^2/(1-1/a^2/x^2)^(1/2)+68/3*c^4*(1-1/a^2/x^2)^(1/2)/a+1/3*c^4*(1-1/a^2/x^2
)^(1/2)/a^3/x^2-7/2*c^4*(1-1/a^2/x^2)^(1/2)/a^2/x+c^4*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6177, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^4x\sqrt{1-\frac{1}{a^2x^2}}+\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{91c^4\csc^{-}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (68*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) + (64*c^4*(a - x^(-1)))/(a^2*Sqrt[1 -
1/(a^2*x^2)]) + (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*x^2) - (7*c^4*Sqrt[1 - 1
/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*x + (91*c^4*ArcCsc[a*x])
/(2*a) - (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^7}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^7+\frac{7c^7x}{a}+\frac{42c^7x^2}{a^2}-\frac{22c^7x^3}{a^3}+\frac{7c^7x^4}{a^4}-\frac{c^7x^5}{a^5}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{7c^7}{a}-\frac{42c^7x}{a^2}+\frac{22c^7x^2}{a^3}-\frac{7c^7x^3}{a^4}+\frac{c^7x^4}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\,\text{Subst}\left(\int \frac{\frac{21c^7}{a^3}+\frac{126c^7x}{a^4}-\frac{68c^7x^2}{a^5}+\frac{21c^7}{a^6}\cdots}{x\sqrt{1-\frac{x^2}{a^2}}}\cdots\right)}{3c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^4\,\text{Subst}\left(\int -\frac{42c^7}{a^5}\cdots\right)}{\cdots}$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x \cdots$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x \cdots$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x \cdots$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x \cdots$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x \cdots$$

Mathematica [C] time = 1.19, size = 567, normalized size = 3.46

$$\frac{c^4\left(2772\sqrt{2}\,a^3x^3(ax+1)(ax-1)^3\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+1980\sqrt{2}\,a^2x^2(ax+1)(ax-1)^4\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]

[Out] (c^4*(2772*Sqrt[2]*a^3*x^3*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2] + 1980*Sqrt[2]*a^2*x^2*(-1 + a*x)^4*(1 + a*x)*Hypergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] + 35*(-198*a^2*Sqrt[1 + 1/(a*x)]*x^2 + 1716*a^3*Sqrt[1 + 1/(a*x)]*x^3 - 7425*a^4*Sqrt[1 + 1/(a*x)]*x^4 + 26268*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 29403*a^6*Sqrt[1 + 1/(a*x)]*x^6 - 50160*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 396*a^8*Sqrt[1 + 1/(a*x)]*x^8 + 66726*a^6*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 66726*a^7*Sqrt[1 - 1/(a*x)]*x^7*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 1980*a^6*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[1/(a*x)] - 1980*a^7*Sqrt[1 - 1/(a*x)]*x^7*ArcSin[1/(a*x)] - 2772*a^7*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]] + 44*Sqrt[2]*a*x*(-1 + a*x)^5*(1 + a*x)*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/(a*x))/2] + 36*Sqrt[2]*(-1 + a*x)^6*(1 + a*x)*Hypergeometric2F1[3/2, 11/2, 13/2, (1 - 1/(a*x))/2])))/(13860*a^7*Sqrt[1 - 1/(a*x)]*x^6*(1 + a*x))

fricas [A] time = 0.58, size = 157, normalized size = 0.96

$$-\frac{546\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^4x^4+526\,a^3c\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] -1/6*(546*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 42*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 42*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (6*a^4*c^4*x^4 + 526*a^3*c^4*x^3 + 115*a^2*c^4*x^2 - 19*a*c^4*x + 2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")

[Out] undef

maple [B] time = 0.06, size = 672, normalized size = 4.10

$$-\frac{\left(-138\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^6a^6+138\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4-549\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5-273\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x)

[Out] -1/6*(-138*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^6*a^6+138*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^4-549*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5-273*arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)*x^5*a^5+138*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+96*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-96*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+255*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-684*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4-546*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+276*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+192*(a^2)^(1/2)*((a*x-1)*(a*x

```
+1))^(3/2)*x^3*a^3+192*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-192*ln((
a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+98*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-273*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-273
*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+138*ln((a^2*x+(a^2*x^2-1)^
(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+96*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2)*x^3*a^3-96*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*
x^3*a^4-17*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
))/a^4*c^4*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x^3/((a*x-1)*(a*x+1))^(1/2)/
(a*x-1)
```

maxima [A] time = 0.43, size = 246, normalized size = 1.50

$$-\frac{1}{3}\left(\frac{273\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{192\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{153\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}{2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(273*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 21*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 21*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
192*c^4*sqrt((a*x - 1)/(a*x + 1))/a^2 + (153*c^4*((a*x - 1)/(a*x + 1))^(7/2
) + 91*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 169*c^4*((a*x - 1)/(a*x + 1))^(3/2
) - 123*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)*a^2/(a*x + 1) - 2*(a*x
- 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 + a^2))*a
```

mupad [B] time = 0.14, size = 211, normalized size = 1.29

$$\frac{41\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{169\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-\frac{91\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{3}-51\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{a+\frac{2\,a\,(a\,x-1)}{a\,x+1}-\frac{2\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}+\frac{64\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{91\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-\cdots}{a\,x+\cdots}}\cdots\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (41*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (169*c^4*((a*x - 1)/(a*x + 1))^(3/2))
/3 - (91*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 51*c^4*((a*x - 1)/(a*x + 1))^
(7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (64*c^4*((a*x - 1)/(a*x + 1))^(1/2))/a - (91*c^
4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^4*atan(((a*x - 1)/(a*x + 1))^(1
/2)*1i)*14i)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{5a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{10a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{10a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{5a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+\cdots}}}{ax+1}\cdots\right.\right.}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Int
egral(5*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(
-10*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(1
0*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-5*a**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

3.430 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=135

$$\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{6c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{33c^3\csc^{-1}(ax)}{2a}$$

```
[Out] 33/2*c^3*arccsc(a*x)/a-6*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+32*c^3*(a-1/x)/
a^2/(1-1/a^2/x^2)^(1/2)+6*c^3*(1-1/a^2/x^2)^(1/2)/a-1/2*c^3*(1-1/a^2/x^2)^(
1/2)/a^2/x+c^3*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6177, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{6c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{33c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (6*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^
2*x^2)]) - (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*Sqrt[1 - 1/(a^2*x^2)
]*x + (33*c^3*ArcCsc[a*x])/(2*a) - (6*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^6}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^6+\frac{6c^6x}{a}+\frac{16c^6x^2}{a^2}-\frac{6c^6x^3}{a^3}+\frac{c^6x^4}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{6c^6}{a}-\frac{16c^6x}{a^2}+\frac{6c^6x^2}{a^3}-\frac{c^6x^3}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\,\text{Subst}\left(\int \frac{\frac{12c^6}{a^3}+\frac{33c^6x}{a^4}-\frac{12c^6x^2}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx\right.}{2c^3}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^4\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\left(33c^3\right)\text{Subst}}{}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{33c^3\csc^{-1}(ax}{2a}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{33c^3\csc^{-1}(ax}{2a}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{33c^3\csc^{-1}(ax}{2a}$$

Mathematica [C] time = 0.46, size = 663, normalized size = 4.91

$$\frac{c^3\left(630a^7x^7\sqrt{\frac{1}{ax}+1}+70\sqrt{2}\,a^6x^6\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)-32340a^6x^6\sqrt{\frac{1}{ax}+1}+44730a^6x^6\sqrt{1-\frac{1}{ax}}\sin^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*(420*a^2*Sqrt[1 + 1/(a*x)]*x^2 - 3465*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 1680
0*a^4*Sqrt[1 + 1/(a*x)]*x^4 + 17955*a^5*Sqrt[1 + 1/(a*x)]*x^5 - 32340*a^6*S
qrt[1 + 1/(a*x)]*x^6 + 630*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 44730*a^5*Sqrt[1 - 1
/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 44730*a^6*Sqrt[1 - 1/(a*x)]
*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2520*a^5*Sqrt[1 - 1/(a*x)]*x^5*Arc
Sin[1/(a*x)] - 2520*a^6*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[1/(a*x)] - 3780*a^6*Sq
rt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]] +
126*Sqrt[2]*a^2*x^2*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5/2, 7/2,
 (1 - 1/(a*x))/2] + 90*Sqrt[2]*a*x*(-1 + a*x)^4*(1 + a*x)*Hypergeometric2F1
[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] - 70*Sqrt[2]*Hypergeometric2F1[3/2, 9/2, 1
1/2, (1 - 1/(a*x))/2] + 280*Sqrt[2]*a*x*Hypergeometric2F1[3/2, 9/2, 11/2, (
1 - 1/(a*x))/2] - 350*Sqrt[2]*a^2*x^2*Hypergeometric2F1[3/2, 9/2, 11/2, (1
- 1/(a*x))/2] + 350*Sqrt[2]*a^4*x^4*Hypergeometric2F1[3/2, 9/2, 11/2, (1 -
1/(a*x))/2] - 280*Sqrt[2]*a^5*x^5*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/
(a*x))/2] + 70*Sqrt[2]*a^6*x^6*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/(a*
x))/2]))/(630*a^6*Sqrt[1 - 1/(a*x)]*x^5*(1 + a*x))
```

fricas [A] time = 0.81, size = 146, normalized size = 1.08

$$-\frac{66\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^3x^3+78\,a^2c^3x^2+\right.}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(66*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 12*a^2*c^3*x^2*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (2*a^3*c^3*x^3 + 78*a^2*c^3*x^2 + 11*a*c^3*x - c^3)*sqrt((a*x -
1)/(a*x + 1)))/(a^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 450, normalized size = 3.33

$$-\frac{\left(-12\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5+12\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^3a^3-57\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4-33a^4x^4\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/2*(-12*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+12*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2)*x^3*a^3-57*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4-33*a^4*x^4*(a^2)^(1/
2)*arctan(1/(a^2*x^2-1)^(1/2))+12*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/
(a^2)^(1/2))*x^4*a^5+32*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+23*(a^2
*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-78*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-
66*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+24*ln((a^2*x+(a^2*x^2-1)
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+10*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x
*a-33*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-33*a^2*x^2*(a^2)^(1/2)*arctan(1
/(a^2*x^2-1)^(1/2))+12*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)
```

```
)*x^2*a^3-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/a^3*c^3*((a*x-1)/(a*x+1))^(3/2)/(a
^2)^(1/2)/x^2/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.41, size = 225, normalized size = 1.67

$$-\left(\frac{33\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{32\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1}{(ax+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(33*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 6*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) + 1)/a^2 - 6*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 32*c^3*
sqrt((a*x - 1)/(a*x + 1))/a^2 + (11*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 6*c^3
*((a*x - 1)/(a*x + 1))^(3/2) - 13*c^3*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)
*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3
+ a^2))*a
```

mupad [B] time = 1.27, size = 190, normalized size = 1.41

$$\frac{13\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,\left(a\,x-1\right)}{a\,x+1}-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{32\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{33\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,12{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (13*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 1
1*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x -
 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (32*c^3*((a*x - 1)/(a*x
 + 1))^(1/2))/a - (33*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^3*atan(
((a*x - 1)/(a*x + 1))^(1/2)*1i)*12i)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^4x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Inte
gral(-4*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(
6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-4*a**
3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**3
```

3.431 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$

Optimal. Leaf size=105

$$\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{5c^2\csc^{-1}(ax)}{a}$$

```
[Out] 5*c^2*arccsc(a*x)/a-5*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+16*c^2*(a-1/x)/a^2
/(1-1/a^2/x^2)^(1/2)+c^2*(1-1/a^2/x^2)^(1/2)/a+c^2*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6177, 1805, 1807, 1809, 844, 216, 266, 63, 208}

$$\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{5c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/a + (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*
x^2)]) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x + (5*c^2*ArcCsc[a*x])/a - (5*c^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^5+\frac{5c^5x}{a}+\frac{5c^5x^2}{a^2}-\frac{c^5x^3}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{5c^5}{a}-\frac{5c^5x}{a^2}+\frac{c^5x^2}{a^3}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\,\text{Subst}\left(\int \frac{\frac{5c^5}{a^3}+\frac{5c^5x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a^2} +$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} + \frac{\left(5c^2\right)\text{Subst}\left(\int\right.}{2}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} - \left(5ac^2\right)\text{Subst}\left(\int\right.$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} - \frac{5c^2\tanh^{-1}\left(\sqrt{1\right.}}{a}$$

Mathematica [C] time = 0.42, size = 424, normalized size = 4.04

$$c^2\left(35a^6x^6\sqrt{\frac{1}{ax}+1} - 595a^5x^5\sqrt{\frac{1}{ax}+1} + 910a^5x^5\sqrt{1-\frac{1}{ax}}\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right) - 105a^5x^5\sqrt{1-\frac{1}{ax}}\sin^{-1}\left(\frac{1}{ax}\right) + 280a^4x\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-35*a^2*Sqrt[1 + 1/(a*x)]*x^2 + 315*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 280*a
^4*Sqrt[1 + 1/(a*x)]*x^4 - 595*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 35*a^6*Sqrt[1 +
1/(a*x)]*x^6 + 910*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[
2]] + 910*a^5*Sqrt[1 - 1/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 105
*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[1/(a*x)] - 105*a^5*Sqrt[1 - 1/(a*x)]*x^5*
ArcSin[1/(a*x)] - 175*a^5*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^5*ArcTa
```

```
nh[Sqrt[1 - 1/(a^2*x^2)]] + 7*Sqrt[2]*a*x*(-1 + a*x)^3*(1 + a*x)*Hypergeome
tric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2] + 5*Sqrt[2]*(-1 + a*x)^4*(1 + a*x)*
Hypergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2]))/(35*a^5*Sqrt[1 - 1/(a*x
)]*x^4*(1 + a*x))
```

fricas [A] time = 0.63, size = 120, normalized size = 1.14

$$-\frac{10\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2c^2x^2+18\,ac^2x+c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(10*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 5*a*c^2*x*log(sqrt((a*x -
1)/(a*x + 1)) + 1) - 5*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^
2*x^2 + 18*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.05, size = 600, normalized size = 5.71

$$-\frac{\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4-4\sqrt{a^2}\,\sqrt{(ax-1)\,(ax+1)}\,x^3a^3+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-7\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-5a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4-4*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1
/2)*x^3*a^3+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-7*(a^2*x^2-1)^(1/2)*(a^2)
^(1/2)*x^3*a^3-5*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+4*ln((a^2*x+((a*x-1)*(a
*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+8*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(3/2)*x*a-8*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+2*(a^2*x^2-1)^(
3/2)*(a^2)^(1/2)*x*a-11*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2-10*a^2*x^2*(a
^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2))*x^2*a^3+8*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))
/(a^2)^(1/2))*x^2*a^3-4*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+(a^2*x^2-1)
^(3/2)*(a^2)^(1/2)-5*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-5*a*x*(a^2)^(1/2)*ar
ctan(1/(a^2*x^2-1)^(1/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))*x*a^2+4*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*
a^2)/a^2*c^2*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/x/((a*x-1)*(a*x+1))^(1/2)/
(a*x-1)
```

maxima [A] time = 0.42, size = 149, normalized size = 1.42

$$-\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{10\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{16\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 10*
c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 5*c^2*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^2 - 5*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 16*c^2*sqrt(
(a*x - 1)/(a*x + 1))/a^2)*a
```

mupad [B] time = 0.10, size = 117, normalized size = 1.11

$$\frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{10\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,10{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (16*c^2*((a*x - 1)/(a*x + 1))^(1/2))/a + (4*c^2*((a*x - 1)/(a*x + 1))^(1/2)
)/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (10*c^2*atan(((a*x - 1)/(a*x + 1))^(1
/2)))/a + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*10i)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{2}\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{3}+x^{2}}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{2}+x}\,dx+\int\left(-\frac{3a^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^{3}x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Int
egral(3*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-3*
a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

3.432 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=75

$$\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c\csc^{-1}(ax)}{a}$$

```
[Out] c*arccsc(a*x)/a-4*c*arctanh((1-1/a^2/x^2)^(1/2))/a+8*c*(a-1/x)/a^2/(1-1/a^2
/x^2)^(1/2)+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6177, 1805, 1807, 844, 216, 266, 63, 208}

$$\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + c*Sqrt[1 - 1/(a^2*x^2)]*x
+ (c*ArcCsc[a*x])/a - (4*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^4+\frac{4c^4x}{a}+\frac{c^4x^2}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{4c^4}{a}-\frac{c^4x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a^2} + \frac{(4c)\,\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} - (4ac)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}dx, x, \sqrt{}\right.$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} - \frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [C] time = 0.54, size = 234, normalized size = 3.12

$$\frac{5a^2cx^2\left((ax+1)\left(\sqrt{\frac{1}{ax}+1}\,\left(a^2x^2-3ax+2\right)+6ax\sqrt{1-\frac{1}{ax}}\,\sin^{-1}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2ax\sqrt{1-\frac{1}{ax}}\,\sin^{-1}\left(\frac{1}{ax}\right)\right)-4a^2x^2\sqrt{}\right.}{5a^4x^3\sqrt{1-\frac{1}{ax}}\,(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (5*a^2*c*x^2*((1 + a*x)*(Sqrt[1 + 1/(a*x)]*(2 - 3*a*x + a^2*x^2) + 6*a*Sqrt
[1 - 1/(a*x)]*x*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*a*Sqrt[1 - 1/(a*x)]*x
*ArcSin[1/(a*x)]) - 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]]) + Sqrt[2]*c*(-1 + a*x)^3*(1 + a*x)*Hypergeometri
c2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2])/(5*a^4*Sqrt[1 - 1/(a*x)]*x^3*(1 + a*x
))
```

fricas [A] time = 0.65, size = 92, normalized size = 1.23

$$-\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(acx+9\,c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*c*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) - 4*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a*c*x + 9*c)*sqrt((a*x -
1)/(a*x + 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.05, size = 376, normalized size = 5.01

$$-\frac{\left(-4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2-a^2x^2\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -(-4*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-(a^2*x^2-1)^(1/2)*(a^2)^(1
/2)*x^2*a^2-a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+4*ln((a^2*x+((a
*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+4*((a*x-1)*(a*x+1))^
(3/2)*(a^2)^(1/2)-8*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-2*(a^2*x^2-1)^(
1/2)*(a^2)^(1/2)*x*a-2*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+8*ln((a^
2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-4*((a*x-1)*(a*x
+1))^(1/2)*(a^2)^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(
1/2))*(a^2)^(1/2)+4*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2)))/a*c*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a*
x-1)
```

maxima [A] time = 0.42, size = 135, normalized size = 1.80

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a
^2 - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 4*c*sqrt((a*x - 1)/(a*x +
 1))/a^2)
```

mupad [B] time = 0.09, size = 107, normalized size = 1.43

$$\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,8\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*atan
(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*
8i)/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int \frac{a^2x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-
2*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a
```

3.433 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=72

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}-\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -2*arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(
1-1/a^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6177, 1805, 807, 266, 63, 208}

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}-\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/
c - (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^2+\frac{2c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{2\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 61, normalized size = 0.85

$$\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax+3)-2(ax+1)\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a(acx+c)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(3 + a*x) - 2*(1 + a*x)*Log[(1 + Sqrt[1 - 1/(a^2
*x^2)])*x])/(a*(c + a*c*x))
```

fricas [A] time = 0.58, size = 69, normalized size = 0.96

$$\frac{(ax+3)\sqrt{\frac{ax-1}{ax+1}}-2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] ((a*x + 3)*sqrt((a*x - 1)/(a*x + 1)) - 2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
 + 2*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.05, size = 250, normalized size = 3.47

$$-\frac{\left(2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3-2\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}\,x^2a^2+4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x\,a^2+((ax-1)(}{a\sqrt{a^2}\,c\sqrt{(ax-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x)
```

```
[Out] -(2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-2*(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+4*ln((a^2*x+((a*x-1)*(a*x+1))^(1
/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-4*(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+2*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2
)*(a^2)^(1/2))/(a^2)^(1/2))-2*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/a*((a*x-
1)/(a*x+1))^(3/2)/(a^2)^(1/2)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.31, size = 120, normalized size = 1.67

$$-2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

mupad [B] time = 0.06, size = 87, normalized size = 1.21

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (2*((a*
x - 1)/(a*x + 1))^(1/2))/(a*c) - (4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*
c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int\left(-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx+\int\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(-x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Inte
gral(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

3.434 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{x\left(a-\frac{1}{x}\right)}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c^2-(a-1/x)*x/a/c^2/(1-1/a^2/x^2)^(1/2)+2*x
*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6177, 823, 807, 266, 63, 208}

$$-\frac{x\left(a-\frac{1}{x}\right)}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - ((a - x^(-1))*x)/(a*c^2*Sqrt[1 - 1/(a^2*x
^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
```

```
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{c-\frac{cx}{a}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a^2\,\text{Subst}\left(\int \frac{\frac{2c}{a^2}-\frac{cx}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.93

$$\frac{a^2x^2 - ax\sqrt{1-\frac{1}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right) + ax - 2}{a^2c^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] (-2 + a*x + a^2*x^2 - a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^2
)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.60, size = 67, normalized size = 0.91

$$\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}} - \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] ((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 250, normalized size = 3.38

$$-\frac{\left(2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3 - 3\sqrt{(ax-1)\,(ax+1)}\ \sqrt{a^2}\ x^2a^2 + 4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}}{\sqrt{a^2}}\right)x\,a^2 + ((ax-1)(}{2a\sqrt{a^2}\ c^2\sqrt{(ax-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x)
```

```
[Out] -1/2*(2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3
-3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+4*ln((a^2*x+((a*x-1)*(a*x+1)
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-6*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+2*a*ln((a^2*x+((a*x-1)*(a*x+1))^
(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/a*((
a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/c^2/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.30, size = 125, normalized size = 1.69

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1} - a^2c^2} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2} - \frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^2) + l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^2) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

mupad [B] time = 1.19, size = 90, normalized size = 1.22

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2 - \frac{a\,c^2\,(a\,x-1)}{a\,x+1}} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2} - \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^2,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + ((a
*x - 1)/(a*x + 1))^(1/2)/(a*c^2) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(
a*c^2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx+\int\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x*
*2 - a*x + 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3
*x**3 - a**2*x**2 - a*x + 1), x))/c**2
```

3.435 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=45

$$\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2}{a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -2/a^2/c^3/x/(1-1/a^2/x^2)^(1/2)+x/c^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6177, 271, 191}

$$\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2}{a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -2/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x) + x/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^3}\\
&= \frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{a^2c^3}\\
&= -\frac{2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.73

$$\frac{a^2x^2-2}{a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
[Out] (-2 + a^2*x^2)/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.53, size = 42, normalized size = 0.93

$$\frac{\left(a^2x^2-2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3x-ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
[Out] (a^2*x^2 - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*x - a*c^3)
```

giac [A] time = 0.13, size = 42, normalized size = 0.93

$$\frac{\left(\frac{\sqrt{a^2x^2-1}}{c^3}-\frac{1}{\sqrt{a^2x^2-1}\,c^3}\right)\operatorname{sgn}\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
[Out] (sqrt(a^2*x^2 - 1)/c^3 - 1/(sqrt(a^2*x^2 - 1)*c^3))*sgn(a*x + 1)/a
```

maple [A] time = 0.04, size = 44, normalized size = 0.98

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(a^2x^2-2\right)\left(ax+1\right)}{a\left(ax-1\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x)
[Out] 1/a*((a*x-1)/(a*x+1))^(3/2)*(a^2*x^2-2)*(a*x+1)/(a*x-1)^2/c^3
```

maxima [B] time = 0.30, size = 92, normalized size = 2.04

$$-\frac{1}{2}a\left(\frac{\frac{5(ax-1)}{ax+1}-1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
[Out] -1/2*a*((5*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

mupad [B] time = 0.07, size = 41, normalized size = 0.91

$$\frac{a^2\,x^2-2}{\left(x\,a^2\,c^3+a\,c^3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^3,x)
```

```
[Out] (a^2*x^2 - 2)/((a*c^3 + a^2*c^3*x)*((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\left(\int\left(-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx+\int\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*(Integral(-x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*
x**3 + 2*a*x - 1), x) + Integral(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

3.436 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=111

$$-\frac{x\left(4a+\frac{3}{x}\right)}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{8x\sqrt{1-\frac{1}{a^2x^2}}}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] arctanh((1-1/a^2/x^2)^(1/2))/a/c^4-1/3*a*x/c^4/(a-1/x)/(1-1/a^2/x^2)^(1/2)-
1/3*(4*a+3/x)*x/a/c^4/(1-1/a^2/x^2)^(1/2)+8/3*x*(1-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A] time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 857, 823, 807, 266, 63, 208}

$$-\frac{x\left(4a+\frac{3}{x}\right)}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{8x\sqrt{1-\frac{1}{a^2x^2}}}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (8*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*c^4) - (a*x)/(3*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a
 - x^(-1))) - ((4*a + 3/x)*x)/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) + ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]/(a*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 857

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[(d*(f + g*x)^(n + 1)*(a + c*x^2)^(p + 1))/(2*a*p*(e*
f - d*g)*(d + e*x)), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} + \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{4c}{a^2}-\frac{3cx}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a^4\,\text{Subst}\left(\int \frac{-\frac{8c}{a^4}-\frac{3cx}{a^5}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\right.}{c^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.85

$$\frac{3a^3x^3-7a^2x^2+3ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-5ax+8}{3a^2c^4x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)
*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a
*x))
```

fricas [A] time = 0.51, size = 134, normalized size = 1.21

$$\frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+8\right)\sqrt{\frac{ax-}{ax-}}}{3\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^2
- 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4, x)
```

maple [B] time = 0.06, size = 523, normalized size = 4.71

$$-\frac{\left(-45\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^5a^5-24\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^5a^6+21\sqrt{a^2}\,\left((ax-1)(ax+1)\right)^{\frac{3}{2}}x^3a^3+45\sqrt{a}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x)
```

```
[Out] -1/24*(-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-24*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+21*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^3*a^3+45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+24*ln
((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+11*(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+90*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
)*x^3*a^3+48*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^
3*a^4-5*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-90*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2)*x^2*a^2-48*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)
^(1/2))*x^2*a^3-19*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-45*((a*x-1)*(a*x+1))
^(1/2)*(a^2)^(1/2)*x*a-24*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a
^2)^(1/2))*x*a^2+45*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+24*a*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(3/2)/(a
^2)^(1/2)/c^4/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)^4
```

maxima [A] time = 0.31, size = 160, normalized size = 1.44

$$\frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^4*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 12*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^4) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4))
```

mupad [B] time = 1.24, size = 128, normalized size = 1.15

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^4}-\frac{\frac{17\,(a\,x-1)}{3\,(a\,x+1)}-\frac{14\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-4\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4,x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^4) - ((17*(a*x - 1))/(3*(a*x + 1)) - (14
*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 4*a
*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/
(a*c^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*
x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x**5*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*
x**2 - 3*a*x + 1), x))/c**4
```

3.437 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5} dx$

Optimal. Leaf size=138

$$-\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^5}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

```
[Out] -2/5*(a+1/x)/a^2/c^5/(1-1/a^2/x^2)^(5/2)+1/15*(-10*a-13/x)/a^2/c^5/(1-1/a^2
/x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c^5+1/15*(-30*a-41/x)/a^2/c^5/
(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^5
```

Rubi [A] time = 0.39, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6177, 852, 1805, 807, 266, 63, 208}

$$-\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^5}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] (-2*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(5/2)) - (10*a + 13/x)/(15*a
^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) - (30*a + 41/x)/(15*a^2*c^5*Sqrt[1 - 1/(a^2
*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^5 + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
)/(a*c^5)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 852

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[((f + g*x)^n*(a + c*x^2)^(m + p)
)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^2-\frac{10c^2x}{a}-\frac{8c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^2+\frac{30c^2x}{a}+\frac{26c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^2-\frac{30c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \ldots\right.}{15c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} - \frac{2\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} - \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} + \frac{(2a)\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} + \frac{2\tanh^{-1}\left(\sqrt{\ldots}\right.}{ac^5}$$

Mathematica [**A**] time = 0.09, size = 104, normalized size = 0.75

$$\frac{15a^4x^4-76a^3x^3+32a^2x^2+30ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+82ax-56}{15a^2c^5x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] (-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*a*Sqrt[1 - 1/(a^2
*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^5*Sqrt[1 -
1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

fricas [A] time = 0.61, size = 170, normalized size = 1.23

$$\frac{30\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-30\left(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(15\,a^{4}x^{4}-7\right.}{15\left(a^{4}c^{5}x^{3}-3\,a^{3}c^{5}x^{2}+3\,a^{2}c^{5}x-ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 76*a^3*x^3 + 32*a^2*x^2 + 82*a*x - 56)*sqrt((a*x - 1)/(a*
x + 1)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [B] time = 0.07, size = 615, normalized size = 4.46

$$-\frac{\left(-75\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}\,x^{6}a^{6}-60\ln\left(\frac{a^{2}x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)x^{6}a^{7}+45\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^{2}}\,x^{4}a^{4}+15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x)
```

```
[Out] -1/30*(-75*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^6*a^6-60*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6*a^7+45*((a*x-1)*(a*x+1))^(3/
2)*(a^2)^(1/2)*x^4*a^4+150*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5+120*
ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+2*(a^2)
^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+75*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/
2)*x^4*a^4+60*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x
^4*a^5-64*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-300*(a^2)^(1/2)*((a*x
-1)*(a*x+1))^(1/2)*x^3*a^3-240*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*x^3*a^4-14*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+75*((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+60*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)
*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+37*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+1
50*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+120*ln((a^2*x+((a*x-1)*(a*x+1))^
(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-75*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2
)-60*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x
-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/c^5/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)^5
```

maxima [A] time = 0.32, size = 176, normalized size = 1.28

$$\frac{1}{120}\,a\left(\frac{\frac{32\,(ax-1)}{ax+1}+\frac{310\,(ax-1)^2}{(ax+1)^2}-\frac{585\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^5}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^5}+\frac{15\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] 1/120*a*((32*(a*x - 1)/(a*x + 1) + 310*(a*x - 1)^2/(a*x + 1)^2 - 585*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^5*((a*x
 - 1)/(a*x + 1))^(5/2)) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^5)
- 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^5) + 15*sqrt((a*x - 1)/(a*x
 + 1))/(a^2*c^5))
```

mupad [B] time = 0.09, size = 144, normalized size = 1.04

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^5}-\frac{\frac{62\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{39\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{32\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{8\,a\,c^5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-8\,a\,c^5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}+\frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^5,x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(8*a*c^5) - ((62*(a*x - 1)^2)/(3*(a*x + 1)^2) -
 (39*(a*x - 1)^3)/(a*x + 1)^3 + (32*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(8*a*c
^5*((a*x - 1)/(a*x + 1))^(5/2) - 8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2)) + (4*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**5,x)
```

```
[Out] Timed out
```

3.438 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=235

$$-\frac{7c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{173c^5\sqrt{1-\frac{1}{a^2x^2}}}{105a\sqrt{c-\frac{c}{ax}}}+\frac{227c^4\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{105a}+\frac{59c^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35a}+\frac{9c^2\sqrt{1-\right.}}{}$$

```
[Out] -7*c^(9/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+59/35*c^3
*(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/a+9/7*c^2*(c-c/a/x)^(5/2)*(1-1/a^2/x^2
)^(1/2)/a+c*(c-c/a/x)^(7/2)*x*(1-1/a^2/x^2)^(1/2)+173/105*c^5*(1-1/a^2/x^2)
^(1/2)/a/(c-c/a/x)^(1/2)+227/105*c^4*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.16, antiderivative size = 279, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6182, 6179, 97, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^4\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(400a-\frac{227}{x}\right.}{105}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] ((400*a - 227/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(105*a^2*(1 - 1/(a*
x))^(9/2)) + (59*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(35*
a^3*(1 - 1/(a*x))^(9/2)) + (9*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x)
)^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*(c
 - c/(a*x))^(9/2)*x)/(a^4*(1 - 1/(a*x))^(9/2)) - (7*(c - c/(a*x))^(9/2)*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
```

```
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(-\frac{7}{2a}-\frac{9x}{2a^2}\right)\left(1-\frac{x}{a}\right)^3}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/}\right.}{}$$

$$= \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^4\sqrt{1}}{a^4\left(\right.}$$

$$= \frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)}{7}$$

$$= \frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)}{7}$$

$$= \frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)}{7}$$

Mathematica [A] time = 0.12, size = 109, normalized size = 0.46

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(105a^4x^4+292a^3x^3-356a^2x^2+162ax-30\right)-735a^3x^3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{105a^4x^3\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-30 + 162*a*x - 356*a^2*x^2 + 29
2*a^3*x^3 + 105*a^4*x^4) - 735*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(105*a^
4*Sqrt[1 - 1/(a*x)]*x^3)
```

fricas [A] time = 0.56, size = 437, normalized size = 1.86

$$\left[\frac{735\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(105a^5c^4x^5+397a^4c^4x^4-64\right.}{420\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(105*a^5*c^4*x^5 + 397*a^4*c^4*x^4 -
64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x - 30*c^4)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/210*(735*(a^4*c^4*x
^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(105*a^5
*c^4*x^5 + 397*a^4*c^4*x^4 - 64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x
 - 30*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^
4*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.07, size = 166, normalized size = 0.71

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(210a^{\frac{9}{2}}\sqrt{(ax+1)\,x}\,x^4+584a^{\frac{7}{2}}x^3\sqrt{(ax+1)\,x}-735\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^4a^4-712a^{\frac{5}{2}}x^2\sqrt{(ax+1)}\right.}{210\sqrt{\frac{ax-1}{ax+1}}\,x^3a^{\frac{9}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x)
```

```
[Out] 1/210/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^4*(210*a^(9/2)*((a*x+
1)*x)^(1/2)*x^4+584*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-735*ln(1/2*(2*((a*x+1)*x)
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^4*a^4-712*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+
324*a^(3/2)*x*((a*x+1)*x)^(1/2)-60*((a*x+1)*x)^(1/2)*a^(1/2))/x^3/a^(9/2)/(
(a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(9/2),x)
[Out] Timed out
```

3.439 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=196

$$-\frac{5c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{49c^4\sqrt{1-\frac{1}{a^2x^2}}}{15a\sqrt{c-\frac{c}{ax}}}+\frac{31c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{15a}+\frac{7c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5a}+cx\sqrt{1-\frac{1}{a^2x^2}}\,($$

```
[Out] -5*c^(7/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+7/5*c^2*(
c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/a+c*(c-c/a/x)^(5/2)*x*(1-1/a^2/x^2)^(1/2
)+49/15*c^4*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+31/15*c^3*(1-1/a^2/x^2)^(
1/2)*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.15, antiderivative size = 221, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6182, 6179, 97, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{7/2}\tan}{a\left(1-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] ((80*a - 31/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2*(1 - 1/(a*x))
^(7/2)) + (7*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*(
1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)
*x)/(a^3*(1 - 1/(a*x))^(7/2)) - (5*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
```

```
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(-\frac{5}{2a}-\frac{7x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{S}}{}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+}}{a^3\left(1-\right.}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+}}{a^3\left(1-\right.}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+}}{a^3\left(1-\right.}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.52

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(15a^3x^3+56a^2x^2-28ax+6\right)-75a^2x^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{15a^3x^2\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(6 - 28*a*x + 56*a^2*x^2 + 15*a^3
*x^3) - 75*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]*x
^2)
```

fricas [A] time = 0.71, size = 415, normalized size = 2.12

$$\left[\frac{75\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(15a^4c^3x^4+71a^3c^3x^3+28a^2c\right.}{60\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 + 28*a
^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)
*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 +
 28*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.07, size = 149, normalized size = 0.76

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{3} \left(30 a^{\frac{7}{2}} x^{3} \sqrt{(ax+1)\,x}\ +112 a^{\frac{5}{2}} x^{2} \sqrt{(ax+1)\,x}\ -75 \ln\left(\frac{2 \sqrt{(ax+1)x}\ \sqrt{a}+2ax+1}{2 \sqrt{a}}\right) x^{3} a^{3}-56 a^{\frac{3}{2}} x \sqrt{(ax+1)\,x}\right.}{30 \sqrt{\frac{ax-1}{ax+1}}\ x^{2} a^{\frac{7}{2}} \sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x)
```

```
[Out] 1/30/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30*a^(7/2)*x^3*((a*
x+1)*x)^(1/2)+112*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-75*ln(1/2*(2*((a*x+1)*x)^(1
/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^3*a^3-56*a^(3/2)*x*((a*x+1)*x)^(1/2)+12*((a
*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

3.440 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=157

$$-\frac{3c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -2/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^4*(1-1/a^2/x^2)^(3/2)*x/(c
-c/a/x)^(3/2)-3*c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
)/a+3*c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 189, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6182, 6179, 89, 80, 50, 63, 208}

$$-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{3\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(a*(1 - 1/(a*x))^(5/2)) - (2*(1 +
 1/(a*x))^(3/2)*(c - c/(a*x))^(5/2))/(3*a*(1 - 1/(a*x))^(5/2)) + ((1 + 1/(a
*x))^(3/2)*(c - c/(a*x))^(5/2)*x)/(1 - 1/(a*x))^(5/2) - (3*(c - c/(a*x))^(5
/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(-\frac{3}{2a}+\frac{x}{a^2}\right)\sqrt{1+\frac{x}{a}}}{x}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(3\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{}{}\right.}{2a\left(1-\frac{1}{ax}\right)^{5/}}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{(3}{}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{(3}{}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3(}{}$$

Mathematica [**A**] time = 0.09, size = 89, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(3a^2x^2+10ax-2\right)-9ax\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{3a^2x\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 10*a*x + 3*a^2*x^2) - 9*a*x
*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

fricas [**A**] time = 0.73, size = 381, normalized size = 2.43

$$\left[\frac{9\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3+13a^2c^2x^2+8ac^2x-\right.}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x
- 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*
x), 1/6*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) + 2*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x - 2*c^2)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.07, size = 132, normalized size = 0.84

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{2} \left(6 a^{\frac{5}{2}} x^{2} \sqrt{(ax+1)\,x} + 20 a^{\frac{3}{2}} x \sqrt{(ax+1)\,x} - 9 \ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right) x^{2} a^{2} - 4\sqrt{(ax+1)\,x}\,\sqrt{a}\right)}{6\sqrt{\frac{ax-1}{ax+1}}\, x\, a^{\frac{5}{2}} \sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x)
```

```
[Out] 1/6/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x^2*((a*x+
1)*x)^(1/2)+20*a^(3/2)*x*((a*x+1)*x)^(1/2)-9*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2))*x^2*a^2-4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(5/2)/((a*
x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

3.441 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$

Optimal. Leaf size=117

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a/x)^(3/2)-c^(3/2)*arctanh(c^(1/2)*(1-1/a^2/
x^2)^(1/2)/(c-c/a/x)^(1/2))/a+c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6177, 879, 865, 875, 208}

$$\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2))^
(3/2)*x)/(c - c/(a*x))^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^
2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
```

```
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=-\left(c\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^2\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^3\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.60

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\,(ax+2)-\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + a*x) - ArcTanh[Sqrt[1 + 1/(a*x
)]]))/(a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.61, size = 313, normalized size = 2.68

$$\left[\frac{(acx-c)\sqrt{c}\,\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2+3acx+2c\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\ \frac{(ac\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*((a*c*x - c)*sqrt(-c)*arctan(2*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*
c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.06, size = 105, normalized size = 0.90

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}\;+\ln\left(\frac{2\sqrt{(ax+1)x}\;\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa-4\sqrt{(ax+1)\,x}\;\sqrt{a}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\;a^{\frac{3}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)*(-2*a^(3/2)*x*
((a*x+1)*x)^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a
-4*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

3.442 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=78

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6177, 863, 875, 208}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\left(c\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{c^2\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.85

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax+\sqrt{\frac{1}{ax}+1}\,\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+1\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])
)/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [B] time = 0.67, size = 295, normalized size = 3.78

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, -\frac{(ax-1)\sqrt{-c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))
/(a^2*x - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.05, size = 87, normalized size = 1.12

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2\sqrt{(ax+1)\,x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.443 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-\frac{c}{ax}}}+\frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 3*arctanh((1+1/a/x)^(1/2))*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-2*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)+x*(1-1/a
/x)^(1/2)*(1+1/a/x)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6182, 6179, 99, 156, 63, 208, 206}

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-\frac{c}{ax}}}+\frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - c/(a*x)] + (3*Sqrt[1 - 1/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) - (2*Sqrt[2]*Sqrt[1
 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c - c/(a*x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\frac{3}{2a}+\frac{x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(2\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}} \cdots\right.}{2a\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(4\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-\cdots}\right.}{a\sqrt{c-\cdots}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.61

$$\frac{\sqrt{1-\frac{1}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}+3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 3*ArcTanh[Sqrt[1 + 1/(a*x)]] -
2*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[c - c/(a*x)])
```

fricas [A] time = 0.76, size = 517, normalized size = 3.40

$$\left[\frac{3(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}+\frac{2\sqrt{2}(acx-c)\cdots}{\cdots}}{4\left(a^2cx-ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)) + 2*sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sq
rt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*c*
x - a*c), 1/2*(2*sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 +
 a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x
^2 - 2*a*x - 1)) - 3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*s
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
 + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*c*x - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.08, size = 151, normalized size = 0.99

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{3}{2}}c\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(3/2)*(1/a)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a
*(1/a)^(1/2)-2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+
1)/(a*x-1))*a^(1/2))/a^(3/2)/c/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))), x)
```

3.444 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 5*(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-7/2*(1-1/a/x)^
(3/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)-2*(1-1
/a/x)^(3/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)+a*(1-1/a/x)^(3/2)*x*(1+
1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6182, 6179, 99, 151, 156, 63, 208, 206}

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*(c - c/(a*x))^(3/2
)) + (a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*x)
)^(3/2)) + (5*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*
x))^(3/2)) - (7*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sq
rt[2]*a*(c - c/(a*x))^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^2}dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\frac{5}{2a}+\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{-\frac{5}{a^2}-\frac{2x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}d\right.}{2\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(7\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}dx\right.}{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}dx, x,\right.}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7(1}{}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 0.57

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\,(ax-2)+10(ax-1)\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-7\sqrt{2}\,(ax-1)\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{2ac(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-2 + a*x) + 10*(-1 + a*x)*ArcT
anh[Sqrt[1 + 1/(a*x)]] - 7*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqr
t[2]]))/(2*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

fricas [A] time = 1.64, size = 594, normalized size = 2.76

$$\left[\frac{7\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+10\left(a^2x^2-2\,ax+\right.}{8\left(a^3c^2x^2-2\,a^2c^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 10*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*
a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)) - c)/(a*x - 1)) + 8*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), 1/4*
(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*
sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2
*a*c*x - c)) - 10*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 4*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.07, size = 259, normalized size = 1.20

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x-7a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x+10\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2\sqrt{\frac{1}{a}}\,x-\cdots\right.}{4\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)\,a^{\frac{3}{2}}c^2\sqrt{(ax\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(5/2)*(1/a
)^(1/2)*((a*x+1)*x)^(1/2)*x-7*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+10*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-8*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2
)-10*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+7*
2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(
1/2))/a^(3/2)/c^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
[Out] Timed out
```

3.445 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{79\left(1\right.}{}$$

```
[Out] 7*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-79/16*(1-1/a/x
)^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-3/2*
a*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)-23/8*(1-1/a/x)^
(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a^2*(1-1/a/x)^(5/2)*x*(1+1/a/
x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)
```

Rubi [A] time = 0.17, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6182, 6179, 99, 151, 156, 63, 208, 206}

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{79\left(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-3*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))^2*(c - c/(a*x)
)^(5/2)) - (23*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(8*(a - x^(-1))*(c -
c/(a*x))^(5/2)) + (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1
))^2*(c - c/(a*x))^(5/2)) + (7*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)
]])/(a*(c - c/(a*x))^(5/2)) - (79*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a
*x)]/Sqrt[2]])/(8*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{-\frac{14}{a^2}-\frac{9x}{a^3}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,d\right.}{4\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{5}\right.}{}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(79\left(1-\frac{1}{ax}\right)^{5}\right.}{}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(7\left(1-\frac{1}{ax}\right)^{5/2}\right.}{}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{7\left(1-\frac{1}{ax}\right)^{5/2}}{a\,(c}$$

Mathematica [A] time = 0.13, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\left(8a^2x^2-35ax+23\right)+112(ax-1)^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-79\sqrt{2}\,(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{16ac^2(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(23 - 35*a*x + 8*a^2*x^2) + 112
*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 79*Sqrt[2]*(-1 + a*x)^2*ArcTanh[
Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(16*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

fricas [A] time = 0.74, size = 668, normalized size = 2.41

$$\left[\frac{79\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+112\left(a^3\right.}{64\left(a^4c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c*
x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)) + 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(8*
a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(8*a^4*x^4 - 27*a^
3*x^3 - 12*a^2*x^2 + 23*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3), 1/32*(79*sqrt(2)*
(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
2*a*c*x - c)) - 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*
a^2*c*x^2 - a*c*x - c)) + 4*(8*a^4*x^4 - 27*a^3*x^3 - 12*a^2*x^2 + 23*a*x)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3
*x^2 + 3*a^2*c^3*x - a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.07, size = 366, normalized size = 1.32

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x^2-79a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^2-140a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/32/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(32*a^(7/2)*
(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2-79*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1
/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-140*a^(5/2)*(1/a)^(1/2)*((a*x
+1)*x)^(1/2)*x+112*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^
3*(1/a)^(1/2)*x^2+158*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)
^(1/2)*a+3*a*x+1)/(a*x-1))*x+92*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-224*l
n(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x+112*
ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-79*2^(1
/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)
)/a^(3/2)/c^3/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c - \frac{c}{ax}\right)^{\frac{5}{2}} \sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c - \frac{c}{a\,x}\right)^{5/2} \sqrt{\frac{a\,x-1}{a\,x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

3.446 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$

Optimal. Leaf size=143

$$-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+x\left(c-\frac{c}{ax}\right)^{9/2}$$

```
[Out] 5/3*c^3*(c-c/a/x)^(3/2)/a+c^2*(c-c/a/x)^(5/2)/a+5/7*c*(c-c/a/x)^(7/2)/a+(c-
c/a/x)^(9/2)*x-5*c^(9/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+5*c^4*(c-c/a/x)
^(1/2)/a
```

Rubi [A] time = 0.24, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 50, 63, 208}

$$\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+x\left(c-\frac{c}{ax}\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (5*c^4*Sqrt[c - c/(a*x)])/a + (5*c^3*(c - c/(a*x))^(3/2))/(3*a) + (c^2*(c -
 c/(a*x))^(5/2))/a + (5*c*(c - c/(a*x))^(7/2))/(7*a) + (c - c/(a*x))^(9/2)*
x - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx$$
$$=-\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}{1-ax}dx$$
$$=\frac{c\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{x}dx}{a}$$
$$=\frac{c\int\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}dx}{a}$$
$$=-\frac{c\,\text{Subst}\left(\int\frac{(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}{x^2}dx,x,\frac{1}{x}\right)}{a}$$
$$=\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{(5c)\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{\left(5c^2\right)\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{\left(5c^3\right)\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right.}{2a}$$
$$=\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{\left(5c^4\right)\text{Subst}}{}$$
$$=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x$$
$$=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x$$
$$=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x$$

Mathematica [A] time = 0.17, size = 91, normalized size = 0.64

$$\frac{c^4\left(21a^4x^4+92a^3x^3+4a^2x^2-18ax+6\right)\sqrt{c-\frac{c}{ax}}}{21a^4x^3}-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(6 - 18*a*x + 4*a^2*x^2 + 92*a^3*x^3 + 21*a^4*x^4))/
(21*a^4*x^3) - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

fricas [A] time = 0.70, size = 234, normalized size = 1.64

$$\left[\frac{105\,a^{3}c^{\frac{9}{2}}x^{3}\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(21\,a^{4}c^{4}x^{4}+92\,a^{3}c^{4}x^{3}+4\,a^{2}c^{4}x^{2}-18\,ac^{4}x+6\,c^{4}\right)\sqrt{\frac{acx-c}{ax}}}{42\,a^{4}x^{3}},\ 10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/42*(105*a^3*c^(9/2)*x^3*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) + 2*(21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x
 + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3), 1/21*(105*a^3*sqrt(-c)*c^4*x^
3*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (21*a^4*c^4*x^4 + 92*a^3*c^4
*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^
3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-97,36.6646323889,7]W
arning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[
0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,
[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-89,7.79369851155,-49]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-64,-30,70]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [22,42,56]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-9,-13,46]Warning, choosing root of [1,0,%%
%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%
%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] a
t parameters values [24,49,-6]Warning, choosing root of [1,0,%%%{-2,[2,1,2]
%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3
]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters
 values [-49,-33,-70]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{
2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%
{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [
8,63,-64]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%
}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%
%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [2,62,-37]War
ning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,
1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1
,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-80,-23,65]Warning, choo
sing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0
,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+
```

%%%{1,[0,2,0]%%%}] at parameters values [-85,28,-44]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-22,93,91]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [31,-21,88]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [76,-66,66]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [5,-23,79]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{2,[1,0,1]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{4,[3,1,3]%%%}+%%%{-2,[3,0,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,1,2]%%%}+%%%{1,[2,0,2]%%%}] at parameters values [-88,9,6]Warning, choosing root of [1,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-69,-8,31]Warning, choosing root of [1,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [89,2,97]Warning, choosing root of [1,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-92,80,-24]Warning, choosing root of [1,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-17,41,64]Warning, choosing root of [1,0,%%%{2,[1,1,1]%%%}+%%%{-2,[0,2,1]%%%}+%%%{-2,[0,0,1]%%%},0,%%%{1,[2,2,2]%%%}+%%%{-2,[1,3,2]%%%}+%%%{2,[1,1,2]%%%}+%%%{1,[0,4,2]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,0,2]%%%}] at parameters values [18,-51,-42]Warning, choosing root of [1,0,%%%{2,[1,1,1]%%%}+%%%{-2,[0,2,1]%%%}+%%%{-2,[0,0,1]%%%},0,%%%{1,[2,2,2]%%%}+%%%{-2,[1,3,2]%%%}+%%%{2,[1,1,2]%%%}+%%%{1,[0,4,2]%%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,0,2]%%%}] at parameters values [-65,22,-94]Sign error (%%%{sqrt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+%%%{-2*sqrt(c)*a^2,1%%%}+%%%{-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*c)*abs(a)/4,5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 3.43Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 0.05, size = 163, normalized size = 1.14

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^4 \left(210 a^{\frac{9}{2}} \sqrt{a\,x^2-x}\; x^5 - 168 a^{\frac{7}{2}} \left(a\,x^2-x\right)^{\frac{3}{2}} x^3 + 16 a^{\frac{5}{2}} \left(a\,x^2-x\right)^{\frac{3}{2}} x^2 - 105 \ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right) x^5 a^4\right.}{42 x^4 \sqrt{(ax-1)\,x}\; a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(9/2),x)

[Out] 1/42*(c*(a*x-1)/a/x)^(1/2)*c^4*(210*a^(9/2)*(a*x^2-x)^(1/2)*x^5-168*a^(7/2)*(a*x^2-x)^(3/2)*x^3+16*a^(5/2)*(a*x^2-x)^(3/2)*x^2-105*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^5*a^4+24*a^(3/2)*(a*x^2-x)^(3/2)*x-12*(a*x^2-x)^(3/2)*a^(1/2))/x^4/((a*x-1)*x)^(1/2)/a^(9/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(9/2)/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(9/2),x)
```

```
[Out] Exception raised: TypeError
```

3.447 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$

Optimal. Leaf size=118

$$-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[Out] c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c/a/x)^(7/2)*x-3*c^(7/2)*a
rctanh((c-c/a/x)^(1/2)/c^(1/2))/a+3*c^3*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.23, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 50, 63, 208}

$$\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (3*c^3*Sqrt[c - c/(a*x)])/a + (c^2*(c - c/(a*x))^(3/2))/a + (3*c*(c - c/(a*
x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a
*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$$
$$=-\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{1-ax}dx$$
$$=\frac{c\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{x}dx}{a}$$
$$=\frac{c\int\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}dx}{a}$$
$$=-\frac{c\,\mathrm{Subst}\left(\int\frac{(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}{x^2}dx,x,\frac{1}{x}\right)}{a}$$
$$=\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c)\,\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\left(3c^2\right)\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\left(3c^3\right)\mathrm{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x}dx,x,\frac{1}{x}\right)}{2a}$$
$$=\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\left(3c^4\right)\mathrm{Subst}\left(\int\right.}{2}$$
$$=\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\left(3c^3\right)\mathrm{Subst}\left(\int\right.$$
$$=\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{3c^{7/2}\tanh^{-1}\left(\surd\right.}{a}$$

Mathematica [A] time = 0.13, size = 83, normalized size = 0.70

$$\frac{c^3\left(5a^3x^3+8a^2x^2+4ax-2\right)\sqrt{c-\frac{c}{ax}}}{5a^3x^2}-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(-2 + 4*a*x + 8*a^2*x^2 + 5*a^3*x^3))/(5*a^3*x^2) -
(3*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

fricas [A] time = 0.47, size = 212, normalized size = 1.80

$$\left[\frac{15\,a^2c^{\frac{7}{2}}x^2\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(5\,a^3c^3x^3+8\,a^2c^3x^2+4\,ac^3x-2\,c^3\right)\sqrt{\frac{acx-c}{ax}}}{10\,a^3x^2},\ \frac{15\,a^2\sqrt{-c}\,c^3x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/10*(15*a^2*c^(7/2)*x^2*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) + 2*(5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*c*
x - c)/(a*x)))/(a^3*x^2), 1/5*(15*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) + (5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3
)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-97,36.6646323889,7]W
arning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[
0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,
[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-89,7.79369851155,-49]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-64,-30,70]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [22,42,56]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-9,-13,46]Warning, choosing root of [1,0,%%
%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%
%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] a
t parameters values [24,49,-6]Warning, choosing root of [1,0,%%%{-2,[2,1,2]
%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3
]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters
 values [-49,-33,-70]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{
2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%
{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [
8,63,-64]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%
}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%
%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [2,62,-37]War
ning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,
1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1
,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-80,-23,65]Warning, choo
sing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0
,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+
%%%{1,[0,2,0]%%%}] at parameters values [-85,28,-44]Warning, choosing root
of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,
2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2
,0]%%%}] at parameters values [-22,93,91]Warning, choosing root of [1,0,%%%
{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%
%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at
 parameters values [31,-21,88]Warning, choosing root of [1,0,%%%{-2,[2,1,2]
%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{2,[1,0,1]%%%},0,%%%{1,[4,2,4]
%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{4,[3,1,3]%
```

%%}+%%%{-2,[3,0,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,1,2]%%%}+%%%{1,[2,0,2]%%
%}] at parameters values [76,-66,66]Warning, choosing root of [1,0,%%%{-2,[
2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%%%{-2,
[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0
,2,0]%%%}] at parameters values [5,-23,79]Warning, choosing root of [1,0,%%
%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%%%}+%
%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%%}+%%
%{1,[0,2,0]%%%}] at parameters values [-88,9,6]Warning, choosing root of [1
,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,0]%
%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [-69,-8,31]Warning, choosing roo
t of [1,0,%%%{-2,[2,1,0]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[
4,2,0]%%%}+%%%{-2,[3,2,1]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,2,0]%%%}+%%%{2,[1
,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [89,2,97]Warning, choosin
g root of [1,0,%%%{2,[1,1,1]%%%}+%%%{-2,[0,2,1]%%%}+%%%{-2,[0,0,1]%%%},0,%%
%{1,[2,2,2]%%%}+%%%{-2,[1,3,2]%%%}+%%%{2,[1,1,2]%%%}+%%%{1,[0,4,2]%%%}+%%%{
-2,[0,2,2]%%%}+%%%{1,[0,0,2]%%%}] at parameters values [-92,80,-24]Warning,
 choosing root of [1,0,%%%{2,[1,1,1]%%%}+%%%{-2,[0,2,1]%%%}+%%%{-2,[0,0,1]%
%%},0,%%%{1,[2,2,2]%%%}+%%%{-2,[1,3,2]%%%}+%%%{2,[1,1,2]%%%}+%%%{1,[0,4,2]%
%%}+%%%{-2,[0,2,2]%%%}+%%%{1,[0,0,2]%%%}] at parameters values [-17,41,64]S
ign error (%%%{sqrt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+%%%{-2*sqrt(
c)*a^2,1%%%}+%%%{-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*c)*abs(a)/4,
5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 0.55Limit: Max order reached or
unable to make series expansion Error: Bad Argument Value

maple [A] time = 0.05, size = 144, normalized size = 1.22

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^3 \left(-30\sqrt{a\,x^2-x}\; a^{\frac{7}{2}} x^4+20 a^{\frac{5}{2}} \left(a\,x^2-x\right)^{\frac{3}{2}} x^2+15\ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right) x^4 a^3+4 a^{\frac{3}{2}} \left(a\,x^2-x\right)^{\frac{3}{2}} x -$$

$$10x^3\sqrt{(ax-1)\,x}\; a^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(7/2),x)
```

```
[Out] -1/10*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3*(-30*(a*x^2-x)^(1/2)*a^(7/2)*x^4+20*a^(
5/2)*(a*x^2-x)^(3/2)*x^2+15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1
/2))*x^4*a^3+4*a^(3/2)*(a*x^2-x)^(3/2)*x-4*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1
)*x)^(1/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(7/2)/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 14.12, size = 777, normalized size = 6.58

$$c^3\left(\begin{cases}\frac{\sqrt{a}\sqrt{c}x^{\frac{3}{2}}}{\sqrt{ax-1}}-\frac{\sqrt{c}\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a}-\frac{\sqrt{c}\sqrt{x}}{\sqrt{a}\sqrt{ax-1}} & \text{for } |ax|>1\\ \frac{i\sqrt{c}\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a}+\frac{i\sqrt{c}\sqrt{x}\sqrt{-ax+1}}{\sqrt{a}} & \text{otherwise}\end{cases}\right)+\frac{2c^4\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^3\left(\begin{cases}0 & \text{for } \\ \frac{2a\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{3c} & \text{oth}\end{cases}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(7/2),x)
```

```
[Out] c**3*Piecewise((sqrt(a)*sqrt(c)*x**(3/2)/sqrt(a*x - 1) - sqrt(c)*acosh(sqrt
(a)*sqrt(x))/a - sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(a*x - 1)), Abs(a*x) > 1), (I
*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)*sqrt(-a*x + 1)/sqrt(a)
, True)) + 2*c**4*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) + 2*c**3*sq
rt(c - c/(a*x))/a - c**3*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**(3/2)
/(3*c), True))/a**2 + c**3*Piecewise((4*I*a**(11/2)*sqrt(c)*x**(7/2)/(-15*I
*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) - 4*I*a**(9/2)*sqrt(c)*x**(5/2
)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) - 4*I*a**5*sqrt(c)*x**
3*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(5/2)) + 2*I*a*
*4*sqrt(c)*x**2*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a**(5/2)*x**(
5/2)) + 8*I*a**3*sqrt(c)*x*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2) + 15*I*a*
*(5/2)*x**(5/2)) - 6*I*a**2*sqrt(c)*sqrt(a*x - 1)/(-15*I*a**(7/2)*x**(7/2)
+ 15*I*a**(5/2)*x**(5/2)), Abs(a*x) > 1), (-4*I*a**(11/2)*sqrt(c)*x**(7/2)/
(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) + 4*I*a**(9/2)*sqrt(c)*x*
*(5/2)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) - 4*a**5*sqrt(c)*x
**3*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(5/2)) + 2*a*
*4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(5/2)*x**(
5/2)) + 8*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15*I*a**(
5/2)*x**(5/2)) - 6*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*I*a**(7/2)*x**(7/2) - 15
*I*a**(5/2)*x**(5/2)), True))/a**3
```

3.448 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$

Optimal. Leaf size=95

$$-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[Out] 1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-c^(5/2)*arctanh((c-c/a/x)^(1/2)/c
^(1/2))/a+c^2*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.20, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 50, 63, 208}

$$\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x))^(
5/2)*x - (c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{1-ax}dx\\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{x}dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}dx}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c^2\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-c^2\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.79

$$
\frac{c^2\left(3a^2x^2-2ax+2\right)\sqrt{c-\frac{c}{ax}}-3ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{3a^2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(2 - 2*a*x + 3*a^2*x^2) - 3*a*c^(5/2)*x*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/(3*a^2*x)
```

fricas [A] time = 0.60, size = 182, normalized size = 1.92

$$
\left[\frac{3\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^2c^2x^2-2\,ac^2x+2\,c^2\right)\sqrt{\frac{acx-c}{ax}}}{6\,a^2x},\ \frac{3\,a\sqrt{-c}\,c^2x\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c) + 2*(3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)
, 1/3*(3*a*sqrt(-c)*c^2*x*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (3*a
^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [86,-97,-82]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [7,-27,26]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-89,63,-49]Warning, choosing root of [1,0,%
%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+
%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]
at parameters values [-86,-64,-30]Warning, choosing root of [1,0,%%%{-2,[2,
1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3
,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parame
ters values [70,22,42]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%
{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%
%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values
[56,-9,-13]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%
%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2
]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [46,24,49]W
arning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[
0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,
[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [18,-49,-33]Warning, ch
oosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%}
,0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%
}+%%%{1,[0,2,0]%%%}] at parameters values [-70,8,63]Warning, choosing root
of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{2,[1,0,
1]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{-2,[3,2
,3]%%%}+%%%{4,[3,1,3]%%%}+%%%{-2,[3,0,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{-2,[2,1,
2]%%%}+%%%{1,[2,0,2]%%%}] at parameters values [-64,2,62]Sign error (%%%{sq
rt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+%%%{-2*sqrt(c)*a^2,1%%%}+%%%{
-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*c)*abs(a)/4,5/2%%%}+%%%{undef
,7/2%%%})Limit: Max order reached or unable to make series expansion Error:
 Bad Argument Value
```

maple [A] time = 0.05, size = 108, normalized size = 1.14

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{2} \left(-6\sqrt{a\,x^{2}-x}\; a^{\frac{5}{2}} x^{3}+3\ln\left(\frac{2\sqrt{a\,x^{2}-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right) x^{3} a^{2}+4\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}\right)}{6x^{2}\sqrt{(ax-1)\,x}\; a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(5/2),x)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3+3*ln(1/2
*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^2+4*(a*x^2-x)^(3/2)*a^(
1/2))/((a*x-1)*x)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c - \frac{c}{ax}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(5/2)/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{a\,x}\right)^{5/2}\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

3.449 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[Out] (c-c/a/x)^(3/2)*x+c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-c*(c-c/a/x)^(1
/2)/a
```

Rubi [A] time = 0.18, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 50, 63, 208}

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x + (c^(3/2)*ArcTanh[Sqrt[
c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$$
$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{1-ax}dx$$
$$= \frac{c\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x}dx}{a}$$
$$= \frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}dx}{a}$$
$$= -\frac{c\,\mathrm{Subst}\left(\int \frac{(a+x)\sqrt{c-\frac{cx}{a}}}{x^2}dx, x, \frac{1}{x}\right)}{a}$$
$$= \left(c-\frac{c}{ax}\right)^{3/2}x - \frac{c\,\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}dx, x, \frac{1}{x}\right)}{2a}$$
$$= -\frac{c\sqrt{c-\frac{c}{ax}}}{a} + \left(c-\frac{c}{ax}\right)^{3/2}x - \frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{2a}$$
$$= -\frac{c\sqrt{c-\frac{c}{ax}}}{a} + \left(c-\frac{c}{ax}\right)^{3/2}x + c\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}dx, x, \sqrt{c-\frac{c}{ax}}\right)$$
$$= -\frac{c\sqrt{c-\frac{c}{ax}}}{a} + \left(c-\frac{c}{ax}\right)^{3/2}x + \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.79

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+c(ax-2)\sqrt{c-\frac{c}{ax}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(-2 + a*x) + c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]
])/a
```

fricas [A] time = 0.68, size = 137, normalized size = 1.96

$$\left[\frac{c^{\frac{3}{2}}\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a}, -\frac{\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c^(3/2)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2
*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(sqrt(-c)*c*arctan(sqrt(-c)*sqr
t((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [86,-97,-82]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [7,-27,26]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-89,63,-49]Warning, choosing root of [1,0,%
%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+
%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]
at parameters values [-86,-64,-30]Warning, choosing root of [1,0,%%%{-2,[2,
1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3
,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parame
ters values [70,22,42]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%
{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%
%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values
[56,-9,-13]Sign error (%%%{sqrt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+
%%%{-2*sqrt(c)*a^2,1%%%}+%%%{-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*
c)*abs(a)/4,5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

maple [A] time = 0.05, size = 103, normalized size = 1.47

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(-2\sqrt{a\,x^{2}-x}\; a^{\frac{3}{2}} x^{2}+4\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}+\ln\left(\frac{2\sqrt{a\,x^{2}-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^{2}a\right)}{2x\sqrt{(ax-1)\,x}\; a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(3/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)/x*c*(-2*(a*x^2-x)^(1/2)*a^(3/2)*x^2+4*(a*x^2-x)^(
3/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a)/((a
*x-1)*x)^(1/2)/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(3/2)/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(3/2),x)
[Out] Exception raised: TypeError
```

3.450 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=50

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 375, 78, 63, 208}

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \sqrt{c-\frac{c}{ax}}\,x-\frac{(3c)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \sqrt{c-\frac{c}{ax}}\,x+3\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= \sqrt{c-\frac{c}{ax}}\,x+\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.00

$$
x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

fricas [A] time = 0.40, size = 124, normalized size = 2.48

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}} + 3\,\sqrt{c}\,\log\left(-2\,acx - 2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}} + c\right)}{2\,a}, \frac{ax\sqrt{\frac{acx-c}{ax}} - 3\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*
x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c
)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

giac [B] time = 0.19, size = 96, normalized size = 1.92

$$\frac{3\,\sqrt{c}\,\log\left(|a||c|\right)\operatorname{sgn}(x)}{2\,a} - \frac{3\,\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x - \sqrt{a^2cx^2 - acx}\right)\sqrt{c}\,|a| + ac\right|\right)}{2\,a\operatorname{sgn}(x)} + \frac{\sqrt{a^2cx^2 - acx}\,|a|}{a^2\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(a^
2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))
```

maple [B] time = 0.05, size = 120, normalized size = 2.40

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^2-x}\,\sqrt{a} - 4\sqrt{(ax-1)\,x}\,\sqrt{a} - \ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) - 2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{(ax-1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2),x)
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(1/2)-4*((a*x-1)*x)^(1/2)
*a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-2*ln(1/2*(2*((
a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c - \frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c - \frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.451 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=70

$$\frac{x}{\sqrt{c-\frac{c}{ax}}}-\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] 5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)-5/a/(c-c/a/x)^(1/2)+x/(c-c/a/x
)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 51, 63, 208}

$$\frac{x}{\sqrt{c-\frac{c}{ax}}}-\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] -5/(a*Sqrt[c - c/(a*x)]) + x/Sqrt[c - c/(a*x)] + (5*ArcTanh[Sqrt[c - c/(a*x
)]/Sqrt[c]])/(a*Sqrt[c])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$= -\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{a}$$

$$= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{(5c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{5\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}$$

$$= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 0.61

$$\frac{ax - 5\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{1}{ax}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (a*x - 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*Sqrt[c - c/(a*x)]
)
```

fricas [A] time = 0.52, size = 176, normalized size = 2.51

$$\left[\frac{5\,(ax-1)\sqrt{c}\,\log\left(-2\,acx - 2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}} + c\right) + 2\left(a^2x^2 - 5\,ax\right)\sqrt{\frac{acx-c}{ax}}}{2\left(a^2cx - ac\right)}, -\frac{5\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right.}{a^2cx - ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(5*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) + 2*(a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c), -(
5*(a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a^2*x^2
- 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

giac [B] time = 0.16, size = 124, normalized size = 1.77

$$-\frac{a\left(\frac{5\,c\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^{2}\sqrt{-c}}+\frac{4\,c^{2}-\frac{5\,(acx-c)c}{ax}}{\left(c\sqrt{\frac{acx-c}{ax}}-\frac{(acx-c)\sqrt{\frac{acx-c}{ax}}}{ax}\right)a^{2}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -a*(5*c*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)) + (4*c^2 -
5*(a*c*x - c)*c/(a*x))/((c*sqrt((a*c*x - c)/(a*x)) - (a*c*x - c)*sqrt((a*c*
x - c)/(a*x))/(a*x))*a^2))/c
```

maple [B] time = 0.05, size = 194, normalized size = 2.77

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(10a^{\frac{5}{2}}\sqrt{(ax-1)\,x}\;x^{2}+5\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^{2}a^{2}-8a^{\frac{3}{2}}\left((ax-1)\,x\right)^{\frac{3}{2}}-20a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\;x-10\ln\left(\right.\right.}{2\sqrt{(ax-1)\,x}\;c\,(ax-1)^{2}\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(10*a^(5/2)*((a*x-1)*x)^(1/2)*x^2+5*ln(1/2*(2*(
(a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2-8*a^(3/2)*((a*x-1)*x)^(3
/2)-20*a^(3/2)*((a*x-1)*x)^(1/2)*x-10*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*x*a+10*((a*x-1)*x)^(1/2)*a^(1/2)+5*ln(1/2*(2*((a*x-1)*x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c/(a*x-1)^2/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a\,x+1}{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)), x)
```

3.452 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -7/3/a/(c-c/a/x)^(3/2)+x/(c-c/a/x)^(3/2)+7*arctanh((c-c/a/x)^(1/2)/c^(1/2))
/a/c^(3/2)-7/a/c/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 51, 63, 208}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -7/(3*a*(c - c/(a*x))^(3/2)) - 7/(a*c*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^
(3/2) + (7*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(3/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx \\
&= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}\,dx \\
&= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}x}\,dx}{a} \\
&= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{a} \\
&= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{a} \\
&= \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(7c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{2a} \\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a} \\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2ac} \\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{c^2} \\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 55, normalized size = 0.58

$$
\frac{x\left(3ax-7\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{1}{ax}\right)\right)}{3c(ax-1)\sqrt{c-\frac{c}{ax}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (x*(3*a*x - 7*Hypergeometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c*Sqrt[c -
 c/(a*x)]*(-1 + a*x))
```

fricas [A] time = 0.54, size = 238, normalized size = 2.51

$$\left[\frac{21\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3\,a^3x^3-28\,a^2x^2+21\,ax\right)\sqrt{\frac{acx-c}{ax}}}{6\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)},-\frac{21\left(a^2x^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(21*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) + 2*(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c
)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(21*(a^2*x^2 - 2*a*x +
1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^3*x^3 - 28*a^
2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2
)]
```

giac [A] time = 0.17, size = 136, normalized size = 1.43

$$-\frac{a\left(\frac{2\left(2\,c+\frac{9\,(acx-c)}{ax}\right)x}{(acx-c)a\sqrt{\frac{acx-c}{ax}}}+\frac{21\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}-\frac{3\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*a*(2*(2*c + 9*(a*c*x - c)/(a*x))*x/((a*c*x - c)*a*sqrt((a*c*x - c)/(a*
x))) + 21*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)) - 3*sqrt(
(a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))))/c
```

maple [B] time = 0.06, size = 260, normalized size = 2.74

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(42a^{\frac{7}{2}}\sqrt{(ax-1)\,x}\;x^3+21\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^3a^3-36a^{\frac{5}{2}}\left((ax-1)\,x\right)^{\frac{3}{2}}x-126a^{\frac{5}{2}}\sqrt{(ax-1)\,x}\;x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)*x*(42*a^(7/2)*((a*x-1)*x)^(1/2)*x^3+21*ln(1/2*(2*
((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3-36*a^(5/2)*((a*x-1)*x)^
(3/2)*x-126*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^
(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2+28*a^(3/2)*((a*x-1)*x)^(3/2)+126*a^(3/2)*((
a*x-1)*x)^(1/2)*x+63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*
x*a-42*((a*x-1)*x)^(1/2)*a^(1/2)-21*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a
*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^2/a^(1/2)/(a*x-1)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

3.453 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] -9/5/a/(c-c/a/x)^(5/2)-3/a/c/(c-c/a/x)^(3/2)+x/(c-c/a/x)^(5/2)+9*arctanh((c
-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)-9/a/c^2/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 51, 63, 208}

$$-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] -9/(5*a*(c - c/(a*x))^(5/2)) - 3/(a*c*(c - c/(a*x))^(3/2)) - 9/(a*c^2*Sqrt[
c - c/(a*x)]) + x/(c - c/(a*x))^(5/2) + (9*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]])/(a*c^(5/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{a}$$

$$= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{(9c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\cdots}\right)}{c^3}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Mathematica [C] time = 0.04, size = 58, normalized size = 0.49

$$\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{1}{ax}\right)}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] x/(c - c/(a*x))^(5/2) - (9*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)])/(
5*a*(c - c/(a*x))^(5/2))
```

fricas [A] time = 0.48, size = 294, normalized size = 2.49

$$\left[\frac{45\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(5\,a^4x^4-69\,a^3x^3+105\,a^2x^2-45\,ax\right)\sqrt{\frac{a}{}}}{10\left(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/10*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt
(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^
2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c
^3*x - a*c^3), -1/5*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^2
- 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)]
```

giac [A] time = 0.14, size = 165, normalized size = 1.40

$$-\frac{a\left(\frac{2\left(2\,c^2+\frac{5\,(acx-c)c}{ax}+\frac{20\,(acx-c)^2}{a^2x^2}\right)x^2}{(acx-c)^2c\sqrt{\frac{acx-c}{ax}}}+\frac{45\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c}-\frac{5\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c}\right)}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/5*a*(2*(2*c^2 + 5*(a*c*x - c)*c/(a*x) + 20*(a*c*x - c)^2/(a^2*x^2))*x^2/
((a*c*x - c)^2*c*sqrt((a*c*x - c)/(a*x))) + 45*arctan(sqrt((a*c*x - c)/(a*x
))/sqrt(-c))/(a^2*sqrt(-c)*c) - 5*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x
- c)/(a*x))*c))/c
```

maple [B] time = 0.06, size = 328, normalized size = 2.78

$$\sqrt{\frac{c(ax-1)}{ax}}\,x\left(90a^{\frac{9}{2}}\sqrt{(ax-1)\,x}\,x^4+45\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^4a^4-80a^{\frac{7}{2}}\left((ax-1)\,x\right)^{\frac{3}{2}}x^2-360a^{\frac{7}{2}}\sqrt{(ax-1)\,x}\,x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/10*(c*(a*x-1)/a/x)^(1/2)*x*(90*a^(9/2)*((a*x-1)*x)^(1/2)*x^4+45*ln(1/2*(2
*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^4*a^4-80*a^(7/2)*((a*x-1)*x)
^(3/2)*x^2-360*a^(7/2)*((a*x-1)*x)^(1/2)*x^3-180*ln(1/2*(2*((a*x-1)*x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3+132*a^(5/2)*((a*x-1)*x)^(3/2)*x+540*a^(
5/2)*((a*x-1)*x)^(1/2)*x^2+270*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*x^2*a^2-60*a^(3/2)*((a*x-1)*x)^(3/2)-360*a^(3/2)*((a*x-1)*x)^(1/2
)*x-180*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*a+90*((a*x-
1)*x)^(1/2)*a^(1/2)+45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
))/((a*x-1)*x)^(1/2)/c^3/a^(1/2)/(a*x-1)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}\left(ax-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(5/2),x)
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

3.454 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] -11/7/a/(c-c/a/x)^(7/2)-11/5/a/c/(c-c/a/x)^(5/2)-11/3/a/c^2/(c-c/a/x)^(3/2)
+x/(c-c/a/x)^(7/2)+11*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(7/2)-11/a/c^3/(
c-c/a/x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 78, 51, 63, 208}

$$-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] -11/(7*a*(c - c/(a*x))^(7/2)) - 11/(5*a*c*(c - c/(a*x))^(5/2)) - 11/(3*a*c^
2*(c - c/(a*x))^(3/2)) - 11/(a*c^3*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^(7/
2) + (11*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{9/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx}{a}$$

$$= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{(11c)\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x\right.}{2ac^2}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\,\text{Subst}\left(\right.}{}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\,\text{Subst}\left(\right.}{}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{11\tanh^{-1}\left(\right.}{ac^{7/}}$$

Mathematica [C] time = 0.05, size = 46, normalized size = 0.32

$$\frac{7x - \frac{11\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};1-\frac{1}{ax}\right)}{a}}{7\left(c-\frac{c}{ax}\right)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] (7*x - (11*Hypergeometric2F1[-7/2, 1, -5/2, 1 - 1/(a*x)])/a)/(7*(c - c/(a*x
))^(7/2))
```

fricas [A] time = 0.60, size = 346, normalized size = 2.39

$$\left[\frac{1155\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{c}\,\log\left(-2acx-2a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(105a^5x^5-1936a^4x^4+44\right.}{210\left(a^5c^4x^4-4a^4c^4x^3+6a^3c^4x^2-4a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/210*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log(-2*a
*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(105*a^5*x^5 - 1936*a
^4*x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(
a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/105*
(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(sqrt(-c
)*sqrt((a*c*x - c)/(a*x))/c) - (105*a^5*x^5 - 1936*a^4*x^4 + 4466*a^3*x^3 -
 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^5*c^4*x^4 - 4*a^4*c^4
*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.14, size = 187, normalized size = 1.29

$$-\frac{a\left(\frac{2\left(30c^3+\frac{63(acx-c)c^2}{ax}+\frac{140(acx-c)^2c}{a^2x^2}+\frac{525(acx-c)^3}{a^3x^3}\right)ax^3}{(acx-c)^3c^2\sqrt{\frac{acx-c}{ax}}}+\frac{1155\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^2}-\frac{105\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^2}\right)}{105c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/105*a*(2*(30*c^3 + 63*(a*c*x - c)*c^2/(a*x) + 140*(a*c*x - c)^2*c/(a^2*x
^2) + 525*(a*c*x - c)^3/(a^3*x^3))*a*x^3/((a*c*x - c)^3*c^2*sqrt((a*c*x - c
)/(a*x))) + 1155*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^2
) - 105*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^2))/c
```

maple [B] time = 0.06, size = 396, normalized size = 2.73

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2310\sqrt{(ax-1)x}\,a^{\frac{11}{2}}x^5+2100\left((ax-1)x\right)^{\frac{3}{2}}a^{\frac{9}{2}}x^3-1155\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^5a^5+11550a^{\frac{9}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a/x)^(7/2),x)
```

```
[Out] -1/210*(c*(a*x-1)/a/x)^(1/2)*x*(-2310*((a*x-1)*x)^(1/2)*a^(11/2)*x^5+2100*(
(a*x-1)*x)^(3/2)*a^(9/2)*x^3-1155*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x
-1)/a^(1/2))*x^5*a^5+11550*a^(9/2)*((a*x-1)*x)^(1/2)*x^4-5368*a^(7/2)*((a*x
-1)*x)^(3/2)*x^2+5775*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*x^4*a^4-23100*a^(7/2)*((a*x-1)*x)^(1/2)*x^3+4928*a^(5/2)*((a*x-1)*x)^(3/2)
*x-11550*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^3*a^3+2310
0*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-1540*a^(3/2)*((a*x-1)*x)^(3/2)+11550*ln(1/2
*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^2-11550*a^(3/2)*((a*x
-1)*x)^(1/2)*x-5775*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x
```

```
*a+2310*((a*x-1)*x)^(1/2)*a^(1/2)+1155*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^4/a^(1/2)/(a*x-1)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\left(c-\frac{c}{a\,x}\right)^{7/2}\,(a\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

3.455 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=268

$$\frac{9\left(a-\frac{1}{x}\right)^2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\sqrt{\frac{1}{a\ldots}}}{\ldots}$$

```
[Out] 3/35*(28*a-17/x)*(1+1/a/x)^(3/2)*(c-c/a/x)^(9/2)/a^2/(1-1/a/x)^(9/2)+9/7*(a
-1/x)^2*(1+1/a/x)^(3/2)*(c-c/a/x)^(9/2)/a^3/(1-1/a/x)^(9/2)+(a-1/x)^3*(1+1/
a/x)^(3/2)*(c-c/a/x)^(9/2)*x/a^3/(1-1/a/x)^(9/2)-3*(c-c/a/x)^(9/2)*arctanh(
(1+1/a/x)^(1/2))/a/(1-1/a/x)^(9/2)+3*(c-c/a/x)^(9/2)*(1+1/a/x)^(1/2)/a/(1-1
/a/x)^(9/2)
```

Rubi [A] time = 0.16, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 97, 153, 147, 50, 63, 208}

$$\frac{9\left(a-\frac{1}{x}\right)^2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\sqrt{\frac{1}{a\ldots}}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(a*(1 - 1/(a*x))^(9/2)) + (3*(28*
a - 17/x)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(35*a^2*(1 - 1/(a*x))^(9
/2)) + (9*(a - x^(-1))^2*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(7*a^3*(1
 - 1/(a*x))^(9/2)) + ((a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2
)*x)/(a^3*(1 - 1/(a*x))^(9/2)) - (3*(c - c/(a*x))^(9/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx = \frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(-\frac{3}{2a}-\frac{9x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x}dx\right.}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^3\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{\left(2a\left(c-\right.\right.}{}$$

$$= \frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(a-\right.}{}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\right.}{7a^3\left(1-\right.}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\right.}{7a^3\left(1-\right.}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{9\left(a-\frac{1}{x}\right)^2\left(1+\right.}{7a^3\left(1-\right.}$$

Mathematica [A] time = 0.11, size = 109, normalized size = 0.41

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(35a^4x^4+164a^3x^3-12a^2x^2-26ax+10\right)-105a^3x^3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{35a^4x^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]

[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(10 - 26*a*x - 12*a^2*x^2 + 164*a
^3*x^3 + 35*a^4*x^4) - 105*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(35*a^4*Sqr
t[1 - 1/(a*x)]*x^3)
```

fricas [A] time = 0.82, size = 437, normalized size = 1.63

$$\left[\frac{105 \left(a^{4} c^{4} x^{4}-a^{3} c^{4} x^{3}\right) \sqrt{c}\, \log\left(-\frac{8 a^{3} c x^{3}-7 a c x-4 \left(2 a^{3} x^{3}+3 a^{2} x^{2}+a x\right) \sqrt{c} \sqrt{\frac{a x-1}{a x+1}} \sqrt{\frac{a c x-c}{a x}}-c}{a x-1}\right)+4 \left(35 a^{5} c^{4} x^{5}+199 a^{4} c^{4} x^{4}+152\right.}{140 \left(a^{5} x^{4}-a^{4} x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(35*a^5*c^4*x^5 + 199*a^4*c^4*x^4 + 1
52*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10*c^4)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/70*(105*(a^4*c^4*x^4
- a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(35*a^5*c^4
*x^5 + 199*a^4*c^4*x^4 + 152*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10
*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{a x}\right)^{\frac{9}{2}}}{\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.07, size = 178, normalized size = 0.66

$$\frac{(a x-1) \sqrt{\frac{c(a x-1)}{a x}}\, c^{4} \left(70 a^{\frac{9}{2}} \sqrt{(a x+1) x}\, x^{4}+328 a^{\frac{7}{2}} x^{3} \sqrt{(a x+1) x}-105 \ln\left(\frac{2 \sqrt{(a x+1) x}\, \sqrt{a}+2 a x+1}{2 \sqrt{a}}\right) x^{4} a^{4}-24 a^{\frac{5}{2}} x^{2} \sqrt{(a\right.}{70 \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}} (a x+1)\, x^{3} a^{\frac{9}{2}} \sqrt{(a x+1) x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x)
```

```
[Out] 1/70/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^4*(70*
a^(9/2)*((a*x+1)*x)^(1/2)*x^4+328*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-105*ln(1/2*
(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^4*a^4-24*a^(5/2)*x^2*((a*x
+1)*x)^(1/2)-52*a^(3/2)*x*((a*x+1)*x)^(1/2)+20*((a*x+1)*x)^(1/2)*a^(1/2))/x
^3/a^(9/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{a x}\right)^{\frac{9}{2}}}{\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

3.456 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=237

$$-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\tanh}{a\left(1-\right.}$$

```
[Out] 1/3*(1+1/a/x)^(3/2)*(c-c/a/x)^(7/2)/a/(1-1/a/x)^(7/2)-2/5*(1+1/a/x)^(5/2)*(
c-c/a/x)^(7/2)/a/(1-1/a/x)^(7/2)+(1+1/a/x)^(5/2)*(c-c/a/x)^(7/2)*x/(1-1/a/x
)^(7/2)-(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)+(c-c/a/x
)^(7/2)*(1+1/a/x)^(1/2)/a/(1-1/a/x)^(7/2)
```

Rubi [A] time = 0.15, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 89, 80, 50, 63, 208}

$$-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\tanh}{a\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(
a*x))^(3/2)*(c - c/(a*x))^(7/2))/(3*a*(1 - 1/(a*x))^(7/2)) - (2*(1 + 1/(a*x
))^(5/2)*(c - c/(a*x))^(7/2))/(5*a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(a*x))^(5
/2)*(c - c/(a*x))^(7/2)*x)/(1 - 1/(a*x))^(7/2) - ((c - c/(a*x))^(7/2)*ArcTa
nh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(-\frac{1}{2a}+\frac{x}{a^2}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x}\cdots\right.}{2a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}+\cdots$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\cdots$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\cdots$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\cdots$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.43

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(15a^3x^3+44a^2x^2+8ax-6\right)-15a^2x^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{15a^3x^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-6 + 8*a*x + 44*a^2*x^2 + 15*a^3
*x^3) - 15*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]*x
^2)
```

fricas [A] time = 0.64, size = 415, normalized size = 1.75

$$\left[\frac{15\left(a^3c^3x^3 - a^2c^3x^2\right)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3 - 7\,acx - 4\left(2\,a^3x^3 + 3\,a^2x^2 + ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}} - c}{ax-1}\right) + 4\left(15\,a^4c^3x^4 + 59\,a^3c^3x^3 + 52\,a\right.}{60\left(a^4x^3 - a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 + 52*a
^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 +
52*a^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.07, size = 161, normalized size = 0.68

$$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\;c^3\left(30a^{\frac{7}{2}}x^3\sqrt{(ax+1)\,x}\;+88a^{\frac{5}{2}}x^2\sqrt{(ax+1)\,x}\;-15\ln\left(\frac{2\sqrt{(ax+1)x}\;\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^3a^3+16a^{\frac{3}{2}}x\sqrt{(ax\right.}{30\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,x^2a^{\frac{7}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x)
```

```
[Out] 1/30/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)+88*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-15*ln(1/2*(2
*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^3*a^3+16*a^(3/2)*x*((a*x+1)*
x)^(1/2)-12*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(7/2),x)
[Out] Timed out
```

3.457 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=156

$$\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^5x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -1/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^5*(1-1/a^2/x^2)^(5/2)*x/(c
-c/a/x)^(5/2)+c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/
a-c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6177, 879, 865, 875, 208}

$$\frac{c^5x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] -(c^4*(1 - 1/(a^2*x^2))^(3/2))/(3*a*(c - c/(a*x))^(3/2)) - (c^3*Sqrt[1 - 1/
(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^5*(1 - 1/(a^2*x^2))^(5/2)*x)/(c - c/
(a*x))^(5/2) + (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/
(a*x)]])/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
```

```
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\left(c^3\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^3\,\mathrm{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^2\,\mathrm{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\,\mathrm{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\ldots\right)}{a^3}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(3a^2x^2+2ax+2\right)+3ax\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{3a^2x\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + 2*a*x + 3*a^2*x^2) + 3*a*x*A
rcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

fricas [A] time = 0.53, size = 381, normalized size = 2.44

$$\left[\frac{3\left(a^2c^2x^2 - ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3 - 7acx + 4\left(2a^3x^3 + 3a^2x^2 + ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}} - c}{ax-1}\right) + 4\left(3a^3c^2x^3 + 5a^2c^2x^2 + 4ac^2x + \right.}{12\left(a^3x^2 - a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x +
 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x
), -1/6*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x + 2*c^2)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.07, size = 144, normalized size = 0.92

$$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c^2\left(6a^{\frac{5}{2}}x^2\sqrt{(ax+1)\,x} + 3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^2a^2 + 4a^{\frac{3}{2}}x\sqrt{(ax+1)\,x} + 4\sqrt{(ax+1)\,x}\,\sqrt{\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,x\,a^{\frac{5}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x)
```

```
[Out] 1/6/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(
5/2)*(6*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*x^2*a^2+4*a^(3/2)*x*((a*x+1)*x)^(1/2)+4*((a*x+1)*x)^(1/2
)*a^(1/2))/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c - \frac{c}{ax}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(5/2),x)
[Out] Timed out
```

3.458 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$

Optimal. Leaf size=118

$$\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a/x)^(3/2)+3*c^(3/2)*arctanh(c^(1/2)*(1-1/a^
2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a-3*c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6177, 863, 865, 875, 208}

$$\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-3*c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2
))^(3/2)*x)/(c - c/(a*x))^(3/2) + (3*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = -\left(c^3\,\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{(3c^2)\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{(3c)\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{(3c^3)\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$=-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [C] time = 0.05, size = 66, normalized size = 0.56

$$-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1+\frac{1}{ax}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
[Out] (-2*(1 + 1/(a*x))^(5/2)*(c - c/(a*x))^(3/2)*Hypergeometric2F1[2, 5/2, 7/2,
 1 + 1/(a*x)])/(5*a*(1 - 1/(a*x))^(3/2))
```

fricas [A] time = 0.89, size = 315, normalized size = 2.67

$$\left[\frac{3\,(acx-c)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2-acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},-\frac{3\,(ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), -1/2*(3*(a*c*x - c)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) - 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.06, size = 118, normalized size = 1.00

$$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}\,+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa-4\sqrt{(ax+1)\,x}\,\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,a^{\frac{3}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)
*(2*a^(3/2)*x*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*x*a-4*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{3/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

3.459 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 5*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+x*(1+1/a
/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 98, 156, 63, 208, 206}

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{-\frac{5}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subs}}{2a\sqrt{}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Su}}{}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}} + \frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.08, size = 93, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}+5\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 5*ArcTanh[Sqrt[1 + 1/(a*x)]] -
4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.73, size = 512, normalized size = 3.37

$$\left[\frac{4\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+5\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-}{}\right.}{4\left(a^2x-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*
x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x
)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) +
 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*
x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.06, size = 160, normalized size = 1.05

$$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\;x\left(2\sqrt{(ax+1)\,x}\;a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-4\sqrt{2}\;\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)\,x}\;a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*
x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x
+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1
/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
[Out] Timed out
```

3.460 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=215

$$\frac{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 7*arctanh((1+1/a/x)^(1/2))*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-5*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-3*(1-1/a
/x)^(1/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(1/2)+a*x*(1-1/a/x)^(1/2)*(1+1/
a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 151, 156, 63, 208, 206}

$$\frac{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (-3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - c/(a*x)]) +
 (a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*Sqrt[c - c/(a*x)])
 + (7*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) -
 (5*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c
 - c/(a*x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{-\frac{7}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(a\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{\frac{7}{a^2}+\frac{3x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(5\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{c-\frac{c}{ax}}} -$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(7\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right.}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{7\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}} - \frac{5\sqrt{2}\sqrt{1-\ldots}}{a\ldots}$$

Mathematica [A] time = 0.10, size = 115, normalized size = 0.53

$$\frac{\sqrt{1-\frac{1}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}\,(ax-3)+7(ax-1)\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-5\sqrt{2}\,(ax-1)\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x*(-3 + a*x) + 7*(-1 + a*x)*ArcTanh
[Sqrt[1 + 1/(a*x)]] - 5*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2
]]))/(a*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

fricas [A] time = 0.72, size = 581, normalized size = 2.70

$$\left[\frac{7\left(a^{2}x^{2}-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^{3}x^{3}-2\,a^{2}x^{2}-3\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{ac}{}}}{4\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(7*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^
3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)) - c)/(a*x - 1)) + 4*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)) + 5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*log(
-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)
*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 -
3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c), 1/2*(5*sqrt
(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*s
qrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x^2 - 2*
a*x - 1)) - 7*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(
-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x
- c)) + 2*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.07, size = 259, normalized size = 1.20

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x-5a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x+7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}\sqrt{\frac{1}{a}}\,x-\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,a^{\frac{3}{2}}c\sqrt{(ax+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^(5/2)*(1/a
)^(1/2)*((a*x+1)*x)^(1/2)*x-5*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+
2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-6*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)
-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+5*2^
```

```
(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/
2))/a^(3/2)/c/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c - \frac{c}{ax}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c - \frac{c}{a\,x}} \left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

3.461 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{a^2 x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-51(1$$

```
[Out] 9*(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-51/8*(1-1/a/x)
^(3/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)-2*a*(
1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(3/2)-15/4*(1-1/a/x)^(3/
2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)+a^2*(1-1/a/x)^(3/2)*x*(1+1/a/x)^
(1/2)/(a-1/x)^2/(c-c/a/x)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 151, 156, 63, 208, 206}

$$\frac{a^2 x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-51(1$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))^2*(c - c/(a*x))^
(3/2)) - (15*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))*(c - c/
(a*x))^(3/2)) + (a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))
^2*(c - c/(a*x))^(3/2)) + (9*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]]
)/(a*(c - c/(a*x))^(3/2)) - (51*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]/Sqrt[2]])/(4*Sqrt[2]*a*(c - c/(a*x))^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{-\frac{9}{2a}-\frac{7x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{\frac{18}{a^2}+\frac{12x}{a^3}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,d\right.}{4\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{3}\right.}{}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(51\left(1-\frac{1}{ax}\right)^{3}\right.}{}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(9\left(1-\frac{1}{ax}\right)^{3/2}\right.}{}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9\left(1-\frac{1}{ax}\right)^{3/2}}{a\,(c}$$

Mathematica [A] time = 0.13, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\left(4a^2x^2-23ax+15\right)+72(ax-1)^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-51\sqrt{2}\,(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{8ac(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(15 - 23*a*x + 4*a^2*x^2) + 72*
(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 51*Sqrt[2]*(-1 + a*x)^2*ArcTanh[S
qrt[1 + 1/(a*x)]/Sqrt[2]]))/(8*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

fricas [A] time = 0.82, size = 668, normalized size = 2.43

$$\left[\frac{51\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+72\left(a^3x\right.}{32\left(a^4c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(51*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c*
x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)) + 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(8*a
^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^4*x^4 - 19*a^3
*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2), 1/16*(51*sqrt(2)*(a
^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*
a*c*x - c)) - 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 4*(4*a^4*x^4 - 19*a^3*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2
 + 3*a^2*c^2*x - a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.07, size = 373, normalized size = 1.36

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,x^2-51a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^2-92a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,x+72\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^
(7/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2-51*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1
/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-92*a^(5/2)*(1/a)^(1/2)*
((a*x+1)*x)^(1/2)*x+72*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*a^3*(1/a)^(1/2)*x^2+102*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1
)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+60*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-1
44*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x+
72*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-51*2
```

```
^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1
/2))/a^(3/2)/c^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c - \frac{c}{ax}\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c - \frac{c}{a\,x}\right)^{3/2} \left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
[Out] Timed out
```

3.462 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=335

$$\frac{a^3x\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11\left(1-\right.}{}$$

```
[Out] 11*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-249/32*(1-1/a
/x)^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-5/
3*a^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^3/(c-c/a/x)^(5/2)-29/12*a*(1-
1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)-73/16*(1-1/a/x)^(5/2
)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a^3*(1-1/a/x)^(5/2)*x*(1+1/a/x)^(
1/2)/(a-1/x)^3/(c-c/a/x)^(5/2)
```

Rubi [A] time = 0.19, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 151, 156, 63, 208, 206}

$$\frac{a^3x\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11\left(1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-5*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(3*(a - x^(-1))^3*(c - c/(a*
x))^(5/2)) - (29*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(12*(a - x^(-1))^
2*(c - c/(a*x))^(5/2)) - (73*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(16*(a
- x^(-1))*(c - c/(a*x))^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]
*x)/((a - x^(-1))^3*(c - c/(a*x))^(5/2)) + (11*(1 - 1/(a*x))^(5/2)*ArcTanh[
Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - (249*(1 - 1/(a*x))^(5/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int \frac{-\frac{11}{2a}-\frac{9x}{2a^2}}{x\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\frac{33}{a^2}+\frac{25x}{a^3}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\ldots}}\right.}{6\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a^2\left(1-\ldots\right.\right.}{\ldots}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\ldots\right.}{\left(a-\frac{1}{x}\ldots\right.}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\ldots\right.}{\left(a-\frac{1}{x}\ldots\right.}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\ldots\right.}{\left(a-\frac{1}{x}\ldots\right.}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\ldots\right.}{\left(a-\frac{1}{x}\ldots\right.}$$

Mathematica [A] time = 0.16, size = 143, normalized size = 0.43

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\left(48a^3x^3-415a^2x^2+554ax-219\right)+1056(ax-1)^3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-747\sqrt{2}\,(ax-1\ldots\right.}{96ac^2(ax-1)^3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-219 + 554*a*x - 415*a^2*x^2 +
 48*a^3*x^3) + 1056*(-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]] - 747*Sqrt[2]*(
```

```
-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(96*a*c^2*Sqrt[c - c/(a*x)
]*(-1 + a*x)^3)
```

fricas [A] time = 1.63, size = 736, normalized size = 2.20

$$\left[\frac{747\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(747*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*l
og(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x
^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a
^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 -
4*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
 1)) + 8*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^3*x^3 + 335*a^2*x^2 - 219*a*x)*s
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), 1/192*(747*sqrt(2)*(a^4*x^4 - 4
*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
2*a*c*x - c)) - 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)
*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^
3*x^3 + 335*a^2*x^2 - 219*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.08, size = 480, normalized size = 1.43

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(192\sqrt{(ax+1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^3-747a^{\frac{7}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^3-1660a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(19
2*((a*x+1)*x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x^3-747*a^(7/2)*2^(1/2)*ln((2*2^(1/
2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^3-1660*a^(7/2)*(1/a)
^(1/2)*((a*x+1)*x)^(1/2)*x^2+1056*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*a^4*(1/a)^(1/2)*x^3+2241*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(
```

```
1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+2216*a^(5/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*x-3168*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*a^3*(1/a)^(1/2)*x^2-2241*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1
)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-876*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+
3168*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*
x-1056*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+
747*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))
*a^(1/2))/a^(3/2)/c^3/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c - \frac{c}{ax}\right)^{\frac{5}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c - \frac{c}{a\,x}\right)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
[Out] Timed out
```

3.463 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=221

$$\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{9\left(c-\frac{c}{ax}\right)^{7/2}\text{tanh}}{a\left(1-\right.}$$

```
[Out] -9*(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)-1/5*(80*a-7/x
)*(c-c/a/x)^(7/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(7/2)+3/5*(a-1/x)^2*(c-c/a/
x)^(7/2)*(1+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)+(a-1/x)^3*(c-c/a/x)^(7/2)*x*(1
+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A] time = 0.14, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 98, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{9\left(c-\frac{c}{ax}\right)^{7/2}\text{tanh}}{a\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -((80*a - 7/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^2*(1 - 1/(a*x))^
(7/2)) + (3*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*(1
 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)*
x)/(a^3*(1 - 1/(a*x))^(7/2)) - (9*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a
*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
```

```
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^2\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(\frac{9}{2a}+\frac{3x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\cdots\right.}{\cdots}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1-\cdots}}{a^3\left(1\cdots\right.}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1-\cdots}}{a^3\left(1\cdots\right.}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1-\cdots}}{a^3\left(1\cdots\right.}$$

Mathematica [A] time = 0.11, size = 101, normalized size = 0.46

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(5a^3x^3-92a^2x^2+16ax-2\right)-45a^2x^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{5a^3x^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 16*a*x - 92*a^2*x^2 + 5*a^3
*x^3) - 45*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(5*a^3*Sqrt[1 - 1/(a*x)]*x^
2)
```

fricas [A] time = 0.59, size = 415, normalized size = 1.88

$$\left[\frac{45\left(a^3c^3x^3-a^2c^3x^2\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(5a^4c^3x^4-87a^3c^3x^3-76a^2c^3\cdots\right.}{20\left(a^4x^3-a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 76*a^
2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^3 - a^3*x^2), 1/10*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 7
6*a^2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 161, normalized size = 0.73

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(10a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-184a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}-45\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^3a^3+32}{10x^2a^{\frac{7}{2}}(ax-1)\sqrt{(ax+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/10*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(10*a^(7/2)*
x^3*((a*x+1)*x)^(1/2)-184*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-45*ln(1/2*(2*((a*x+
1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^3*a^3+32*a^(3/2)*x*((a*x+1)*x)^(1/2
)-4*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/a^(7/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a\,x}\right)}^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.464 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=161

$$\frac{x\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -7*(c-c/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(5/2)-1/3*(16*a+1/x
)*(c-c/a/x)^(5/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)+(a-1/x)^2*(c-c/a/x)^(
5/2)*x*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)
```

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6182, 6179, 98, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] -((16*a + x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(3*a^2*(1 - 1/(a*x
))^(5/2)) + ((a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)*x)/(a^2*(
1 - 1/(a*x))^(5/2)) - (7*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a
*(1 - 1/(a*x))^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
```

```
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^2\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(\frac{7}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(7\left(c-\frac{c}{ax}\right)^{5/2}\right.}{}$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(7\left(c-\frac{c}{ax}\right)^{5/2}\right.}{}$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\text{t}}{a(1}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 0.55

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\left(3a^2x^2-22ax+2\right)-21ax\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{3a^2x\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 - 22*a*x + 3*a^2*x^2) - 21*a*x
*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

fricas [A] time = 0.66, size = 381, normalized size = 2.37

$$\left[\frac{21\left(a^2c^2x^2 - ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3 - 7acx - 4\left(2a^3x^3 + 3a^2x^2 + ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}} - c}{ax-1}\right) + 4\left(3a^3c^2x^3 - 19a^2c^2x^2 - 20ac^2x\right.}{12\left(a^3x^2 - a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*
x + 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^
2*x), 1/6*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*x + 2*c^2)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 144, normalized size = 0.89

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(6a^{\frac{5}{2}}x^2\sqrt{(ax+1)x} - 44a^{\frac{3}{2}}x\sqrt{(ax+1)x} - 21\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^2a^2 + 4\sqrt{(ax}\right.}{6x\,a^{\frac{5}{2}}\,(ax-1)\sqrt{(ax+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)-44*a^(3/2)*x*((a*x+1)*x)^(1/2)-21*ln(1/2*(2*((a*x+1)*x)
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^2*a^2+4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(
5/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-\frac{c}{a\,x}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.465 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -5*(c-c/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(3/2)-2*(c-c/a/x)^(
3/2)*(1+1/a/x)^(1/2)/a/(1-1/a/x)^(3/2)+(c-c/a/x)^(3/2)*x*(1+1/a/x)^(1/2)/(1
-1/a/x)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6182, 6179, 89, 80, 63, 208}

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))/(a*(1 - 1/(a*x))^(3/2)) + (Sqrt[
1 + 1/(a*x)]*(c - c/(a*x))^(3/2)*x)/(1 - 1/(a*x))^(3/2) - (5*(c - c/(a*x))^
(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx &= \frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}} \\
&= -\frac{\left(c-\frac{c}{ax}\right)^{3/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}} \\
&= \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{\left(c-\frac{c}{ax}\right)^{3/2}\operatorname{Subst}\left(\int\frac{-\frac{5}{2a}+\frac{x}{a^2}}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}} \\
&= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\left(5\left(c-\frac{c}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\right.}{2a\left(1-\frac{1}{ax}\right)^{3/2}} \\
&= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\left(5\left(c-\frac{c}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{-a+ax^2}\right.}{\left(1-\frac{1}{ax}\right)^{3/2}} \\
&= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.50

$$
\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{\frac{1}{ax}+1}\,(ax-2)-5\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 5*ArcTanh[Sqrt[1 + 1/(
a*x)]]))/(a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.47, size = 315, normalized size = 2.25

$$\left[\frac{5\,(acx-c)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2-acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\ 5\,(a\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(5*(a*c*x - c)*sqrt(-c)*arctan(2*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^
2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 118, normalized size = 0.84

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}-5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa-4\sqrt{(ax+1)\,x}\,\sqrt{a}\right)}{2a^{\frac{3}{2}}\,(ax-1)\,\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^(3/2)*x*((
a*x+1)*x)^(1/2)-5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a
-4*((a*x+1)*x)^(1/2)*a^(1/2))/a^(3/2)/(a*x-1)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-\frac{c}{a\,x}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

3.466 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=79

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)

Rubi [A] time = 0.16, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6177, 879, 875, 208}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]

[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.82

$$\frac{\sqrt{c-\frac{c}{ax}}\left(x\sqrt{\frac{1}{ax}+1}-\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a)
)/Sqrt[1 - 1/(a*x)]
```

fricas [B] time = 0.50, size = 297, normalized size = 3.76

$$\left[\frac{3\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, \frac{3\,(ax-1)\sqrt{-c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*x - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{ax}} \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.06, size = 101, normalized size = 1.28

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,\sqrt{a} - 3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(1/2)-3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x
-1)/((a*x+1)*x)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{ax}} \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c - \frac{c}{a\,x}} \sqrt{\frac{a\,x - 1}{a\,x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

3.467 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=78

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[Out] -arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)+x*(1-1/a^2/
x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6177, 873, 875, 208}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{c\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.85

$$\frac{\sqrt{1-\frac{1}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}-\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x - ArcTanh[Sqrt[1 + 1/(a*x)]]))/(a
*Sqrt[c - c/(a*x)])
```

fricas [B] time = 0.75, size = 299, normalized size = 3.83

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2cx-ac\right)}, \frac{(ax-1)\sqrt{-c}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*c*x - a*c), 1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*c*x - a*c)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.05, size = 104, normalized size = 1.33

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)\,x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{a}\,c\,(ax-1)\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(1/2)-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2
)/c/(a*x-1)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))), x)
```

3.468 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-(1-1/a/x)^(3/2)*
arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)+(1-1/a/x)^(3
/2)*x*(1+1/a/x)^(1/2)/(c-c/a/x)^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 103, 21, 83, 63, 208, 206}

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] ((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(c - c/(a*x))^(3/2) + ((1 - 1/(a*
x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(3/2)) - (Sqrt[2]*(1
 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*(c - c/(a*x))^(3/2
))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{-\frac{1}{2a}-\frac{x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int -\right.}{2a\left(c-\frac{c}{ax}\right.}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(2\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subs}}{a\,(}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [A] time = 0.08, size = 91, normalized size = 0.60

$$\frac{\left(1-\frac{1}{ax}\right)^{3/2}\left(ax\sqrt{\frac{1}{ax}+1}+\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] ((1 - 1/(a*x))^(3/2)*(a*Sqrt[1 + 1/(a*x)]*x + ArcTanh[Sqrt[1 + 1/(a*x)]] - Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*(c - c/(a*x))^(3/2))
```

fricas [A] time = 0.94, size = 522, normalized size = 3.46

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}+\frac{\sqrt{2}\,(acx-c)\log}{}}{4\left(a^2c^2x-ac^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)) + sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2
)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*c^2*x
- a*c^2), 1/2*(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 + a
*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x^2
 - 2*a*x - 1)) - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2
*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*c^
2*x - a*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.07, size = 162, normalized size = 1.07

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\;x\left(2\sqrt{(ax+1)\,x}\;a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{(ax+1)x}\;\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-\sqrt{2}\;\ln\left(\frac{2\sqrt{2}\;\sqrt{\frac{1}{a}}\;\sqrt{(ax+1)x}\;a+3ax+\cdots}{ax-1}\right.\right.}{2a^{\frac{3}{2}}c^{2}\,(ax-1)\sqrt{(ax+1)\,x}\;\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(3/2)*(1/a)^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1
/2))*a*(1/a)^(1/2)-2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*
a*x+1)/(a*x-1))*a^(1/2))/a^(3/2)/c^2/(a*x-1)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

3.469 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{2\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 3*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-9/4*(1-1/a/x)^
(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-3/2*(1
-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a*(1-1/a/x)^(5/2)*x*(
1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)
```

Rubi [A] time = 0.15, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6182, 6179, 103, 21, 99, 156, 63, 208, 206}

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{2\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (-3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))*(c - c/(a*x))^(5
/2)) + (a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*
x))^(5/2)) + (3*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(
a*x))^(5/2)) - (9*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(
2*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\mathrm{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\mathrm{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\mathrm{Subst}\left(\int \frac{-1-\frac{x}{2a}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x\right.}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(9\left(1-\frac{1}{ax}\right)^{5/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x\right.}{4a^2\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\mathrm{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1\right.}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\left(1-\right.}{2}$$

Mathematica [A] time = 0.11, size = 123, normalized size = 0.56

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\,(2ax-3)+12(ax-1)\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-9\sqrt{2}\,(ax-1)\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{4ac^2(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-3 + 2*a*x) + 12*(-1 + a*x)*Ar
cTanh[Sqrt[1 + 1/(a*x)]] - 9*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/S
qrt[2]]))/(4*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

fricas [A] time = 0.79, size = 596, normalized size = 2.72

$$\left[\frac{9\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+12\left(a^2x^2-2\,ax+\right.}{16\left(a^3c^3x^2-2\,a^2c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
- 1)) + 12*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 8*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), 1
/8*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2
- 2*a*c*x - c)) - 12*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 4*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x),sign(a*x+1)]s
ym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad
 Argument Value
```

maple [A] time = 0.07, size = 264, normalized size = 1.21

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(8a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x-9a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x-12\sqrt{(ax+1)\,x}\,a^{\frac{3}{2}}\right.}{8a^{\frac{3}{2}}c^3\,(ax-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(8*a^(5/2)*(1/a
)^(1/2)*((a*x+1)*x)^(1/2)*x-9*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a
*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-12*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/
2)+12*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)
*x-12*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+9
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^
(1/2))/a^(3/2)/c^3/(a*x-1)^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

3.470 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{5a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}}-115$$

```
[Out] 5*(1-1/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(7/2)-115/32*(1-1/a/
x)^(7/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(7/2)*2^(1/2)-5/4
*a*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(7/2)-35/16*(1-1/a/x
)^(7/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(7/2)+a^2*(1-1/a/x)^(7/2)*x*(1+1/
a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(7/2)
```

Rubi [A] time = 0.17, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6182, 6179, 103, 21, 99, 151, 156, 63, 208, 206}

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{5a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}}-115$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (-5*a*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))^2*(c - c/(a*x)
)^(7/2)) - (35*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(16*(a - x^(-1))*(c -
 c/(a*x))^(7/2)) + (a^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-
1))^2*(c - c/(a*x))^(7/2)) + (5*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]])/(a*(c - c/(a*x))^(7/2)) - (115*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(7/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
```

```
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{-\frac{5}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{-2-\frac{3x}{2a}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\cdots}}\right.}{4a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right.\right.}{\cdots}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(115\left(1-\cdots\right.\right.}{\cdots}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right.\right.}{\cdots}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{5\left(1-\frac{1}{ax}\right)\cdots}{\cdots}$$

Mathematica [A] time = 0.14, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax\sqrt{\frac{1}{ax}+1}\left(16a^2x^2-55ax+35\right)+160(ax-1)^2\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-115\sqrt{2}(ax-1)^2\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}\cdots}}{\sqrt{\cdots}}\right.\right.}{32ac^3(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(35 - 55*a*x + 16*a^2*x^2) + 16
0*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 115*Sqrt[2]*(-1 + a*x)^2*ArcTan
h[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(32*a*c^3*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

fricas [A] time = 0.70, size = 668, normalized size = 2.41

$$\left[\frac{115\sqrt{2}\left(a^{3}x^{3}-3a^{2}x^{2}+3ax-1\right)\sqrt{c}\log\left(-\frac{17a^{3}cx^{3}-3a^{2}cx^{2}-13acx-4\sqrt{2}\left(3a^{3}x^{3}+4a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3a^{2}x^{2}+3ax-1}\right)+160\left(a^{3}x^{3}\right.}{128\left(a^{4}c^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/128*(115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*
c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a
^2*x^2 + 3*a*x - 1)) + 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(
8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(16*a^4*x^4 - 39
*a^3*x^3 - 20*a^2*x^2 + 35*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4), 1/64*(115*sqrt
(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) - 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2
*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) + 4*(16*a^4*x^4 - 39*a^3*x^3 - 20*a^2*x^2 + 35*
a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^4*x^3 - 3*a^
3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.08, size = 371, normalized size = 1.34

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(64a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x^{2}-115a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{2}-220a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/64*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(64*a^(7/2)*(1
/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2-115*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/
2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-220*a^(5/2)*(1/a)^(1/2)*((a*x+
1)*x)^(1/2)*x+160*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3
*(1/a)^(1/2)*x^2+230*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^
(1/2)*a+3*a*x+1)/(a*x-1))*x+140*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-320*l
n(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x+160*
ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-115*2^(
```

```
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2
))/a^(3/2)/c^4/(a*x-1)^3/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(7/2),x)
[Out] Timed out
```

3.471 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=163

$$-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

```
[Out] -5/3*c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c/a/x)^(7/2)*x-11*c^(
7/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+32*c^(7/2)*arctanh(1/2*(c-c/a/x)^(1
/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-21*c^3*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.28, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-21*c^3*Sqrt[c - c/(a*x)])/a - (5*c^2*(c - c/(a*x))^(3/2))/(3*a) + (3*c*(c
 - c/(a*x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (11*c^(7/2)*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$$
$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}{1+ax}dx$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}x}{1+ax}dx}{c}$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{9/2}}{a+\frac{1}{x}}dx}{c}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{9/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$= \left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}\left(\frac{11c^2}{2}+\frac{3c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$= \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{2\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{55c^3}{4}-\frac{25c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{5c}$$
$$= -\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{4\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}\left(\frac{165c^4}{8}-\frac{315c^4x}{8a}\right)}{x(a+x)}\right.}{15c}$$
$$= -\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{8\,\text{Subst}\left(\int \right.}{}$$
$$= -\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\left(11c^4\right)\text{Subst}}{}$$
$$= -\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\left(11c^3\right)\text{Subst}$$
$$= -\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{11c^{7/2}\tanh^{-1}}{a}$$

Mathematica [A] time = 0.27, size = 125, normalized size = 0.77

$$\frac{c^3\left(15a^3x^3-376a^2x^2+52ax-6\right)\sqrt{c-\frac{c}{ax}}}{15a^3x^2}-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(-6 + 52*a*x - 376*a^2*x^2 + 15*a^3*x^3))/(15*a^3*x^
2) - (11*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2
)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.59, size = 323, normalized size = 1.98

$$\left[\frac{480\sqrt{2}a^2c^{\frac{7}{2}}x^2\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+165a^2c^{\frac{7}{2}}x^2\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(15a^3c^3x^3-\right.}{30a^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/30*(480*sqrt(2)*a^2*c^(7/2)*x^2*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 165*a^2*c^(7/2)*x^2*log(-2*a*c*x +
2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(15*a^3*c^3*x^3 - 376*a^2*c^
3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), -1/15*(480*
sqrt(2)*a^2*sqrt(-c)*c^3*x^2*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(
a*x))/c) - 165*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))
/c) - (15*a^3*c^3*x^3 - 376*a^2*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x -
 c)/(a*x)))/(a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [B] time = 0.08, size = 281, normalized size = 1.72

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(-1110\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4+480a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\,x^4+660a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2\sqrt{\frac{1}{a}}+555\ln\left(\frac{2\sqrt{a\,x^2}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/30*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3/a^(7/2)*(-1110*(a*x^2-x)^(1/2)*a^(7/2)*(
1/a)^(1/2)*x^4+480*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^4+660*a^(5/2)*(a
*x^2-x)^(3/2)*x^2*(1/a)^(1/2)+555*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))*(1/a)^(1/2)*x^4*a^3-720*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*
x-1)/a^(1/2))*(1/a)^(1/2)*x^4*a^3-480*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)
^(1/2)*a-3*a*x+1)/(a*x+1))*a^(5/2)*2^(1/2)*x^4-92*a^(3/2)*(a*x^2-x)^(3/2)*x
*(1/a)^(1/2)+12*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/(1/a
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)/(a*x + 1), x)
```

3.472 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=138

$$-\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

```
[Out] 1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-9*c^(5/2)*arctanh((c-c/a/x)^(1/2)
/c^(1/2))/a+16*c^(5/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)
/a-7*c^2*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.25, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}-\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x)
)^(5/2)*x - (9*c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (16*Sqrt[2]*
c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$$
$$= -\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}{1+ax}dx$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{1+ax}dx}{c}$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx}{c}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x^2(a+x)}dx, x, \frac{1}{x}\right)}{c}$$
$$= \left(c-\frac{c}{ax}\right)^{5/2}x + \frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{9c^2}{2}+\frac{c^2x}{2a}\right)}{x(a+x)}dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}\left(\frac{27c^3}{4}-\frac{21c^3x}{4a}\right)}{x(a+x)}dx, x, \frac{1}{x}\right)}{3c}$$
$$= -\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a} + \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x + \frac{4\,\text{Subst}\left(\int \frac{\frac{27c^4}{8}-\frac{69c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{3c}$$
$$= -\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a} + \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x + \frac{(9c^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}dx, x, \frac{1}{x}\right)}{2a}$$
$$= -\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a} + \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x - (9c^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}dx, x,\right.$$
$$= -\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a} + \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a} + \left(c-\frac{c}{ax}\right)^{5/2}x - \frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{16\sqrt{2}}{}$$

Mathematica [A] time = 0.11, size = 116, normalized size = 0.84

$$\frac{c^2\left(3a^2x^2-26ax+2\right)\sqrt{c-\frac{c}{ax}} - 27ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) + 48\sqrt{2}\,ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(2 - 26*a*x + 3*a^2*x^2) - 27*a*c^(5/2)*x*ArcTanh[Sq
rt[c - c/(a*x)]/Sqrt[c]] + 48*Sqrt[2]*a*c^(5/2)*x*ArcTanh[Sqrt[c - c/(a*x)]
/(Sqrt[2]*Sqrt[c])])/(3*a^2*x)
```

fricas [A] time = 0.73, size = 285, normalized size = 2.07

$$\left[\frac{48\sqrt{2}ac^{\frac{5}{2}}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+27ac^{\frac{5}{2}}x\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\left(3a^{2}c^{2}x^{2}-26ac^{2}x+\right.}{6a^{2}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(48*sqrt(2)*a*c^(5/2)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(
a*x)) + 3*a*c*x - c)/(a*x + 1)) + 27*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*sq
rt((a*c*x - c)/(a*x)))/(a^2*x), -1/3*(48*sqrt(2)*a*sqrt(-c)*c^2*x*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 27*a*sqrt(-c)*c^2*x*arctan(
sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*
sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [B] time = 0.05, size = 257, normalized size = 1.86

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^{2}\left(-90\sqrt{a\,x^{2}-x}\;a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\;x^{3}+48a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\;\sqrt{(ax-1)\,x}\;x^{3}+48a^{\frac{3}{2}}\left(a\,x^{2}-x\right)^{\frac{3}{2}}x\sqrt{\frac{1}{a}}\;+45\ln\left(\frac{2\sqrt{a\,x^{2}-x}\;\sqrt{a}+2ax-}{2\sqrt{a}}\right.\right.}{6x^{2}a^{\frac{5}{2}}\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2/a^(5/2)*(-90*(a*x^2-x)^(1/2)*a^(5/2)*(1/a
)^(1/2)*x^3+48*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3+48*a^(3/2)*(a*x^2-
x)^(3/2)*x*(1/a)^(1/2)+45*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2
))*(1/a)^(1/2)*x^3*a^2-48*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1
)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3-72*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^3*a^2-4*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))
/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)*(a*x-1)/(a*x+1),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)/(a*x + 1), x)
```

3.473 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=113

$$-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

```
[Out] (c-c/a/x)^(3/2)*x-7*c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+8*c^(3/2)*ar
ctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-c*(c-c/a/x)^(1/2)/a
```

Rubi [A] time = 0.24, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 98, 154, 156, 63, 208}

$$-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x - (7*c^(3/2)*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/a + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx$$
$$=-\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}{1+ax}dx$$
$$=\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x}{1+ax}dx}{c}$$
$$=\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}dx}{c}$$
$$=-\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}\left(\frac{7c^2}{2}-\frac{c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{2\,\text{Subst}\left(\int \frac{\frac{7c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{c}$$
$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{(7c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}-\frac{(8c^2)\,\text{Subst}\left(\int \cdots\right.}{}$$
$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-(7c)\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)+(16c)\,\text{Sub}\cdots$$
$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.84

$$\frac{-7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)+c(ax-2)\sqrt{c-\frac{c}{ax}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(-2 + a*x) - 7*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[
c]] + 8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.43, size = 235, normalized size = 2.08

$$\left[\frac{8\sqrt{2}\,c^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+7\,c^{\frac{3}{2}}\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},-\frac{8\sqrt{2}\sqrt{\cdots}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(8*sqrt(2)*c^(3/2)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 + 3*a*c*x - c)/(a*x + 1)) + 7*c^(3/2)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) + 2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(8*sqr
t(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 7*
sqrt(-c)*c*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt(
(a*c*x - c)/(a*x)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [B] time = 0.06, size = 229, normalized size = 2.03

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c \left(-10\sqrt{a x^2-x}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\, x^2+8a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\, x^2+4\left(a x^2-x\right)^{\frac{3}{2}}\sqrt{a}\,\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{a x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right.\right.}{2x a^{\frac{3}{2}}\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)/x*c/a^(3/2)*(-10*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1
/2)*x^2+8*a^(3/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2+4*(a*x^2-x)^(3/2)*a^(1/
2)*(1/a)^(1/2)+5*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^
(1/2)*x^2*a-8*a^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a
-3*a*x+1)/(a*x+1))*x^2-12*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1
/2))*(1/a)^(1/2)*x^2*a)/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x-1)}{a\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)/(a*x + 1), x)
```

3.474 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=92

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+4*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 98, 156, 63, 208}

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{1+ax}\,dx$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \sqrt{c-\frac{c}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \sqrt{c-\frac{c}{ax}}\,x + \frac{(5c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{(4c)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= \sqrt{c-\frac{c}{ax}}\,x - 5\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) + 8\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x,$$
$$= \sqrt{c-\frac{c}{ax}}\,x - \frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 1.00

$$x\sqrt{c-\frac{c}{ax}} - \frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.87, size = 219, normalized size = 2.38

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}} + 4\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right) + 5\sqrt{c}\log\left(-2\,acx + 2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}} + c\right)}{2\,a}, \frac{ax\sqrt{\frac{acx-c}{ax}} -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x
- c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.05, size = 190, normalized size = 2.07

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(2\sqrt{a x^{2}-x}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-4\sqrt{(ax-1)x}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-\ln\left(\frac{2\sqrt{a x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a\sqrt{\frac{1}{a}}+4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\, a \cdots}{ax+1}\right) \cdots\right)}{2\sqrt{(ax-1)x}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x
-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*a*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/
2)*a-3*a*x+1)/(a*x+1))*a^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.475 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=95

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -3*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)+2*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)+x*(c-c/a/x)^(1/2)/c
```

Rubi [A] time = 0.21, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 99, 156, 63, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$= -\int \frac{1-ax}{\sqrt{c-\frac{c}{ax}}(1+ax)}\,dx$$

$$= \frac{a\int \frac{\sqrt{c-\frac{c}{ax}}\,x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}+\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c} + \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{2\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c} - \frac{3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c} + \frac{4\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.05, size = 95, normalized size = 1.00

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

fricas [A] time = 0.61, size = 234, normalized size = 2.46

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}} + 2\sqrt{2}\sqrt{c}\,\log\left(-\frac{\frac{2\sqrt{2}\,ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right) + 3\sqrt{c}\,\log\left(-2\,acx + 2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}} + c\right)}{2\,ac}, -\frac{2\sqrt{2}\,c\sqrt{-\frac{1}{c}}\,\text{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 2*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x
*sqrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + 3*sqrt(c)*log(-2
*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c), -(2*sqrt(2)*c*s
qrt(-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1))
- a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)
/(a*x))/c))/(a*c)]
```

giac [A] time = 0.16, size = 131, normalized size = 1.38

$$-ac\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c}-\frac{3\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c}-\frac{\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] -a*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*s
qrt(-c)*c) - 3*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c) -
sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c))
```

maple [A] time = 0.04, size = 136, normalized size = 1.43

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2\sqrt{(ax-1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+3\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}+2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}\right)}{2\sqrt{(ax-1)x}\,c\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(-2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+3*ln
(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+2*2^(1/2)
*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/(
(a*x-1)*x)^(1/2)/c/a^(3/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)), x)
```

3.476 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}+\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}$$

```
[Out] -arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(3/2)+arctanh(1/2*(c-c/a/x)^(1/2)*2^(
1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)+x*(c-c/a/x)^(1/2)/c^2
```

Rubi [A] time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 103, 21, 83, 63, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$$
$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx$$
$$= \frac{a\int \frac{x}{\sqrt{c-\frac{c}{ax}}\,(1+ax)}\,dx}{c}$$
$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}\,dx}{c}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{\frac{c}{2}-\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{2c^2}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2}$$
$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [A] time = 0.06, size = 94, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}} + \frac{x\sqrt{c-\frac{c}{ax}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

fricas [A] time = 0.58, size = 231, normalized size = 2.46

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}} + \sqrt{2}\sqrt{c}\log\left(-\frac{\frac{2\sqrt{2}\,ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right) + \sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac^2}, -\frac{\sqrt{2}\,c\sqrt{-\frac{1}{c}}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x*s
qrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + sqrt(c)*log(-2*a*c
*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c^2), -(sqrt(2)*c*sqrt(
-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1)) - a*
x*sqrt((a*c*x - c)/(a*x)) - sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x)
)/c))/(a*c^2)]
```

giac [A] time = 0.15, size = 130, normalized size = 1.38

$$-ac\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^2}-\frac{\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^2}-\frac{\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] -a*c*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqr
t(-c)*c^2) - arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^2) -
sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^2))
```

maple [A] time = 0.05, size = 134, normalized size = 1.43

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}+\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\sqrt{a}\right)}{2a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\,c^2\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1
/2)+ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2
))/((a*x-1)*x)^(1/2)/c^2/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a\,x-1}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

3.477 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}$$

```
[Out] arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)+1/2*arctanh(1/2*(c-c/a/x)^(1/2)*
2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)-2/a/c^2/(c-c/a/x)^(1/2)+x/c^2/(c-c/a/x)^
(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] -2/(a*c^2*Sqrt[c - c/(a*x)]) + x/(c^2*Sqrt[c - c/(a*x)]) + ArcTanh[Sqrt[c -
 c/(a*x)]/Sqrt[c]]/(a*c^(5/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c]
)]/(Sqrt[2]*a*c^(5/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{c}{2}-\frac{3cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\frac{c^2}{2}+\frac{c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3} + \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\right.}{c^3}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}$$

Mathematica [C] time = 0.06, size = 70, normalized size = 0.60

$$\frac{-{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a-\frac{1}{x}}{2a}\right) - {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{1}{ax}\right) + ax}{ac^2\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (a*x - Hypergeometric2F1[-1/2, 1, 1/2, (a - x^(-1))/(2*a)] - Hypergeometric
2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*c^2*Sqrt[c - c/(a*x)])
```

fricas [A] time = 0.60, size = 287, normalized size = 2.47

$$\left[\frac{\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\,(ax-1)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+4\left(a^2x^2-2\right.}{4\left(a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 2*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a
*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(a^2*x^2 - 2*a*x)*sqrt((a*c*x -
 c)/(a*x)))/(a^2*c^3*x - a*c^3), -1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*(a*x - 1)*sqrt(-c)*arctan
(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(a^2*x^2 - 2*a*x)*sqrt((a*c*x - c)
/(a*x)))/(a^2*c^3*x - a*c^3)]
```

giac [A] time = 0.14, size = 166, normalized size = 1.43

$$-\frac{1}{2}\,ac\left(\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{acx-c}{ax}}}{2\,\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^3}+\frac{2\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^3}+\frac{2\left(c-\frac{2\,(acx-c)}{ax}\right)}{\left(c\sqrt{\frac{acx-c}{ax}}-\frac{(acx-c)\sqrt{\frac{acx-c}{ax}}}{ax}\right)a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*a*c*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2
*sqrt(-c)*c^3) + 2*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c
^3) + 2*(c - 2*(a*c*x - c)/(a*x))/((c*sqrt((a*c*x - c)/(a*x)) - (a*c*x - c)
*sqrt((a*c*x - c)/(a*x))/(a*x))*a^2*c^3))
```

maple [B] time = 0.06, size = 370, normalized size = 3.19

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(8\sqrt{\frac{1}{a}}\;a^{\frac{7}{2}}\sqrt{(ax-1)\,x}\;x^2+2\sqrt{\frac{1}{a}}\;\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^2a^3-4\sqrt{\frac{1}{a}}\;a^{\frac{5}{2}}\left((ax-1)\,x\right)^{\frac{3}{2}}-\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(5/2),x)
```

```
[Out] 1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(8*(1/a)^(1/2)*a^(7/2)*((a*x-1)*x)^(1/2
)*x^2+2*(1/a)^(1/2)*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x
^2*a^3-4*(1/a)^(1/2)*a^(5/2)*((a*x-1)*x)^(3/2)-ln((2*2^(1/2)*(1/a)^(1/2)*((
a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(5/2)*2^(1/2)*x^2-16*(1/a)^(1/2)*a^(5
/2)*((a*x-1)*x)^(1/2)*x-4*(1/a)^(1/2)*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*x*a^2+2*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*
x+1)/(a*x+1))*a^(3/2)*2^(1/2)*x+8*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+2*l
n(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/2)*
ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((
a*x-1)*x)^(1/2)/c^3/(a*x-1)^2/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax - 1}{\left(- c \left(-1 + \frac{1}{ax}\right)\right)^{\frac{5}{2}} \left(ax + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

3.478 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}} dx$

Optimal. Leaf size=147

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -4/3/a/c^2/(c-c/a/x)^(3/2)+x/c^2/(c-c/a/x)^(3/2)+3*arctanh((c-c/a/x)^(1/2)/
c^(1/2))/a/c^(7/2)+1/4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a/c^(7/
2)*2^(1/2)-7/2/a/c^3/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] -4/(3*a*c^2*(c - c/(a*x))^(3/2)) - 7/(2*a*c^3*Sqrt[c - c/(a*x)]) + x/(c^2*(
c - c/(a*x))^(3/2)) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2)) +
ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}-\frac{5cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{9c^2}{2}+\frac{6c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^4}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{9c^3}{2}-\frac{21c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c^6}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4ac^3} - \frac{3\,\text{Su}}{}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{2c^4} + \frac{3}{}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.54

$$\frac{x\left(-{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a-\frac{1}{x}}{2a}\right)-3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{1}{ax}\right)+3ax\right)}{3c^3(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (x*(3*a*x - Hypergeometric2F1[-3/2, 1, -1/2, (a - x^(-1))/(2*a)] - 3*Hyperg
eometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c^3*Sqrt[c - c/(a*x)]*(-1 + a*
x))
```

fricas [A] time = 0.53, size = 359, normalized size = 2.44

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+36\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}\right.}{24\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*
sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 36*(a^2*x^2 - 2*a*x + 1
)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(6*
a^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^
2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 36*(a^2*x^2 - 2*a*x + 1)*sq
rt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(6*a^3*x^3 - 29*a^2*x
^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

giac [A] time = 0.14, size = 187, normalized size = 1.27

$$-\frac{1}{12}\,ac\left(\frac{2\left(2\,c+\frac{15\,(acx-c)}{ax}\right)x}{(acx-c)ac^4\sqrt{\frac{acx-c}{ax}}}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^4}+\frac{36\,\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^4}-\frac{12\sqrt{\frac{acx-c}{ax}}}{a^2\left(c-\frac{acx-c}{ax}\right)c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/12*a*c*(2*(2*c + 15*(a*c*x - c)/(a*x))*x/((a*c*x - c)*a*c^4*sqrt((a*c*x
- c)/(a*x))) + 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c
))/(a^2*sqrt(-c)*c^4) + 36*arctan(sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sq
rt(-c)*c^4) - 12*sqrt((a*c*x - c)/(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^4))
```

maple [B] time = 0.06, size = 497, normalized size = 3.38

$$\sqrt{\frac{c(ax-1)}{ax}}\,x\left(84a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\,x^3+36\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,x^3a^4-3a^{\frac{7}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,a-3ax+1}{ax+1}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/24*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(84*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1
/2)*x^3+36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2
)*x^3*a^4-3*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3
*a*x+1)/(a*x+1))*x^3-60*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(3/2)*x-252*(1/a)^(
1/2)*a^(7/2)*((a*x-1)*x)^(1/2)*x^2-108*(1/a)^(1/2)*ln(1/2*(2*((a*x-1)*x)^(1
/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^3+9*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*
x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(5/2)*2^(1/2)*x^2+52*(1/a)^(1/2)*a^(5/2)*((a
*x-1)*x)^(3/2)+252*(1/a)^(1/2)*a^(5/2)*((a*x-1)*x)^(1/2)*x+108*(1/a)^(1/2)*
ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*a^2-9*ln((2*2^(1/2)
*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(3/2)*2^(1/2)*x-84*((a
*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)
```

```
*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^3/(1/a
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a\,x}\right)}^{7/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(7/2),x)
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

3.479 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

Optimal. Leaf size=172

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] -6/5/a/c^2/(c-c/a/x)^(5/2)-11/6/a/c^3/(c-c/a/x)^(3/2)+x/c^2/(c-c/a/x)^(5/2)
+5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(9/2)+1/8*arctanh(1/2*(c-c/a/x)^(1/
2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)-21/4/a/c^4/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6167, 6133, 25, 514, 375, 103, 152, 156, 63, 208}

$$\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] -6/(5*a*c^2*(c - c/(a*x))^(5/2)) - 11/(6*a*c^3*(c - c/(a*x))^(3/2)) - 21/(4
*a*c^4*Sqrt[c - c/(a*x)]) + x/(c^2*(c - c/(a*x))^(5/2)) + (5*ArcTanh[Sqrt[c
 - c/(a*x)]/Sqrt[c]])/(a*c^(9/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt
[c])]/(4*Sqrt[2]*a*c^(9/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{5c}{2}-\frac{7cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{\frac{25c^2}{2}+\frac{15c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^4}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{75c^3}{2}-\frac{165c^3x}{4a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^6}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{\frac{75c^4}{2}+\frac{315c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\right.}{15c^8}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\right.}{8ac^4}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x\right.}{4c^5}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}} +$$

Mathematica [C] time = 0.07, size = 82, normalized size = 0.48

$$\frac{ax^2\left(-{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{a-\frac{1}{x}}{2a}\right)-5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{1}{ax}\right)+5ax\right)}{5c^4(ax-1)^2\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] (a*x^2*(5*a*x - Hypergeometric2F1[-5/2, 1, -3/2, (a - x^(-1))/(2*a)] - 5*Hy
pergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]))/(5*c^4*Sqrt[c - c/(a*x)]*(-1
+ a*x)^2)
```

fricas [A] time = 0.67, size = 431, normalized size = 2.51

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+600\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-2\right.}{240\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(2*sqrt(2
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 600*(a^3*
x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c
*x - c)/(a*x)) + c) + 4*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*
sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5
), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/2
*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 600*(a^3*x^3 - 3*a^2*x^2 + 3
*a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(60*a^4*x
^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^5
*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

giac [A] time = 0.14, size = 207, normalized size = 1.20

$$-\frac{1}{120}ac\left(\frac{2\left(12c^2+\frac{50(acx-c)c}{ax}+\frac{255(acx-c)^2}{a^2x^2}\right)x^2}{(acx-c)^2c^5\sqrt{\frac{acx-c}{ax}}}+\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}c^5}+\frac{600\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^5}-\frac{120\sqrt{\frac{ac}{}}}{a^2\left(c-\frac{acx}{a}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/120*a*c*(2*(12*c^2 + 50*(a*c*x - c)*c/(a*x) + 255*(a*c*x - c)^2/(a^2*x^2
))*x^2/((a*c*x - c)^2*c^5*sqrt((a*c*x - c)/(a*x))) + 15*sqrt(2)*arctan(1/2*
sqrt(2)*sqrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^5) + 600*arctan(s
qrt((a*c*x - c)/(a*x))/sqrt(-c))/(a^2*sqrt(-c)*c^5) - 120*sqrt((a*c*x - c)/
(a*x))/(a^2*(c - (a*c*x - c)/(a*x))*c^5))
```

maple [B] time = 0.06, size = 626, normalized size = 3.64

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-1260\sqrt{(ax-1)x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^4+1020\left((ax-1)x\right)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^2-600\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,x^4a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a/x)^(9/2),x)
```

```
[Out] -1/240*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-1260*((a*x-1)*x)^(1/2)*a^(11/2)*(1
/a)^(1/2)*x^4+1020*((a*x-1)*x)^(3/2)*a^(9/2)*(1/a)^(1/2)*x^2-600*ln(1/2*(2*
((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^4*a^5+15*a^(9/2)*
2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^4
```

```
+5040*a^(9/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3-1792*a^(7/2)*(1/a)^(1/2)*((
a*x-1)*x)^(3/2)*x+2400*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*(1/a)^(1/2)*x^3*a^4-60*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)
*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3-7560*(1/a)^(1/2)*a^(7/2)*((a*x-1)*x)^(1/2
)*x^2+820*(1/a)^(1/2)*a^(5/2)*((a*x-1)*x)^(3/2)-3600*(1/a)^(1/2)*ln(1/2*(2*
((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a^3+90*ln((2*2^(1/2)*(1/a)^
(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(5/2)*2^(1/2)*x^2+5040*(1/a)^
(1/2)*a^(5/2)*((a*x-1)*x)^(1/2)*x+2400*(1/a)^(1/2)*ln(1/2*(2*((a*x-1)*x)^(1
/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x*a^2-60*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x
)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(3/2)*2^(1/2)*x-1260*((a*x-1)*x)^(1/2)*a^(3/2
)*(1/a)^(1/2)-600*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(
1/a)^(1/2)+15*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1
)/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^5/(a*x-1)^4/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{9/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{9}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(9/2),x)
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(9/2)*(a*x + 1)), x)
```

3.480 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=335

$$\frac{x\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{65\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

```
[Out] -15*(c-c/a/x)^(9/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(9/2)+10*(a-1/x)^4
*(c-c/a/x)^(9/2)/a^5/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)+(a-1/x)^5*(c-c/a/x)^(9
/2)*x/a^5/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)+5/7*(304*a-65/x)*(c-c/a/x)^(9/2)*
(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(9/2)+135/7*(a-1/x)^2*(c-c/a/x)^(9/2)*(1+1/a/
x)^(1/2)/a^3/(1-1/a/x)^(9/2)+65/7*(a-1/x)^3*(c-c/a/x)^(9/2)*(1+1/a/x)^(1/2)
/a^4/(1-1/a/x)^(9/2)
```

Rubi [A] time = 0.20, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 150, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{65\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^4*(c - c/(a*x))^(9/2))/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1
/(a*x)]) + (5*(304*a - 65/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(7*a^2*
(1 - 1/(a*x))^(9/2)) + (135*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(9/2))/(7*a^3*(1 - 1/(a*x))^(9/2)) + (65*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(
c - c/(a*x))^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^5*(c - c/(a
*x))^(9/2)*x)/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]) - (15*(c - c/(a*x
))^(9/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^6}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(\frac{15}{2a}+\frac{5x}{2a^2}\right)\left(1-\frac{x}{a}\right)^4}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{Subst}\left(\int\right.}{\left(1-\frac{1}{ax}\right.}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right.}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1}}{7a^4\left(1-\right.}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2}{7a^3\left(1\right.}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2}{7a^3\left(1\right.}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2}{7a^3\left(1\right.}$$

Mathematica [C] time = 0.13, size = 140, normalized size = 0.42

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(7a^5x^5+70a^4x^4\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)+1685a^4x^4-35a^4x^4\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+720a^3x^3\right.}{7a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(-2 + 20*a*x - 110*a^2*x^2 + 720*a^3*x^3 + 1685*a^4*
x^4 + 7*a^5*x^5 - 35*a^4*Sqrt[1 + 1/(a*x)]*x^4*ArcTanh[Sqrt[1 + 1/(a*x)]] +
```

```
70*a^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(7*a^5*Sqrt[1 -
1/(a^2*x^2)]*x^4)
```

fricas [A] time = 0.69, size = 437, normalized size = 1.30

$$\left[\frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(7\,a^5c^4x^5+1755\,a^4c^4x^4+720\right.}{28\left(a^5x^4-a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/28*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 + 72
0*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt((a*x - 1)/(a*x +
 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/14*(105*(a^4*c^4*x^4 -
 a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(7*a^5*c^4*x
^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*
c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.08, size = 229, normalized size = 0.68

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(14\sqrt{(ax+1)\,x}\,a^{\frac{11}{2}}x^5+3510a^{\frac{9}{2}}\sqrt{(ax+1)\,x}\,x^4+1440a^{\frac{7}{2}}x^3\sqrt{(ax+1)\,x}-105\ln\left(\frac{2\sqrt{(}\right.\right.}{14\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/14*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^4*(1
4*((a*x+1)*x)^(1/2)*a^(11/2)*x^5+3510*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+1440*a^
(7/2)*x^3*((a*x+1)*x)^(1/2)-105*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1
)/a^(1/2))*x^5*a^5-105*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*x^4*a^4-220*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+40*a^(3/2)*x*((a*x+1)*x)^(1/2)-
4*((a*x+1)*x)^(1/2)*a^(1/2))/x^3/a^(9/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c-\frac{c}{a\,x}\right)^{9/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

3.481 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=277

$$\frac{x\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -13*(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)+10*(a-1/x)^3
*(c-c/a/x)^(7/2)/a^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+(a-1/x)^4*(c-c/a/x)^(7
/2)*x/a^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+1/15*(1360*a-311/x)*(c-c/a/x)^(7/
2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(7/2)+47/5*(a-1/x)^2*(c-c/a/x)^(7/2)*(1+1/
a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A] time = 0.17, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 98, 150, 153, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^3*(c - c/(a*x))^(7/2))/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1
/(a*x)]) + ((1360*a - 311/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2
*(1 - 1/(a*x))^(7/2)) + (47*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(7/2))/(5*a^3*(1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^4*(c - c/(a*x))^(7/2)*x)
/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - (13*(c - c/(a*x))^(7/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
```

```
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(\frac{13}{2a}+\frac{3x}{2a^2}\right)\left(1-\frac{x}{a}\right)^3}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{1}{4a}\right.}{}\right.}{\left(1-\frac{1}{ax}\right)^{7/}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+}}{5a^3\left(1-\right.}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+}}{5a^3\left(1-\right.}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+}}{5a^3\left(1-\right.}$$

Mathematica [C] time = 0.11, size = 132, normalized size = 0.48

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(15a^4x^4+150a^3x^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)+1441a^3x^3-45a^3x^3\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+548a^2x^2-\right.}{15a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(6 - 62*a*x + 548*a^2*x^2 + 1441*a^3*x^3 + 15*a^4*x^
4 - 45*a^3*Sqrt[1 + 1/(a*x)]*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]] + 150*a^3*x^3*H
ypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(15*a^4*Sqrt[1 - 1/(a^2*x^2)]
*x^3)
```

fricas [A] time = 0.51, size = 415, normalized size = 1.50

$$\left[\frac{195\left(a^3c^3x^3 - a^2c^3x^2\right)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3 - 7\,acx - 4\left(2\,a^3x^3 + 3\,a^2x^2 + ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}} - c}{ax-1}\right) + 4\left(15\,a^4c^3x^4 + 1591\,a^3c^3x^3 + \right.}{60\left(a^4x^3 - a^3x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3 + 5
48*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqr
t(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 1591*a^3*c^
3*x^3 + 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.08, size = 212, normalized size = 0.77

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(30a^{\frac{9}{2}}\sqrt{(ax+1)\,x}\,x^4 + 3182a^{\frac{7}{2}}x^3\sqrt{(ax+1)\,x} - 195\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^4a^4 + \right.}{30\,(ax-1)^2\,x^2a^{\frac{7}{2}}\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/30*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^3*(3
0*a^(9/2)*((a*x+1)*x)^(1/2)*x^4+3182*a^(7/2)*x^3*((a*x+1)*x)^(1/2)-195*ln(1
/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^4*a^4+1096*a^(5/2)*x^2*
((a*x+1)*x)^(1/2)-195*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*x^3*a^3-124*a^(3/2)*x*((a*x+1)*x)^(1/2)+12*((a*x+1)*x)^(1/2)*a^(1/2))/x^2/
a^(7/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c-\frac{c}{a\,x}\right)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

3.482 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=219

$$\frac{x\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{11\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}\right.}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -11*(c-c/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(5/2)+10*(a-1/x)^2
*(c-c/a/x)^(5/2)/a^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+(a-1/x)^3*(c-c/a/x)^(5
/2)*x/a^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+1/3*(112*a-29/x)*(c-c/a/x)^(5/2)*
(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)
```

Rubi [A] time = 0.15, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 98, 150, 147, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{11\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}\right.}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^2*(c - c/(a*x))^(5/2))/(a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1
/(a*x)]) + ((112*a - 29/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(3*a^2*(1
 - 1/(a*x))^(5/2)) + ((a - x^(-1))^3*(c - c/(a*x))^(5/2)*x)/(a^3*(1 - 1/(a*
x))^(5/2)*Sqrt[1 + 1/(a*x)]) - (11*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
```

```
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx = \frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(\frac{11}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2a\left(c-\frac{c}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \cdots\right.}{\left(1-\frac{1}{ax}\cdots\right.}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\cdots\right.}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1\cdots}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\cdots\right.}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1\cdots}}$$

$$= \frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\cdots\right.}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1\cdots}}$$

Mathematica [C] time = 0.10, size = 124, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(3a^3x^3+30a^2x^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)+103a^2x^2-3a^2x^2\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+32ax-2\right)}{3a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 32*a*x + 103*a^2*x^2 + 3*a^3*x^3 - 3*a^2*Sqrt[
1 + 1/(a*x)]*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]] + 30*a^2*x^2*Hypergeometric2F1[
-1/2, 1, 1/2, 1 + 1/(a*x)]))/(3*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

fricas [A] time = 0.78, size = 381, normalized size = 1.74

$$\left[\frac{33\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3+133a^2c^2x^2+32ac\cdots\right.}{12\left(a^3x^2-a^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2
*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a
^2*x), 1/6*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sq
rt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c
*x - c)) + 2*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.08, size = 195, normalized size = 0.89

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1) \sqrt{\frac{c(ax-1)}{ax}}\, c^2 \left(6a^{\frac{7}{2}}x^3\sqrt{(ax+1)x} + 266a^{\frac{5}{2}}x^2\sqrt{(ax+1)x} - 33\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^3a^3 + 64a^{\frac{3}{2}}x\right.}{6\,(ax-1)^2\, x\, a^{\frac{5}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)+266*a^(5/2)*x^2*((a*x+1)*x)^(1/2)-33*ln(1/2*(
2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^3*a^3+64*a^(3/2)*x*((a*x+1)
*x)^(1/2)-33*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^2*a^2-
4*((a*x+1)*x)^(1/2)*a^(1/2))/x/a^(5/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^{\frac{5}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a\,x}\right)}^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.483 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=158

$$\frac{x\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -9*(c-c/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(3/2)+(21*a+1/x)*(c
-c/a/x)^(3/2)/a^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+(a-1/x)^2*(c-c/a/x)^(3/2)
*x/a^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6182, 6179, 98, 146, 63, 208}

$$\frac{x\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((21*a + x^(-1))*(c - c/(a*x))^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(
a*x)]) + ((a - x^(-1))^2*(c - c/(a*x))^(3/2)*x)/(a^2*(1 - 1/(a*x))^(3/2)*Sq
rt[1 + 1/(a*x)]) - (9*(c - c/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1
 - 1/(a*x))^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(\frac{9}{2a}-\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(9\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\cdots\right.}{2a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(9\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subst}\left(\int\cdots\right.}{\left(1-\frac{1}{ax}\right)\cdots}$$

$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\cdots}\right.}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.45

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(a^2x^2+9ax\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)+10ax+2\right)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(2 + 10*a*x + a^2*x^2 + 9*a*x*Hypergeometric2F1[-1/2,
1, 1/2, 1 + 1/(a*x)]))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.80, size = 315, normalized size = 1.99

$$\left[\frac{9\,(acx-c)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2cx^2+19\,acx+2\,c\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\ \frac{9\,(a\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(9*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(9*(a*c*x - c)*sqrt(-c)*arctan(2*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2
*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.08, size = 169, normalized size = 1.07

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(2a^{\frac{5}{2}}x^2\sqrt{(ax+1)\,x}+38a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}-9\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^2a^2-9\ln\left(\frac{2\sqrt{(ax\ldots}}{\ldots}\right.\ldots\right.}{2\,(ax-1)^2\,a^{\frac{3}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^
(5/2)*x^2*((a*x+1)*x)^(1/2)+38*a^(3/2)*x*((a*x+1)*x)^(1/2)-9*ln(1/2*(2*((a*
x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x^2*a^2-9*ln(1/2*(2*((a*x+1)*x)^(1/
```

```
2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a+4*((a*x+1)*x)^(1/2)*a^(1/2))/a^(3/2)/((a*x
+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-\frac{c}{a\,x}\right)}^{3/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

3.484 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=140

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -7*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+9*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+x*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6182, 6179, 89, 78, 63, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || IntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\operatorname{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{x}{a^2}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\operatorname{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1 \cdots}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax-7\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+9\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]
]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.67, size = 299, normalized size = 2.14

$$\left[\frac{7\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+9\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},\,\frac{7\,(ax-1)\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 146, normalized size = 1.04

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}\,-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa+18\sqrt{(ax+1)\,x}\,\sqrt{a}\,-7\ln\left(\frac{2\sqrt{(ax+1)}}{2}\right.\right.}{2\,(ax-1)^2\,\sqrt{a}\,\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^
(3/2)*x*((a*x+1)*x)^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*x*a+18*((a*x+1)*x)^(1/2)*a^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2)))/a^(1/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.485 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=118

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[Out] -5*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)+5*(c-c/a/
x)^(1/2)/a/c/(1-1/a^2/x^2)^(1/2)+x*(c-c/a/x)^(1/2)/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6177, 879, 869, 875, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (5*Sqrt[c - c/(a*x)])/(a*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a*x)]*x)/(
c*Sqrt[1 - 1/(a^2*x^2)]) - (5*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[
c - c/(a*x)]])/(a*Sqrt[c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 869

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*(p + 1)*(e*f + d*g)), x] + Dist[(e^2*g*(m - n - 2))/(c*(p +
 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
```

```
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{5\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(5c)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Mathematica [C] time = 0.05, size = 69, normalized size = 0.58

$$\frac{\sqrt{1-\frac{1}{ax}}\left(5\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)+ax\right)}{a\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*x + 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/
(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])
```

fricas [A] time = 1.16, size = 303, normalized size = 2.57

$$\left[\frac{5\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+5\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^2cx-ac\right)},\ \frac{5\,(ax-1)\sqrt{-}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c*x - a*c), 1/2*(5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 2*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^2*c*x - a*c)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep)]Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 integration of abs or sign assumes constant sign by intervals (correct if
the argument is real):Check [abs(t_nostep)]Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, integration of abs
or sign assumes constant sign by intervals (correct if the argument is real
):Check [abs(t_nostep)]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%
%{2,[1,1,1]%%%}+%%%{2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%
%{3,[2,2,2]%%%}+%%%{-2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values
[86,-97,-82]Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[0,1]%%%},
0,%%%{1,[2,2]%%%}+%%%{-2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters values [-
27,-94.616423693]Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[0,1]
%%%},0,%%%{1,[2,2]%%%}+%%%{-2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters valu
es [-49,-82.3579015951]Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2
,[0,1]%%%},0,%%%{1,[2,2]%%%}+%%%{-2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameter
s values [70,-29.292030761]Undef/Unsigned Inf encountered in limitLimit: Ma
x order reached or unable to make series expansion Error: Bad Argument Valu
e
```

maple [A] time = 0.06, size = 149, normalized size = 1.26

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa-10\sqrt{(ax+1)\,x}\,\sqrt{a}+5\ln\left(\frac{2\sqrt{(a\cdots}{\cdots}\right.\cdots\right.}{2\,(ax-1)^2\sqrt{a}\,c\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1
/2)/c*(-2*a^(3/2)*x*((a*x+1)*x)^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*x*a-10*((a*x+1)*x)^(1/2)*a^(1/2)+5*ln(1/2*(2*((a*x+1)*x)
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{ax}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{\sqrt{c-\frac{c}{a\,x}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

3.486 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}-\frac{2x\sqrt{c-\frac{c}{ax}}}{c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(3/2)+3*x*(1-1/
a^2/x^2)^(1/2)/c/(c-c/a/x)^(1/2)-2*x*(c-c/a/x)^(1/2)/c^2/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6177, 869, 873, 875, 208}

$$-\frac{2x\sqrt{c-\frac{c}{ax}}}{c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (3*Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a*x)]) - (2*Sqrt[c - c/(a*x)]*x)
/(c^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/S
qrt[c - c/(a*x)]])/(a*c^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 869

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*(p + 1)*(e*f + d*g)), x] + Dist[(e^2*g*(m - n - 2))/(c*(p +
 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && RationalQ[n]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
```

```
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

Mathematica [C] time = 0.05, size = 64, normalized size = 0.55

$$\frac{2\left(1-\frac{1}{ax}\right)^{3/2} {}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1+\frac{1}{ax}\right)}{a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (2*(1 - 1/(a*x))^(3/2)*Hypergeometric2F1[-1/2, 2, 1/2, 1 + 1/(a*x)])/(a*Sqr
t[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))
```

fricas [A] time = 0.67, size = 311, normalized size = 2.66

$$\left[\frac{3\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+3\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^2c^2x-ac^2\right)},\ \frac{3\,(ax-1)\sqrt{-}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c^2*x - a*c^2), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2
+ a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*
x^2 - a*c*x - c)) + 2*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)))/(a^2*c^2*x - a*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.06, size = 149, normalized size = 1.27

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa-6\sqrt{(ax+1)\,x}\,\sqrt{a}+3\ln\left(\frac{2\sqrt{(ax+}\cdots}{\cdots}\right.\right.}{2\,(ax-1)^2\sqrt{a}\,c^2\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(1
/2)/c^2*(-2*a^(3/2)*x*((a*x+1)*x)^(1/2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2))*x*a-6*((a*x+1)*x)^(1/2)*a^(1/2)+3*ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
[Out] Timed out
```

3.487 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] -(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-1/2*(1-1/a/x)^(
5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)+2*(1-1/
a/x)^(5/2)/a/(c-c/a/x)^(5/2)/(1+1/a/x)^(1/2)+(1-1/a/x)^(5/2)*x/(c-c/a/x)^(5
/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6182, 6179, 103, 152, 156, 63, 208, 206}

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (2*(1 - 1/(a*x))^(5/2))/(a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) + ((1 - 1
/(a*x))^(5/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\frac{1}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\frac{1}{2a^2}-\frac{x}{a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}} + \cdots$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}} - \cdots$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{ta}\cdots}{\sqrt{2}\,a\,(\cdots}$$

Mathematica [**C**] time = 0.07, size = 90, normalized size = 0.45

$$\frac{\sqrt{1-\frac{1}{ax}}\left({}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+\frac{1}{x}}{2a}\right)+{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)+ax\right)}{ac^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*x + Hypergeometric2F1[-1/2, 1, 1/2, (a + x^(-1))/(2*a
)] + Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(a*c^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - c/(a*x)])
```

fricas [**A**] time = 0.69, size = 524, normalized size = 2.63

$$\left[\frac{\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx\cdots}{\cdots}\right)\cdots}{8\left(a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x
 - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1)
)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(a*x
- 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) +
 8*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*c^3*x - a*c^3), 1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2
 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c
*x^2 - 2*a*c*x - c)) + 2*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) + 4*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*c^3*x - a*c^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep)]Warning, replacing 0
 by ` u`, a substitution variable should perhaps be purged.Warning, replaci
ng 0 by ` u`, a substitution variable should perhaps be purged.Warning, rep
lacing 0 by ` u`, a substitution variable should perhaps be purged.Warning,
 integration of abs or sign assumes constant sign by intervals (correct if
the argument is real):Check [abs(t_nostep)]Warning, replacing 0 by ` u`, a
substitution variable should perhaps be purged.Warning, integration of abs
or sign assumes constant sign by intervals (correct if the argument is real
):Check [abs(t_nostep)]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%
%{2,[1,1,1]%%%}+%%%{2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%
%{3,[2,2,2]%%%}+%%%{-2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values
[86,-97,-82]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]
%%%}+%%%{2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{3,[2,2,2]
%%%}+%%%{-2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [-89,63,-49
]Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[0,1]%%%},0,%%%{1,[2,
2]%%%}+%%%{-2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters values [-64,-15.6438
432182]Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[0,1]%%%},0,%%%
{1,[2,2]%%%}+%%%{-2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters values [42,-55
.0901457258]Warning, choosing root of [1,0,%%%{2,[1,1]%%%}+%%%{2,[0,1]%%%},
0,%%%{1,[2,2]%%%}+%%%{-2,[1,2]%%%}+%%%{1,[0,2]%%%}] at parameters values [4
6,-26.2290649475]Undef/Unsigned Inf encountered in limitEvaluation time: 0.
42Limit: Max order reached or unable to make series expansion Error: Bad Ar
gument Value
```

maple [A] time = 0.07, size = 262, normalized size = 1.32

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1) \sqrt{\frac{c(ax-1)}{ax}}\, x \left(-4a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\, x + a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right) x + 2\ln\left(\frac{2\sqrt{(ax+1)x}\cdots}{2\cdots}\right.\right.\cdots}{4\,(ax-1)^2\, a^{\frac{3}{2}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3
/2)/c^3*(-4*a^(5/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x+a^(3/2)*2^(1/2)*ln((2*2
^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x+2*ln(1/2*(2*((a*
x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*(1/a)^(1/2)*x-8*((a*x+1)*x)^(1/
2)*a^(3/2)*(1/a)^(1/2)+2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*
a+3*a*x+1)/(a*x-1))*a^(1/2)+2*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/
a^(1/2))*a*(1/a)^(1/2))/(1/a)^(1/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

3.488 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=267

$$\frac{ax\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}\right.}{a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] (1-1/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(7/2)-11/8*(1-1/a/x)^(
7/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(7/2)*2^(1/2)+7/4*(1-
1/a/x)^(7/2)/a/(c-c/a/x)^(7/2)/(1+1/a/x)^(1/2)-3/2*(1-1/a/x)^(7/2)/(a-1/x)/
(c-c/a/x)^(7/2)/(1+1/a/x)^(1/2)+a*(1-1/a/x)^(7/2)*x/(a-1/x)/(c-c/a/x)^(7/2)
/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6182, 6179, 103, 151, 152, 156, 63, 208, 206}

$$\frac{ax\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}\right.}{a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (7*(1 - 1/(a*x))^(7/2))/(4*a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) - (3*(1
 - 1/(a*x))^(7/2))/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) +
 (a*(1 - 1/(a*x))^(7/2)*x)/((a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7
/2)) + ((1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(7
/2)) - (11*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(4*Sqrt[
2]*a*(c - c/(a*x))^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{Subst}\left(\int \frac{-\frac{1}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int\right.}{2\left(c-\right.}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} -$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} -$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} -$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} +$$

Mathematica [C] time = 0.09, size = 121, normalized size = 0.45

$$\frac{\sqrt{1-\frac{1}{ax}}\left(11(ax-1)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+\frac{1}{x}}{2a}\right)+(4-4ax)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)+2ax(2ax-3)\right)}{4ac^3\sqrt{\frac{1}{ax}+1}\,(ax-1)\sqrt{c-\frac{c}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*x*(-3 + 2*a*x) + 11*(-1 + a*x)*Hypergeometric2F1[-1
/2, 1, 1/2, (a + x^(-1))/(2*a)] + (4 - 4*a*x)*Hypergeometric2F1[-1/2, 1, 1/
2, 1 + 1/(a*x)]))/(4*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

fricas [A] time = 0.77, size = 594, normalized size = 2.22

$$\left[\frac{11\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+8\left(a^2x^2-2\,ax+1\right)}{32\left(a^3c^4x^2-2\,a^2c^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/32*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)) + 8*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1
/16*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) - 8*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 4*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.07, size = 290, normalized size = 1.09

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x^2-11a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^2+4a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x)
```

```
[Out] 1/16*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^3*(c*(a*x-1)/a/x)^(1/2)*x*(16*
a^(7/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2-11*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*
(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+4*a^(5/2)*(1/a)^(1/2)
*((a*x+1)*x)^(1/2)*x+8*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*a^3*(1/a)^(1/2)*x^2-28*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-8*ln(1/2*(2*
((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+11*2^(1/2)*ln((2*
2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/a^(3/2)/
c^4/(1/a)^(1/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{ax}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(7/2),x)
[Out] Timed out
```

3.489 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx$

Optimal. Leaf size=60

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\;{}_2F_1\left(-\frac{1}{2},-m-1;-m;-\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] x^(1+m)*hypergeom([-1/2, -1-m],[-m],-1/a/x)*(c-c/a/x)^(1/2)/(1+m)/(1-1/a/x)
^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6182, 6181, 64}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\;{}_2F_1\left(-\frac{1}{2},-m-1;-m;-\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^m\,dx}{\sqrt{1-\frac{1}{ax}}}$$
$$=-\frac{\left(\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-2-m}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$=\frac{\sqrt{c-\frac{c}{ax}}\,x^{1+m}\,{}_2F_1\left(-\frac{1}{2},-1-m;-m;-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.00

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(-\frac{1}{2},-m-1;-m;-\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a
*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{ax}}\,x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{x^m\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}\,x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\sqrt{c - \frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^m*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***m*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

3.490 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=164

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[Out] 1/8*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^3-1/8*c*
x*(1-1/a^2/x^2)^(1/2)/a^2/(c-c/a/x)^(1/2)+1/12*c*x^2*(1-1/a^2/x^2)^(1/2)/a/
(c-c/a/x)^(1/2)+1/3*c*x^3*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6178, 863, 873, 875, 208}

$$\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -(c*Sqrt[1 - 1/(a^2*x^2)]*x)/(8*a^2*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2
*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*Sqr
t[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c -
 c/(a*x)]])/(8*a^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
```

```
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx &= -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{6a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{8a^2}\\
&= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{16a^3}\\
&= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{c^2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^5}\\
&= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}
\end{aligned}
$$

Mathematica [A] time = 0.51, size = 147, normalized size = 0.90

$$
\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(8a^2x^2+2ax-3\right)\sqrt{c-\frac{c}{ax}}}{ax-1}+3\sqrt{c}\log\left(2a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)-3\sqrt{c}\log(1-ax)}{48a^3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] ((2*a^2*x^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3 + 2*a*x + 8*a^2*x^2
))/(-1 + a*x) - 3*Sqrt[c]*Log[1 - a*x] + 3*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1
 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a^3)
```

fricas [A] time = 0.65, size = 337, normalized size = 2.05

$$\left[\frac{3\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax\right)\sqrt{\frac{ax-1}{ax+1}}}{96\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/96*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), -1/48*(3*(a*x - 1)*sqrt(
-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x
^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.06, size = 121, normalized size = 0.74

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{5}{2}}x^2\sqrt{(ax+1)\,x}\,+4a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}\,-6\sqrt{(ax+1)\,x}\,\sqrt{a}\,+3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{48\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{5}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/48/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(5/2)*(16*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)+4*a^(3/2)*x*((a*x+1)*x)^(1/2)-6*((a*x+1)*x)^(1/2)*a^(1/
2)+3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a/x)**(1/2),x)
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.491 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=124

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}+\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

```
[Out] -1/4*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^2+1/4*c
*x*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/2*c*x^2*(1-1/a^2/x^2)^(1/2)/(c-c
/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6178, 863, 873, 875, 208}

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}+\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^
2)]*x^2)/(2*Sqrt[c - c/(a*x)]) - (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*
x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx &= -\left(c\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{4a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{8a^2}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{c^2\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^4}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 148, normalized size = 1.19

$$\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}(2ax+1)\sqrt{c-\frac{c}{ax}}+\sqrt{c}(1-ax)\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)+\sqrt{c}(ax}{8a^2(ax-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(1 + 2*a*x) + Sqrt[c]*(-
1 + a*x)*Log[1 - a*x] + Sqrt[c]*(1 - a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2*(-1 + a*x)
)
```

fricas [A] time = 0.67, size = 317, normalized size = 2.56

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)}, (ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt
((a*c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.05, size = 102, normalized size = 0.82

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-4a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}\,-2\sqrt{(ax+1)\,x}\,\sqrt{a}\,+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{8\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{3}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/8/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-4*a^(3/2)*x*
((a*x+1)*x)^(1/2)-2*((a*x+1)*x)^(1/2)*a^(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a
^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.492 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=78

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6177, 863, 875, 208}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 863

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g*
(n + 1)), x] + Dist[(c*m)/(e*g*(n + 1)), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\left(c\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{c^2\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.85

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax+\sqrt{\frac{1}{ax}+1}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+1\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])
)/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [B] time = 0.73, size = 295, normalized size = 3.78

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)},-\frac{(ax-1)\sqrt{-c}\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))
/(a^2*x - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.05, size = 87, normalized size = 1.12

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2\sqrt{(ax+1)\,x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^(1/2)*a^
(1/2)+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.493 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=76

$$2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 2*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)-2*c*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6178, 865, 875, 208}

$$2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[((d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p)/(g
*(m - n - 1)), x] - Dist[(c*m*(e*f + d*g))/(e^2*g*(m - n - 1)), Int[(d + e*
x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f,
 g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Int
egerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^2}$$

$$= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

Mathematica [A] time = 0.24, size = 132, normalized size = 1.74

$$\frac{-2ax\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \sqrt{c}(ax-1)\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) + \sqrt{c}(1-ax)\log}{ax-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
 a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
 c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

fricas [B] time = 0.65, size = 275, normalized size = 3.62

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right) - 4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{-c}\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/
(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]Warning,
choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%
%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%
%}+%%%{-2,[3,2,3]%%%}+%%%{2,[3,1,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{2,[2,1,2]%%%}
+%%%{-2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,0,0]%%%}] at parameters val
ues [-89,63,-49]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2
,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[
4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[3,2,3]%%%}+%%%{-2,[3,1,3]%%%}+%%%{1,[2,
2,2]%%%}+%%%{2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{1,[0,0,
0]%%%}] at parameters values [-86,-64,-30]Evaluation time: 0.47sym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
alue
```

maple [A] time = 0.06, size = 88, normalized size = 1.16

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\left(\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa-2\sqrt{(ax+1)\,x}\,\sqrt{a}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*((a*x+1)*x)^(1/2
)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a-2*((a*x+1)*x)^(1/2)*a^(1/2))/((a*x+1)*x)^(1
/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{ax}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a\,x}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.494 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6178, 649}

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2))
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\ &= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 1.22

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\,(ax+1)\sqrt{c-\frac{c}{ax}}}{3ax-3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + a*x))/(-3 + 3*a*x)
```

fricas [A] time = 0.68, size = 58, normalized size = 1.57

$$-\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] -2/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
)/(a*x^2 - x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,
[1,1,1]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,
[4,0,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{2,[3,1,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{2,[2
,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,0,0]%%%}] at param
eters values [86,-97,-82]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+
%%%{-2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}
+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[3,2,3]%%%}+%%%{-2,[3,1,3]%%%}+
%%%{1,[2,2,2]%%%}+%%%{2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%
%{1,[0,0,0]%%%}] at parameters values [7,-27,26]Warning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(a*t_nostep+1)]sym2poly/r2sym(const gen & e,const index_m & i,
const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 41, normalized size = 1.11

$$-\frac{2\left(ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] -2/3*(a*x+1)/x/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [B] time = 1.37, size = 47, normalized size = 1.27

$$-\frac{2\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] -(2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*x*(a*x
- 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

3.495 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=77

$$\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -2/15*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)+2/5*a^2*c*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6178, 795, 649}

$$\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-2*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) + (2*a^2*c*(1
 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\left(c\,\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}} - \frac{1}{5}(ac)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}$$

Mathematica [**A**] time = 0.11, size = 58, normalized size = 0.75

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-ax-3\right)\sqrt{c-\frac{c}{ax}}}{15x(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-3 - a*x + 2*a^2*x^2))/(15*x*
(-1 + a*x))
```

fricas [**A**] time = 0.51, size = 68, normalized size = 0.88

$$\frac{2\left(2\,a^3x^3+a^2x^2-4\,ax-3\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] 2/15*(2*a^3*x^3 + a^2*x^2 - 4*a*x - 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x))/(a*x^3 - x^2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]Warning,
choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%
%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%
%}+%%%{-2,[3,2,3]%%%}+%%%{2,[3,1,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{2,[2,1,2]%%%}
+%%%{-2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,0,0]%%%}] at parameters val
ues [-89,63,-49]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2
,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[
4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[3,2,3]%%%}+%%%{-2,[3,1,3]%%%}+%%%{1,[2,
```

```
2,2]%%%}+%%%{2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{1,[0,0,
0]%%%}] at parameters values [-86,-64,-30]sym2poly/r2sym(const gen & e,cons
t index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 47, normalized size = 0.61

$$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] 2/15*(a*x+1)*(2*a*x-3)*(c*(a*x-1)/a/x)^(1/2)/x^2/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [B] time = 1.39, size = 53, normalized size = 0.69

$$\frac{2\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^2\,\left(2\,a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,x^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))
/(15*x^2*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

3.496 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=117

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7x^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 8/105*a^3*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-2/7*a*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)/x^2-8/35*a^3*c*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6178, 871, 795, 649}

$$\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (8*a^3*c^2*(1 - 1/(a^2*x^2))^(3/2))/(105*(c - c/(a*x))^(3/2)) - (8*a^3*c*(1
 - 1/(a^2*x^2))^(3/2))/(35*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^2))^
(3/2))/(7*(c - c/(a*x))^(3/2)*x^2)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
```

```
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}+\frac{1}{7}(4ac)\,\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}+\frac{1}{35}\left(4a^2c\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 66, normalized size = 0.56

$$
-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(8a^3x^3-4a^2x^2+3ax+15\right)\sqrt{c-\frac{c}{ax}}}{105x^2(ax-1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(15 + 3*a*x - 4*a^2*x^2 + 8*a
^3*x^3))/(105*x^2*(-1 + a*x))
```

fricas [A] time = 0.53, size = 77, normalized size = 0.66

$$
-\frac{2\left(8\,a^4x^4+4\,a^3x^3-a^2x^2+18\,ax+15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] -2/105*(8*a^4*x^4 + 4*a^3*x^3 - a^2*x^2 + 18*a*x + 15)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,
[1,1,1]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,
[4,0,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{2,[3,1,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{2,[2
,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,0,0]%%%}] at param
eters values [86,-97,-82]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+
%%%{-2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}
+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[3,2,3]%%%}+%%%{-2,[3,1,3]%%%}+
%%%{1,[2,2,2]%%%}+%%%{2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%
%{1,[0,0,0]%%%}] at parameters values [7,-27,26]Warning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(a*t_nostep+1)]sym2poly/r2sym(const gen & e,const index_m & i,
const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 55, normalized size = 0.47

$$-\frac{2\,(ax+1)\left(8a^2x^2-12ax+15\right)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] -2/105*(a*x+1)*(8*a^2*x^2-12*a*x+15)*(c*(a*x-1)/a/x)^(1/2)/x^3/((a*x-1)/(a*
x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^4\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [B] time = 1.41, size = 100, normalized size = 0.85

$$-\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(8\,a^3\,x^3+12\,a^2\,x^2+11\,a\,x+29\right)\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3}-\frac{88\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*(11*a*x + 12*a^2*x^2 + 8*a^3*x^3 + 29)*((c
*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (88*((a*x - 1)/(a*x + 1))^(1/2)*((c*(
a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.497 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=159

$$\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9x^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -16/315*a^4*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-2/9*a*c^2*(1-1/a^2/x^2)
^(3/2)/(c-c/a/x)^(3/2)/x^3+4/21*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)
/x^2+16/105*a^4*c*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6178, 871, 795, 649}

$$-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9x^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (-16*a^4*c^2*(1 - 1/(a^2*x^2))^(3/2))/(315*(c - c/(a*x))^(3/2)) + (16*a^4*c
*(1 - 1/(a^2*x^2))^(3/2))/(105*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^
2))^(3/2))/(9*(c - c/(a*x))^(3/2)*x^3) + (4*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)
)/(21*(c - c/(a*x))^(3/2)*x^2)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
```

```
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\left(c\,\text{Subst}\left(\int \frac{x^3\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{1}{3}(2ac)\,\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2} - \frac{1}{21}\left(8a^2c\right)\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2} - \frac{1}{105}\left(8a^3c\right)\text{Subst}\left(\int \right.$$

$$= -\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

Mathematica [A] time = 0.13, size = 74, normalized size = 0.47

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(16a^4x^4-8a^3x^3+6a^2x^2-5ax-35\right)\sqrt{c-\frac{c}{ax}}}{315x^3(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-35 - 5*a*x + 6*a^2*x^2 - 8*a
^3*x^3 + 16*a^4*x^4))/(315*x^3*(-1 + a*x))
```

fricas [A] time = 0.48, size = 84, normalized size = 0.53

$$\frac{2\left(16\,a^5x^5+8\,a^4x^4-2\,a^3x^3+a^2x^2-40\,ax-35\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{315\left(ax^5-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] 2/315*(16*a^5*x^5 + 8*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 40*a*x - 35)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]Warning,
choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%
%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[4,0,4]%%
%}+%%%{-2,[3,2,3]%%%}+%%%{2,[3,1,3]%%%}+%%%{1,[2,2,2]%%%}+%%%{2,[2,1,2]%%%}
+%%%{-2,[2,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,0,0]%%%}] at parameters val
ues [-89,63,-49]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2
,0,2]%%%}+%%%{-2,[1,1,1]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[
4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{2,[3,2,3]%%%}+%%%{-2,[3,1,3]%%%}+%%%{1,[2,
2,2]%%%}+%%%{2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{1,[0,0,
0]%%%}] at parameters values [-86,-64,-30]sym2poly/r2sym(const gen & e,cons
t index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 63, normalized size = 0.40

$$\frac{2\left(ax+1\right)\left(16x^{3}a^{3}-24a^{2}x^{2}+30ax-35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^{4}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 2/315*(a*x+1)*(16*a^3*x^3-24*a^2*x^2+30*a*x-35)*(c*(a*x-1)/a/x)^(1/2)/x^4/(
(a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^{5}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [B] time = 1.44, size = 108, normalized size = 0.68

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}\,\left(16\,a^4\,x^4+24\,a^3\,x^3+22\,a^2\,x^2+23\,a\,x-17\right)}{315\,x^4}-\frac{104\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{315\,x^4\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(23*a*x + 22*a^2
*x^2 + 24*a^3*x^3 + 16*a^4*x^4 - 17))/(315*x^4) - (104*((a*x - 1)/(a*x + 1)
)^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(315*x^4*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

3.498 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=130

$$\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

```
[Out] 75/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4+75/64*x*(c-c/a/x)^(1/2)/
a^3+25/32*x^2*(c-c/a/x)^(1/2)/a^2+5/8*x^3*(c-c/a/x)^(1/2)/a+1/4*x^4*(c-c/a/
x)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 78, 51, 63, 208}

$$\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}+\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{75\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (75*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (25*Sqrt[c - c/(a*x)]*x^2)/(32*a^2) + (
5*Sqrt[c - c/(a*x)]*x^3)/(8*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (75*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx=-\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$$

$$=-\int\frac{\sqrt{c-\frac{c}{ax}}\,x^3(1+ax)}{1-ax}\,dx$$

$$=\frac{c\int\frac{x^2(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=\frac{c\int\frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=-\frac{c\,\text{Subst}\left(\int\frac{a+x}{x^5\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{(15c)\,\text{Subst}\left(\int\frac{1}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a}$$

$$=\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{(25c)\,\text{Subst}\left(\int\frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^2}$$

$$=\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{(75c)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{64a^3}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{(75c)\,\text{Subst}\left(\int\frac{}{x\sqrt{}}\right.}{128a^4}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{75\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\right.}{64a}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{75\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.}{64a^4}$$

Mathematica [C] time = 0.04, size = 50, normalized size = 0.38

$$\frac{\sqrt{c-\frac{c}{ax}}\left(a^4x^4+15\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\frac{1}{ax}\right)\right)}{4a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a^4*x^4 + 15*Hypergeometric2F1[1/2, 4, 3/2, 1 - 1/(a*x)
]))/(4*a^4)
```

fricas [A] time = 0.51, size = 179, normalized size = 1.38

$$\left[\frac{2\left(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax\right)\sqrt{\frac{acx-c}{ax}}+75\,\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{128\,a^4}, \frac{\left(16\,a^4x^4+40\,a^3x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x - c)/
(a*x)) + 75*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c))/a^4, 1/64*((16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x
- c)/(a*x)) - 75*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

giac [A] time = 0.17, size = 142, normalized size = 1.09

$$\frac{1}{64}\,\sqrt{a^2cx^2-acx}\left(2\left(4\,x\left(\frac{2\,x|a|}{a^2\mathrm{sgn}(x)}+\frac{5\,|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{25\,|a|}{a^4\mathrm{sgn}(x)}\right)x+\frac{75\,|a|}{a^5\mathrm{sgn}(x)}\right)+\frac{75\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{128\,a^4}-\frac{75\,\sqrt{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(a^2*c*x^2 - a*c*x)*(2*(4*x*(2*x*abs(a)/(a^2*sgn(x)) + 5*abs(a)/(a
^3*sgn(x))) + 25*abs(a)/(a^4*sgn(x)))*x + 75*abs(a)/(a^5*sgn(x))) + 75/128*
sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^4 - 75/128*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^4*sgn(x))
```

maple [A] time = 0.05, size = 172, normalized size = 1.32

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32x\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{7}{2}}+112\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{5}{2}}+212\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-106\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+256a^{\frac{3}{2}}\sqrt{(ax-1)}\right.}{128\sqrt{(ax-1)\,x}\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/128*(c*(a*x-1)/a/x)^(1/2)*x*(32*x*(a*x^2-x)^(3/2)*a^(7/2)+112*(a*x^2-x)^(
3/2)*a^(5/2)+212*(a*x^2-x)^(1/2)*a^(5/2)*x-106*(a*x^2-x)^(1/2)*a^(3/2)+256*
a^(3/2)*((a*x-1)*x)^(1/2)+128*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))-53*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x-
1)*x)^(1/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x^3}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^3/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.499 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=105

$$\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[Out] 11/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3+11/8*x*(c-c/a/x)^(1/2)/a^
2+11/12*x^2*(c-c/a/x)^(1/2)/a+1/3*x^3*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 78, 51, 63, 208}

$$\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (11*Sqrt[c - c/(a*x)]*x)/(8*a^2) + (11*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 + (11*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^2\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}x^2(1+ax)}{1-ax}\,dx$$
$$= \frac{c\int \frac{x(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$= \frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$= -\frac{c\,\mathrm{Subst}\left(\int \frac{a+x}{x^4\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{(11c)\,\mathrm{Subst}\left(\int \frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6a}$$
$$= \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{(11c)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$
$$= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 - \frac{(11c)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$
$$= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{11\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{a}}\right.}{8a^2}$$
$$= \frac{11\sqrt{c-\frac{c}{ax}}x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}x^3 + \frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}$$

Mathematica [C] time = 0.04, size = 50, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(a^3x^3+11\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{1}{ax}\right)\right)}{3a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a^3*x^3 + 11*Hypergeometric2F1[1/2, 3, 3/2, 1 - 1/(a*x)
]))/(3*a^3)
```

fricas [A] time = 0.44, size = 163, normalized size = 1.55

$$\left[\frac{2\left(8\,a^3x^3+22\,a^2x^2+33\,ax\right)\sqrt{\frac{acx-c}{ax}}+33\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{48\,a^3}, \frac{\left(8\,a^3x^3+22\,a^2x^2+33\,ax\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) + 33*sqr
t(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^3, 1/24*(
(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sqrt(-c)*arc
tan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

giac [A] time = 0.44, size = 127, normalized size = 1.21

$$\frac{1}{24}\sqrt{a^2cx^2-acx}\left(2x\left(\frac{4x|a|}{a^2\mathrm{sgn}(x)}+\frac{11|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{33|a|}{a^4\mathrm{sgn}(x)}\right)+\frac{11\sqrt{c}\log(|a||c|)\,\mathrm{sgn}(x)}{16a^3}-\frac{11\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{\right.\right.\right.}{16a^3\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a^2*c*x^2 - a*c*x)*(2*x*(4*x*abs(a)/(a^2*sgn(x)) + 11*abs(a)/(a^3
*sgn(x))) + 33*abs(a)/(a^4*sgn(x))) + 11/16*sqrt(c)*log(abs(a)*abs(c))*sgn(
x)/a^3 - 11/16*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))
*sqrt(c)*abs(a) + a*c))/(a^3*sgn(x))
```

maple [A] time = 0.05, size = 155, normalized size = 1.48

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{5}{2}}+60\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-30\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+96a^{\frac{3}{2}}\sqrt{(ax-1)\,x}+48a\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-}{2\sqrt{a}}\right.\right.}{48\sqrt{(ax-1)\,x}\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(a*x^2-x)^(3/2)*a^(5/2)+60*(a*x^2-x)^(1/2)
*a^(5/2)*x-30*(a*x^2-x)^(1/2)*a^(3/2)+96*a^(3/2)*((a*x-1)*x)^(1/2)+48*a*ln(
1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-15*ln(1/2*(2*(a*x^2-x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x-1)*x)^(1/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x^2}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.500 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=80

$$\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[Out] 7/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2+7/4*x*(c-c/a/x)^(1/2)/a+1/
2*x^2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6167, 6133, 25, 434, 446, 78, 51, 63, 208}

$$\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (7*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (7*Sqrt[c]*ArcT
anh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=-\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$$
$$=-\int \frac{\sqrt{c-\frac{c}{ax}}\,x(1+ax)}{1-ax}\,dx$$
$$=\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=\frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$
$$=-\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$
$$=\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{(7c)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{4a}$$
$$=\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{(7c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$
$$=\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{7\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{4a}$$
$$=\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.96

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax\sqrt{1-\frac{1}{ax}}\,(2ax+7)+7\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\right)\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 - 1/(a*x)]*x*(7 + 2*a*x) + 7*ArcTanh[Sqrt[1 -
1/(a*x)]]))/(4*a^2*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.62, size = 147, normalized size = 1.84

$$\left[\frac{2\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}+7\,\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{8\,a^2},\frac{\left(2\,a^2x^2+7\,ax\right)\sqrt{\frac{acx-c}{ax}}-7\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\cdots}{\cdots}\right.}{4\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(2*a^2*x^2 + 7*a*x)*sqrt((a*c*x - c)/(a*x)) + 7*sqrt(c)*log(-2*a*c*
x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2, 1/4*((2*a^2*x^2 + 7*a*
x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a
*x))/c))/a^2]
```

giac [A] time = 0.18, size = 112, normalized size = 1.40

$$\frac{1}{4}\sqrt{a^2cx^2-acx}\left(\frac{2x|a|}{a^2\mathrm{sgn}(x)}+\frac{7|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{7\sqrt{c}\log(|a||c|)\mathrm{sgn}(x)}{8a^2}-\frac{7\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\right|\right.}{8a^2\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a^2*c*x^2 - a*c*x)*(2*x*abs(a)/(a^2*sgn(x)) + 7*abs(a)/(a^3*sgn(x)
)) + 7/8*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^2 - 7/8*sqrt(c)*log(abs(-2*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^2*sgn(x))
```

maple [B] time = 0.04, size = 139, normalized size = 1.74

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+16a^{\frac{3}{2}}\sqrt{(ax-1)\,x}+8a\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}}{2\sqrt{a}}\right.\right.}{8\sqrt{(ax-1)\,x}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*(a*x^2-x)^(1/2)*a^(5/2)*x-2*(a*x^2-x)^(1/2)*
a^(3/2)+16*a^(3/2)*((a*x-1)*x)^(1/2)+8*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2
)+2*a*x-1)/a^(1/2))-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/
((a*x-1)*x)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}\,x}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.501 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=50

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 375, 78, 63, 208}

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx \\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{1-ax}\,dx \\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a} \\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a} \\
&= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \sqrt{c-\frac{c}{ax}}\,x - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= \sqrt{c-\frac{c}{ax}}\,x + 3\,\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) \\
&= \sqrt{c-\frac{c}{ax}}\,x + \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.00

$$x\sqrt{c-\frac{c}{ax}} + \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]

[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a

fricas [A] time = 0.64, size = 124, normalized size = 2.48

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\,\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")

[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]

giac [B] time = 0.19, size = 96, normalized size = 1.92

$$\frac{3\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}\left(x\right)}{2\,a}-\frac{3\,\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,|a|+ac\right|\right)}{2\,a\mathrm{sgn}\left(x\right)}+\frac{\sqrt{a^2cx^2-acx}\,|a|}{a^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")

[Out] 3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))

maple [B] time = 0.05, size = 118, normalized size = 2.36

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax-1)\,x}\,\sqrt{a}-2\sqrt{a\,x^2-x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)+2\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right)}{2\sqrt{(ax-1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2),x)

[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a*x-1)*x)^(1/2)*a^(1/2)-2*(a*x^2-x)^(1/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))+2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")

[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.502 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=47

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

```
[Out] 2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)+2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6133, 25, 514, 446, 80, 63, 208}

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^2}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{a+x}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= 2\sqrt{c-\frac{c}{ax}} - c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= 2\sqrt{c-\frac{c}{ax}} + (2a)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)\\
&= 2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.00

$$
2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

fricas [A] time = 0.63, size = 111, normalized size = 2.36

$$\left[\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,\sqrt{\frac{acx-c}{ax}},-2\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)+2\,\sqrt{\frac{acx-c}{ax}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*sqrt
((a*c*x - c)/(a*x)), -2*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c)
 + 2*sqrt((a*c*x - c)/(a*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,
[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2
,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values [86,-97,-82]Warning, c
hoosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%
},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%
%}+%%%{1,[0,2,0]%%%}] at parameters values [7,-27,26]Warning, choosing root
 of [1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4
,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,
2,0]%%%}] at parameters values [-89,63,-49]Warning, choosing root of [1,0,%
%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+
%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]
at parameters values [-86,-64,-30]Warning, choosing root of [1,0,%%%{-2,[2,
1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3
,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parame
ters values [70,22,42]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%
{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%
%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}] at parameters values
[56,-9,-13]Sign error (%%%{sqrt(c)*a,0%%%}+%%%{2*sqrt(-a*c)*abs(a),1/2%%%}+
%%%{-2*sqrt(c)*a^2,1%%%}+%%%{-a*sqrt(-a*c)*abs(a),3/2%%%}+%%%{-a^2*sqrt(-a*
c)*abs(a)/4,5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

maple [B] time = 0.05, size = 99, normalized size = 2.11

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,\left(-2a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\;x^{2}+2\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}\;-\ln\left(\frac{2\sqrt{(ax-1)x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)x^{2}a\right)}{x\sqrt{(ax-1)\,x}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] -(c*(a*x-1)/a/x)^(1/2)/x*(-2*a^(3/2)*((a*x-1)*x)^(1/2)*x^2+2*(a*x^2-x)^(3/2
)*a^(1/2)-ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*x^2*a)/((a*
x-1)*x)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{x\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

sympy [A] time = 10.69, size = 39, normalized size = 0.83

$$-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x,x)
[Out] -2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) + 2*sqrt(c - c/(a*x))
```

3.503 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=42

$$4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

```
[Out] -2/3*a*(c-c/a/x)^(3/2)/c+4*a*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6133, 25, 514, 444, 43}

$$4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] 4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^2(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^3}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^2}\,dx}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{a+x}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \left(\frac{2a}{\sqrt{c-\frac{cx}{a}}}-\frac{a\sqrt{c-\frac{cx}{a}}}{c}\right)dx, x, \frac{1}{x}\right)}{a}\\
&= 4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.67

$$\frac{2(5ax+1)\sqrt{c-\frac{c}{ax}}}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
[Out] (2*Sqrt[c - c/(a*x)]*(1 + 5*a*x))/(3*x)
```

fricas [A] time = 0.44, size = 28, normalized size = 0.67

$$\frac{2\,(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")
[Out] 2/3*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%%{6,[2,1,5]%%%}+%%%{-6
,[1,1,4]%%%}+%%%{-6,[0,1,3]%%%},[4]%%%}+%%%{%%{[%%%{-9,[2,0,4]%%%}+%%%{9,[1
,0,3]%%%}+%%%{9,[0,0,2]%%%},0,%%%{9,[4,1,6]%%%}+%%%{-18,[3,1,5]%%%}+%%%{9,[
2,1,4]%%%}+%%%{-9,[0,1,2]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%
%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+
%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[3]%%%}+%%%{%%%{18,[4,1,5]%%%}+%%%{
-36,[3,1,4]%%%}+%%%{18,[1,1,2]%%%},[2]%%%}+%%%{%%{[%%%{-3,[4,0,4]%%%}+%%%{6
,[3,0,3]%%%}+%%%{-3,[1,0,1]%%%},0,%%%{3,[6,1,6]%%%}+%%%{-9,[5,1,5]%%%}+%%%{
9,[4,1,4]%%%}+%%%{-3,[3,1,3]%%%}+%%%{-3,[2,1,2]%%%}+%%%{3,[1,1,1]%%%}]:[1,0
,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%
}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}
]%%},[1]%%%} / %%%{%%%{256,[8,8,8]%%%}+%%%{-1024,[7,8,7]%%%}+%%%{1536,[6,8,
6]%%%}+%%%{-1024,[5,8,5]%%%}+%%%{256,[4,8,4]%%%},[4]%%%}+%%%{%%{[%%%{-384,[
8,7,7]%%%}+%%%{1536,[7,7,6]%%%}+%%%{-2304,[6,7,5]%%%}+%%%{1536,[5,7,4]%%%}+
%%%{-384,[4,7,3]%%%},0,%%%{384,[10,8,9]%%%}+%%%{-1920,[9,8,8]%%%}+%%%{4224,
[8,8,7]%%%}+%%%{-5376,[7,8,6]%%%}+%%%{4224,[6,8,5]%%%}+%%%{-1920,[5,8,4]%%%
}+%%%{384,[4,8,3]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,
1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1
,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[3]%%%}+%%%{%%%{768,[10,8,8]%%%}+%%%{-3840,
[9,8,7]%%%}+%%%{7680,[8,8,6]%%%}+%%%{-7680,[7,8,5]%%%}+%%%{3840,[6,8,4]%%%}
+%%%{-768,[5,8,3]%%%},[2]%%%}+%%%{%%{[%%%{-128,[10,7,7]%%%}+%%%{640,[9,7,6]
%%%}+%%%{-1280,[8,7,5]%%%}+%%%{1280,[7,7,4]%%%}+%%%{-640,[6,7,3]%%%}+%%%{12
8,[5,7,2]%%%},0,%%%{128,[12,8,9]%%%}+%%%{-768,[11,8,8]%%%}+%%%{2048,[10,8,7
]%%%}+%%%{-3200,[9,8,6]%%%}+%%%{3200,[8,8,5]%%%}+%%%{-2048,[7,8,4]%%%}+%%%{
768,[6,8,3]%%%}+%%%{-128,[5,8,2]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]
%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,
2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[1]%%%} Error: Bad Argument
Value
```

maple [A] time = 0.04, size = 27, normalized size = 0.64

$$\frac{2\left(5ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] 2/3*(5*a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^2), x)
```

mupad [B] time = 1.26, size = 24, normalized size = 0.57

$$\frac{2\sqrt{c-\frac{c}{a\,x}}\,\left(5\,a\,x+1\right)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*(5*a*x + 1))/(3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

3.504 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=69

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}+4a^2\sqrt{c-\frac{c}{ax}}$$

```
[Out] -2*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2+4*a^2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6133, 25, 514, 446, 77}

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}+4a^2\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] - (2*a^2*(c - c/(a*x))^(3/2))/c + (2*a^2*(c - c/(a*
x))^(5/2))/(5*c^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
```

```
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}(1+ax)}{x^3(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^4}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^3}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{x(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \left(\frac{2a^2}{\sqrt{c-\frac{cx}{a}}}-\frac{3a^2\sqrt{c-\frac{cx}{a}}}{c}+\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}\right)dx, x, \frac{1}{x}\right)}{a}\\
&= 4a^2\sqrt{c-\frac{c}{ax}}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 36, normalized size = 0.52

$$\frac{2\left(6a^2x^2+3ax+1\right)\sqrt{c-\frac{c}{ax}}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
[Out] (2*Sqrt[c - c/(a*x)]*(1 + 3*a*x + 6*a^2*x^2))/(5*x^2)
```

fricas [A] time = 0.42, size = 36, normalized size = 0.52

$$\frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{\frac{acx-c}{ax}}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fricas")
[Out] 2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%%{%%{[-5,0]:[1,0,%%%{-
1,[1]%%%}]%%},[0,5]%%%},[6]%%%}+%%%{%%{[%%%{25,[0,4]%%%},0]:[1,0,%%%{%%%{-1
,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[5]%%%}+%%%{%%%{%%{[-50,
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,5]%%%}+%%%{%%{[50,0]:[1,0,%%%{-1,[1]%%%}]%%},
[1,4]%%%},[4]%%%}+%%%{%%{[%%%{50,[2,4]%%%}+%%%{-50,[1,3]%%%},0]:[1,0,%%%{%%
%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[3]%%%}+%%%{%%%{%%{[
-25,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,5]%%%}+%%%{%%{[50,0]:[1,0,%%%{-1,[1]%%%}]
%%},[3,4]%%%}+%%%{%%{[-25,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3]%%%},[2]%%%}+%%%{
%%{[%%%{5,[4,4]%%%}+%%%{-10,[3,3]%%%}+%%%{5,[2,2]%%%},0]:[1,0,%%%{%%%{-1,[1
]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[1]%%%} / %%%{%%%{%%{poly1[
%%%{5,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,12]%%%}+%%%{%%{poly1[%%%{-30,[
6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,11]%%%}+%%%{%%{poly1[%%%{75,[6]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[10,10]%%%}+%%%{%%{poly1[%%%{-100,[6]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[9,9]%%%}+%%%{%%{poly1[%%%{75,[6]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[8,8]%%%}+%%%{%%{poly1[%%%{-30,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[7,7]%%%}+%%%{%%{poly1[%%%{5,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,6]%%%},
[6]%%%}+%%%{%%{[%%%{%%%{-25,[6]%%%},[12,11]%%%}+%%%{%%%{150,[6]%%%},[11,10]
%%%}+%%%{%%%{-375,[6]%%%},[10,9]%%%}+%%%{%%%{500,[6]%%%},[9,8]%%%}+%%%{%%%{
-375,[6]%%%},[8,7]%%%}+%%%{%%%{150,[6]%%%},[7,6]%%%}+%%%{%%%{-25,[6]%%%},[6
,5]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%
},[5]%%%}+%%%{%%%{%%{poly1[%%%{50,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,12
]%%%}+%%%{%%{poly1[%%%{-350,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,11]%%%}+
%%%{%%{poly1[%%%{1050,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,10]%%%}+%%%{%%
{poly1[%%%{-1750,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,9]%%%}+%%%{%%{poly1
[%%%{1750,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,8]%%%}+%%%{%%{poly1[%%%{-1
050,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,7]%%%}+%%%{%%{poly1[%%%{350,[6]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,6]%%%}+%%%{%%{poly1[%%%{-50,[6]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[7,5]%%%},[4]%%%}+%%%{%%{[%%%{%%%{-50,[6]%%%},[14,11]%
%%}+%%%{%%%{350,[6]%%%},[13,10]%%%}+%%%{%%%{-1050,[6]%%%},[12,9]%%%}+%%%{%%
%{1750,[6]%%%},[11,8]%%%}+%%%{%%%{-1750,[6]%%%},[10,7]%%%}+%%%{%%%{1050,[6]
%%%},[9,6]%%%}+%%%{%%%{-350,[6]%%%},[8,5]%%%}+%%%{%%%{50,[6]%%%},[7,4]%%%},
0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[3]%%%
}+%%%{%%%{%%{poly1[%%%{25,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[16,12]%%%}+%%
%{%%{poly1[%%%{-200,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,11]%%%}+%%%{%%{p
oly1[%%%{700,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,10]%%%}+%%%{%%{poly1[%%
%{-1400,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,9]%%%}+%%%{%%{poly1[%%%{1750
,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,8]%%%}+%%%{%%{poly1[%%%{-1400,[6]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,7]%%%}+%%%{%%{poly1[%%%{700,[6]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[10,6]%%%}+%%%{%%{poly1[%%%{-200,[6]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[9,5]%%%}+%%%{%%{poly1[%%%{25,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[8,4]%%%},[2]%%%}+%%%{%%{[%%%{%%%{-5,[6]%%%},[16,11]%%%}+%%%{%%%{40,[6
]%%%},[15,10]%%%}+%%%{%%%{-140,[6]%%%},[14,9]%%%}+%%%{%%%{280,[6]%%%},[13,8
]%%%}+%%%{%%%{-350,[6]%%%},[12,7]%%%}+%%%{%%%{280,[6]%%%},[11,6]%%%}+%%%{%%
%{-140,[6]%%%},[10,5]%%%}+%%%{%%%{40,[6]%%%},[9,4]%%%}+%%%{%%%{-5,[6]%%%},[
8,3]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%
%},[1]%%%} Error: Bad Argument Value
```

maple [A] time = 0.04, size = 35, normalized size = 0.51

$$\frac{2\left(6a^{2}x^{2}+3ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}}{5x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x^3,x)
[Out] 2/5*(6*a^2*x^2+3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^3), x)
```

mupad [B] time = 1.29, size = 32, normalized size = 0.46

$$\frac{2\sqrt{c-\frac{c}{a\,x}}\,\left(6\,a^2\,x^2+3\,a\,x+1\right)}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
[Out] (2*(c - c/(a*x))^(1/2)*(3*a*x + 6*a^2*x^2 + 1))/(5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**3,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

3.505 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}$$

```
[Out] -10/3*a^3*(c-c/a/x)^(3/2)/c+8/5*a^3*(c-c/a/x)^(5/2)/c^2-2/7*a^3*(c-c/a/x)^(
7/2)/c^3+4*a^3*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6133, 25, 514, 446, 77}

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] 4*a^3*Sqrt[c - c/(a*x)] - (10*a^3*(c - c/(a*x))^(3/2))/(3*c) + (8*a^3*(c -
c/(a*x))^(5/2))/(5*c^2) - (2*a^3*(c - c/(a*x))^(7/2))/(7*c^3)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^4(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^5}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^4}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{x^2(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \left(\frac{2a^3}{\sqrt{c-\frac{cx}{a}}}-\frac{5a^3\sqrt{c-\frac{cx}{a}}}{c}+\frac{4a^3\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}-\frac{a^3\left(c-\frac{cx}{a}\right)^{5/2}}{c^3}\right)dx, x, \frac{1}{x}\right)}{a}\\
&= 4a^3\sqrt{c-\frac{c}{ax}}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.46

$$\frac{2\left(104a^3x^3+52a^2x^2+39ax+15\right)\sqrt{c-\frac{c}{ax}}}{105x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
[Out] (2*Sqrt[c - c/(a*x)]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*x^3)
```

fricas [A] time = 0.66, size = 44, normalized size = 0.46

$$\frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{\frac{acx-c}{ax}}}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fricas")
[Out] 2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt((a*c*x - c)/(a*x))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Evaluation time: 0.92Unable to divide, perhaps due to rounding error%%%{%%%
{210,[2,1,9]%%%}+%%%{-210,[1,1,8]%%%}+%%%{-210,[0,1,7]%%%},[8]%%%}+%%%{%%{[
%%%{-735,[2,0,8]%%%}+%%%{735,[1,0,7]%%%}+%%%{735,[0,0,6]%%%},0,%%%{735,[4,1
,10]%%%}+%%%{-1470,[3,1,9]%%%}+%%%{735,[2,1,8]%%%}+%%%{-735,[0,1,6]%%%}]:[1
,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%
%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%
%}]%%},[7]%%%}+%%%{%%%{4410,[4,1,9]%%%}+%%%{-8820,[3,1,8]%%%}+%%%{4410,[1,1
,6]%%%},[6]%%%}+%%%{%%{[%%%{-3675,[4,0,8]%%%}+%%%{7350,[3,0,7]%%%}+%%%{-367
5,[1,0,5]%%%},0,%%%{3675,[6,1,10]%%%}+%%%{-11025,[5,1,9]%%%}+%%%{11025,[4,1
,8]%%%}+%%%{-3675,[3,1,7]%%%}+%%%{-3675,[2,1,6]%%%}+%%%{3675,[1,1,5]%%%}]:[
1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]
%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%
%%}]%%},[5]%%%}+%%%{%%%{7350,[6,1,9]%%%}+%%%{-22050,[5,1,8]%%%}+%%%{14700,[
4,1,7]%%%}+%%%{7350,[3,1,6]%%%}+%%%{-7350,[2,1,5]%%%},[4]%%%}+%%%{%%{[%%%{-
2205,[6,0,8]%%%}+%%%{6615,[5,0,7]%%%}+%%%{-4410,[4,0,6]%%%}+%%%{-2205,[3,0,
5]%%%}+%%%{2205,[2,0,4]%%%},0,%%%{2205,[8,1,10]%%%}+%%%{-8820,[7,1,9]%%%}+%
%%{13230,[6,1,8]%%%}+%%%{-8820,[5,1,7]%%%}+%%%{4410,[3,1,5]%%%}+%%%{-2205,[
2,1,4]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,
%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%
%%{1,[0,2,0]%%%}]%%},[3]%%%}+%%%{%%%{1470,[8,1,9]%%%}+%%%{-5880,[7,1,8]%%%}
+%%%{7350,[6,1,7]%%%}+%%%{-1470,[5,1,6]%%%}+%%%{-2940,[4,1,5]%%%}+%%%{1470,
[3,1,4]%%%},[2]%%%}+%%%{%%{[%%%{-105,[8,0,8]%%%}+%%%{420,[7,0,7]%%%}+%%%{-5
25,[6,0,6]%%%}+%%%{105,[5,0,5]%%%}+%%%{210,[4,0,4]%%%}+%%%{-105,[3,0,3]%%%}
,0,%%%{105,[10,1,10]%%%}+%%%{-525,[9,1,9]%%%}+%%%{1050,[8,1,8]%%%}+%%%{-105
0,[7,1,7]%%%}+%%%{420,[6,1,6]%%%}+%%%{210,[5,1,5]%%%}+%%%{-315,[4,1,4]%%%}+
%%%{105,[3,1,3]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,
0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2
,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[1]%%%} / %%%{%%%{65536,[16,16,16]%%%}+%%%{-5
24288,[15,16,15]%%%}+%%%{1835008,[14,16,14]%%%}+%%%{-3670016,[13,16,13]%%%}
+%%%{4587520,[12,16,12]%%%}+%%%{-3670016,[11,16,11]%%%}+%%%{1835008,[10,16,
10]%%%}+%%%{-524288,[9,16,9]%%%}+%%%{65536,[8,16,8]%%%},[8]%%%}+%%%{%%{[%%%
{-229376,[16,15,15]%%%}+%%%{1835008,[15,15,14]%%%}+%%%{-6422528,[14,15,13]%
%%}+%%%{12845056,[13,15,12]%%%}+%%%{-16056320,[12,15,11]%%%}+%%%{12845056,[
11,15,10]%%%}+%%%{-6422528,[10,15,9]%%%}+%%%{1835008,[9,15,8]%%%}+%%%{-2293
76,[8,15,7]%%%},0,%%%{229376,[18,16,17]%%%}+%%%{-2064384,[17,16,16]%%%}+%%%
{8486912,[16,16,15]%%%}+%%%{-21102592,[15,16,14]%%%}+%%%{35323904,[14,16,13
]%%%}+%%%{-41746432,[13,16,12]%%%}+%%%{35323904,[12,16,11]%%%}+%%%{-2110259
2,[11,16,10]%%%}+%%%{8486912,[10,16,9]%%%}+%%%{-2064384,[9,16,8]%%%}+%%%{22
9376,[8,16,7]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]
%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1
]%%%}+%%%{1,[0,2,0]%%%}]%%},[7]%%%}+%%%{%%%{1376256,[18,16,16]%%%}+%%%{-123
86304,[17,16,15]%%%}+%%%{49545216,[16,16,14]%%%}+%%%{-115605504,[15,16,13]%
%%}+%%%{173408256,[14,16,12]%%%}+%%%{-173408256,[13,16,11]%%%}+%%%{11560550
4,[12,16,10]%%%}+%%%{-49545216,[11,16,9]%%%}+%%%{12386304,[10,16,8]%%%}+%%%
{-1376256,[9,16,7]%%%},[6]%%%}+%%%{%%{[%%%{-1146880,[18,15,15]%%%}+%%%{1032
1920,[17,15,14]%%%}+%%%{-41287680,[16,15,13]%%%}+%%%{96337920,[15,15,12]%%%
}+%%%{-144506880,[14,15,11]%%%}+%%%{144506880,[13,15,10]%%%}+%%%{-96337920,
[12,15,9]%%%}+%%%{41287680,[11,15,8]%%%}+%%%{-10321920,[10,15,7]%%%}+%%%{11
46880,[9,15,6]%%%},0,%%%{1146880,[20,16,17]%%%}+%%%{-11468800,[19,16,16]%%%
}+%%%{52756480,[18,16,15]%%%}+%%%{-147947520,[17,16,14]%%%}+%%%{282132480,[
16,16,13]%%%}+%%%{-385351680,[15,16,12]%%%}+%%%{385351680,[14,16,11]%%%}+%%
%{-282132480,[13,16,10]%%%}+%%%{147947520,[12,16,9]%%%}+%%%{-52756480,[11,1
6,8]%%%}+%%%{11468800,[10,16,7]%%%}+%%%{-1146880,[9,16,6]%%%}]:[1,0,%%%{-2,
```

```
[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2
,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[5]
%%%}+%%%{%%%{2293760,[20,16,16]%%%}+%%%{-22937600,[19,16,15]%%%}+%%%{103219
200,[18,16,14]%%%}+%%%{-275251200,[17,16,13]%%%}+%%%{481689600,[16,16,12]%%
%}+%%%{-578027520,[15,16,11]%%%}+%%%{481689600,[14,16,10]%%%}+%%%{-27525120
0,[13,16,9]%%%}+%%%{103219200,[12,16,8]%%%}+%%%{-22937600,[11,16,7]%%%}+%%%
{2293760,[10,16,6]%%%},[4]%%%}+%%%{%%{[%%%{-688128,[20,15,15]%%%}+%%%{68812
80,[19,15,14]%%%}+%%%{-30965760,[18,15,13]%%%}+%%%{82575360,[17,15,12]%%%}+
%%%{-144506880,[16,15,11]%%%}+%%%{173408256,[15,15,10]%%%}+%%%{-144506880,[
14,15,9]%%%}+%%%{82575360,[13,15,8]%%%}+%%%{-30965760,[12,15,7]%%%}+%%%{688
1280,[11,15,6]%%%}+%%%{-688128,[10,15,5]%%%},0,%%%{688128,[22,16,17]%%%}+%%
%{-7569408,[21,16,16]%%%}+%%%{38535168,[20,16,15]%%%}+%%%{-120422400,[19,16
,14]%%%}+%%%{258048000,[18,16,13]%%%}+%%%{-400490496,[17,16,12]%%%}+%%%{462
422016,[16,16,11]%%%}+%%%{-400490496,[15,16,10]%%%}+%%%{258048000,[14,16,9]
%%%}+%%%{-120422400,[13,16,8]%%%}+%%%{38535168,[12,16,7]%%%}+%%%{-7569408,[
11,16,6]%%%}+%%%{688128,[10,16,5]%%%}]:[1,0,%%%{-2,[2,1,2]%%%}+%%%{2,[1,1,1
]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[3,2,3]%%%}+%%%{-1,[2,2
,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[3]%%%}+%%%{%%%{458752,[22,
16,16]%%%}+%%%{-5046272,[21,16,15]%%%}+%%%{25231360,[20,16,14]%%%}+%%%{-756
94080,[19,16,13]%%%}+%%%{151388160,[18,16,12]%%%}+%%%{-211943424,[17,16,11]
%%%}+%%%{211943424,[16,16,10]%%%}+%%%{-151388160,[15,16,9]%%%}+%%%{75694080
,[14,16,8]%%%}+%%%{-25231360,[13,16,7]%%%}+%%%{5046272,[12,16,6]%%%}+%%%{-4
58752,[11,16,5]%%%},[2]%%%}+%%%{%%{[%%%{-32768,[22,15,15]%%%}+%%%{360448,[2
1,15,14]%%%}+%%%{-1802240,[20,15,13]%%%}+%%%{5406720,[19,15,12]%%%}+%%%{-10
813440,[18,15,11]%%%}+%%%{15138816,[17,15,10]%%%}+%%%{-15138816,[16,15,9]%%
%}+%%%{10813440,[15,15,8]%%%}+%%%{-5406720,[14,15,7]%%%}+%%%{1802240,[13,15
,6]%%%}+%%%{-360448,[12,15,5]%%%}+%%%{32768,[11,15,4]%%%},0,%%%{32768,[24,1
6,17]%%%}+%%%{-393216,[23,16,16]%%%}+%%%{2195456,[22,16,15]%%%}+%%%{-756940
8,[21,16,14]%%%}+%%%{18022400,[20,16,13]%%%}+%%%{-31358976,[19,16,12]%%%}+%
%%{41091072,[18,16,11]%%%}+%%%{-41091072,[17,16,10]%%%}+%%%{31358976,[16,16
,9]%%%}+%%%{-18022400,[15,16,8]%%%}+%%%{7569408,[14,16,7]%%%}+%%%{-2195456,
[13,16,6]%%%}+%%%{393216,[12,16,5]%%%}+%%%{-32768,[11,16,4]%%%}]:[1,0,%%%{-
2,[2,1,2]%%%}+%%%{2,[1,1,1]%%%}+%%%{-2,[0,1,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{
-2,[3,2,3]%%%}+%%%{-1,[2,2,2]%%%}+%%%{2,[1,2,1]%%%}+%%%{1,[0,2,0]%%%}]%%},[
1]%%%} Error: Bad Argument Value
```

maple [A] time = 0.04, size = 43, normalized size = 0.45

$$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,\left(104x^3a^3+52a^2x^2+39ax+15\right)}{105x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] 2/105*(c*(a*x-1)/a/x)^(1/2)*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^4), x)
```

mupad [B] time = 1.28, size = 77, normalized size = 0.80

$$\frac{208\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{105}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{7\,x^3}+\frac{26\,a\,\sqrt{c-\frac{c}{a\,x}}}{35\,x^2}+\frac{104\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

```
[Out] (208*a^3*(c - c/(a*x))^(1/2))/105 + (2*(c - c/(a*x))^(1/2))/(7*x^3) + (26*a
*(c - c/(a*x))^(1/2))/(35*x^2) + (104*a^2*(c - c/(a*x))^(1/2))/(105*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

3.506 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=121

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}$$

```
[Out] -14/3*a^4*(c-c/a/x)^(3/2)/c+18/5*a^4*(c-c/a/x)^(5/2)/c^2-10/7*a^4*(c-c/a/x)
^(7/2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4+4*a^4*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6133, 25, 514, 446, 77}

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] - (14*a^4*(c - c/(a*x))^(3/2))/(3*c) + (18*a^4*(c -
 c/(a*x))^(5/2))/(5*c^2) - (10*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c
 - c/(a*x))^(9/2))/(9*c^4)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
```

```
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^5(1-ax)}\,dx\\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^6}\,dx}{a}\\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^5}\,dx}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \frac{x^3(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{c\,\mathrm{Subst}\left(\int \left(\frac{2a^4}{\sqrt{c-\frac{cx}{a}}}-\frac{7a^4\sqrt{c-\frac{cx}{a}}}{c}+\frac{9a^4\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}-\frac{5a^4\left(c-\frac{cx}{a}\right)^{5/2}}{c^3}+\frac{a^4\left(c-\frac{cx}{a}\right)^{7/2}}{c^4}\right)dx, x, \frac{1}{x}\right)}{a}\\
&= 4a^4\sqrt{c-\frac{c}{ax}}-\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{a\ldots\right.}{9c^4}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 52, normalized size = 0.43

$$\frac{2\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)\sqrt{c-\frac{c}{ax}}}{315x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*a^4*x^4
))/(315*x^4)
```

fricas [A] time = 0.54, size = 52, normalized size = 0.43

$$\frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{\frac{acx-c}{ax}}}{315\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt((a*c*x -
 c)/(a*x))/x^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%%{%%{[-315,0]:[1,0,%%%
{-1,[1]%%%}]%%},[0,9]%%%},[10]%%%}+%%%{%%{[%%%{2835,[0,8]%%%},0]:[1,0,%%%{%
%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[9]%%%}+%%%{%%%{%%{
[-11340,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,9]%%%}+%%%{%%{[11340,0]:[1,0,%%%{-1,[
1]%%%}]%%},[1,8]%%%},[8]%%%}+%%%{%%{[%%%{26460,[2,8]%%%}+%%%{-26460,[1,7]%%
%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[7]
%%%}+%%%{%%%{%%{[-39690,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,9]%%%}+%%%{%%{[79380,
0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8]%%%}+%%%{%%{[-39690,0]:[1,0,%%%{-1,[1]%%%}]
%%},[2,7]%%%},[6]%%%}+%%%{%%{[%%%{39690,[4,8]%%%}+%%%{-79380,[3,7]%%%}+%%%{
39690,[2,6]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1
]%%%}]%%},[5]%%%}+%%%{%%%{%%{[-26460,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,9]%%%}+%
%%{%%{[79380,0]:[1,0,%%%{-1,[1]%%%}]%%},[5,8]%%%}+%%%{%%{[-79380,0]:[1,0,%%
%{-1,[1]%%%}]%%},[4,7]%%%}+%%%{%%{[26460,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,6]%%
%},[4]%%%}+%%%{%%{[%%%{11340,[6,8]%%%}+%%%{-34020,[5,7]%%%}+%%%{34020,[4,6]
%%%}+%%%{-11340,[3,5]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1
]%%%},[1,1]%%%}]%%},[3]%%%}+%%%{%%%{%%{[-2835,0]:[1,0,%%%{-1,[1]%%%}]%%},[8
,9]%%%}+%%%{%%{[11340,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,8]%%%}+%%%{%%{[-17010,0
]:[1,0,%%%{-1,[1]%%%}]%%},[6,7]%%%}+%%%{%%{[11340,0]:[1,0,%%%{-1,[1]%%%}]%%
},[5,6]%%%}+%%%{%%{[-2835,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,5]%%%},[2]%%%}+%%%{
%%{[%%%{315,[8,8]%%%}+%%%{-1260,[7,7]%%%}+%%%{1890,[6,6]%%%}+%%%{-1260,[5,5
]%%%}+%%%{315,[4,4]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%
%%},[1,1]%%%}]%%},[1]%%%} / %%%{%%%{%%{poly1[%%%{315,[10]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[20,20]%%%}+%%%{%%{poly1[%%%{-3150,[10]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[19,19]%%%}+%%%{%%{poly1[%%%{14175,[10]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[18,18]%%%}+%%%{%%{poly1[%%%{-37800,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[17,17]%%%}+%%%{%%{poly1[%%%{66150,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
16,16]%%%}+%%%{%%{poly1[%%%{-79380,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,
15]%%%}+%%%{%%{poly1[%%%{66150,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,14]%
%%}+%%%{%%{poly1[%%%{-37800,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,13]%%%}
+%%%{%%{poly1[%%%{14175,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,12]%%%}+%%%
{%%{poly1[%%%{-3150,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,11]%%%}+%%%{%%{
poly1[%%%{315,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,10]%%%},[10]%%%}+%%%{
%%{[%%%{%%%{-2835,[10]%%%},[20,19]%%%}+%%%{%%%{28350,[10]%%%},[19,18]%%%}+%
%%{%%%{-127575,[10]%%%},[18,17]%%%}+%%%{%%%{340200,[10]%%%},[17,16]%%%}+%%%
{%%%{-595350,[10]%%%},[16,15]%%%}+%%%{%%%{714420,[10]%%%},[15,14]%%%}+%%%{%
%%{-595350,[10]%%%},[14,13]%%%}+%%%{%%%{340200,[10]%%%},[13,12]%%%}+%%%{%%%
{-127575,[10]%%%},[12,11]%%%}+%%%{%%%{28350,[10]%%%},[11,10]%%%}+%%%{%%%{-2
835,[10]%%%},[10,9]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%
%%},[1,1]%%%}]%%},[9]%%%}+%%%{%%%{%%{poly1[%%%{11340,[10]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[22,20]%%%}+%%%{%%{poly1[%%%{-124740,[10]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[21,19]%%%}+%%%{%%{poly1[%%%{623700,[10]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[20,18]%%%}+%%%{%%{poly1[%%%{-1871100,[10]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[19,17]%%%}+%%%{%%{poly1[%%%{3742200,[10]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[18,16]%%%}+%%%{%%{poly1[%%%{-5239080,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[17,15]%%%}+%%%{%%{poly1[%%%{5239080,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[16,14]%%%}+%%%{%%{poly1[%%%{-3742200,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[15,13]%%%}+%%%{%%{poly1[%%%{1871100,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[14,12]%%%}+%%%{%%{poly1[%%%{-623700,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1
3,11]%%%}+%%%{%%{poly1[%%%{124740,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,1
```

0]%%%}+%%%{%%{poly1[%%%{-11340,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[11,9]%%
%},[8]%%%}+%%%{%%{[%%%{%%%{-26460,[10]%%%},[22,19]%%%}+%%%{%%%{291060,[10]%
%%},[21,18]%%%}+%%%{%%%{-1455300,[10]%%%},[20,17]%%%}+%%%{%%%{4365900,[10]%
%%},[19,16]%%%}+%%%{%%%{-8731800,[10]%%%},[18,15]%%%}+%%%{%%%{12224520,[10]
%%%},[17,14]%%%}+%%%{%%%{-12224520,[10]%%%},[16,13]%%%}+%%%{%%%{8731800,[10
]%%%},[15,12]%%%}+%%%{%%%{-4365900,[10]%%%},[14,11]%%%}+%%%{%%%{1455300,[10
]%%%},[13,10]%%%}+%%%{%%%{-291060,[10]%%%},[12,9]%%%}+%%%{%%%{26460,[10]%%%
},[11,8]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%
%}]%%},[7]%%%}+%%%{%%%{%%{poly1[%%%{39690,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[24,20]%%%}+%%%{%%{poly1[%%%{-476280,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[23,19]%%%}+%%%{%%{poly1[%%%{2619540,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
22,18]%%%}+%%%{%%{poly1[%%%{-8731800,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2
1,17]%%%}+%%%{%%{poly1[%%%{19646550,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[20
,16]%%%}+%%%{%%{poly1[%%%{-31434480,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[19
,15]%%%}+%%%{%%{poly1[%%%{36673560,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[18,
14]%%%}+%%%{%%{poly1[%%%{-31434480,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[17,
13]%%%}+%%%{%%{poly1[%%%{19646550,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[16,1
2]%%%}+%%%{%%{poly1[%%%{-8731800,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[15,11
]%%%}+%%%{%%{poly1[%%%{2619540,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,10]%
%%}+%%%{%%{poly1[%%%{-476280,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[13,9]%%%}
+%%%{%%{poly1[%%%{39690,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[12,8]%%%},[6]%
%%}+%%%{%%{[%%%{%%%{-39690,[10]%%%},[24,19]%%%}+%%%{%%%{476280,[10]%%%},[23
,18]%%%}+%%%{%%%{-2619540,[10]%%%},[22,17]%%%}+%%%{%%%{8731800,[10]%%%},[21
,16]%%%}+%%%{%%%{-19646550,[10]%%%},[20,15]%%%}+%%%{%%%{31434480,[10]%%%},[
19,14]%%%}+%%%{%%%{-36673560,[10]%%%},[18,13]%%%}+%%%{%%%{31434480,[10]%%%}
,[17,12]%%%}+%%%{%%%{-19646550,[10]%%%},[16,11]%%%}+%%%{%%%{8731800,[10]%%%
},[15,10]%%%}+%%%{%%%{-2619540,[10]%%%},[14,9]%%%}+%%%{%%%{476280,[10]%%%},
[13,8]%%%}+%%%{%%%{-39690,[10]%%%},[12,7]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2
,2]%%%}+%%%{%%%{1,[1]%%%},[1,1]%%%}]%%},[5]%%%}+%%%{%%%{%%{poly1[%%%{26460,
[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[26,20]%%%}+%%%{%%{poly1[%%%{-343980,[1
0]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[25,19]%%%}+%%%{%%{poly1[%%%{2063880,[10]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[24,18]%%%}+%%%{%%{poly1[%%%{-7567560,[10]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[23,17]%%%}+%%%{%%{poly1[%%%{18918900,[10]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[22,16]%%%}+%%%{%%{poly1[%%%{-34054020,[10]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[21,15]%%%}+%%%{%%{poly1[%%%{45405360,[10]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[20,14]%%%}+%%%{%%{poly1[%%%{-45405360,[10]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[19,13]%%%}+%%%{%%{poly1[%%%{34054020,[10]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[18,12]%%%}+%%%{%%{poly1[%%%{-18918900,[10]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[17,11]%%%}+%%%{%%{poly1[%%%{7567560,[10]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[16,10]%%%}+%%%{%%{poly1[%%%{-2063880,[10]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[15,9]%%%}+%%%{%%{poly1[%%%{343980,[10]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[14,8]%%%}+%%%{%%{poly1[%%%{-26460,[10]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[13,7]%%%},[4]%%%}+%%%{%%{[%%%{%%%{-11340,[10]%%%},[26,19]
%%%}+%%%{%%%{147420,[10]%%%},[25,18]%%%}+%%%{%%%{-884520,[10]%%%},[24,17]%%
%}+%%%{%%%{3243240,[10]%%%},[23,16]%%%}+%%%{%%%{-8108100,[10]%%%},[22,15]%%
%}+%%%{%%%{14594580,[10]%%%},[21,14]%%%}+%%%{%%%{-19459440,[10]%%%},[20,13]
%%%}+%%%{%%%{19459440,[10]%%%},[19,12]%%%}+%%%{%%%{-14594580,[10]%%%},[18,1
1]%%%}+%%%{%%%{8108100,[10]%%%},[17,10]%%%}+%%%{%%%{-3243240,[10]%%%},[16,9
]%%%}+%%%{%%%{884520,[10]%%%},[15,8]%%%}+%%%{%%%{-147420,[10]%%%},[14,7]%%%
}+%%%{%%%{11340,[10]%%%},[13,6]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%
%{%%%{1,[1]%%%},[1,1]%%%}]%%},[3]%%%}+%%%{%%%{%%{poly1[%%%{2835,[10]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[28,20]%%%}+%%%{%%{poly1[%%%{-39690,[10]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[27,19]%%%}+%%%{%%{poly1[%%%{257985,[10]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[26,18]%%%}+%%%{%%{poly1[%%%{-1031940,[10]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[25,17]%%%}+%%%{%%{poly1[%%%{2837835,[10]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[24,16]%%%}+%%%{%%{poly1[%%%{-5675670,[10]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[23,15]%%%}+%%%{%%{poly1[%%%{8513505,[10]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[22,14]%%%}+%%%{%%{poly1[%%%{-9729720,[10]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[21,13]%%%}+%%%{%%{poly1[%%%{8513505,[10]%%%},0]:[1,0,%%%{-1,[1

```
]%%%}]%%},[20,12]%%%}+%%%{%%{poly1[%%%{-5675670,[10]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[19,11]%%%}+%%%{%%{poly1[%%%{2837835,[10]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[18,10]%%%}+%%%{%%{poly1[%%%{-1031940,[10]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[17,9]%%%}+%%%{%%{poly1[%%%{257985,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[16,8]%%%}+%%%{%%{poly1[%%%{-39690,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1
5,7]%%%}+%%%{%%{poly1[%%%{2835,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[14,6]%%
%},[2]%%%}+%%%{%%{[%%%{%%%{-315,[10]%%%},[28,19]%%%}+%%%{%%%{4410,[10]%%%},
[27,18]%%%}+%%%{%%%{-28665,[10]%%%},[26,17]%%%}+%%%{%%%{114660,[10]%%%},[25
,16]%%%}+%%%{%%%{-315315,[10]%%%},[24,15]%%%}+%%%{%%%{630630,[10]%%%},[23,1
4]%%%}+%%%{%%%{-945945,[10]%%%},[22,13]%%%}+%%%{%%%{1081080,[10]%%%},[21,12
]%%%}+%%%{%%%{-945945,[10]%%%},[20,11]%%%}+%%%{%%%{630630,[10]%%%},[19,10]%
%%}+%%%{%%%{-315315,[10]%%%},[18,9]%%%}+%%%{%%%{114660,[10]%%%},[17,8]%%%}+
%%%{%%%{-28665,[10]%%%},[16,7]%%%}+%%%{%%%{4410,[10]%%%},[15,6]%%%}+%%%{%%%
{-315,[10]%%%},[14,5]%%%},0]:[1,0,%%%{%%%{-1,[1]%%%},[2,2]%%%}+%%%{%%%{1,[1
]%%%},[1,1]%%%}]%%},[1]%%%} Error: Bad Argument Value
```

maple [A] time = 0.04, size = 51, normalized size = 0.42

$$\frac{2\left(272x^{4}a^{4}+136x^{3}a^{3}+102a^{2}x^{2}+85ax+35\right)\sqrt{\frac{c(ax-1)}{ax}}}{315x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 2/315*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35)*(c*(a*x-1)/a/x)^(1/2)
/x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{ax}}}{(ax-1)x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^5), x)
```

mupad [B] time = 1.37, size = 98, normalized size = 0.81

$$\frac{544\,a^{4}\,\sqrt{c-\frac{c}{a\,x}}}{315}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{9\,x^{4}}+\frac{34\,a\,\sqrt{c-\frac{c}{a\,x}}}{63\,x^{3}}+\frac{68\,a^{2}\,\sqrt{c-\frac{c}{a\,x}}}{105\,x^{2}}+\frac{272\,a^{3}\,\sqrt{c-\frac{c}{a\,x}}}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

```
[Out] (544*a^4*(c - c/(a*x))^(1/2))/315 + (2*(c - c/(a*x))^(1/2))/(9*x^4) + (34*a
*(c - c/(a*x))^(1/2))/(63*x^3) + (68*a^2*(c - c/(a*x))^(1/2))/(105*x^2) + (
272*a^3*(c - c/(a*x))^(1/2))/(315*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^{5}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

3.507 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=313

$$\frac{363\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}+\frac{149x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\cdots}}{96a^2\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 363/64*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)-4*arcta
nh(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)
+149/64*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+107/96*x^2*(1
+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+17/24*x^3*(1+1/a/x)^(1/2)
*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/4*x^4*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/
(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6182, 6180, 98, 151, 156, 63, 208, 206}

$$\frac{107x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{149x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{363\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+\cdots}}{\sqrt{2}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (149*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(64*a^3*Sqrt[1 - 1/(a*x)]) + (1
07*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]) + (1
7*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]) + (Sqrt
[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]) + (363*Sqrt[c -
c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2
]*Sqrt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^4*Sqrt[1 - 1/(a*
x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^3\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^5\left(1-\frac{x}{a}\right)}dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{-\frac{17}{2a}-\frac{15x}{2a^2}}{x^4\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\frac{107}{4a^2}+\frac{85x}{4a^3}}{x^3\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\right.}{12\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-}}{}$$

$$= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{}}{}$$

$$= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{}}{}$$

$$= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{}}{}$$

$$= \frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} + \frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} + \frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{}}{}$$

Mathematica [A] time = 0.99, size = 252, normalized size = 0.81

$$\frac{1089\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)-768\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(447 + 214*a*x + 136*a^
2*x^2 + 48*a^3*x^3))/(-1 + a*x) - 1089*Sqrt[c]*Log[1 - a*x] + 768*Sqrt[2]*S
qrt[c]*Log[(-1 + a*x)^2] + 1089*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x
^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 768*Sqrt[2]*Sqrt[c
]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c
*(-1 - 2*a*x + 3*a^2*x^2)])/(384*a^4)
```

fricas [A] time = 0.88, size = 568, normalized size = 1.81

$$\left[\frac{768\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+1089\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3c\cdots}{\cdots}\right)\cdots}{768\left(a\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/768*(768*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13
*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1
089*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^
2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*
x - 1)) + 4*(48*a^5*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x
)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4), 1/384*(
768*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)
) - 1089*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(48*a^5
*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\,x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 224, normalized size = 0.72

$$-\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-96\sqrt{(ax+1)\,x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^3-272a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x^2-428a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\,x-894\sqrt{(a\cdots}\right.}{384\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,a^{\frac{9}{2}}\sqrt{(a\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x)
```

```
[Out] -1/384/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(9
/2)*(-96*((a*x+1)*x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x^3-272*a^(7/2)*(1/a)^(1/2)*
((a*x+1)*x)^(1/2)*x^2-428*a^(5/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x-894*((a*x
+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+768*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*
x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)-1089*ln(1/2*(2*((a*x+1)*x)^(1/2)*
a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}\,x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

3.508 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=261

$$\frac{45\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}+\frac{19x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{13x^2}{}$$

```
[Out] 45/8*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)-4*arctanh
(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+1
9/8*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+13/12*x^2*(1+1/a/
x)^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/3*x^3*(1+1/a/x)^(1/2)*(c-c/a/x
)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6182, 6180, 98, 151, 156, 63, 208, 206}

$$\frac{19x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{45\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}+\frac{x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{13x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (19*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(8*a^2*Sqrt[1 - 1/(a*x)]) + (13*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]) + (45*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^3*Sqrt[1 - 1/(a*x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^2\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^4\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{13}{2a}-\frac{11x}{2a^2}}{x^3\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\frac{57}{4a^2}+\frac{39x}{4a^3}}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx \cdots\right.}{6\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\cdots}{\cdots}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\cdots\right.}{\cdots}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\cdots\right.}{\cdots}$$

$$= \frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}} + \frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}} + \frac{45\sqrt{c-\cdots}}{8\cdots}$$

Mathematica [A] time = 0.66, size = 244, normalized size = 0.93

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(8a^2x^2+26ax+57\right)\sqrt{c-\frac{c}{ax}}}{ax-1} + 135\sqrt{c}\,\log\left(2a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) - 96\sqrt{2}\sqrt{c}\,\log\cdots}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(57 + 26*a*x + 8*a^2*x^
2))/(-1 + a*x) - 135*Sqrt[c]*Log[1 - a*x] + 96*Sqrt[2]*Sqrt[c]*Log[(-1 + a*
x)^2] + 135*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x
)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 96*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*S
qrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*
x^2)])/(48*a^3)
```

fricas [A] time = 0.85, size = 552, normalized size = 2.11

$$\left[\frac{96\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+135(ax-1)\sqrt{c}\log\left(-\frac{8a^3}{}\right.}{96\left(a^4x-a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/96*(96*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a
*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 135
*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
 1)) + 4*(8*a^4*x^4 + 34*a^3*x^3 + 83*a^2*x^2 + 57*a*x)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(96*sqrt(2)*(a*x - 1)*s
qrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 135*(a*x - 1)*sqrt(
-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 + 34*a^3*x^3 + 83*a^
2*x^2 + 57*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x -
 a^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.06, size = 202, normalized size = 0.77

$$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,x^2+52a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,x+114\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-96\sqrt{2}\,\ln\left(\frac{2}{-}\right.\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,a^{\frac{7}{2}}\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^
(7/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x^2+52*a^(5/2)*(1/a)^(1/2)*((a*x+1)*x)^
(1/2)*x+114*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-96*2^(1/2)*ln((2*2^(1/2)*
(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+135*ln(1/2*(2*((a
*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(7/2)/((a*x+1)*x)
^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\sqrt{c - \frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

3.509 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=209

$$\frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}}+\frac{9x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 23/4*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)-4*arctanh
(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+9
/4*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/2*x^2*(1+1/a/x)^(1
/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6182, 6180, 98, 151, 156, 63, 208, 206}

$$\frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}}+\frac{9x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (9*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]) + (23*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^2*Sqrt[1 - 1/(a*x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{9}{2a}-\frac{7x}{2a^2}}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\frac{23}{4a^2}+\frac{9x}{4a^3}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,\cdots\right.}{2\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,d\cdots\right.}{a^3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{\cdots}\right.}{a^2\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}+\frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\cdots}{\cdots}$$

Mathematica [A] time = 0.58, size = 236, normalized size = 1.13

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}(2ax+9)\sqrt{c-\frac{c}{ax}}}{ax-1}+23\sqrt{c}\log\left(2a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)-16\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}\cdots\right.}{8a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(9 + 2*a*x))/(-1 + a*x)
 - 23*Sqrt[c]*Log[1 - a*x] + 16*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + 23*Sqrt
[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 -
 a*x + 2*a^2*x^2)] - 16*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 -
1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]])/(8*a^2)
```

fricas [A] time = 0.89, size = 536, normalized size = 2.56

$$\left[\frac{16\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+23(ax-1)\sqrt{c}\log\left(-\frac{8a^3c\cdots}{\cdots}\right.}{16\left(a^3x-a^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(16*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a
*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 23*
(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) + 4*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*(16*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2
*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 23*(a*x - 1)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.06, size = 180, normalized size = 0.86

$$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,x+18\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+23\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-16\sqrt{2}\,\ln\left(\right.\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,a^{\frac{5}{2}}\sqrt{(ax+1)x}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/8/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^(5
/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*x+18*((a*x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2
)+23*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-16
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^
(1/2))/a^(5/2)/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\,x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a/x)**(1/2),x)
[Out] Timed out
```

3.510 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 5*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+x*(1+1/a
/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6182, 6179, 98, 156, 63, 208, 206}

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{5}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Su}\ldots}{2a\ldots}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\ldots}{\ldots}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{a\ldots}}}{\sqrt{2}}\ldots\right.}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax\sqrt{\frac{1}{ax}+1}+5\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 5*ArcTanh[Sqrt[1 + 1/(a*x)]] -
4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[1 - 1/(a*x)])
```

fricas [A] time = 0.75, size = 512, normalized size = 3.37

$$\left[\frac{4\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+5(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7ac}{}\right.}{4\left(a^2x-a\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*
x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x
)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) +
 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*
x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.07, size = 160, normalized size = 1.05

$$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)\sqrt{a}+5\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*
x+1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x
+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+5*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1
/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{a\,x}}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
[Out] Timed out
```

3.511 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=146

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(1/2*(1
+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2*(1+1/a/x)^
(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6182, 6180, 84, 156, 63, 208, 206}

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)] - (4*Sqrt[2]*Sqrt[c - c
/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{}{(}\right.}{a\sqrt{1-}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\right)\text{S}}{}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right.}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.40, size = 218, normalized size = 1.49

$$\frac{2ax\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{ax-1}+\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)-2\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x)/(-1 + a*x) - Sqrt[c]*Log[1
- a*x] + 2*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sq
rt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 2*S
qrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(
a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

fricas [A] time = 1.52, size = 490, normalized size = 3.36

$$\left[\frac{2\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+}{}\right.}{2(ax-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/2*(2*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + (a*x
- 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) +
 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1),
(2*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c))
 - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*x + 1)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.06, size = 161, normalized size = 1.10

$$-\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{a}\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x-\ln\left(\frac{2\sqrt{(ax+1)x}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,xa-2\sqrt{(ax+1)x}\sqrt{a}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)x}\sqrt{a}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x)
```

```
[Out] -1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(2*a^(1/2)
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x-
ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*(1/a)^(1/2)*x*a-2*((a
*x+1)*x)^(1/2)*a^(1/2)*(1/a)^(1/2))/((a*x+1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Timed out
```

3.512 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=125

$$\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-4*a*arctanh(1/2*c^(1/2)*(1-1/
a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)*c^(1/2)+4*a*c*(1-1/a^2/x^2)
^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6178, 665, 661, 208}

$$\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a*c*Sqrt[1 -
 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - (4c)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \frac{\left(8c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{2c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Mathematica [A] time = 0.25, size = 155, normalized size = 1.24

$$\frac{2a\left(\sqrt{1-\frac{1}{a^2x^2}}(7ax+1)\sqrt{c-\frac{c}{ax}} - 3\sqrt{2}\sqrt{c}(ax-1)\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(3a^2x^2-2ax-1\right)\right)\right.}{3ax-3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*a*(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + 7*a*x) + 3*Sqrt[2]*Sqrt[
c]*(-1 + a*x)*Log[(-1 + a*x)^2] - 3*Sqrt[2]*Sqrt[c]*(-1 + a*x)*Log[2*Sqrt[2
]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x +
 3*a^2*x^2)]))/(-3 + 3*a*x)
```

fricas [A] time = 0.59, size = 353, normalized size = 2.82

$$\left[\frac{3\sqrt{2}\left(a^2x^2-ax\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\left(7a^2x^2+8ax+1\right)\sqrt{}}{3\left(ax^2-x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] [1/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 -
13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) +
 2*(7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a*x^2 - x), 2/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*arctan(2*sqrt(2)*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3
*a^2*c*x^2 - 2*a*c*x - c)) + (7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.08, size = 141, normalized size = 1.13

$$-\frac{2\,(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\left(3a\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{2}-7a\sqrt{\frac{1}{a}}\,x\sqrt{(ax+1)\,x}-\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)\,x}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,x\sqrt{(ax+1)\,x}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x)
```

```
[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x*(3*a*2
^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-
7*a*(1/a)^(1/2)*x*((a*x+1)*x)^(1/2)-(1/a)^(1/2)*((a*x+1)*x)^(1/2))/((a*x+1)
*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

3.513 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=170

$$\frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2/5*a^2*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^2*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)-4*a^2*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/
(c-c/a/x)^(1/2))*2^(1/2)*c^(1/2)+4*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2
)
```

Rubi [A] time = 0.30, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6178, 795, 665, 661, 208}

$$\frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) + (2*a^2*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a^2*c*Sqrt[1 - 1/(a^2*x
^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} - \left(ac^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - (4ac)\,\text{Subst}\left(\int \frac{\ }{\sqrt{c-\ }}\right.$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \left(8c^2\right)\text{Subst}\left(\int \frac{\ }{-\frac{2c}{a^2}}\right.$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.28, size = 162, normalized size = 0.95

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(38a^2x^2+11ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x(ax-1)} - 2\sqrt{2}\,a^2\sqrt{c}\,\log\left(2\sqrt{2}\,a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(3a^2x^2-2a\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 11*a*x + 38*a^2*x^2))/(15
*x*(-1 + a*x)) + 2*Sqrt[2]*a^2*Sqrt[c]*Log[(-1 + a*x)^2] - 2*Sqrt[2]*a^2*Sq
rt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2
 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

fricas [A] time = 0.53, size = 381, normalized size = 2.24

$$\left[\frac{15\sqrt{2}\left(a^3x^3-a^2x^2\right)\sqrt{c}\,\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\left(38a^3x^3+49a^2x^2\right.}{15\left(ax^3-x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] [1/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 2*(38*a^3*x^3 + 49*a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a*x^3 - x^2), 2/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)
*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (38*a^3*x^3 + 49*
a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a
*x^3 - x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.07, size = 165, normalized size = 0.97

$$\frac{2\left(ax-1\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(-15a^{2}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{3}+38x^{2}\sqrt{\left(ax+1\right)x}\,a^{2}\sqrt{\frac{1}{a}}+11a\sqrt{\frac{1}{a}}\,x\sqrt{\left(ax+1\right)x}\,-}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)x^{2}\sqrt{\left(ax+1\right)x}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x)
```

```
[Out] 2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-15*a^2
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^
3+38*x^2*((a*x+1)*x)^(1/2)*a^2*(1/a)^(1/2)+11*a*(1/a)^(1/2)*x*((a*x+1)*x)^(
1/2)+3*(1/a)^(1/2)*((a*x+1)*x)^(1/2))/x^2/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{x^{3} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**3,x)
[Out] Timed out
```

3.514 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=209

$$\frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 4/7*a^3*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^3*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)-2/7*a^3*c^2*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(3/2)-4*a^3*
arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)*c^
(1/2)+4*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6178, 1639, 795, 665, 661, 208}

$$-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (4*a^3*c^3*(1 - 1/(a^2*x^2))^(5/2))/(7*(c - c/(a*x))^(5/2)) + (2*a^3*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) - (2*a^3*c^2*(1 - 1/(a^2*x^2
))^(5/2))/(7*(c - c/(a*x))^(3/2)) + (4*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c
 - c/(a*x)] - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 661

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\left(c^3\,\mathrm{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{1}{7}\left(2a^4c\right)\mathrm{Subst}\left(\int \frac{\left(\frac{3c^2}{2a^2}-\frac{5c^2x}{a^3}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} - \left(a^2c^3\right)\mathrm{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2a^2c^2\right)\mathrm{Subst}\left(\int$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} -$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} +$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} -$$

Mathematica [A] time = 0.31, size = 170, normalized size = 0.81

$$2\sqrt{2}\,a^3\sqrt{c}\,\log\left((ax-1)^2\right)-2\sqrt{2}\,a^3\sqrt{c}\,\log\left(2\sqrt{2}\,a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}+c\left(3a^2x^2-2ax-1\right)\right)+\frac{2a\sqrt{1-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 9*a*x + 16*a^2*x^2 + 52*a
^3*x^3))/(21*x^2*(-1 + a*x)) + 2*Sqrt[2]*a^3*Sqrt[c]*Log[(-1 + a*x)^2] - 2*
Sqrt[2]*a^3*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c
- c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

fricas [A] time = 0.95, size = 397, normalized size = 1.90

$$\left[\frac{21\sqrt{2}\left(a^4x^4 - a^3x^3\right)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3 - 3\,a^2cx^2 - 13\,acx - 4\,\sqrt{2}\left(3\,a^3x^3 + 4\,a^2x^2 + ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}} - c}{a^3x^3 - 3\,a^2x^2 + 3\,ax - 1}\right) + 2\left(52\,a^4x^4 + 68\,a^3x^3 + \right.}{21\left(ax^4 - x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] [1/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 2*(52*a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3), 2/21*(21*sqrt(2)*(a^4*x
^4 - a^3*x^3)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (52*
a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.07, size = 187, normalized size = 0.89

$$\frac{2\,(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-21a^3\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^4 + 52x^3\sqrt{(ax+1)\,x}\,a^3\sqrt{\frac{1}{a}} + 16x^2\sqrt{(ax+1)\,x}\,a^2\sqrt{\frac{1}{a}}\right.}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,x^3\sqrt{(ax+1)\,x}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x)
```

```
[Out] 2/21/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-21*a^3
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*x^
4+52*x^3*((a*x+1)*x)^(1/2)*a^3*(1/a)^(1/2)+16*x^2*((a*x+1)*x)^(1/2)*a^2*(1/
a)^(1/2)+9*a*(1/a)^(1/2)*x*((a*x+1)*x)^(1/2)+3*(1/a)^(1/2)*((a*x+1)*x)^(1/2
))/x^3/((a*x+1)*x)^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}}{x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c - \frac{c}{a\,x}}}{x^4 \left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.515 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=303

$$\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4a^4\sqrt{\frac{1}{ax}+}}{\sqrt{1}}$$

```
[Out] 2/3*a^4*(1+1/a/x)^(3/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2/5*a^4*(1+1/a/x)^(
5/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-2/7*a^4*(1+1/a/x)^(7/2)*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)+2/9*a^4*(1+1/a/x)^(9/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-4
*a^4*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)
^(1/2)+4*a^4*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6182, 6180, 88, 50, 63, 206}

$$\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4a^4\sqrt{\frac{1}{ax}+}}{\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (4*a^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*a^4*(1 +
 1/(a*x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a
*x))^(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) - (2*a^4*(1 + 1/(a*x))^
(7/2)*Sqrt[c - c/(a*x)])/(7*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(9/2)
*Sqrt[c - c/(a*x)])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*a^4*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^5}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \left(-a^3\left(1+\frac{x}{a}\right)^{3/2}+\frac{a^3\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}+a^3\left(1+\frac{x}{a}\right)^{5/2}-a^3\left(1+\frac{x}{a}\right)^{7/2}\right)dx, x,}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}} - \frac{\left(a^3\right.}{}$$

$$= \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4}{}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1\right.}{}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1\right.}{}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1\right.}{}$$

Mathematica [**A**] time = 0.31, size = 178, normalized size = 0.59

$$2\sqrt{2}a^4\sqrt{c}\log\left((ax-1)^2\right)-2\sqrt{2}a^4\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(3a^2x^2-2ax-1\right)\right)+\frac{2a\sqrt{1-\frac{1}{a^2x^2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(35 + 95*a*x + 138*a^2*x^2 + 2
36*a^3*x^3 + 788*a^4*x^4))/(315*x^3*(-1 + a*x)) + 2*Sqrt[2]*a^4*Sqrt[c]*Log
[(-1 + a*x)^2] - 2*Sqrt[2]*a^4*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

fricas [**A**] time = 0.75, size = 413, normalized size = 1.36

$$\left[\frac{315\sqrt{2}\left(a^5x^5-a^4x^4\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\left(788a^5x^5+1024a^4\right.}{315\left(ax^5-x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] [1/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)) + 2*(788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2 + 130*a*
x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5 - x^4), 2
/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) + (788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2
 + 130*a*x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5
- x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1)]sym2poly/
r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argumen
t Value
```

maple [A] time = 0.07, size = 209, normalized size = 0.69

$$\frac{2\left(ax-1\right)\sqrt{\frac{c(ax-1)}{ax}}\left(-315a^{4}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax+1)x}\,a+3ax+1}{ax-1}\right)x^{5}+788x^{4}\sqrt{(ax+1)\,x}\,a^{4}\sqrt{\frac{1}{a}}+236x^{3}\sqrt{(ax+1)}\right.}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\,x^{4}\sqrt{(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x)
```

```
[Out] 2/315/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-315*a
^4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x+1)*x)^(1/2)*a+3*a*x+1)/(a*x-1))*
x^5+788*x^4*((a*x+1)*x)^(1/2)*a^4*(1/a)^(1/2)+236*x^3*((a*x+1)*x)^(1/2)*a^3
*(1/a)^(1/2)+138*x^2*((a*x+1)*x)^(1/2)*a^2*(1/a)^(1/2)+95*a*(1/a)^(1/2)*x*(
(a*x+1)*x)^(1/2)+35*(1/a)^(1/2)*((a*x+1)*x)^(1/2))/x^4/((a*x+1)*x)^(1/2)/(1
/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}}{x^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

3.516 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt{\frac{1}{ax}+1}\,x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)\sqrt{1-\frac{1}{ax}}}-\frac{(4m+3)x^m\sqrt{c-\frac{c}{ax}}\;{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)}{2am(m+1)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -1/2*(3+4*m)*x^m*hypergeom([1/2, -m],[1-m],-1/a/x)*(c-c/a/x)^(1/2)/a/m/(1+m
)/(1-1/a/x)^(1/2)+x^(1+m)*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1+m)/(1-1/a/x)^(
1/2)
```

Rubi [A] time = 0.27, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6182, 6181, 79, 64}

$$\frac{\sqrt{\frac{1}{ax}+1}\,x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)\sqrt{1-\frac{1}{ax}}}-\frac{(4m+3)x^m\sqrt{c-\frac{c}{ax}}\;{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)}{2am(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a*x)])
 - ((3 + 4*m)*Sqrt[c - c/(a*x)]*x^m*Hypergeometric2F1[1/2, -m, 1 - m, -(1/(
a*x))])/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 6181

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))
/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p},
x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]
) && !IntegerQ[m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx &= \frac{\sqrt{c-\frac{c}{ax}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^m\,dx}{\sqrt{1-\frac{1}{ax}}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}}+\frac{\left((3+4m)\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a(1+m)\sqrt{1-\frac{1}{ax}}}\\
&= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}}-\frac{(3+4m)\sqrt{c-\frac{c}{ax}}\,x^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.74

$$
\frac{x^m\sqrt{c-\frac{c}{ax}}\left(2amx\sqrt{\frac{1}{ax}+1}-(4m+3)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)\right)}{2am(m+1)\sqrt{1-\frac{1}{ax}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^m*(2*a*m*Sqrt[1 + 1/(a*x)]*x - (3 + 4*m)*Hypergeometri
c2F1[1/2, -m, 1 - m, -(1/(a*x))]))/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$
\text{integral}\left(x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int\sqrt{c-\frac{c}{ax}}\,x^m\sqrt{\frac{ax-1}{ax+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$
\int x^m\sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{ax}} x^m \sqrt{\frac{ax - 1}{ax + 1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \sqrt{c - \frac{c}{a\,x}} \sqrt{\frac{a\,x - 1}{a\,x + 1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^m*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.517 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=164

$$-\frac{11cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}+\frac{11cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[Out] -11/8*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^3+11/8
*c*x*(1-1/a^2/x^2)^(1/2)/a^2/(c-c/a/x)^(1/2)-11/12*c*x^2*(1-1/a^2/x^2)^(1/2
)/a/(c-c/a/x)^(1/2)+1/3*c*x^3*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6178, 879, 873, 875, 208}

$$\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{11cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}+\frac{11cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] (11*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(8*a^2*Sqrt[c - c/(a*x)]) - (11*c*Sqrt[1 - 1
/(a^2*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(
3*Sqrt[c - c/(a*x)]) - (11*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
Sqrt[c - c/(a*x)]])/(8*a^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
```

```
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{11\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{6a}$$

$$=-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$

$$=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{11\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x\cdots\right)}{16a^3}$$

$$=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{\left(11c^2\right)\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}\cdots}\cdots\right)}{8a^5}$$

$$=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\cdots}{\sqrt{c-\frac{c}{ax}}\cdots}\cdots\right)}{8a^3}$$

Mathematica [A] time = 0.51, size = 147, normalized size = 0.90

$$\frac{\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(8a^2x^2-22ax+33\right)\sqrt{c-\frac{c}{ax}}}{ax-1}-33\sqrt{c}\log\left(2a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)+33\sqrt{c}\log(1\cdots}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(33 - 22*a*x + 8*a^2*x^
2))/(-1 + a*x) + 33*Sqrt[c]*Log[1 - a*x] - 33*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a
^3)
```

fricas [A] time = 0.72, size = 337, normalized size = 2.05

$$\left[\frac{33\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(8\,a^4x^4-14\,a^3x^3+11\,a^2x^2+33\,ax\right)\sqrt{\frac{ax-}{ax+}}}{96\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(33*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(8*a^4*x^4 - 14*a^3*x^3 + 11*a^2*x^2 + 33*a*x)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(33*(a*x - 1)*
sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 14*a^3*x^3 +
11*a^2*x^2 + 33*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
4*x - a^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 133, normalized size = 0.81

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{5}{2}}x^2\sqrt{(ax+1)\,x}-44a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}+66\sqrt{(ax+1)\,x}\,\sqrt{a}-33\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2}{2\sqrt{a}}\right.\right.}{48a^{\frac{5}{2}}\,(ax-1)\,\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^(5/2)*x^
2*((a*x+1)*x)^(1/2)-44*a^(3/2)*x*((a*x+1)*x)^(1/2)+66*((a*x+1)*x)^(1/2)*a^(
1/2)-33*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(5/2)/(a*x
-1)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{ax}}\,x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.518 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=124

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}-\frac{7cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

```
[Out] 7/4*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^2-7/4*c*
x*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/2*c*x^2*(1-1/a^2/x^2)^(1/2)/(c-c/
a/x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6178, 879, 873, 875, 208}

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}-\frac{7cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] (-7*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2
*x^2)]*x^2)/(2*Sqrt[c - c/(a*x)]) + (7*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/
(a^2*x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 873

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*(a + c*
x^2)^(p + 1))/((n + 1)*(c*e*f + c*d*g)), x] - Dist[(e*(m - n - 2))/((n + 1)
*(e*f + d*g)), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2
, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
```

```
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{7\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{4a}$$

$$=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{7\,\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$

$$=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\left(7c^2\right)\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^4}$$

$$=-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Mathematica [A] time = 0.49, size = 139, normalized size = 1.12

$$\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}\,(2ax-7)\sqrt{c-\frac{c}{ax}}}{4ax-4}+\frac{7\sqrt{c}\,\log\left(2a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)}{8a^2}-\frac{7\sqrt{c}\,\log(1-ax)}{8a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-7 + 2*a*x))/(-4 + 4*a*x) - (
7*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (7*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2)
```

fricas [A] time = 0.87, size = 321, normalized size = 2.59

$$\left[\frac{7\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(2\,a^3x^3-5\,a^2x^2-7\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)},\ 7\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1))*
sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(7*(a*x - 1)*sqrt(-c)*arctan(2
*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{ax}}\,x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.06, size = 116, normalized size = 0.94

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}-14\sqrt{(ax+1)\,x}\,\sqrt{a}+7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{8a^{\frac{3}{2}}\,(ax-1)\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(4*a^(3
/2)*x*((a*x+1)*x)^(1/2)-14*((a*x+1)*x)^(1/2)*a^(1/2)+7*ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{ax}}\,x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

3.519 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=79

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/
a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6177, 879, 875, 208}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
 1)*(a + c*x^2)^(p + 1))/(c*g*(n + 1)*(e*f + d*g)), x] - Dist[(e*(e*f*(p +
1) - d*g*(2*n + p + 3)))/(g*(n + 1)*(e*f + d*g)), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6177

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^2, x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\left(3c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 0.82

$$\frac{\sqrt{c-\frac{c}{ax}}\left(x\sqrt{\frac{1}{ax}+1}-\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a)
)/Sqrt[1 - 1/(a*x)]
```

fricas [B] time = 0.64, size = 297, normalized size = 3.76

$$\left[\frac{3\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^2x^2+ax\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^2x-a\right)}, \frac{3\,(ax-1)\sqrt{\cdot}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*x - a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.06, size = 101, normalized size = 1.28

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax+1)x}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\,(ax-1)\sqrt{(ax+1)x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x+1)*x)^
(1/2)*a^(1/2)-3*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x
-1)/((a*x+1)*x)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

3.520 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=76

$$\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)+2*c*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6178, 881, 875, 208}

$$\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*
Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 875

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^2}$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

Mathematica [A] time = 0.25, size = 132, normalized size = 1.74

$$\frac{2ax\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \sqrt{c}(ax-1)\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + c\left(2a^2x^2-ax-1\right)\right) + \sqrt{c}(1-ax)\log(1-}{ax-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

fricas [B] time = 0.45, size = 275, normalized size = 3.62

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{-c}\,\text{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/
(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

maple [A] time = 0.06, size = 100, normalized size = 1.32

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa+2\sqrt{(ax+1)\,x}\,\sqrt{a}\right)}{(ax-1)\sqrt{(ax+1)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*((a*x+1)*x
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a+2*((a*x+1)*x)^(1/2)*a^(1/2))/(a*x-1)/
((a*x+1)*x)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a\,x}}\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
[Out] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x, x)
```

3.521 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -8/3*a*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-2/3*a*(1-1/a^2/x^2)^(1/2)*(c-c
/a/x)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6178, 657, 649}

$$-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (-8*a*c*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) - (2*a*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[c - c/(a*x)])/3
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{4}{3}\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)$$
$$= -\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}} - \frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.66

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}(5ax-1)\sqrt{c-\frac{c}{ax}}}{3ax-3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 5*a*x))/(-3 + 3*a*x)
```

fricas [A] time = 0.55, size = 59, normalized size = 0.84

$$-\frac{2\left(5\,a^2x^2+4\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas"
)
[Out] -2/3*(5*a^2*x^2 + 4*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x))/(a*x^2 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

maple [A] time = 0.04, size = 54, normalized size = 0.77

$$-\frac{2\,(ax+1)\,(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{3\,(ax-1)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] -2/3*(a*x+1)*(5*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

mupad [B] time = 1.33, size = 54, normalized size = 0.77

$$-\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^2\,x^2+4\,a\,x-1\right)}{3\,x\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

```
[Out] -(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + 5*a^2*x^2 - 1)
)/(3*x*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

3.522 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=113

$$\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 2/5*a^2*(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/c+8/5*a^2*c*(1-1/a^2/x^2)^(1/2)
/(c-c/a/x)^(1/2)+2/5*a^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6178, 795, 657, 649}

$$\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (8*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(5*Sqrt[c - c/(a*x)]) + (2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*Sqrt[c - c/(a*x)])/5 + (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))
^(3/2))/(5*c)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} + \frac{(3a)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{5c}$$

$$= \frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} + \frac{1}{5}(4a)\,\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx\right.$$

$$= \frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}} + \frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}$$

Mathematica [A] time = 0.12, size = 58, normalized size = 0.51

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(6a^2x^2-3ax+1\right)\sqrt{c-\frac{c}{ax}}}{5x(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 - 3*a*x + 6*a^2*x^2))/(5*x*
(-1 + a*x))
```

fricas [A] time = 0.51, size = 69, normalized size = 0.61

$$\frac{2\left(6\,a^3x^3+3\,a^2x^2-2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{5\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] 2/5*(6*a^3*x^3 + 3*a^2*x^2 - 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(a*x^3 - x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

maple [A] time = 0.04, size = 62, normalized size = 0.55

$$\frac{2\left(ax+1\right)\left(6a^2x^2-3ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{5x^2\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x)
```

```
[Out] 2/5*(a*x+1)*(6*a^2*x^2-3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/
2)/x^2/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

mupad [B] time = 1.35, size = 62, normalized size = 0.55

$$\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(6\,a^3\,x^3+3\,a^2\,x^2-2\,a\,x+1\right)}{5\,x^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^3,x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 2*a*x + 6*a
^3*x^3 + 1))/(5*x^2*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

3.523 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35x^2\sqrt{c-\frac{c}{ax}}}+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}-\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -104/105*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2/7*c*(1-1/a^2/x^2)^(1/2
)/x^3/(c-c/a/x)^(1/2)-26/35*a*c*(1-1/a^2/x^2)^(1/2)/x^2/(c-c/a/x)^(1/2)-104
/105*a^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6178, 881, 871, 795, 649}

$$-\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}-\frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35x^2\sqrt{c-\frac{c}{ax}}}+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] (-104*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) - (104*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (2*c*Sqrt[1 - 1/(a^2*x^2)])/(7*Sq
rt[c - c/(a*x)]*x^3) - (26*a*c*Sqrt[1 - 1/(a^2*x^2)])/(35*Sqrt[c - c/(a*x)]
*x^2)
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 871

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> -Simp[(e*(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p
+ 1))/(c*(m - n - 1)), x] - Dist[(n*(e*f + d*g))/(e*(m - n - 1)), Int[(d +
e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g,
m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&
 EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Integ
erQ[n])
```

Rule 881

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*(a + c*x
^2)^(p + 1))/(c*g*(n + p + 2)), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
```

```
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3} - \frac{13}{7}\text{Subst}\left(\int \frac{x^2\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}\,x^2} + \frac{1}{35}(52a)\,\text{Subst}\left(\int \frac{x\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}\,x^2} - \frac{1}{105}\left(52a^2\right)\text{Sub}$$

$$= -\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} - \frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}\,x^2}$$

Mathematica [A] time = 0.13, size = 66, normalized size = 0.44

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(104a^3x^3-52a^2x^2+39ax-15\right)\sqrt{c-\frac{c}{ax}}}{105x^2(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 39*a*x - 52*a^2*x^2 +
104*a^3*x^3))/(105*x^2*(-1 + a*x))
```

fricas [A] time = 0.51, size = 77, normalized size = 0.52

$$-\frac{2\left(104\,a^4x^4+52\,a^3x^3-13\,a^2x^2+24\,ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] -2/105*(104*a^4*x^4 + 52*a^3*x^3 - 13*a^2*x^2 + 24*a*x - 15)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

maple [A] time = 0.04, size = 70, normalized size = 0.47

$$-\frac{2\,(ax+1)\left(104x^3a^3-52a^2x^2+39ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105x^3\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x)
```

```
[Out] -2/105*(a*x+1)*(104*a^3*x^3-52*a^2*x^2+39*a*x-15)*(c*(a*x-1)/a/x)^(1/2)*((a
*x-1)/(a*x+1))^(1/2)/x^3/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{ax}}\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

mupad [B] time = 1.41, size = 100, normalized size = 0.67

$$-\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(104\,a^3\,x^3+156\,a^2\,x^2+143\,a\,x+167\right)\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3}-\frac{304\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,(a\,x-1)}{a\,x}}}{105\,x^3\,(a\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^4,x)
```

```
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*(143*a*x + 156*a^2*x^2 + 104*a^3*x^3 + 167
)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (304*((a*x - 1)/(a*x + 1))^(1/2)
*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.524 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=172

$$\frac{363\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{17x^3\sqrt{c-}}{24a}$$

```
[Out] 363/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4-4*arctanh(1/2*(c-c/a/x)
^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4-149/64*x*(c-c/a/x)^(1/2)/a^3+10
7/96*x^2*(c-c/a/x)^(1/2)/a^2-17/24*x^3*(c-c/a/x)^(1/2)/a+1/4*x^4*(c-c/a/x)^
(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6167, 6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}-\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{363\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{17x^3\sqrt{c-}}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-149*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (107*Sqrt[c - c/(a*x)]*x^2)/(96*a^2)
- (17*Sqrt[c - c/(a*x)]*x^3)/(24*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (363*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4) - (4*Sqrt[2]*Sqrt[c]*ArcTa
nh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^4
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,x^3(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^5(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 + \frac{\text{Subst}\left(\int \frac{\frac{17c^2}{2}-\frac{15c^2x}{2a}}{x^4(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$= -\frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 - \frac{\text{Subst}\left(\int \frac{\frac{107c^3}{4}-\frac{85c^3x}{4a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{12ac^2}$$

$$= \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 + \frac{\text{Subst}\left(\int \frac{\frac{447c^4}{8}-\frac{321c^4x}{8a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x,\right.}{24a^2c^3}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 - \frac{(363c)\,\text{Su}}{}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 + \frac{363\,\text{Subs}}{}$$

$$= -\frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3} + \frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2} - \frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a} + \frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4 + \frac{363\sqrt{c}\,\text{ta}}{}$$

Mathematica [A] time = 0.16, size = 116, normalized size = 0.67

$$\frac{ax\left(48a^3x^3-136a^2x^2+214ax-447\right)\sqrt{c-\frac{c}{ax}}+1089\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-768\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-447 + 214*a*x - 136*a^2*x^2 + 48*a^3*x^3) + 1089*S
qrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 768*Sqrt[2]*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(192*a^4)
```

fricas [A] time = 0.89, size = 271, normalized size = 1.58

$$\left[\frac{768 \sqrt{2} \sqrt{c} \log\left(\frac{2 \sqrt{2} a \sqrt{c} x \sqrt{\frac{acx-c}{ax}} - 3 acx + c}{ax+1}\right) + 2 \left(48 a^4x^4 - 136 a^3x^3 + 214 a^2x^2 - 447 ax\right)\sqrt{\frac{acx-c}{ax}} + 1089 \sqrt{c} \log\left(-2\right.}{384 a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/384*(768*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) - 3*a*c*x + c)/(a*x + 1)) + 2*(48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2 -
 447*a*x)*sqrt((a*c*x - c)/(a*x)) + 1089*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c))/a^4, 1/192*(768*sqrt(2)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (48*a^4*x^4 - 136*a^3*x^3 +
 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) - 1089*sqrt(-c)*arctan(sqrt
(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [A] time = 0.05, size = 259, normalized size = 1.51

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(-96x \left(a x^2 - x\right)^{\frac{3}{2}} a^{\frac{9}{2}} \sqrt{\frac{1}{a}} + 176\sqrt{\frac{1}{a}}\, \left(a x^2 - x\right)^{\frac{3}{2}} a^{\frac{7}{2}} - 252\sqrt{\frac{1}{a}}\, \sqrt{a x^2 - x}\, a^{\frac{7}{2}} x + 126\sqrt{\frac{1}{a}}\, \sqrt{a x^2 - x}\, a^{\frac{5}{2}} +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/384*(c*(a*x-1)/a/x)^(1/2)*x*(-96*x*(a*x^2-x)^(3/2)*a^(9/2)*(1/a)^(1/2)+1
76*(1/a)^(1/2)*(a*x^2-x)^(3/2)*a^(7/2)-252*(1/a)^(1/2)*(a*x^2-x)^(1/2)*a^(7
/2)*x+126*(1/a)^(1/2)*(a*x^2-x)^(1/2)*a^(5/2)+768*(1/a)^(1/2)*a^(5/2)*((a*x
-1)*x)^(1/2)-1152*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/
a)^(1/2)*a^2-768*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*
x+1)/(a*x+1))*a^(3/2)+63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(11/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c - \frac{c}{ax}}\, x^3}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x^3/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{- c \left(-1 + \frac{1}{a x}\right)} \left(a x - 1\right)}{a x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.525 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=147

$$-\frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}+\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

```
[Out] -45/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3+4*arctanh(1/2*(c-c/a/x)^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3+19/8*x*(c-c/a/x)^(1/2)/a^2-13/12
*x^2*(c-c/a/x)^(1/2)/a+1/3*x^3*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6167, 6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}-\frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (19*Sqrt[c - c/(a*x)]*x)/(8*a^2) - (13*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 - (45*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^3
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,x^2(1-ax)}{1+ax}\,dx$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{1+ax}\,dx}{c}$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}\,dx}{c}$$
$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 + \frac{\mathrm{Subst}\left(\int \frac{\frac{13c^2}{2}-\frac{11c^2x}{2a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$
$$= -\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 - \frac{\mathrm{Subst}\left(\int \frac{\frac{57c^3}{4}-\frac{39c^3x}{4a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6ac^2}$$
$$= \frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 + \frac{\mathrm{Subst}\left(\int \frac{\frac{135c^4}{8}-\frac{57c^4x}{8a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6a^2c^3}$$
$$= \frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 + \frac{(45c)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{16a^3} -$$
$$= \frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 - \frac{45\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right.}{8a^2}$$
$$= \frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2} - \frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 - \frac{45\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3} + \frac{4\sqrt{2}\sqrt{c}}{}$$

Mathematica [A] time = 0.12, size = 108, normalized size = 0.73

$$\frac{ax\left(8a^2x^2-26ax+57\right)\sqrt{c-\frac{c}{ax}}-135\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+96\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(57 - 26*a*x + 8*a^2*x^2) - 135*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]] + 96*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt
[2]*Sqrt[c])])/(24*a^3)
```

fricas [A] time = 0.53, size = 259, normalized size = 1.76

$$\left[\frac{96\,\sqrt{2}\sqrt{c}\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\left(8\,a^3x^3-26\,a^2x^2+57\,ax\right)\sqrt{\frac{acx-c}{ax}}+135\,\sqrt{c}\log\left(-2\,acx+2\,a\sqrt{c}\right.}{48\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x
)) + 3*a*c*x - c)/(a*x + 1)) + 2*(8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*
c*x - c)/(a*x)) + 135*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)
/(a*x)) + c))/a^3, -1/24*(96*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*s
qrt((a*c*x - c)/(a*x))/c) - (8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*c*x -
 c)/(a*x)) - 135*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [A] time = 0.05, size = 237, normalized size = 1.61

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{\frac{1}{a}}\,\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{7}{2}}-36\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}x+18\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}+96\sqrt{\frac{1}{a}}\,a^{\frac{5}{2}}\sqrt{(ax-1)\,x}-96\right.}{48\sqrt{(ax-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(1/a)^(1/2)*(a*x^2-x)^(3/2)*a^(7/2)-36*(1/
a)^(1/2)*(a*x^2-x)^(1/2)*a^(7/2)*x+18*(1/a)^(1/2)*(a*x^2-x)^(1/2)*a^(5/2)+9
6*(1/a)^(1/2)*a^(5/2)*((a*x-1)*x)^(1/2)-96*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2
)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(3/2)-144*ln(1/2*(2*((a*x-1)*x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2+9*ln(1/2*(2*(a*x^2-x)^(1/2)*
a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(9/2)/(1/a)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}\,x^2}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.526 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=122

$$\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

```
[Out] 23/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2-4*arctanh(1/2*(c-c/a/x)^(
1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2-9/4*x*(c-c/a/x)^(1/2)/a+1/2*x^2*(
c-c/a/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6167, 6133, 25, 514, 446, 98, 151, 156, 63, 208}

$$\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-9*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (23*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=-\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$$
$$=-\int\frac{\sqrt{c-\frac{c}{ax}}\,x(1-ax)}{1+ax}dx$$
$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{1+ax}dx}{c}$$
$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}dx}{c}$$
$$=-\frac{a\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{\text{Subst}\left(\int\frac{\frac{9c^2}{2}-\frac{7c^2x}{2a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2c}$$
$$=-\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{\text{Subst}\left(\int\frac{\frac{23c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2ac^2}$$
$$=-\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{(23c)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{8a^2}+\frac{(4c)\,\text{Subst}\Big(}{}$$
$$=-\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{23\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)}{4a}-\frac{8\,\text{Subst}\Big(}{}$$
$$=-\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\Big(\frac{\sqrt{c-}}{\sqrt{2}}}{a^2}$$

Mathematica [**A**] time = 0.10, size = 100, normalized size = 0.82

$$\frac{ax(2ax-9)\sqrt{c-\frac{c}{ax}}+23\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-16\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
[Out] (a*Sqrt[c - c/(a*x)]*x*(-9 + 2*a*x) + 23*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/
Sqrt[c]] - 16*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])
/(4*a^2)
```

fricas [**A**] time = 0.47, size = 239, normalized size = 1.96

$$\left[\frac{16\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+2\left(2a^2x^2-9ax\right)\sqrt{\frac{acx-c}{ax}}+23\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8a^2},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 - 3*a*c*x + c)/(a*x + 1)) + 2*(2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x))
+ 23*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2
, 1/4*(16*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) + (2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)) - 23*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.04, size = 216, normalized size = 1.77

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(4\sqrt{\frac{1}{a}}\, \sqrt{a\,x^{2}-x}\, a^{\frac{7}{2}} x-2\sqrt{\frac{1}{a}}\, \sqrt{a\,x^{2}-x}\, a^{\frac{5}{2}}-16\sqrt{\frac{1}{a}}\, a^{\frac{5}{2}} \sqrt{(ax-1)\,x}\, +16\sqrt{2}\, \ln\left(\frac{2\sqrt{2}\, \sqrt{\frac{1}{a}}\, \sqrt{(ax-1)x}\, a-3ax+1}{ax+1}\right.\right.}{8\sqrt{(ax-1)\,x}\, a^{\frac{7}{2}} \sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*(1/a)^(1/2)*(a*x^2-x)^(1/2)*a^(7/2)*x-2*(1/a
)^(1/2)*(a*x^2-x)^(1/2)*a^(5/2)-16*(1/a)^(1/2)*a^(5/2)*((a*x-1)*x)^(1/2)+16
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^
(3/2)+24*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*
a^2-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/((
a*x-1)*x)^(1/2)/a^(7/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}\, x}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.527 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=92

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+4*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6133, 25, 514, 375, 98, 156, 63, 208}

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=-\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$
$$=-\int\frac{\sqrt{c-\frac{c}{ax}}(1-ax)}{1+ax}dx$$
$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}dx}{c}$$
$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}dx}{c}$$
$$=-\frac{a\operatorname{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$
$$=\sqrt{c-\frac{c}{ax}}x+\frac{\operatorname{Subst}\left(\int\frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{c}$$
$$=\sqrt{c-\frac{c}{ax}}x+\frac{(5c)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}-\frac{(4c)\operatorname{Subst}\left(\int\frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{a}$$
$$=\sqrt{c-\frac{c}{ax}}x-5\operatorname{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}dx,x,\sqrt{c-\frac{c}{ax}}\right)+8\operatorname{Subst}\left(\int\frac{1}{2a-\frac{ax^2}{c}}dx,x,\sqrt{c\right.$$
$$=\sqrt{c-\frac{c}{ax}}x-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 1.00

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

fricas [A] time = 0.42, size = 219, normalized size = 2.38

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\,\sqrt{c}\,\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\frac{ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x
- c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.05, size = 190, normalized size = 2.07

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(2\sqrt{a\,x^2-x}\; a^{\frac{3}{2}}\sqrt{\frac{1}{a}} - 4\sqrt{(ax-1)\,x}\; a^{\frac{3}{2}}\sqrt{\frac{1}{a}} - \ln\left(\frac{2\sqrt{a\,x^2-x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right) a\sqrt{\frac{1}{a}} + 4\sqrt{2}\;\ln\left(\frac{2\sqrt{2}\;\sqrt{\frac{1}{a}}\;\sqrt{(ax-}}{ax+1}\right.\right.}{2\sqrt{(ax-1)\,x}\; a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x
-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*a*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/
2)*a-3*a*x+1)/(a*x+1))*a^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\;(a\,x-1)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\;(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.528 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=86

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(c-c/a/x)^(1/2)*2^
(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 434, 446, 84, 156, 63, 208}

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x(1+ax)}\,dx\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{1+ax}\,dx}{c}\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}\,dx}{c}\\
&= -\frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c}\\
&= 2\sqrt{c-\frac{c}{ax}}-\frac{a\,\mathrm{Subst}\left(\int \frac{c^2-\frac{3c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= 2\sqrt{c-\frac{c}{ax}}-c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)+(4c)\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\right.\\
&= 2\sqrt{c-\frac{c}{ax}}+(2a)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)-(8a)\,\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,\right.\\
&= 2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 86, normalized size = 1.00

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.53, size = 203, normalized size = 2.36

$$\left[2\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)+2\sqrt{\frac{acx-c}{ax}},4\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a
*c*x + c)/(a*x + 1)) + sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x)) + c) + 2*sqrt((a*c*x - c)/(a*x)), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sq
rt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*sqrt(-c)*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) + 2*sqrt((a*c*x - c)/(a*x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [B] time = 0.06, size = 228, normalized size = 2.65

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-4\sqrt{a x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,x^2+2a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)x}\,x^2+2\left(a x^2-x\right)^{\frac{3}{2}}\sqrt{a}\,\sqrt{\frac{1}{a}}+2\ln\left(\frac{2\sqrt{a x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\right.}{x\sqrt{(ax-1)x}\,\sqrt{a}\,\sqrt{\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] -(c*(a*x-1)/a/x)^(1/2)/x*(-4*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)*x^2+2*a^(3
/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^2+2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2)
+2*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^2*a-2*
a^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x
+1))*x^2-3*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2
)*x^2*a)/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

sympy [A] time = 12.75, size = 80, normalized size = 0.93

$$-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}\,c\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-\frac{c}{ax}}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] -2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*atan(sqrt(2)*s
qrt(c - c/(a*x))/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(c - c/(a*x))
```

3.529 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -2/3*a*(c-c/a/x)^(3/2)/c+4*a*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)-4*a*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6133, 25, 514, 444, 50, 63, 208}

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] -4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c) + 4*Sqrt[2]*a*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx\\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^2(1+ax)}\,dx\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x(1+ax)}\,dx}{c}\\
&= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}\,dx}{c}\\
&= -\frac{a\,\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}\\
&= -\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - (2a)\,\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)\\
&= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - (4ac)\,\mathrm{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\\
&= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \left(8a^2\right)\mathrm{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)\\
&= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.84

$$\frac{2(1-7ax)\sqrt{c-\frac{c}{ax}}}{3x}+4\sqrt{2}a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 - 7*a*x))/(3*x) + 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[Sqrt[
c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.58, size = 161, normalized size = 1.96

$$\left[\frac{2\left(3\sqrt{2}a\sqrt{c}x\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)-(7ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3x},-\frac{2\left(6\sqrt{2}a\sqrt{-c}x\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2c}\right)+(\right.}{3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a*sqrt(c)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + 3*a*c*x - c)/(a*x + 1)) - (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x, -2
/3*(6*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m operator + Error: Bad Argument Value
```

maple [B] time = 0.05, size = 254, normalized size = 3.10

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-18\sqrt{a\,x^{2}-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^{3}+6a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\,x^{3}+12a^{\frac{3}{2}}\left(a\,x^{2}-x\right)^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+9\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right.\right.}{3x^{2}\sqrt{(ax-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] 1/3*(c*(a*x-1)/a/x)^(1/2)/x^2*(-18*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x^3+
6*a^(5/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^3+12*a^(3/2)*(a*x^2-x)^(3/2)*x*(1
/a)^(1/2)+9*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)
*x^3*a^2-6*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*
a*x+1)/(a*x+1))*x^3-9*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*(1/a)^(1/2)*x^3*a^2-2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/
2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^2\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

3.530 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=113

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^2\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2/3*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2-4*a^2*arctanh(1/2*(c-
c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^2*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 80, 50, 63, 208}

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^2\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/(3*c) + (2*a^2*(c - c
/(a*x))^(5/2))/(5*c^2) - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^3(1+ax)}\,dx$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^2(1+ax)}\,dx}{c}$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}\,dx}{c}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{a^2\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$
$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(4a^2c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-}}\right.$$
$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \left(8a^3\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x\right.$$
$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - 4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Mathematica [A] time = 0.09, size = 79, normalized size = 0.70

$$\frac{2\left(38a^2x^2-11ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x^2} - 4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(3 - 11*a*x + 38*a^2*x^2))/(15*x^2) - 4*Sqrt[2]*a^2*Sq
rt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.44, size = 181, normalized size = 1.60

$$\left[\frac{2\left(15\sqrt{2}\,a^2\sqrt{c}\,x^2\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(38\,a^2x^2-11\,ax+3\right)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2}, \frac{2\left(30\sqrt{2}\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{2}}{}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*a^2*sqrt(c)*x^2*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x
 - c)/(a*x)))/x^2, 2/15*(30*sqrt(2)*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x))/c) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x -
c)/(a*x)))/x^2]
```

giac [B] time = 0.88, size = 278, normalized size = 2.46

$$\frac{4\sqrt{2}\,a^3c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}\,|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}\,|a|\mathrm{sgn}(x)}+\frac{2\left(60\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^4a^5c-45\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-ac\cdots}\right.\right.}{15\left(\sqrt{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/15*(60*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c - 45*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - a*c*x))^3*a^4*c^(3/2)*abs(a) + 35*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - a*c*x))^2*a^5*c^2 - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^4*c
^(5/2)*abs(a) + 3*a^5*c^3)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*a^2
*abs(a)*sgn(x))
```

maple [B] time = 0.05, size = 278, normalized size = 2.46

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-90\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^4+30a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\,x}\,x^4+60a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2\sqrt{\frac{1}{a}}+45\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax\cdots}{2\sqrt{a}}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] -1/15*(c*(a*x-1)/a/x)^(1/2)/x^3*(-90*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^
4+30*a^(7/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^4+60*a^(5/2)*(a*x^2-x)^(3/2)*x
^2*(1/a)^(1/2)+45*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)
^(1/2)*x^4*a^3-30*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a
*x+1))*a^(5/2)*2^(1/2)*x^4-45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/
a^(1/2))*(1/a)^(1/2)*x^4*a^3-16*a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)+6*(a*
x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^3\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

3.531 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=113

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] -2/3*a^3*(c-c/a/x)^(3/2)/c-2/7*a^3*(c-c/a/x)^(7/2)/c^3+4*a^3*arctanh(1/2*(c
-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-4*a^3*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 88, 50, 63, 208}

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] -4*a^3*Sqrt[c - c/(a*x)] - (2*a^3*(c - c/(a*x))^(3/2))/(3*c) - (2*a^3*(c -
c/(a*x))^(7/2))/(7*c^3) + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^4(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^3(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^4}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x} - \frac{a\left(c-\frac{cx}{a}\right)^{5/2}}{c}\right)dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{a^3\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \left(2a^3\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \left(4a^3c\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\ldots\right.$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \left(8a^4\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \ldots\right.$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + 4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Mathematica [A] time = 0.14, size = 87, normalized size = 0.77

$$4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right) + \frac{2\left(-52a^3x^3+16a^2x^2-9ax+3\right)\sqrt{c-\frac{c}{ax}}}{21x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(3 - 9*a*x + 16*a^2*x^2 - 52*a^3*x^3))/(21*x^3) + 4*Sq
rt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

fricas [A] time = 0.60, size = 201, normalized size = 1.78

$$\left[\frac{2\left(21\sqrt{2}\,a^3\sqrt{c}\,x^3\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-\left(52\,a^3x^3-16\,a^2x^2+9\,ax-3\right)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3}, -\frac{2\left(42\sqrt{2}\,a^3\sqrt{-c}\,x^3\,\text{ar}\ldots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*a^3*sqrt(c)*x^3*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (52*a^3*x^3 - 16*a^2*x^2 + 9*a*x - 3
)*sqrt((a*c*x - c)/(a*x)))/x^3, -2/21*(42*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1
/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (52*a^3*x^3 - 16*a^2*x^2 +
 9*a*x - 3)*sqrt((a*c*x - c)/(a*x)))/x^3]
```

giac [B] time = 1.06, size = 356, normalized size = 3.15

$$-\frac{4\sqrt{2}\,a^4c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}\,|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}\,|a|\mathrm{sgn}(x)}-\frac{2\left(84\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^6a^7c-84\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/21*(84*(
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a^7*c - 84*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^5*a^6*c^(3/2)*abs(a) + 112*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - a*c*x))^4*a^7*c^2 - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*
a^6*c^(5/2)*abs(a) + 63*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^7*c^3
 - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^6*c^(7/2)*abs(a) + 3*a^7*
c^4)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^3*abs(a)*sgn(x))
```

maple [B] time = 0.06, size = 302, normalized size = 2.67

$$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-126\sqrt{a\,x^2-x}\,\sqrt{\frac{1}{a}}\,a^{\frac{9}{2}}x^5+42\sqrt{\frac{1}{a}}\,a^{\frac{9}{2}}\sqrt{(ax-1)\,x}\,x^5+84\left(a\,x^2-x\right)^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,a^{\frac{7}{2}}x^3+63\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] 1/21*(c*(a*x-1)/a/x)^(1/2)/x^4*(-126*(a*x^2-x)^(1/2)*(1/a)^(1/2)*a^(9/2)*x^
5+42*(1/a)^(1/2)*a^(9/2)*((a*x-1)*x)^(1/2)*x^5+84*(a*x^2-x)^(3/2)*(1/a)^(1/
2)*a^(7/2)*x^3+63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)
^(1/2)*x^5*a^4-42*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a
*x+1))*2^(1/2)*a^(7/2)*x^5-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/
a^(1/2))*(1/a)^(1/2)*x^5*a^4-20*a^(5/2)*(a*x^2-x)^(3/2)*x^2*(1/a)^(1/2)+12*
a^(3/2)*(a*x^2-x)^(3/2)*x*(1/a)^(1/2)-6*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2)
)/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^4\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

3.532 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=163

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2/3*a^4*(c-c/a/x)^(3/2)/c+2/5*a^4*(c-c/a/x)^(5/2)/c^2-2/7*a^4*(c-c/a/x)^(7/
2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4-4*a^4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2
)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^4*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6133, 25, 514, 446, 88, 50, 63, 208}

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] + (2*a^4*(c - c/(a*x))^(3/2))/(3*c) + (2*a^4*(c - c
/(a*x))^(5/2))/(5*c^2) - (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c -
c/(a*x))^(9/2))/(9*c^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^5(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^4(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^5}\,dx}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{x^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(a^2\left(c-\frac{cx}{a}\right)^{3/2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x} - \frac{a^2\left(c-\frac{cx}{a}\right)^{5/2}}{c} + \frac{a^2\left(c-\frac{cx}{a}\right)^{7/2}}{c^2}\right)dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + \frac{a^4\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right.}{c}$$

$$= \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + \left(2a^4\right)\text{Subs}$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)}{9c^4}$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)}{9c^4}$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)}{9c^4}$$

<u>Mathematica</u> [A] time = 0.15, size = 95, normalized size = 0.58

$$\frac{2\left(788a^4x^4 - 236a^3x^3 + 138a^2x^2 - 95ax + 35\right)\sqrt{c-\frac{c}{ax}}}{315x^4} - 4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(35 - 95*a*x + 138*a^2*x^2 - 236*a^3*x^3 + 788*a^4*x^4
))/(315*x^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqr
t[c])]
```

fricas [A] time = 0.47, size = 213, normalized size = 1.31

$$\left[\frac{2\left(315\sqrt{2}\,a^4\sqrt{c}\,x^4\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35\right)\sqrt{\frac{acx-c}{ax}}\right)}{315\,x^4}, \frac{2\left(630\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*a^4*sqrt(c)*x^4*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (788*a^4*x^4 - 236*a^3*x^3 + 138*a^
2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4, 2/315*(630*sqrt(2)*a^4*s
qrt(-c)*x^4*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (788*a
^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/
x^4]
```

giac [B] time = 1.33, size = 434, normalized size = 2.66

$$\frac{4\sqrt{2}\,a^5c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}\,|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}\,|a|\mathrm{sgn}(x)}+\frac{2\left(1260\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^8a^9c-1260\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^5*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/315*(1260
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^8*a^9*c - 1260*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - a*c*x))^7*a^8*c^(3/2)*abs(a) + 2100*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - a*c*x))^6*a^9*c^2 - 3150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))^5*a^8*c^(5/2)*abs(a) + 3528*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4
*a^9*c^3 - 2625*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^8*c^(7/2)*abs
(a) + 1215*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^9*c^4 - 315*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^8*c^(9/2)*abs(a) + 35*a^9*c^5)/((sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*a^4*abs(a)*sgn(x))
```

maple [B] time = 0.06, size = 326, normalized size = 2.00

$$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-1890\sqrt{a\,x^2-x}\;a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\;x^6+630a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\;\sqrt{(ax-1)\,x}\;x^6+1260\left(a\,x^2-x\right)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\;x^4+945\ln\left(\frac{2\sqrt{a\,x^2}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)/(a*x+1)*(a*x-1)/x^5,x)
```

```
[Out] -1/315*(c*(a*x-1)/a/x)^(1/2)/x^5*(-1890*(a*x^2-x)^(1/2)*a^(11/2)*(1/a)^(1/2
)*x^6+630*a^(11/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*x^6+1260*(a*x^2-x)^(3/2)*a
^(9/2)*(1/a)^(1/2)*x^4+945*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/
2))*(1/a)^(1/2)*x^6*a^5-630*a^(9/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x
-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^6-945*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2
)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*x^6*a^5-316*(a*x^2-x)^(3/2)*(1/a)^(1/2)*a^(
7/2)*x^3+156*a^(5/2)*(a*x^2-x)^(3/2)*x^2*(1/a)^(1/2)-120*a^(3/2)*(a*x^2-x)^
(3/2)*x*(1/a)^(1/2)+70*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/
2)/a^(1/2)/(1/a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{ax}}}{(ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^5\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

3.533 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=303

$$-\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{1115\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}-\frac{1115x\sqrt{c-\frac{c}{ax}}}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{223x^2\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{}{4\sqrt{}}$$

```
[Out] 1115/64*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)-1115/6
4*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1115/192*x*(c-c/a/x)^
(1/2)/a^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+223/96*x^2*(c-c/a/x)^(1/2)/a^2/(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-25/24*x^3*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)/(
1+1/a/x)^(1/2)+1/4*x^4*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 306, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6182, 6180, 89, 78, 51, 63, 208}

$$\frac{1115x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}}-\frac{223x^2\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{1115x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{1115\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}+\frac{}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-1115*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(64*a^3*Sqrt[1 - 1/(a*x)]) -
(223*Sqrt[c - c/(a*x)]*x^2)/(24*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) +
(1115*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]) -
 (25*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (S
qrt[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1115*Sqrt[
c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}x^3\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}x^3\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^5\left(1+\frac{x}{a}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{-\frac{25}{2a}+\frac{4x}{a^2}}{x^4\left(1+\frac{x}{a}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(223\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\right.}{48a^2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(1115\sqrt{\right.}{}$$

$$= -\frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} - \frac{25\sqrt{c-\frac{c}{ax}}x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{}{4}$$

$$= -\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} - \frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} -$$

$$= -\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} - \frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} -$$

$$= -\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x}{64a^3\sqrt{1-\frac{1}{ax}}} - \frac{223\sqrt{c-\frac{c}{ax}}x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}x^2}{96a^2\sqrt{1-\frac{1}{ax}}} -$$

Mathematica [A] time = 0.63, size = 167, normalized size = 0.55

$$\frac{3345\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)+\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(48a^4x^4-200a^3x^3+446a^2x^2-1115ax-3345\right)\sqrt{}}{a^2x^2-1}}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3345 - 1115*a*x + 446
*a^2*x^2 - 200*a^3*x^3 + 48*a^4*x^4))/(-1 + a^2*x^2) - 3345*Sqrt[c]*Log[1 -
 a*x] + 3345*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*
x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(384*a^4)
```

fricas [A] time = 0.73, size = 353, normalized size = 1.17

$$\left[\frac{3345\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-11\right.}{768\left(a^5x-a^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3345*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 +
 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) + 4*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2
 - 3345*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^
4), -1/384*(3345*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2
*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2 - 3345*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 197, normalized size = 0.65

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\;x\left(96a^{\frac{9}{2}}\sqrt{(ax+1)\,x}\;x^4-400a^{\frac{7}{2}}x^3\sqrt{(ax+1)\,x}\;+892a^{\frac{5}{2}}x^2\sqrt{(ax+1)\,x}\;-2230a^{\frac{3}{2}}x\sqrt{(a\right.}{384\,(ax-1)^2\,a^{\frac{7}{2}}\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/384*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(96
*a^(9/2)*((a*x+1)*x)^(1/2)*x^4-400*a^(7/2)*x^3*((a*x+1)*x)^(1/2)+892*a^(5/2
)*x^2*((a*x+1)*x)^(1/2)-2230*a^(3/2)*x*((a*x+1)*x)^(1/2)+3345*ln(1/2*(2*((a
*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a-6690*((a*x+1)*x)^(1/2)*a^(1/2)
+3345*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(7/2)/((a*x+
1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{ax}}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.534 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=251

$$\frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{119\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}+\frac{119x\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{x^3\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{19x^2}{12a\sqrt{1-}}$$

```
[Out] -119/8*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+119/8*(
c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+119/24*x*(c-c/a/x)^(1/2)
/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-19/12*x^2*(c-c/a/x)^(1/2)/a/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)+1/3*x^3*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/
2)
```

Rubi [A] time = 0.29, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6182, 6180, 89, 78, 51, 63, 208}

$$\frac{119x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{119x\sqrt{c-\frac{c}{ax}}}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{119\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}+\frac{x^3\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{19x^2}{12a\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-119*Sqrt[c - c/(a*x)]*x)/(12*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (
119*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(8*a^2*Sqrt[1 - 1/(a*x)]) - (19*
Sqrt[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c
 - c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (119*Sqrt[c - c/
(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^2\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^4\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{19}{2a}+\frac{3x}{a^2}}{x^3\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{19\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(119\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x^2\left(1+\frac{x}{a}\right.}\right.}{24a^2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}\,x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{(119}{}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}\,x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{}{3}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}\,x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{}{3}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}\,x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{}{3}$$

Mathematica [A] time = 0.54, size = 159, normalized size = 0.63

$$\frac{-357\sqrt{c}\log\left(2a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)+\frac{2a^2x^2\sqrt{1-\frac{1}{a^2x^2}}\left(8a^3x^3-38a^2x^2+119ax+357\right)\sqrt{c-\frac{c}{ax}}}{a^2x^2-1}+3}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(357 + 119*a*x - 38*a^2
*x^2 + 8*a^3*x^3))/(-1 + a^2*x^2) + 357*Sqrt[c]*Log[1 - a*x] - 357*Sqrt[c]*
Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x
 + 2*a^2*x^2)])/(48*a^3)
```

fricas [A] time = 0.60, size = 337, normalized size = 1.34

$$\left[\frac{357\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(8\,a^4x^4-38\,a^3x^3+119\,a^2x^2+357\,a\right.}{96\left(a^4x-a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(357*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(357*(a*x -
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 38*a^3*x^
3 + 119*a^2*x^2 + 357*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^4*x - a^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 180, normalized size = 0.72

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{7}{2}}x^3\sqrt{(ax+1)x}-76a^{\frac{5}{2}}x^2\sqrt{(ax+1)x}+238a^{\frac{3}{2}}x\sqrt{(ax+1)x}-357\ln\left(\frac{2\sqrt{(ax+1)x}\ \cdots}{2\sqrt{\cdots}}\right.\right.}{48\,(ax-1)^2\,a^{\frac{5}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(16*
a^(7/2)*x^3*((a*x+1)*x)^(1/2)-76*a^(5/2)*x^2*((a*x+1)*x)^(1/2)+238*a^(3/2)*
x*((a*x+1)*x)^(1/2)-357*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2
))*x*a+714*((a*x+1)*x)^(1/2)*a^(1/2)-357*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2
)+2*a*x+1)/a^(1/2)))/a^(5/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{ax}}\,x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

3.535 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=199

$$-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{13x\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

```
[Out] 47/4*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)-47/4*(c-c
/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-13/4*x*(c-c/a/x)^(1/2)/a/(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1/2*x^2*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/
a/x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6182, 6180, 89, 78, 51, 63, 208}

$$-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{13x\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-47*Sqrt[c - c/(a*x)])/(4*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (13*S
qrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (47*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^3\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\operatorname{Subst}\left(\int\frac{-\frac{13}{2a}+\frac{2x}{a^2}}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(47\sqrt{c-\frac{c}{ax}}\right)\operatorname{Subst}\left(\int\frac{1}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,\right.}{8a^2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(47\sqrt{c-\frac{c}{ax}}\right.}{}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(47\sqrt{c-\frac{c}{ax}}\right.}{}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{47\sqrt{c-\frac{c}{ax}}}{4a}$$

Mathematica [A] time = 0.54, size = 151, normalized size = 0.76

$$\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-13ax-47\right)\sqrt{c-\frac{c}{ax}}}{4a^2x^2-4}+\frac{47\sqrt{c}\log\left(2a^2\sqrt{c}\,x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)}{8a^2}-\frac{47\sqrt{c}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-47 - 13*a*x + 2*a^2*x^2))/(-
4 + 4*a^2*x^2) - (47*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (47*Sqrt[c]*Log[2*a^2*
Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x
^2)])/(8*a^2)
```

fricas [A] time = 0.53, size = 321, normalized size = 1.61

$$\left[\frac{47\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(2\,a^3x^3-13\,a^2x^2-47\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3x-a^2\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(47*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(47*(a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 163, normalized size = 0.82

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1) \sqrt{\frac{c(ax-1)}{ax}}\, x \left(4a^{\frac{5}{2}}x^2\sqrt{(ax+1)\,x} - 26a^{\frac{3}{2}}x\sqrt{(ax+1)\,x} + 47\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa - 94\sqrt{(ax}\right.}{8\,(ax-1)^2\, a^{\frac{3}{2}}\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^
(5/2)*x^2*((a*x+1)*x)^(1/2)-26*a^(3/2)*x*((a*x+1)*x)^(1/2)+47*ln(1/2*(2*((a
*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a-94*((a*x+1)*x)^(1/2)*a^(1/2)+4
7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(3/2)/((a*x+1)*x
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{ax}}\, x \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.536 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=140

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -7*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+9*(c-c/a/x)^(
1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+x*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6182, 6179, 89, 78, 63, 208}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}\\
&= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{7}{2a}+\frac{x}{a^2}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2a\sqrt{1-\frac{1}{ax}}}\\
&= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}\\
&= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(ax-7\sqrt{\frac{1}{ax}+1}\,\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)+9\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]
]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.84, size = 299, normalized size = 2.14

$$\left[\frac{7\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^{2}x^{2}+9\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^{2}x-a\right)},\ \frac{7\,(ax-1)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 146, normalized size = 1.04

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{(ax+1)\,x}\,-7\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa+18\sqrt{(ax+1)\,x}\,\sqrt{a}\,-7\ln\left(\frac{2\sqrt{(ax}\cdots}{\cdots}\right.\right.}{2\,(ax-1)^{2}\sqrt{a}\,\sqrt{(ax+1)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^
(3/2)*x*((a*x+1)*x)^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*x*a+18*((a*x+1)*x)^(1/2)*a^(1/2)-7*ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2)))/a^(1/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

3.537 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=134

$$-\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-8*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/
x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6182, 6180, 87, 63, 208}

$$-\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-8*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (2*Sqrt[1 +
1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(a*x)]*ArcTan
h[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \left(-\frac{4}{a\left(1+\frac{x}{a}\right)^{3/2}}+\frac{1}{a\sqrt{1+\frac{x}{a}}}+\frac{1}{x\sqrt{1+\frac{x}{a}}}\right)dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A] time = 0.41, size = 131, normalized size = 0.98

$$-\frac{2ax\sqrt{1-\frac{1}{a^2x^2}}(5ax+1)\sqrt{c-\frac{c}{ax}}}{a^2x^2-1}+\sqrt{c}\log\left(2a^2\sqrt{c}x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+c\left(2a^2x^2-ax-1\right)\right)-\sqrt{c}\log(1-ax)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x*(1 + 5*a*x))/(-1 + a^2*x^2)
 - Sqrt[c]*Log[1 - a*x] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*S
qrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)]
```

fricas [A] time = 0.65, size = 277, normalized size = 2.07

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4(5ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(ax-1)}, -\frac{(ax-1)\sqrt{-c}\,\text{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
)/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(
5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 151, normalized size = 1.13

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(10a^{\frac{3}{2}}x\sqrt{(ax+1)x}-\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)x^{2}a^{2}-\ln\left(\frac{2\sqrt{(ax+1)x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)xa+2\right.}{(ax-1)^{2}\sqrt{a}\,\sqrt{(ax+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*(10*a^(3/2
)*x*((a*x+1)*x)^(1/2)-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*x^2*a^2-ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x*a+2*((a*x+
1)*x)^(1/2)*a^(1/2))/a^(1/2)/((a*x+1)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

3.538 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -16/3*a*(c-c/a/x)^(3/2)/c/(1-1/a^2/x^2)^(1/2)-2/3*a*(c-c/a/x)^(5/2)/c^2/(1-
1/a^2/x^2)^(1/2)+64/3*a*(c-c/a/x)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6178, 657, 649}

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (64*a*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a^2*x^2)]) - (16*a*(c - c/(a*x))^(3
/2))/(3*c*Sqrt[1 - 1/(a^2*x^2)]) - (2*a*(c - c/(a*x))^(5/2))/(3*c^2*Sqrt[1
- 1/(a^2*x^2)])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{8\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{32\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.10, size = 58, normalized size = 0.53

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(23a^2x^2+10ax-1\right)\sqrt{c-\frac{c}{ax}}}{3a^2x^2-3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 10*a*x + 23*a^2*x^2))/(-
3 + 3*a^2*x^2)
```

fricas [A] time = 0.50, size = 59, normalized size = 0.54

$$\frac{2\left(23\,a^2x^2+10\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] 2/3*(23*a^2*x^2 + 10*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x))/(a*x^2 - x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
```

```
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 62, normalized size = 0.57

$$\frac{2\left(ax+1\right)\left(23a^{2}x^{2}+10ax-1\right)\sqrt{\frac{c\left(ax-1\right)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] 2/3*(a*x+1)*(23*a^2*x^2+10*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(
3/2)/x/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

mupad [B] time = 1.36, size = 54, normalized size = 0.50

$$\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(23\,a^{2}\,x^{2}+10\,a\,x-1\right)}{3\,x\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(10*a*x + 23*a^2*x^2 - 1
))/(3*x*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

3.539 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=150

$$-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -a^2*(c-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)-7/5*a^2*(c-c/a/x)^(3/2)*(1-1/a
^2/x^2)^(1/2)/c-224/15*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-56/15*a^2*
(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6178, 789, 657, 649}

$$-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-224*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(15*Sqrt[c - c/(a*x)]) - (56*a^2*Sqrt[1
 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/15 - (7*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/
(a*x))^(3/2))/(5*c) - (a^2*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 789

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d*g + e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*
(p + 1)), x] - Dist[(e*(m*(d*g + e*f) + 2*e*f*(p + 1)))/(2*c*d*(p + 1)), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{(7a)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{(28a)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5c}$$

$$= -\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{15}(11$$

$$= -\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}} - \frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2}{c^3}$$

Mathematica [A] time = 0.12, size = 70, normalized size = 0.47

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(158a^3x^3+79a^2x^2-16ax+3\right)\sqrt{c-\frac{c}{ax}}}{15x\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 - 16*a*x + 79*a^2*x^2 + 15
8*a^3*x^3))/(15*x*(-1 + a^2*x^2))
```

fricas [A] time = 0.50, size = 69, normalized size = 0.46

$$-\frac{2\left(158\,a^3x^3+79\,a^2x^2-16\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] -2/15*(158*a^3*x^3 + 79*a^2*x^2 - 16*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x))/(a*x^3 - x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 70, normalized size = 0.47

$$-\frac{2\left(ax+1\right)\left(158x^{3}a^{3}+79a^{2}x^{2}-16ax+3\right)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^{2}\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] -2/15*(a*x+1)*(158*a^3*x^3+79*a^2*x^2-16*a*x+3)*(c*(a*x-1)/a/x)^(1/2)*((a*x
-1)/(a*x+1))^(3/2)/x^2/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

mupad [B] time = 1.44, size = 62, normalized size = 0.41

$$-\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(158\,a^{3}\,x^{3}+79\,a^{2}\,x^{2}-16\,a\,x+3\right)}{15\,x^{2}\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

```
[Out] -(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a^2*x^2 - 16*a*x +
158*a^3*x^3 + 3))/(15*x^2*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

3.540 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=188

$$\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{1888a^3c\sqrt{1\cdots}}{105\sqrt{c\cdots}}$$

```
[Out] a^3*(c-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)+59/35*a^3*(c-c/a/x)^(3/2)*(1-1/
a^2/x^2)^(1/2)/c+2/7*a^3*(c-c/a/x)^(5/2)*(1-1/a^2/x^2)^(1/2)/c^2+1888/105*a
^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+472/105*a^3*(1-1/a^2/x^2)^(1/2)*(c
-c/a/x)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6178, 1635, 795, 657, 649}

$$\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{1888a^3c\sqrt{1\cdots}}{105\sqrt{c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (1888*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) + (472*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (59*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c
- c/(a*x))^(3/2))/(35*c) + (2*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(5/2)
)/(7*c^2) + (a^3*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 649

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*Simplify[m + p])/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1635

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, -Simp[(d*f*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*e*(
p + 1)), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
```

```
1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a,
c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0] &
& GtQ[m, 0]
```

Rule 6178

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^n, Subst[Int[((c + d*x)^(p - n)*(1 - x^2/a^2)^(n/2))/x^(m
 + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{7a^2}{2}-ax\right)\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\left(59a^2\right)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{14c^2}$$

$$= \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\left(236a^2\right)\text{Su}}{}$$

$$= \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}$$

$$= \frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} + \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a}{}$$

Mathematica [A] time = 0.13, size = 78, normalized size = 0.41

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15\right)\sqrt{c-\frac{c}{ax}}}{105x^2\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 66*a*x - 167*a^2*x^2 +
668*a^3*x^3 + 1336*a^4*x^4))/(105*x^2*(-1 + a^2*x^2))
```

fricas [A] time = 0.82, size = 77, normalized size = 0.41

$$\frac{2\left(1336\,a^4x^4+668\,a^3x^3-167\,a^2x^2+66\,ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] 2/105*(1336*a^4*x^4 + 668*a^3*x^3 - 167*a^2*x^2 + 66*a*x - 15)*sqrt((a*x -
1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 78, normalized size = 0.41

$$\frac{2\left(ax+1\right)\left(1336x^4a^4+668x^3a^3-167a^2x^2+66ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] 2/105*(a*x+1)*(1336*a^4*x^4+668*a^3*x^3-167*a^2*x^2+66*a*x-15)*(c*(a*x-1)/a
/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

mupad [B] time = 1.41, size = 100, normalized size = 0.53

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(1336\,a^3\,x^3+2004\,a^2\,x^2+1837\,a\,x+1903\right)\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3}+\frac{3776\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2)*(1837*a*x + 2004*a^2*x^2 + 1336*a^3*x^3 + 19
03)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) + (3776*((a*x - 1)/(a*x + 1))^(1
/2)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

3.541 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=289

$$-\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-32a$$

```
[Out] 50/3*a^4*(1+1/a/x)^(3/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-38/5*a^4*(1+1/a/x)
^(5/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2*a^4*(1+1/a/x)^(7/2)*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)-2/9*a^4*(1+1/a/x)^(9/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-8
*a^4*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-32*a^4*(1+1/a/x)^(1/2)
*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6182, 6180, 88}

$$-\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-32a$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-8*a^4*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (32*a^4*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (50*a^4*(1 + 1/(a*
x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) - (38*a^4*(1 + 1/(a*x))^
(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(7/2)
*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] - (2*a^4*(1 + 1/(a*x))^(9/2)*Sqrt[c -
 c/(a*x)])/(9*Sqrt[1 - 1/(a*x)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6180

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> -Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^(m + 2)
*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^5}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \left(-\frac{4a^3}{\left(1+\frac{x}{a}\right)^{3/2}}+\frac{16a^3}{\sqrt{1+\frac{x}{a}}}-25a^3\sqrt{1+\frac{x}{a}}+19a^3\left(1+\frac{x}{a}\right)^{3/2}-7a^3\left(1+\right.\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{32a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(1\right.}{5}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.30

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\left(656a^5x^5+328a^4x^4-82a^3x^3+41a^2x^2-20ax+5\right)\sqrt{c-\frac{c}{ax}}}{45x^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(5 - 20*a*x + 41*a^2*x^2 - 82
*a^3*x^3 + 328*a^4*x^4 + 656*a^5*x^5))/(45*x^3*(-1 + a^2*x^2))
```

fricas [A] time = 0.55, size = 85, normalized size = 0.29

$$-\frac{2\left(656\,a^5x^5+328\,a^4x^4-82\,a^3x^3+41\,a^2x^2-20\,ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{45\left(ax^5-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas"
)
```

```
[Out] -2/45*(656*a^5*x^5 + 328*a^4*x^4 - 82*a^3*x^3 + 41*a^2*x^2 - 20*a*x + 5)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x),
abs(a*x+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l
) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 86, normalized size = 0.30

$$-\frac{2\left(ax+1\right)\left(656x^{5}a^{5}+328x^{4}a^{4}-82x^{3}a^{3}+41a^{2}x^{2}-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45x^{4}\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] -2/45*(a*x+1)*(656*a^5*x^5+328*a^4*x^4-82*a^3*x^3+41*a^2*x^2-20*a*x+5)*(c*(
a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

mupad [B] time = 1.41, size = 108, normalized size = 0.37

$$-\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}\,\left(656\,a^4\,x^4+984\,a^3\,x^3+902\,a^2\,x^2+943\,a\,x+923\right)}{45\,x^4}-\frac{1856\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{45\,x^4\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

```
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(943*a*x + 902
*a^2*x^2 + 984*a^3*x^3 + 656*a^4*x^4 + 923))/(45*x^4) - (1856*((a*x - 1)/(a
*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(45*x^4*(a*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.542 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx$

Optimal. Leaf size=185

$$-\frac{c2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}-\frac{2c(1-n)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{an}+cx\left(\frac{1}{ax}+\right.$$

```
[Out] c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x-2*c*(1-n)*(1+1/a/x)^(1/2*n)*hyper
geom([1, 1/2*n],[1+1/2*n],(a+1/x)/(a-1/x))/a/n/((1-1/a/x)^(1/2*n))-2^(1/2*n
)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([1-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/
a)/a/(2-n)
```

Rubi [C] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 0.44, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6179, 136}

$$-\frac{c2^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-2}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] -((2^(2 - n/2)*c*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 + n)/2, (-2 + n)/2,
2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n)))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx&=-\left(c\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx,x,\frac{1}{x}\right)\right)\\&=-\frac{2^{2-\frac{n}{2}}c\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{1}{2}(-2+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}\end{aligned}$$

Mathematica [A] time = 0.38, size = 155, normalized size = 0.84

$$\frac{ce^{n\coth^{-1}(ax)}\left(n\left(-e^{2\coth^{-1}(ax)}\right){}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;-e^{2\coth^{-1}(ax)}\right)+(n-1)ne^{2\coth^{-1}(ax)}{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(\right.\right.}{an(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2,
 2 + n/2, -E^(2*ArcCoth[a*x])]) + E^(2*ArcCoth[a*x])*(-1 + n)*n*Hypergeomet
ric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n*x + Hypergeo
metric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])] + (-1 + n)*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*n*(2 + n))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(acx - c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] integral((a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)\left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arccoth}(ax)} \left(c - \frac{c}{ax}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)\left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a\,x)} \left(c - \frac{c}{a\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x)),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int ae^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-\frac{e^{n\,\mathrm{acoth}\,(ax)}}{x}\right)\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x),x)
```

```
[Out] c*(Integral(a*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x, x))/a
```

3.543 $\int \frac{e^{n \coth^{-1}(ax)}}{c-\frac{c}{ax}} dx$

Optimal. Leaf size=113

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}-\frac{2(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[Out] (1+1/a/x)^(1+1/2*n)*x/c/((1-1/a/x)^(1/2*n))-2*(1+n)*(1+1/a/x)^(1/2*n)*hyper
geom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6179, 96, 131}

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}-\frac{2(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] ((1 + 1/(a*x))^((2 + n)/2)*x)/(c*(1 - 1/(a*x))^(n/2)) - (2*(1 + n)*(1 + 1/(
a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a + x^(-1))])
/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x}{c} - \frac{(1+n)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x}{c} - \frac{2(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Mathematica [A] time = 0.41, size = 97, normalized size = 0.86

$$\frac{e^{n\coth^{-1}(ax)}\left(n(n+1)e^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(ax)}\right)+(n+2)\left((n+1)\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+a\right.\right.}{acn(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(1 + n)*Hypergeometric2F1[1, 1 +
n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + (1 + n)*Hypergeom
etric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="fricas")
[Out] integral(a*x*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*x - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x)),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x),x)
[Out] a*Integral(x*exp(n*acoth(a*x))/(a*x - 1), x)/c
```

3.544 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{2(n+2)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n}-\frac{(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^2(n+2)}+\frac{x\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{a\right.}}{c^2}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)+(1-1/a/x)^(-1-1
/2*n)*(1+1/a/x)^(1+1/2*n)*x/c^2-2*(2+n)*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/
2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c^2/n/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6179, 129, 155, 12, 131}

$$-\frac{2(n+2)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2} {}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n}-\frac{(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^2(n+2)}+\frac{x\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{a\right.}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 +
n))) + ((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x)/c^2 - (2*(2 +
 n)*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -n/2, 1 - n/2, (a - x^(-1))/(a
 + x^(-1))])/(a*c^2*n*(1 - 1/(a*x))^(n/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{2+n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{a\,\text{Subst}\left(\int \frac{(2+n)^2\left(1-\frac{x}{a}\right)^{-1\cdots}}{a^2x}\right.}{c^2(2+\cdots}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{(2+n)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1\cdots}}{\cdots}\right.}{ac^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2} - \frac{2(2+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.07, size = 113, normalized size = 0.68

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}\left(n(ax+1)(n(ax-1)+2ax-3)-2(n+2)^2(ax-1)\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{ax-1}{ax+1}\right)\right)}{ac^2n(n+2)(ax-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*(n*(1 + a*x)*(-3 + 2*a*x + n*(-1 + a*x)) - 2*(2 + n)^2
*(-1 + a*x)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (-1 + a*x)/(1 + a*x)]))/(
a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2c^2x^2-2\,ac^2x+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*c^2*x + c
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^2,x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2 e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

3.545 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -2^(5/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(c-c/a/x)^(3/2)*AppellF1(1+1/2*n,-3/2+1/
2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/(1-1/a/x)^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6182, 6179, 136}

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] -((2^(5/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^(3/2)*AppellF1[(2
+ n)/2, (-3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(1 - 1/(a*x))^(3/2)))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}F_1\left(\frac{2+n}{2};\frac{1}{2}(-3+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(1-\frac{1}{ax}\right)^{3/2}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(acx-c)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="fricas")
[Out] integral((a*c*x - c)*((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/
(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{ax}\right)^{\frac{3}{2}} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)} \left(c - \frac{c}{a\,x}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**(3/2),x)
[Out] Timed out
```

3.546 $\int e^{n \coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}}\, dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}} \sqrt{c-\frac{c}{ax}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} F_1\left(\frac{n+2}{2};\frac{n-1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{1-\frac{1}{ax}}}$$

[Out] -2^(3/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,-1/2+1/2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)*(c-c/a/x)^(1/2)/a/(2+n)/(1-1/a/x)^(1/2)

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6182, 6179, 136}

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}} \sqrt{c-\frac{c}{ax}} \left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} F_1\left(\frac{n+2}{2};\frac{n-1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]

[Out] -((2^(3/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[c - c/(a*x)]*AppellF1[(2 + n)/2, (-1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n)*Sqrt[1 - 1/(a*x)]))

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{ax}}\,F_1\left(\frac{2+n}{2};\frac{1}{2}(-1+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
[Out] $Aborted
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{ax}}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{ax}} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a x)} \sqrt{c - \frac{c}{a x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c \left(-1 + \frac{1}{ax}\right)} e^{n \operatorname{acoth}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

3.547 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n+1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -2^(1/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,1/2+1/2*n,2,2+1/2*n,1/2
*(a+1/x)/a,1+1/a/x)*(1-1/a/x)^(1/2)/a/(2+n)/(c-c/a/x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6182, 6179, 136}

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n+1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] -((2^(1/2 - n/2)*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 +
n)/2, (1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n)
*Sqrt[c - c/(a*x)]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$
$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$
$$= -\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} F_1\left(\frac{2+n}{2};\frac{1+n}{2},2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{c-\frac{c}{ax}}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
[Out] $Aborted
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ax\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{acx-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] integral(a*x*((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*c*x -
 c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\text{arccoth}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))), x)
```

3.548 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{-\frac{n}{2}-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n+3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -2^(-1/2-1/2*n)*(1-1/a/x)^(3/2)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,3/2+1/
2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/(c-c/a/x)^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6182, 6179, 136}

$$-\frac{2^{-\frac{n}{2}-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n+3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -((2^(-1/2 - n/2)*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2
 + n)/2, (3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(c - c/(a*x))^(3/2)))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2^{-\frac{1}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{3+n}{2},2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{acx-c}{ax}}}{a^2c^2x^2-2\,ac^2x+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a^2
*c^2*x^2 - 2*a*c^2*x + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**(3/2),x)
[Out] Integral(exp(n*acoth(a*x))/(-c*(-1 + 1/(a*x)))**(3/2), x)
```

3.549 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=110

$$-\frac{2^{-\frac{n}{2}+p+1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{n+2}{2};\frac{1}{2}(n-2p),2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

```
[Out] -2^(1-1/2*n+p)*(1+1/a/x)^(1+1/2*n)*(c-c/a/x)^p*AppellF1(1+1/2*n,1/2*n-p,2,2
+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/((1-1/a/x)^p)
```

Rubi [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6182, 6179, 136}

$$-\frac{2^{-\frac{n}{2}+p+1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{n+2}{2};\frac{1}{2}(n-2p),2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -((2^(1 - n/2 + p)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^p*AppellF1[(2 +
n)/2, (n - 2*p)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
n)*(1 - 1/(a*x))^p))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$
$$= -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{2+n}{2};\frac{1}{2}(n-2p),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}$$

Mathematica [F] time = 0.97, size = 0, normalized size = 0.00

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
[Out] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p, x]
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\left(\frac{acx-c}{ax}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)*((a*c*x - c)/(a*x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^p,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

3.550 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=67

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(2,p+1;p+2;1+\frac{1}{ax}\right)}{a(p+1)}$$

```
[Out] -(1+1/a/x)^(1+p)*(c-c/a/x)^p*hypergeom([2, 1+p],[2+p],1+1/a/x)/a/(1+p)/((1-
1/a/x)^p)
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6182, 6179, 65}

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(2,p+1;p+2;1+\frac{1}{ax}\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx\\
&= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^p}{x^2}dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(2,1+p;2+p;1+\frac{1}{ax}\right)}{a(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.00

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^{p}{}_2F_1\left(2,p+1;p+2;1+\frac{1}{ax}\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{p}\left(\frac{acx-c}{ax}\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral(((a*x - 1)/(a*x + 1))^p*((a*c*x - c)/(a*x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^{p}\left(\frac{ax-1}{ax+1}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int e^{2p\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
[Out] int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^{p}\left(\frac{ax-1}{ax+1}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{2\,p\,\mathrm{acoth}(a\,x)}\left(c-\frac{c}{a\,x}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p,x)
```

```
[Out] int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} e^{2p\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

3.551 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=93

$$-\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{1-p}F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a(1-p)}$$

```
[Out] -4^p*(1+1/a/x)^(1-p)*(c-c/a/x)^p*AppellF1(1-p,-2*p,2,2-p,1/2*(a+1/x)/a,1+1/
a/x)/a/(1-p)/((1-1/a/x)^p)
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6182, 6179, 136}

$$-\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{1-p}F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] -((4^p*(1 + 1/(a*x))^(1 - p)*(c - c/(a*x))^p*AppellF1[1 - p, -2*p, 2, 2 - p
, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - p)*(1 - 1/(a*x))^p))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2p}\left(1+\frac{x}{a}\right)^{-p}}{x^2}dx,x,\frac{1}{x}\right)\right)$$
$$= -\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1-p}\left(c-\frac{c}{ax}\right)^p F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(1-p)}$$

Mathematica [F] time = 0.66, size = 0, normalized size = 0.00

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
[Out] Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]), x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{acx-c}{ax}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a*c*x - c)/(a*x))^p/((a*x - 1)/(a*x + 1))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^p e^{-2p\,\mathrm{arccoth}(ax)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
[Out] int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{ax}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{-2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p,x)
```

```
[Out] int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^p e^{-2p\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p/exp(2*p*acoth(a*x)),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

3.552 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=57

$$\frac{(2-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}+x\left(c-\frac{c}{ax}\right)^p$$

[Out] (c-c/a/x)^p*x+(2-p)*(c-c/a/x)^p*hypergeom([1, p],[1+p],1-1/a/x)/a/p

Rubi [A] time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6167, 6133, 25, 514, 375, 78, 65}

$$\frac{(2-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}+x\left(c-\frac{c}{ax}\right)^p$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] (c - c/(a*x))^p*x + ((2 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 + p,
 1 - 1/(a*x)])/(a*p)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^p(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{-1+p}(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{-1+p}\,dx}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{-1+p}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^p x-\frac{(c(2-p))\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{-1+p}}{x}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^p x+\frac{(2-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)}{ap}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.81

$$\frac{\left(c-\frac{c}{ax}\right)^p\left(apx-(p-2)\,{}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] ((c - c/(a*x))^p*(a*p*x - (-2 + p)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*
x)]))/(a*p)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)\left(\frac{acx-c}{ax}\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a/x)^p,x)
[Out] int((a*x+1)/(a*x-1)*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^p\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 7.83, size = 272, normalized size = 4.77

$$a\left(\left(\begin{cases} \frac{0^p x}{a} + \frac{0^p \log(ax-1)}{a^2} - \frac{a^{-p}c^p p x^2 x^{-p} e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{for } |ax|>1 \\ \frac{0^p x}{a} + \frac{0^p \log(-ax+1)}{a^2} - \frac{a^{-p}c^p p x^2 x^{-p} e^{i\pi p}\Gamma(p)\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{otherwise}\end{cases}\right) + \begin{cases} \frac{0^p\log(ax-1)}{a} - \frac{a^{-p}c^p p x x^{-p} e^{i\pi p}\Gamma(p)\Gamma(1\cdots}{\Gamma(2-\cdots} \\ \frac{0^p \log(-ax+1)}{a} - \frac{a^{-p}c^p p x x^{-p}e^{i\pi p}\Gamma(p)\Gamma(\cdots}{\Gamma(2\cdots}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**p,x)
```

```
[Out] a*Piecewise((0**p*x/a + 0**p*log(a*x - 1)/a**2 - a**(-p)*c**p*p*x**2*x**(-p
)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(g
amma(3 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*x/a + 0**p*log(-a*x + 1)/a*
*2 - a**(-p)*c**p*p*x**2*x**(-p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1
 - p, 2 - p), (3 - p,), a*x)/(gamma(3 - p)*gamma(p + 1)), True)) + Piecewis
e((0**p*log(a*x - 1)/a - a**(-p)*c**p*p*x*x**(-p)*exp(I*pi*p)*gamma(p)*gamm
a(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(gamma(2 - p)*gamma(p + 1)),
Abs(a*x) > 1), (0**p*log(-a*x + 1)/a - a**(-p)*c**p*p*x*x**(-p)*exp(I*pi*p)
*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(gamma(2 - p)*g
amma(p + 1)), True))
```

3.553 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=90

$$-\frac{2^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{3a}$$

```
[Out] -1/3*2^(1/2+p)*(1+1/a/x)^(3/2)*(c-c/a/x)^p*AppellF1(3/2,1/2-p,2,5/2,1/2*(a+
1/x)/a,1+1/a/x)/a/((1-1/a/x)^p)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6182, 6179, 136}

$$-\frac{2^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

```
[Out] -(2^(1/2 + p)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^p*AppellF1[3/2, 1/2 - p, 2,
 5/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(3*a*(1 - 1/(a*x))^p)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)\right)$$
$$= -\frac{2^{\frac{1}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{3a}$$

Mathematica [F] time = 1.08, size = 0, normalized size = 0.00

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
[Out] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p, x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)\left(\frac{acx-c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral((a*x + 1)*((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c - \frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{a\,x}\right)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1 + \frac{1}{ax}\right)\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.554 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{2^{p+\frac{3}{2}}\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{1}{2};-p-\frac{1}{2},2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a}$$

```
[Out] -2^(3/2+p)*(c-c/a/x)^p*AppellF1(1/2,-1/2-p,2,3/2,1/2*(a+1/x)/a,1+1/a/x)*(1+
1/a/x)^(1/2)/a/((1-1/a/x)^p)
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6182, 6179, 136}

$$-\frac{2^{p+\frac{3}{2}}\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{1}{2};-p-\frac{1}{2},2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

```
[Out] -((2^(3/2 + p)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^p*AppellF1[1/2, -1/2 - p, 2,
 3/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6179

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> -
Dist[c^p, Subst[Int[((1 + (d*x)/c)^p*(1 + x/a)^(n/2))/(x^2*(1 - x/a)^(n/2))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6182

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}}{x^2\sqrt{1+\frac{x}{a}}}dx, x, \frac{1}{x}\right)\right)$$
$$= -\frac{2^{\frac{3}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{1}{2};-\frac{1}{2}-p,2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a}$$

Mathematica [F] time = 1.23, size = 0, normalized size = 0.00

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
[Out] Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x], x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{acx-c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral(((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{ax}\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-\frac{c}{a\,x}\right)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\left(-c\left(-1+\frac{1}{ax}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(-1 + 1/(a*x)))**p, x)
```

3.555 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=114

$$\frac{\left(c-\frac{c}{ax}\right)^{p+2}{}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}{}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)}{ac^2}+\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{c^2}$$

```
[Out] (c-c/a/x)^(2+p)*x/c^2+1/2*(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1/2*(a-1
/x)/a)/a/c^2/(2+p)-(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1-1/a/x)/a/c^2
```

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6167, 6133, 25, 514, 375, 103, 156, 65, 68}

$$\frac{\left(c-\frac{c}{ax}\right)^{p+2}{}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}{}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)}{ac^2}+\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a*x))^(2 + p)*x)/c^2 + ((c - c/(a*x))^(2 + p)*Hypergeometric2F1[1,
 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*a*c^2*(2 + p)) - ((c - c/(a*x))^(2 +
 p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/(a*c^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6133

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[(u*(c + d/x)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$= -\int \frac{\left(c-\frac{c}{ax}\right)^p(1-ax)}{1+ax}\,dx$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}x}{1+ax}\,dx}{c}$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{1+p}}{a+\frac{1}{x}}\,dx}{c}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$
$$= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}\left(c(2+p)+\frac{c(1+p)x}{a}\right)}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c^2}$$
$$= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}-\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{a+x}\,dx,x,\frac{1}{x}\right)}{ac}+\frac{(2+p)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x}\,dx,x,\frac{1}{x}\right)}{ac}$$
$$= \frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{ax}\right)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)}-\frac{\left(c-\frac{c}{ax}\right)^{2+p}\,{}_2F_1\left(1,2+p;3\right.}{ac^2}$$

Mathematica [A] time = 0.05, size = 87, normalized size = 0.76

$$\frac{(ax-1)^2\left(c-\frac{c}{ax}\right)^p\left({}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)+2(p+2)\left(ax-{}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)\right)\right)}{2a^3(p+2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a*x))^p*(-1 + a*x)^2*(Hypergeometric2F1[1, 2 + p, 3 + p, (a - x^(-
1))/(2*a)] + 2*(2 + p)*(a*x - Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x
)])))/(2*a^3*(2 + p)*x^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax-1)\left(\frac{acx-c}{ax}\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^{p} (ax-1)}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/(a*x+1)*(a*x-1),x)
```

```
[Out] int((c-c/a/x)^p/(a*x+1)*(a*x-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(c-\frac{c}{ax}\right)^{p}}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{p} (a\,x-1)}{a\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} (ax-1)}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p*(a*x - 1)/(a*x + 1), x)
```

3.556 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=393

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{7}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{5}{72}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{5}{144}a^5c$$

```
[Out] 5/72*a^6*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)*x^7-7/72*a^7*c^4*(1-1/a/x)^(5
/2)*(1+1/a/x)^(11/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(11/2)*x^9+3
5/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+35/384*a*c^4*(1+1/a/x)
^(3/2)*x^2*(1-1/a/x)^(1/2)+7/192*a^2*c^4*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2
)+1/64*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)+1/144*a^4*c^4*(1+1/a/x)^
(9/2)*x^5*(1-1/a/x)^(1/2)-5/144*a^5*c^4*(1+1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2
)+35/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{7}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{5}{72}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{5}{144}a^5c$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (35*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(5/2)*x^3)/192 + (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/6
4 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/144 - (5*a^5*c^4*Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/144 + (5*a^6*c^4*(1 - 1/(a*x))^(3
/2)*(1 + 1/(a*x))^(11/2)*x^7)/72 - (7*a^7*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a
*x))^(11/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(11/2)*x^9
)/9 + (35*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9+\frac{1}{9}\left(7a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^9}\,dx,x\right.$$

$$=-\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9-\frac{1}{72}\left(35\right.$$

$$=\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8+\frac{1}{9}a^8c^4\left(\right.$$

$$=-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{7}{72}a^7c^4$$

$$=\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6+\frac{5}{72}a^6c^4\left(1\right.$$

$$=\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{144}a^5c^4\sqrt{1}$$

$$=\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{144}a^4c^4\sqrt{1}$$

$$=\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3c^4\sqrt{1-}$$

$$=\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}$$

$$=\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}$$

$$=\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}$$

Mathematica [A] time = 0.20, size = 111, normalized size = 0.28

$$\frac{c^4\left(315\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(128a^8x^8+144a^7x^7-512a^6x^6-600a^5x^5+768a^4x^4+978a^3x^3-\right.\right.}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 - 837*a*x - 512*a^2*x^2 + 978*a^3*x^3
+ 768*a^4*x^4 - 600*a^5*x^5 - 512*a^6*x^6 + 144*a^7*x^7 + 128*a^8*x^8) + 31
5*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

fricas [A] time = 0.55, size = 169, normalized size = 0.43

$$\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(128\,a^9c^4x^9+272\,a^8c^4x^8-368\,a^7c^4x^7-1112\,a^6c^4x^6+16\right.}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (128*a^9*c^4*x^9 + 272*a^8*c^4*x^8 - 368*a^7*c^4*x^7
 - 1112*a^6*c^4*x^6 + 168*a^5*c^4*x^5 + 1746*a^4*c^4*x^4 + 466*a^3*c^4*x^3
- 1349*a^2*c^4*x^2 - 709*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.19, size = 340, normalized size = 0.87

$$\frac{1}{1152}ac^4\left(\frac{315\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{2730\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{10458\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{23202\,(ax-1}{(ax+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/1152*a*c^4*(315*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*log(abs(sqrt
((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(2730*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1
))/(a*x + 1) - 10458*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 23
202*(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 32768*(a*x - 1)^4*s
qrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 23202*(a*x - 1)^5*sqrt((a*x - 1)/(a*
x + 1))/(a*x + 1)^5 + 10458*(a*x - 1)^6*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)
^6 - 2730*(a*x - 1)^7*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^7 + 315*(a*x - 1)
^8*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^8 - 315*sqrt((a*x - 1)/(a*x + 1)))/(
a^2*((a*x - 1)/(a*x + 1) - 1)^9))
```

maple [A] time = 0.06, size = 279, normalized size = 0.71

$$-\frac{(ax-1)\,c^4\left(-128\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^6a^6-144\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^5a^5+384\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4+456\left(a^2x^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/1152*(a*x-1)*c^4/a*(-128*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6-144*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5+384*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^4+
456*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-384*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)
*x^2*a^2-522*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+384*((a*x-1)*(a*x+1))^(3/2)*
(a^2)^(1/2)-256*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+315*(a^2*x^2-1)^(1/2)*(a^2)^(
1/2)*x*a-315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x
-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.32, size = 415, normalized size = 1.06

$$\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10458\,c^4\left(\frac{ax-1}{ax+1}\right.\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax-1)^2a^2}{(ax+1)^2}+\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) - 27
30*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(13/2
) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 32768*c^4*((a*x - 1)/(a*x + 1)
)^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)/(a*x
 + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a*x - 1
)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x + 1)^2 +
 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 + 126*(a*
x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*x - 1)^7*
a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2/(a*x + 1)
^9 - a^2))*a
```

mupad [B] time = 1.37, size = 362, normalized size = 0.92

$$\frac{\frac{455\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96}-\frac{35\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{581\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32}+\frac{1289\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{32}+\frac{512\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{9}-\frac{1289\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{32}+\frac{581\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{32}-\dots}{a-\frac{9\,a\,(a\,x-1)}{a\,x+1}+\frac{36\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{84\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{126\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{126\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{84\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{36\,a\,(a\,x-1)^7}{(a\,x+1)^7}+\frac{9\,a\,(a\,x-\dots}{(a\,x+1\dots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((455*c^4*((a*x - 1)/(a*x + 1))^(3/2))/96 - (35*c^4*((a*x - 1)/(a*x + 1))^(
1/2))/64 - (581*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (1289*c^4*((a*x - 1)/
(a*x + 1))^(7/2))/32 + (512*c^4*((a*x - 1)/(a*x + 1))^(9/2))/9 - (1289*c^4*
((a*x - 1)/(a*x + 1))^(11/2))/32 + (581*c^4*((a*x - 1)/(a*x + 1))^(13/2))/3
2 - (455*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 + (35*c^4*((a*x - 1)/(a*x + 1
))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a*x + 1
)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^4 - (1
26*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*a*(a*x
 - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9)/(a*x
 + 1)^9) + (35*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^8x^8}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{\dots}{\sqrt{\frac{ax}{ax+1\dots}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(6*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**6*x**6
/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**8*x**8/sqrt(a*x/(a*x +
 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.557 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=313

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{5}{42}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{14}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{56}a^3c^3x$$

```
[Out] 5/42*a^5*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x^6-1/7*a^6*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(9/2)*x^7+5/16*c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+5
/48*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/24*a^2*c^3*(1+1/a/x)^(5/2)*
x^3*(1-1/a/x)^(1/2)+1/56*a^3*c^3*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)-1/14*a
^4*c^3*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)+5/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/
a/x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{5}{42}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{14}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{56}a^3c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (5*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/48 + (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/56 - (a^
4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/14 + (5*a^5*c^3*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/42 - (a^6*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1
/(a*x))^(9/2)*x^7)/7 + (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])
/(16*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
```

```
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\left(\left(a^6c^3\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$=\left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7-\frac{1}{7}\left(5a^5c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^7}\,dx,x\right.$$

$$=\frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7+\frac{1}{14}\left(5a^4c^3\right)$$

$$=-\frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6-\frac{1}{7}a^6c^3\left(1-\right.$$

$$=\frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{5}{42}a^5c^3\left(1-\right.$$

$$=\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{14}a^4c^3\sqrt{1-\right.$$

$$=\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{56}a^3c^3\sqrt{1-\right.$$

$$=\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1-\right.$$

$$=\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1-\right.$$

$$=\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1-\right.$$

Mathematica [A] time = 0.18, size = 95, normalized size = 0.30

$$\frac{c^3\left(105\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(-48a^6x^6-56a^5x^5+144a^4x^4+182a^3x^3-144a^2x^2-231ax+48\right.\right.}{336a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 - 231*a*x - 144*a^2*x^2 + 182*a^3*x^3 +
 144*a^4*x^4 - 56*a^5*x^5 - 48*a^6*x^6) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)
])*x]))/(336*a)
```

fricas [A] time = 0.57, size = 148, normalized size = 0.47

$$\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(48\,a^7c^3x^7+104\,a^6c^3x^6-88\,a^5c^3x^5-326\,a^4c^3x^4-38\,a^3c\right.}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (48*a^7*c^3*x^7 + 104*a^6*c^3*x^6 - 88*a^5*c^3*x^5 -
326*a^4*c^3*x^4 - 38*a^3*c^3*x^3 + 375*a^2*c^3*x^2 + 183*a*c^3*x - 48*c^3)*
sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.19, size = 278, normalized size = 0.89

$$\frac{1}{336}\,ac^3\left(\frac{105\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{700\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{1981\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{3072\,(ax-1)^3\sqrt{\cdots}}{(ax+1)^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/336*a*c^3*(105*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*log(abs(sqrt(
(a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(700*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))
/(a*x + 1) - 1981*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 3072*
(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 1981*(a*x - 1)^4*sqrt((
a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 700*(a*x - 1)^5*sqrt((a*x - 1)/(a*x + 1))
/(a*x + 1)^5 + 105*(a*x - 1)^6*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^6 - 105*
sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^7))
```

maple [A] time = 0.05, size = 231, normalized size = 0.74

$$-\frac{(ax-1)\,c^3\left(48\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^4a^4+56\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^3a^3-96\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^2a^2-126\left(a^2x^2-1\right)^{\frac{3}{2}}\right.}{336a\sqrt{\frac{ax-1}{ax+1}}\;\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/336*(a*x-1)*c^3/a*(48*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^4+56*(a^2*x^2-
1)^(3/2)*(a^2)^(1/2)*x^3*a^3-96*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-126*(
a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+105*(a^2*
x^2-1)^(1/2)*(a^2)^(1/2)*x*a+112*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-105*ln
((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1
/2)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.32, size = 337, normalized size = 1.08

$$\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}\cdots\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+\frac{35\,(ax-1)^3a^2}{(ax+1)^3}-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 700
*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2) -
3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(5/2)
 + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1)))
/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x - 1)
^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/(a
*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 - a
^2))*a
```

mupad [B] time = 1.31, size = 289, normalized size = 0.92

$$\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{25\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}+\frac{283\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{24}+\frac{128\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{7}-\frac{283\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{24}+\frac{25\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{6}-\cdots}{a-\frac{7\,a\,(a\,x-1)}{a\,x+1}+\frac{21\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{35\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{35\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{21\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{7\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - ((5*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/8 - (25*c^3*((a*x - 1)/(a*x + 1))^(3/2))/6 + (283*c^3*((a*x -
1)/(a*x + 1))^(5/2))/24 + (128*c^3*((a*x - 1)/(a*x + 1))^(7/2))/7 - (283*c^
3*((a*x - 1)/(a*x + 1))^(9/2))/24 + (25*c^3*((a*x - 1)/(a*x + 1))^(11/2))/6
 - (5*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (7*a*(a*x - 1))/(a*x + 1) +
 (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)^3)/(a*x + 1)^3 + (35*a*(a
*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x + 1)^5 + (7*a*(a*x - 1)^6)
/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(-3*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6/s
qrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)), x))
```

3.558 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{3}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{20}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{8}ac^2x^2\sqrt{1$$

```
[Out] 1/5*a^4*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5+3/8*c^2*arctanh((1-1/a/x)^(
1/2)*(1+1/a/x)^(1/2))/a+1/8*a*c^2*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/20*
a^2*c^2*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-3/20*a^3*c^2*(1+1/a/x)^(7/2)*x^
4*(1-1/a/x)^(1/2)+3/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{3}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{20}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{8}ac^2x^2\sqrt{1$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 + (a*c^2*Sqrt[1 - 1/(a*x)]*
(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)
*x^3)/20 - (3*a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (a^4*
c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/5 + (3*c^2*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{5}\left(3a^3c^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}{x^5}\,dx,x,\frac{1}{x}\right.$$

$$=-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{1}{20}\left(3a^2c^2\right)$$

$$=\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right.$$

$$=\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

Mathematica [A] time = 0.11, size = 79, normalized size = 0.34

$$\frac{c^2\left(15\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4+10a^3x^3-16a^2x^2-25ax+8\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 25*a*x - 16*a^2*x^2 + 10*a^3*x^3 + 8*a
^4*x^4) + 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

fricas [A] time = 0.57, size = 125, normalized size = 0.54

$$\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(8\,a^5c^2x^5+18\,a^4c^2x^4-6\,a^3c^2x^3-41\,a^2c^2x^2-17\,ac^2x+8\,c^2\right)\sqrt{}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) + (8*a^5*c^2*x^5 + 18*a^4*c^2*x^4 - 6*a^3*c^2*x^3 - 41*a^2
*c^2*x^2 - 17*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.18, size = 216, normalized size = 0.93

$$\frac{1}{40}\,ac^2\left(\frac{15\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{70\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{128\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\frac{70\,(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}+\cdots\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/40*a*c^2*(15*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*log(abs(sqrt((a*
x - 1)/(a*x + 1)) - 1))/a^2 - 2*(70*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*
x + 1) + 128*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 70*(a*x -
1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 15*(a*x - 1)^4*sqrt((a*x - 1)/
(a*x + 1))/(a*x + 1)^4 - 15*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x
 + 1) - 1)^5))
```

maple [A] time = 0.05, size = 183, normalized size = 0.79

$$\frac{(ax-1)\,c^2\left(24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^2a^2+30\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;xa+16\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;-45\sqrt{a^2x^2-1}\;\sqrt{a^2}\;xa-\cdots\right.}{120a\sqrt{\frac{ax-1}{ax+1}}\;\sqrt{(ax-1)(ax+1)}\;\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/120*(a*x-1)*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+30*(a^2*x^2-1
)^(3/2)*(a^2)^(1/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-45*(a^2*x^2-1)^(1/
2)*(a^2)^(1/2)*x*a-40*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+45*ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)
*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.31, size = 259, normalized size = 1.11

$$\frac{1}{40}\,a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+128\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\cdots\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4\cdots}{(ax+1)^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^2
*((a*x - 1)/(a*x + 1))^(7/2) + 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^2
*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x -
1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*x
```

```
+ 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^2
))
```

mupad [B] time = 1.26, size = 214, normalized size = 0.92

$$\frac{\frac{7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}-\frac{3\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{32\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{5}-\frac{7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{2}+\frac{3\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}+\frac{3\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((7*c^2*((a*x - 1)/(a*x + 1))^(3/2))/2 - (3*c^2*((a*x - 1)/(a*x + 1))^(1/2)
)/4 + (32*c^2*((a*x - 1)/(a*x + 1))^(5/2))/5 - (7*c^2*((a*x - 1)/(a*x + 1))
^(7/2))/2 + (3*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (3*c^2*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{2a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*
x + 1) - 1/(a*x + 1)), x))
```

3.559 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=145

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}\right.}{2a}$$

```
[Out] 1/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+1/6*a*c*(1+1/a/x)^(3/2)*x^
2*(1-1/a/x)^(1/2)-1/3*a^2*c*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+1/2*c*x*(1-
1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (a*c*Sqrt[1 - 1/(a*x)]*(1 + 1
/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^3)/3
+ (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
```

```
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx=-\left(\left(a^2c\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$
$$=\left(a^2c\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^4}dx,x,\frac{1}{x}\right)$$
$$=-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{3}(ac)\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$
$$=\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{2}c\,\text{Subst}\left(\int\frac{\sqrt{1+}}{x^2\sqrt{1-}}\right.$$
$$=\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$
$$=\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$
$$=\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}$$

Mathematica [A] time = 0.10, size = 61, normalized size = 0.42

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-2a^2x^2-3ax+2\right)+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(6*a)
```

fricas [A] time = 0.56, size = 91, normalized size = 0.63

$$\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3cx^3+5\,a^2cx^2+acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x +
 1)) - 1) - (2*a^3*c*x^3 + 5*a^2*c*x^2 + a*c*x - 2*c)*sqrt((a*x - 1)/(a*x +
 1)))/a
```

giac [A] time = 0.16, size = 152, normalized size = 1.05

$$\frac{1}{6}\,ac\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{8\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{3\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+3\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/6*a*c*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(abs(sqrt((a*x - 1
)/(a*x + 1)) - 1))/a^2 + 2*(8*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)
 - 3*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 3*sqrt((a*x - 1)/(
a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [A] time = 0.04, size = 119, normalized size = 0.82

$$-\frac{(ax-1)\,c\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+2\,((ax-1)\,(ax+1))^{\frac{3}{2}}\,\sqrt{a^2}\,-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)\,(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x)
```

```
[Out] -1/6*(a*x-1)*c*(3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+2*((a*x-1)*(a*x+1))^(3/
2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/(
(a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

maxima [A] time = 0.30, size = 171, normalized size = 1.18

$$-\frac{1}{6}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) - 8*c*((a*x - 1)/(a*x + 1))^(3/2
) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)
^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 0.07, size = 131, normalized size = 0.90

$$\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2) +
 (8*c*((a*x - 1)/(a*x + 1))^(3/2))/3 - c*((a*x - 1)/(a*x + 1))^(5/2))/(a -
```

```
(3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)
/(a*x + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/
sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.560 $\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2} dx$

Optimal. Leaf size=13

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6183}

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] E^ArcCoth[a*x]/(a*c)
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2} dx = \frac{e^{\coth^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.05, size = 13, normalized size = 1.00

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] E^ArcCoth[a*x]/(a*c)
```

fricas [A] time = 0.79, size = 34, normalized size = 2.62

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] (a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

giac [A] time = 0.15, size = 22, normalized size = 1.69

$$\frac{1}{ac\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/(a*c*sqrt((a*x - 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 23, normalized size = 1.77

$$\frac{1}{\sqrt{\frac{ax-1}{ax+1}}\, ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

maxima [A] time = 0.30, size = 22, normalized size = 1.69

$$\frac{1}{ac\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/(a*c*sqrt((a*x - 1)/(a*x + 1)))
```

mupad [B] time = 1.22, size = 22, normalized size = 1.69

$$\frac{1}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 1/(a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x +
1) - 1/(a*x + 1))), x)/c
```

3.561 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)/a/c^2-1/3/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/
c^2/(-a^2*x^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6185, 6183}

$$\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^ArcCoth[a*x])/(3*a*c^2) - (E^ArcCoth[a*x]*(1 - 2*a*x))/(3*a*c^2*(1 - a
^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx&=-\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2(1-a^2x^2)}+\frac{2\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{3c}\\&=\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2(1-a^2x^2)}\end{aligned}$$

Mathematica [A] time = 0.16, size = 50, normalized size = 0.98

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-2ax-1\right)}{3c^2(ax-1)^2(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^2*(-1 + a*x)^2*(1 +
 a*x))
```

fricas [A] time = 0.48, size = 58, normalized size = 1.14

$$\frac{\left(2\,a^2x^2-2\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*
c^2*x + a*c^2)
```

giac [A] time = 0.13, size = 70, normalized size = 1.37

$$\frac{\frac{(ax+1)\left(\frac{6\,(ax-1)}{ax+1}-1\right)}{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}+3\,\sqrt{\frac{ax-1}{ax+1}}}{12\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*((a*x + 1)*(6*(a*x - 1)/(a*x + 1) - 1)/((a*x - 1)*sqrt((a*x - 1)/(a*x
+ 1))) + 3*sqrt((a*x - 1)/(a*x + 1)))/(a*c^2)
```

maple [A] time = 0.04, size = 49, normalized size = 0.96

$$\frac{2a^2x^2-2ax-1}{3\left(a^2x^2-1\right)c^2\sqrt{\frac{ax-1}{ax+1}}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/3*(2*a^2*x^2-2*a*x-1)/(a^2*x^2-1)/c^2/((a*x-1)/(a*x+1))^(1/2)/a
```

maxima [A] time = 0.30, size = 65, normalized size = 1.27

$$\frac{1}{12}\,a\left(\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2) + (6*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)))
```

mupad [B] time = 0.07, size = 50, normalized size = 0.98

$$\frac{-2\,a^2\,x^2+2\,a\,x+1}{\left(3\,a\,c^2-3\,a^3\,c^2\,x^2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a*x - 2*a^2*x^2 + 1)/((3*a*c^2 - 3*a^3*c^2*x^2)*((a*x - 1)/(a*x + 1))^(1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x**2*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

3.562 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3} dx$

Optimal. Leaf size=85

$$-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}-\frac{4(1-2ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)}+\frac{8e^{\coth^{-1}(ax)}}{15ac^3}$$

```
[Out] 8/15/((a*x-1)/(a*x+1))^(1/2)/a/c^3-1/15/((a*x-1)/(a*x+1))^(1/2)*(-4*a*x+1)/
a/c^3/(-a^2*x^2+1)^2-4/15/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/c^3/(-a^2*x^
2+1)
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6185, 6183}

$$-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}-\frac{4(1-2ax)e^{\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)}+\frac{8e^{\coth^{-1}(ax)}}{15ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8*E^ArcCoth[a*x])/(15*a*c^3) - (E^ArcCoth[a*x]*(1 - 4*a*x))/(15*a*c^3*(1 -
 a^2*x^2)^2) - (4*E^ArcCoth[a*x]*(1 - 2*a*x))/(15*a*c^3*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3} dx &= -\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3(1-a^2x^2)^2}+\frac{4\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx}{5c} \\
&= -\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3(1-a^2x^2)^2}-\frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3(1-a^2x^2)}+\frac{8\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2} dx}{15c^2} \\
&= \frac{8e^{\coth^{-1}(ax)}}{15ac^3}-\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3(1-a^2x^2)^2}-\frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3(1-a^2x^2)}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 66, normalized size = 0.78

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-8a^3x^3-12a^2x^2+12ax+3\right)}{15c^3(ax-1)^3(ax+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(3 + 12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4))
/(15*c^3*(-1 + a*x)^3*(1 + a*x)^2)
```

fricas [A] time = 0.52, size = 86, normalized size = 1.01

$$\frac{\left(8\,a^4x^4-8\,a^3x^3-12\,a^2x^2+12\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*a^4*x^4 - 8*a^3*x^3 - 12*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x
+ 1))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.16, size = 117, normalized size = 1.38

$$-\frac{\frac{(ax+1)^2\left(\frac{20\,(ax-1)}{ax+1}-\frac{90\,(ax-1)^2}{(ax+1)^2}-3\right)}{(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{5\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-60\sqrt{\frac{ax-1}{ax+1}}}{240\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/240*((a*x + 1)^2*(20*(a*x - 1)/(a*x + 1) - 90*(a*x - 1)^2/(a*x + 1)^2 -
3)/((a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))) + 5*(a*x - 1)*sqrt((a*x - 1)/(a*
x + 1))/(a*x + 1) - 60*sqrt((a*x - 1)/(a*x + 1)))/(a*c^3)
```

maple [A] time = 0.04, size = 65, normalized size = 0.76

$$\frac{8x^4a^4-8x^3a^3-12a^2x^2+12ax+3}{15\left(a^2x^2-1\right)^2c^3\sqrt{\frac{ax-1}{ax+1}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/15*(8*a^4*x^4-8*a^3*x^3-12*a^2*x^2+12*a*x+3)/(a^2*x^2-1)^2/c^3/((a*x-1)/(
a*x+1))^(1/2)/a
```

maxima [A] time = 0.31, size = 99, normalized size = 1.16

$$-\frac{1}{240}\,a\left(\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-12\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{\frac{20\,(ax-1)}{ax+1}-\frac{90\,(ax-1)^2}{(ax+1)^2}-3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/240*a*(5*(((a*x - 1)/(a*x + 1))^(3/2) - 12*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*c^3) + (20*(a*x - 1)/(a*x + 1) - 90*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a^2*c^
3*((a*x - 1)/(a*x + 1))^(5/2)))
```

mupad [B] time = 0.07, size = 60, normalized size = 0.71

$$\frac{8\,a^4\,x^4-8\,a^3\,x^3-12\,a^2\,x^2+12\,a\,x+3}{15\,a\,c^3\,{\left(a\,x+1\right)}^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4 + 3)/(15*a*c^3*(a*x + 1)^4*((a
*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

3.563 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=119

$$-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}-\frac{8(1-2ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)}-\frac{2(1-4ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^2}+\frac{16e^{\coth^{-1}(ax)}}{35ac^4}$$

```
[Out] 16/35/((a*x-1)/(a*x+1))^(1/2)/a/c^4-1/35/((a*x-1)/(a*x+1))^(1/2)*(-6*a*x+1)
/a/c^4/(-a^2*x^2+1)^3-2/35/((a*x-1)/(a*x+1))^(1/2)*(-4*a*x+1)/a/c^4/(-a^2*x
^2+1)^2-8/35/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/c^4/(-a^2*x^2+1)
```

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6185, 6183}

$$-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}-\frac{8(1-2ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)}-\frac{2(1-4ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^2}+\frac{16e^{\coth^{-1}(ax)}}{35ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (16*E^ArcCoth[a*x])/(35*a*c^4) - (E^ArcCoth[a*x]*(1 - 6*a*x))/(35*a*c^4*(1
- a^2*x^2)^3) - (2*E^ArcCoth[a*x]*(1 - 4*a*x))/(35*a*c^4*(1 - a^2*x^2)^2) -
 (8*E^ArcCoth[a*x]*(1 - 2*a*x))/(35*a*c^4*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3} + \frac{6\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{7c}$$

$$= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3} - \frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2} + \frac{24\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{35c^2}$$

$$= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3} - \frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2} - \frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4\left(1-a^2x^2\right)} + \frac{16\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^3}$$

$$= \frac{16e^{\coth^{-1}(ax)}}{35ac^4} - \frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3} - \frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2} - \frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.28, size = 82, normalized size = 0.69

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5\right)}{35c^4(ax-1)^4(ax+1)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-5 - 30*a*x + 30*a^2*x^2 + 40*a^3*x^3 - 40*a^4*x^
4 - 16*a^5*x^5 + 16*a^6*x^6))/(35*c^4*(-1 + a*x)^4*(1 + a*x)^3)
```

fricas [A] time = 0.55, size = 134, normalized size = 1.13

$$\frac{\left(16\,a^6x^6-16\,a^5x^5-40\,a^4x^4+40\,a^3x^3+30\,a^2x^2-30\,ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(16*a^6*x^6 - 16*a^5*x^5 - 40*a^4*x^4 + 40*a^3*x^3 + 30*a^2*x^2 - 30*a
*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^
4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.16, size = 164, normalized size = 1.38

$$\frac{\frac{(ax+1)^3\left(\frac{42\,(ax-1)}{ax+1}-\frac{175\,(ax-1)^2}{(ax+1)^2}+\frac{700\,(ax-1)^3}{(ax+1)^3}-5\right)}{(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}} - \frac{70\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} + \frac{7\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2} + 525\sqrt{\frac{ax-1}{ax+1}}}{2240\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/2240*((a*x + 1)^3*(42*(a*x - 1)/(a*x + 1) - 175*(a*x - 1)^2/(a*x + 1)^2 +
 700*(a*x - 1)^3/(a*x + 1)^3 - 5)/((a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))) -
 70*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 7*(a*x - 1)^2*sqrt((a*x
 - 1)/(a*x + 1))/(a*x + 1)^2 + 525*sqrt((a*x - 1)/(a*x + 1)))/(a*c^4)
```

maple [A] time = 0.04, size = 81, normalized size = 0.68

$$\frac{16x^6a^6 - 16x^5a^5 - 40x^4a^4 + 40x^3a^3 + 30a^2x^2 - 30ax - 5}{35\left(a^2x^2 - 1\right)^3 c^4 \sqrt{\frac{ax-1}{ax+1}}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/35*(16*a^6*x^6-16*a^5*x^5-40*a^4*x^4+40*a^3*x^3+30*a^2*x^2-30*a*x-5)/(a^2
*x^2-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)/a
```

maxima [A] time = 0.31, size = 132, normalized size = 1.11

$$\frac{1}{2240}a\left(\frac{7\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 75\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4} + \frac{\frac{42(ax-1)}{ax+1} - \frac{175(ax-1)^2}{(ax+1)^2} + \frac{700(ax-1)^3}{(ax+1)^3} - 5}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/2240*a*(7*(((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) +
 75*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + (42*(a*x - 1)/(a*x + 1) - 175*(a
*x - 1)^2/(a*x + 1)^2 + 700*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^4*((a*x - 1
)/(a*x + 1))^(7/2)))
```

mupad [B] time = 0.06, size = 142, normalized size = 1.19

$$\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4} - \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{32\,a\,c^4} + \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4} - \frac{\frac{5\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{20\,(a\,x-1)^3}{(a\,x+1)^3} - \frac{6\,(a\,x-1)}{5\,(a\,x+1)} + \frac{1}{7}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((a*x - 1)/(a*x + 1))^(3/2)/(
32*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - ((5*(a*x - 1)^2)/(a*x
 + 1)^2 - (20*(a*x - 1)^3)/(a*x + 1)^3 - (6*(a*x - 1))/(5*(a*x + 1)) + 1/7)
/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^8x^8\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}} - 4a^6x^6\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}} + 6a^4x^4\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}} - 4a^2x^2\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}} + \sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Integral(1/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x**6*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))), x)/c**4
```

3.564 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

Optimal. Leaf size=84

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{4c^5(ax+1)^{10}}{5a}-\frac{8c^5(ax+1)^9}{3a}+\frac{4c^5(ax+1)^8}{a}-\frac{16c^5(ax+1)^7}{7a}$$

```
[Out] -16/7*c^5*(a*x+1)^7/a+4*c^5*(a*x+1)^8/a-8/3*c^5*(a*x+1)^9/a+4/5*c^5*(a*x+1)
^10/a-1/11*c^5*(a*x+1)^11/a
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{4c^5(ax+1)^{10}}{5a}-\frac{8c^5(ax+1)^9}{3a}+\frac{4c^5(ax+1)^8}{a}-\frac{16c^5(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (-16*c^5*(1 + a*x)^7)/(7*a) + (4*c^5*(1 + a*x)^8)/a - (8*c^5*(1 + a*x)^9)/(
3*a) + (4*c^5*(1 + a*x)^10)/(5*a) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx\\
&= -\left(c^5\int(1-ax)^4(1+ax)^6\,dx\right)\\
&= -\left(c^5\int\left(16(1+ax)^6-32(1+ax)^7+24(1+ax)^8-8(1+ax)^9+(1+ax)^{10}\right)dx\right)\\
&= -\frac{16c^5(1+ax)^7}{7a}+\frac{4c^5(1+ax)^8}{a}-\frac{8c^5(1+ax)^9}{3a}+\frac{4c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.56

$$-\frac{c^5(ax+1)^7\left(105a^4x^4-504a^3x^3+938a^2x^2-812ax+281\right)}{1155a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] -1/1155*(c^5*(1 + a*x)^7*(281 - 812*a*x + 938*a^2*x^2 - 504*a^3*x^3 + 105*a
^4*x^4))/a
```

fricas [A] time = 0.45, size = 113, normalized size = 1.35

$$-\frac{1}{11}a^{10}c^5x^{11}-\frac{1}{5}a^9c^5x^{10}+\frac{1}{3}a^8c^5x^9+a^7c^5x^8-\frac{2}{7}a^6c^5x^7-2a^5c^5x^6-\frac{2}{5}a^4c^5x^5+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

giac [A] time = 0.14, size = 113, normalized size = 1.35

$$-\frac{1}{11}a^{10}c^5x^{11}-\frac{1}{5}a^9c^5x^{10}+\frac{1}{3}a^8c^5x^9+a^7c^5x^8-\frac{2}{7}a^6c^5x^7-2a^5c^5x^6-\frac{2}{5}a^4c^5x^5+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

maple [A] time = 0.03, size = 85, normalized size = 1.01

$$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{1}{5}a^9x^{10}+\frac{1}{3}x^9a^8+a^7x^8-\frac{2}{7}a^6x^7-2x^6a^5-\frac{2}{5}a^4x^5+2x^4a^3+x^3a^2-a\,x^2-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^5,x)
```

```
[Out] c^5*(-1/11*a^10*x^11-1/5*a^9*x^10+1/3*x^9*a^8+a^7*x^8-2/7*a^6*x^7-2*x^6*a^5
-2/5*a^4*x^5+2*x^4*a^3+x^3*a^2-a*x^2-x)
```

maxima [A] time = 0.31, size = 113, normalized size = 1.35

$$-\frac{1}{11}a^{10}c^5x^{11}-\frac{1}{5}a^9c^5x^{10}+\frac{1}{3}a^8c^5x^9+a^7c^5x^8-\frac{2}{7}a^6c^5x^7-2a^5c^5x^6-\frac{2}{5}a^4c^5x^5+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

mupad [B] time = 1.25, size = 113, normalized size = 1.35

$$-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{a^9\,c^5\,x^{10}}{5}+\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8-\frac{2\,a^6\,c^5\,x^7}{7}-2\,a^5\,c^5\,x^6-\frac{2\,a^4\,c^5\,x^5}{5}+2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3-a\,c^5\,x^2-c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] a^2*c^5*x^3 - a*c^5*x^2 - c^5*x + 2*a^3*c^5*x^4 - (2*a^4*c^5*x^5)/5 - 2*a^5
*c^5*x^6 - (2*a^6*c^5*x^7)/7 + a^7*c^5*x^8 + (a^8*c^5*x^9)/3 - (a^9*c^5*x^1
0)/5 - (a^10*c^5*x^11)/11
```

sympy [A] time = 0.10, size = 119, normalized size = 1.42

$$-\frac{a^{10}c^5x^{11}}{11}-\frac{a^9c^5x^{10}}{5}+\frac{a^8c^5x^9}{3}+a^7c^5x^8-\frac{2a^6c^5x^7}{7}-2a^5c^5x^6-\frac{2a^4c^5x^5}{5}+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - a**9*c**5*x**10/5 + a**8*c**5*x**9/3 + a**7*c**5*x**
8 - 2*a**6*c**5*x**7/7 - 2*a**5*c**5*x**6 - 2*a**4*c**5*x**5/5 + 2*a**3*c**
5*x**4 + a**2*c**5*x**3 - a*c**5*x**2 - c**5*x
```

3.565 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=69

$$\frac{c^4(ax+1)^9}{9a}-\frac{3c^4(ax+1)^8}{4a}+\frac{12c^4(ax+1)^7}{7a}-\frac{4c^4(ax+1)^6}{3a}$$

```
[Out] -4/3*c^4*(a*x+1)^6/a+12/7*c^4*(a*x+1)^7/a-3/4*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+
1)^9/a
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^4(ax+1)^9}{9a}-\frac{3c^4(ax+1)^8}{4a}+\frac{12c^4(ax+1)^7}{7a}-\frac{4c^4(ax+1)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (-4*c^4*(1 + a*x)^6)/(3*a) + (12*c^4*(1 + a*x)^7)/(7*a) - (3*c^4*(1 + a*x)^
8)/(4*a) + (c^4*(1 + a*x)^9)/(9*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx\\
&= -\left(c^4\int(1-ax)^3(1+ax)^5\,dx\right)\\
&= -\left(c^4\int\left(8(1+ax)^5-12(1+ax)^6+6(1+ax)^7-(1+ax)^8\right)dx\right)\\
&= -\frac{4c^4(1+ax)^6}{3a}+\frac{12c^4(1+ax)^7}{7a}-\frac{3c^4(1+ax)^8}{4a}+\frac{c^4(1+ax)^9}{9a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.57

$$\frac{c^4(ax+1)^6\left(28a^3x^3-105a^2x^2+138ax-65\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(1 + a*x)^6*(-65 + 138*a*x - 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

fricas [A] time = 0.53, size = 82, normalized size = 1.19

$$\frac{1}{9}\,a^{8}c^{4}x^{9}+\frac{1}{4}\,a^{7}c^{4}x^{8}-\frac{2}{7}\,a^{6}c^{4}x^{7}-a^{5}c^{4}x^{6}+\frac{3}{2}\,a^{3}c^{4}x^{4}+\frac{2}{3}\,a^{2}c^{4}x^{3}-ac^{4}x^{2}-c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

giac [A] time = 0.14, size = 82, normalized size = 1.19

$$\frac{1}{9}\,a^{8}c^{4}x^{9}+\frac{1}{4}\,a^{7}c^{4}x^{8}-\frac{2}{7}\,a^{6}c^{4}x^{7}-a^{5}c^{4}x^{6}+\frac{3}{2}\,a^{3}c^{4}x^{4}+\frac{2}{3}\,a^{2}c^{4}x^{3}-ac^{4}x^{2}-c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

maple [A] time = 0.04, size = 63, normalized size = 0.91

$$c^{4}\left(\frac{1}{9}x^{9}a^{8}+\frac{1}{4}a^{7}x^{8}-\frac{2}{7}a^{6}x^{7}-x^{6}a^{5}+\frac{3}{2}x^{4}a^{3}+\frac{2}{3}x^{3}a^{2}-a\,x^{2}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] c^4*(1/9*x^9*a^8+1/4*a^7*x^8-2/7*a^6*x^7-x^6*a^5+3/2*x^4*a^3+2/3*x^3*a^2-a*
x^2-x)
```

maxima [A] time = 0.31, size = 82, normalized size = 1.19

$$\frac{1}{9}\,a^{8}c^{4}x^{9}+\frac{1}{4}\,a^{7}c^{4}x^{8}-\frac{2}{7}\,a^{6}c^{4}x^{7}-a^{5}c^{4}x^{6}+\frac{3}{2}\,a^{3}c^{4}x^{4}+\frac{2}{3}\,a^{2}c^{4}x^{3}-ac^{4}x^{2}-c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

mupad [B] time = 0.04, size = 82, normalized size = 1.19

$$\frac{a^{8}\,c^{4}\,x^{9}}{9}+\frac{a^{7}\,c^{4}\,x^{8}}{4}-\frac{2\,a^{6}\,c^{4}\,x^{7}}{7}-a^{5}\,c^{4}\,x^{6}+\frac{3\,a^{3}\,c^{4}\,x^{4}}{2}+\frac{2\,a^{2}\,c^{4}\,x^{3}}{3}-a\,c^{4}\,x^{2}-c^{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*a^2*c^4*x^3)/3 - a*c^4*x^2 - c^4*x + (3*a^3*c^4*x^4)/2 - a^5*c^4*x^6 - (
2*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

sympy [A] time = 0.09, size = 87, normalized size = 1.26

$$\frac{a^8c^4x^9}{9} + \frac{a^7c^4x^8}{4} - \frac{2a^6c^4x^7}{7} - a^5c^4x^6 + \frac{3a^3c^4x^4}{2} + \frac{2a^2c^4x^3}{3} - ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 - a**5*c**4*x**6 +
 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 - a*c**4*x**2 - c**4*x
```

3.566 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=52

$$-\frac{c^3(ax+1)^7}{7a}+\frac{2c^3(ax+1)^6}{3a}-\frac{4c^3(ax+1)^5}{5a}$$

```
[Out] -4/5*c^3*(a*x+1)^5/a+2/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$-\frac{c^3(ax+1)^7}{7a}+\frac{2c^3(ax+1)^6}{3a}-\frac{4c^3(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-4*c^3*(1 + a*x)^5)/(5*a) + (2*c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/
(7*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx\\
&= -\left(c^3\int(1-ax)^2(1+ax)^4\,dx\right)\\
&= -\left(c^3\int\left(4(1+ax)^4-4(1+ax)^5+(1+ax)^6\right)dx\right)\\
&= -\frac{4c^3(1+ax)^5}{5a}+\frac{2c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.60

$$-\frac{c^3(ax+1)^5\left(15a^2x^2-40ax+29\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/105*(c^3*(1 + a*x)^5*(29 - 40*a*x + 15*a^2*x^2))/a
```

fricas [A] time = 0.50, size = 70, normalized size = 1.35

$$-\frac{1}{7}a^6c^3x^7 - \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 + a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 - ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

giac [A] time = 0.14, size = 70, normalized size = 1.35

$$-\frac{1}{7}a^6c^3x^7 - \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 + a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 - ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

maple [A] time = 0.03, size = 54, normalized size = 1.04

$$c^3\left(-\frac{1}{7}a^6x^7 - \frac{1}{3}x^6a^5 + \frac{1}{5}a^4x^5 + x^4a^3 + \frac{1}{3}x^3a^2 - a\,x^2 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(-1/7*a^6*x^7-1/3*x^6*a^5+1/5*a^4*x^5+x^4*a^3+1/3*x^3*a^2-a*x^2-x)
```

maxima [A] time = 0.30, size = 70, normalized size = 1.35

$$-\frac{1}{7}a^6c^3x^7 - \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 + a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 - ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

mupad [B] time = 0.04, size = 70, normalized size = 1.35

$$-\frac{a^6\,c^3\,x^7}{7} - \frac{a^5\,c^3\,x^6}{3} + \frac{a^4\,c^3\,x^5}{5} + a^3\,c^3\,x^4 + \frac{a^2\,c^3\,x^3}{3} - a\,c^3\,x^2 - c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (a^2*c^3*x^3)/3 - a*c^3*x^2 - c^3*x + a^3*c^3*x^4 + (a^4*c^3*x^5)/5 - (a^5*
c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

sympy [A] time = 0.08, size = 70, normalized size = 1.35

$$-\frac{a^6c^3x^7}{7} - \frac{a^5c^3x^6}{3} + \frac{a^4c^3x^5}{5} + a^3c^3x^4 + \frac{a^2c^3x^3}{3} - ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**6*c**3*x**7/7 - a**5*c**3*x**6/3 + a**4*c**3*x**5/5 + a**3*c**3*x**4 +
a**2*c**3*x**3/3 - a*c**3*x**2 - c**3*x
```

3.567 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=35

$$\frac{c^2(ax+1)^5}{5a}-\frac{c^2(ax+1)^4}{2a}$$

```
[Out] -1/2*c^2*(a*x+1)^4/a+1/5*c^2*(a*x+1)^5/a
```

Rubi [A] time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^2(ax+1)^5}{5a}-\frac{c^2(ax+1)^4}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] -(c^2*(1 + a*x)^4)/(2*a) + (c^2*(1 + a*x)^5)/(5*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx&=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx\\&=-\left(c^2\int(1-ax)(1+ax)^3\,dx\right)\\&=-\left(c^2\int\left(2(1+ax)^3-(1+ax)^4\right)\,dx\right)\\&=-\frac{c^2(1+ax)^4}{2a}+\frac{c^2(1+ax)^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.66

$$\frac{c^2(ax+1)^4(2ax-3)}{10a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^4*(-3 + 2*a*x))/(10*a)
```

fricas [A] time = 0.52, size = 38, normalized size = 1.09

$$\frac{1}{5}a^4c^2x^5 + \frac{1}{2}a^3c^2x^4 - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

giac [A] time = 0.12, size = 38, normalized size = 1.09

$$\frac{1}{5}a^4c^2x^5 + \frac{1}{2}a^3c^2x^4 - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

maple [A] time = 0.03, size = 31, normalized size = 0.89

$$c^2\left(\frac{1}{5}a^4x^5 + \frac{1}{2}x^4a^3 - a\,x^2 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^2,x)
[Out] c^2*(1/5*a^4*x^5+1/2*x^4*a^3-a*x^2-x)
```

maxima [A] time = 0.30, size = 38, normalized size = 1.09

$$\frac{1}{5}a^4c^2x^5 + \frac{1}{2}a^3c^2x^4 - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

mupad [B] time = 0.05, size = 38, normalized size = 1.09

$$\frac{a^4\,c^2\,x^5}{5} + \frac{a^3\,c^2\,x^4}{2} - a\,c^2\,x^2 - c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1))/(a*x - 1),x)
[Out] (a^3*c^2*x^4)/2 - a*c^2*x^2 - c^2*x + (a^4*c^2*x^5)/5
```

sympy [A] time = 0.07, size = 36, normalized size = 1.03

$$\frac{a^4c^2x^5}{5} + \frac{a^3c^2x^4}{2} - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**2,x)
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4/2 - a*c**2*x**2 - c**2*x
```

3.568 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=15

$$-\frac{c(ax+1)^3}{3a}$$

```
[Out] -1/3*c*(a*x+1)^3/a
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6167, 6140, 32}

$$-\frac{c(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -(c*(1 + a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx\\ &= -\left(c\int (1+ax)^2\,dx\right)\\ &= -\frac{c(1+ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.33

$$-c\left(\frac{a^2x^3}{3}+ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -(c*(x + a*x^2 + (a^2*x^3)/3))
```

fricas [A] time = 0.52, size = 21, normalized size = 1.40

$$-\frac{1}{3}a^2cx^3 - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

giac [A] time = 0.12, size = 21, normalized size = 1.40

$$-\frac{1}{3}a^2cx^3 - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

maple [A] time = 0.03, size = 14, normalized size = 0.93

$$-\frac{c(ax+1)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c),x)
[Out] -1/3*c*(a*x+1)^3/a
```

maxima [A] time = 0.31, size = 21, normalized size = 1.40

$$-\frac{1}{3}a^2cx^3 - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

mupad [B] time = 0.03, size = 17, normalized size = 1.13

$$-\frac{c\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1))/(a*x - 1),x)
[Out] -(c*x*(3*a*x + a^2*x^2 + 3))/3
```

sympy [A] time = 0.06, size = 20, normalized size = 1.33

$$-\frac{a^2cx^3}{3} - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c),x)
[Out] -a**2*c*x**3/3 - a*c*x**2 - c*x
```

3.569 $\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{ac(1-ax)}$$

```
[Out] -1/a/c/(-a*x+1)
```

Rubi [A] time = 0.07, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 32}

$$-\frac{1}{ac(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] -(1/(a*c*(1 - a*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^2}\,dx}{c}\\
&= -\frac{1}{ac(1-ax)}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 18, normalized size = 1.12

$$\frac{e^{2\coth^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(2*ArcCoth[a*x])/(2*a*c)
```

fricas [A] time = 0.49, size = 13, normalized size = 0.81

$$\frac{1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/(a^2*c*x - a*c)
```

giac [A] time = 0.14, size = 14, normalized size = 0.88

$$\frac{1}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/((a*x - 1)*a*c)
```

maple [A] time = 0.03, size = 15, normalized size = 0.94

$$\frac{1}{ca\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c),x)
[Out] 1/c/a/(a*x-1)
```

maxima [A] time = 0.30, size = 13, normalized size = 0.81

$$\frac{1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/(a^2*c*x - a*c)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$-\frac{1}{a\ (c-a\,c\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)*(a*x - 1)),x)
[Out] -1/(a*(c - a*c*x))
```

sympy [A] time = 0.12, size = 10, normalized size = 0.62

$$\frac{1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c),x)
[Out] 1/(a**2*c*x - a*c)
```

3.570 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] -1/4/a/c^2/(-a*x+1)^2-1/4/a/c^2/(-a*x+1)-1/4*arctanh(a*x)/a/c^2
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{1}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/(4*a*c^2*(1 - a*x)^2) - 1/(4*a*c^2*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= -\frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^2}$$
$$= -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}$$
$$= -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.69

$$\frac{ax+(ax-1)^2\left(-\tanh^{-1}(ax)\right)-2}{4ac^2(ax-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^2*(-1 + a*x)^2)
```

fricas [A] time = 0.43, size = 76, normalized size = 1.49

$$\frac{2\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)-4}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.14, size = 51, normalized size = 1.00

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}+\frac{ax-2}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x -
 2)/((a*x - 1)^2*a*c^2)
```

maple [A] time = 0.04, size = 60, normalized size = 1.18

$$-\frac{1}{4c^2a\,(ax-1)^2}+\frac{1}{4a\,c^2\,(ax-1)}+\frac{\ln\left(ax-1\right)}{8c^2a}-\frac{\ln\left(ax+1\right)}{8a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/4/c^2/a/(a*x-1)^2+1/4/a/c^2/(a*x-1)+1/8/c^2/a*ln(a*x-1)-1/8*ln(a*x+1)/a/
c^2
```

maxima [A] time = 0.31, size = 63, normalized size = 1.24

$$\frac{ax-2}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^2
) + 1/8*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 1.26, size = 46, normalized size = 0.90

$$\frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}-\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^2*(a*x - 1)),x)
```

```
[Out] (x/4 - 1/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

sympy [A] time = 0.27, size = 54, normalized size = 1.06

$$\frac{ax-2}{4a^3c^2x^2-8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (a*x - 2)/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**2)
```

3.571 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}-\frac{1}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] -1/12/a/c^3/(-a*x+1)^3-1/8/a/c^3/(-a*x+1)^2-3/16/a/c^3/(-a*x+1)+1/16/a/c^3/
(a*x+1)-1/4*arctanh(a*x)/a/c^3
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}-\frac{1}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/(12*a*c^3*(1 - a*x)^3) - 1/(8*a*c^3*(1 - a*x)^2) - 3/(16*a*c^3*(1 - a*x)
) + 1/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$
$$= -\frac{\int \left(\frac{1}{4(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^3}$$
$$= -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}$$
$$= -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.73

$$\frac{3a^3x^3-6a^2x^2+ax-3(ax-1)^3(ax+1)\tanh^{-1}(ax)+4}{12ac^3(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (4 + a*x - 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x])/(
12*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.43, size = 121, normalized size = 1.41

$$\frac{6\,a^3x^3-12\,a^2x^2+2\,ax-3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)+3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax-1\right)+}{24\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^5*c
^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.13, size = 74, normalized size = 0.86

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^3}+\frac{\log\left(|ax-1|\right)}{8\,ac^3}+\frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(ax+1)(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a^
3*x^3 - 6*a^2*x^2 + a*x + 4)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

maple [A] time = 0.04, size = 90, normalized size = 1.05

$$\frac{1}{12c^3a\,(ax-1)^3}-\frac{1}{8c^3a\,(ax-1)^2}+\frac{3}{16a\,c^3\,(ax-1)}+\frac{\ln\left(ax-1\right)}{8c^3a}+\frac{1}{16a\,c^3\,(ax+1)}-\frac{\ln\left(ax+1\right)}{8a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/12/c^3/a/(a*x-1)^3-1/8/c^3/a/(a*x-1)^2+3/16/a/c^3/(a*x-1)+1/8/c^3/a*ln(a*
x-1)+1/16/a/c^3/(a*x+1)-1/8*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.31, size = 91, normalized size = 1.06

$$\frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}-\frac{\log\left(ax+1\right)}{8\,ac^3}+\frac{\log\left(ax-1\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^3 - 6*a^2*x^2 + a*x + 4)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2
*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.09, size = 73, normalized size = 0.85

$$-\frac{\frac{x}{12}-\frac{a\,x^2}{2}+\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}-\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^3*(a*x - 1)),x)
```

```
[Out] - (x/12 - (a*x^2)/2 + 1/(3*a) + (a^2*x^3)/4)/(c^3 + 2*a^3*c^3*x^3 - a^4*c^3
*x^4 - 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

sympy [A] time = 0.40, size = 85, normalized size = 0.99

$$-\frac{-3a^3x^3+6a^2x^2-ax-4}{12a^5c^3x^4-24a^4c^3x^3+24a^2c^3x-12ac^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(-3*a**3*x**3 + 6*a**2*x**2 - a*x - 4)/(12*a**5*c**3*x**4 - 24*a**4*c**3*x
**3 + 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c
**3)
```

3.572 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=121

$$-\frac{5}{32ac^4(1-ax)}+\frac{5}{64ac^4(ax+1)}-\frac{3}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}-\frac{1}{16ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}-\frac{15\tanh^{-1}(}{64ac^4}$$

```
[Out] -1/32/a/c^4/(-a*x+1)^4-1/16/a/c^4/(-a*x+1)^3-3/32/a/c^4/(-a*x+1)^2-5/32/a/c
^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2+5/64/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/c^
4
```

Rubi [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{5}{32ac^4(1-ax)}+\frac{5}{64ac^4(ax+1)}-\frac{3}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}-\frac{1}{16ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}-\frac{15\tanh^{-1}(}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/(32*a*c^4*(1 - a*x)^4) - 1/(16*a*c^4*(1 - a*x)^3) - 3/(32*a*c^4*(1 - a*x
)^2) - 5/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) + 5/(64*a*c^4*(1 +
 a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$
$$= -\frac{\int \left(-\frac{1}{8(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{5}{32(-1+ax)^2}+\frac{1}{32(1+ax)^3}+\frac{5}{64(1+ax)^2}-\frac{15}{64\left(-1+a^2x^2\right)}\right)dx}{c^4}$$
$$= -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+ \frac{}{64}$$
$$= -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+ \frac{}{64}$$

Mathematica [A] time = 0.07, size = 82, normalized size = 0.68

$$-\frac{-15a^5x^5+30a^4x^4+10a^3x^3-50a^2x^2+17ax+15(ax-1)^4(ax+1)^2\tanh^{-1}(ax)+16}{64ac^4(ax-1)^4(ax+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/64*(16 + 17*a*x - 50*a^2*x^2 + 10*a^3*x^3 + 30*a^4*x^4 - 15*a^5*x^5 + 15
*(-1 + a*x)^4*(1 + a*x)^2*ArcTanh[a*x])/(a*c^4*(-1 + a*x)^4*(1 + a*x)^2)
```

fricas [B] time = 0.60, size = 217, normalized size = 1.79

$$\frac{30\,a^5x^5-60\,a^4x^4-20\,a^3x^3+100\,a^2x^2-34\,ax-15\left(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1\right)\log\left(ax+1\right)}{128\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(30*a^5*x^5 - 60*a^4*x^4 - 20*a^3*x^3 + 100*a^2*x^2 - 34*a*x - 15*(a^
6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1)
 + 15*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 32)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3
- a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.12, size = 91, normalized size = 0.75

$$-\frac{15\,\log\left(|ax+1|\right)}{128\,ac^4}+\frac{15\,\log\left(|ax-1|\right)}{128\,ac^4}+\frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\left(ax+1\right)^2\left(ax-1\right)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/64
*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/((a*x +
1)^2*(a*x - 1)^4*a*c^4)
```

maple [A] time = 0.04, size = 120, normalized size = 0.99

$$-\frac{1}{32c^4a\left(ax-1\right)^4}+\frac{1}{16c^4a\left(ax-1\right)^3}-\frac{3}{32c^4a\left(ax-1\right)^2}+\frac{5}{32a\,c^4\left(ax-1\right)}+\frac{15\ln\left(ax-1\right)}{128c^4a}+\frac{1}{64a\,c^4\left(ax+1\right)^2}+\frac{5}{64a\,c^4\left(ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/32/c^4/a/(a*x-1)^4+1/16/c^4/a/(a*x-1)^3-3/32/c^4/a/(a*x-1)^2+5/32/a/c^4/
(a*x-1)+15/128/c^4/a*ln(a*x-1)+1/64/a/c^4/(a*x+1)^2+5/64/a/c^4/(a*x+1)-15/1
28*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.30, size = 140, normalized size = 1.16

$$\frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}-\frac{15\,\log\left(ax+1\right)}{128\,ac^4}+\frac{15\,\log\left(ax-1\right)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/(a^7
*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^
2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*c^4
)
```

mupad [B] time = 1.30, size = 121, normalized size = 1.00

$$\frac{\frac{17\,x}{64}-\frac{25\,a\,x^2}{32}+\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}+\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5+a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+a^2\,c^4\,x^2+2\,a\,c^4\,x-c^4}-\frac{15\,\mathrm{atanh}\left(a\,x\right)}{64\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^4*(a*x - 1)),x)
```

```
[Out] ((17*x)/64 - (25*a*x^2)/32 + 1/(4*a) + (5*a^2*x^3)/32 + (15*a^3*x^4)/32 - (
15*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 + 2*a^5*c^
4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

sympy [A] time = 0.56, size = 141, normalized size = 1.17

$$\frac{15a^5x^5-30a^4x^4-10a^3x^3+50a^2x^2-17ax-16}{64a^7c^4x^6-128a^6c^4x^5-64a^5c^4x^4+256a^4c^4x^3-64a^3c^4x^2-128a^2c^4x+64ac^4}+\frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (15*a**5*x**5 - 30*a**4*x**4 - 10*a**3*x**3 + 50*a**2*x**2 - 17*a*x - 16)/(
64*a**7*c**4*x**6 - 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 + 256*a**4*c**4*
x**3 - 64*a**3*c**4*x**2 - 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/a)/
128 - 15*log(x + 1/a)/128)/(a*c**4)
```

3.573 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=393

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{13/2}+\frac{5}{168}a^6c^4x^7\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5a^5c^4x^6}{}$$

```
[Out] -5/72*a^7*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(13/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(5
/2)*(1+1/a/x)^(13/2)*x^9-55/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
)/a-55/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-11/192*a^2*c^4*(1+1/a/
x)^(5/2)*x^3*(1-1/a/x)^(1/2)-11/448*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(
1/2)-11/1008*a^4*c^4*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)-5/1008*a^5*c^4*(1+
1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2)+5/168*a^6*c^4*(1+1/a/x)^(13/2)*x^7*(1-1/a
/x)^(1/2)-55/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{13/2}+\frac{5}{168}a^6c^4x^7\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5a^5c^4x^6}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (-55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (55*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/
(a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x
^4)/448 - (11*a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/1008 - (5*
a^5*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/1008 + (5*a^6*c^4*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(13/2)*x^7)/168 - (5*a^7*c^4*(1 - 1/(a*x))^(3/2)
*(1 + 1/(a*x))^(13/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(13/2)*x^9)/9 - (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*
a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9+\frac{1}{9}\left(5a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^9}\,dx\right.$$

$$=-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9-\frac{1}{24}\left(5\right.$$

$$=\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(\right.$$

$$=-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\right.$$

$$=-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008}-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}+\frac{5}{168}a^6c^4\sqrt{1-\right.}$$

$$=-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008}-\frac{5a^5c^4\sqrt{1-\right.}}{}$$

$$=-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{11a^4c^4\sqrt{}}{}$$

$$=-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{}$$

$$=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-}$$

$$=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-}$$

$$=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-}$$

Mathematica [A] time = 0.20, size = 111, normalized size = 0.28

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(896a^8x^8+3024a^7x^7+1024a^6x^6-7224a^5x^5-8448a^4x^4+3066a^3x^3+10240a^2x^2+4599ax-\right.\right.}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 + 4599*a*x + 10240*a^2*x^2 + 3066*a^
3*x^3 - 8448*a^4*x^4 - 7224*a^5*x^5 + 1024*a^6*x^6 + 3024*a^7*x^7 + 896*a^8
*x^8) - 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

fricas [A] time = 0.71, size = 170, normalized size = 0.43

$$-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(896\,a^9c^4x^9+3920\,a^8c^4x^8+4048\,a^7c^4x^7-6200\,a^6c^4\right.}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1) - (896*a^9*c^4*x^9 + 3920*a^8*c^4*x^8 + 4048*a^7*c^
4*x^7 - 6200*a^6*c^4*x^6 - 15672*a^5*c^4*x^5 - 5382*a^4*c^4*x^4 + 13306*a^3
*c^4*x^3 + 14839*a^2*c^4*x^2 + 887*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

giac [A] time = 0.20, size = 340, normalized size = 0.87

$$-\frac{1}{8064}\,ac^4\left(\frac{3465\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{2\left(\frac{30030\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{115038\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/8064*a*c^4*(3465*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*log(abs(s
qrt((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(30030*(a*x - 1)*sqrt((a*x - 1)/(a*x
 + 1))/(a*x + 1) - 115038*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2
 - 334602*(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 360448*(a*x -
 1)^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 255222*(a*x - 1)^5*sqrt((a*x
- 1)/(a*x + 1))/(a*x + 1)^5 + 115038*(a*x - 1)^6*sqrt((a*x - 1)/(a*x + 1))/
(a*x + 1)^6 - 30030*(a*x - 1)^7*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^7 + 346
5*(a*x - 1)^8*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^8 - 3465*sqrt((a*x - 1)/(
a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^9))
```

maple [A] time = 0.06, size = 288, normalized size = 0.73

$$\frac{(ax-1)^2\,c^4\left(896\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^6a^6+3024\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^5a^5+1920\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^4a^4-4200\left(a^2x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/8064*(a*x-1)^2*c^4/a*(896*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6+3024*(a^2
*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5+1920*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^
4-4200*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-6528*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^2*a^2-1134*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+8064*((a*x-1)*(a*x+1))^
(3/2)*(a^2)^(1/2)-4352*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+3465*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*x*a-3465*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*a)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.32, size = 415, normalized size = 1.06

$$-\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+11503\ldots\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(\ldots}{(a\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqr
t((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2)
- 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1))
^(13/2) - 255222*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 360448*c^4*((a*x - 1)/(
a*x + 1))^(9/2) - 334602*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*((a*x
 - 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*c^4*s
qrt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(
a*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)
^4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*
(a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a
^2/(a*x + 1)^9 - a^2))*a
```

mupad [B] time = 0.21, size = 362, normalized size = 0.92

$$\frac{\frac{55\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{715\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96}+\frac{913\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32}+\frac{18589\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224}-\frac{5632\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{63}+\frac{14179\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{224}-\frac{913\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{32}+\ldots}{a-\frac{9\,a\,(a\,x-1)}{a\,x+1}+\frac{36\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{84\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{126\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{126\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{84\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{36\,a\,(a\,x-1)^7}{(a\,x+1)^7}+\frac{9\,a\,(a\,x\ldots}{(a\,x+\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((55*c^4*((a*x - 1)/(a*x + 1))^(1/2))/64 - (715*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/96 + (913*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (18589*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/224 - (5632*c^4*((a*x - 1)/(a*x + 1))^(9/2))/63 + (14179
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/224 - (913*c^4*((a*x - 1)/(a*x + 1))^(13
/2))/32 + (715*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 - (55*c^4*((a*x - 1)/(a
*x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a
*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^
4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*
a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9
)/(a*x + 1)^9) - (55*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^6x^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4*x**4
/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6*x**6/(a*x*sqrt(a*x/(a*x + 1) -
```

```
 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)
+ Integral(a**8*x**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x))
```

3.574 $\int e^{3\coth^{-1}(ax)} \left(c - a^2cx^2\right)^3 \, dx$

Optimal. Leaf size=313

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{1}{14}a^5c^3x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{1}{70}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{9}{280}a^3c^3x^4$$

```
[Out] -1/7*a^6*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)*x^7-9/16*c^3*arctanh((1-1/a/x
)^(1/2)*(1+1/a/x)^(1/2))/a-3/16*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-3
/40*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-9/280*a^3*c^3*(1+1/a/x)^(7/
2)*x^4*(1-1/a/x)^(1/2)-1/70*a^4*c^3*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)+1/1
4*a^5*c^3*(1+1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2)-9/16*c^3*x*(1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{1}{14}a^5c^3x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{1}{70}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{9}{280}a^3c^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (3*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x)
)^(5/2)*x^3)/40 - (9*a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/280
 - (a^4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/70 + (a^5*c^3*Sqrt[1
 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(3/2)*(1
+ 1/(a*x))^(11/2)*x^7)/7 - (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x
)]])/(16*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
```

```
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\left(\left(a^6c^3\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$=\left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{1}{7}\left(3a^5c^3\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}{x^7}\,d\right.$$

$$=\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7+\frac{1}{14}\left(a^4\right.$$

$$=-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(\right.$$

$$=-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3$$

$$=-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{70}a^4c^3$$

$$=-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3c^3$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{a}}$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{a}}$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{a}}$$

Mathematica [A] time = 0.15, size = 95, normalized size = 0.30

$$-\frac{c^3\left(315\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(80a^6x^6+280a^5x^5+208a^4x^4-350a^3x^3-656a^2x^2-245ax\right.\right.}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/560*(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 - 245*a*x - 656*a^2*x^2 - 350*a
^3*x^3 + 208*a^4*x^4 + 280*a^5*x^5 + 80*a^6*x^6) + 315*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x]))/a
```

fricas [A] time = 0.68, size = 147, normalized size = 0.47

$$-\frac{315\,c^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(80\,a^{7}c^{3}x^{7}+360\,a^{6}c^{3}x^{6}+488\,a^{5}c^{3}x^{5}-142\,a^{4}c^{3}x^{4}-1006\,a}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (80*a^7*c^3*x^7 + 360*a^6*c^3*x^6 + 488*a^5*c^3*x^5
- 142*a^4*c^3*x^4 - 1006*a^3*c^3*x^3 - 901*a^2*c^3*x^2 + 123*a*c^3*x + 368*
c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.20, size = 278, normalized size = 0.89

$$-\frac{1}{560}\,ac^{3}\left(\frac{315\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}}-\frac{315\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^{2}}-\frac{2\left(\frac{2100\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{8393\,(ax-1)^{2}\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^{2}}-\frac{9216\,(ax-1)^{3}\sqrt{\frac{a}{a}}}{(ax+1)^{3}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/560*a*c^3*(315*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*log(abs(sqrt
((a*x - 1)/(a*x + 1)) - 1))/a^2 - 2*(2100*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1
))/(a*x + 1) + 8393*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 921
6*(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 + 5943*(a*x - 1)^4*sqrt
((a*x - 1)/(a*x + 1))/(a*x + 1)^4 - 2100*(a*x - 1)^5*sqrt((a*x - 1)/(a*x +
1))/(a*x + 1)^5 + 315*(a*x - 1)^6*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^6 - 3
15*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^7))
```

maple [A] time = 0.05, size = 240, normalized size = 0.77

$$-\frac{(ax-1)^{2}\,c^{3}\left(80\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,x^{4}a^{4}+280\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,x^{3}a^{3}+288\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,x^{2}a^{2}-70\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\right.}{560a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/560*(a*x-1)^2*c^3/a*(80*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^4+280*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+288*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-7
0*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+192*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-315*(
a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-560*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+31
5*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1)
)^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.32, size = 337, normalized size = 1.08

$$-\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+5943\,c^3\left(\frac{a}{a}\right.\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+\frac{35\,(ax-}{(ax+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 21
00*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 5943*c^3*((a*x - 1)/(a*x + 1))^(9/2)
- 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 8393*c^3*((a*x - 1)/(a*x + 1))^(5/
2) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x -
 1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2
/(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7
- a^2))*a
```

mupad [B] time = 0.12, size = 289, normalized size = 0.92

$$-\frac{\frac{15\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2}-\frac{9\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{1199\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}-\frac{1152\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{35}+\frac{849\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{40}-\frac{15\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{2}+\frac{9\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{8}}{a-\frac{7\,a\,(a\,x-1)}{a\,x+1}+\frac{21\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{35\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{35\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{21\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{7\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{a\,(a\,x-1)^7}{(a\,x+1)^7}}-\frac{9\,c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - ((15*c^3*((a*x - 1)/(a*x + 1))^(3/2))/2 - (9*c^3*((a*x - 1)/(a*x + 1))^(1
/2))/8 + (1199*c^3*((a*x - 1)/(a*x + 1))^(5/2))/40 - (1152*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/35 + (849*c^3*((a*x - 1)/(a*x + 1))^(9/2))/40 - (15*c^3*((
a*x - 1)/(a*x + 1))^(11/2))/2 + (9*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a
- (7*a*(a*x - 1))/(a*x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x -
 1)^3)/(a*x + 1)^3 + (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a
*x + 1)^5 + (7*a*(a*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7) -
(9*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^6x^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4*x**
4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1)), x) + Integral(a**6*x**6/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
Integral(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

3.575 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{1}{5}a^4c^2x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{7}{60}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{7}{24}ac^2x^2\sqrt{1-\frac{1}{ax}}$$

```
[Out] -7/8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/24*a*c^2*(1+1/a/x)^(3
/2)*x^2*(1-1/a/x)^(1/2)-7/60*a^2*c^2*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-1/
20*a^3*c^2*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)+1/5*a^4*c^2*(1+1/a/x)^(9/2)*
x^5*(1-1/a/x)^(1/2)-7/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{5}a^4c^2x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{7}{60}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{7}{24}ac^2x^2\sqrt{1-\frac{1}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (-7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/24 - (7*a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/60 - (a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (
a^4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/5 - (7*c^2*ArcTanh[Sqrt[
1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{5}\left(a^3c^2\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{7/2}}{x^5\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{20}\left(7a^2c\right.$$

$$=-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\sqrt{}$$

$$=-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{}$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}$$

Mathematica [A] time = 0.11, size = 79, normalized size = 0.34

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(24a^4x^4+90a^3x^3+112a^2x^2+15ax-136\right)-105\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 + 15*a*x + 112*a^2*x^2 + 90*a^3*x^3 +
 24*a^4*x^4) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

fricas [A] time = 0.47, size = 126, normalized size = 0.54

$$-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(24\,a^5c^2x^5+114\,a^4c^2x^4+202\,a^3c^2x^3+127\,a^2c^2x^2-121\right.}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) - (24*a^5*c^2*x^5 + 114*a^4*c^2*x^4 + 202*a^3*c^2*x^3
+ 127*a^2*c^2*x^2 - 121*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.16, size = 216, normalized size = 0.93

$$-\frac{1}{120}ac^2\left(\frac{105\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{790(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{896(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}+\frac{490(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^3}\right.}{a^2\left(\frac{ax-1}{ax+1}-1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/120*a*c^2*(105*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*log(abs(sqrt
((a*x - 1)/(a*x + 1)) - 1))/a^2 + 2*(790*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1)
)/(a*x + 1) - 896*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 + 490*(
a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3 - 105*(a*x - 1)^4*sqrt((a*
x - 1)/(a*x + 1))/(a*x + 1)^4 + 105*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x -
 1)/(a*x + 1) - 1)^5))
```

maple [A] time = 0.05, size = 192, normalized size = 0.82

$$\frac{(ax-1)^2c^2\left(24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2+90\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,xa+16\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}+105\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+1\right.}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/120*(a*x-1)^2*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+90*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+105*(a^2*x^2-1)^
(1/2)*(a^2)^(1/2)*x*a+120*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-105*ln((a^2*x
+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(3/2)/(a*
x+1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.31, size = 259, normalized size = 1.11

$$-\frac{1}{120}a\left(\frac{105c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-490c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+896c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}\right.}{\frac{5(ax-1)a^2}{ax+1}-\frac{10(ax-1)^2a^2}{(ax+1)^2}+\frac{10(ax-1)^3a^2}{(ax+1)^3}-5(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 4
90*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 896*c^2*((a*x - 1)/(a*x + 1))^(5/2) -
790*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5
*(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*
```

```
a^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)
^5 - a^2))
```

mupad [B] time = 1.25, size = 214, normalized size = 0.92

$$\frac{\frac{7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{79\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6}-\frac{224\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{15}+\frac{49\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{6}-\frac{7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}-\frac{7\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((7*c^2*((a*x - 1)/(a*x + 1))^(1/2))/4 + (79*c^2*((a*x - 1)/(a*x + 1))^(3/2
))/6 - (224*c^2*((a*x - 1)/(a*x + 1))^(5/2))/15 + (49*c^2*((a*x - 1)/(a*x +
 1))^(7/2))/6 - (7*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (7*c^2*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{2a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4*x**4/(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a
*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

3.576 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=145

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{5}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+\right.}{2a}$$

```
[Out] -5/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-5/6*a*c*(1+1/a/x)^(3/2)*x
^2*(1-1/a/x)^(1/2)-1/3*a^2*c*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-5/2*c*x*(1
-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{5}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 - (5*a*c*Sqrt[1 - 1/(a*x)]*(
1 + 1/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^
3)/3 - (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
```

```
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + 
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx = -\left(\left(a^2c\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{3}(5ac)\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{2}(5c)\,\text{Subst}\Big($$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\Big(1+$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\Big(1+$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\Big(1+$$

Mathematica [A] time = 0.08, size = 61, normalized size = 0.42

$$-\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+9ax+22\right)+15\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -1/6*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 + 9*a*x + 2*a^2*x^2) + 15*Log[(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/a
```

fricas [A] time = 0.60, size = 91, normalized size = 0.63

$$-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3cx^3+11\,a^2cx^2+31\,acx+22\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) + (2*a^3*c*x^3 + 11*a^2*c*x^2 + 31*a*c*x + 22*c)*sqrt((a*x - 1
)/(a*x + 1)))/a
```

giac [A] time = 0.17, size = 152, normalized size = 1.05

$$-\frac{1}{6}\,ac\left(\frac{15\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{2\left(\frac{40\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\frac{15\,(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2}-33\,\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/6*a*c*(15*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*log(abs(sqrt((a*x
- 1)/(a*x + 1)) - 1))/a^2 + 2*(40*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x
+ 1) - 15*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^2 - 33*sqrt((a*x
- 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) - 1)^3))
```

maple [A] time = 0.05, size = 183, normalized size = 1.26

$$-\frac{(ax-1)^2\,c\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+2\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+24\sqrt{(ax-1)(ax+1)}\,\cdots\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x)
```

```
[Out] -1/6*(a*x-1)^2*c*(9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+2*((a*x-1)*(a*x+1))^(
3/2)*(a^2)^(1/2)-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+
24*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+24*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/
2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x
+1))^(1/2)/a/(a^2)^(1/2)
```

maxima [A] time = 0.30, size = 171, normalized size = 1.18

$$\frac{1}{6}\,a\left(\frac{2\left(15\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+33\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(15*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3/
2) + 33*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 15*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 0.07, size = 133, normalized size = 0.92

$$-\frac{11\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+5\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}}-\frac{5\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (11*c*((a*x - 1)/(a*x + 1))^(1/2) - (40*c*((a*x - 1)/(a*x + 1))^(3/2))/3
+ 5*c*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a
*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c*atanh(((a*x -
1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
x + 1)), x))
```

3.577 $\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

[Out] 1/3/((a*x-1)/(a*x+1))^(3/2)/a/c

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6183}

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]

[Out] E^(3*ArcCoth[a*x])/(3*a*c)

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Mathematica [A] time = 0.05, size = 18, normalized size = 1.00

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]

[Out] E^(3*ArcCoth[a*x])/(3*a*c)

fricas [B] time = 0.59, size = 51, normalized size = 2.83

$$\frac{\left(a^2x^2+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")

[Out] 1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c*x^2 - 2*a^2*c*x + a*c)

giac [A] time = 0.14, size = 35, normalized size = 1.94

$$\frac{ax+1}{3\,(ax-1)ac\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/3*(a*x + 1)/((a*x - 1)*a*c*sqrt((a*x - 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 24, normalized size = 1.33

$$\frac{1}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x)
[Out] 1/3/((a*x-1)/(a*x+1))^(3/2)/a/c
```

maxima [A] time = 0.30, size = 23, normalized size = 1.28

$$\frac{1}{3\,ac\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/3/(a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

mupad [B] time = 0.03, size = 23, normalized size = 1.28

$$\frac{1}{3\,a\,c\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] 1/(3*a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
[Out] -Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c
```

3.578 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}$$

```
[Out] -2/15/((a*x-1)/(a*x+1))^(3/2)/a/c^2+1/5/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/
a/c^2/(-a^2*x^2+1)
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-2*E^(3*ArcCoth[a*x]))/(15*a*c^2) + (E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(5*a*
c^2*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx &= \frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)}-\frac{2\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{5c}\\ &= -\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}+\frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)}\end{aligned}$$

Mathematica [A] time = 0.16, size = 43, normalized size = 0.78

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2-6ax+7\right)}{15c^2(ax-1)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^2*(-1 + a*x)^3)
```

fricas [A] time = 0.59, size = 77, normalized size = 1.40

$$-\frac{\left(2\,a^{3}x^{3}-4\,a^{2}x^{2}+ax+7\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*
x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [A] time = 0.15, size = 69, normalized size = 1.25

$$\frac{(ax+1)^{2}\left(\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^{2}}{(ax+1)^{2}}-3\right)}{60\,(ax-1)^{2}ac^{2}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/60*(a*x + 1)^2*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/
((a*x - 1)^2*a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

maple [A] time = 0.04, size = 49, normalized size = 0.89

$$-\frac{2a^{2}x^{2}-6ax+7}{15\left(a^{2}x^{2}-1\right)c^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/15*(2*a^2*x^2-6*a*x+7)/(a^2*x^2-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

maxima [A] time = 0.30, size = 55, normalized size = 1.00

$$\frac{\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^{2}}{(ax+1)^{2}}-3}{60\,ac^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^2*((a*x
 - 1)/(a*x + 1))^(5/2))
```

mupad [B] time = 0.04, size = 55, normalized size = 1.00

$$-\frac{\frac{(a\,x-1)^{2}}{(a\,x+1)^{2}}-\frac{2\,(a\,x-1)}{3\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^{2}\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^2*((a
*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**4*x*
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

3.579 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=91

$$-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12(3-2ax)e^{3\coth^{-1}(ax)}}{35ac^3\left(1-a^2x^2\right)}-\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}$$

```
[Out] -8/35/((a*x-1)/(a*x+1))^(3/2)/a/c^3-1/7/((a*x-1)/(a*x+1))^(3/2)*(-4*a*x+3)/
a/c^3/(-a^2*x^2+1)^2+12/35/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/a/c^3/(-a^2*x
^2+1)
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12(3-2ax)e^{3\coth^{-1}(ax)}}{35ac^3\left(1-a^2x^2\right)}-\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-8*E^(3*ArcCoth[a*x]))/(35*a*c^3) - (E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(7*a*
c^3*(1 - a^2*x^2)^2) + (12*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(35*a*c^3*(1 - a
^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx &= -\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{7c}\\
&= -\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3\left(1-a^2x^2\right)}-\frac{24\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^2}\\
&= -\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}-\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3\left(1-a^2x^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 66, normalized size = 0.73

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13\right)}{35c^3(ax-1)^4(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a
^4*x^4))/(c^3*(-1 + a*x)^4*(1 + a*x))
```

fricas [A] time = 0.47, size = 96, normalized size = 1.05

$$-\frac{\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.18, size = 104, normalized size = 1.14

$$-\frac{\frac{(ax+1)^3\left(\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5\right)}{(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}+35\sqrt{\frac{ax-1}{ax+1}}}{560\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/560*((a*x + 1)^3*(28*(a*x - 1)/(a*x + 1) - 70*(a*x - 1)^2/(a*x + 1)^2 +
140*(a*x - 1)^3/(a*x + 1)^3 - 5)/((a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))) +
35*sqrt((a*x - 1)/(a*x + 1)))/(a*c^3)
```

maple [A] time = 0.04, size = 65, normalized size = 0.71

$$-\frac{8x^4a^4-24x^3a^3+20a^2x^2+4ax-13}{35\left(a^2x^2-1\right)^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/35*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/(a^2*x^2-1)^2/c^3/((a*x-1)
/(a*x+1))^(3/2)/a
```

maxima [A] time = 0.30, size = 97, normalized size = 1.07

$$-\frac{1}{560}a\left(\frac{35\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + (28*(a*x - 1)/(a*x + 1)
- 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^3*((
a*x - 1)/(a*x + 1))^(7/2)))
```

mupad [B] time = 1.26, size = 60, normalized size = 0.66

$$-\frac{8\,a^4\,x^4-24\,a^3\,x^3+20\,a^2\,x^2+4\,a\,x-13}{35\,a\,c^3\,{\left(a\,x+1\right)}^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -(4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^3*(a*x + 1)^4*(
(a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^{7}x^{7}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^{6}x^{6}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^{5}x^{5}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^{4}x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(1/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**6*x
**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3
*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x)/c**3
```

3.580 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=127

$$-\frac{10(3-4ax)e^{3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8(3-2ax)e^{3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{16e^{3\coth^{-1}(ax)}}{63ac^4}$$

```
[Out] -16/63/((a*x-1)/(a*x+1))^(3/2)/a/c^4-1/9/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+1)
/a/c^4/(-a^2*x^2+1)^3-10/63/((a*x-1)/(a*x+1))^(3/2)*(-4*a*x+3)/a/c^4/(-a^2*
x^2+1)^2+8/21/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/a/c^4/(-a^2*x^2+1)
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{10(3-4ax)e^{3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8(3-2ax)e^{3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{16e^{3\coth^{-1}(ax)}}{63ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-16*E^(3*ArcCoth[a*x]))/(63*a*c^4) - (E^(3*ArcCoth[a*x])*(1 - 2*a*x))/(9*a
*c^4*(1 - a^2*x^2)^3) - (10*E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(63*a*c^4*(1 -
a^2*x^2)^2) + (8*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(21*a*c^4*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}+\frac{10\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx}{9c}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}-\frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{40\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{21c^2}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}-\frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4\left(1-a^2x^2\right)}-\frac{16\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}}{21c^3}$$

$$= -\frac{16e^{3\coth^{-1}(ax)}}{63ac^4}-\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}-\frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.28, size = 82, normalized size = 0.65

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(16a^6x^6-48a^5x^5+24a^4x^4+56a^3x^3-66a^2x^2+6ax+19\right)}{63c^4(ax-1)^5(ax+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/63*(Sqrt[1 - 1/(a^2*x^2)]*x*(19 + 6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a
^4*x^4 - 48*a^5*x^5 + 16*a^6*x^6))/(c^4*(-1 + a*x)^5*(1 + a*x)^2)
```

fricas [A] time = 0.63, size = 124, normalized size = 0.98

$$-\frac{\left(16\,a^6x^6-48\,a^5x^5+24\,a^4x^4+56\,a^3x^3-66\,a^2x^2+6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/63*(16*a^6*x^6 - 48*a^5*x^5 + 24*a^4*x^4 + 56*a^3*x^3 - 66*a^2*x^2 + 6*a
*x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4
*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.16, size = 149, normalized size = 1.17

$$\frac{\frac{(ax+1)^4\left(\frac{54\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{420\,(ax-1)^3}{(ax+1)^3}-\frac{945\,(ax-1)^4}{(ax+1)^4}-7\right)}{(ax-1)^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{21\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-378\sqrt{\frac{ax-1}{ax+1}}}{4032\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/4032*((a*x + 1)^4*(54*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 +
 420*(a*x - 1)^3/(a*x + 1)^3 - 945*(a*x - 1)^4/(a*x + 1)^4 - 7)/((a*x - 1)^
4*sqrt((a*x - 1)/(a*x + 1))) + 21*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x
+ 1) - 378*sqrt((a*x - 1)/(a*x + 1)))/(a*c^4)
```

maple [A] time = 0.04, size = 81, normalized size = 0.64

$$-\frac{16x^6a^6-48x^5a^5+24x^4a^4+56x^3a^3-66a^2x^2+6ax+19}{63\left(a^2x^2-1\right)^3c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/63*(16*a^6*x^6-48*a^5*x^5+24*a^4*x^4+56*a^3*x^3-66*a^2*x^2+6*a*x+19)/(a^
2*x^2-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)/a
```

maxima [A] time = 0.31, size = 131, normalized size = 1.03

$$\frac{1}{4032}a\left(\frac{21\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-18\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{54(ax-1)}{ax+1}-\frac{189(ax-1)^2}{(ax+1)^2}+\frac{420(ax-1)^3}{(ax+1)^3}-\frac{945(ax-1)^4}{(ax+1)^4}-7}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4032*a*(21*(((a*x - 1)/(a*x + 1))^(3/2) - 18*sqrt((a*x - 1)/(a*x + 1)))/(
a^2*c^4) + (54*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 420*(a*x
 - 1)^3/(a*x + 1)^3 - 945*(a*x - 1)^4/(a*x + 1)^4 - 7)/(a^2*c^4*((a*x - 1)/
(a*x + 1))^(9/2)))
```

mupad [B] time = 1.26, size = 76, normalized size = 0.60

$$-\frac{16\,a^6\,x^6-48\,a^5\,x^5+24\,a^4\,x^4+56\,a^3\,x^3-66\,a^2\,x^2+6\,a\,x+19}{63\,a\,c^4\,(a\,x+1)^6\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -(6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a^4*x^4 - 48*a^5*x^5 + 16*a^6*x^6 +
19)/(63*a*c^4*(a*x + 1)^6*((a*x - 1)/(a*x + 1))^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.581 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$

Optimal. Leaf size=66

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

```
[Out] c^5*(a*x+1)^8/a-4/3*c^5*(a*x+1)^9/a+3/5*c^5*(a*x+1)^10/a-1/11*c^5*(a*x+1)^1
1/a
```

Rubi [A] time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (c^5*(1 + a*x)^8)/a - (4*c^5*(1 + a*x)^9)/(3*a) + (3*c^5*(1 + a*x)^10)/(5*a
) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx\\
&= c^5\int(1-ax)^3(1+ax)^7\,dx\\
&= c^5\int\left(8(1+ax)^7-12(1+ax)^8+6(1+ax)^9-(1+ax)^{10}\right)dx\\
&= \frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.59

$$-\frac{c^5(ax+1)^8\left(15a^3x^3-54a^2x^2+67ax-29\right)}{165a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] -1/165*(c^5*(1 + a*x)^8*(-29 + 67*a*x - 54*a^2*x^2 + 15*a^3*x^3))/a
```

fricas [A] time = 0.55, size = 101, normalized size = 1.53

$$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

giac [A] time = 0.13, size = 102, normalized size = 1.55

$$-\frac{\left(15c^5+\frac{231c^5}{ax-1}+\frac{1540c^5}{(ax-1)^2}+\frac{5775c^5}{(ax-1)^3}+\frac{13200c^5}{(ax-1)^4}+\frac{18480c^5}{(ax-1)^5}+\frac{14784c^5}{(ax-1)^6}+\frac{5280c^5}{(ax-1)^7}\right)(ax-1)^{11}}{165a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/165*(15*c^5 + 231*c^5/(a*x - 1) + 1540*c^5/(a*x - 1)^2 + 5775*c^5/(a*x -
 1)^3 + 13200*c^5/(a*x - 1)^4 + 18480*c^5/(a*x - 1)^5 + 14784*c^5/(a*x - 1)
^6 + 5280*c^5/(a*x - 1)^7)*(a*x - 1)^11/a
```

maple [A] time = 0.03, size = 75, normalized size = 1.14

$$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{2}{5}a^9x^{10}-\frac{1}{3}x^9a^8+a^7x^8+2a^6x^7-\frac{14}{5}a^4x^5-2x^4a^3+x^3a^2+2a\,x^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x)
```

```
[Out] c^5*(-1/11*a^10*x^11-2/5*a^9*x^10-1/3*x^9*a^8+a^7*x^8+2*a^6*x^7-14/5*a^4*x^
5-2*x^4*a^3+x^3*a^2+2*a*x^2+x)
```

maxima [A] time = 0.30, size = 101, normalized size = 1.53

$$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

mupad [B] time = 1.24, size = 101, normalized size = 1.53

$$-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{2\,a^9\,c^5\,x^{10}}{5}-\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8+2\,a^6\,c^5\,x^7-\frac{14\,a^4\,c^5\,x^5}{5}-2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3+2\,a\,c^5\,x^2+c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^5*x + 2*a*c^5*x^2 + a^2*c^5*x^3 - 2*a^3*c^5*x^4 - (14*a^4*c^5*x^5)/5 + 2*
a^6*c^5*x^7 + a^7*c^5*x^8 - (a^8*c^5*x^9)/3 - (2*a^9*c^5*x^10)/5 - (a^10*c^
5*x^11)/11
```

sympy [B] time = 0.90, size = 109, normalized size = 1.65

$$-\frac{a^{10}c^5x^{11}}{11}-\frac{2a^9c^5x^{10}}{5}-\frac{a^8c^5x^9}{3}+a^7c^5x^8+2a^6c^5x^7-\frac{14a^4c^5x^5}{5}-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - 2*a**9*c**5*x**10/5 - a**8*c**5*x**9/3 + a**7*c**5*x
**8 + 2*a**6*c**5*x**7 - 14*a**4*c**5*x**5/5 - 2*a**3*c**5*x**4 + a**2*c**5
*x**3 + 2*a*c**5*x**2 + c**5*x
```

3.582 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=52

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

```
[Out] 4/7*c^4*(a*x+1)^7/a-1/2*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+1)^9/a
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (4*c^4*(1 + a*x)^7)/(7*a) - (c^4*(1 + a*x)^8)/(2*a) + (c^4*(1 + a*x)^9)/(9*
a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx\\
&= c^4\int (1-ax)^2(1+ax)^6\,dx\\
&= c^4\int \left(4(1+ax)^6-4(1+ax)^7+(1+ax)^8\right)dx\\
&= \frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.60

$$\frac{c^4(ax+1)^7\left(14a^2x^2-35ax+23\right)}{126a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(1 + a*x)^7*(23 - 35*a*x + 14*a^2*x^2))/(126*a)
```

fricas [A] time = 0.54, size = 92, normalized size = 1.77

$$\frac{1}{9}a^8c^4x^9 + \frac{1}{2}a^7c^4x^8 + \frac{4}{7}a^6c^4x^7 - \frac{2}{3}a^5c^4x^6 - 2a^4c^4x^5 - a^3c^4x^4 + \frac{4}{3}a^2c^4x^3 + 2ac^4x^2 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

giac [A] time = 0.12, size = 90, normalized size = 1.73

$$\frac{\left(14\,c^4 + \frac{189\,c^4}{ax-1} + \frac{1080\,c^4}{(ax-1)^2} + \frac{3360\,c^4}{(ax-1)^3} + \frac{6048\,c^4}{(ax-1)^4} + \frac{6048\,c^4}{(ax-1)^5} + \frac{2688\,c^4}{(ax-1)^6}\right)(ax-1)^9}{126\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/126*(14*c^4 + 189*c^4/(a*x - 1) + 1080*c^4/(a*x - 1)^2 + 3360*c^4/(a*x -
1)^3 + 6048*c^4/(a*x - 1)^4 + 6048*c^4/(a*x - 1)^5 + 2688*c^4/(a*x - 1)^6)*
(a*x - 1)^9/a
```

maple [A] time = 0.03, size = 69, normalized size = 1.33

$$c^4\left(\frac{1}{9}x^9a^8 + \frac{1}{2}a^7x^8 + \frac{4}{7}a^6x^7 - \frac{2}{3}x^6a^5 - 2a^4x^5 - x^4a^3 + \frac{4}{3}x^3a^2 + 2a\,x^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x)
```

```
[Out] c^4*(1/9*x^9*a^8+1/2*a^7*x^8+4/7*a^6*x^7-2/3*x^6*a^5-2*a^4*x^5-x^4*a^3+4/3*
x^3*a^2+2*a*x^2+x)
```

maxima [A] time = 0.30, size = 92, normalized size = 1.77

$$\frac{1}{9}a^8c^4x^9 + \frac{1}{2}a^7c^4x^8 + \frac{4}{7}a^6c^4x^7 - \frac{2}{3}a^5c^4x^6 - 2a^4c^4x^5 - a^3c^4x^4 + \frac{4}{3}a^2c^4x^3 + 2ac^4x^2 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

mupad [B] time = 0.05, size = 92, normalized size = 1.77

$$\frac{a^8\,c^4\,x^9}{9} + \frac{a^7\,c^4\,x^8}{2} + \frac{4\,a^6\,c^4\,x^7}{7} - \frac{2\,a^5\,c^4\,x^6}{3} - 2\,a^4\,c^4\,x^5 - a^3\,c^4\,x^4 + \frac{4\,a^2\,c^4\,x^3}{3} + 2\,a\,c^4\,x^2 + c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^4*x + 2*a*c^4*x^2 + (4*a^2*c^4*x^3)/3 - a^3*c^4*x^4 - 2*a^4*c^4*x^5 - (2*
a^5*c^4*x^6)/3 + (4*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/2 + (a^8*c^4*x^9)/9
```

sympy [**B**] time = 0.11, size = 100, normalized size = 1.92

$$\frac{a^8c^4x^9}{9} + \frac{a^7c^4x^8}{2} + \frac{4a^6c^4x^7}{7} - \frac{2a^5c^4x^6}{3} - 2a^4c^4x^5 - a^3c^4x^4 + \frac{4a^2c^4x^3}{3} + 2ac^4x^2 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/2 + 4*a**6*c**4*x**7/7 - 2*a**5*c**4*x**6
/3 - 2*a**4*c**4*x**5 - a**3*c**4*x**4 + 4*a**2*c**4*x**3/3 + 2*a*c**4*x**2
 + c**4*x
```

3.583 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=35

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

```
[Out] 1/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/(7*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx&=\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx\\&=c^3\int(1-ax)(1+ax)^5\,dx\\&=c^3\int\left(2(1+ax)^5-(1+ax)^6\right)\,dx\\&=\frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.66

$$-\frac{c^3(ax+1)^6(3ax-4)}{21a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/21*(c^3*(1 + a*x)^6*(-4 + 3*a*x))/a
```

fricas [A] time = 0.42, size = 59, normalized size = 1.69

$$-\frac{1}{7}a^6c^3x^7 - \frac{2}{3}a^5c^3x^6 - a^4c^3x^5 + \frac{5}{3}a^2c^3x^3 + 2ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

giac [B] time = 0.13, size = 78, normalized size = 2.23

$$-\frac{\left(3c^3 + \frac{35c^3}{ax-1} + \frac{168c^3}{(ax-1)^2} + \frac{420c^3}{(ax-1)^3} + \frac{560c^3}{(ax-1)^4} + \frac{336c^3}{(ax-1)^5}\right)(ax-1)^7}{21a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/21*(3*c^3 + 35*c^3/(a*x - 1) + 168*c^3/(a*x - 1)^2 + 420*c^3/(a*x - 1)^3
 + 560*c^3/(a*x - 1)^4 + 336*c^3/(a*x - 1)^5)*(a*x - 1)^7/a
```

maple [A] time = 0.03, size = 45, normalized size = 1.29

$$c^3\left(-\frac{1}{7}a^6x^7 - \frac{2}{3}x^6a^5 - a^4x^5 + \frac{5}{3}x^3a^2 + 2ax^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x)
```

```
[Out] c^3*(-1/7*a^6*x^7-2/3*x^6*a^5-a^4*x^5+5/3*x^3*a^2+2*a*x^2+x)
```

maxima [A] time = 0.30, size = 59, normalized size = 1.69

$$-\frac{1}{7}a^6c^3x^7 - \frac{2}{3}a^5c^3x^6 - a^4c^3x^5 + \frac{5}{3}a^2c^3x^3 + 2ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

mupad [B] time = 0.03, size = 59, normalized size = 1.69

$$-\frac{a^6c^3x^7}{7} - \frac{2a^5c^3x^6}{3} - a^4c^3x^5 + \frac{5a^2c^3x^3}{3} + 2ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^3*x + 2*a*c^3*x^2 + (5*a^2*c^3*x^3)/3 - a^4*c^3*x^5 - (2*a^5*c^3*x^6)/3 -
 (a^6*c^3*x^7)/7
```

sympy [B] time = 0.09, size = 63, normalized size = 1.80

$$-\frac{a^6c^3x^7}{7} - \frac{2a^5c^3x^6}{3} - a^4c^3x^5 + \frac{5a^2c^3x^3}{3} + 2ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**6*c**3*x**7/7 - 2*a**5*c**3*x**6/3 - a**4*c**3*x**5 + 5*a**2*c**3*x**3/
3 + 2*a*c**3*x**2 + c**3*x
```

3.584 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(ax+1)^5}{5a}$$

```
[Out] 1/5*c^2*(a*x+1)^5/a
```

Rubi [A] time = 0.06, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 32}

$$\frac{c^2(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(1 + a*x)^5)/(5*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx\\ &= c^2\int(1+ax)^4\,dx\\ &= \frac{c^2(1+ax)^5}{5a}\end{aligned}$$

Mathematica [B] time = 0.02, size = 37, normalized size = 2.18

$$c^2\left(\frac{a^4x^5}{5}+a^3x^4+2a^2x^3+2ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] c^2*(x + 2*a*x^2 + 2*a^2*x^3 + a^3*x^4 + (a^4*x^5)/5)
```

fricas [B] time = 0.44, size = 47, normalized size = 2.76

$$\frac{1}{5}\,a^{4}c^{2}x^{5}+a^{3}c^{2}x^{4}+2\,a^{2}c^{2}x^{3}+2\,ac^{2}x^{2}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

giac [B] time = 0.14, size = 64, normalized size = 3.76

$$\frac{\left(c^{2}+\frac{10\,c^{2}}{ax-1}+\frac{40\,c^{2}}{(ax-1)^{2}}+\frac{80\,c^{2}}{(ax-1)^{3}}+\frac{80\,c^{2}}{(ax-1)^{4}}\right)(ax-1)^{5}}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] 1/5*(c^2 + 10*c^2/(a*x - 1) + 40*c^2/(a*x - 1)^2 + 80*c^2/(a*x - 1)^3 + 80*
c^2/(a*x - 1)^4)*(a*x - 1)^5/a
```

maple [A] time = 0.03, size = 16, normalized size = 0.94

$$\frac{c^{2}\left(ax+1\right)^{5}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x)
[Out] 1/5*c^2*(a*x+1)^5/a
```

maxima [B] time = 0.30, size = 47, normalized size = 2.76

$$\frac{1}{5}\,a^{4}c^{2}x^{5}+a^{3}c^{2}x^{4}+2\,a^{2}c^{2}x^{3}+2\,ac^{2}x^{2}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

mupad [B] time = 0.03, size = 47, normalized size = 2.76

$$\frac{a^{4}\,c^{2}\,x^{5}}{5}+a^{3}\,c^{2}\,x^{4}+2\,a^{2}\,c^{2}\,x^{3}+2\,a\,c^{2}\,x^{2}+c^{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] c^2*x + 2*a*c^2*x^2 + 2*a^2*c^2*x^3 + a^3*c^2*x^4 + (a^4*c^2*x^5)/5
```

sympy [B] time = 0.09, size = 48, normalized size = 2.82

$$\frac{a^{4}c^{2}x^{5}}{5}+a^{3}c^{2}x^{4}+2a^{2}c^{2}x^{3}+2ac^{2}x^{2}+c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**2,x)
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4 + 2*a**2*c**2*x**3 + 2*a*c**2*x**2 + c**2
*x
```

3.585 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=46

$$-\frac{c(ax+1)^3}{3a}-\frac{c(ax+1)^2}{a}-\frac{8c\log(1-ax)}{a}-4cx$$

```
[Out] -4*c*x-c*(a*x+1)^2/a-1/3*c*(a*x+1)^3/a-8*c*ln(-a*x+1)/a
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6167, 6140, 43}

$$-\frac{c(ax+1)^3}{3a}-\frac{c(ax+1)^2}{a}-\frac{8c\log(1-ax)}{a}-4cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -4*c*x - (c*(1 + a*x)^2)/a - (c*(1 + a*x)^3)/(3*a) - (8*c*Log[1 - a*x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx\\
&= c\int \frac{(1+ax)^3}{1-ax}\,dx\\
&= c\int \left(-4+\frac{8}{1-ax}-2(1+ax)-(1+ax)^2\right)dx\\
&= -4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.78

$$-\frac{1}{3}a^2cx^3-2acx^2-\frac{8c\log(1-ax)}{a}-7cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
[Out] -7*c*x - 2*a*c*x^2 - (a^2*c*x^3)/3 - (8*c*Log[1 - a*x])/a
```

fricas [A] time = 0.64, size = 37, normalized size = 0.80

$$-\frac{a^3cx^3 + 6\,a^2cx^2 + 21\,acx + 24\,c\log(ax-1)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/3*(a^3*c*x^3 + 6*a^2*c*x^2 + 21*a*c*x + 24*c*log(a*x - 1))/a
```

giac [A] time = 0.14, size = 60, normalized size = 1.30

$$-\frac{(ax-1)^3\left(c + \frac{9\,c}{ax-1} + \frac{36\,c}{(ax-1)^2}\right)}{3\,a} + \frac{8\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] -1/3*(a*x - 1)^3*(c + 9*c/(a*x - 1) + 36*c/(a*x - 1)^2)/a + 8*c*log(abs(a*x
 - 1)/((a*x - 1)^2*abs(a)))/a
```

maple [A] time = 0.04, size = 34, normalized size = 0.74

$$-\frac{a^2c\,x^3}{3} - 2ac\,x^2 - 7cx - \frac{8c\ln(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x)
[Out] -1/3*a^2*c*x^3-2*a*c*x^2-7*c*x-8*c/a*ln(a*x-1)
```

maxima [A] time = 0.30, size = 33, normalized size = 0.72

$$-\frac{1}{3}\,a^2cx^3 - 2\,acx^2 - 7\,cx - \frac{8\,c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 - 2*a*c*x^2 - 7*c*x - 8*c*log(a*x - 1)/a
```

mupad [B] time = 0.05, size = 33, normalized size = 0.72

$$-7\,c\,x - \frac{a^2\,c\,x^3}{3} - \frac{8\,c\,\ln(a\,x-1)}{a} - 2\,a\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] - 7*c*x - (a^2*c*x^3)/3 - (8*c*log(a*x - 1))/a - 2*a*c*x^2
```

sympy [A] time = 0.17, size = 36, normalized size = 0.78

$$-\frac{a^2cx^3}{3} - 2acx^2 - 7cx - \frac{8c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c),x)
[Out] -a**2*c*x**3/3 - 2*a*c*x**2 - 7*c*x - 8*c*log(a*x - 1)/a
```

3.586 $\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{x}{c(1-ax)^2}$$

[Out] x/c/(-a*x+1)^2

Rubi [A] time = 0.06, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 34}

$$\frac{x}{c(1-ax)^2}$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]

[Out] x/(c*(1 - a*x)^2)

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx\\ &= \frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c}\\ &= \frac{x}{c(1-ax)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.92

$$\frac{(ax+1)^2}{4ac(1-ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]

[Out] (1 + a*x)^2/(4*a*c*(1 - a*x)^2)

fricas [**A**] time = 1.01, size = 19, normalized size = 1.46

$$\frac{x}{a^2cx^2 - 2\,acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

giac [**B**] time = 0.14, size = 27, normalized size = 2.08

$$\frac{\frac{1}{(ax-1)a} + \frac{1}{(ax-1)^2a}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] (1/((a*x - 1)*a) + 1/((a*x - 1)^2*a))/c
```

maple [**B**] time = 0.04, size = 28, normalized size = 2.15

$$\frac{\frac{1}{a(ax-1)^2} + \frac{1}{a(ax-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x)
[Out] 1/c*(1/a/(a*x-1)^2+1/a/(a*x-1))
```

maxima [**A**] time = 0.31, size = 19, normalized size = 1.46

$$\frac{x}{a^2cx^2 - 2\,acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

mupad [**B**] time = 0.05, size = 12, normalized size = 0.92

$$\frac{x}{c\,(a\,x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)*(a*x - 1)^2),x)
[Out] x/(c*(a*x - 1)^2)
```

sympy [**B**] time = 0.18, size = 17, normalized size = 1.31

$$\frac{x}{a^2cx^2 - 2acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c),x)
[Out] x/(a**2*c*x**2 - 2*a*c*x + c)
```

3.587 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3ac^2(1-ax)^3}$$

```
[Out] 1/3/a/c^2/(-a*x+1)^3
```

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 32}

$$\frac{1}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] 1/(3*a*c^2*(1 - a*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx\\ &= \frac{\int \frac{1}{(1-ax)^4}\,dx}{c^2}\\ &= \frac{1}{3ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 0.94

$$-\frac{1}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/3*1/(a*c^2*(-1 + a*x)^3)
```

fricas [B] time = 0.69, size = 41, normalized size = 2.28

$$-\frac{1}{3\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [A] time = 0.12, size = 15, normalized size = 0.83

$$-\frac{1}{3\,(ax-1)^{3}ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] -1/3/((a*x - 1)^3*a*c^2)
```

maple [A] time = 0.03, size = 16, normalized size = 0.89

$$-\frac{1}{3c^{2}a\,(ax-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x)
[Out] -1/3/c^2/a/(a*x-1)^3
```

maxima [B] time = 0.30, size = 41, normalized size = 2.28

$$-\frac{1}{3\left(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

mupad [B] time = 0.06, size = 40, normalized size = 2.22

$$\frac{1}{-3\,a^{4}\,c^{2}\,x^{3}+9\,a^{3}\,c^{2}\,x^{2}-9\,a^{2}\,c^{2}\,x+3\,a\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^2*(a*x - 1)^2),x)
[Out] 1/(3*a*c^2 - 9*a^2*c^2*x + 9*a^3*c^2*x^2 - 3*a^4*c^2*x^3)
```

sympy [B] time = 0.23, size = 42, normalized size = 2.33

$$-\frac{1}{3a^{4}c^{2}x^{3}-9a^{3}c^{2}x^{2}+9a^{2}c^{2}x-3ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**2,x)
[Out] -1/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3*a*c**2)
```

3.588 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=87

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

```
[Out] 1/8/a/c^3/(-a*x+1)^4+1/12/a/c^3/(-a*x+1)^3+1/16/a/c^3/(-a*x+1)^2+1/16/a/c^3
/(-a*x+1)+1/16*arctanh(a*x)/a/c^3
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(8*a*c^3*(1 - a*x)^4) + 1/(12*a*c^3*(1 - a*x)^3) + 1/(16*a*c^3*(1 - a*x)^
2) + 1/(16*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(16*a*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$$
$$= \frac{\int \frac{1}{(1-ax)^5(1+ax)}\,dx}{c^3}$$
$$= \frac{\int \left(-\frac{1}{2(-1+ax)^5}+\frac{1}{4(-1+ax)^4}-\frac{1}{8(-1+ax)^3}+\frac{1}{16(-1+ax)^2}-\frac{1}{16\left(-1+a^2x^2\right)}\right)dx}{c^3}$$
$$= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{16c^3}$$
$$= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.60

$$\frac{-3a^3x^3+12a^2x^2-19ax+3(ax-1)^4\tanh^{-1}(ax)+16}{48ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (16 - 19*a*x + 12*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^4*ArcTanh[a*x])/(48*a*
c^3*(-1 + a*x)^4)
```

fricas [B] time = 0.56, size = 147, normalized size = 1.69

$$-\frac{6\,a^3x^3-24\,a^2x^2+38\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax+1\right)+3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,a\right.}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(6*a^3*x^3 - 24*a^2*x^2 + 38*a*x - 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2
 - 4*a*x + 1)*log(a*x + 1) + 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1
)*log(a*x - 1) - 32)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c
^3*x + a*c^3)
```

giac [A] time = 0.14, size = 91, normalized size = 1.05

$$\frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3}-\frac{\frac{3\,a^3c^9}{ax-1}-\frac{3\,a^3c^9}{(ax-1)^2}+\frac{4\,a^3c^9}{(ax-1)^3}-\frac{6\,a^3c^9}{(ax-1)^4}}{48\,a^4c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(3*a^3*c^9/(a*x - 1) - 3*a^3
*c^9/(a*x - 1)^2 + 4*a^3*c^9/(a*x - 1)^3 - 6*a^3*c^9/(a*x - 1)^4)/(a^4*c^12
)
```

maple [A] time = 0.04, size = 90, normalized size = 1.03

$$\frac{1}{8c^3a\left(ax-1\right)^4}-\frac{1}{12c^3a\left(ax-1\right)^3}+\frac{1}{16c^3a\left(ax-1\right)^2}-\frac{1}{16a\,c^3\left(ax-1\right)}-\frac{\ln\left(ax-1\right)}{32c^3a}+\frac{\ln\left(ax+1\right)}{32a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/8/c^3/a/(a*x-1)^4-1/12/c^3/a/(a*x-1)^3+1/16/c^3/a/(a*x-1)^2-1/16/a/c^3/(a
*x-1)-1/32/c^3/a*ln(a*x-1)+1/32*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.30, size = 102, normalized size = 1.17

$$-\frac{3\,a^3x^3-12\,a^2x^2+19\,ax-16}{48\,\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}+\frac{\log\left(ax+1\right)}{32\,ac^3}-\frac{\log\left(ax-1\right)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 +
 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + 1/32*log(a*x + 1)/(a*c^3) - 1/32*lo
g(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.09, size = 83, normalized size = 0.95

$$\frac{\mathrm{atanh}\left(a\,x\right)}{16\,a\,c^3}-\frac{\frac{19\,x}{48}-\frac{a\,x^2}{4}-\frac{1}{3\,a}+\frac{a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^3*(a*x - 1)^2),x)
```

```
[Out] atanh(a*x)/(16*a*c^3) - ((19*x)/48 - (a*x^2)/4 - 1/(3*a) + (a^2*x^3)/16)/(c
^3 + 6*a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x)
```

sympy [A] time = 0.50, size = 99, normalized size = 1.14

$$-\frac{3a^3x^3-12a^2x^2+19ax-16}{48a^5c^3x^4-192a^4c^3x^3+288a^3c^3x^2-192a^2c^3x+48ac^3}-\frac{\frac{\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 - 12*a**2*x**2 + 19*a*x - 16)/(48*a**5*c**3*x**4 - 192*a**4*c
**3*x**3 + 288*a**3*c**3*x**2 - 192*a**2*c**3*x + 48*a*c**3) - (log(x - 1/a
)/32 - log(x + 1/a)/32)/(a*c**3)
```

3.589 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=122

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax}{32ac^4}$$

```
[Out] 1/20/a/c^4/(-a*x+1)^5+1/16/a/c^4/(-a*x+1)^4+1/16/a/c^4/(-a*x+1)^3+1/16/a/c^
4/(-a*x+1)^2+5/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+3/32*arctanh(a*x)/a/c^4
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax}{32ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] 1/(20*a*c^4*(1 - a*x)^5) + 1/(16*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)
^3) + 1/(16*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 +
a*x)) + (3*ArcTanh[a*x])/(32*a*c^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$$
$$= \frac{\int \frac{1}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$
$$= \frac{\int \left(\frac{1}{4(-1+ax)^6}-\frac{1}{4(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{1}{8(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{64(1+ax)^2}-\frac{3}{32\left(-1+a^2x^2\right)}\right)dx}{c^4}$$
$$= \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{}{64a}$$
$$= \frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{}{64a}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.66

$$\frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax+15(ax-1)^5(ax+1)\tanh^{-1}(ax)-48}{160ac^4(ax-1)^5(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-48 + 47*a*x + 20*a^2*x^2 - 80*a^3*x^3 + 60*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^5*(1 + a*x)*ArcTanh[a*x])/(160*a*c^4*(-1 + a*x)^5*(1 + a*x))
```

fricas [A] time = 0.58, size = 191, normalized size = 1.57

$$-\frac{30\,a^5x^5-120\,a^4x^4+160\,a^3x^3-40\,a^2x^2-94\,ax-15\left(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1\right)\log\left(ax+1\right)+}{320\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/320*(30*a^5*x^5 - 120*a^4*x^4 + 160*a^3*x^3 - 40*a^2*x^2 - 94*a*x - 15*(
a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 15*
(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) + 96
)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*
x - a*c^4)
```

giac [A] time = 0.14, size = 127, normalized size = 1.04

$$\frac{3\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4}+\frac{1}{128\,ac^4\left(\frac{2}{ax-1}+1\right)}-\frac{\frac{25\,a^9c^{16}}{ax-1}-\frac{20\,a^9c^{16}}{(ax-1)^2}+\frac{20\,a^9c^{16}}{(ax-1)^3}-\frac{20\,a^9c^{16}}{(ax-1)^4}+\frac{16\,a^9c^{16}}{(ax-1)^5}}{320\,a^{10}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 3/64*log(abs(-2/(a*x - 1) - 1))/(a*c^4) + 1/128/(a*c^4*(2/(a*x - 1) + 1)) -
 1/320*(25*a^9*c^16/(a*x - 1) - 20*a^9*c^16/(a*x - 1)^2 + 20*a^9*c^16/(a*x
- 1)^3 - 20*a^9*c^16/(a*x - 1)^4 + 16*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

maple [A] time = 0.04, size = 120, normalized size = 0.98

$$-\frac{1}{20c^4a\,(ax-1)^5}+\frac{1}{16c^4a\,(ax-1)^4}-\frac{1}{16c^4a\,(ax-1)^3}+\frac{1}{16c^4a\,(ax-1)^2}-\frac{5}{64a\,c^4\,(ax-1)}-\frac{3\ln(ax-1)}{64c^4a}-\frac{1}{64a\,c^4\,(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/20/c^4/a/(a*x-1)^5+1/16/c^4/a/(a*x-1)^4-1/16/c^4/a/(a*x-1)^3+1/16/c^4/a/
(a*x-1)^2-5/64/a/c^4/(a*x-1)-3/64/c^4/a*ln(a*x-1)-1/64/a/c^4/(a*x+1)+3/64*l
n(a*x+1)/a/c^4
```

maxima [A] time = 0.32, size = 130, normalized size = 1.07

$$-\frac{15\,a^5x^5-60\,a^4x^4+80\,a^3x^3-20\,a^2x^2-47\,ax+48}{160\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}+\frac{3\,\log(ax+1)}{64\,ac^4}-\frac{3\,\log(ax-1)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/160*(15*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/(a
^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x -
a*c^4) + 3/64*log(a*x + 1)/(a*c^4) - 3/64*log(a*x - 1)/(a*c^4)
```

mupad [B] time = 1.30, size = 111, normalized size = 0.91

$$\frac{3\,\mathrm{atanh}\left(a\,x\right)}{32\,a\,c^4}-\frac{\frac{47\,x}{160}+\frac{a\,x^2}{8}-\frac{3}{10\,a}-\frac{a^2\,x^3}{2}+\frac{3\,a^3\,x^4}{8}-\frac{3\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^4*(a*x - 1)^2),x)
```

```
[Out] (3*atanh(a*x))/(32*a*c^4) - ((47*x)/160 + (a*x^2)/8 - 3/(10*a) - (a^2*x^3)/
2 + (3*a^3*x^4)/8 - (3*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 +
4*a^5*c^4*x^5 - a^6*c^4*x^6 - 4*a*c^4*x)
```

sympy [A] time = 0.60, size = 129, normalized size = 1.06

$$\frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax-48}{160a^7c^4x^6-640a^6c^4x^5+800a^5c^4x^4-800a^3c^4x^2+640a^2c^4x-160ac^4}+\frac{-\frac{3\log\left(x-\frac{1}{a}\right)}{64}+\frac{3\log\left(x+\frac{1}{a}\right)}{64}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (-15*a**5*x**5 + 60*a**4*x**4 - 80*a**3*x**3 + 20*a**2*x**2 + 47*a*x - 48)/
(160*a**7*c**4*x**6 - 640*a**6*c**4*x**5 + 800*a**5*c**4*x**4 - 800*a**3*c*
*4*x**2 + 640*a**2*c**4*x - 160*a*c**4) + (-3*log(x - 1/a)/64 + 3*log(x + 1
/a)/64)/(a*c**4)
```

3.590 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=393

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{8}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{8}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{5}{48}a^5c^4x^6\Big(1$$

```
[Out] -5/48*a^5*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x^6+1/8*a^6*c^4*(1-1/a/x)^(5/
2)*(1+1/a/x)^(9/2)*x^7-1/8*a^7*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(9/2)*x^8+1/9*
a^8*c^4*(1-1/a/x)^(9/2)*(1+1/a/x)^(9/2)*x^9-35/128*c^4*arctanh((1-1/a/x)^(1
/2)*(1+1/a/x)^(1/2))/a-35/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-7/1
92*a^2*c^4*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-1/64*a^3*c^4*(1+1/a/x)^(7/2)
*x^4*(1-1/a/x)^(1/2)+1/16*a^4*c^4*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)-35/12
8*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{8}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{8}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{5}{48}a^5c^4x^6\Big(1$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```

```
[Out] (-35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (35*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/
64 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/16 - (5*a^5*c^4*(1
 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/48 + (a^6*c^4*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(9/2)*x^7)/8 - (a^7*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
9/2)*x^8)/8 + (a^8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(9/2)*x^9)/9 - (35
*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4dx=\left(a^8c^4\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}x^9+\left(a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^9}\,dx,x,\frac{1}{x}\right.$$

$$=-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}x^9-\frac{1}{8}\left(7a^6c^4\right.$$

$$=\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8+\frac{1}{9}a^8c^4\left(1-\right.$$

$$=-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7-\frac{1}{8}a^7c^4\left(\right.$$

$$=\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\right.$$

$$=-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\right.$$

$$=-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{16}a^4c^4\sqrt{\cdots}$$

$$=-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{64}a^3c^4\sqrt{\cdots}$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\cdots}$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\cdots}$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\cdots}$$

Mathematica [A] time = 0.20, size = 111, normalized size = 0.28

$$\frac{c^4\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(128a^8x^8-144a^7x^7-512a^6x^6+600a^5x^5+768a^4x^4-978a^3x^3-512a^2x^2+837ax+128\right)-315\cdots\right.}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 + 837*a*x - 512*a^2*x^2 - 978*a^3*x^3
+ 768*a^4*x^4 + 600*a^5*x^5 - 512*a^6*x^6 - 144*a^7*x^7 + 128*a^8*x^8) - 31
5*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

fricas [A] time = 0.57, size = 170, normalized size = 0.43

$$-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(128\,a^9c^4x^9-16\,a^8c^4x^8-656\,a^7c^4x^7+88\,a^6c^4x^6+1368\right.}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) - (128*a^9*c^4*x^9 - 16*a^8*c^4*x^8 - 656*a^7*c^4*x^7
 + 88*a^6*c^4*x^6 + 1368*a^5*c^4*x^5 - 210*a^4*c^4*x^4 - 1490*a^3*c^4*x^3 +
 325*a^2*c^4*x^2 + 965*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.18, size = 196, normalized size = 0.50

$$\frac{35\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{128\,|a|}+\frac{1}{1152}\sqrt{a^2x^2-1}\left(\frac{128\,c^4\operatorname{sgn}(ax+1)}{a}+\left(837\,c^4\operatorname{sgn}(ax+1)-2\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 35/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/
1152*sqrt(a^2*x^2 - 1)*(128*c^4*sgn(a*x + 1)/a + (837*c^4*sgn(a*x + 1) - 2*
(256*a*c^4*sgn(a*x + 1) + (489*a^2*c^4*sgn(a*x + 1) - 4*(96*a^3*c^4*sgn(a*x
 + 1) + (75*a^4*c^4*sgn(a*x + 1) - 2*(32*a^5*c^4*sgn(a*x + 1) - (8*a^7*c^4*
x*sgn(a*x + 1) - 9*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)
```

maple [A] time = 0.07, size = 279, normalized size = 0.71

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^4\left(-128\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^6a^6+144\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^5a^5+384\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4-45\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/1152*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4/a*(-128*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2)*x^6*a^6+144*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5+384*(a^2*x^2-1)^(3
/2)*(a^2)^(1/2)*x^4*a^4-456*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-384*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+522*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+384*
((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-256*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-315*(
a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*a)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.33, size = 415, normalized size = 1.06

$$-\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10458\,c^4\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax-1)^2a}{(ax+1)^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) - 2
730*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(13/
2) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 32768*c^4*((a*x - 1)/(a*x + 1
))^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)/(a*
x + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a*x -
1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x + 1)^2
+ 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 + 126*(a
*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*x - 1)^7
*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2/(a*x + 1
)^9 - a^2))*a
```

mupad [B] time = 1.34, size = 362, normalized size = 0.92

$$\frac{\frac{35\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{455\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96}+\frac{581\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32}-\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{32}+\frac{512\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{9}+\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{32}-\frac{581\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{32}+\frac{455}{}}{a-\frac{9\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{36\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{84\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{126\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{126\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{84\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{36\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}+\frac{9\,a\,\left(a\,x-\right.}{\left(a\,x+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((35*c^4*((a*x - 1)/(a*x + 1))^(1/2))/64 - (455*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/96 + (581*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 - (1289*c^4*((a*x - 1)/
(a*x + 1))^(7/2))/32 + (512*c^4*((a*x - 1)/(a*x + 1))^(9/2))/9 + (1289*c^4*
((a*x - 1)/(a*x + 1))^(11/2))/32 - (581*c^4*((a*x - 1)/(a*x + 1))^(13/2))/3
2 + (455*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 - (35*c^4*((a*x - 1)/(a*x + 1
))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a*x + 1
)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^4 - (1
26*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*a*(a*x
 - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9)/(a*x
 + 1)^9) - (35*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int 6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**6*x**6
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**8*x**8*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.591 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=313

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{6}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{6}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{8}a^3c^3x^4$$

```
[Out] -1/6*a^4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5+1/6*a^5*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(7/2)*x^6-1/7*a^6*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)*x^7-5/16*
c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-5/48*a*c^3*(1+1/a/x)^(3/2)*x
^2*(1-1/a/x)^(1/2)-1/24*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+1/8*a^3
*c^3*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)-5/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{6}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{6}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{8}a^3c^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

```
[Out] (-5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (5*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/48 - (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/8 - (a^
4*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/6 + (a^5*c^3*(1 - 1/(a*x
))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/6 - (a^6*c^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(
a*x))^(7/2)*x^7)/7 - (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(
16*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
```

```
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\left(\left(a^6c^3\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$=\left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\left(a^5c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^7}\,dx,x,\right.$$

$$=\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7+\frac{1}{6}\left(5a^4c^3\right)$$

$$=-\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6-\frac{1}{7}a^6c^3\left(1\right.$$

$$=\frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{6}a^5c^3\left(1-\right.$$

$$=-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{6}a^4c^3\left(1-\right.$$

$$=-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{8}a^3c^3\sqrt{1-\right.$$

$$=-\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}$$

$$=-\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}$$

$$=-\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}$$

Mathematica [A] time = 0.20, size = 95, normalized size = 0.30

$$\frac{c^3\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-48a^6x^6+56a^5x^5+144a^4x^4-182a^3x^3-144a^2x^2+231ax+48\right)-105\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right.\right.}{336a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 231*a*x - 144*a^2*x^2 - 182*a^3*x^3 +
 144*a^4*x^4 + 56*a^5*x^5 - 48*a^6*x^6) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)
])*x]))/(336*a)
```

fricas [A] time = 0.58, size = 147, normalized size = 0.47

$$-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(48\,a^7c^3x^7-8\,a^6c^3x^6-200\,a^5c^3x^5+38\,a^4c^3x^4+326\,a^3c\right.}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (48*a^7*c^3*x^7 - 8*a^6*c^3*x^6 - 200*a^5*c^3*x^5 +
38*a^4*c^3*x^4 + 326*a^3*c^3*x^3 - 87*a^2*c^3*x^2 - 279*a*c^3*x - 48*c^3)*s
qrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.15, size = 161, normalized size = 0.51

$$\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{16\,|a|}+\frac{1}{336}\,\sqrt{a^2x^2-1}\left(\frac{48\,c^3\mathrm{sgn}\left(ax+1\right)}{a}+\left(231\,c^3\mathrm{sgn}\left(ax+1\right)-2\left(72\,a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 5/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/33
6*sqrt(a^2*x^2 - 1)*(48*c^3*sgn(a*x + 1)/a + (231*c^3*sgn(a*x + 1) - 2*(72*
a*c^3*sgn(a*x + 1) + (91*a^2*c^3*sgn(a*x + 1) - 4*(18*a^3*c^3*sgn(a*x + 1)
- (6*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

maple [A] time = 0.05, size = 231, normalized size = 0.74

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^3\left(48\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^4a^4-56\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^3a^3-96\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\;x^2a^2+126\left(a^2\right.\right.}{336a\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/336*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3/a*(48*(a^2*x^2-1)^(3/2)*(a^2)^(1
/2)*x^4*a^4-56*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-96*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*x^2*a^2+126*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-64*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)-105*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+112*((a*x-1)*(a*x+1))^
(3/2)*(a^2)^(1/2)+105*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*a)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.33, size = 337, normalized size = 1.08

$$-\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\,c^3\left(\frac{ax-}{ax-}\right.\right.}{\frac{7\,(ax-1)a^2}{ax+1}-\frac{21\,(ax-1)^2a^2}{(ax+1)^2}+\frac{35\,(ax-1)}{(ax+1)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 70
0*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2) +
 3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(5/2
) + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x - 1
)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/(
a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 -
a^2))*a
```

mupad [B] time = 0.11, size = 289, normalized size = 0.92

$$-\frac{\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6}-\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{24}+\frac{128\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{7}+\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{24}-\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{6}+\frac{5\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{8}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}-\frac{5\,c^3\,\mathrm{atan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - ((25*c^3*((a*x - 1)/(a*x + 1))^(3/2))/6 - (5*c^3*((a*x - 1)/(a*x + 1))^(1
/2))/8 - (283*c^3*((a*x - 1)/(a*x + 1))^(5/2))/24 + (128*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/7 + (283*c^3*((a*x - 1)/(a*x + 1))^(9/2))/24 - (25*c^3*((a*x
 - 1)/(a*x + 1))^(11/2))/6 + (5*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (
7*a*(a*x - 1))/(a*x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)
^3)/(a*x + 1)^3 + (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x
+ 1)^5 + (7*a*(a*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7) - (5*
c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int 3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(-3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6*s
qrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*
x + 1)), x))
```

3.592 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{4}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{4}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{8}ac^2x^2\sqrt{1-}$$

```
[Out] -1/4*a^3*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x^4+1/5*a^4*c^2*(1-1/a/x)^(5/2
)*(1+1/a/x)^(5/2)*x^5-3/8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-1/
8*a*c^2*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/4*a^2*c^2*(1+1/a/x)^(5/2)*x^3
*(1-1/a/x)^(1/2)-3/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{4}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{4}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{8}ac^2x^2\sqrt{1-}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```

```
[Out] (-3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (a*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
)*x^3)/4 - (a^3*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/4 + (a^4*c
^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/5 - (3*c^2*ArcTanh[Sqrt[1 -
 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimpl
erQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5+\left(a^3c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^5}\,dx,x,\frac{1}{x}\right.$$

$$=-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5-\frac{1}{4}\left(3a^2c^2\right)$$

$$=\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4+\frac{1}{5}a^4c^2\left(1-\right.$$

$$=-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right.$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\right.$$

Mathematica [A] time = 0.11, size = 79, normalized size = 0.34

$$\frac{c^2\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8\right)-15\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a
^4*x^4) - 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

fricas [A] time = 1.24, size = 126, normalized size = 0.54

$$-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(8\,a^5c^2x^5-2\,a^4c^2x^4-26\,a^3c^2x^3+9\,a^2c^2x^2+33\,ac^2x+8\,c^2\right)\sqrt{}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (8*a^5*c^2*x^5 - 2*a^4*c^2*x^4 - 26*a^3*c^2*x^3 + 9*a^2
*c^2*x^2 + 33*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.15, size = 126, normalized size = 0.54

$$\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{8\,|a|}+\frac{1}{40}\,\sqrt{a^2x^2-1}\left(\left(25\,c^2\operatorname{sgn}(ax+1)-2\left(8\,ac^2\operatorname{sgn}(ax+1)-\left(4\,a^3c^2\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/40*
sqrt(a^2*x^2 - 1)*((25*c^2*sgn(a*x + 1) - 2*(8*a*c^2*sgn(a*x + 1) - (4*a^3*
c^2*x*sgn(a*x + 1) - 5*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 8*c^2*sgn(a*x + 1)/a
)
```

maple [A] time = 0.05, size = 183, normalized size = 0.79

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^2\left(24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-30\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,xa+16\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}+45\sqrt{a^2x^2-1}\,\sqrt{a}\right.}{120a\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/120*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*x^2*a^2-30*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2)+45*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-40*((a*x-1)*(a*x+1))^(3/2)*(a^2)
^(1/2)-45*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)
*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.34, size = 259, normalized size = 1.11

$$-\frac{1}{40}\,a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+7\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)}{(ax+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^
2*((a*x - 1)/(a*x + 1))^(7/2) - 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^
2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x -
 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*
x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^
2))
```

mupad [B] time = 0.08, size = 214, normalized size = 0.92

$$\frac{\frac{3\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}+\frac{32\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{5}+\frac{7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{2}-\frac{3\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}-\frac{3\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((3*c^2*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^2*((a*x - 1)/(a*x + 1))^(3/2)
)/2 + (32*c^2*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^2*((a*x - 1)/(a*x + 1))
^(7/2))/2 - (3*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^2*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1)), x))
```

3.593 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=145

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\cdot}\right.}{2a}$$

```
[Out] -1/3*a^2*c*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x^3-1/2*c*arctanh((1-1/a/x)^(1/2
)*(1+1/a/x)^(1/2))/a+1/2*a*c*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-1/2*c*x*(1
-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\cdot}\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] -(c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (a*c*Sqrt[1 - 1/(a*x)]*(1 +
1/(a*x))^(3/2)*x^2)/2 - (a^2*c*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x^3)
/3 - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
```

```
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx=-\left(\left(a^2c\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$=\left(a^2c\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^4}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-(ac)\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}{x^3}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3+\frac{1}{2}c\,\text{Subst}\left(\int\frac{\sqrt{\ }}{x^2}\right.$$

$$=-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

$$=-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

$$=-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.$$

Mathematica [A] time = 0.10, size = 61, normalized size = 0.42

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-2a^2x^2+3ax+2\right)-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(6*a)
```

fricas [A] time = 0.46, size = 91, normalized size = 0.63

$$-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3cx^3-a^2cx^2-5\,acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (2*a^3*c*x^3 - a^2*c*x^2 - 5*a*c*x - 2*c)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

giac [A] time = 0.17, size = 82, normalized size = 0.57

$$\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2|a|}-\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(2\,acx\operatorname{sgn}(ax+1)-3\,c\operatorname{sgn}(ax+1)\right)x-\frac{2\,c\operatorname{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sqr
t(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 3*c*sgn(a*x + 1))*x - 2*c*sgn(a*x +
 1)/a)
```

maple [A] time = 0.05, size = 119, normalized size = 0.82

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c\left(3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa-2\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-
2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2))*a)/((a*x-1)*(a*x+1))^(1/2)/a/(a^2)^(1/2)
```

maxima [A] time = 0.31, size = 171, normalized size = 1.18

$$\frac{1}{6}a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) + 8*c*((a*x - 1)/(a*x + 1))^(3/2)
 - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^
2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 1.24, size = 132, normalized size = 0.91

$$-\frac{\frac{8\,c\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-c\sqrt{\frac{a\,x-1}{a\,x+1}}+c\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}}-\frac{c\,\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - ((8*c*((a*x - 1)/(a*x + 1))^(3/2))/3 - c*((a*x - 1)/(a*x + 1))^(1/2) + c*
((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1
)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c*atanh(((a*x - 1)/(a*x
+ 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c*(Integral(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sq
rt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

3.594 $\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

[Out] -1/a/c*((a*x-1)/(a*x+1))^(1/2)

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6183}

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]

[Out] -(1/(a*c*E^ArcCoth[a*x]))

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Mathematica [A] time = 0.05, size = 16, normalized size = 1.00

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]

[Out] -(1/(a*c*E^ArcCoth[a*x]))

fricas [A] time = 0.90, size = 23, normalized size = 1.44

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")

[Out] -sqrt((a*x - 1)/(a*x + 1))/(a*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.04, size = 24, normalized size = 1.50

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c*((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.31, size = 23, normalized size = 1.44

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

mupad [B] time = 0.03, size = 23, normalized size = 1.44

$$-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2),x)
```

```
[Out] -((a*x - 1)/(a*x + 1))^(1/2)/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

3.595 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}-\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}$$

```
[Out] -2/3/a/c^2*((a*x-1)/(a*x+1))^(1/2)+1/3*(2*a*x+1)/a/c^2*((a*x-1)/(a*x+1))^(1
/2)/(-a^2*x^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}-\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] -2/(3*a*c^2*E^ArcCoth[a*x]) + (1 + 2*a*x)/(3*a*c^2*E^ArcCoth[a*x]*(1 - a^2*
x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx &= \frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2(1-a^2x^2)}+\frac{2\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{3c}\\ &= -\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}+\frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2(1-a^2x^2)}\end{aligned}$$

Mathematica [A] time = 0.16, size = 48, normalized size = 0.87

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+2ax-1\right)}{3(ax-1)(acx+c)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x + 2*a^2*x^2))/((-1 + a*x)*(c + a*
c*x)^2)
```

fricas [A] time = 0.64, size = 50, normalized size = 0.91

$$-\frac{\left(2\,a^{2}x^{2}+2\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\,\left(a^{3}c^{2}x^{2}-ac^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a^2*x^2 + 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - a*c^2
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(a^{2}cx^{2}-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^2, x)
```

maple [A] time = 0.04, size = 49, normalized size = 0.89

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(2a^{2}x^{2}+2ax-1\right)}{3\left(a^{2}x^{2}-1\right)a\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/3*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2+2*a*x-1)/(a^2*x^2-1)/a/c^2
```

maxima [A] time = 0.32, size = 67, normalized size = 1.22

$$\frac{1}{12}\,a\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-6\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{2}}-\frac{3}{a^{2}c^{2}\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*((((a*x - 1)/(a*x + 1))^(3/2) - 6*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^
2) - 3/(a^2*c^2*sqrt((a*x - 1)/(a*x + 1))))
```

mupad [B] time = 0.05, size = 55, normalized size = 1.00

$$-\frac{\frac{6\,(a\,x-1)}{a\,x+1}-\frac{(a\,x-1)^{2}}{(a\,x+1)^{2}}+3}{12\,a\,c^{2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^2,x)
```

```
[Out] -((6*(a*x - 1))/(a*x + 1) - (a*x - 1)^2/(a*x + 1)^2 + 3)/(12*a*c^2*((a*x -
1)/(a*x + 1))^(1/2))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x**2 + 1), x
)/c**2
```

3.596 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=91

$$\frac{4(2ax+1)e^{-\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}$$

```
[Out] -8/15/a/c^3*((a*x-1)/(a*x+1))^(1/2)+1/15*(4*a*x+1)/a/c^3*((a*x-1)/(a*x+1))^
(1/2)/(-a^2*x^2+1)^2+4/15*(2*a*x+1)/a/c^3*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^2
+1)
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{4(2ax+1)e^{-\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3(1-a^2x^2)^2}-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -8/(15*a*c^3*E^ArcCoth[a*x]) + (1 + 4*a*x)/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^
2*x^2)^2) + (4*(1 + 2*a*x))/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx &= \frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3(1-a^2x^2)^2}+\frac{4\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{5c}\\
&= \frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3(1-a^2x^2)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3(1-a^2x^2)}+\frac{8\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{15c^2}\\
&= -\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}+\frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3(1-a^2x^2)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3(1-a^2x^2)}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 64, normalized size = 0.70

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3\right)}{15(ax-1)^2(acx+c)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(3 - 12*a*x - 12*a^2*x^2 + 8*a^3*x^3 + 8*a^4
*x^4))/((-1 + a*x)^2*(c + a*c*x)^3)
```

fricas [A] time = 0.51, size = 76, normalized size = 0.84

$$-\frac{\left(8\,a^4x^4+8\,a^3x^3-12\,a^2x^2-12\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(8*a^4*x^4 + 8*a^3*x^3 - 12*a^2*x^2 - 12*a*x + 3)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^3, x)
```

maple [A] time = 0.04, size = 65, normalized size = 0.71

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\left(8x^4a^4+8x^3a^3-12a^2x^2-12ax+3\right)}{15\left(a^2x^2-1\right)^2c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -1/15*((a*x-1)/(a*x+1))^(1/2)*(8*a^4*x^4+8*a^3*x^3-12*a^2*x^2-12*a*x+3)/(a^
2*x^2-1)^2/c^3/a
```

maxima [A] time = 0.31, size = 102, normalized size = 1.12

$$-\frac{1}{240}\,a\left(\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-20\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+90\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{5\left(\frac{12\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/240*a*((3*((a*x - 1)/(a*x + 1))^(5/2) - 20*((a*x - 1)/(a*x + 1))^(3/2) +
 90*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 5*(12*(a*x - 1)/(a*x + 1) - 1)/(
a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)))
```

mupad [B] time = 1.24, size = 109, normalized size = 1.20

$$\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12\,a\,c^3}-\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{80\,a\,c^3}-\frac{\frac{4\,(a\,x-1)}{a\,x+1}-\frac{1}{3}}{16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^3,x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^3) - (3*((a*x - 1)/(a*x + 1))^(1/2))/(8
*a*c^3) - ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) - ((4*(a*x - 1))/(a*x + 1)
 - 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x**4 + 3*a*
*2*x**2 - 1), x)/c**3
```

3.597 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=127

$$\frac{8(2ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)}+\frac{2(4ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^2}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}-\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}$$

```
[Out] -16/35/a/c^4*((a*x-1)/(a*x+1))^(1/2)+1/35*(6*a*x+1)/a/c^4*((a*x-1)/(a*x+1))
^(1/2)/(-a^2*x^2+1)^3+2/35*(4*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^
2+1)^2+8/35*(2*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^2+1)
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{8(2ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)}+\frac{2(4ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^2}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}-\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -16/(35*a*c^4*E^ArcCoth[a*x]) + (1 + 6*a*x)/(35*a*c^4*E^ArcCoth[a*x]*(1 - a
^2*x^2)^3) + (2*(1 + 4*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2)^2) + (8
*(1 + 2*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{6\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{7c}$$

$$= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{24\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{35c^2}$$

$$= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4(1-a^2x^2)} + \frac{16\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^3}$$

$$= -\frac{16e^{-\coth^{-1}(ax)}}{35ac^4} + \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4(1-a^2x^2)}$$

Mathematica [A] time = 0.29, size = 80, normalized size = 0.63

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5\right)}{35(ax-1)^3(acx+c)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + 30*a*x + 30*a^2*x^2 - 40*a^3*x^3 - 40*
a^4*x^4 + 16*a^5*x^5 + 16*a^6*x^6))/((-1 + a*x)^3*(c + a*c*x)^4)
```

fricas [A] time = 0.63, size = 104, normalized size = 0.82

$$-\frac{\left(16\,a^6x^6+16\,a^5x^5-40\,a^4x^4-40\,a^3x^3+30\,a^2x^2+30\,ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/35*(16*a^6*x^6 + 16*a^5*x^5 - 40*a^4*x^4 - 40*a^3*x^3 + 30*a^2*x^2 + 30*
a*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4
*x^2 - a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^4, x)
```

maple [A] time = 0.04, size = 81, normalized size = 0.64

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\left(16x^6a^6+16x^5a^5-40x^4a^4-40x^3a^3+30a^2x^2+30ax-5\right)}{35\left(a^2x^2-1\right)^3c^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/35*((a*x-1)/(a*x+1))^(1/2)*(16*a^6*x^6+16*a^5*x^5-40*a^4*x^4-40*a^3*x^3+
30*a^2*x^2+30*a*x-5)/(a^2*x^2-1)^3/c^4/a
```

maxima [A] time = 0.31, size = 135, normalized size = 1.06

$$\frac{1}{2240}a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+175\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-700\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{7\left(\frac{10(ax-1)}{ax+1}-\frac{75(ax-1)^2}{(ax+1)^2}-1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/2240*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 42*((a*x - 1)/(a*x + 1))^(5/2) +
 175*((a*x - 1)/(a*x + 1))^(3/2) - 700*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4)
 + 7*(10*(a*x - 1)/(a*x + 1) - 75*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^4*((a
*x - 1)/(a*x + 1))^(5/2)))
```

mupad [B] time = 0.04, size = 148, normalized size = 1.17

$$\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{64\,a\,c^4}-\frac{5\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{160\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{448\,a\,c^4}-\frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{a\,x+1}+\frac{1}{5}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^4,x)
```

```
[Out] (5*((a*x - 1)/(a*x + 1))^(3/2))/(64*a*c^4) - (5*((a*x - 1)/(a*x + 1))^(1/2)
)/(16*a*c^4) - (3*((a*x - 1)/(a*x + 1))^(5/2))/(160*a*c^4) + ((a*x - 1)/(a*
x + 1))^(7/2)/(448*a*c^4) - ((15*(a*x - 1)^2)/(a*x + 1)^2 - (2*(a*x - 1))/(
a*x + 1) + 1/5)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.598 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=73

$$-\frac{c^4(1-ax)^9}{9a}+\frac{3c^4(1-ax)^8}{4a}-\frac{12c^4(1-ax)^7}{7a}+\frac{4c^4(1-ax)^6}{3a}$$

```
[Out] 4/3*c^4*(-a*x+1)^6/a-12/7*c^4*(-a*x+1)^7/a+3/4*c^4*(-a*x+1)^8/a-1/9*c^4*(-a
*x+1)^9/a
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$-\frac{c^4(1-ax)^9}{9a}+\frac{3c^4(1-ax)^8}{4a}-\frac{12c^4(1-ax)^7}{7a}+\frac{4c^4(1-ax)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*c^4*(1 - a*x)^6)/(3*a) - (12*c^4*(1 - a*x)^7)/(7*a) + (3*c^4*(1 - a*x)^8
)/(4*a) - (c^4*(1 - a*x)^9)/(9*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx\\ &= -\left(c^4\int(1-ax)^5(1+ax)^3\,dx\right)\\ &= -\left(c^4\int\left(8(1-ax)^5-12(1-ax)^6+6(1-ax)^7-(1-ax)^8\right)dx\right)\\ &= \frac{4c^4(1-ax)^6}{3a}-\frac{12c^4(1-ax)^7}{7a}+\frac{3c^4(1-ax)^8}{4a}-\frac{c^4(1-ax)^9}{9a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.53

$$\frac{c^4(ax-1)^6\left(28a^3x^3+105a^2x^2+138ax+65\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(-1 + a*x)^6*(65 + 138*a*x + 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

fricas [A] time = 0.72, size = 80, normalized size = 1.10

$$\frac{1}{9}a^8c^4x^9 - \frac{1}{4}a^7c^4x^8 - \frac{2}{7}a^6c^4x^7 + a^5c^4x^6 - \frac{3}{2}a^3c^4x^4 + \frac{2}{3}a^2c^4x^3 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

giac [A] time = 0.14, size = 80, normalized size = 1.10

$$\frac{1}{9}a^8c^4x^9 - \frac{1}{4}a^7c^4x^8 - \frac{2}{7}a^6c^4x^7 + a^5c^4x^6 - \frac{3}{2}a^3c^4x^4 + \frac{2}{3}a^2c^4x^3 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

maple [A] time = 0.03, size = 61, normalized size = 0.84

$$c^4\left(\frac{1}{9}x^9a^8 - \frac{1}{4}a^7x^8 - \frac{2}{7}a^6x^7 + x^6a^5 - \frac{3}{2}x^4a^3 + \frac{2}{3}x^3a^2 + a\,x^2 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4/(a*x+1)*(a*x-1),x)
```

```
[Out] c^4*(1/9*x^9*a^8-1/4*a^7*x^8-2/7*a^6*x^7+x^6*a^5-3/2*x^4*a^3+2/3*x^3*a^2+a*
x^2-x)
```

maxima [A] time = 0.30, size = 80, normalized size = 1.10

$$\frac{1}{9}a^8c^4x^9 - \frac{1}{4}a^7c^4x^8 - \frac{2}{7}a^6c^4x^7 + a^5c^4x^6 - \frac{3}{2}a^3c^4x^4 + \frac{2}{3}a^2c^4x^3 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

mupad [B] time = 0.05, size = 80, normalized size = 1.10

$$\frac{a^8\,c^4\,x^9}{9} - \frac{a^7\,c^4\,x^8}{4} - \frac{2\,a^6\,c^4\,x^7}{7} + a^5\,c^4\,x^6 - \frac{3\,a^3\,c^4\,x^4}{2} + \frac{2\,a^2\,c^4\,x^3}{3} + a\,c^4\,x^2 - c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] a*c^4*x^2 - c^4*x + (2*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + a^5*c^4*x^6 - (
2*a^6*c^4*x^7)/7 - (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

sympy [A] time = 0.31, size = 87, normalized size = 1.19

$$\frac{a^8c^4x^9}{9} - \frac{a^7c^4x^8}{4} - \frac{2a^6c^4x^7}{7} + a^5c^4x^6 - \frac{3a^3c^4x^4}{2} + \frac{2a^2c^4x^3}{3} + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] a**8*c**4*x**9/9 - a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 + a**5*c**4*x**6 -
 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 + a*c**4*x**2 - c**4*x
```

3.599 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=55

$$\frac{c^3(1-ax)^7}{7a}-\frac{2c^3(1-ax)^6}{3a}+\frac{4c^3(1-ax)^5}{5a}$$

```
[Out] 4/5*c^3*(-a*x+1)^5/a-2/3*c^3*(-a*x+1)^6/a+1/7*c^3*(-a*x+1)^7/a
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^3(1-ax)^7}{7a}-\frac{2c^3(1-ax)^6}{3a}+\frac{4c^3(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*c^3*(1 - a*x)^5)/(5*a) - (2*c^3*(1 - a*x)^6)/(3*a) + (c^3*(1 - a*x)^7)/(
7*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx\\
&= -\left(c^3\int(1-ax)^4(1+ax)^2\,dx\right)\\
&= -\left(c^3\int\left(4(1-ax)^4-4(1-ax)^5+(1-ax)^6\right)dx\right)\\
&= \frac{4c^3(1-ax)^5}{5a}-\frac{2c^3(1-ax)^6}{3a}+\frac{c^3(1-ax)^7}{7a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.56

$$-\frac{c^3(ax-1)^5\left(15a^2x^2+40ax+29\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/105*(c^3*(-1 + a*x)^5*(29 + 40*a*x + 15*a^2*x^2))/a
```

fricas [A] time = 0.52, size = 70, normalized size = 1.27

$$-\frac{1}{7}a^6c^3x^7 + \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 - a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

giac [A] time = 0.13, size = 70, normalized size = 1.27

$$-\frac{1}{7}a^6c^3x^7 + \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 - a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

maple [A] time = 0.03, size = 54, normalized size = 0.98

$$c^3\left(-\frac{1}{7}a^6x^7 + \frac{1}{3}x^6a^5 + \frac{1}{5}a^4x^5 - x^4a^3 + \frac{1}{3}x^3a^2 + a\,x^2 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/(a*x+1)*(a*x-1),x)
```

```
[Out] c^3*(-1/7*a^6*x^7+1/3*x^6*a^5+1/5*a^4*x^5-x^4*a^3+1/3*x^3*a^2+a*x^2-x)
```

maxima [A] time = 0.31, size = 70, normalized size = 1.27

$$-\frac{1}{7}a^6c^3x^7 + \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 - a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

mupad [B] time = 0.04, size = 70, normalized size = 1.27

$$-\frac{a^6\,c^3\,x^7}{7} + \frac{a^5\,c^3\,x^6}{3} + \frac{a^4\,c^3\,x^5}{5} - a^3\,c^3\,x^4 + \frac{a^2\,c^3\,x^3}{3} + a\,c^3\,x^2 - c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] a*c^3*x^2 - c^3*x + (a^2*c^3*x^3)/3 - a^3*c^3*x^4 + (a^4*c^3*x^5)/5 + (a^5*
c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

sympy [A] time = 0.09, size = 70, normalized size = 1.27

$$-\frac{a^6c^3x^7}{7} + \frac{a^5c^3x^6}{3} + \frac{a^4c^3x^5}{5} - a^3c^3x^4 + \frac{a^2c^3x^3}{3} + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**6*c**3*x**7/7 + a**5*c**3*x**6/3 + a**4*c**3*x**5/5 - a**3*c**3*x**4 +
a**2*c**3*x**3/3 + a*c**3*x**2 - c**3*x
```

3.600 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=37

$$\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

```
[Out] 1/2*c^2*(-a*x+1)^4/a-1/5*c^2*(-a*x+1)^5/a
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 43}

$$\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(1 - a*x)^4)/(2*a) - (c^2*(1 - a*x)^5)/(5*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx\\
&= -\left(c^2\int (1-ax)^3(1+ax)\,dx\right)\\
&= -\left(c^2\int \left(2(1-ax)^3-(1-ax)^4\right)\,dx\right)\\
&= \frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.81

$$\frac{1}{10}c^2x\left(2a^4x^4-5a^3x^3+10ax-10\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

fricas [A] time = 0.59, size = 37, normalized size = 1.00

$$\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

giac [A] time = 0.14, size = 37, normalized size = 1.00

$$\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

maple [A] time = 0.04, size = 30, normalized size = 0.81

$$c^2\left(\frac{1}{5}a^4x^5 - \frac{1}{2}x^4a^3 + a\,x^2 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/(a*x+1)*(a*x-1),x)
[Out] c^2*(1/5*a^4*x^5-1/2*x^4*a^3+a*x^2-x)
```

maxima [A] time = 0.31, size = 37, normalized size = 1.00

$$\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

mupad [B] time = 0.05, size = 37, normalized size = 1.00

$$\frac{a^4\,c^2\,x^5}{5} - \frac{a^3\,c^2\,x^4}{2} + a\,c^2\,x^2 - c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x - 1))/(a*x + 1),x)
[Out] a*c^2*x^2 - c^2*x - (a^3*c^2*x^4)/2 + (a^4*c^2*x^5)/5
```

sympy [A] time = 0.08, size = 36, normalized size = 0.97

$$\frac{a^4c^2x^5}{5} - \frac{a^3c^2x^4}{2} + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*(a*x-1)/(a*x+1),x)
[Out] a**4*c**2*x**5/5 - a**3*c**2*x**4/2 + a*c**2*x**2 - c**2*x
```

3.601 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=16

$$\frac{c(1-ax)^3}{3a}$$

```
[Out] 1/3*c*(-a*x+1)^3/a
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6167, 6140, 32}

$$\frac{c(1-ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*(1 - a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)dx\\ &= -\left(c\int(1-ax)^2\,dx\right)\\ &= \frac{c(1-ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.31

$$-c\left(\frac{a^2x^3}{3}-ax^2+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(c*(x - a*x^2 + (a^2*x^3)/3))
```

fricas [A] time = 0.47, size = 20, normalized size = 1.25

$$-\frac{1}{3}a^2cx^3 + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

giac [A] time = 0.14, size = 20, normalized size = 1.25

$$-\frac{1}{3}a^2cx^3 + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

maple [A] time = 0.03, size = 14, normalized size = 0.88

$$-\frac{c(ax-1)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/(a*x+1)*(a*x-1),x)
[Out] -1/3*c*(a*x-1)^3/a
```

maxima [A] time = 0.30, size = 20, normalized size = 1.25

$$-\frac{1}{3}a^2cx^3 + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

mupad [B] time = 0.04, size = 17, normalized size = 1.06

$$-\frac{c\,x\,\left(a^2\,x^2-3\,a\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x - 1))/(a*x + 1),x)
[Out] -(c*x*(a^2*x^2 - 3*a*x + 3))/3
```

sympy [A] time = 0.07, size = 19, normalized size = 1.19

$$-\frac{a^2cx^3}{3} + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*(a*x-1)/(a*x+1),x)
[Out] -a**2*c*x**3/3 + a*c*x**2 - c*x
```

3.602 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2} dx$

Optimal. Leaf size=14

$$\frac{1}{ac(ax+1)}$$

[Out] 1/a/c/(a*x+1)

Rubi [A] time = 0.06, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6167, 6140, 32}

$$\frac{1}{ac(ax+1)}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]

[Out] 1/(a*c*(1 + a*x))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2} dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-a^2cx^2} dx\\ &= -\frac{\int \frac{1}{(1+ax)^2} dx}{c}\\ &= \frac{1}{ac(1+ax)}\end{aligned}$$

Mathematica [C] time = 0.02, size = 18, normalized size = 1.29

$$-\frac{e^{-2\coth^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]

[Out] -1/2*1/(a*c*E^(2*ArcCoth[a*x]))

fricas [A] time = 0.45, size = 12, normalized size = 0.86

$$\frac{1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/(a^2*c*x + a*c)
```

giac [A] time = 0.15, size = 14, normalized size = 1.00

$$\frac{1}{(ax+1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/((a*x + 1)*a*c)
```

maple [A] time = 0.03, size = 15, normalized size = 1.07

$$\frac{1}{ac\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c),x)
[Out] 1/a/c/(a*x+1)
```

maxima [A] time = 0.30, size = 12, normalized size = 0.86

$$\frac{1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/(a^2*c*x + a*c)
```

mupad [B] time = 0.05, size = 12, normalized size = 0.86

$$\frac{1}{a\,\left(c+a\,c\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)*(a*x + 1)),x)
[Out] 1/(a*(c + a*c*x))
```

sympy [A] time = 0.12, size = 10, normalized size = 0.71

$$\frac{1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c),x)
[Out] 1/(a**2*c*x + a*c)
```

3.603 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] 1/4/a/c^2/(a*x+1)^2+1/4/a/c^2/(a*x+1)-1/4*arctanh(a*x)/a/c^2
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$\frac{1}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] 1/(4*a*c^2*(1 + a*x)^2) + 1/(4*a*c^2*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)(1+ax)^3}\,dx}{c^2}$$
$$= -\frac{\int \left(\frac{1}{2(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^2}$$
$$= \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}$$
$$= \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.67

$$\frac{ax+(ax+1)^2\left(-\tanh^{-1}(ax)\right)+2}{4a(acx+c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] (2 + a*x - (1 + a*x)^2*ArcTanh[a*x])/(4*a*(c + a*c*x)^2)
```

fricas [A] time = 0.59, size = 76, normalized size = 1.55

$$\frac{2\,ax-\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2+2\,ax+1\right)\log\left(ax-1\right)+4}{8\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 + 2*a*x + 1)*log
(a*x - 1) + 4)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.13, size = 51, normalized size = 1.04

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}+\frac{ax+2}{4\,(ax+1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x +
 2)/((a*x + 1)^2*a*c^2)
```

maple [A] time = 0.04, size = 60, normalized size = 1.22

$$\frac{\ln\left(ax-1\right)}{8c^2a}+\frac{1}{4a\,c^2\,(ax+1)^2}+\frac{1}{4a\,c^2\,(ax+1)}-\frac{\ln\left(ax+1\right)}{8a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/8/c^2/a*ln(a*x-1)+1/4/a/c^2/(a*x+1)^2+1/4/a/c^2/(a*x+1)-1/8*ln(a*x+1)/a/c
^2
```

maxima [A] time = 0.30, size = 63, normalized size = 1.29

$$\frac{ax+2}{4\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^2
) + 1/8*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.07, size = 46, normalized size = 0.94

$$\frac{\frac{x}{4}+\frac{1}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2}-\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^2*(a*x + 1)),x)
```

```
[Out] (x/4 + 1/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

sympy [A] time = 0.28, size = 54, normalized size = 1.10

$$\frac{ax+2}{4a^3c^2x^2+8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (a*x + 2)/(4*a**3*c**2*x**2 + 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**2)
```

3.604 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{16ac^3(1-ax)}+\frac{3}{16ac^3(ax+1)}+\frac{1}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] -1/16/a/c^3/(-a*x+1)+1/12/a/c^3/(a*x+1)^3+1/8/a/c^3/(a*x+1)^2+3/16/a/c^3/(a
*x+1)-1/4*arctanh(a*x)/a/c^3
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{1}{16ac^3(1-ax)}+\frac{3}{16ac^3(ax+1)}+\frac{1}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/(16*a*c^3*(1 - a*x)) + 1/(12*a*c^3*(1 + a*x)^3) + 1/(8*a*c^3*(1 + a*x)^2
) + 3/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)^2(1+ax)^4}\,dx}{c^3}$$
$$= -\frac{\int \left(\frac{1}{16(-1+ax)^2}+\frac{1}{4(1+ax)^4}+\frac{1}{4(1+ax)^3}+\frac{3}{16(1+ax)^2}-\frac{1}{4\left(-1+a^2x^2\right)}\right)dx}{c^3}$$
$$= -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}$$
$$= -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.73

$$\frac{3a^3x^3+6a^2x^2+ax-3(ax-1)(ax+1)^3\tanh^{-1}(ax)-4}{12a(ax-1)(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (-4 + a*x + 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)*(1 + a*x)^3*ArcTanh[a*x])/
(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

fricas [A] time = 0.54, size = 121, normalized size = 1.44

$$\frac{6\,a^3x^3+12\,a^2x^2+2\,ax-3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log(ax+1)+3\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log(ax-1)-8}{24\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 + 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) - 8)/(a^5*c
^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.15, size = 74, normalized size = 0.88

$$-\frac{\log(|ax+1|)}{8\,ac^3}+\frac{\log(|ax-1|)}{8\,ac^3}+\frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(ax+1)^3(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a^
3*x^3 + 6*a^2*x^2 + a*x - 4)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

maple [A] time = 0.04, size = 90, normalized size = 1.07

$$\frac{1}{16a\,c^3\,(ax-1)}+\frac{\ln(ax-1)}{8c^3a}+\frac{1}{12a\,c^3\,(ax+1)^3}+\frac{1}{8a\,c^3\,(ax+1)^2}+\frac{3}{16a\,c^3\,(ax+1)}-\frac{\ln(ax+1)}{8a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/16/a/c^3/(a*x-1)+1/8/c^3/a*ln(a*x-1)+1/12/a/c^3/(a*x+1)^3+1/8/a/c^3/(a*x+
1)^2+3/16/a/c^3/(a*x+1)-1/8*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.30, size = 91, normalized size = 1.08

$$\frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}-\frac{\log\left(ax+1\right)}{8\,ac^3}+\frac{\log\left(ax-1\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^3 + 6*a^2*x^2 + a*x - 4)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2
*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 1.26, size = 73, normalized size = 0.87

$$-\frac{\frac{x}{12}+\frac{a\,x^2}{2}-\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}-\frac{\mathrm{atanh}\left(a\,x\right)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^3*(a*x + 1)),x)
```

```
[Out] - (x/12 + (a*x^2)/2 - 1/(3*a) + (a^2*x^3)/4)/(c^3 - 2*a^3*c^3*x^3 - a^4*c^3
*x^4 + 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

sympy [A] time = 0.42, size = 85, normalized size = 1.01

$$-\frac{-3a^3x^3-6a^2x^2-ax+4}{12a^5c^3x^4+24a^4c^3x^3-24a^2c^3x-12ac^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(-3*a**3*x**3 - 6*a**2*x**2 - a*x + 4)/(12*a**5*c**3*x**4 + 24*a**4*c**3*x
**3 - 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c
**3)
```

3.605 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=119

$$-\frac{5}{64ac^4(1-ax)}+\frac{5}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{3}{32ac^4(ax+1)^2}+\frac{1}{16ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}-\frac{15\tanh^{-1}(ax}{64ac^4}$$

```
[Out] -1/64/a/c^4/(-a*x+1)^2-5/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4+1/16/a/c^4/
(a*x+1)^3+3/32/a/c^4/(a*x+1)^2+5/32/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/c^4
```

Rubi [A] time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6140, 44, 207}

$$-\frac{5}{64ac^4(1-ax)}+\frac{5}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{3}{32ac^4(ax+1)^2}+\frac{1}{16ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}-\frac{15\tanh^{-1}(ax}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] -1/(64*a*c^4*(1 - a*x)^2) - 5/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1 + a*x)^
4) + 1/(16*a*c^4*(1 + a*x)^3) + 3/(32*a*c^4*(1 + a*x)^2) + 5/(32*a*c^4*(1 +
 a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6140

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$$

$$= -\frac{\int \frac{1}{(1-ax)^3(1+ax)^5}\,dx}{c^4}$$

$$= -\frac{\int \left(-\frac{1}{32(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{8(1+ax)^5}+\frac{3}{16(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{5}{32(1+ax)^2}-\frac{15}{64\left(-1+a^2x^2\right)}\right) dx}{c^4}$$

$$= -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+ax)^2}$$

$$= -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+ax)^2}$$

Mathematica [A] time = 0.07, size = 80, normalized size = 0.67

$$\frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax-15(ax-1)^2(ax+1)^4\tanh^{-1}(ax)+16}{64a(ax-1)^2(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (16 - 17*a*x - 50*a^2*x^2 - 10*a^3*x^3 + 30*a^4*x^4 + 15*a^5*x^5 - 15*(-1 +
 a*x)^2*(1 + a*x)^4*ArcTanh[a*x])/(64*a*(-1 + a*x)^2*(c + a*c*x)^4)
```

fricas [B] time = 0.58, size = 217, normalized size = 1.82

$$\frac{30\,a^5x^5+60\,a^4x^4-20\,a^3x^3-100\,a^2x^2-34\,ax-15\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\log\left(ax\right.}{128\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(30*a^5*x^5 + 60*a^4*x^4 - 20*a^3*x^3 - 100*a^2*x^2 - 34*a*x - 15*(a^
6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
 + 15*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log
(a*x - 1) + 32)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3
- a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.15, size = 91, normalized size = 0.76

$$-\frac{15\,\log\left(|ax+1|\right)}{128\,ac^4}+\frac{15\,\log\left(|ax-1|\right)}{128\,ac^4}+\frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(ax+1)^4(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/64
*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/((a*x +
1)^4*(a*x - 1)^2*a*c^4)
```

maple [A] time = 0.05, size = 120, normalized size = 1.01

$$-\frac{1}{64c^4a\,(ax-1)^2}+\frac{5}{64a\,c^4\,(ax-1)}+\frac{15\ln\left(ax-1\right)}{128c^4a}+\frac{1}{32a\,c^4\,(ax+1)^4}+\frac{1}{16a\,c^4\,(ax+1)^3}+\frac{3}{32a\,c^4\,(ax+1)^2}+\frac{}{32a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -1/64/c^4/a/(a*x-1)^2+5/64/a/c^4/(a*x-1)+15/128/c^4/a*ln(a*x-1)+1/32/a/c^4/
(a*x+1)^4+1/16/a/c^4/(a*x+1)^3+3/32/a/c^4/(a*x+1)^2+5/32/a/c^4/(a*x+1)-15/1
28*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.31, size = 140, normalized size = 1.18

$$\frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}-\frac{15\,\log\left(ax+1\right)}{128\,ac^4}+\frac{15\,\log\left(ax-1\right)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/(a^7
*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^
2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*c^4
)
```

mupad [B] time = 1.28, size = 121, normalized size = 1.02

$$-\frac{\frac{17\,x}{64}+\frac{25\,a\,x^2}{32}-\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}-\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4}-\frac{15\,\mathrm{atanh}\left(a\,x\right)}{64\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^4*(a*x + 1)),x)
```

```
[Out] - ((17*x)/64 + (25*a*x^2)/32 - 1/(4*a) + (5*a^2*x^3)/32 - (15*a^3*x^4)/32 -
 (15*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^4*x^4 + 2*a^5*
c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

sympy [A] time = 0.60, size = 141, normalized size = 1.18

$$\frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax+16}{64a^7c^4x^6+128a^6c^4x^5-64a^5c^4x^4-256a^4c^4x^3-64a^3c^4x^2+128a^2c^4x+64ac^4}+\frac{\frac{15\log\left(x-\frac{1}{a}\right)}{128}-\frac{15\log\left(x+\frac{1}{a}\right)}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (15*a**5*x**5 + 30*a**4*x**4 - 10*a**3*x**3 - 50*a**2*x**2 - 17*a*x + 16)/(
64*a**7*c**4*x**6 + 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 - 256*a**4*c**4*
x**3 - 64*a**3*c**4*x**2 + 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/a)/
128 - 15*log(x + 1/a)/128)/(a*c**4)
```

3.606 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=393

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{11/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{11}{56}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{48}a^5c$$

```
[Out] 11/48*a^4*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5-11/48*a^5*c^4*(1-1/a/x)^(
5/2)*(1+1/a/x)^(7/2)*x^6+11/56*a^6*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)*x^7-
11/72*a^7*c^4*(1-1/a/x)^(9/2)*(1+1/a/x)^(7/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(11
/2)*(1+1/a/x)^(7/2)*x^9+55/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))
/a+55/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+11/192*a^2*c^4*(1+1/a/x
)^(5/2)*x^3*(1-1/a/x)^(1/2)-11/64*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/
2)+55/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{11/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{11}{56}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{48}a^5c$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (55*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^
4)/64 + (11*a^4*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/48 - (11*a
^5*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/48 + (11*a^6*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(7/2)*x^7)/56 - (11*a^7*c^4*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(7/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^
(7/2)*x^9)/9 + (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a
)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}x^9+\frac{1}{9}\left(11a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{5/}}{x^9}\right.$$

$$=-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}x^9-\frac{1}{8}\left(\right.$$

$$=\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^8+\frac{1}{9}a^8$$

$$=-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\frac{11}{72}$$

$$=\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a$$

$$=-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5$$

$$=\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{11}{48}a^4c^4$$

$$=\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{64}a^3c^4$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1}$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1}$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1}$$

Mathematica [A] time = 0.21, size = 111, normalized size = 0.28

$$\frac{c^4\left(3465\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(896a^8x^8-3024a^7x^7+1024a^6x^6+7224a^5x^5-8448a^4x^4-3\right.\right.}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 - 4599*a*x + 10240*a^2*x^2 - 3066*a^
```

```
3*x^3 - 8448*a^4*x^4 + 7224*a^5*x^5 + 1024*a^6*x^6 - 3024*a^7*x^7 + 896*a^8
*x^8) + 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

fricas [A] time = 0.94, size = 169, normalized size = 0.43

$$\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(896\,a^9c^4x^9-2128\,a^8c^4x^8-2000\,a^7c^4x^7+8248\,a^6c^4x^6-\right.}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) + (896*a^9*c^4*x^9 - 2128*a^8*c^4*x^8 - 2000*a^7*c^4
*x^7 + 8248*a^6*c^4*x^6 - 1224*a^5*c^4*x^5 - 11514*a^4*c^4*x^4 + 7174*a^3*c
^4*x^3 + 5641*a^2*c^4*x^2 - 8311*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*x +
1)))/a
```

giac [A] time = 0.21, size = 198, normalized size = 0.50

$$-\frac{55\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\,(ax+1)}{128\,|a|}-\frac{1}{8064}\sqrt{a^2x^2-1}\left(\frac{3712\,c^4\mathrm{sgn}\,(ax+1)}{a}+\left(4599\,c^4\mathrm{sgn}\,(ax+1)-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -55/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1
/8064*sqrt(a^2*x^2 - 1)*(3712*c^4*sgn(a*x + 1)/a + (4599*c^4*sgn(a*x + 1) -
 2*(5120*a*c^4*sgn(a*x + 1) - (1533*a^2*c^4*sgn(a*x + 1) + 4*(1056*a^3*c^4*
sgn(a*x + 1) - (903*a^4*c^4*sgn(a*x + 1) + 2*(64*a^5*c^4*sgn(a*x + 1) + 7*(
8*a^7*c^4*x*sgn(a*x + 1) - 27*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)
```

maple [A] time = 0.07, size = 288, normalized size = 0.73

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^4\left(896\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^6a^6-3024\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^5a^5+1920\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4+420\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/8064*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^4/a*(896*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2)*x^6*a^6-3024*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5+1920*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2)*x^4*a^4+4200*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-6528*(
a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+1134*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a
+8064*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-4352*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
)-3465*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+3465*ln((a^2*x+(a^2*x^2-1)^(1/2)*(
a^2)^(1/2))/(a^2)^(1/2))*a)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 1.39, size = 415, normalized size = 1.06

$$\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+115038\right.}{\frac{9\,(ax-1)a^2}{ax+1}-\frac{36\,(ax}{(ax-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2) -
 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1))^
(13/2) + 334602*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 360448*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 255222*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*((a*x
- 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*c^4*sq
rt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a
*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^
4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(
a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^
2/(a*x + 1)^9 - a^2))*a
```

mupad [B] time = 0.17, size = 362, normalized size = 0.92

$$\frac{\frac{715\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96}-\frac{55\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{913\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32}+\frac{14179\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224}-\frac{5632\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{63}+\frac{18589\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{224}+\frac{913\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{32}}{a-\frac{9\,a\,(a\,x-1)}{a\,x+1}+\frac{36\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{84\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{126\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{126\,a\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{84\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{36\,a\,(a\,x-1)^7}{(a\,x+1)^7}+9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((715*c^4*((a*x - 1)/(a*x + 1))^(3/2))/96 - (55*c^4*((a*x - 1)/(a*x + 1))^(
1/2))/64 - (913*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (14179*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/224 - (5632*c^4*((a*x - 1)/(a*x + 1))^(9/2))/63 + (18589
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/224 + (913*c^4*((a*x - 1)/(a*x + 1))^(13
/2))/32 - (715*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 + (55*c^4*((a*x - 1)/(a
*x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a
*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^
4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*
a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9
)/(a*x + 1)^9) + (55*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.607 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=313

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{14}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3}{10}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{8}a^3c^3x^4$$

```
[Out] 3/8*a^3*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x^4-3/10*a^4*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(5/2)*x^5+3/14*a^5*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^6-1/7*
a^6*c^3*(1-1/a/x)^(9/2)*(1+1/a/x)^(5/2)*x^7+9/16*c^3*arctanh((1-1/a/x)^(1/2
)*(1+1/a/x)^(1/2))/a+3/16*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-3/8*a^2
*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+9/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{14}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3}{10}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{8}a^3c^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (3*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/8 + (3*a^3*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/8 -
 (3*a^4*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/10 + (3*a^5*c^3*(1
 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(9/2
)*(1 + 1/(a*x))^(5/2)*x^7)/7 + (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/(16*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
```

```
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3 dx = -\left(\left(a^6c^3\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x^7 - \frac{1}{7}\left(9a^5c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^7}\right.$$

$$= \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x^7 + \frac{1}{2}(3a$$

$$= -\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6 - \frac{1}{7}a$$

$$= \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 - \frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5 + \frac{3}{14}a^5$$

$$= -\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4 - \frac{3}{10}a^4c^3$$

$$= \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{3}{8}a^3c^3\left(1-\right.$$

$$= \frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}$$

$$= \frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}$$

$$= \frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}$$

Mathematica [A] time = 0.16, size = 95, normalized size = 0.30

$$-\frac{c^3\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(80a^6x^6-280a^5x^5+208a^4x^4+350a^3x^3-656a^2x^2+245ax+368\right)-315\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}\right.\right.\right.}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/560*(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 + 245*a*x - 656*a^2*x^2 + 350*a
^3*x^3 + 208*a^4*x^4 - 280*a^5*x^5 + 80*a^6*x^6) - 315*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x]))/a
```

fricas [A] time = 0.53, size = 148, normalized size = 0.47

$$\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(80\,a^7c^3x^7-200\,a^6c^3x^6-72\,a^5c^3x^5+558\,a^4c^3x^4-306\,a^3c^3x\right.}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (80*a^7*c^3*x^7 - 200*a^6*c^3*x^6 - 72*a^5*c^3*x^5 +
558*a^4*c^3*x^4 - 306*a^3*c^3*x^3 - 411*a^2*c^3*x^2 + 613*a*c^3*x + 368*c^3
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.21, size = 162, normalized size = 0.52

$$-\frac{9\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{16\,|a|}-\frac{1}{560}\sqrt{a^2x^2-1}\left(\frac{368\,c^3\mathrm{sgn}\left(ax+1\right)}{a}+\left(245\,c^3\mathrm{sgn}\left(ax+1\right)-2\left(328\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -9/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/5
60*sqrt(a^2*x^2 - 1)*(368*c^3*sgn(a*x + 1)/a + (245*c^3*sgn(a*x + 1) - 2*(3
28*a*c^3*sgn(a*x + 1) - (175*a^2*c^3*sgn(a*x + 1) + 4*(26*a^3*c^3*sgn(a*x +
 1) + 5*(2*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

maple [A] time = 0.05, size = 240, normalized size = 0.77

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(80\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4-280\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^3a^3+288\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2+70\left(a^2\right.\right.}{560a\left(ax-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/560*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^3/a*(80*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2)*x^4*a^4-280*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+288*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^2*a^2+70*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+192*(a^2*x^2-1)
^(3/2)*(a^2)^(1/2)+315*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-560*((a*x-1)*(a*x+
1))^(3/2)*(a^2)^(1/2)-315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))*a)/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.97, size = 337, normalized size = 1.08

$$\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-8393\,c^3\left(\frac{ax-1}{ax+1}\right)\right.}{\frac{7(ax-1)a^2}{ax+1}-\frac{21(ax-1)^2a^2}{(ax+1)^2}+\frac{35(ax-1)^3a^2}{(ax+1)^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 210
0*c^3*((a*x - 1)/(a*x + 1))^(11/2) - 8393*c^3*((a*x - 1)/(a*x + 1))^(9/2) +
 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 5943*c^3*((a*x - 1)/(a*x + 1))^(5/2
) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x + 1)
))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x -
1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/
(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 -
 a^2))*a
```

mupad [B] time = 0.13, size = 289, normalized size = 0.92

$$\frac{9\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{\frac{9\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}+\frac{849\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}-\frac{1152\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{35}+\frac{1199\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{40}+\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/}}{2}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a}{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - ((9*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/8 - (15*c^3*((a*x - 1)/(a*x + 1))^(3/2))/2 + (849*c^3*((a*x -
1)/(a*x + 1))^(5/2))/40 - (1152*c^3*((a*x - 1)/(a*x + 1))^(7/2))/35 + (1199
*c^3*((a*x - 1)/(a*x + 1))^(9/2))/40 + (15*c^3*((a*x - 1)/(a*x + 1))^(11/2)
)/2 - (9*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (7*a*(a*x - 1))/(a*x + 1
) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)^3)/(a*x + 1)^3 + (35*a
*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x + 1)^5 + (7*a*(a*x - 1)
^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.608 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{20}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{7}{12}a^2c^2x^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{8}ac^2x^2\sqrt{1}$$

```
[Out] 7/12*a^2*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x^3-7/20*a^3*c^2*(1-1/a/x)^(5/
2)*(1+1/a/x)^(3/2)*x^4+1/5*a^4*c^2*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^5+7/8*
c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/8*a*c^2*(1+1/a/x)^(3/2)*x^
2*(1-1/a/x)^(1/2)+7/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6191, 6195, 94, 92, 208}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{20}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{7}{12}a^2c^2x^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{8}ac^2x^2\sqrt{1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x)
]*(1 + 1/(a*x))^(3/2)*x^2)/8 + (7*a^2*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))
^(3/2)*x^3)/12 - (7*a^3*c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^4)/20
 + (a^4*c^2*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^5)/5 + (7*c^2*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5+\frac{1}{5}\left(7a^3c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}{x^5}\,dx\right.$$

$$=-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5-\frac{1}{4}($$

$$=\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4$$

$$=-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{7}{20}a^3c^2$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/}$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/}$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/}$$

Mathematica [A] time = 0.12, size = 79, normalized size = 0.34

$$\frac{c^2\left(105\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\left(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
 24*a^4*x^4) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

fricas [A] time = 0.52, size = 125, normalized size = 0.54

$$\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(24\,a^5c^2x^5-66\,a^4c^2x^4+22\,a^3c^2x^3+97\,a^2c^2x^2-151\,ac^2x\right.}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (24*a^5*c^2*x^5 - 66*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 9
7*a^2*c^2*x^2 - 151*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

giac [A] time = 0.18, size = 126, normalized size = 0.54

$$-\frac{7\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{8\,|a|}-\frac{1}{120}\sqrt{a^2x^2-1}\left(\left(15\,c^2\operatorname{sgn}(ax+1)-2\left(56\,ac^2\operatorname{sgn}(ax+1)+3\left(4\,a^3\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -7/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/12
0*sqrt(a^2*x^2 - 1)*((15*c^2*sgn(a*x + 1) - 2*(56*a*c^2*sgn(a*x + 1) + 3*(4
*a^3*c^2*x*sgn(a*x + 1) - 15*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 136*c^2*sgn(a*
x + 1)/a)
```

maple [A] time = 0.05, size = 192, normalized size = 0.82

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^2\left(24\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-90\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,xa+16\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-105\sqrt{a^2x^2-1}\,\sqrt{a}\right.}{120a\,(ax-1)\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/120*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^2*a^2-90*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+16*(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)-105*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+120*((a*x-1)*(a*x+1))^(3/2)*(
a^2)^(1/2)+105*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/(a*
x-1)/((a*x-1)*(a*x+1))^(1/2)/(a^2)^(1/2)
```

maxima [A] time = 0.87, size = 259, normalized size = 1.11

$$\frac{1}{120}a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+\right.}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-}{(ax-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) + 79
0*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 4
90*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*
(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a
^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^
5 - a^2))
```

mupad [B] time = 1.27, size = 214, normalized size = 0.92

$$\frac{\frac{49\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{7\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{79\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}+\frac{7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}+\frac{7\,c^2\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((49*c^2*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^2*((a*x - 1)/(a*x + 1))^(1/2
))/4 - (224*c^2*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^2*((a*x - 1)/(a*x +
 1))^(7/2))/6 + (7*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*c^2*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(2*a**2*x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2*a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**4*x**4*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x**5*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.609 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$

Optimal. Leaf size=145

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}+\frac{5}{6}acx^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

```
[Out] 5/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+5/6*a*c*(1-1/a/x)^(3/2)*x^
2*(1+1/a/x)^(1/2)-1/3*a^2*c*(1-1/a/x)^(5/2)*x^3*(1+1/a/x)^(1/2)-5/2*c*x*(1-
1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6191, 6195, 94, 92, 208}

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}+\frac{5}{6}acx^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (5*a*c*(1 - 1/(a*x))^(3/2)
*Sqrt[1 + 1/(a*x)]*x^2)/6 - (a^2*c*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^
3)/3 + (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
```

```
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx = -\left(\left(a^2c\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$
$$= \left(a^2c\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3 - \frac{1}{3}(5ac)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3 + \frac{1}{2}(5c)\,\text{Subst}\Bigg(\int$$
$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{}$$
$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{}$$
$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{}$$

Mathematica [A] time = 0.11, size = 61, normalized size = 0.42

$$\frac{c\left(ax\sqrt{1-\frac{1}{a^2x^2}}\left(-2a^2x^2+9ax-22\right)+15\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-22 + 9*a*x - 2*a^2*x^2) + 15*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/(6*a)
```

fricas [A] time = 0.56, size = 92, normalized size = 0.63

$$\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3cx^3-7\,a^2cx^2+13\,acx+22\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (2*a^3*c*x^3 - 7*a^2*c*x^2 + 13*a*c*x + 22*c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

giac [A] time = 0.18, size = 82, normalized size = 0.57

$$-\frac{5\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}-\frac{1}{6}\,\sqrt{a^2x^2-1}\left(\left(2\,acx\operatorname{sgn}(ax+1)-9\,c\operatorname{sgn}(ax+1)\right)x+\frac{22\,c\operatorname{sgn}(ax+\cdots}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -5/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sq
rt(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 9*c*sgn(a*x + 1))*x + 22*c*sgn(a*x
 + 1)/a)
```

maple [A] time = 0.05, size = 183, normalized size = 1.26

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2\,c\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa-2\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-24\sqrt{(ax-1)(ax\cdots}\right.}{6\,(ax-1)\,\sqrt{(ax-1)(ax+1)}\,a\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c*(9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*
a-2*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)
^(1/2))/(a^2)^(1/2))*a-24*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+24*a*ln((a^2*
x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/(a*x-1)/((a*x-1)*(a*x+
1))^(1/2)/a/(a^2)^(1/2)
```

maxima [A] time = 0.31, size = 171, normalized size = 1.18

$$\frac{1}{6}\,a\left(\frac{2\left(33\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(33*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3/
2) + 15*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) + 15*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 0.07, size = 133, normalized size = 0.92

$$\frac{5\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{5\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+11\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (5*c*((a*x - 1)/(a*x + 1))^(1/
2) - (40*c*((a*x - 1)/(a*x + 1))^(3/2))/3 + 11*c*((a*x - 1)/(a*x + 1))^(5/2
```

```
))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x
 - 1)^3)/(a*x + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**2*x**2*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

3.610 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

```
[Out] -1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6183}

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/(3*a*c*E^(3*ArcCoth[a*x]))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Mathematica [A] time = 0.05, size = 18, normalized size = 1.00

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/3*1/(a*c*E^(3*ArcCoth[a*x]))
```

fricas [A] time = 0.41, size = 34, normalized size = 1.89

$$-\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x + a*c)
```

giac [B] time = 0.19, size = 49, normalized size = 2.72

$$\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x + 1)^
3*a*c)
```

maple [A] time = 0.04, size = 24, normalized size = 1.33

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x)
```

```
[Out] -1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.44, size = 23, normalized size = 1.28

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/3*((a*x - 1)/(a*x + 1))^(3/2)/(a*c)
```

mupad [B] time = 1.21, size = 23, normalized size = 1.28

$$-\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2),x)
```

```
[Out] -((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x**2 - a*x
- 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2
*x**2 - a*x - 1), x))/c
```

3.611 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$\frac{2e^{-3\coth^{-1}(ax)}}{15ac^2}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2\left(1-a^2x^2\right)}$$

```
[Out] 2/15/a/c^2*((a*x-1)/(a*x+1))^(3/2)+1/5*(-2*a*x-3)/a/c^2*((a*x-1)/(a*x+1))^(
3/2)/(-a^2*x^2+1)
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{2e^{-3\coth^{-1}(ax)}}{15ac^2}-\frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] 2/(15*a*c^2*E^(3*ArcCoth[a*x])) - (3 + 2*a*x)/(5*a*c^2*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= -\frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2\left(1-a^2x^2\right)}-\frac{2\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{5c}\\ &= \frac{2e^{-3\coth^{-1}(ax)}}{15ac^2}-\frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2\left(1-a^2x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.16, size = 43, normalized size = 0.78

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(2a^2x^2+6ax+7\right)}{15c^2(ax+1)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(7 + 6*a*x + 2*a^2*x^2))/(15*c^2*(1 + a*x)^3)
```

fricas [A] time = 0.65, size = 58, normalized size = 1.05

$$\frac{\left(2\,a^2x^2+6\,ax+7\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(2*a^2*x^2 + 6*a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 + 2*a^2
*c^2*x + a*c^2)
```

giac [A] time = 0.23, size = 65, normalized size = 1.18

$$-\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^5ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x + 1)^5*a*c^2)
```

maple [A] time = 0.04, size = 49, normalized size = 0.89

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(2a^2x^2+6ax+7\right)}{15\left(a^2x^2-1\right)a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] 1/15*((a*x-1)/(a*x+1))^(3/2)*(2*a^2*x^2+6*a*x+7)/(a^2*x^2-1)/a/c^2
```

maxima [A] time = 0.30, size = 60, normalized size = 1.09

$$\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\sqrt{\frac{ax-1}{ax+1}}}{60\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(3*((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 15*s
qrt((a*x - 1)/(a*x + 1)))/(a*c^2)
```

mupad [B] time = 0.05, size = 60, normalized size = 1.09

$$\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}-10\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}+3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{60\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^2,x)
```

```
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x
- 1)/(a*x + 1))^(5/2))/(60*a*c^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.612 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=91

$$-\frac{12(2ax+3)e^{-3\coth^{-1}(ax)}}{35ac^3\left(1-a^2x^2\right)}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}$$

```
[Out] 8/35/a/c^3*((a*x-1)/(a*x+1))^(3/2)+1/7*(4*a*x+3)/a/c^3*((a*x-1)/(a*x+1))^(3
/2)/(-a^2*x^2+1)^2-12/35*(2*a*x+3)/a/c^3*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2+
1)
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{12(2ax+3)e^{-3\coth^{-1}(ax)}}{35ac^3\left(1-a^2x^2\right)}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] 8/(35*a*c^3*E^(3*ArcCoth[a*x])) + (3 + 4*a*x)/(7*a*c^3*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2)^2) - (12*(3 + 2*a*x))/(35*a*c^3*E^(3*ArcCoth[a*x])*(1 - a^2*x^
2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx &= \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{7c}\\
&= \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}-\frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3\left(1-a^2x^2\right)}-\frac{24\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^2}\\
&= \frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}+\frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}-\frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3\left(1-a^2x^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 66, normalized size = 0.73

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(8a^4x^4+24a^3x^3+20a^2x^2-4ax-13\right)}{35c^3(ax-1)(ax+1)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-13 - 4*a*x + 20*a^2*x^2 + 24*a^3*x^3 + 8*a^4*x^4
))/(35*c^3*(-1 + a*x)*(1 + a*x)^4)
```

fricas [A] time = 0.54, size = 86, normalized size = 0.95

$$\frac{\left(8\,a^4x^4+24\,a^3x^3+20\,a^2x^2-4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^4*x^4 + 24*a^3*x^3 + 20*a^2*x^2 - 4*a*x - 13)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^3, x)
```

maple [A] time = 0.04, size = 65, normalized size = 0.71

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(8x^4a^4+24x^3a^3+20a^2x^2-4ax-13\right)}{35\left(a^2x^2-1\right)^2c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] 1/35*((a*x-1)/(a*x+1))^(3/2)*(8*a^4*x^4+24*a^3*x^3+20*a^2*x^2-4*a*x-13)/(a^
2*x^2-1)^2/c^3/a
```

maxima [A] time = 0.30, size = 103, normalized size = 1.13

$$-\frac{1}{560}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-28\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-140\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{35}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 28*((a*x - 1)/(a*x + 1))^(5/2) +
 70*((a*x - 1)/(a*x + 1))^(3/2) - 140*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3)
- 35/(a^2*c^3*sqrt((a*x - 1)/(a*x + 1))))
```

mupad [B] time = 1.21, size = 116, normalized size = 1.27

$$\frac{1}{16\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^3}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8\,a\,c^3}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{20\,a\,c^3}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{112\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^3,x)
```

```
[Out] 1/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2)) + ((a*x - 1)/(a*x + 1))^(1/2)/(4*a
*c^3) - ((a*x - 1)/(a*x + 1))^(3/2)/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)
/(20*a*c^3) - ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.613 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=127

$$\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{8(2ax+3)e^{-3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}+\frac{10(4ax+3)e^{-3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}$$

```
[Out] 16/63/a/c^4*((a*x-1)/(a*x+1))^(3/2)+1/9*(2*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(
3/2)/(-a^2*x^2+1)^3+10/63*(4*a*x+3)/a/c^4*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2
+1)^2-8/21*(2*a*x+3)/a/c^4*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2+1)
```

Rubi [A] time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{8(2ax+3)e^{-3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}+\frac{10(4ax+3)e^{-3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] 16/(63*a*c^4*E^(3*ArcCoth[a*x])) + (1 + 2*a*x)/(9*a*c^4*E^(3*ArcCoth[a*x])*
(1 - a^2*x^2)^3) + (10*(3 + 4*a*x))/(63*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x
^2)^2) - (8*(3 + 2*a*x))/(21*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{9c}$$
$$= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2} + \frac{40\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{21c^2}$$
$$= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2} - \frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4(1-a^2x^2)} - \frac{16\int \frac{e^{-3\coth^{-1}}}{c-a^2cx}}{21c^3}$$
$$= \frac{16e^{-3\coth^{-1}(ax)}}{63ac^4} + \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2} - \frac{8e^{-3\coth^{-1}(ax)}(3+2a}{21ac^4(1-a^2x^2)}$$

Mathematica [A] time = 0.28, size = 82, normalized size = 0.65

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19\right)}{63c^4(ax-1)^2(ax+1)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 6*a*x - 66*a^2*x^2 - 56*a^3*x^3 + 24*a^4*x^4
 + 48*a^5*x^5 + 16*a^6*x^6))/(63*c^4*(-1 + a*x)^2*(1 + a*x)^5)
```

fricas [A] time = 0.48, size = 134, normalized size = 1.06

$$\frac{\left(16\,a^6x^6+48\,a^5x^5+24\,a^4x^4-56\,a^3x^3-66\,a^2x^2-6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/63*(16*a^6*x^6 + 48*a^5*x^5 + 24*a^4*x^4 - 56*a^3*x^3 - 66*a^2*x^2 - 6*a*
x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^
4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^4, x)
```

maple [A] time = 0.04, size = 81, normalized size = 0.64

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(16x^6a^6+48x^5a^5+24x^4a^4-56x^3a^3-66a^2x^2-6ax+19\right)}{63\left(a^2x^2-1\right)^3c^4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x)
```

```
[Out] 1/63*((a*x-1)/(a*x+1))^(3/2)*(16*a^6*x^6+48*a^5*x^5+24*a^4*x^4-56*a^3*x^3-6
6*a^2*x^2-6*a*x+19)/(a^2*x^2-1)^3/c^4/a
```

maxima [A] time = 0.48, size = 136, normalized size = 1.07

$$\frac{1}{4032}a\left(\frac{7\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-54\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+189\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-420\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+945\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{21\left(\frac{18\,(ax-1)}{ax+1}-1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4032*a*((7*((a*x - 1)/(a*x + 1))^(9/2) - 54*((a*x - 1)/(a*x + 1))^(7/2) +
 189*((a*x - 1)/(a*x + 1))^(5/2) - 420*((a*x - 1)/(a*x + 1))^(3/2) + 945*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + 21*(18*(a*x - 1)/(a*x + 1) - 1)/(a^2*c
^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

mupad [B] time = 0.04, size = 155, normalized size = 1.22

$$\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^4}+\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{64\,a\,c^4}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{576\,a\,c^4}+\frac{\frac{6\,(a\,x-1)}{a\,x+1}-\frac{1}{3}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^4,x)
```

```
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - (5*((a*x - 1)/(a*x + 1))^(3/2
))/(48*a*c^4) + (3*((a*x - 1)/(a*x + 1))^(5/2))/(64*a*c^4) - (3*((a*x - 1)/
(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*c^4) + (
(6*(a*x - 1))/(a*x + 1) - 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.614 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=229

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(a\ldots}{3\ldots}$$

```
[Out] 8/3*(a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(a*x+1
)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(a*x+1)^8*(-a^2*c*x
^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(a*x+1)^9*(-a^2*c*x^2+c)^(9/2)
/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c)^(9/2)/a^10/(1-
1/a^2/x^2)^(9/2)/x^9
```

Rubi [A] time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(a\ldots}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (8*(1 + a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 - (32*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^4(1+ax)^5\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(16(1+ax)^5-32(1+ax)^6+24(1+ax)^7-8(1+ax)^8+(1+ax\right.}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{8(1+ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.34

$$\frac{c^4(ax+1)^6\left(63a^4x^4-308a^3x^3+588a^2x^2-528ax+193\right)\sqrt{c-a^2cx^2}}{630a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*(1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 - 528*a*x + 588*a^2*x^2 - 308*a^3
*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.49, size = 117, normalized size = 0.51

$$\frac{\left(63\,a^9c^4x^{10}+70\,a^8c^4x^9-315\,a^7c^4x^8-360\,a^6c^4x^7+630\,a^5c^4x^6+756\,a^4c^4x^5-630\,a^3c^4x^4-840\,a^2c^4x^3+315\,ac^4\right.}{630\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fric
as")
```

```
[Out] 1/630*(63*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 - 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 + 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 - 840*a^2*c^4*x^3 +
315*a*c^4*x^2 + 630*c^4*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 116, normalized size = 0.51

$$\frac{x\left(63a^{9}x^{9}+70x^{8}a^{8}-315a^{7}x^{7}-360x^{6}a^{6}+630x^{5}a^{5}+756x^{4}a^{4}-630x^{3}a^{3}-840a^{2}x^{2}+315ax+630\right)\left(-a^{2}c\,x^{2}\right.}{630\left(ax-1\right)^{4}\left(ax+1\right)^{5}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 1/630*x*(63*a^9*x^9+70*a^8*x^8-315*a^7*x^7-360*a^6*x^6+630*a^5*x^5+756*a^4*
x^4-630*a^3*x^3-840*a^2*x^2+315*a*x+630)*(-a^2*c*x^2+c)^(9/2)/(a*x-1)^4/(a*
x+1)^5/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^{2}cx^{2}+c\right)^{\frac{9}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-a^{2}\,c\,x^{2}\right)^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

3.615 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=183

$$\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(ax+1)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[Out] -8/5*(a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(a*x+1)^6
*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(a*x+1)^7*(-a^2*c*x^2
+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*x^2+c)^(7/2)/a^
8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(ax+1)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-8*(1 + a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (6*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx=\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=\frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^3(1+ax)^4\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=\frac{\left(c-a^2cx^2\right)^{7/2}\int\left(-8(1+ax)^4+12(1+ax)^5-6(1+ax)^6+(1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=-\frac{8(1+ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1+ax)^7\left(c-a^2cx^2\right.}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^{\cdot}}$$

<u>**Mathematica**</u> [**A**] time = 0.06, size = 71, normalized size = 0.39

$$-\frac{c^3(ax+1)^5\left(35a^3x^3-135a^2x^2+185ax-93\right)\sqrt{c-a^2cx^2}}{280a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/280*(c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(-93 + 185*a*x - 135*a^2*x^2 +
35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

<u>**fricas**</u> [**A**] time = 0.54, size = 95, normalized size = 0.52

$$-\frac{\left(35\,a^7c^3x^8+40\,a^6c^3x^7-140\,a^5c^3x^6-168\,a^4c^3x^5+210\,a^3c^3x^4+280\,a^2c^3x^3-140\,ac^3x^2-280\,c^3x\right)\sqrt{-a^2c}}{280\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 + 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 - 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 + 280*a^2*c^3*x^3 - 140*a*c^3*x^2 - 280*c^3*x)*sqrt(-a^2
*c)/a
```

<u>**giac**</u> [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 100, normalized size = 0.55

$$\frac{x\left(35a^{7}x^{7}+40x^{6}a^{6}-140x^{5}a^{5}-168x^{4}a^{4}+210x^{3}a^{3}+280a^{2}x^{2}-140ax-280\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}{280\left(ax-1\right)^{3}\left(ax+1\right)^{4}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/280*x*(35*a^7*x^7+40*a^6*x^6-140*a^5*x^5-168*a^4*x^4+210*a^3*x^3+280*a^2*
x^2-140*a*x-280)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^3/(a*x+1)^4/((a*x-1)/(a*x+1))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c-a^{2}\,c\,x^{2}\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.616 $\int e^{\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{5/2} \, dx$

Optimal. Leaf size=136

$$\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^4\left(c-a^2cx^2\right)^{5/2}}{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] (a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(a*x+1)^5*(-
a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1)^6*(-a^2*c*x^2+c)
^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^4\left(c-a^2cx^2\right)^{5/2}}{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx=\frac{\left(c-a^2cx^2\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$=\frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^2(1+ax)^3\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$=\frac{\left(c-a^2cx^2\right)^{5/2}\int\left(4(1+ax)^3-4(1+ax)^4+(1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.46

$$\frac{c^2(ax+1)^4\left(5a^2x^2-14ax+11\right)\sqrt{c-a^2cx^2}}{30a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*(1 + a*x)^4*(11 - 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.63, size = 73, normalized size = 0.54

$$\frac{\left(5\,a^5c^2x^6+6\,a^4c^2x^5-15\,a^3c^2x^4-20\,a^2c^2x^3+15\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 + 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 84, normalized size = 0.62

$$\frac{x\left(5x^5a^5+6x^4a^4-15x^3a^3-20a^2x^2+15ax+30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30\,(ax-1)^2\,(ax+1)^3\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5+6*a^4*x^4-15*a^3*x^3-20*a^2*x^2+15*a*x+30)*(-a^2*c*x^2+c)
^(5/2)/(a*x-1)^2/(a*x+1)^3/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.617 $\int e^{\coth^{-1}(ax)} \left(c - a^2cx^2\right)^{3/2} dx$

Optimal. Leaf size=93

$$\frac{(ax+1)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^3 \left(c-a^2cx^2\right)^{3/2}}{3a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] -2/3*(a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(a*x+1)
^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.17, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^3 \left(c-a^2cx^2\right)^{3/2}}{3a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*(1 + a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int(-1+ax)(1+ax)^2\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int\left(-2(1+ax)^2+(1+ax)^3\right)dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=-\frac{2(1+ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.57

$$-\frac{c(ax+1)^3(3ax-5)\sqrt{c-a^2cx^2}}{12a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
[Out] -1/12*(c*(1 + a*x)^3*(-5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

fricas [A] time = 0.57, size = 43, normalized size = 0.46

$$-\frac{\left(3\,a^3cx^4+4\,a^2cx^3-6\,acx^2-12\,cx\right)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
[Out] -1/12*(3*a^3*c*x^4 + 4*a^2*c*x^3 - 6*a*c*x^2 - 12*c*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 68, normalized size = 0.73

$$\frac{x\left(3x^3a^3+4a^2x^2-6ax-12\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{12\,(ax-1)\,(ax+1)^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/12*x*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*(-a^2*c*x^2+c)^(3/2)/(a*x-1)/(a*x+1)^
2/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.618 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=68

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6192, 6193}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx&=\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\&=\frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\&=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.60

$$\frac{(ax+2)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.46, size = 22, normalized size = 0.32

$$\frac{\sqrt{-a^2c}\left(ax^2+2\,x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 44, normalized size = 0.65

$$\frac{x\,(ax+2)\,\sqrt{-a^2c\,x^2+c}}{2\,(ax+1)\,\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.619 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=38

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

[Out] x*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.15, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6193, 31}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]

[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{1}{-1+ax}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

fricas [A] time = 0.63, size = 22, normalized size = 0.58

$$-\frac{\sqrt{-a^2c}\log(ax-1)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] -sqrt(-a^2*c)*log(a*x - 1)/(a^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 51, normalized size = 1.34

$$-\frac{\ln(ax-1)\sqrt{-c\left(a^2x^2-1\right)}}{ac(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -ln(a*x-1)*(-c*(a^2*x^2-1))^(1/2)/a/c/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

3.620 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1/
a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} - \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.62

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left((ax-1)\tanh^{-1}(ax)-1\right)}{2c(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (-1 + a*x)*ArcTanh[a*x]))/(c*(-1 + a*x)
*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 1.06, size = 86, normalized size = 0.95

$$-\frac{\left(a^2x-a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\,\sqrt{-a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 84, normalized size = 0.92

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-\ln\left(ax-1\right)+\ln\left(ax+1\right)+2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^{2}x^{2}-1\right)c^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-a*x*ln(a
*x+1)-ln(a*x-1)+ln(a*x+1)+2)/(a^2*x^2-1)/c^2/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(c-a^{2}\,c\,x^{2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.621 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4(-1+ax)^3}-\frac{1}{4(-1+ax)^2}-\frac{1}{8(1+ax)^2}+\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{\left(3a^5\right.}{}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^4\left(\right.}{}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.45

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2+3ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)+2\right)}{8c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 +
a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.50, size = 136, normalized size = 0.74

$$-\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\left(3\,a^2x^2-3\,ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 169, normalized size = 0.92

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(3\ln\left(ax-1\right)x^{3}a^{3}-3a^{3}x^{3}\ln\left(ax+1\right)-3\ln\left(ax-1\right)x^{2}a^{2}+3\ln\left(ax+1\right)x^{2}a^{2}+6a^{2}x^{2}-3\ln\left(a\right.\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\left(a^{2}x^{2}-1\right)c^{3}a\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x
^3*a^3-3*a^3*x^3*ln(a*x+1)-3*ln(a*x-1)*x^2*a^2+3*ln(a*x+1)*x^2*a^2+6*a^2*x^
2-3*ln(a*x-1)*x*a+3*a*x*ln(a*x+1)-6*a*x+3*ln(a*x-1)-3*ln(a*x+1)-4)/(a^2*x^2
-1)/c^3/a/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c-a^{2}\,c\,x^{2}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.622 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{}{24(}$$

```
[Out] 1/24*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*(
1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/x^
2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7
/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a
^2*c*x^2+c)^(7/2)+5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2
+c)^(7/2)
```

Rubi [A] time = 0.21, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{}{24(}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
+ (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (
5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^4(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{8(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{16\left(-1+a^2x^2\right)}\right.}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a}{32(1}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a}{32(1}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.36

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-15a^4x^4+15a^3x^3+25a^2x^2-25ax+15(ax-1)^3(ax+1)^2\tanh^{-1}(ax)-8\right)}{48c^3(ax-1)^3(ax+1)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/48*(Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 25*a*x + 25*a^2*x^2 + 15*a^3*x^3 - 15*
a^4*x^4 + 15*(-1 + a*x)^3*(1 + a*x)^2*ArcTanh[a*x]))/(c^3*(-1 + a*x)^3*(1 +
 a*x)^2*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.59, size = 191, normalized size = 0.69

$$-\frac{15\left(a^6x^5-a^5x^4-2\,a^4x^3+2\,a^3x^2+a^2x-a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\left(15\,a^4x^4-15\,a^3x^3-25\,a^2x^2+2\right.}{96\left(a^7c^4x^5-a^6c^4x^4-2\,a^5c^4x^3+2\,a^4c^4x^2+a^3c^4x-a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/96*(15*(a^6*x^5 - a^5*x^4 - 2*a^4*x^3 + 2*a^3*x^2 + a^2*x - a)*sqrt(-c)*
log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15*a^4*
x^4 - 15*a^3*x^3 - 25*a^2*x^2 + 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 - a^
6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 + a^3*c^4*x - a^2*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 241, normalized size = 0.87

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(15\ln\left(ax-1\right)x^5a^5-15\ln\left(ax+1\right)x^5a^5-15\ln\left(ax-1\right)x^4a^4+15\ln\left(ax+1\right)x^4a^4+30x^4a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/96/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x-1
)*x^5*a^5-15*ln(a*x+1)*x^5*a^5-15*ln(a*x-1)*x^4*a^4+15*ln(a*x+1)*x^4*a^4+30
*x^4*a^4-30*ln(a*x-1)*x^3*a^3+30*a^3*x^3*ln(a*x+1)-30*x^3*a^3+30*ln(a*x-1)*
x^2*a^2-30*ln(a*x+1)*x^2*a^2-50*a^2*x^2+15*ln(a*x-1)*x*a-15*a*x*ln(a*x+1)+5
0*a*x-15*ln(a*x-1)+15*ln(a*x+1)+16)/(a^2*x^2-1)/c^4/a/(a*x+1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.623 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=176

$$-\frac{77c^{9/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+$$

```
[Out] -77/384*c^3*x*(-a^2*c*x^2+c)^(3/2)-77/480*c^2*x*(-a^2*c*x^2+c)^(5/2)-11/80*
c*x*(-a^2*c*x^2+c)^(7/2)+11/90*(-a^2*c*x^2+c)^(9/2)/a+1/10*(a*x+1)*(-a^2*c*
x^2+c)^(9/2)/a-77/256*c^(9/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-77
/256*c^4*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6141, 671, 641, 195, 217, 203}

$$-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{77c^{9/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{256a}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-77*c^4*x*Sqrt[c - a^2*c*x^2])/256 - (77*c^3*x*(c - a^2*c*x^2)^(3/2))/384
- (77*c^2*x*(c - a^2*c*x^2)^(5/2))/480 - (11*c*x*(c - a^2*c*x^2)^(7/2))/80
+ (11*(c - a^2*c*x^2)^(9/2))/(90*a) + ((1 + a*x)*(c - a^2*c*x^2)^(9/2))/(10
*a) - (77*c^(9/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(256*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
```

```
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx = -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx$$
$$= -\left(c\int (1+ax)^2\left(c-a^2cx^2\right)^{7/2}dx\right)$$
$$= \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{10}(11c)\int (1+ax)\left(c-a^2cx^2\right)^{7/2}dx$$
$$= \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{10}(11c)\int \left(c-a^2cx^2\right)^{7/2}dx$$
$$= -\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a} - \frac{1}{80}\left(77c\right.$$
$$= -\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11\left(c-a^2cx^2\right)^{9/2}}{90a} + \frac{(1+ax)}{}$$
$$= -\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2} - \frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2} + \frac{11(}{}$$
$$= -\frac{77}{256}c^4x\sqrt{c-a^2cx^2} - \frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2} - \frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx$$
$$= -\frac{77}{256}c^4x\sqrt{c-a^2cx^2} - \frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2} - \frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx$$
$$= -\frac{77}{256}c^4x\sqrt{c-a^2cx^2} - \frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2} - \frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2} - \frac{11}{80}cx$$

Mathematica [A] time = 0.16, size = 167, normalized size = 0.95

$$\frac{c^4\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(-1152a^{10}x^{10}-1408a^9x^9+5584a^8x^8+7216a^7x^7-10552a^6x^6-15048a^5x^5+9210a^4\right.\right.}{11520a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(2560 - 10615*a*x - 2185*a^2*x^2 +
16390*a^3*x^3 + 9210*a^4*x^4 - 15048*a^5*x^5 - 10552*a^6*x^6 + 7216*a^7*x^7
 + 5584*a^8*x^8 - 1408*a^9*x^9 - 1152*a^10*x^10) + 6930*Sqrt[1 - a*x]*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(11520*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.70, size = 329, normalized size = 1.87

$$\left[\frac{3465\,\sqrt{-c}\,c^4\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(1152\,a^9c^4x^9+2560\,a^8c^4x^8-3024\,a^7c^4x^7-10240\,a^6\right.}{23040\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(3465*sqrt(-c)*c^4*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) + 2*(1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 1
0240*a^6*c^4*x^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 -
 10240*a^2*c^4*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a, 1/11
520*(3465*c^(9/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c))
+ (1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 10240*a^6*c^4*x
^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 - 10240*a^2*c^4
*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a]
```

giac [A] time = 0.19, size = 164, normalized size = 0.93

$$\frac{77\,c^5\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{256\,\sqrt{-c}\,|a|}+\frac{1}{11520}\,\sqrt{-a^2cx^2+c}\left(\frac{2560\,c^4}{a}-\left(8055\,c^4+2\left(5120\,ac^4-\left(3075\,a^2c^4+4\right(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 77/256*c^5*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a
)) + 1/11520*sqrt(-a^2*c*x^2 + c)*(2560*c^4/a - (8055*c^4 + 2*(5120*a*c^4 -
 (3075*a^2*c^4 + 4*(1920*a^3*c^4 + (39*a^4*c^4 - 2*(640*a^5*c^4 + (189*a^6*
c^4 - 8*(9*a^8*c^4*x + 20*a^7*c^4)*x)*x)*x)*x)*x)*x)*x)*x)
```

maple [B] time = 0.04, size = 350, normalized size = 1.99

$$\frac{x\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}{10}+\frac{9cx\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{80}+\frac{21c^2x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{160}+\frac{21c^3x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{128}+\frac{63c^4x\sqrt{-a^2c\,x^2+c}}{256}+\frac{63c^5\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 1/10*x*(-a^2*c*x^2+c)^(9/2)+9/80*c*x*(-a^2*c*x^2+c)^(7/2)+21/160*c^2*x*(-a^
2*c*x^2+c)^(5/2)+21/128*c^3*x*(-a^2*c*x^2+c)^(3/2)+63/256*c^4*x*(-a^2*c*x^2
+c)^(1/2)+63/256*c^5/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1
/2))+2/9/a*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(9/2)-1/4*c*(-(x-1/a)^2*a^2*c-2
*a*c*(x-1/a))^(7/2)*x-7/24*c^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(5/2)*x-35/
96*c^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x-35/64*c^4*(-(x-1/a)^2*a^2*c
-2*a*c*(x-1/a))^(1/2)*x-35/64*c^5/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x
-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.41, size = 192, normalized size = 1.09

$$\frac{1}{10}\left(-a^2cx^2+c\right)^{\frac{9}{2}}x-\frac{11}{80}\left(-a^2cx^2+c\right)^{\frac{7}{2}}cx-\frac{77}{480}\left(-a^2cx^2+c\right)^{\frac{5}{2}}c^2x-\frac{77}{384}\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^3x-\frac{35}{64}\sqrt{a^2cx^2-4\,acx+3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/10*(-a^2*c*x^2 + c)^(9/2)*x - 11/80*(-a^2*c*x^2 + c)^(7/2)*c*x - 77/480*(
-a^2*c*x^2 + c)^(5/2)*c^2*x - 77/384*(-a^2*c*x^2 + c)^(3/2)*c^3*x - 35/64*s
qrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^4*x + 63/256*sqrt(-a^2*c*x^2 + c)*c^4*x -
35/64*c^6*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 63/256*c^(9/2)*arcsin(a*x)/a + 2
/9*(-a^2*c*x^2 + c)^(9/2)/a + 35/32*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^4/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{9/2}\,\left(a\,x+1\right)}{a\,x-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 36.35, size = 1341, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] a**8*c**4*Piecewise((I*a**2*sqrt(c)*x**11/(10*sqrt(a**2*x**2 - 1)) - 9*I*sq
rt(c)*x**9/(80*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**7/(480*a**2*sqrt(a**2*x*
*2 - 1)) - 7*I*sqrt(c)*x**5/(1920*a**4*sqrt(a**2*x**2 - 1)) - 7*I*sqrt(c)*x
**3/(768*a**6*sqrt(a**2*x**2 - 1)) + 7*I*sqrt(c)*x/(256*a**8*sqrt(a**2*x**2
 - 1)) - 7*I*sqrt(c)*acosh(a*x)/(256*a**9), Abs(a**2*x**2) > 1), (-a**2*sqr
t(c)*x**11/(10*sqrt(-a**2*x**2 + 1)) + 9*sqrt(c)*x**9/(80*sqrt(-a**2*x**2 +
 1)) + sqrt(c)*x**7/(480*a**2*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**5/(1920*
a**4*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**3/(768*a**6*sqrt(-a**2*x**2 + 1))
 - 7*sqrt(c)*x/(256*a**8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*asin(a*x)/(256*a
**9), True)) + 2*a**7*c**4*Piecewise((x**8*sqrt(-a**2*c*x**2 + c)/9 - x**6*
sqrt(-a**2*c*x**2 + c)/(63*a**2) - 2*x**4*sqrt(-a**2*c*x**2 + c)/(105*a**4)
 - 8*x**2*sqrt(-a**2*c*x**2 + c)/(315*a**6) - 16*sqrt(-a**2*c*x**2 + c)/(31
5*a**8), Ne(a, 0)), (sqrt(c)*x**8/8, True)) - 2*a**6*c**4*Piecewise((I*a**2
*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqrt(c)*x**7/(48*sqrt(a**2*x**2
 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**3/(
384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(128*a**6*sqrt(a**2*x**2 - 1)
) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*
x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/(48*sqrt(-a**2*x**2 + 1)) +
sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**3/(384*a**4*sqr
t(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c
)*asin(a*x)/(128*a**7), True)) - 6*a**5*c**4*Piecewise((x**6*sqrt(-a**2*c*x
**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a**2) - 4*x**2*sqrt(-a**2*c*x*
*2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(105*a**6), Ne(a, 0)), (sqrt(
c)*x**6/6, True)) + 6*a**3*c**4*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 -
x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4),
 Ne(a, 0)), (sqrt(c)*x**4/4, True)) + 2*a**2*c**4*Piecewise((I*a**2*sqrt(c)
*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) +
I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), A
bs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c
)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) +
 sqrt(c)*asin(a*x)/(8*a**3), True)) - 2*a*c**4*Piecewise((0, Eq(c, 0)), (sq
rt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True))
 - c**4*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x
/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1),
 (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

3.624 $\int e^{2\coth^{-1}(ax)} \left(c - a^2cx^2\right)^{7/2} dx$

Optimal. Leaf size=153

$$-\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}+$$

```
[Out] -15/64*c^2*x*(-a^2*c*x^2+c)^(3/2)-3/16*c*x*(-a^2*c*x^2+c)^(5/2)+9/56*(-a^2*
c*x^2+c)^(7/2)/a+1/8*(a*x+1)*(-a^2*c*x^2+c)^(7/2)/a-45/128*c^(7/2)*arctan(a
*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-45/128*c^3*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6141, 671, 641, 195, 217, 203}

$$-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{45c^{7/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{128a}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-45*c^3*x*Sqrt[c - a^2*c*x^2])/128 - (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 -
 (3*c*x*(c - a^2*c*x^2)^(5/2))/16 + (9*(c - a^2*c*x^2)^(7/2))/(56*a) + ((1
+ a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) - (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(128*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
```

```
+ 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx\\
&= -\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{5/2}dx\right)\\
&= \frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{8}(9c)\int(1+ax)\left(c-a^2cx^2\right)^{5/2}dx\\
&= \frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{8}(9c)\int\left(c-a^2cx^2\right)^{5/2}dx\\
&= -\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{16}\left(15c^2\right.\\
&= -\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c\right.}{8}\\
&= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2\right.}{56}\\
&= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2\right.}{56}\\
&= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2\right.}{56}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 151, normalized size = 0.99

$$\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(112a^8x^8+144a^7x^7-424a^6x^6-600a^5x^5+558a^4x^4+978a^3x^3-187a^2x^2-837ax+\right.\right.}{896a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(256 - 837*a*x - 187*a^2*x^2 + 978*
a^3*x^3 + 558*a^4*x^4 - 600*a^5*x^5 - 424*a^6*x^6 + 144*a^7*x^7 + 112*a^8*x
^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(896*a*Sqrt[1 - a*x
]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.59, size = 286, normalized size = 1.87

$$\left[\frac{315\sqrt{-c}\,c^3\log\left(2\,a^2cx^2 - 2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x - c\right) - 2\left(112\,a^7c^3x^7 + 256\,a^6c^3x^6 - 168\,a^5c^3x^5 - 768\,a^4c^3x^4 - 2\right.}{1792\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 2*(112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 - 768*a^
4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256*c^3)*sqrt
(-a^2*c*x^2 + c))/a, 1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(
c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^
5 - 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256
*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

giac [A] time = 0.19, size = 141, normalized size = 0.92

$$\frac{45\,c^4\log\left(\left|-\sqrt{-a^2c}\,x + \sqrt{-a^2cx^2+c}\right|\right)}{128\sqrt{-c}\,|a|} + \frac{1}{896}\sqrt{-a^2cx^2+c}\left(\frac{256\,c^3}{a} - \left(581\,c^3 + 2\left(384\,ac^3 - \left(105\,a^2c^3 + 4\left(96\,a^3c^3\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 45/128*c^4*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a
)) + 1/896*sqrt(-a^2*c*x^2 + c)*(256*c^3/a - (581*c^3 + 2*(384*a*c^3 - (105
*a^2*c^3 + 4*(96*a^3*c^3 + (21*a^4*c^3 - 2*(7*a^6*c^3*x + 16*a^5*c^3)*x)*x)
*x)*x)*x)*x)
```

maple [B] time = 0.04, size = 296, normalized size = 1.93

$$\frac{x\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{8} + \frac{7cx\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{48} + \frac{35c^2x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{192} + \frac{35c^3x\sqrt{-a^2c\,x^2+c}}{128} + \frac{35c^4\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{128\sqrt{a^2c}} + \frac{2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/8*x*(-a^2*c*x^2+c)^(7/2)+7/48*c*x*(-a^2*c*x^2+c)^(5/2)+35/192*c^2*x*(-a^2
*c*x^2+c)^(3/2)+35/128*c^3*x*(-a^2*c*x^2+c)^(1/2)+35/128*c^4/(a^2*c)^(1/2)*
arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/7/a*(-(x-1/a)^2*a^2*c-2*a*c*
(x-1/a))^(7/2)-1/3*c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(5/2)*x-5/12*c^2*(-(x
-1/a)^2*a^2*c-2*a*c*(x-1/a))^(3/2)*x-5/8*c^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a
))^(1/2)*x-5/8*c^4/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2
*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.42, size = 173, normalized size = 1.13

$$\frac{1}{8}\left(-a^2cx^2+c\right)^{\frac{7}{2}}x - \frac{3}{16}\left(-a^2cx^2+c\right)^{\frac{5}{2}}cx - \frac{15}{64}\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^2x - \frac{5}{8}\sqrt{a^2cx^2-4\,acx+3\,c}\,c^3x + \frac{35}{128}\sqrt{-a^2cx^2+c}\,c^3x -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(-a^2*c*x^2 + c)^(7/2)*x - 3/16*(-a^2*c*x^2 + c)^(5/2)*c*x - 15/64*(-a^
2*c*x^2 + c)^(3/2)*c^2*x - 5/8*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^3*x + 35/1
28*sqrt(-a^2*c*x^2 + c)*c^3*x - 5/8*c^5*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 35
/128*c^(7/2)*arcsin(a*x)/a + 2/7*(-a^2*c*x^2 + c)^(7/2)/a + 5/4*sqrt(a^2*c*
x^2 - 4*a*c*x + 3*c)*c^3/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{7/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 21.83, size = 1091, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] -a**6*c**3*Piecewise((I*a**2*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqr
t(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**
2 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(
128*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2
*x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/
(48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) +
5*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt
(-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128*a**7), True)) - 2*a**5*c**3*Pi
ecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a*
*2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(
105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) + a**4*c**3*Piecewise((I*a**2
*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2*x**2
 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(16*a**
4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1
), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*sqrt(-
a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(
16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5), True)) + 4*a**
3*c**3*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 +
c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4
/4, True)) + a**2*c**3*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1
)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a*
*2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*
sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 +
 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3)
, True)) - 2*a*c**3*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)),
 (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)) - c**3*Piecewise((I*a**2*sq
rt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) -
I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2
 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

3.625 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=130

$$-\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

```
[Out] -7/24*c*x*(-a^2*c*x^2+c)^(3/2)+7/30*(-a^2*c*x^2+c)^(5/2)/a+1/6*(a*x+1)*(-a^
2*c*x^2+c)^(5/2)/a-7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-
7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6141, 671, 641, 195, 217, 203}

$$-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 + (7*(
c - a^2*c*x^2)^(5/2))/(30*a) + ((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$=-\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx\right)$$
$$=\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int(1+ax)\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{8}\left(7c^2\right)\int$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-\right.}{6a}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-\right.}{6a}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-\right.}{6a}$$

Mathematica [A] time = 0.13, size = 135, normalized size = 1.04

$$\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(-40a^6x^6-56a^5x^5+106a^4x^4+182a^3x^3-57a^2x^2-231ax+96\right)+210\sqrt{1-ax}\sin^{-1}\right.}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(96 - 231*a*x - 57*a^2*x^2 + 182*a^
3*x^3 + 106*a^4*x^4 - 56*a^5*x^5 - 40*a^6*x^6) + 210*Sqrt[1 - a*x]*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.64, size = 241, normalized size = 1.85

$$\left[\frac{105\sqrt{-c}\,c^2\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(40\,a^5c^2x^5+96\,a^4c^2x^4-10\,a^3c^2x^3-192\,a^2c^2x^2-1\right.}{480\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) + 2*(40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^
2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2)*a
rctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5 +
 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*c^2)*
sqrt(-a^2*c*x^2 + c))/a]
```

giac [A] time = 0.18, size = 116, normalized size = 0.89

$$\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}\,|a|}-\frac{1}{240}\,\sqrt{-a^2cx^2+c}\left(\left(135\,c^2+2\left(96\,ac^2+\left(5\,a^2c^2-4\left(5\,a^4c^2x+12\,a^3c^2\right)x\right)x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
 - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 + 2*(96*a*c^2 + (5*a^2*c^2 - 4*(5*a
^4*c^2*x + 12*a^3*c^2)*x)*x)*x)*x - 96*c^2/a)
```

maple [B] time = 0.05, size = 242, normalized size = 1.86

$$\frac{x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{6}+\frac{5cx\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{24}+\frac{5c^2x\sqrt{-a^2c\,x^2+c}}{16}+\frac{5c^3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{16\sqrt{a^2c}}+\frac{2\left(-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\right.\right.}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/6*x*(-a^2*c*x^2+c)^(5/2)+5/24*c*x*(-a^2*c*x^2+c)^(3/2)+5/16*c^2*x*(-a^2*c
*x^2+c)^(1/2)+5/16*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^
(1/2))+2/5/a*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(5/2)-1/2*c*(-(x-1/a)^2*a^2*c
-2*a*c*(x-1/a))^(3/2)*x-3/4*c^2*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x-3/
4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))
^(1/2))
```

maxima [A] time = 0.42, size = 154, normalized size = 1.18

$$\frac{1}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}x-\frac{7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx-\frac{3}{4}\sqrt{a^2cx^2-4\,acx+3\,c}\,c^2x+\frac{5}{16}\sqrt{-a^2cx^2+c}\,c^2x-\frac{3\,c^4\arcsin\left(ax-2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(a
^2*c*x^2 - 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x - 3/4*c^4
*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a + 2/5*(-a^2*c*
x^2 + c)^(5/2)/a + 3/2*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 12.02, size = 478, normalized size = 3.68

$$a^4c^2\left(\begin{cases} \frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,x^3}{48a^2\sqrt{a^2x^2-1}} + \frac{i\sqrt{c}\,x}{16a^4\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{16a^5} & \text{for } \left|a^2x^2\right| > 1 \\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,x^3}{48a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{c}\,x}{16a^4\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,\operatorname{asin}(ax)}{16a^5} & \text{otherwise} \end{cases}\right) + 2a^3c^2\left(\begin{cases} \frac{x^4\sqrt{-a^2c}}{5} \\ \frac{\sqrt{c}\,x^4}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt
(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
- 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(1
6*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
+ 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a*
*2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x
)/(16*a**5), True)) + 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
- x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4
), Ne(a, 0)), (sqrt(c)*x**4/4, True)) - 2*a*c**2*Piecewise((0, Eq(c, 0)), (
sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)
) - c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)
*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1
), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

3.626 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=107

$$-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[Out] 5/12*(-a^2*c*x^2+c)^(3/2)/a+1/4*(a*x+1)*(-a^2*c*x^2+c)^(3/2)/a-5/8*c^(3/2)*
arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6141, 671, 641, 195, 217, 203}

$$-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{3/2}}{4a}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 + (5*(c - a^2*c*x^2)^(3/2))/(12*a) + ((1 + a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\left(c\int (1+ax)^2\sqrt{c-a^2cx^2}\,dx\right)\\
&= \frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int (1+ax)\sqrt{c-a^2cx^2}\,dx\\
&= \frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int \sqrt{c-a^2cx^2}\,dx\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\int \frac{}{\sqrt{}}\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\text{Subs}\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\tan^{-1}\left(\right.}{8a}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 117, normalized size = 1.09

$$
\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(6a^4x^4+10a^3x^3-7a^2x^2-25ax+16\right)+30\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(16 - 25*a*x - 7*a^2*x^2 + 10*a^3*x^3
 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqrt
[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.60, size = 180, normalized size = 1.68

$$
\left[\frac{15\sqrt{-c}\,c\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\left(6\,a^3cx^3+16\,a^2cx^2+9\,acx-16\,c\right)\sqrt{-a^2cx^2+c}}{48\,a},\ \frac{15\,c^{\frac{3}{2}}}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) - 2*(6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c)
```

```
)/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
c)) - (6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c))/a
]
```

giac [A] time = 0.16, size = 85, normalized size = 0.79

$$-\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx+8\,ac\right)x+9\,c\right)x-\frac{16\,c}{a}\right)+\frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x + 8*a*c)*x + 9*c)*x - 16*c/a) + 5
/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

maple [B] time = 0.04, size = 188, normalized size = 1.76

$$\frac{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{4}+\frac{3cx\sqrt{-a^2c\,x^2+c}}{8}+\frac{3c^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{8\sqrt{a^2c}}+\frac{2\left(-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a}-c\sqrt{-\left(x-\frac{1}{a}\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)+3/8*c*x*(-a^2*c*x^2+c)^(1/2)+3/8*c^2/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3/a*(-(x-1/a)^2*a^2*c-2*a
*c*(x-1/a))^(3/2)-c*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)*x-c^2/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.42, size = 131, normalized size = 1.22

$$\frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x-\sqrt{a^2cx^2-4\,acx+3\,c}\,cx+\frac{3}{8}\sqrt{-a^2cx^2+c}\,cx-\frac{c^3\arcsin\left(ax-2\right)}{a\left(-c\right)^{\frac{3}{2}}}+\frac{3\,c^{\frac{3}{2}}\arcsin\left(ax\right)}{8\,a}+\frac{2\left(-a^2cx^2+\right.}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c*x + 3/8*sq
rt(-a^2*c*x^2 + c)*c*x - c^3*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)*a
rcsin(a*x)/a + 2/3*(-a^2*c*x^2 + c)^(3/2)/a + 2*sqrt(a^2*c*x^2 - 4*a*c*x +
3*c)*c/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 7.80, size = 342, normalized size = 3.20

$$-a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)-2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0\\ -\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c
)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
 I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4
*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*
x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) - 2*a*
c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
 c)**(3/2)/(3*a**2*c), True)) - c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a*
*2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/
(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asi
n(a*x)/(2*a), True))
```

3.627 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=86

$$\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)^
(1/2)/a+1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6141, 671, 641, 217, 203}

$$\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\ &= -\left(c\int\frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\ &= \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\,\text{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2}}\right.\\ &= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.88

$$\frac{\sqrt{c-a^2cx^2}\left(\sqrt{1-a^2x^2}(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.42, size = 134, normalized size = 1.56

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4a},\ \frac{\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{c}\,\text{arc}}{2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x + 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

giac [A] time = 0.15, size = 62, normalized size = 0.72

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)+\frac{3c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))

maple [A] time = 0.04, size = 134, normalized size = 1.56

$$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}+\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2c-2ac\left(x-\frac{1}{a}\right)}}{a}-\frac{2c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2 a^2c-2ac\left(x-\frac{1}{a}\right)}}\right)}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2),x)

[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))

maxima [A] time = 0.42, size = 47, normalized size = 0.55

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\,x-\frac{3\sqrt{c}\,\arcsin\left(ax\right)}{2\,a}+\frac{2\sqrt{-a^2cx^2+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2 + c)/a

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)

[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)

[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)

3.628 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)-2*(a*x+1)/a/(-a^2*c*x^2+
c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6141, 653, 217, 203}

$$\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*(1 + a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*
c*x^2]]/(a*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] && !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\right)$$
$$= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}} + \int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}} + \text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx, x, \frac{x}{\sqrt{c-a^2cx^2}}\right)$$
$$= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}} + \frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.39

$$-\frac{2\sqrt{1-a^2x^2}\left(\sqrt{ax+1}+\sqrt{1-ax}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\,\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*(Sqrt[1 + a*x] + Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/S
qrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.46, size = 153, normalized size = 2.59

$$\left[-\frac{(ax-1)\sqrt{-c}\,\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\,\left(a^2cx-ac\right)}, -\frac{(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{a^2cx-ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x - a*c), -((a*x - 1)*sqrt(c)*ar
ctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2
+ c))/(a^2*c*x - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.04, size = 79, normalized size = 1.34

$$\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}} + \frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{a^2c\left(x-\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x-1/a
)*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)
```

maxima [A] time = 0.40, size = 40, normalized size = 0.68

$$\frac{2\sqrt{-a^2cx^2+c}}{a^2cx-ac}+\frac{\arcsin(ax)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x - a*c) + arcsin(a*x)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a\,x+1}{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-c(ax-1)(ax+1)}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)), x)
```

3.629 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -2/3*(a*x+1)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6141, 653, 191}

$$-\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*(1 + a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\right)$$
$$= -\frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$
$$= -\frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}} - \frac{x}{3c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 1.25

$$-\frac{(2-ax)\sqrt{ax+1}\,\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/2}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/3*((2 - a*x)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)^(3/2)*Sqrt[
c - a^2*c*x^2])
```

fricas [A] time = 0.88, size = 47, normalized size = 0.92

$$\frac{\sqrt{-a^2cx^2+c}\,(ax-2)}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [B] time = 0.23, size = 148, normalized size = 2.90

$$\frac{\left(ac-3\sqrt{-a^2c}\sqrt{c}\right)\mathrm{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}-\sqrt{-a^2c}\,ac^2\right)} - \frac{2\left(2\,a^2c+3\,a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}+\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^3 c\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*c - 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) - sqrt(-a^2*c)*a*c^2
) - 2/3*(2*a^2*c + 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqrt
(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) + sqrt(-a^2*c + c/x^2) - sqrt(
c)/x)^3*c*sgn(x))
```

maple [A] time = 0.04, size = 31, normalized size = 0.61

$$\frac{(ax+1)^2\,(ax-2)}{3a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/3*(a*x+1)^2*(a*x-2)/a/(-a^2*c*x^2+c)^(3/2)
```

maxima [A] time = 0.30, size = 61, normalized size = 1.20

$$-\frac{x}{3\sqrt{-a^2cx^2+c}\,c}+\frac{2}{3\left(\sqrt{-a^2cx^2+c}\,a^2cx-\sqrt{-a^2cx^2+c}\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x - sqrt(
-a^2*c*x^2 + c)*a*c)
```

mupad [B] time = 1.29, size = 33, normalized size = 0.65

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-2\right)}{3\,a\,c^2\,{\left(a\,x-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(3/2)*(a*x - 1)),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x - 2))/(3*a*c^2*(a*x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x - 1)), x)
```

3.630 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -2/5*(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c^2/
(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6141, 653, 192, 191}

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*(1 + a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2))
- (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx\right) \\
&= -\frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}-\frac{3}{5}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx \\
&= -\frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{5c} \\
&= -\frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.72

$$
-\frac{2a^3x^3-4a^2x^2+ax+2}{5ac^2(ax-1)^2\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/5*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(a*c^2*(-1 + a*x)^2*Sqrt[c - a^2*c*x
^2])
```

fricas [A] time = 0.74, size = 75, normalized size = 1.01

$$
\frac{\left(2\,a^3x^3-4\,a^2x^2+ax+2\right)\sqrt{-a^2cx^2+c}}{5\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 - 2
*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax-1)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x - 1)), x)
```

maple [A] time = 0.04, size = 47, normalized size = 0.64

$$
-\frac{(ax+1)^2\left(2x^3a^3-4a^2x^2+ax+2\right)}{5a\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/5*(a*x+1)^2*(2*a^3*x^3-4*a^2*x^2+a*x+2)/a/(-a^2*c*x^2+c)^(5/2)
```

maxima [A] time = 0.32, size = 80, normalized size = 1.08

$$\frac{2}{5\left(\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^2cx-\left(-a^2cx^2+c\right)^{\frac{3}{2}}ac\right)}-\frac{2\,x}{5\,\sqrt{-a^2cx^2+c}\,c^2}-\frac{x}{5\left(-a^2cx^2+c\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x - (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/(
sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

mupad [B] time = 1.38, size = 56, normalized size = 0.76

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2\right)}{5\,a\,c^3\,{\left(a\,x-1\right)}^3\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(5/2)*(a*x - 1)),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2))/(5*a*c^3*(a*x - 1
)^3*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x - 1)), x)
```

3.631 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] -2/7*(a*x+1)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/c^2
/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6141, 653, 192, 191}

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-2*(1 + a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2))
- (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{9/2}}\,dx\right)$$
$$= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{5}{7}\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx$$
$$= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{7c}$$
$$= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} - \frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{21c^2}$$
$$= -\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.99

$$-\frac{\sqrt{1-a^2x^2}\left(-8a^5x^5+16a^4x^4+4a^3x^3-24a^2x^2+9ax+6\right)}{21ac^3(1-ax)^{7/2}(ax+1)^{3/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/21*(Sqrt[1 - a^2*x^2]*(6 + 9*a*x - 24*a^2*x^2 + 4*a^3*x^3 + 16*a^4*x^4 -
 8*a^5*x^5))/(a*c^3*(1 - a*x)^(7/2)*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.82, size = 124, normalized size = 1.28

$$\frac{\left(8\,a^5x^5-16\,a^4x^4-4\,a^3x^3+24\,a^2x^2-9\,ax-6\right)\sqrt{-a^2cx^2+c}}{21\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(8*a^5*x^5 - 16*a^4*x^4 - 4*a^3*x^3 + 24*a^2*x^2 - 9*a*x - 6)*sqrt(-a^
2*c*x^2 + c)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a
^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x - 1)), x)
```

maple [A] time = 0.04, size = 64, normalized size = 0.66

$$\frac{(ax+1)^2\left(8x^5a^5-16x^4a^4-4x^3a^3+24a^2x^2-9ax-6\right)}{21a\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/21*(a*x+1)^2*(8*a^5*x^5-16*a^4*x^4-4*a^3*x^3+24*a^2*x^2-9*a*x-6)/a/(-a^2*
c*x^2+c)^(7/2)
```

maxima [A] time = 0.32, size = 99, normalized size = 1.02

$$\frac{2}{7\left(\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^2cx-\left(-a^2cx^2+c\right)^{\frac{5}{2}}ac\right)}-\frac{8\,x}{21\,\sqrt{-a^2cx^2+c}\,c^3}-\frac{4\,x}{21\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^2}-\frac{x}{7\left(-a^2cx^2+c\right)^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x - (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x/
(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/((
-a^2*c*x^2 + c)^(5/2)*c)
```

mupad [B] time = 1.45, size = 134, normalized size = 1.38

$$\frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,(a\,x-1)^3}-\frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,(a\,x-1)^4}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}+\frac{5}{28\,a\,c^4}\right)}{(a\,x-1)^2\,(a\,x+1)^2}+\frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(7/2)*(a*x - 1)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x - 1)^3) - (c - a^2*c*x^2)^(1/2)/(28*a*
c^4*(a*x - 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) + 5/(28*a*c^4)))
/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x - 1)*
(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{7}{2}}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x - 1)), x)
```

3.632 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2(ax+1)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

```
[Out] -2/9*(a*x+1)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/c^2
/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2*c*x
^2+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6141, 653, 192, 191}

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2(ax+1)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-2*(1 + a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2))
- (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2
)) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{\left(c-a^2cx^2\right)^{11/2}}\,dx\right)$$
$$= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{7}{9}\int \frac{1}{\left(c-a^2cx^2\right)^{9/2}}\,dx$$
$$= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx}{3c}$$
$$= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{15c^2}$$
$$= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{16\int -}{}$$
$$= -\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{}{45c^4\sqrt{}}$$

Mathematica [A] time = 0.07, size = 112, normalized size = 0.93

$$\frac{\sqrt{1-a^2x^2}\left(-16a^7x^7+32a^6x^6+24a^5x^5-80a^4x^4+10a^3x^3+60a^2x^2-25ax-10\right)}{45ac^4(1-ax)^{9/2}(ax+1)^{5/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-10 - 25*a*x + 60*a^2*x^2 + 10*a^3*x^3 - 80*a^4*x^4 + 2
4*a^5*x^5 + 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(9/2)*(1 + a*x)^(
5/2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 1.84, size = 152, normalized size = 1.27

$$\frac{\left(16\,a^7x^7-32\,a^6x^6-24\,a^5x^5+80\,a^4x^4-10\,a^3x^3-60\,a^2x^2+25\,ax+10\right)\sqrt{-a^2cx^2+c}}{45\left(a^9c^5x^8-2\,a^8c^5x^7-2\,a^7c^5x^6+6\,a^6c^5x^5-6\,a^4c^5x^3+2\,a^3c^5x^2+2\,a^2c^5x-ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/45*(16*a^7*x^7 - 32*a^6*x^6 - 24*a^5*x^5 + 80*a^4*x^4 - 10*a^3*x^3 - 60*a
^2*x^2 + 25*a*x + 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 - 2
*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5*x
- a*c^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x - 1)), x)
```

maple [A] time = 0.04, size = 80, normalized size = 0.67

$$-\frac{(ax+1)^2\left(16a^7x^7-32x^6a^6-24x^5a^5+80x^4a^4-10x^3a^3-60a^2x^2+25ax+10\right)}{45a\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] -1/45*(a*x+1)^2*(16*a^7*x^7-32*a^6*x^6-24*a^5*x^5+80*a^4*x^4-10*a^3*x^3-60*
a^2*x^2+25*a*x+10)/a/(-a^2*c*x^2+c)^(9/2)
```

maxima [A] time = 0.32, size = 118, normalized size = 0.98

$$\frac{2}{9\left(\left(-a^2cx^2+c\right)^{\frac{7}{2}}a^2cx-\left(-a^2cx^2+c\right)^{\frac{7}{2}}ac\right)}-\frac{16\,x}{45\,\sqrt{-a^2cx^2+c}\,c^4}-\frac{8\,x}{45\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^3}-\frac{2\,x}{15\left(-a^2cx^2+c\right)^{\frac{5}{2}}c^2}-\frac{}{9\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x - (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*x
/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*x/
((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

mupad [B] time = 1.51, size = 177, normalized size = 1.48

$$\frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,(a\,x-1)^5}-\frac{5\,\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,(a\,x-1)^4}+\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}+\frac{5}{24\,a\,c^5}\right)}{(a\,x-1)^3\,(a\,x+1)^3}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}-\frac{5}{144\,a\,c^5}\right)}{(a\,x-1)^2\,(a\,x+1)^2}+\frac{16\,x\,\sqrt{c}}{45\,c^5\,(a\,x-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(9/2)*(a*x - 1)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(72*a*c^5*(a*x - 1)^5) - (5*(c - a^2*c*x^2)^(1/2))/(1
44*a*c^5*(a*x - 1)^4) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) + 5/(24*a*
c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5) -
 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2))/(
45*c^5*(a*x - 1)*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{9}{2}}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x - 1)), x)
```

3.633 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=185

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[Out] -8/7*(a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(a*x+1
)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(a*x+1)^9*(-a^2*c
*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c)^(
9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A] time = 0.20, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-8*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (2*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^3(1+ax)^6\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(-8(1+ax)^6+12(1+ax)^7-6(1+ax)^8+(1+ax)^9\right)dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=-\frac{8(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1+ax)^9\left(c-a^2cx\right.}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.38

$$\frac{c^4(ax+1)^7\left(21a^3x^3-77a^2x^2+98ax-44\right)\sqrt{c-a^2cx^2}}{210a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(-44 + 98*a*x - 77*a^2*x^2 + 21*a^3*x^
3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.67, size = 95, normalized size = 0.51

$$\frac{\left(21\,a^9c^4x^{10}+70\,a^8c^4x^9-240\,a^6c^4x^7-210\,a^5c^4x^6+252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2-210\,c^4x\right)\sqrt{-a^2c}}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fric
as")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 + 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 - 210*c^4*x)*sqrt(-a^2
*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 100, normalized size = 0.54

$$\frac{x\left(21a^9x^9+70x^8a^8-240x^6a^6-210x^5a^5+252x^4a^4+420x^3a^3-315ax-210\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}{210\left(ax-1\right)^3\left(ax+1\right)^6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] 1/210*x*(21*a^9*x^9+70*a^8*x^8-240*a^6*x^6-210*a^5*x^5+252*a^4*x^4+420*a^3*
x^3-315*a*x-210)*(-a^2*c*x^2+c)^(9/2)/(a*x-1)^3/(a*x+1)^6/((a*x-1)/(a*x+1))
^(3/2)
```

maxima [A] time = 0.36, size = 204, normalized size = 1.10

$$\frac{\left(21\,a^{11}\sqrt{-c}\,c^4x^{11}+49\,a^{10}\sqrt{-c}\,c^4x^{10}-70\,a^9\sqrt{-c}\,c^4x^9-240\,a^8\sqrt{-c}\,c^4x^8+30\,a^7\sqrt{-c}\,c^4x^7+462\,a^6\sqrt{-c}\,c^4x^6+168}{210\left(a^3x^2+2\,a^2x+a\right)(ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(21*a^11*sqrt(-c)*c^4*x^11 + 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt(
-c)*c^4*x^9 - 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 + 462*a^6*
sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 - 420*a^4*sqrt(-c)*c^4*x^4 - 31
5*a^3*sqrt(-c)*c^4*x^3 + 105*a^2*sqrt(-c)*c^4*x^2 + 210*sqrt(-c)*c^4)*(a*x
+ 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{9/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

3.634 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=139

$$\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[Out] 2/3*(a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(a*x+1)^
7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*x^
2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx=\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=\frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^2(1+ax)^5\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=\frac{\left(c-a^2cx^2\right)^{7/2}\int\left(4(1+ax)^5-4(1+ax)^6+(1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.45

$$-\frac{c^3(ax+1)^6\left(21a^2x^2-54ax+37\right)\sqrt{c-a^2cx^2}}{168a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/168*(c^3*(1 + a*x)^6*(37 - 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a^
2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.40, size = 95, normalized size = 0.68

$$-\frac{\left(21\,a^7c^3x^8+72\,a^6c^3x^7+28\,a^5c^3x^6-168\,a^4c^3x^5-210\,a^3c^3x^4+56\,a^2c^3x^3+252\,ac^3x^2+168\,c^3x\right)\sqrt{-a^2c}}{168\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 + 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 - 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 + 56*a^2*c^3*x^3 + 252*a*c^3*x^2 + 168*c^3*x)*sqrt(-a^2*c
)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.04, size = 100, normalized size = 0.72

$$\frac{x\left(21a^7x^7+72x^6a^6+28x^5a^5-168x^4a^4-210x^3a^3+56a^2x^2+252ax+168\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{168\left(ax-1\right)^2\left(ax+1\right)^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/168*x*(21*a^7*x^7+72*a^6*x^6+28*a^5*x^5-168*a^4*x^4-210*a^3*x^3+56*a^2*x^
2+252*a*x+168)*(-a^2*c*x^2+c)^(7/2)/(a*x-1)^2/(a*x+1)^5/((a*x-1)/(a*x+1))^(
3/2)
```

maxima [A] time = 0.34, size = 172, normalized size = 1.24

$$-\frac{\left(21\,a^9\sqrt{-c}\,c^3x^9+51\,a^8\sqrt{-c}\,c^3x^8-44\,a^7\sqrt{-c}\,c^3x^7-196\,a^6\sqrt{-c}\,c^3x^6-42\,a^5\sqrt{-c}\,c^3x^5+266\,a^4\sqrt{-c}\,c^3x^4+196}{168\left(a^3x^2+2\,a^2x+a\right)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] -1/168*(21*a^9*sqrt(-c)*c^3*x^9 + 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c)
*c^3*x^7 - 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 + 266*a^4*sqr
t(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 - 84*a^2*sqrt(-c)*c^3*x^2 - 168*sq
rt(-c)*c^3)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.635 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=93

$$\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] -2/5*(a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1)
^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx=\frac{\left(c-a^2cx^2\right)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$=\frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)(1+ax)^4\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$=\frac{\left(c-a^2cx^2\right)^{5/2}\int\left(-2(1+ax)^4+(1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$=-\frac{2(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.59

$$\frac{c^2(ax+1)^5(5ax-7)\sqrt{c-a^2cx^2}}{30a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
[Out] (c^2*(1 + a*x)^5*(-7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2*
x^2)]*x)
```

fricas [A] time = 0.55, size = 73, normalized size = 0.78

$$\frac{\left(5\,a^5c^2x^6+18\,a^4c^2x^5+15\,a^3c^2x^4-20\,a^2c^2x^3-45\,ac^2x^2-30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
[Out] 1/30*(5*a^5*c^2*x^6 + 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.04, size = 84, normalized size = 0.90

$$\frac{x\left(5x^5a^5+18x^4a^4+15x^3a^3-20a^2x^2-45ax-30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30\,(ax-1)\,(ax+1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5+18*a^4*x^4+15*a^3*x^3-20*a^2*x^2-45*a*x-30)*(-a^2*c*x^2+c
)^(5/2)/(a*x-1)/(a*x+1)^4/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.34, size = 140, normalized size = 1.51

$$\frac{\left(5\,a^7\sqrt{-c}\,c^2x^7+13\,a^6\sqrt{-c}\,c^2x^6-3\,a^5\sqrt{-c}\,c^2x^5-35\,a^4\sqrt{-c}\,c^2x^4-25\,a^3\sqrt{-c}\,c^2x^3+15\,a^2\sqrt{-c}\,c^2x^2+30\,\sqrt{-c}\,c^2\right)(a}{30\left(a^3x^2+2\,a^2x+a\right)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] 1/30*(5*a^7*sqrt(-c)*c^2*x^7 + 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^2
*x^5 - 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 + 15*a^2*sqrt(-c)*
c^2*x^2 + 30*sqrt(-c)*c^2)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.636 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=46

$$\frac{(ax+1)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

[Out] 1/4*(a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3

Rubi [A] time = 0.18, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 32}

$$\frac{(ax+1)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]

[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx&=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\&=\frac{\left(c-a^2cx^2\right)^{3/2}\int(1+ax)^3\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\&=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.26

$$-\frac{c\left(a^3x^3+4a^2x^2+6ax+4\right)\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/4*(c*Sqrt[c - a^2*c*x^2]*(4 + 6*a*x + 4*a^2*x^2 + a^3*x^3))/(a*Sqrt[1 -
1/(a^2*x^2)])
```

fricas [A] time = 0.91, size = 42, normalized size = 0.91

$$-\frac{\left(a^3cx^4+4\,a^2cx^3+6\,acx^2+4\,cx\right)\sqrt{-a^2c}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*(a^3*c*x^4 + 4*a^2*c*x^3 + 6*a*c*x^2 + 4*c*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.04, size = 60, normalized size = 1.30

$$\frac{x\left(x^3a^3+4a^2x^2+6ax+4\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{4\left(ax+1\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*x*(a^3*x^3+4*a^2*x^2+6*a*x+4)*(-a^2*c*x^2+c)^(3/2)/(a*x+1)^3/((a*x-1)/(
a*x+1))^(3/2)
```

maxima [B] time = 0.34, size = 97, normalized size = 2.11

$$-\frac{\left(a^5\sqrt{-c}\,cx^5+3\,a^4\sqrt{-c}\,cx^4+2\,a^3\sqrt{-c}\,cx^3-2\,a^2\sqrt{-c}\,cx^2-4\,\sqrt{-c}\,c\right)(ax+1)^2}{4\left(a^3x^2+2\,a^2x+a\right)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -1/4*(a^5*sqrt(-c)*c*x^5 + 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 - 2*
a^2*sqrt(-c)*c*x^2 - 4*sqrt(-c)*c)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*
x - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.637 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=113

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(3+ax+\frac{4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.50

$$\frac{\sqrt{c-a^2cx^2}\,(ax(ax+6)+8\log(1-ax))}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/(a
^2*x^2)]*x)
```

fricas [A] time = 0.66, size = 33, normalized size = 0.29

$$\frac{\left(a^2x^2+6\,ax+8\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 67, normalized size = 0.59

$$\frac{\left(a^2x^2+6ax+8\ln\left(ax-1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\;(ax-1)}{2a\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - a^2\,c\,x^2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.638 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] 2*x*(1-1/a^2/x^2)^(1/2)/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+x*ln(-a*x+1)*(1-1/a^2
/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{1+ax}{(-1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{(1-ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.67

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,((ax-1)\log(1-ax)-2)}{(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + (-1 + a*x)*Log[1 - a*x]))/((-1 + a*x)*Sqrt[c
 - a^2*c*x^2])
```

fricas [A] time = 0.56, size = 39, normalized size = 0.49

$$-\frac{\sqrt{-a^2c}\left((ax-1)\log(ax-1)-2\right)}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] -sqrt(-a^2*c)*((a*x - 1)*log(a*x - 1) - 2)/(a^3*c*x - a^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 64, normalized size = 0.81

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax-1)\,xa-\ln(ax-1)-2\right)}{ac\,(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-ln(a*x-1)-2)/a/c/(a*x+1)^2/((a*x-1)/
(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2cx^2 + c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c - a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

3.639 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 32}

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 1.09

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-a^2cx^2}}{2c^2(ax-1)^3(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)^3*(1 + a
*x))
```

fricas [A] time = 0.46, size = 39, normalized size = 0.83

$$-\frac{\sqrt{-a^2c}}{2\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.04, size = 39, normalized size = 0.83

$$-\frac{ax-1}{2a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(a*x-1)/a/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [B] time = 1.55, size = 90, normalized size = 1.91

$$\frac{\left(\frac{1}{2\,a^3\,c}+\frac{x}{2\,a^2\,c}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}+x^2\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x\,\sqrt{c-a^2\,c\,x^2}}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((1/(2*a^3*c) + x/(2*a^2*c))*((a*x - 1)/(a*x + 1))^(1/2))/((c - a^2*c*x^2)^
(1/2)/a^2 + x^2*(c - a^2*c*x^2)^(1/2) - (2*x*(c - a^2*c*x^2)^(1/2))/a)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.640 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] 1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-
1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^
(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arc
tanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.20, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 - a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^4(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{2(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{1}{8(-1+ax)^2}-\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^5\left(1-\right.\right.}{8}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\right.}{8}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.38

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2+9ax+3(ax-1)^3\tanh^{-1}(ax)-10\right)}{24c^2(ax-1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh[
a*x]))/(24*c^2*(-1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.50, size = 139, normalized size = 0.75

$$-\frac{3\left(a^4x^3-3\,a^3x^2+3\,a^2x-a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)-2\left(3\,a^2x^2-9\,ax+10\right)\sqrt{-a^2c}}{48\left(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/48*(3*(a^4*x^3 - 3*a^3*x^2 + 3*a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sq
rt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 - 9*a*x + 10)*sqrt
(-a^2*c))/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")

[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

maple [A] time = 0.07, size = 169, normalized size = 0.91

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(3\ln\left(ax-1\right)x^3a^3-3a^3x^3\ln\left(ax+1\right)-9\ln\left(ax-1\right)x^2a^2+9\ln\left(ax+1\right)x^2a^2+6a^2x^2+9\ln\left(\right.\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax-1\right)\left(ax+1\right)\left(a^2x^2-1\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x)

[Out] 1/48/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a
*x-1)*x^3*a^3-3*a^3*x^3*ln(a*x+1)-9*ln(a*x-1)*x^2*a^2+9*ln(a*x+1)*x^2*a^2+6
*a^2*x^2+9*ln(a*x-1)*x*a-9*a*x*ln(a*x+1)-18*a*x-3*ln(a*x-1)+3*ln(a*x+1)+20)
/(a^2*x^2-1)/c^3/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")

[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)

[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)

[Out] Timed out

3.641 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=278

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a}{16(1}$$

```
[Out] -1/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6*
(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2/x
^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x
^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(
-a^2*c*x^2+c)^(7/2)-5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x
^2+c)^(7/2)
```

Rubi [A] time = 0.23, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a}{16(1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^4*(c - a^2*c*x^2)^(7/2)) -
 (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2)) -
 (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
 - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) +
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) - (5
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^5(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int\left(\frac{1}{4(-1+ax)^5}-\frac{1}{4(-1+ax)^4}+\frac{3}{16(-1+ax)^3}-\frac{1}{8(-1+ax)^2}-\frac{1}{32(1+ax)^2}+\frac{5}{32(-1+a}\right.}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-$$

$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-$$

Mathematica [A] time = 0.09, size = 99, normalized size = 0.36

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-15a^4x^4+45a^3x^3-35a^2x^2-15ax+15(ax-1)^4(ax+1)\tanh^{-1}(ax)+32\right)}{96c^3(ax-1)^4(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(32 - 15*a*x - 35*a^2*x^2 + 45*a^3*x^3 - 15*a^4*x^
4 + 15*(-1 + a*x)^4*(1 + a*x)*ArcTanh[a*x]))/(96*c^3*(-1 + a*x)^4*(1 + a*x)
*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.61, size = 190, normalized size = 0.68

$$-\frac{15\left(a^6x^5-3\,a^5x^4+2\,a^4x^3+2\,a^3x^2-3\,a^2x+a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)-2\left(15\,a^4x^4-45\,a^3x^3+35\,a^2\right.}{192\left(a^7c^4x^5-3\,a^6c^4x^4+2\,a^5c^4x^3+2\,a^4c^4x^2-3\,a^3c^4x+a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/192*(15*(a^6*x^5 - 3*a^5*x^4 + 2*a^4*x^3 + 2*a^3*x^2 - 3*a^2*x + a)*sqrt
(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15
*a^4*x^4 - 45*a^3*x^3 + 35*a^2*x^2 + 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4*x^
5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.07, size = 241, normalized size = 0.87

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(15\ln\left(ax-1\right)x^5a^5-15\ln\left(ax+1\right)x^5a^5-45\ln\left(ax-1\right)x^4a^4+45\ln\left(ax+1\right)x^4a^4+30x^4a^4+30\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(1
5*ln(a*x-1)*x^5*a^5-15*ln(a*x+1)*x^5*a^5-45*ln(a*x-1)*x^4*a^4+45*ln(a*x+1)*
x^4*a^4+30*x^4*a^4+30*ln(a*x-1)*x^3*a^3-30*a^3*x^3*ln(a*x+1)-90*x^3*a^3+30*
ln(a*x-1)*x^2*a^2-30*ln(a*x+1)*x^2*a^2+70*a^2*x^2-45*ln(a*x-1)*x*a+45*a*x*l
n(a*x+1)+30*a*x+15*ln(a*x-1)-15*ln(a*x+1)-64)/(a^2*x^2-1)/c^4/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.642 $\int e^{-\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{9/2} dx$

Optimal. Leaf size=234

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(1\cdots}{3\cdots}$$

```
[Out] 8/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(-a*x
+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(-a*x+1)^8*(-a^2*
c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(-a*x+1)^9*(-a^2*c*x^2+c)^(
9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x^2+c)^(9/2)/a^1
0/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A] time = 0.21, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(1\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (8*(1 - a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^5(1+ax)^4\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(16(-1+ax)^5+32(-1+ax)^6+24(-1+ax)^7+8(-1+ax)^8+\right.}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{8(1-ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.34

$$\frac{c^4(ax-1)^6\left(63a^4x^4+308a^3x^3+588a^2x^2+528ax+193\right)\sqrt{c-a^2cx^2}}{630a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(-1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 + 528*a*x + 588*a^2*x^2 + 308*a^
3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.65, size = 117, normalized size = 0.50

$$\frac{\left(63\,a^9c^4x^{10}-70\,a^8c^4x^9-315\,a^7c^4x^8+360\,a^6c^4x^7+630\,a^5c^4x^6-756\,a^4c^4x^5-630\,a^3c^4x^4+840\,a^2c^4x^3+315\,ac^{4}\right.}{630\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/630*(63*a^9*c^4*x^10 - 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 + 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 - 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 + 840*a^2*c^4*x^3 +
315*a*c^4*x^2 - 630*c^4*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 116, normalized size = 0.50

$$\frac{x\left(63a^9x^9-70x^8a^8-315a^7x^7+360x^6a^6+630x^5a^5-756x^4a^4-630x^3a^3+840a^2x^2+315ax-630\right)\left(-a^2c\,x^2+\right.}{630\,(ax+1)^4\,(ax-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/630*x*(63*a^9*x^9-70*a^8*x^8-315*a^7*x^7+360*a^6*x^6+630*a^5*x^5-756*a^4*
x^4-630*a^3*x^3+840*a^2*x^2+315*a*x-630)*(-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x
+1))^(1/2)/(a*x+1)^4/(a*x-1)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-a^2\,c\,x^2\right)}^{9/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.643 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=187

$$\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(1-ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[Out] -8/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(-a*x+1)
^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(-a*x+1)^7*(-a^2*c*
x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2*c*x^2+c)^(7/2
)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(1-ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-8*(1 - a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (6*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{7/2} dx = \frac{\left(c-a^2cx^2\right)^{7/2} \int e^{-\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7 dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7}$$
$$= \frac{\left(c-a^2cx^2\right)^{7/2} \int (-1+ax)^4(1+ax)^3 dx}{a^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7}$$
$$= \frac{\left(c-a^2cx^2\right)^{7/2} \int \left(8(-1+ax)^4+12(-1+ax)^5+6(-1+ax)^6+(-1+ax)^7\right) dx}{a^7 \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7}$$
$$= -\frac{8(1-ax)^5 \left(c-a^2cx^2\right)^{7/2}}{5a^8 \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7} + \frac{2(1-ax)^6 \left(c-a^2cx^2\right)^{7/2}}{a^8 \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7} - \frac{6(1-ax)^7 \left(c-a^2cx^2\right)^{7/2}}{7a^8 \left(1-\frac{1}{a^2x^2}\right)^{7/2} x^7}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.38

$$-\frac{c^3(ax-1)^5 \left(35a^3x^3+135a^2x^2+185ax+93\right) \sqrt{c-a^2cx^2}}{280a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/280*(c^3*(-1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(93 + 185*a*x + 135*a^2*x^2 +
35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.61, size = 95, normalized size = 0.51

$$-\frac{\left(35\,a^7c^3x^8-40\,a^6c^3x^7-140\,a^5c^3x^6+168\,a^4c^3x^5+210\,a^3c^3x^4-280\,a^2c^3x^3-140\,ac^3x^2+280\,c^3x\right)\sqrt{-a^2c}}{280\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 - 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 + 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 - 280*a^2*c^3*x^3 - 140*a*c^3*x^2 + 280*c^3*x)*sqrt(-a^2
*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^{\frac{7}{2}} \sqrt{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 100, normalized size = 0.53

$$\frac{x\left(35a^7x^7-40x^6a^6-140x^5a^5+168x^4a^4+210x^3a^3-280a^2x^2-140ax+280\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}{280\,(ax+1)^3\,(ax-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/280*x*(35*a^7*x^7-40*a^6*x^6-140*a^5*x^5+168*a^4*x^4+210*a^3*x^3-280*a^2*
x^2-140*a*x+280)*(-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^3/(a*
x-1)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-a^2\,c\,x^2\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.644 $\int e^{-\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{5/2} dx$

Optimal. Leaf size=139

$$\frac{(1-ax)^6 \left(c-a^2cx^2\right)^{5/2}}{6a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(1-ax)^5 \left(c-a^2cx^2\right)^{5/2}}{5a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^4 \left(c-a^2cx^2\right)^{5/2}}{a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] (-a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(-a*x+1)^5*
(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x+1)^6*(-a^2*c*x^2
+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^6 \left(c-a^2cx^2\right)^{5/2}}{6a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(1-ax)^5 \left(c-a^2cx^2\right)^{5/2}}{5a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^4 \left(c-a^2cx^2\right)^{5/2}}{a^6x^5 \left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx = \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^3(1+ax)^2\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(4(-1+ax)^3+4(-1+ax)^4+(-1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5} - \frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5} + \frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.45

$$\frac{c^2(ax-1)^4\left(5a^2x^2+14ax+11\right)\sqrt{c-a^2cx^2}}{30a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sq
rt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.43, size = 73, normalized size = 0.53

$$\frac{\left(5\,a^5c^2x^6-6\,a^4c^2x^5-15\,a^3c^2x^4+20\,a^2c^2x^3+15\,ac^2x^2-30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 - 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 84, normalized size = 0.60

$$\frac{x\left(5x^5a^5-6x^4a^4-15x^3a^3+20a^2x^2+15ax-30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{30\,(ax+1)^2\,(ax-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*(-a^2*c*x^2+c)
^(5/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^2/(a*x-1)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.645 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] -2/3*(-a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(-a*x+
1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*(1 - a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int(-1+ax)^2(1+ax)\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int\left(2(-1+ax)^2+(-1+ax)^3\right)dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.56

$$-\frac{c(ax-1)^3(3ax+5)\sqrt{c-a^2cx^2}}{12a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
[Out] -1/12*(c*(-1 + a*x)^3*(5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

fricas [A] time = 0.77, size = 43, normalized size = 0.45

$$-\frac{\left(3\,a^3cx^4-4\,a^2cx^3-6\,acx^2+12\,cx\right)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
[Out] -1/12*(3*a^3*c*x^4 - 4*a^2*c*x^3 - 6*a*c*x^2 + 12*c*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 68, normalized size = 0.72

$$\frac{x\left(3x^3a^3-4a^2x^2-6ax+12\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}{12\,(ax+1)\,(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/12*x*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*(-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1)
)^(1/2)/(a*x+1)/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.646 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6192, 6193}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx&=\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\&=\frac{\sqrt{c-a^2cx^2}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\&=-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.59

$$\frac{(ax-2)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] ((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 1.17, size = 22, normalized size = 0.32

$$\frac{\sqrt{-a^2c}\left(ax^2-2x\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 44, normalized size = 0.64

$$\frac{x(ax-2)\sqrt{-a^2cx^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-a^2cx^2}\,\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.647 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=37

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

```
[Out] x*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 31}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{1}{1+ax}\,dx}{\sqrt{c-a^2cx^2}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

fricas [A] time = 0.49, size = 22, normalized size = 0.59

$$-\frac{\sqrt{-a^2c}\log(ax+1)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c)*log(a*x + 1)/(a^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

maple [A] time = 0.05, size = 51, normalized size = 1.38

$$-\frac{\ln(ax+1)\sqrt{-c\left(a^2x^2-1\right)}\sqrt{\frac{ax-1}{ax+1}}}{ac(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -ln(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/a/c/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.648 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a^2*(1-1/a
^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 + a*x)*(c - a^2*c*x^2)^(3/2)) - (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(-\frac{1}{2(1+ax)^2}+\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}} - \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.60

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left((ax+1)\tanh^{-1}(ax)-1\right)}{2(acx+c)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (1 + a*x)*ArcTanh[a*x]))/(2*(c + a*c*x)*Sqrt
[c - a^2*c*x^2])
```

fricas [A] time = 0.59, size = 83, normalized size = 0.92

$$-\frac{\left(a^2x+a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}\,x+c}{a^2x^2-1}\right)-2\sqrt{-a^2c}}{4\left(a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/4*((a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) - 2*sqrt(-a^2*c))/(a^3*c^2*x + a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.06, size = 84, normalized size = 0.93

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)+\ln\left(ax-1\right)-\ln\left(ax+1\right)+2\right)}{4\left(a^2x^2-1\right)c^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-a*x*ln(a*
x+1)+ln(a*x-1)-ln(a*x+1)+2)/(a^2*x^2-1)/c^2/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.649 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] 1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1/
a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1-1/a^2/x^2)^(5/
2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arctanh
(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.20, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) - (a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (3*a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^2(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{8(-1+ax)^2}+\frac{1}{4(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(3a^5\left(1\right.\right.}{}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{3a^4\left(1\right.}{}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.44

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2-3ax+3(ax-1)(ax+1)^2\tanh^{-1}(ax)+2\right)}{8(ax-1)(acx+c)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)*(1 + a*x)^2*
ArcTanh[a*x]))/(8*(-1 + a*x)*(c + a*c*x)^2*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.52, size = 137, normalized size = 0.75

$$-\frac{3\left(a^4x^3+a^3x^2-a^2x-a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(3a^2x^2+3ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/16*(3*(a^4*x^3 + a^3*x^2 - a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 + 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.07, size = 169, normalized size = 0.92

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(3\ln\left(ax-1\right)x^3a^3-3a^3x^3\ln\left(ax+1\right)+3\ln\left(ax-1\right)x^2a^2-3\ln\left(ax+1\right)x^2a^2+6a^2x^2-3\right.}{16\left(ax+1\right)\left(a^2x^2-1\right)c^3a\left(ax-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x^
3*a^3-3*a^3*x^3*ln(a*x+1)+3*ln(a*x-1)*x^2*a^2-3*ln(a*x+1)*x^2*a^2+6*a^2*x^2
-3*ln(a*x-1)*x*a+3*a*x*ln(a*x+1)+6*a*x-3*ln(a*x-1)+3*ln(a*x+1)-4)/(a^2*x^2-
1)/c^3/a/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.650 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=276

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+$$

```
[Out] -1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(
1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/24*a^6*(1-1/a^2/x^2)
^(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/
(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a
^2*c*x^2+c)^(7/2)-5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2
+c)^(7/2)
```

Rubi [A] time = 0.22, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2)) -
 (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) + (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) + (3
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) -
(5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$
$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^3(1+ax)^4}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$
$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{16(-1+ax)^3}-\frac{1}{8(-1+ax)^2}-\frac{1}{8(1+ax)^4}-\frac{3}{16(1+ax)^3}-\frac{3}{16(1+ax)^2}+\frac{5}{16\left(-1+a^2x^2\right)}\right.}{\left(c-a^2cx^2\right)^{7/2}}$$
$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3}{32(1}$$
$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3}{32(1}$$

Mathematica [A] time = 0.10, size = 99, normalized size = 0.36

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-15a^4x^4-15a^3x^3+25a^2x^2+25ax+15(ax-1)^2(ax+1)^3\tanh^{-1}(ax)-8\right)}{48(ax-1)^2(acx+c)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-8 + 25*a*x + 25*a^2*x^2 - 15*a^3*x^3 - 15*a^4*x^
4 + 15*(-1 + a*x)^2*(1 + a*x)^3*ArcTanh[a*x]))/(48*(-1 + a*x)^2*(c + a*c*x)
^3*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.43, size = 186, normalized size = 0.67

$$-\frac{15\left(a^6x^5+a^5x^4-2\,a^4x^3-2\,a^3x^2+a^2x+a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)-2\left(15\,a^4x^4+15\,a^3x^3-25\,a^2x^2-2\right.}{96\left(a^7c^4x^5+a^6c^4x^4-2\,a^5c^4x^3-2\,a^4c^4x^2+a^3c^4x+a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/96*(15*(a^6*x^5 + a^5*x^4 - 2*a^4*x^3 - 2*a^3*x^2 + a^2*x + a)*sqrt(-c)*
log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15*a^4*
x^4 + 15*a^3*x^3 - 25*a^2*x^2 - 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 + a^
6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 + a^3*c^4*x + a^2*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

maple [A] time = 0.07, size = 241, normalized size = 0.87

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(a^2x^2-1\right)}\,\left(15\ln\left(ax-1\right)x^5a^5-15\ln\left(ax+1\right)x^5a^5+15\ln\left(ax-1\right)x^4a^4-15\ln\left(ax+1\right)x^4a^4+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/96*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x-1)
*x^5*a^5-15*ln(a*x+1)*x^5*a^5+15*ln(a*x-1)*x^4*a^4-15*ln(a*x+1)*x^4*a^4+30*
x^4*a^4-30*ln(a*x-1)*x^3*a^3+30*a^3*x^3*ln(a*x+1)+30*x^3*a^3-30*ln(a*x-1)*x
^2*a^2+30*ln(a*x+1)*x^2*a^2-50*a^2*x^2+15*ln(a*x-1)*x*a-15*a*x*ln(a*x+1)-50
*a*x+15*ln(a*x-1)-15*ln(a*x+1)+16)/(a^2*x^2-1)/c^4/a/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.651 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=131

$$-\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

```
[Out] -7/24*c*x*(-a^2*c*x^2+c)^(3/2)-7/30*(-a^2*c*x^2+c)^(5/2)/a-1/6*(-a*x+1)*(-a
^2*c*x^2+c)^(5/2)/a-7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a
-7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6142, 671, 641, 195, 217, 203}

$$-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7c^{5/2}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 - (7*(
c - a^2*c*x^2)^(5/2))/(30*a) - ((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx\\
&= -\left(c\int(1-ax)^2\left(c-a^2cx^2\right)^{3/2}dx\right)\\
&= -\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int(1-ax)\left(c-a^2cx^2\right)^{3/2}dx\\
&= -\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int\left(c-a^2cx^2\right)^{3/2}dx\\
&= -\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{8}\left(7c^2\right)\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-\right.}{6a}\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-\right.}{6a}\\
&= -\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-\right.}{6a}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 136, normalized size = 1.04

$$\frac{c^2\sqrt{c-a^2cx^2}\left(210\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\sqrt{ax+1}\left(40a^6x^6-136a^5x^5+86a^4x^4+202a^3x^3-327a^2x^2+39a\right.\right.}{240a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(96 + 39*a*x - 327*a^2*x^2 + 202*
a^3*x^3 + 86*a^4*x^4 - 136*a^5*x^5 + 40*a^6*x^6)) + 210*Sqrt[1 - a*x]*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.76, size = 241, normalized size = 1.84

$$\left[\frac{105\sqrt{-c}\,c^2\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\left(40\,a^5c^2x^5-96\,a^4c^2x^4-10\,a^3c^2x^3+192\,a^2c^2x^2-1\right.}{480\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) + 2*(40*a^5*c^2*x^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^
2*x^2 - 135*a*c^2*x - 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2)*a
rctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5 -
 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^2*x^2 - 135*a*c^2*x - 96*c^2)*
sqrt(-a^2*c*x^2 + c))/a]
```

giac [A] time = 0.17, size = 117, normalized size = 0.89

$$\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{16\,\sqrt{-c}\,|a|}-\frac{1}{240}\,\sqrt{-a^2cx^2+c}\left(\left(135\,c^2-2\left(96\,ac^2-\left(5\,a^2c^2-4\left(5\,a^4c^2x-12\,a^3c^2\right)x\right)x\right)x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
 - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 - 2*(96*a*c^2 - (5*a^2*c^2 - 4*(5*a
^4*c^2*x - 12*a^3*c^2)*x)*x)*x)*x + 96*c^2/a)
```

maple [B] time = 0.05, size = 226, normalized size = 1.73

$$\frac{x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{6}+\frac{5cx\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{24}+\frac{5c^2x\sqrt{-a^2c\,x^2+c}}{16}+\frac{5c^3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{16\sqrt{a^2c}}-\frac{2\left(-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)\right.}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/6*x*(-a^2*c*x^2+c)^(5/2)+5/24*c*x*(-a^2*c*x^2+c)^(3/2)+5/16*c^2*x*(-a^2*c
*x^2+c)^(1/2)+5/16*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^
(1/2))-2/5/a*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(5/2)-1/2*c*(-(x+1/a)^2*a^2*c
+2*(x+1/a)*a*c)^(3/2)*x-3/4*c^2*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)*x-3/
4*c^3/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)
^(1/2))
```

maxima [A] time = 0.41, size = 154, normalized size = 1.18

$$\frac{1}{6}\left(-a^2cx^2+c\right)^{\frac{5}{2}}x-\frac{7}{24}\left(-a^2cx^2+c\right)^{\frac{3}{2}}cx-\frac{3}{4}\sqrt{a^2cx^2+4\,acx+3\,c}\,c^2x+\frac{5}{16}\sqrt{-a^2cx^2+c}\,c^2x+\frac{3\,c^4\arcsin\left(ax+2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(a
^2*c*x^2 + 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x + 3/4*c^4
*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a - 2/5*(-a^2*c*
x^2 + c)^(5/2)/a - 3/2*sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c^2/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [C] time = 10.88, size = 478, normalized size = 3.65

$$a^4c^2\left(\begin{cases} \frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,x^3}{48a^2\sqrt{a^2x^2-1}} + \frac{i\sqrt{c}\,x}{16a^4\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{16a^5} & \text{for } |a^2x^2| > 1 \\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,x^3}{48a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{c}\,x}{16a^4\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,\operatorname{asin}(ax)}{16a^5} & \text{otherwise} \end{cases}\right) - 2a^3c^2\left(\begin{cases} \frac{x^4\sqrt{-a^2c}}{5} \\ \frac{\sqrt{c}\,x^4}{4} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt
(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
- 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(1
6*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
+ 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a*
*2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x
)/(16*a**5), True)) - 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
- x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4
), Ne(a, 0)), (sqrt(c)*x**4/4, True)) + 2*a*c**2*Piecewise((0, Eq(c, 0)), (
sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)
) - c**2*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)
*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1
), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asin(a*x)/(2*a), True))
```

3.652 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=108

$$-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

```
[Out] -5/12*(-a^2*c*x^2+c)^(3/2)/a-1/4*(-a*x+1)*(-a^2*c*x^2+c)^(3/2)/a-5/8*c^(3/2
)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6142, 671, 641, 195, 217, 203}

$$-\frac{5c^{3/2}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 - (5*(c - a^2*c*x^2)^(3/2))/(12*a) - ((1 - a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\left(c\int (1-ax)^2\sqrt{c-a^2cx^2}\,dx\right)\\
&= -\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int (1-ax)\sqrt{c-a^2cx^2}\,dx\\
&= -\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int \sqrt{c-a^2cx^2}\,dx\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\int \frac{}{\sqrt{}}\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\text{Sub}\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\tan^{-1}\left(\right.}{8a}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 117, normalized size = 1.08

$$\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{ax+1}\left(6a^4x^4-22a^3x^3+25a^2x^2+7ax-16\right)+30\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-16 + 7*a*x + 25*a^2*x^2 - 22*a^3*x^
3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqr
t[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.63, size = 180, normalized size = 1.67

$$\left[\frac{15\sqrt{-c}\,c\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\left(6a^3cx^3-16a^2cx^2+9acx+16c\right)\sqrt{-a^2cx^2+c}}{48a},\ \frac{15c^{\frac{3}{2}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) - 2*(6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c)
```

```
)/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
c)) - (6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c))/a
]
```

giac [A] time = 0.18, size = 85, normalized size = 0.79

$$-\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3a^2cx-8ac\right)x+9c\right)x+\frac{16c}{a}\right)+\frac{5c^2\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x - 8*a*c)*x + 9*c)*x + 16*c/a) + 5
/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

maple [A] time = 0.05, size = 176, normalized size = 1.63

$$\frac{x\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{4}+\frac{3cx\sqrt{-a^2cx^2+c}}{8}+\frac{3c^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{8\sqrt{a^2c}}-\frac{2\left(-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}{3a}-c\sqrt{-\left(x+\frac{1}{a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)+3/8*c*x*(-a^2*c*x^2+c)^(1/2)+3/8*c^2/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3/a*(-(x+1/a)^2*a^2*c+2*(
x+1/a)*a*c)^(3/2)-c*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)*x-c^2/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [A] time = 0.41, size = 130, normalized size = 1.20

$$\frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x-\sqrt{a^2cx^2+4acx+3c}\,cx+\frac{3}{8}\sqrt{-a^2cx^2+c}\,cx+\frac{c^3\arcsin\left(ax+2\right)}{a\left(-c\right)^{\frac{3}{2}}}+\frac{3c^{\frac{3}{2}}\arcsin\left(ax\right)}{8a}-\frac{2\left(-a^2cx^2+\right.}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c*x + 3/8*sq
rt(-a^2*c*x^2 + c)*c*x + c^3*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)*a
rcsin(a*x)/a - 2/3*(-a^2*c*x^2 + c)^(3/2)/a - 2*sqrt(a^2*c*x^2 + 4*a*c*x +
3*c)*c/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c-a^2cx^2\right)^{3/2}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [C] time = 8.16, size = 340, normalized size = 3.15

$$-a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0\\ -\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c
)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
 I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4
*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*
x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) + 2*a*
c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
 c)**(3/2)/(3*a**2*c), True)) - c*Piecewise((I*a**2*sqrt(c)*x**3/(2*sqrt(a*
*2*x**2 - 1)) - I*sqrt(c)*x/(2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/
(2*a), Abs(a**2*x**2) > 1), (sqrt(c)*x*sqrt(-a**2*x**2 + 1)/2 + sqrt(c)*asi
n(a*x)/(2*a), True))
```

3.653 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=87

$$-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)^
(1/2)/a-1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{6167, 6142, 671, 641, 217, 203\}$

$$-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= -\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{}{\sqrt{c-}}\right.\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 1.15

$$\frac{\sqrt{c-a^2cx^2}\left(6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\sqrt{ax+1}\left(a^2x^2-5ax+4\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 - a
*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.48, size = 134, normalized size = 1.54

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,(ax-4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}\,(ax-4)+3\sqrt{c}\,\text{arct}}{2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x - 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

giac [A] time = 0.15, size = 62, normalized size = 0.71

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

maple [A] time = 0.05, size = 126, normalized size = 1.45

$$\frac{x\sqrt{-a^{2}c\,x^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}-\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2\left(x+\frac{1}{a}\right)ac}}{a}-\frac{2c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^{2}c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2
*c*x^2+c)^(1/2))-2/a*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)-2*c/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [A] time = 0.41, size = 47, normalized size = 0.54

$$\frac{1}{2}\sqrt{-a^{2}cx^{2}+c}\,x-\frac{3\sqrt{c}\arcsin\left(ax\right)}{2\,a}-\frac{2\sqrt{-a^{2}cx^{2}+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2
+ c)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^{2}\,c\,x^{2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax-1\right)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.654 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=60

$$\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)+2*(-a*x+1)/a/(-a^2*c*x^2
+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6142, 653, 217, 203}

$$\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*(1 - a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/(a*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\right)\\ &= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\ &= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 1.67

$$-\frac{2\sqrt{1-a^2x^2}\left(\sqrt{ax+1}\,(ax-1)+\sqrt{1-ax}\,(ax+1)\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\,(ax+1)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*((-1 + a*x)*Sqrt[1 + a*x] + Sqrt[1 - a*x]*(1 + a*x)*A
rcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*(1 + a*x)*Sqrt[c - a^2*c*x^
2])
```

fricas [A] time = 0.57, size = 151, normalized size = 2.52

$$\left[-\frac{(ax+1)\sqrt{-c}\,\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\left(a^2cx+ac\right)},-\frac{(ax+1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{a^2cx+ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x + a*c), -((a*x + 1)*sqrt(c)*ar
ctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2
+ c))/(a^2*c*x + a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

maple [A] time = 0.04, size = 73, normalized size = 1.22

$$\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}}{a^2c\left(x+\frac{1}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x+1/a
)*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)
```

maxima [A] time = 0.40, size = 39, normalized size = 0.65

$$\frac{2\sqrt{-a^2cx^2+c}}{a^2cx+ac}+\frac{\arcsin(ax)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x + a*c) + arcsin(a*x)/(a*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a\,x-1}{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\sqrt{-c(ax-1)(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x - 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

3.655 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] 2/3*(-a*x+1)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6167, 6142, 653, 191}

$$\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*(1 - a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\ &= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\right)\\ &= \frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx\\ &= \frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 63, normalized size = 1.21

$$\frac{\sqrt{1-ax}\,(ax+2)\sqrt{1-a^2x^2}}{3ac(ax+1)^{3/2}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a*x]*(2 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 + a*x)^(3/2)*Sqrt[c -
 a^2*c*x^2])
```

fricas [**A**] time = 0.58, size = 47, normalized size = 0.90

$$\frac{\sqrt{-a^2cx^2+c}\,(ax+2)}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

giac [**B**] time = 0.23, size = 148, normalized size = 2.85

$$-\frac{\left(ac+3\sqrt{-a^2c}\sqrt{c}\right)\mathrm{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}+\sqrt{-a^2c}\,ac^2\right)}+\frac{2\left(2\,a^2c-3\,a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}-\sqrt{-a^2c+\frac{c}{x^2}}+\frac{\sqrt{c}}{x}\right)^3c\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*c + 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) + sqrt(-a^2*c)*a*c^
2) + 2/3*(2*a^2*c - 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqr
t(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) - sqrt(-a^2*c + c/x^2) + sqrt
(c)/x)^3*c*sgn(x))
```

maple [**A**] time = 0.04, size = 31, normalized size = 0.60

$$\frac{(ax-1)^2\,(ax+2)}{3a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/3*(a*x-1)^2*(a*x+2)/a/(-a^2*c*x^2+c)^(3/2)
```

maxima [A] time = 0.31, size = 60, normalized size = 1.15

$$-\frac{x}{3\sqrt{-a^2cx^2+c}\,c}+\frac{2}{3\left(\sqrt{-a^2cx^2+c}\,a^2cx+\sqrt{-a^2cx^2+c}\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x + sqrt(
-a^2*c*x^2 + c)*a*c)
```

mupad [B] time = 1.28, size = 33, normalized size = 0.63

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+2\right)}{3\,a\,c^2\,{\left(a\,x+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x + 2))/(3*a*c^2*(a*x + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax-1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)), x)
```

3.656 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] 2/5*(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c^2/
(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6142, 653, 192, 191}

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*(1 - a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2)) -
 (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx\right)$$
$$= \frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{3}{5}\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx$$
$$= \frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{x}{5c\left(c-a^2cx^2\right)^{3/2}} - \frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{5c}$$
$$= \frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}} - \frac{x}{5c\left(c-a^2cx^2\right)^{3/2}} - \frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 1.05

$$-\frac{\sqrt{1-a^2x^2}\left(2a^3x^3+4a^2x^2+ax-2\right)}{5ac^2\sqrt{1-ax}\,(ax+1)^{5/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/5*(Sqrt[1 - a^2*x^2]*(-2 + a*x + 4*a^2*x^2 + 2*a^3*x^3))/(a*c^2*Sqrt[1 - 
 a*x]*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.57, size = 75, normalized size = 1.00

$$\frac{\left(2\,a^3x^3+4\,a^2x^2+ax-2\right)\sqrt{-a^2cx^2+c}}{5\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 + 4*a^2*x^2 + a*x - 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 + 2
*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)), x)
```

maple [A] time = 0.04, size = 47, normalized size = 0.63

$$-\frac{(ax-1)^2\left(2x^3a^3+4a^2x^2+ax-2\right)}{5a\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/5*(a*x-1)^2*(2*a^3*x^3+4*a^2*x^2+a*x-2)/a/(-a^2*c*x^2+c)^(5/2)
```

maxima [A] time = 0.32, size = 79, normalized size = 1.05

$$\frac{2}{5\left(\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}a^{2}cx+\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}ac\right)}-\frac{2\,x}{5\,\sqrt{-a^{2}cx^{2}+c}\,c^{2}}-\frac{x}{5\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x + (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/(
sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

mupad [B] time = 1.40, size = 56, normalized size = 0.75

$$\frac{\sqrt{c-a^{2}\,c\,x^{2}}\,\left(2\,a^{3}\,x^{3}+4\,a^{2}\,x^{2}+a\,x-2\right)}{5\,a\,c^{3}\,\left(a\,x-1\right)\,{\left(a\,x+1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x + 4*a^2*x^2 + 2*a^3*x^3 - 2))/(5*a*c^3*(a*x - 1
)*(a*x + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)), x)
```

3.657 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] 2/7*(-a*x+1)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/c^2
/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6142, 653, 192, 191}

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (2*(1 - a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2)) -
 (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{9/2}}\,dx\right) \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{5}{7}\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{7c} \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} - \frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{21c^2} \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 96, normalized size = 0.98

$$
-\frac{\sqrt{1-a^2x^2}\left(-8a^5x^5-16a^4x^4+4a^3x^3+24a^2x^2+9ax-6\right)}{21ac^3(1-ax)^{3/2}(ax+1)^{7/2}\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -1/21*(Sqrt[1 - a^2*x^2]*(-6 + 9*a*x + 24*a^2*x^2 + 4*a^3*x^3 - 16*a^4*x^4
- 8*a^5*x^5))/(a*c^3*(1 - a*x)^(3/2)*(1 + a*x)^(7/2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 1.20, size = 124, normalized size = 1.27

$$
\frac{\left(8\,a^5x^5+16\,a^4x^4-4\,a^3x^3-24\,a^2x^2-9\,ax+6\right)\sqrt{-a^2cx^2+c}}{21\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(8*a^5*x^5 + 16*a^4*x^4 - 4*a^3*x^3 - 24*a^2*x^2 - 9*a*x + 6)*sqrt(-a^
2*c*x^2 + c)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a
^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{ax-1}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}(ax+1)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)), x)
```

maple [A] time = 0.04, size = 64, normalized size = 0.65

$$\frac{(ax-1)^2\left(8x^5a^5+16x^4a^4-4x^3a^3-24a^2x^2-9ax+6\right)}{21a\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/21*(a*x-1)^2*(8*a^5*x^5+16*a^4*x^4-4*a^3*x^3-24*a^2*x^2-9*a*x+6)/a/(-a^2*
c*x^2+c)^(7/2)
```

maxima [A] time = 0.32, size = 98, normalized size = 1.00

$$\frac{2}{7\left(\left(-a^2cx^2+c\right)^{\frac{5}{2}}a^2cx+\left(-a^2cx^2+c\right)^{\frac{5}{2}}ac\right)}-\frac{8\,x}{21\,\sqrt{-a^2cx^2+c}\,c^3}-\frac{4\,x}{21\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^2}-\frac{x}{7\left(-a^2cx^2+c\right)^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x + (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x/
(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/((
-a^2*c*x^2 + c)^(5/2)*c)
```

mupad [B] time = 1.45, size = 134, normalized size = 1.37

$$\frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,(a\,x+1)^3}+\frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,(a\,x+1)^4}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}-\frac{5}{28\,a\,c^4}\right)}{(a\,x-1)^2\,(a\,x+1)^2}+\frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,(a\,x-1)\,(a\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x + 1)^3) + (c - a^2*c*x^2)^(1/2)/(28*a*
c^4*(a*x + 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) - 5/(28*a*c^4)))
/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x - 1)*
(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax-1}{(-c\,(ax-1)\,(ax+1))^{\frac{7}{2}}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)), x)
```

3.658 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}+\frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

```
[Out] 2/9*(-a*x+1)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/c^2
/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2*c*x
^2+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6167, 6142, 653, 192, 191}

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}+\frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

```
[Out] (2*(1 - a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2)) -
 (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2)
) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{9/2}}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{11/2}}\,dx\right)$$
$$= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{7}{9}\int \frac{1}{\left(c-a^2cx^2\right)^{9/2}}\,dx$$
$$= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2\int \frac{1}{\left(c-a^2cx^2\right)^{7/2}}\,dx}{3c}$$
$$= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{15c^2}$$
$$= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{16\int \frac{}{(c-}}{4}$$
$$= \frac{2(1-ax)}{9a\left(c-a^2cx^2\right)^{9/2}} - \frac{x}{9c\left(c-a^2cx^2\right)^{7/2}} - \frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}} - \frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}} - \frac{16}{45c^4\sqrt{c}}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.93

$$\frac{\sqrt{1-a^2x^2}\left(-16a^7x^7-32a^6x^6+24a^5x^5+80a^4x^4+10a^3x^3-60a^2x^2-25ax+10\right)}{45ac^4(1-ax)^{5/2}(ax+1)^{9/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(10 - 25*a*x - 60*a^2*x^2 + 10*a^3*x^3 + 80*a^4*x^4 + 24
*a^5*x^5 - 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(5/2)*(1 + a*x)^(9
/2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 1.32, size = 152, normalized size = 1.26

$$\frac{\left(16\,a^7x^7+32\,a^6x^6-24\,a^5x^5-80\,a^4x^4-10\,a^3x^3+60\,a^2x^2+25\,ax-10\right)\sqrt{-a^2cx^2+c}}{45\left(a^9c^5x^8+2\,a^8c^5x^7-2\,a^7c^5x^6-6\,a^6c^5x^5+6\,a^4c^5x^3+2\,a^3c^5x^2-2\,a^2c^5x-ac^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/45*(16*a^7*x^7 + 32*a^6*x^6 - 24*a^5*x^5 - 80*a^4*x^4 - 10*a^3*x^3 + 60*a
^2*x^2 + 25*a*x - 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 + 2*a^8*c^5*x^7 - 2
*a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 - 2*a^2*c^5*x
- a*c^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x + 1)), x)
```

maple [A] time = 0.04, size = 80, normalized size = 0.66

$$-\frac{(ax-1)^2\left(16a^7x^7+32x^6a^6-24x^5a^5-80x^4a^4-10x^3a^3+60a^2x^2+25ax-10\right)}{45a\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(-a^2*c*x^2+c)^(9/2),x)
```

```
[Out] -1/45*(a*x-1)^2*(16*a^7*x^7+32*a^6*x^6-24*a^5*x^5-80*a^4*x^4-10*a^3*x^3+60*
a^2*x^2+25*a*x-10)/a/(-a^2*c*x^2+c)^(9/2)
```

maxima [A] time = 0.33, size = 117, normalized size = 0.97

$$\frac{2}{9\left(\left(-a^2cx^2+c\right)^{\frac{7}{2}}a^2cx+\left(-a^2cx^2+c\right)^{\frac{7}{2}}ac\right)}-\frac{16\,x}{45\,\sqrt{-a^2cx^2+c}\,c^4}-\frac{8\,x}{45\left(-a^2cx^2+c\right)^{\frac{3}{2}}c^3}-\frac{2\,x}{15\left(-a^2cx^2+c\right)^{\frac{5}{2}}c^2}-\frac{}{9\,(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x + (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*x
/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*x/
((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

mupad [B] time = 1.49, size = 177, normalized size = 1.46

$$\frac{5\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,(a\,x+1)^4}+\frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,(a\,x+1)^5}+\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}-\frac{5}{24\,a\,c^5}\right)}{(a\,x-1)^3\,(a\,x+1)^3}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}+\frac{5}{144\,a\,c^5}\right)}{(a\,x-1)^2\,(a\,x+1)^2}+\frac{16\,x\,\sqrt{c}}{45\,c^5\,(a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(9/2)*(a*x + 1)),x)
```

```
[Out] (5*(c - a^2*c*x^2)^(1/2))/(144*a*c^5*(a*x + 1)^4) + (c - a^2*c*x^2)^(1/2)/(
72*a*c^5*(a*x + 1)^5) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) - 5/(24*a*
c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5) +
 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2))/(
45*c^5*(a*x - 1)*(a*x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax-1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{9}{2}}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x + 1)), x)
```

3.659 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=189

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[Out] -8/7*(-a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(-a*x
+1)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(-a*x+1)^9*(-a^
2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x^2+
c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A] time = 0.20, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-8*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (2*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}dx = \frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$= \frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^6(1+ax)^3\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$= \frac{\left(c-a^2cx^2\right)^{9/2}\int\left(8(-1+ax)^6+12(-1+ax)^7+6(-1+ax)^8+(-1+ax)^9\right)d}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$= -\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1-ax)^9\left(c-a^2c\right.}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.38

$$\frac{c^4(ax-1)^7\left(21a^3x^3+77a^2x^2+98ax+44\right)\sqrt{c-a^2cx^2}}{210a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(-1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(44 + 98*a*x + 77*a^2*x^2 + 21*a^3*x^
3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.98, size = 95, normalized size = 0.50

$$\frac{\left(21\,a^9c^4x^{10}-70\,a^8c^4x^9+240\,a^6c^4x^7-210\,a^5c^4x^6-252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2+210\,c^4x\right)\sqrt{-a^2c}}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 - 70*a^8*c^4*x^9 + 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 - 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 + 210*c^4*x)*sqrt(-a^2
*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.04, size = 100, normalized size = 0.53

$$\frac{x\left(21a^9x^9-70x^8a^8+240x^6a^6-210x^5a^5-252x^4a^4+420x^3a^3-315ax+210\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210\,(ax+1)^3\,(ax-1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/210*x*(21*a^9*x^9-70*a^8*x^8+240*a^6*x^6-210*a^5*x^5-252*a^4*x^4+420*a^3*
x^3-315*a*x+210)*(-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^3/(a*
x-1)^6
```

maxima [A] time = 0.34, size = 204, normalized size = 1.08

$$\frac{\left(21\,a^{11}\sqrt{-c}\,c^4x^{11} - 49\,a^{10}\sqrt{-c}\,c^4x^{10} - 70\,a^9\sqrt{-c}\,c^4x^9 + 240\,a^8\sqrt{-c}\,c^4x^8 + 30\,a^7\sqrt{-c}\,c^4x^7 - 462\,a^6\sqrt{-c}\,c^4x^6 + 168\right.}{210\left(a^3x^2 - 2\,a^2x + a\right)(ax +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/210*(21*a^11*sqrt(-c)*c^4*x^11 - 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt(
-c)*c^4*x^9 + 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 - 462*a^6*
sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 + 420*a^4*sqrt(-c)*c^4*x^4 - 31
5*a^3*sqrt(-c)*c^4*x^3 - 105*a^2*sqrt(-c)*c^4*x^2 - 210*sqrt(-c)*c^4)*(a*x
- 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-a^2\,c\,x^2\right)}^{9/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.660 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=142

$$\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

```
[Out] 2/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(-a*x+1
)^7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2*c
*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}dx=\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=\frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^5(1+ax)^2\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=\frac{\left(c-a^2cx^2\right)^{7/2}\int\left(4(-1+ax)^5+4(-1+ax)^6+(-1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$
$$=\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.44

$$-\frac{c^3(ax-1)^6\left(21a^2x^2+54ax+37\right)\sqrt{c-a^2cx^2}}{168a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/168*(c^3*(-1 + a*x)^6*(37 + 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a
^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.62, size = 95, normalized size = 0.67

$$-\frac{\left(21\,a^7c^3x^8-72\,a^6c^3x^7+28\,a^5c^3x^6+168\,a^4c^3x^5-210\,a^3c^3x^4-56\,a^2c^3x^3+252\,ac^3x^2-168\,c^3x\right)\sqrt{-a^2c}}{168\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 - 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 + 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 - 56*a^2*c^3*x^3 + 252*a*c^3*x^2 - 168*c^3*x)*sqrt(-a^2*c
)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.04, size = 100, normalized size = 0.70

$$\frac{x\left(21a^7x^7-72x^6a^6+28x^5a^5+168x^4a^4-210x^3a^3-56a^2x^2+252ax-168\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168\,(ax+1)^2\,(ax-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/168*x*(21*a^7*x^7-72*a^6*x^6+28*a^5*x^5+168*a^4*x^4-210*a^3*x^3-56*a^2*x^
2+252*a*x-168)*(-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*x-
1)^5
```

maxima [A] time = 0.34, size = 172, normalized size = 1.21

$$-\frac{\left(21\,a^9\sqrt{-c}\,c^3x^9 - 51\,a^8\sqrt{-c}\,c^3x^8 - 44\,a^7\sqrt{-c}\,c^3x^7 + 196\,a^6\sqrt{-c}\,c^3x^6 - 42\,a^5\sqrt{-c}\,c^3x^5 - 266\,a^4\sqrt{-c}\,c^3x^4 + 196}{168\left(a^3x^2 - 2\,a^2x + a\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/168*(21*a^9*sqrt(-c)*c^3*x^9 - 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c)
*c^3*x^7 + 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 - 266*a^4*sqr
t(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 + 84*a^2*sqrt(-c)*c^3*x^2 + 168*sq
rt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-a^2\,c\,x^2\right)}^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.661 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

```
[Out] -2/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x+
1)^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}dx = \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^4(1+ax)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(2(-1+ax)^4+(-1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$
$$= -\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.58

$$\frac{c^2(ax-1)^5(5ax+7)\sqrt{c-a^2cx^2}}{30a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-1 + a*x)^5*(7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2*
x^2)]*x)
```

fricas [A] time = 0.54, size = 73, normalized size = 0.77

$$\frac{\left(5\,a^5c^2x^6-18\,a^4c^2x^5+15\,a^3c^2x^4+20\,a^2c^2x^3-45\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 - 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.04, size = 84, normalized size = 0.88

$$\frac{x\left(5x^5a^5-18x^4a^4+15x^3a^3+20a^2x^2-45ax+30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30\,(ax+1)\,(ax-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/30*x*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*(-a^2*c*x^2+c
)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(a*x-1)^4
```

maxima [A] time = 0.34, size = 140, normalized size = 1.47

$$\frac{\left(5\,a^{7}\sqrt{-c}\,c^{2}x^{7}-13\,a^{6}\sqrt{-c}\,c^{2}x^{6}-3\,a^{5}\sqrt{-c}\,c^{2}x^{5}+35\,a^{4}\sqrt{-c}\,c^{2}x^{4}-25\,a^{3}\sqrt{-c}\,c^{2}x^{3}-15\,a^{2}\sqrt{-c}\,c^{2}x^{2}-30\,\sqrt{-c}\,c^{2}\right)(a}{30\left(a^{3}x^{2}-2\,a^{2}x+a\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/30*(5*a^7*sqrt(-c)*c^2*x^7 - 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^2
*x^5 + 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 - 15*a^2*sqrt(-c)*
c^2*x^2 - 30*sqrt(-c)*c^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-a^2\,c\,x^2\right)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.662 $\int e^{-3\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{3/2} \, dx$

Optimal. Leaf size=47

$$\frac{(1-ax)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] 1/4*(-a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.17, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 32}

$$\frac{(1-ax)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 - 
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)} \left(c-a^2cx^2\right)^{3/2} \, dx &= \frac{\left(c-a^2cx^2\right)^{3/2} \int e^{-3\coth^{-1}(ax)} \left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3 \, dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3}\\
&= \frac{\left(c-a^2cx^2\right)^{3/2} \int (-1+ax)^3 \, dx}{a^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3}\\
&= \frac{(1-ax)^4 \left(c-a^2cx^2\right)^{3/2}}{4a^4 \left(1-\frac{1}{a^2x^2}\right)^{3/2} x^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 1.23

$$-\frac{c\left(a^3x^3-4a^2x^2+6ax-4\right)\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/4*(c*Sqrt[c - a^2*c*x^2]*(-4 + 6*a*x - 4*a^2*x^2 + a^3*x^3))/(a*Sqrt[1 -
 1/(a^2*x^2)])
```

fricas [A] time = 0.49, size = 42, normalized size = 0.89

$$-\frac{\left(a^3cx^4-4\,a^2cx^3+6\,acx^2-4\,cx\right)\sqrt{-a^2c}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/4*(a^3*c*x^4 - 4*a^2*c*x^3 + 6*a*c*x^2 - 4*c*x)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.04, size = 60, normalized size = 1.28

$$\frac{x\left(x^3a^3-4a^2x^2+6ax-4\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4\left(ax-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/4*x*(a^3*x^3-4*a^2*x^2+6*a*x-4)*(-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3
/2)/(a*x-1)^3
```

maxima [B] time = 0.33, size = 97, normalized size = 2.06

$$-\frac{\left(a^5\sqrt{-c}\,cx^5-3\,a^4\sqrt{-c}\,cx^4+2\,a^3\sqrt{-c}\,cx^3+2\,a^2\sqrt{-c}\,cx^2+4\,\sqrt{-c}\,c\right)(ax-1)^2}{4\left(a^3x^2-2\,a^2x+a\right)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/4*(a^5*sqrt(-c)*c*x^5 - 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 + 2*
a^2*sqrt(-c)*c*x^2 + 4*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*
x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c-a^2\,c\,x^2\right)^{3/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

3.663 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\frac{(-1+ax)^2}{1+ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\left(-3+ax+\frac{4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.50

$$\frac{\sqrt{c-a^2cx^2}\,(ax(ax-6)+8\log(ax+1))}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*x)
```

fricas [A] time = 0.59, size = 33, normalized size = 0.29

$$\frac{\left(a^2x^2-6\,ax+8\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 67, normalized size = 0.60

$$\frac{\left(a^2x^2-6ax+8\ln\left(ax+1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*
x+1))^(3/2)/a/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.664 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=77

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(ax+1)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

```
[Out] 2*x*(1-1/a^2/x^2)^(1/2)/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+x*ln(a*x+1)*(1-1/a^2/x
^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(ax+1)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{-1+ax}{(1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \left(-\frac{2}{(1+ax)^2}+\frac{1}{1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.68

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,((ax+1)\log(ax+1)+2)}{(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 + (1 + a*x)*Log[1 + a*x]))/((1 + a*x)*Sqrt[c -
a^2*c*x^2])
```

fricas [A] time = 0.62, size = 38, normalized size = 0.49

$$-\frac{\sqrt{-a^2c}\left((ax+1)\log\left(ax+1\right)+2\right)}{a^3cx+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
[Out] -sqrt(-a^2*c)*((a*x + 1)*log(a*x + 1) + 2)/(a^3*c*x + a^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

maple [A] time = 0.06, size = 62, normalized size = 0.81

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax\ln\left(ax+1\right)+\ln\left(ax+1\right)+2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{ac\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(a*x*ln(a*x+1)+ln(a*x+1)+2)*((a*x-1)/(a*x+1))^(3/2)
/a/c/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.665 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 32}

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 + a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(1+ax)^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.06, size = 51, normalized size = 1.11

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-a^2cx^2}}{2c^2(ax-1)(ax+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)*(1 + a*x
)^3)
```

fricas [A] time = 0.53, size = 39, normalized size = 0.85

$$-\frac{\sqrt{-a^2c}}{2\left(a^4c^2x^2+2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
[Out] -1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.04, size = 39, normalized size = 0.85

$$-\frac{(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 1.47, size = 58, normalized size = 1.26

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,c\,\left(x\,\sqrt{c-a^2\,c\,x^2}+\frac{\sqrt{c-a^2\,c\,x^2}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(2*a^2*c*(x*(c - a^2*c*x^2)^(1/2) + (c - a^2*c*
x^2)^(1/2)/a))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.666 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(ax+1)^3\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] 1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1
/a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(5
/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arctan
h(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.20, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(ax+1)^3\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 + a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)(1+ax)^4}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{2(1+ax)^4}-\frac{1}{4(1+ax)^3}-\frac{1}{8(1+ax)^2}+\frac{1}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(a^5\left(1-\right.\right.}{}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\right.}{8}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.39

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2-9ax+3(ax+1)^3\tanh^{-1}(ax)-10\right)}{24c^2(ax+1)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/24*(Sqrt[1 - 1/(a^2*x^2)]*x*(-10 - 9*a*x - 3*a^2*x^2 + 3*(1 + a*x)^3*Arc
Tanh[a*x]))/(c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.56, size = 136, normalized size = 0.75

$$-\frac{3\left(a^4x^3+3\,a^3x^2+3\,a^2x+a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\left(3\,a^2x^2+9\,ax+10\right)\sqrt{-a^2c}}{48\left(a^5c^3x^3+3\,a^4c^3x^2+3\,a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/48*(3*(a^4*x^3 + 3*a^3*x^2 + 3*a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sq
rt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 + 9*a*x + 10)*sqrt
(-a^2*c))/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.06, size = 169, normalized size = 0.93

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\left(a^2x^2-1\right)}\left(3\ln\left(ax-1\right)x^3a^3-3a^3x^3\ln\left(ax+1\right)+9\ln\left(ax-1\right)x^2a^2-9\ln\left(ax+1\right)x^2a^2+6a^2\right.}{48\left(ax+1\right)\left(ax-1\right)\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/48*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(
a*x-1)*x^3*a^3-3*a^3*x^3*ln(a*x+1)+9*ln(a*x-1)*x^2*a^2-9*ln(a*x+1)*x^2*a^2+
6*a^2*x^2+9*ln(a*x-1)*x*a-9*a*x*ln(a*x+1)+18*a*x+3*ln(a*x-1)-3*ln(a*x+1)+20
)/(a^2*x^2-1)/c^3/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.667 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=275

$$\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(ax+1)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6}{16(ax}$$

```
[Out] 1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/16*a^6*(1-
1/a^2/x^2)^(7/2)*x^7/(a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6*(1-1/a^2/x^2)^
(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(
a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a^2
*c*x^2+c)^(7/2)+5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2+c
)^(7/2)
```

Rubi [A] time = 0.22, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6192, 6193, 44, 207}

$$\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(ax+1)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6}{16(ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)^4*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) - (3
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2)) -
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (5*
a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
```

```
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^2(1+ax)^5}\,dx}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{32(-1+ax)^2}+\frac{1}{4(1+ax)^5}+\frac{1}{4(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{5}{32\left(-1+a^2x^2\right)}\right.}{\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{}{32}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{}{32}$$

Mathematica [A] time = 0.10, size = 99, normalized size = 0.36

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-15a^4x^4-45a^3x^3-35a^2x^2+15ax+15(ax-1)(ax+1)^4\tanh^{-1}(ax)+32\right)}{96c^3(ax-1)(ax+1)^4\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -1/96*(Sqrt[1 - 1/(a^2*x^2)]*x*(32 + 15*a*x - 35*a^2*x^2 - 45*a^3*x^3 - 15*
a^4*x^4 + 15*(-1 + a*x)*(1 + a*x)^4*ArcTanh[a*x]))/(c^3*(-1 + a*x)*(1 + a*x
)^4*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 1.30, size = 193, normalized size = 0.70

$$-\frac{15\left(a^6x^5+3\,a^5x^4+2\,a^4x^3-2\,a^3x^2-3\,a^2x-a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\left(15\,a^4x^4+45\,a^3x^3+35\,a^2\right.}{192\left(a^7c^4x^5+3\,a^6c^4x^4+2\,a^5c^4x^3-2\,a^4c^4x^2-3\,a^3c^4x-a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/192*(15*(a^6*x^5 + 3*a^5*x^4 + 2*a^4*x^3 - 2*a^3*x^2 - 3*a^2*x - a)*sqrt
(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15
*a^4*x^4 + 45*a^3*x^3 + 35*a^2*x^2 - 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4*x^
5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

maple [A] time = 0.07, size = 241, normalized size = 0.88

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\left(a^2x^2-1\right)}\,\left(15\ln\left(ax-1\right)x^5a^5-15\ln\left(ax+1\right)x^5a^5+45\ln\left(ax-1\right)x^4a^4-45\ln\left(ax+1\right)x^4a^4+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x)
[Out] -1/192*((a*x-1)/(a*x+1))^(3/2)/(a*x+1)^2/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(
15*ln(a*x-1)*x^5*a^5-15*ln(a*x+1)*x^5*a^5+45*ln(a*x-1)*x^4*a^4-45*ln(a*x+1)
*x^4*a^4+30*x^4*a^4+30*ln(a*x-1)*x^3*a^3-30*a^3*x^3*ln(a*x+1)+90*x^3*a^3-30
*ln(a*x-1)*x^2*a^2+30*ln(a*x+1)*x^2*a^2+70*a^2*x^2-45*ln(a*x-1)*x*a+45*a*x*
ln(a*x+1)-30*a*x-15*ln(a*x-1)+15*ln(a*x+1)-64)/(a^2*x^2-1)/c^4/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima
")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
[Out] Timed out
```

3.668 $\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=76

$$\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)^(
1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 43}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c
*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^2(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(x^2+ax^3\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.59

$$\frac{x^2(3ax+4)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
[Out] (x^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.68, size = 25, normalized size = 0.33

$$\frac{\left(3\,ax^4+4\,x^3\right)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
[Out] 1/12*(3*a*x^4 + 4*x^3)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 47, normalized size = 0.62

$$\frac{x^3\,(3ax+4)\,\sqrt{-a^2c\,x^2+c}}{12\,(ax+1)\,\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*a*x+4)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.669 $\int e^{\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/
2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6192, 6193, 43}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x
^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int x(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.58

$$\frac{x(2ax+3)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
[Out] (x*(3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.46, size = 25, normalized size = 0.34

$$\frac{\left(2\,ax^3+3\,x^2\right)\sqrt{-a^2c}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
[Out] 1/6*(2*a*x^3 + 3*x^2)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.03, size = 47, normalized size = 0.64

$$\frac{x^2\,(2ax+3)\,\sqrt{-a^2c\,x^2+c}}{6\,(ax+1)\,\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*x^2*(2*a*x+3)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.670 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=68

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6192, 6193}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx&=\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\&=\frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\&=\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.60

$$\frac{(ax+2)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.54, size = 22, normalized size = 0.32

$$\frac{\sqrt{-a^{2}c}\left(ax^{2}+2\,x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.03, size = 44, normalized size = 0.65

$$\frac{x\left(ax+2\right)\sqrt{-a^{2}c\,x^{2}+c}}{2\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(a*x+2)*(-a^2*c*x^2+c)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^{2}\,c\,x^{2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.671 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 43}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.61

$$\frac{\sqrt{c-a^2cx^2}\,(ax+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.52, size = 18, normalized size = 0.26

$$\frac{\sqrt{-a^2c}\left(ax+\log(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x + log(x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 44, normalized size = 0.64

$$\frac{(ax+\ln(x))\sqrt{-c\left(a^2x^2-1\right)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] (a*x+ln(x))*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.672 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=73

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/
x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 43}

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) + (Sqrt[c - a^2*c*x^2]
*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2} + \frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.59

$$\frac{\sqrt{c-a^2cx^2}\,(ax\log(x)-1)}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1 + a*x*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

fricas [A] time = 0.59, size = 22, normalized size = 0.30

$$\frac{\sqrt{-a^2c}\left(ax\log(x)-1\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*c)*(a*x*log(x) - 1)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.05, size = 48, normalized size = 0.66

$$\frac{(a\ln(x)x-1)\sqrt{-c\left(a^2x^2-1\right)}}{x\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] (a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)/x/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

3.673 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] -3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+3/5*x^2*(-a^2*c*x
^2+c)^(1/2)/a^2+1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(1/2)
+3/20*(5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

Rubi [A] time = 0.41, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6151, 1809, 833, 780, 217, 203}

$$\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) + (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) - (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
```

```
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx \\
&= -\left(c\int\frac{x^3(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int\frac{x^3\left(-9a^2c-10a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2} \\
&= \frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{\int\frac{x^2\left(30a^3c^2+36a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c} \\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int\frac{x\left(-72a^4c^3-90a^5c^3x\right)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2} \\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4} \\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4} \\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 96, normalized size = 0.70

$$
\frac{15\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\left(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24\right)\sqrt{c-a^2cx^2}}{20a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 10*a^3*x^3 + 4*a^4*x^4) +
15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20*
a^4)
```

fricas [A] time = 0.72, size = 184, normalized size = 1.34

$$\left[\frac{2\left(4\,a^4x^4+10\,a^3x^3+12\,a^2x^2+15\,ax+24\right)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\,\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{40\,a^4},\ \frac{\left(4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/40*(2*(4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-a^2*c*x^
2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c))/a^4, 1/20*((4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-
a^2*c*x^2 + c) + 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*
x^2 - c)))/a^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 210, normalized size = 1.53

$$-\frac{x^2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{5a^2c}-\frac{4\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{5c\,a^4}-\frac{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{2a^3c}+\frac{5x\sqrt{-a^2c\,x^2+c}}{4a^3}+\frac{5c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{4a^3\sqrt{a^2c}}+\frac{2\sqrt{-\left(x-\frac{1}{a}\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c-4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)-1/2/a^3*
x*(-a^2*c*x^2+c)^(3/2)/c+5/4/a^3*x*(-a^2*c*x^2+c)^(1/2)+5/4/a^3*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^4*(-(x-1/a)^2*a^2*c-2
*a*c*(x-1/a))^(1/2)-2/a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^
2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.42, size = 117, normalized size = 0.85

$$-\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2}{5\,a^2c}+\frac{5\,\sqrt{-a^2cx^2+c}\,x}{4\,a^3}-\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{2\,a^3c}-\frac{3\,\sqrt{c}\,\arcsin\left(ax\right)}{4\,a^4}+\frac{2\,\sqrt{-a^2cx^2+c}}{a^4}-\frac{4\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{5\,a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) + 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 -
1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) - 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sqrt
(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}\,\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.674 $\int e^{2\coth^{-1}(ax)} x^2 \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] -7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3+2/3*x^2*(-a^2*c*x
^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)+1/24*(21*a*x+32)*(-a^2*c*x^2+c)^
(1/2)/a^3
```

Rubi [A] time = 0.39, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6167, 6151, 1809, 833, 780, 217, 203}

$$\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 + ((32 + 21
*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(8*a^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
```

```
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = -\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^2(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{\int \frac{x^2\left(-7a^2c-8a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{\int \frac{x\left(16a^3c^2+21a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c)\int \frac{1}{\sqrt{c-a^2cx^2}}}{8a^2}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c)\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{7\sqrt{c}\tan^{-1}\left(\right.}{8a^3}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.79

$$\frac{21\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\left(6a^3x^3+16a^2x^2+21ax+32\right)\sqrt{c-a^2cx^2}}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*Ar
cTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

fricas [A] time = 0.57, size = 168, normalized size = 1.50

$$\left[\frac{2\left(6a^3x^3+16a^2x^2+21ax+32\right)\sqrt{-a^2cx^2+c}+21\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{48a^3}, \frac{\left(6a^3x^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*s
qrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, 1/
24*((6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(c
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

giac [A] time = 0.15, size = 84, normalized size = 0.75

$$\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3x+\frac{8}{a}\right)x+\frac{21}{a^2}\right)x+\frac{32}{a^3}\right)+\frac{7c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8a^2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x + 8/a)*x + 21/a^2)*x + 32/a^3) + 7/8*c*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

maple [B] time = 0.05, size = 186, normalized size = 1.66

$$-\frac{x\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{4a^2c}+\frac{9x\sqrt{-a^2cx^2+c}}{8a^2}+\frac{9c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{8a^2\sqrt{a^2c}}-\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^3c}+\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c+9/8/a^2*x*(-a^2*c*x^2+c)^(1/2)+9/8/a^2*c/
(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3/a^3*(-a^2*c*
x^2+c)^(3/2)/c+2/a^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-2/a^2*c/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.42, size = 93, normalized size = 0.83

$$\frac{9\sqrt{-a^2cx^2+c}\,x}{8a^2}-\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{4a^2c}-\frac{7\sqrt{c}\arcsin(ax)}{8a^3}+\frac{2\sqrt{-a^2cx^2+c}}{a^3}-\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/8
*sqrt(c)*arcsin(a*x)/a^3 + 2*sqrt(-a^2*c*x^2 + c)/a^3 - 2/3*(-a^2*c*x^2 + c
)^(3/2)/(a^3*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\sqrt{c-a^2cx^2}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}\,\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.675 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=85

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] -arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2+1/3*x^2*(-a^2*c*x^2+c
)^(1/2)+1/3*(3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A] time = 0.25, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6167, 6151, 1809, 780, 217, 203}

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) - (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx\\ &= -\left(c\int\frac{x(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\ &= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{\int\frac{x\left(-5a^2c-6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\ &= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\ &= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\,\mathrm{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\ &= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.09, size = 79, normalized size = 0.93

$$\frac{\left(a^2x^2+3ax+5\right)\sqrt{c-a^2cx^2}+3\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((5 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c -
 a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

fricas [A] time = 0.69, size = 150, normalized size = 1.76

$$\left[\frac{2\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,ax+5\right)+3\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{6\,a^2},\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2+3\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*c
*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^2
+ c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c
)*x/(a^2*c*x^2 - c)))/a^2]
```

giac [A] time = 0.14, size = 72, normalized size = 0.85

$$\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x+\frac{3}{a}\right)x+\frac{5}{a^2}\right)+\frac{c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*((x + 3/a)*x + 5/a^2) + c*log(abs(-sqrt(-a^2*c)*x
+ sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

maple [B] time = 0.05, size = 162, normalized size = 1.91

$$-\frac{\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{3 a^{2} c}+\frac{x\sqrt{-a^{2} c\,x^{2}+c}}{a}+\frac{c \arctan\left(\frac{\sqrt{a^{2} c}\,x}{\sqrt{-a^{2} c\,x^{2}+c}}\right)}{a\sqrt{a^{2} c}}+\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2 a c\left(x-\frac{1}{a}\right)}}{a^{2}}-\frac{2 c \arctan\left(\frac{\sqrt{a^{2} c}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^{2} a^{2} c-2 a c\left(x-\frac{1}{a}\right)}}\right)}{a\sqrt{a^{2} c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c+x/a*(-a^2*c*x^2+c)^(1/2)+1/a*c/(a^2*c)^(1/2
)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2*(-(x-1/a)^2*a^2*c-2*a*
c*(x-1/a))^(1/2)-2/a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2
*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.42, size = 70, normalized size = 0.82

$$\frac{\sqrt{-a^{2} c x^{2}+c}\,x}{a}-\frac{\sqrt{c}\,\arcsin\left(a x\right)}{a^{2}}+\frac{2\,\sqrt{-a^{2} c x^{2}+c}}{a^{2}}-\frac{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}}}{3\,a^{2} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*x/a - sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 + c)
/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.676 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=86

$$\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)^
(1/2)/a+1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6141, 671, 641, 217, 203}

$$\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int\frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\,\mathrm{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right.\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.88

$$\frac{\sqrt{c-a^2cx^2}\left(\sqrt{1-a^2x^2}\,(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.58, size = 134, normalized size = 1.56

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{-c}\,\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4\,a},\ \frac{\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{c}\,\arctan\right.}{2\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x + 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

giac [A] time = 0.15, size = 62, normalized size = 0.72

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

maple [A] time = 0.04, size = 134, normalized size = 1.56

$$\frac{x\sqrt{-a^{2}c\,x^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}+\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^{2}a^{2}c-2ac\left(x-\frac{1}{a}\right)}}{a}-\frac{2c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^{2}a^{2}c-2ac\left(x-\frac{1}{a}\right)}}\right)}{\sqrt{a^{2}c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2
*c*x^2+c)^(1/2))+2/a*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-2*c/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.42, size = 47, normalized size = 0.55

$$\frac{1}{2}\sqrt{-a^{2}cx^{2}+c}\,x-\frac{3\sqrt{c}\arcsin\left(ax\right)}{2\,a}+\frac{2\sqrt{-a^{2}cx^{2}+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
+ c)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^{2}\,c\,x^{2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.677 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=75

$$\sqrt{c-a^2cx^2}-2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+arctanh((-a^2*c*x^2+c)^
(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6151, 1809, 844, 217, 203, 266, 63, 208}

$$\sqrt{c-a^2cx^2}-2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \sqrt{c-a^2cx^2} + \frac{\int \frac{-a^2c-2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2}\\
&= \sqrt{c-a^2cx^2} - c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - (2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \sqrt{c-a^2cx^2} - \frac{1}{2}c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - (2ac)\,\mathrm{Subst}\left(\int \frac{1}{1+a^2cx^2}\right.\\
&= \sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)}{a^2}\\
&= \sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 97, normalized size = 1.29

$$\sqrt{c-a^2cx^2} + \sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2} + c\right) + 2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right) - \sqrt{c}\log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(
-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x
^2]]
```

fricas [A] time = 0.71, size = 191, normalized size = 2.55

$$\left[2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c},\sqrt{-c}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*s
qrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt(
-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 -
 c)) + sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
+ sqrt(-a^2*c*x^2 + c)]
```

giac [A] time = 0.17, size = 95, normalized size = 1.27

$$-\frac{2c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) - 2
*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqrt(
-a^2*c*x^2 + c)
```

maple [B] time = 0.05, size = 129, normalized size = 1.72

$$-\sqrt{-a^2c\,x^2+c}+\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)+2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac\left(x-\frac{1}{a}\right)}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2c-2ac}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] -(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2*
(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [A] time = 0.44, size = 90, normalized size = 1.20

$$-a^2\left(\frac{\sqrt{c}\arcsin(ax)}{a^2}-\frac{\sqrt{-a^2cx^2+c}}{a^2}\right)-a\left(\frac{\sqrt{c}\arcsin(ax)}{a}-\frac{\sqrt{c}\log\left(\frac{2\sqrt{-a^2cx^2+c}\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a^2*(sqrt(c)*arcsin(a*x)/a^2 - sqrt(-a^2*c*x^2 + c)/a^2) - a*(sqrt(c)*arcs
in(a*x)/a - sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x))
/a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\,\left(a\,x+1\right)}{x\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)}{x\left(ax-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x*(a*x - 1)), x)
```

3.678 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+2*a*arctanh((-a^2*c*x^2
+c)^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.35, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6151, 1807, 844, 217, 203, 266, 63, 208}

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 + 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{x}+\int \frac{-2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-(2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx-\left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-(ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)-\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\right.\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a}\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 104, normalized size = 1.27

$$
\frac{\sqrt{c-a^2cx^2}}{x}+2a\sqrt{c}\,\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+a\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-2a\sqrt{c}\,\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]
*(-1 + a^2*x^2))] - 2*a*Sqrt[c]*Log[x] + 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c
 - a^2*c*x^2]]
```

fricas [A] time = 0.54, size = 209, normalized size = 2.55

$$\left[\frac{a\sqrt{c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+a\sqrt{c}\,x\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},\ \frac{4\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}}{a^2cx^2-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a*
sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c))/x, 1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqrt
(-c)/(a^2*c*x^2 - c)) + a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 +
c)*a*sqrt(-c)*x - c) + 2*sqrt(-a^2*c*x^2 + c))/x]
```

giac [A] time = 0.16, size = 134, normalized size = 1.63

$$-\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\,\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\,a^2\sqrt{-c}\,c}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
 a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*a
^2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

maple [B] time = 0.05, size = 208, normalized size = 2.54

$$2\sqrt{c}\ \ln\left(\frac{2c+2\sqrt{c}\ \sqrt{-a^2c\,x^2+c}}{x}\right)a-2\sqrt{-a^2c\,x^2+c}\ a+\frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}+a^2x\sqrt{-a^2c\,x^2+c}+\frac{a^2c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] 2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a-2*(-a^2*c*x^2+c)^(1/
2)*a+1/c/x*(-a^2*c*x^2+c)^(3/2)+a^2*x*(-a^2*c*x^2+c)^(1/2)+a^2*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*a*(-(x-1/a)^2*a^2*c-2*a*
c*(x-1/a))^(1/2)-2*a^2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a
^2*c-2*a*c*(x-1/a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)}{(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^2\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

3.679 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a^2*c*x^2+c)^(1
/2)/x^2+2*a*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.34, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6151, 1807, 807, 266, 63, 208}

$$\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) + (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{1}{2}\int \frac{-4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{1}{4}\left(3a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.13, size = 76, normalized size = 0.97

$$\frac{1}{2}\left(\frac{(4ax+1)\sqrt{c-a^2cx^2}}{x^2}+3a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-3a^2\sqrt{c}\log(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] (((1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt[
c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

fricas [A] time = 0.69, size = 148, normalized size = 1.90

$$\left[\frac{3\,a^2\sqrt{c}\,x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}\,(4\,ax+1)}{4\,x^2},\ \frac{3\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-c}}{2\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2, 1/2*(3*a^2*sqrt(-c)*x^
2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2 +
 c)*(4*a*x + 1))/x^2]
```

giac [B] time = 0.16, size = 200, normalized size = 2.56

$$-\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^3a^2c-4\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}\,c|a|+\left(\cdots\right.}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c - 4*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))*a^2*c^2 + 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^2 - c)^2
```

maple [B] time = 0.06, size = 239, normalized size = 3.06

$$\frac{3\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\sqrt{c}\,a^2}{2}-\frac{3\sqrt{-a^2c\,x^2+c}\,a^2}{2}+\frac{2a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}+2a^3x\sqrt{-a^2c\,x^2+c}+\frac{2a^3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] 3/2*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*c^(1/2)*a^2-3/2*(-a^2*c*x^2+
c)^(1/2)*a^2+2*a/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^3*x*(-a^2*c*x^2+c)^(1/2)+2*a^
3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2*a^2*(-(x-1
/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-2*a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)
*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))+1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2cx^2+c}\,(ax+1)}{(ax-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^3\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\;(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

3.680 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=99

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2)/
x^3+a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.38, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6151, 1807, 835, 807, 266, 63, 208}

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) + (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) + a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{1}{3}\int \frac{-6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\int \frac{10a^2c^2+6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\right.\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c}\right.\\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.13, size = 82, normalized size = 0.83

$$a^3\left(-\sqrt{c}\right)\log(x)+\frac{\left(5a^2x^2+3ax+1\right)\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\,\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] ((1 + 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) - a^3*Sqrt[c]*Log[x]
+ a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

fricas [A] time = 0.49, size = 164, normalized size = 1.66

$$\left[\frac{3\,a^{3}\sqrt{c}\,x^{3}\log\left(-\frac{a^{2}cx^{2}-2\,\sqrt{-a^{2}cx^{2}+c}\,\sqrt{c}-2\,c}{x^{2}}\right)+2\,\sqrt{-a^{2}cx^{2}+c}\left(5\,a^{2}x^{2}+3\,ax+1\right)}{6\,x^{3}},\frac{3\,a^{3}\sqrt{-c}\,x^{3}\arctan\left(\frac{\sqrt{-a^{2}cx^{2}+c}\,\sqrt{-c}}{a^{2}cx^{2}-c}\right)}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3, 1/3*(3*a^3
*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(
-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3]
```

giac [B] time = 0.16, size = 250, normalized size = 2.53

$$-\frac{2\,a^{3}c\arctan\left(-\frac{\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\left(3\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)^{5}a^{3}c-3\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)^{4}a^{2}\sqrt{-c}\,c|a\cdots}{3\left(\left(\sqrt{-\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c - 3*(sqrt(-a^2*c)
*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) + 12*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-
a^2*c*x^2 + c))*a^3*c^3 - 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqr
t(-a^2*c*x^2 + c))^2 - c)^3
```

maple [B] time = 0.06, size = 261, normalized size = 2.64

$$\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^{2}c\,x^{2}+c}}{x}\right)a^{3}-\sqrt{-a^{2}c\,x^{2}+c}\,a^{3}+\frac{2a^{2}\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}{cx}+2a^{4}x\sqrt{-a^{2}c\,x^{2}+c}+\frac{2a^{4}c\arctan\left(\frac{\sqrt{\cdots}}{\sqrt{-a^{2}\cdots}}\right.}{\sqrt{a^{2}c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^3-(-a^2*c*x^2+c)^(1/2)
*a^3+2*a^2/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^4*x*(-a^2*c*x^2+c)^(1/2)+2*a^4*c/(a
^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+1/3/c/x^3*(-a^2*c*
x^2+c)^(3/2)+2*a^3*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-2*a^4*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2))+a/c/x^2
*(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}\,(ax+1)}{(ax-1)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^4\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

3.681 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}$$

```
[Out] 7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a^2*c*x^2+c)^(1
/2)/x^4+2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2+4/3
*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.40, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6151, 1807, 835, 807, 266, 63, 208}

$$\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) + (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{1}{4}\int \frac{-8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}-\frac{\int \frac{21a^2c^2+16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{\int \frac{-32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{1}{8}\left(7a^4c\right)\int$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}-\frac{1}{16}\left(7a^4c\right)S$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{1}{8}\left(7a^2\right)\mathrm{Su}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\,\mathrm{tan}$$

Mathematica [A] time = 0.15, size = 95, normalized size = 0.73

$$-\frac{7}{8}a^4\sqrt{c}\,\log(x)+\frac{7}{8}a^4\sqrt{c}\,\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)+\frac{\left(32a^3x^3+21a^2x^2+16ax+6\right)\sqrt{c-a^2cx^2}}{24x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) - (7*
a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/8
```

fricas [A] time = 0.66, size = 180, normalized size = 1.38

$$\left[\frac{21\,a^4\sqrt{c}\,x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+2\left(32\,a^3x^3+21\,a^2x^2+16\,ax+6\right)\sqrt{-a^2cx^2+c}}{48\,x^4},\ \frac{21\,a^4\sqrt{-c}\,x^4\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*
c*x^2 - c)) + (32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c))/
x^4]
```

giac [B] time = 0.15, size = 324, normalized size = 2.49

$$-\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}+\frac{21\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2+96}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 + 96*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^3*a^4*c^3 - 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(
-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 + 32*a^
3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

maple [B] time = 0.06, size = 287, normalized size = 2.21

$$\frac{7\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)a^4}{8}-\frac{7\sqrt{-a^2c\,x^2+c}\,a^4}{8}+\frac{2a^3\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}+2a^5x\sqrt{-a^2c\,x^2+c}+\frac{2a^5c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

```
[Out] 7/8*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^4-7/8*(-a^2*c*x^2+
c)^(1/2)*a^4+2*a^3/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^5*x*(-a^2*c*x^2+c)^(1/2)+2*
a^5*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3*a/c/x^
3*(-a^2*c*x^2+c)^(3/2)+2*a^4*(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2)-2*a^5*c
/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x-1/a)^2*a^2*c-2*a*c*(x-1/a))^(1/2
))+9/8*a^2/c/x^2*(-a^2*c*x^2+c)^(3/2)+1/4/c/x^4*(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax+1)}{(ax-1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^5\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^5\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

3.682 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=228

$$\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^3
/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3
/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^5/x/(1-1/a^2/x^
2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) + (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*
x^2]*Log[1 - a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^4\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=\frac{\sqrt{c-a^2cx^2}\int\frac{x^3(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=\frac{\sqrt{c-a^2cx^2}\int\left(\frac{4}{a^3}+\frac{4x}{a^2}+\frac{4x^2}{a}+3x^3+ax^4+\frac{4}{a^3(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-}}{5\sqrt{1-}}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4\log(1-ax)}{a^4}+\frac{4x}{a^3}+\frac{2x^2}{a^2}+\frac{ax^5}{5}+\frac{4x^3}{3a}+\frac{3x^4}{4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 + (2*x^2)/a^2 + (4*x^3)/(3*a) + (3*x^4)/4 +
 (a*x^5)/5 + (4*Log[1 - a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.54, size = 58, normalized size = 0.25

$$\frac{\left(12\,a^5x^5+45\,a^4x^4+80\,a^3x^3+120\,a^2x^2+240\,ax+240\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/60*(12*a^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x + 240*lo
g(a*x - 1))*sqrt(-a^2*c)/a^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 92, normalized size = 0.40

$$\frac{\left(12x^5a^5+45x^4a^4+80x^3a^3+120a^2x^2+240ax+240\ln\left(ax-1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax-1\right)}{60a^4\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/60*(12*x^5*a^5+45*x^4*a^4+80*x^3*a^3+120*a^2*x^2+240*a*x+240*ln(a*x-1))*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\, x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\, \sqrt{c - a^2\, c\, x^2}}{{\left(\frac{a\, x-1}{a\, x+1}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.683 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=186

$$\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^2
/(1-1/a^2/x^2)^(1/2)+x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3
*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)
/a^4/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/
(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt
[c - a^2*c*x^2]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^2}+\frac{4x}{a}+3x^2+ax^3+\frac{4}{a^2(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-}}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.40

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4\log(1-ax)}{a^3}+\frac{4x}{a^2}+\frac{ax^4}{4}+\frac{2x^2}{a}+x^3\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^2 + (2*x^2)/a + x^3 + (a*x^4)/4 + (4*Log[1 - a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.56, size = 49, normalized size = 0.26

$$\frac{\left(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(-a^2*c)/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 83, normalized size = 0.45

$$\frac{\left(x^4a^4+4x^3a^3+8a^2x^2+16ax+16\ln\left(ax-1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax-1\right)}{4a^3\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(x^4*a^4+4*x^3*a^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2
)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\,x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.684 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=152

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x*(-a^2*c*x^2+c)^(1/2)/a
/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(
-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^3/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 77}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a}+3x+ax^2+\frac{4}{a(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\left(ax\left(2a^2x^2+9ax+24\right)+24\log(1-ax)\right)}{6a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6*a
^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.55, size = 42, normalized size = 0.28

$$\frac{\left(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(-a^2*c)/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 76, normalized size = 0.50

$$\frac{\left(2x^3a^3+9a^2x^2+24ax+24\ln\left(ax-1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax-1\right)}{6a^2\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(2*x^3*a^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1
)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}\,x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.685 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=113

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(3+ax+\frac{4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.50

$$\frac{\sqrt{c-a^2cx^2}\,(ax(ax+6)+8\log(1-ax))}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/(a
^2*x^2)]*x)
```

fricas [A] time = 0.63, size = 33, normalized size = 0.29

$$\frac{\left(a^2x^2+6\,ax+8\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 67, normalized size = 0.59

$$\frac{\left(a^2x^2+6ax+8\ln\left(ax-1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax-1\right)}{2a\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.686 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 72}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.46

$$\frac{\sqrt{c-a^2cx^2}\,(ax+4\log(1-ax)-\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x)
```

fricas [A] time = 0.68, size = 28, normalized size = 0.25

$$\frac{\sqrt{-a^2c}\left(ax+4\log(ax-1)-\log(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 59, normalized size = 0.52

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(-ax+\ln(x)-4\ln(ax-1))\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x)-4*ln(a*x-1))*(a*x-1)/(a*x+1)^2/((a*x-1)
/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

3.687 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(-a^2*c*x^2+c)^(1/2)
/x/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A] time = 0.24, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*Sqrt[c - a^2*c*x^2]*
Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(S
qrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^2(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2} - \frac{3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.48

$$\frac{\sqrt{c-a^2cx^2}\left(-3a\log(x)+4a\log(1-ax)+\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(x^(-1) - 3*a*Log[x] + 4*a*Log[1 - a*x]))/(a*Sqrt[1 -
1/(a^2*x^2)]*x)
```

fricas [A] time = 0.55, size = 33, normalized size = 0.29

$$\frac{\sqrt{-a^2c}\left(4\,ax\log\left(ax-1\right)-3\,ax\log(x)+1\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 65, normalized size = 0.57

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\;(3a\ln(x)x-4\ln\left(ax-1\right)xa-1)\left(ax-1\right)}{x\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(3*a*ln(x)*x-4*ln(a*x-1)*x*a-1)*(a*x-1)/x/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

3.688 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=153

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*(-a^2*c*x^2+c)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3*(-a^2*c*x^2+c)^(1/2)/x
^2/(1-1/a^2/x^2)^(1/2)-4*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
+4*a*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*x^2
])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 -
 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^3(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^3}-\frac{3a}{x^2}-\frac{4a^2}{x}+\frac{4a^3}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3} + \frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2} - \frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{4a\sqrt{c-a^2cx^2}\log(1}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^2\log(x)+4a^2\log(1-ax)+\frac{3a}{x}+\frac{1}{2x^2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(2*x^2) + (3*a)/x - 4*a^2*Log[x] + 4*a^2*Log[1 - a*
x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.58, size = 90, normalized size = 0.59

$$\frac{8\,a^3\sqrt{-c}\,x^2\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}\,(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+\sqrt{-a^2c}\,(6\,ax+1)}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
fricas")
```

```
[Out] 1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + sqrt(-a^2*c)*(6*a*x + 1))/(a*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 77, normalized size = 0.50

$$-\frac{\left(8a^{2}\ln(x)x^{2}-8\ln\left(ax-1\right)x^{2}a^{2}-6ax-1\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{2x^{2}\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(8*a^2*ln(x)*x^2-8*ln(a*x-1)*x^2*a^2-6*a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(
a*x-1)/x^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^{2}cx^{2}+c}}{x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a^{2}\,c\,x^{2}}}{x^{3}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

3.689 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=194

$$\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/3*(-a^2*c*x^2+c)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+3/2*(-a^2*c*x^2+c)^(1/2)
/x^3/(1-1/a^2/x^2)^(1/2)+4*a*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4
*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(-a*x+1)*(-a^
2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*x^2
])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x
) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^4(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^4}-\frac{3a}{x^3}-\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+4a^2$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.41

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^3\log(x)+4a^3\log(1-ax)+\frac{4a^2}{x}+\frac{3a}{2x^2}+\frac{1}{3x^3}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(3*x^3) + (3*a)/(2*x^2) + (4*a^2)/x - 4*a^3*Log[x]
+ 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.64, size = 98, normalized size = 0.51

$$\frac{24\,a^4\sqrt{-c}\,x^3\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}\,(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+\left(24\,a^2x^2+9\,ax+2\right)\sqrt{-a^2c}}{6\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
fricas")
```

```
[Out] 1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(-a^2*c))
/(a*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 85, normalized size = 0.44

$$-\frac{\left(24a^{3}\ln(x)x^{3}-24\ln\left(ax-1\right)x^{3}a^{3}-24a^{2}x^{2}-9ax-2\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\ \left(ax-1\right)}{6x^{3}\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*ln(a*x-1)*x^3*a^3-24*a^2*x^2-9*a*x-2)*(-c*(a^2*x^
2-1))^(1/2)*(a*x-1)/x^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}}{x^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^{2}\,c\,x^{2}}}{x^{4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.690 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=228

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-a^2cx^2}\,\log(1-a}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/4*(-a^2*c*x^2+c)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)/x^4
/(1-1/a^2/x^2)^(1/2)+2*a*(-a^2*c*x^2+c)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+4*a^2
*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(x)*(-a^2*c*x^2+c)^(1
/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/
x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6192, 6193, 88}

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-a^2cx^2}\,\log(1-a}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2]/(
Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2
)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a^3*
Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a^3*Sqrt[c - a^2
*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^5(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{4a^2}{x^3}-\frac{4a^3}{x^2}-\frac{4a^4}{x}+\frac{4a^5}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5} + \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4} + \frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3} + \frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2} - \frac{4a^3\sqrt{c-}}{\sqrt{1-}}$$

Mathematica [A] time = 0.06, size = 84, normalized size = 0.37

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^4\log(x)+4a^4\log(1-ax)+\frac{4a^3}{x}+\frac{2a^2}{x^2}+\frac{a}{x^3}+\frac{1}{4x^4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(4*x^4) + a/x^3 + (2*a^2)/x^2 + (4*a^3)/x - 4*a^4*L
og[x] + 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.55, size = 106, normalized size = 0.46

$$\frac{16\,a^5\sqrt{-c}\,x^4\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}\,(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+\left(16\,a^3x^3+8\,a^2x^2+4\,ax+1\right)\sqrt{-a^2c}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
fricas")
```

```
[Out] 1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*s
qrt(-a^2*c))/(a*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 93, normalized size = 0.41

$$-\frac{\left(16a^{4}\ln(x)x^{4}-16\ln\left(ax-1\right)x^{4}a^{4}-16x^{3}a^{3}-8a^{2}x^{2}-4ax-1\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{4x^{4}\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -1/4*(16*a^4*ln(x)*x^4-16*ln(a*x-1)*x^4*a^4-16*x^3*a^3-8*a^2*x^2-4*a*x-1)*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/x^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}}{x^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a^{2}\,c\,x^{2}}}{x^{5}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

3.691 $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=211

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{a\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a^2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{7x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\text{lo}}{4a^2\left(c-a^2cx\right.}$$

```
[Out] (1-1/a^2/x^2)^(3/2)*x^4/a/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^5/
(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a^2/(-a*x+1)/(-a^2*c*x^2+c
)^(3/2)+7/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)+1/4
*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.23, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{a\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a^2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{7x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\text{lo}}{4a^2\left(c-a^2cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(a*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2)
)^(3/2)*x^5)/(2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*a
^2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (7*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
- a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
+ a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{(c-a^2cx^2)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^4}{(-1+ax)^2(1+ax)}\,dx}{(c-a^2cx^2)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{a^4}+\frac{x}{a^3}+\frac{1}{2a^4(-1+ax)^2}+\frac{7}{4a^4(-1+ax)}+\frac{1}{4a^4(1+ax)}\right)dx}{(c-a^2cx^2)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{2(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a^2(1-ax)(c-a^2cx^2)^{3/2}} + \frac{7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-}{4a^2(c-a^2cx^2)^{3/2}}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 0.36

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2\left(a^2x^2+2ax+\frac{1}{1-ax}\right)+7\log(1-ax)+\log(ax+1)\right)}{4a^2(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(2*(2*a*x + a^2*x^2 + (1 - a*x)^(-1)) + 7*Log[
1 - a*x] + Log[1 + a*x]))/(4*a^2*(c - a^2*c*x^2)^(3/2))
```

fricas [A] time = 0.61, size = 76, normalized size = 0.36

$$\frac{\left(2\,a^3x^3+2\,a^2x^2-4\,ax+(ax-1)\log(ax+1)+7\,(ax-1)\log(ax-1)-2\right)\sqrt{-a^2c}}{4\left(a^7c^2x-a^6c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*a^3*x^3 + 2*a^2*x^2 - 4*a*x + (a*x - 1)*log(a*x + 1) + 7*(a*x - 1)*l
og(a*x - 1) - 2)*sqrt(-a^2*c)/(a^7*c^2*x - a^6*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 106, normalized size = 0.50

$$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(2x^{3}a^{3}+2a^{2}x^{2}+7\ln\left(ax-1\right)xa+ax\ln\left(ax+1\right)-4ax-7\ln\left(ax-1\right)-\ln\left(ax+1\right)-2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^{2}x^{2}-1\right)c^{2}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3+2*a^2*x^2+7*l
n(a*x-1)*x*a+a*x*ln(a*x+1)-4*a*x-7*ln(a*x-1)-ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a
^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***4/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.692 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a\left(c-a^2cx^2\right)^{3/2}}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4a\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] (1-1/a^2/x^2)^(3/2)*x^4/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a/
(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+5/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a/(-a^
2*c*x^2+c)^(3/2)-1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a/(-a^2*c*x^2+c)^(3/
2)
```

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a\left(c-a^2cx^2\right)^{3/2}}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(c - a^2*c*x^2)^(3/2) + ((1 - 1/(a^2*x^2))^(3
/2)*x^3)/(2*a*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (5*(1 - 1/(a^2*x^2))^(3/2)
*x^3*Log[1 - a*x])/(4*a*(c - a^2*c*x^2)^(3/2)) - ((1 - 1/(a^2*x^2))^(3/2)*x
^3*Log[1 + a*x])/(4*a*(c - a^2*c*x^2)^(3/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^3}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a\left(c-a^2cx^2\right)^{3/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)}{4a\left(c\right.}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.41

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(ax+1)\right)}{4a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1
+ a*x]))/(4*a*(c - a^2*c*x^2)^(3/2))
```

fricas [A] time = 0.59, size = 69, normalized size = 0.40

$$\frac{\left(4\,a^2x^2-4\,ax-(ax-1)\log\left(ax+1\right)+5\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{-a^2c}}{4\left(a^6c^2x-a^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)*sqrt(-a^2*c)/(a^6*c^2*x - a^5*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 97, normalized size = 0.56

$$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(4a^{2}x^{2}+5\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-4ax-5\ln\left(ax-1\right)+\ln\left(ax+1\right)-2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^{2}x^{2}-1\right)c^{2}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a^2*x^2+5*ln(a*x-1)*x
*a-a*x*ln(a*x+1)-4*a*x-5*ln(a*x-1)+ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^3/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}}{\left(c-a^{2}\,c\,x^{2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.693 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+3/4*(1-1/a^2/x^2)
^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a
*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (3*(1 -
 1/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(
a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^2}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.58

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(3(ax-1)\log(1-ax)+(ax-1)\log(ax+1)-2)}{4a^2c(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*(-1 + a*x)*Log[1 - a*x] + (-1 + a*x)*
Log[1 + a*x]))/(a^2*c*(-1 + a*x)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.57, size = 56, normalized size = 0.43

$$\frac{\sqrt{-a^2c}\left((ax-1)\log(ax+1)+3(ax-1)\log(ax-1)-2\right)}{4\left(a^5c^2x-a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 2)/(a
^5*c^2*x - a^4*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 86, normalized size = 0.66

$$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(3\ln\left(ax-1\right)xa+ax\ln\left(ax+1\right)-3\ln\left(ax-1\right)-\ln\left(ax+1\right)-2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^{2}x^{2}-1\right)c^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x*a+a*x*ln(
a*x+1)-3*ln(a*x-1)-ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{2}}{\left(c-a^{2}\,c\,x^{2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.694 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a*(1-1/a^2/
x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6192, 6193, 77, 207}

$$\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) - (a*(1
 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2a(-1+ax)^2}+\frac{1}{2a\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} - \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.68

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left((ax-1)\tanh^{-1}(ax)+1\right)}{2ac(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(1 + (-1 + a*x)*ArcTanh[a*x]))/(2*a*c*(-1 + a*x)*S
qrt[c - a^2*c*x^2])
```

fricas [A] time = 0.57, size = 86, normalized size = 0.99

$$-\frac{\left(a^2x-a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\sqrt{-a^2c}}{4\left(a^4c^2x-a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^4*c^2*x - a^3*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 84, normalized size = 0.97

$$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-\ln\left(ax-1\right)+\ln\left(ax+1\right)-2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^{2}x^{2}-1\right)c^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-a*x*ln(a*
x+1)-ln(a*x-1)+ln(a*x+1)-2)/(a^2*x^2-1)/c^2/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x}{{\left(c-a^{2}\,c\,x^{2}\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.695 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1/
a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6192, 6193, 44, 207}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} - \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.62

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left((ax-1)\tanh^{-1}(ax)-1\right)}{2c(ax-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (-1 + a*x)*ArcTanh[a*x]))/(c*(-1 + a*x)
*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.58, size = 86, normalized size = 0.95

$$-\frac{\left(a^2x-a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\,\sqrt{-a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 84, normalized size = 0.92

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-\ln\left(ax-1\right)+\ln\left(ax+1\right)+2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^{2}x^{2}-1\right)c^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x*a-a*x*ln(a
*x+1)-ln(a*x-1)+ln(a*x+1)+2)/(a^2*x^2-1)/c^2/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(c-a^{2}\,c\,x^{2}\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.696 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{3a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2/
x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-3/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3*l
n(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-
a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 72}

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{3a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
3*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (3*a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x}+\frac{a}{2(-1+ax)^2}-\frac{3a}{4(-1+ax)}-\frac{a}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 0.38

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{2-2ax}-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(ax+1)+\log(x)\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*((2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a*
x])/4 - Log[1 + a*x]/4))/(c - a^2*c*x^2)^(3/2)
```

fricas [A] time = 0.50, size = 63, normalized size = 0.36

$$-\frac{\sqrt{-a^2c}\left((ax-1)\log(ax+1)+3(ax-1)\log(ax-1)-4(ax-1)\log(x)+2\right)}{4\left(a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 4*(a
*x - 1)*log(x) + 2)/(a^2*c^2*x - a*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 92, normalized size = 0.52

$$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(4a\ln(x)x-3\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-4\ln(x)+3\ln\left(ax-1\right)+\ln\left(ax+1\right)-2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^{2}x^{2}-1\right)c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a*ln(x)*x-3*ln(a*x-1)
*x*a-a*x*ln(a*x+1)-4*ln(x)+3*ln(a*x-1)+ln(a*x+1)-2)/(a^2*x^2-1)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(c-a^{2}\,c\,x^{2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.697 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{5a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -a^3*(1-1/a^2/x^2)^(3/2)*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^4*(1-1/a^2/x^2)^(3/
2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^
2*c*x^2+c)^(3/2)-5/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^
(3/2)+1/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.27, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{5a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4x^3(1}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*
x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (5*a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x^2(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x^2}+\frac{a}{x}+\frac{a^2}{2(-1+ax)^2}-\frac{5a^2}{4(-1+ax)}+\frac{a^2}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}} - \frac{5a^4\left(1-\frac{1}{a^2}\right.}{4\left(c\right.}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.37

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{2a}{1-ax}+4a\log(x)-5a\log(1-ax)+a\log(ax+1)-\frac{4}{x}\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-4/x + (2*a)/(1 - a*x) + 4*a*Log[x] - 5*a
*Log[1 - a*x] + a*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/2))
```

fricas [A] time = 0.63, size = 92, normalized size = 0.43

$$-\frac{\sqrt{-a^2c}\left(6\,ax-\left(a^2x^2-ax\right)\log\left(ax+1\right)+5\left(a^2x^2-ax\right)\log\left(ax-1\right)-4\left(a^2x^2-ax\right)\log(x)-4\right)}{4\left(a^2c^2x^2-ac^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/4*sqrt(-a^2*c)*(6*a*x - (a^2*x^2 - a*x)*log(a*x + 1) + 5*(a^2*x^2 - a*x)
*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 4)/(a^2*c^2*x^2 - a*c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.07, size = 118, normalized size = 0.55

$$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(4a^{2}\ln(x)x^{2}-5\ln\left(ax-1\right)x^{2}a^{2}+\ln\left(ax+1\right)x^{2}a^{2}-4a\ln(x)x+5\ln\left(ax-1\right)xa-ax\ln\left(ax+\right.\right.}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^{2}x^{2}-1\right)c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(4*a^2*ln(x)*x^2-5*ln(a*
x-1)*x^2*a^2+ln(a*x+1)*x^2*a^2-4*a*ln(x)*x+5*ln(a*x-1)*x*a-a*x*ln(a*x+1)-6*
a*x+4)/(a^2*x^2-1)/c^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}x^{2}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{2}\left(c-a^{2}\,c\,x^{2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.698 $\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=252

$$\frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{7a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/2*a^3*(1-1/a^2/x^2)^(3/2)*x/(-a^2*c*x^2+c)^(3/2)-a^4*(1-1/a^2/x^2)^(3/2)
*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^5*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*
x^2+c)^(3/2)+2*a^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-7/4*a
^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^5*(1-1/a^2
/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.27, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$\frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{7a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -(a^3*(1 - 1/(a^2*x^2))^(3/2)*x)/(2*(c - a^2*c*x^2)^(3/2)) - (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2) + (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^
3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (2*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3
*Log[x])/(c - a^2*c*x^2)^(3/2) - (7*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 -
 a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 +
 a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x^3(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{2a^2}{x}+\frac{a^3}{2(-1+ax)^2}-\frac{7a^3}{4(-1+ax)}-\frac{a^3}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{2\left(c-a^2cx^2\right)^{3/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}} + \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}} + \frac{2a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right.}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.37

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(ax+1)-\frac{4a}{x}-\frac{2}{x^2}\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8*
a^2*Log[x] - 7*a^2*Log[1 - a*x] - a^2*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/
2))
```

fricas [A] time = 0.54, size = 113, normalized size = 0.45

$$-\frac{\left(6\,a^2x^2-2\,ax+\left(a^3x^3-a^2x^2\right)\log\left(ax+1\right)+7\left(a^3x^3-a^2x^2\right)\log\left(ax-1\right)-8\left(a^3x^3-a^2x^2\right)\log(x)-2\right)\sqrt{-a^2c}}{4\left(a^2c^2x^3-ac^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/4*(6*a^2*x^2 - 2*a*x + (a^3*x^3 - a^2*x^2)*log(a*x + 1) + 7*(a^3*x^3 - a
^2*x^2)*log(a*x - 1) - 8*(a^3*x^3 - a^2*x^2)*log(x) - 2)*sqrt(-a^2*c)/(a^2*
c^2*x^3 - a*c^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 138, normalized size = 0.55

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(8a^3\ln(x)x^3-7\ln\left(ax-1\right)x^3a^3-a^3x^3\ln\left(ax+1\right)-8a^2\ln(x)x^2+7\ln\left(ax-1\right)x^2a^2+\ln\left(ax+1\right.\right.}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^2x^2-1\right)c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(8*a^3*ln(x)*x^3-7*ln(a*
x-1)*x^3*a^3-a^3*x^3*ln(a*x+1)-8*a^2*ln(x)*x^2+7*ln(a*x-1)*x^2*a^2+ln(a*x+1
)*x^2*a^2-6*a^2*x^2+2*a*x+2)/(a^2*x^2-1)/c^2/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.699 $\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=262

$$\frac{x^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{a(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{23x^5\left(1-\frac{1}{a^2x^2}\right.}{16a\left(c-\right.}$$

```
[Out] (1-1/a^2/x^2)^(5/2)*x^6/(-a^2*c*x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a/
(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+(1-1/a^2/x^2)^(5/2)*x^5/a/(-a*x+1)/(-a^2*c*
x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+23/
16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-7/16*(1-1/a^2/
x^2)^(5/2)*x^5*ln(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.24, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$\frac{x^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{a(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{23x^5\left(1-\frac{1}{a^2x^2}\right.}{16a\left(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^6)/(c - a^2*c*x^2)^(5/2) - ((1 - 1/(a^2*x^2))^(5
/2)*x^5)/(8*a*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + ((1 - 1/(a^2*x^2))^(5/2)
*x^5)/(a*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - ((1 - 1/(a^2*x^2))^(5/2)*x^5)/(
8*a*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (23*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log[
1 - a*x])/(16*a*(c - a^2*c*x^2)^(5/2)) - (7*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log
[1 + a*x])/(16*a*(c - a^2*c*x^2)^(5/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^5}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{a^5}+\frac{1}{4a^5(-1+ax)^3}+\frac{1}{a^5(-1+ax)^2}+\frac{23}{16a^5(-1+ax)}+\frac{1}{8a^5(1+ax)^2}-\frac{7}{16a^5(1+ax)}\right) d}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}{(c-a^2cx^2)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1-ax)^2(c-a^2cx^2)^{5/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{a(1-ax)(c-a^2cx^2)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(1+ax)(c-a^2}$$

Mathematica [A] time = 0.11, size = 89, normalized size = 0.34

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(16ax+\frac{16}{1-ax}-\frac{2}{ax+1}-\frac{2}{(ax-1)^2}+23\log(1-ax)-7\log(ax+1)\right)}{16a(c-a^2cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^5*(16*a*x + 16/(1 - a*x) - 2/(-1 + a*x)^2 - 2/(1
 + a*x) + 23*Log[1 - a*x] - 7*Log[1 + a*x]))/(16*a*(c - a^2*c*x^2)^(5/2))
```

fricas [A] time = 0.51, size = 138, normalized size = 0.53

$$-\frac{\left(16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax\right.\right.}{16\left(a^{10}c^3x^3-a^9c^3x^2-a^8c^3x+a^7c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) +
12)*sqrt(-a^2*c)/(a^10*c^3*x^3 - a^9*c^3*x^2 - a^8*c^3*x + a^7*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 185, normalized size = 0.71

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(16x^{4}a^{4}+23\ln\left(ax-1\right)x^{3}a^{3}-7a^{3}x^{3}\ln\left(ax+1\right)-16x^{3}a^{3}-23\ln\left(ax-1\right)x^{2}a^{2}+7\ln\left(ax+\right.\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\left(a^{2}x^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(16*x^4*a^4+23
*ln(a*x-1)*x^3*a^3-7*a^3*x^3*ln(a*x+1)-16*x^3*a^3-23*ln(a*x-1)*x^2*a^2+7*ln
(a*x+1)*x^2*a^2-34*a^2*x^2-23*ln(a*x-1)*x*a+7*a*x*ln(a*x+1)+18*a*x+23*ln(a*
x-1)-7*ln(a*x+1)+12)/(a^2*x^2-1)/c^3/a^6/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^5/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**5/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.700 $\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{11x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5x^5\left(\right.}{}$$

```
[Out] -1/8*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+3/4*(1-1/a^2/x
^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*(1-1/a^2/x^2)^(5/2)*x^5/(a*
x+1)/(-a^2*c*x^2+c)^(5/2)+11/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*
x^2+c)^(5/2)+5/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.28, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$\frac{3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{11x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5x^5\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (3*(
1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + ((1 - 1/(
a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (11*(1 - 1/(a^2*
x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*(1 - 1/(a^2*x
^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^4}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^4(-1+ax)^3}+\frac{3}{4a^4(-1+ax)^2}+\frac{11}{16a^4(-1+ax)}-\frac{1}{8a^4(1+ax)^2}+\frac{5}{16a^4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{11\left(1\right.}{}$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.39

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(-\frac{2\left(5a^2x^2+3ax-6\right)}{(ax-1)^2(ax+1)}+11\log(1-ax)+5\log(ax+1)\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^5*((-2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*(
1 + a*x)) + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5/2))
```

fricas [A] time = 0.47, size = 122, normalized size = 0.56

$$\frac{\left(10\,a^2x^2+6\,ax-5\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)-11\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)-12\right)\sqrt{-a^2c}}{16\left(a^9c^3x^3-a^8c^3x^2-a^7c^3x+a^6c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/16*(10*a^2*x^2 + 6*a*x - 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) - 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 12)*sqrt(-a^2*c)/(a^9*c^3*x^
3 - a^8*c^3*x^2 - a^7*c^3*x + a^6*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 169, normalized size = 0.78

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(11\ln\left(ax-1\right)x^{3}a^{3}+5a^{3}x^{3}\ln\left(ax+1\right)-11\ln\left(ax-1\right)x^{2}a^{2}-5\ln\left(ax+1\right)x^{2}a^{2}-10a^{2}x^{2}-11\ln\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\left(a^{2}x^{2}-1\right)c^{3}a^{5}\left(ax+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(11*ln(a*x-1)*
x^3*a^3+5*a^3*x^3*ln(a*x+1)-11*ln(a*x-1)*x^2*a^2-5*ln(a*x+1)*x^2*a^2-10*a^2
*x^2-11*ln(a*x-1)*x*a-5*a*x*ln(a*x+1)-6*a*x+11*ln(a*x-1)+5*ln(a*x+1)+12)/(a
^2*x^2-1)/c^3/a^5/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x***4/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.701 $\int \frac{e^{\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/2*a*(1-1/a
^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a*(1-1/a^2/x^2)^(5/2)*x
^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a*(1-1/a^2/x^2)^(5/2)*x^5*arctanh(a*x)/
(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.27, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6192, 6193, 88, 207}

$$\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a*(1 -
 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a*(1 - 1/
(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^3}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^3(-1+ax)^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{1}{8a^3(1+ax)^2}+\frac{3}{8a^3\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(3a^2\left(1-\right.\right.}{}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a\left(1-\right.}{8}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.49

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(5a^2x^2-ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)-2\right)}{8a^3c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - a*x + 5*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a
*x)*ArcTanh[a*x]))/(a^3*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.62, size = 136, normalized size = 0.77

$$-\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\sqrt{-a^2c}\sqrt{-c}\,x+c}{a^2x^2-1}\right)-2\left(5\,a^2x^2-ax-2\right)\sqrt{-a^2c}}{16\left(a^8c^3x^3-a^7c^3x^2-a^6c^3x+a^5c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(5*a^2*x^2 - a*x - 2)*sqrt(-a^2*c
))/(a^8*c^3*x^3 - a^7*c^3*x^2 - a^6*c^3*x + a^5*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 169, normalized size = 0.96

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(3\ln\left(ax-1\right)x^{3}a^{3}-3a^{3}x^{3}\ln\left(ax+1\right)-3\ln\left(ax-1\right)x^{2}a^{2}+3\ln\left(ax+1\right)x^{2}a^{2}-10a^{2}x^{2}-3\ln\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\left(a^{2}x^{2}-1\right)c^{3}a^{4}\left(ax+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x
^3*a^3-3*a^3*x^3*ln(a*x+1)-3*ln(a*x-1)*x^2*a^2+3*ln(a*x+1)*x^2*a^2-10*a^2*x
^2-3*ln(a*x-1)*x*a+3*a*x*ln(a*x+1)+2*a*x+3*ln(a*x-1)-3*ln(a*x+1)+4)/(a^2*x^
2-1)/c^3/a^4/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^3/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.702 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/4*a^2*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.27, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6192, 6193, 88, 207}

$$\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) +
(a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (a^2*(
1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^3}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^2}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^2(-1+ax)^3}+\frac{1}{4a^2(-1+ax)^2}-\frac{1}{8a^2(1+ax)^2}-\frac{1}{8a^2\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^3\left(\right.\right.}{}$$

$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1\right.}{}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.46

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-a^2x^2-3ax+(ax-1)^2(ax+1)\tanh^{-1}(ax)+2\right)}{8a^2c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - a^2*x^2 + (-1 + a*x)^2*(1 + a*x)*ArcT
anh[a*x]))/(8*a^2*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.58, size = 134, normalized size = 0.73

$$-\frac{\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)-2\left(a^2x^2+3\,ax-2\right)\sqrt{-a^2c}}{16\left(a^7c^3x^3-a^6c^3x^2-a^5c^3x+a^4c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^
2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 + 3*a*x - 2)*sqrt(-a^2*c))
/(a^7*c^3*x^3 - a^6*c^3*x^2 - a^5*c^3*x + a^4*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.07, size = 164, normalized size = 0.89

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln\left(ax-1\right)x^3a^3-a^3x^3\ln\left(ax+1\right)-\ln\left(ax-1\right)x^2a^2+\ln\left(ax+1\right)x^2a^2+2a^2x^2-\ln\left(ax-1\right)xa\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\left(a^2x^2-1\right)c^3a^3\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x^3*
a^3-a^3*x^3*ln(a*x+1)-ln(a*x-1)*x^2*a^2+ln(a*x+1)*x^2*a^2+2*a^2*x^2-ln(a*x-
1)*x*a+a*x*ln(a*x+1)+6*a*x+ln(a*x-1)-ln(a*x+1)-4)/(a^2*x^2-1)/c^3/a^3/(a*x+
1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.703 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^3*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/8*a^3*(1
-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^3*(1-1/a^2/x^2)^(5
/2)*x^5*arctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.22, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6192, 6193, 77, 207}

$$-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
3*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a(-1+ax)^3}+\frac{1}{8a(1+ax)^2}-\frac{1}{8a\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a^2x^2}\,dx}{8\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.44

$$-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax+1}+\frac{1}{(ax-1)^2}-\tanh^{-1}(ax)\right)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5*((-1 + a*x)^(-2) + (1 + a*x)^(-1) - A
rcTanh[a*x]))/(c - a^2*c*x^2)^(5/2)
```

fricas [A] time = 0.67, size = 134, normalized size = 0.98

$$-\frac{\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\log\left(\frac{a^2cx^2-2\sqrt{-a^2c}\sqrt{-c}x+c}{a^2x^2-1}\right)-2\left(a^2x^2-ax+2\right)\sqrt{-a^2c}}{16\left(a^6c^3x^3-a^5c^3x^2-a^4c^3x+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] -1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^
2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 - a*x + 2)*sqrt(-a^2*c))/(
a^6*c^3*x^3 - a^5*c^3*x^2 - a^4*c^3*x + a^3*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 164, normalized size = 1.20

$$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(\ln\left(ax-1\right)x^{3}a^{3}-a^{3}x^{3}\ln\left(ax+1\right)-\ln\left(ax-1\right)x^{2}a^{2}+\ln\left(ax+1\right)x^{2}a^{2}+2a^{2}x^{2}-\ln\left(ax-1\right)x\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\left(a^{2}x^{2}-1\right)c^{3}a^{2}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x-1)*x^3*
a^3-a^3*x^3*ln(a*x+1)-ln(a*x-1)*x^2*a^2+ln(a*x+1)*x^2*a^2+2*a^2*x^2-ln(a*x-
1)*x*a+a*x*ln(a*x+1)-2*a*x+ln(a*x-1)-ln(a*x+1)+4)/(a^2*x^2-1)/c^3/a^2/(a*x+
1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(c-a^{2}\,c\,x^{2}\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.704 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6192, 6193, 44, 207}

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4(-1+ax)^3}-\frac{1}{4(-1+ax)^2}-\frac{1}{8(1+ax)^2}+\frac{3}{8\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(3a^5\right.}{}$$

$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4\left(\right.}{}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.45

$$-\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(-3a^2x^2+3ax+3(ax-1)^2(ax+1)\tanh^{-1}(ax)+2\right)}{8c^2(ax-1)^2(ax+1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.71, size = 136, normalized size = 0.74

$$-\frac{3\left(a^4x^3-a^3x^2-a^2x+a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\left(3\,a^2x^2-3\,ax-2\right)\sqrt{-a^2c}}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a^2*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)

maple [A] time = 0.07, size = 169, normalized size = 0.92

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(3\ln\left(ax-1\right)x^{3}a^{3}-3a^{3}x^{3}\ln\left(ax+1\right)-3\ln\left(ax-1\right)x^{2}a^{2}+3\ln\left(ax+1\right)x^{2}a^{2}+6a^{2}x^{2}-3\ln\left(a\right.\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\left(a^{2}x^{2}-1\right)c^{3}a\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x)

[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x-1)*x^3*a^3-3*a^3*x^3*ln(a*x+1)-3*ln(a*x-1)*x^2*a^2+3*ln(a*x+1)*x^2*a^2+6*a^2*x^2-3*ln(a*x-1)*x*a+3*a*x*ln(a*x+1)-6*a*x+3*ln(a*x-1)-3*ln(a*x+1)-4)/(a^2*x^2-1)/c^3/a/(a*x+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)

[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)

[Out] Timed out

3.705 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{\left(c-a^2cx^2\right)^{5/2}}+\frac{11a^5x}{}$$

```
[Out] -1/8*a^5*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/2*a^5*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^5*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/(-a
^2*c*x^2+c)^(5/2)+11/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*x^2+
c)^(5/2)+5/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.28, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{\left(c-a^2cx^2\right)^{5/2}}+\frac{11a^5x}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) -
(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a^
5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^5*(
1 - 1/(a^2*x^2))^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (11*a^5*(1 - 1/(
a^2*x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*a^5*(1 -
1/(a^2*x^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$
$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{x(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$
$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{x}+\frac{a}{4(-1+ax)^3}-\frac{a}{2(-1+ax)^2}+\frac{11a}{16(-1+ax)}+\frac{a}{8(1+ax)^2}+\frac{5a}{16(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{5/2}}$$
$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\right.}{\left(c\right.}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.32

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{8}{ax-1}-\frac{2}{ax+1}-\frac{2}{(ax-1)^2}+11\log(1-ax)+5\log(ax+1)-16\log(x)\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-2/(-1 + a*x)^2 + 8/(-1 + a*x) - 2/(1 + a
*x) - 16*Log[x] + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5
/2))
```

fricas [A] time = 0.50, size = 145, normalized size = 0.54

$$-\frac{\left(6a^2x^2+2ax+5\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+1)+11\left(a^3x^3-a^2x^2-ax+1\right)\log(ax-1)-16\left(a^3x^3-a^2x^2\right.\right.}{16\left(a^4c^3x^3-a^3c^3x^2-a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] -1/16*(6*a^2*x^2 + 2*a*x + 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 16*(a^3*x^3 - a^2*x^2 - a*x
+ 1)*log(x) - 12)*sqrt(-a^2*c)/(a^4*c^3*x^3 - a^3*c^3*x^2 - a^2*c^3*x + a*c
^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 196, normalized size = 0.72

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(16a^3\ln(x)x^3-11\ln\left(ax-1\right)x^3a^3-5a^3x^3\ln\left(ax+1\right)-16a^2\ln(x)x^2+11\ln\left(ax-1\right)x^2a^2+5\right.}{16\sqrt{\frac{ax-1}{ax+1}}\ \left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(16*a^3*ln(x)*x
^3-11*ln(a*x-1)*x^3*a^3-5*a^3*x^3*ln(a*x+1)-16*a^2*ln(x)*x^2+11*ln(a*x-1)*x
^2*a^2+5*ln(a*x+1)*x^2*a^2-6*a^2*x^2-16*a*ln(x)*x+11*ln(a*x-1)*x*a+5*a*x*ln
(a*x+1)-2*a*x+16*ln(x)-11*ln(a*x-1)-5*ln(a*x+1)+12)/(a^2*x^2-1)/c^3/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.706 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{3a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{\left(c-a^2cx^2\right)^{5/2}}+\frac{23a^6x^5\left(\right.}{1}$$

```
[Out] a^5*(1-1/a^2/x^2)^(5/2)*x^4/(-a^2*c*x^2+c)^(5/2)-1/8*a^6*(1-1/a^2/x^2)^(5/2
)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-3/4*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x
+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c*x^
2+c)^(5/2)-a^6*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*c*x^2+c)^(5/2)+23/16*a^6
*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-7/16*a^6*(1-1/a^2/
x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.29, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 88}

$$-\frac{3a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^5x^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-a^2cx^2\right)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)}{\left(c-a^2cx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^4)/(c - a^2*c*x^2)^(5/2) - (a^6*(1 - 1/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) - (3*a^6*(1 - 1/(a^2
*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^6*(1 - 1/(a^2*x^
2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^6*(1 - 1/(a^2*x^2))
^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (23*a^6*(1 - 1/(a^2*x^2))^(5/2)*
x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) - (7*a^6*(1 - 1/(a^2*x^2))^(5/
2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{x^2(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{x^2}-\frac{a}{x}+\frac{a^2}{4(-1+ax)^3}-\frac{3a^2}{4(-1+ax)^2}+\frac{23a^2}{16(-1+ax)}-\frac{a^2}{8(1+ax)^2}-\frac{7a^2}{16(1+ax}\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}{\left(c-a^2cx^2\right)^{5/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^6\left(1-\right.}{8(1+ax)}$$

Mathematica [A] time = 0.10, size = 99, normalized size = 0.32

$$\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{12a}{ax-1}+\frac{2a}{ax+1}-\frac{2a}{(ax-1)^2}-16a\log(x)+23a\log(1-ax)-7a\log(ax+1)+\frac{16}{x}\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(16/x - (2*a)/(-1 + a*x)^2 + (12*a)/(-1 +
a*x) + (2*a)/(1 + a*x) - 16*a*Log[x] + 23*a*Log[1 - a*x] - 7*a*Log[1 + a*x]
))/(16*(c - a^2*c*x^2)^(5/2))
```

fricas [A] time = 0.69, size = 174, normalized size = 0.57

$$-\frac{\left(30\,a^3x^3-22\,a^2x^2-28\,ax-7\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax+1\right)+23\left(a^4x^4-a^3x^3-a^2x^2+ax\right)\log\left(ax\right.\right.}{16\left(a^4c^3x^4-a^3c^3x^3-a^2c^3x^2+ac^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(30*a^3*x^3 - 22*a^2*x^2 - 28*a*x - 7*(a^4*x^4 - a^3*x^3 - a^2*x^2 +
a*x)*log(a*x + 1) + 23*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(a*x - 1) - 1
6*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(x) + 16)*sqrt(-a^2*c)/(a^4*c^3*x^
4 - a^3*c^3*x^3 - a^2*c^3*x^2 + a*c^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.06, size = 225, normalized size = 0.73

$$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(16a^4\ln(x)x^4-23\ln\left(ax-1\right)x^4a^4+7\ln\left(ax+1\right)x^4a^4-16a^3\ln(x)x^3+23\ln\left(ax-1\right)x^3a^3-7a^3\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(16*a^4*ln(x)*x
^4-23*ln(a*x-1)*x^4*a^4+7*ln(a*x+1)*x^4*a^4-16*a^3*ln(x)*x^3+23*ln(a*x-1)*x
^3*a^3-7*a^3*x^3*ln(a*x+1)-30*x^3*a^3-16*a^2*ln(x)*x^2+23*ln(a*x-1)*x^2*a^2
-7*ln(a*x+1)*x^2*a^2+22*a^2*x^2+16*a*ln(x)*x-23*ln(a*x-1)*x*a+7*a*x*ln(a*x+
1)+28*a*x-16)/(a^2*x^2-1)/c^3/x/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.707 $\int e^{-\coth^{-1}(ax)} x^2 \sqrt{c - a^2 c x^2} \, dx$

Optimal. Leaf size=76

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)^
(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 43}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -(x^2*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*
c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^2(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\left(-x^2+ax^3\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.59

$$\frac{x^2(3ax-4)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] (x^2*(-4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.69, size = 25, normalized size = 0.33

$$\frac{\left(3\,ax^4-4\,x^3\right)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*a*x^4 - 4*x^3)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a^2cx^2+c}\,x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 47, normalized size = 0.62

$$\frac{x^3\,(3ax-4)\,\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{12ax-12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/12*x^3*(3*a*x-4)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\,x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.708 $\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 43}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -(x*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*
x^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\left(-x+ax^2\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.58

$$\frac{x(2ax-3)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
[Out] (x*(-3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 1.42, size = 25, normalized size = 0.34

$$\frac{\left(2\,ax^3-3\,x^2\right)\sqrt{-a^2c}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fric
as")
[Out] 1/6*(2*a*x^3 - 3*x^2)*sqrt(-a^2*c)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a^2cx^2+c}\,x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 47, normalized size = 0.64

$$\frac{x^2\,(2ax-3)\,\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*x^2*(2*a*x-3)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2+c}\,x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.709 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6192, 6193}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.59

$$\frac{(ax-2)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] ((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.66, size = 22, normalized size = 0.32

$$\frac{\sqrt{-a^2c}\left(ax^2 - 2x\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\sqrt{\frac{ax - 1}{ax + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.03, size = 44, normalized size = 0.64

$$\frac{x\left(ax - 2\right)\sqrt{-a^2c\,x^2 + c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/2*x*(a*x-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\sqrt{\frac{ax - 1}{ax + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c - a^2\,c\,x^2}\,\sqrt{\frac{a\,x - 1}{a\,x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.710 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 43}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(a-\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.63

$$\frac{\sqrt{c-a^2cx^2}\,(ax-\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.58, size = 20, normalized size = 0.29

$$\frac{\sqrt{-a^2c}\left(ax-\log(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fric
as")
[Out] sqrt(-a^2*c)*(a*x - log(x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

maple [A] time = 0.05, size = 46, normalized size = 0.66

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(-ax+\ln(x))\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x))*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

3.711 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/x
/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 43}

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (Sqrt[c - a^2*c*x^2]*Lo
g[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{-1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2} + \frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.61

$$\frac{\sqrt{c-a^2cx^2}\left(a\log(x)+\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.71, size = 22, normalized size = 0.31

$$\frac{\sqrt{-a^2c}\left(ax\log(x)+1\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x*log(x) + 1)/(a*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [A] time = 0.06, size = 48, normalized size = 0.67

$$\frac{\left(a\ln(x)x+1\right)\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{\left(ax-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

```
[Out] (a*ln(x)*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\,(ax-1)\,(ax+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x**2, x)
```

3.712 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] 3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+3/5*x^2*(-a^2*c*x^
2+c)^(1/2)/a^2-1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(1/2)+
3/20*(-5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

Rubi [A] time = 0.43, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6152, 1809, 833, 780, 217, 203}

$$\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) - (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 - 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) + (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
```

```
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x^3(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int \frac{x^3\left(-9a^2c+10a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}\\
&= -\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{\int \frac{x^2\left(-30a^3c^2+36a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{\int \frac{x\left(-72a^4c^3+90a^5c^3x\right)}{\sqrt{c-a^2cx^2}}\,dx}{60a^6c^2}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}\\
&= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 96, normalized size = 0.70

$$
\frac{\left(4a^4x^4-10a^3x^3+12a^2x^2-15ax+24\right)\sqrt{c-a^2cx^2}-15\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{20a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(24 - 15*a*x + 12*a^2*x^2 - 10*a^3*x^3 + 4*a^4*x^4) -
15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20*
a^4)
```

fricas [A] time = 0.53, size = 184, normalized size = 1.34

$$\left[\frac{2\left(4\,a^4x^4-10\,a^3x^3+12\,a^2x^2-15\,ax+24\right)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\,\log\left(2\,a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{40\,a^4},\ \ldots\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/40*(2*(4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x^
2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c))/a^4, 1/20*((4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-
a^2*c*x^2 + c) - 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*
x^2 - c)))/a^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 202, normalized size = 1.47

$$-\frac{x^2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{5a^2c}-\frac{4\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{5c\,a^4}+\frac{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{2a^3c}-\frac{5x\sqrt{-a^2c\,x^2+c}}{4a^3}-\frac{5c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{4a^3\sqrt{a^2c}}+\frac{2\sqrt{-\left(x+\frac{1}{a}\right)\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c-4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)+1/2/a^3*
x*(-a^2*c*x^2+c)^(3/2)/c-5/4/a^3*x*(-a^2*c*x^2+c)^(1/2)-5/4/a^3*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^4*(-(x+1/a)^2*a^2*c+2
*(x+1/a)*a*c)^(1/2)+2/a^3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^
2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [A] time = 0.41, size = 117, normalized size = 0.85

$$-\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x^2}{5\,a^2c}-\frac{5\,\sqrt{-a^2cx^2+c}\,x}{4\,a^3}+\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{2\,a^3c}+\frac{3\,\sqrt{c}\,\arcsin\left(ax\right)}{4\,a^4}+\frac{2\,\sqrt{-a^2cx^2+c}}{a^4}-\frac{4\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{5\,a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) - 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 +
1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) + 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sqrt
(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.713 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] -7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3-2/3*x^2*(-a^2*c*x
^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)-1/24*(-21*a*x+32)*(-a^2*c*x^2+c)
^(1/2)/a^3
```

Rubi [A] time = 0.40, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6152, 1809, 833, 780, 217, 203}

$$\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 - ((32 - 2
1*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(8*a^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
```

```
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int\frac{x^2(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{\int\frac{x^2\left(-7a^2c+8a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{\int\frac{x\left(-16a^3c^2+21a^4c^2x\right)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{(7c)\int\frac{1}{\sqrt{c-a^2}}}{8a^2}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{(7c)\,\mathrm{Subst}\Big(}{}\\
&= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\tan^{-1}\Big(}{8a}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.79

$$\frac{21\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)+\left(6a^3x^3-16a^2x^2+21ax-32\right)\sqrt{c-a^2cx^2}}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*A
rcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

fricas [A] time = 1.29, size = 168, normalized size = 1.50

$$\left[\frac{2\left(6\,a^3x^3-16\,a^2x^2+21\,ax-32\right)\sqrt{-a^2cx^2+c}+21\sqrt{-c}\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{48\,a^3},\ \frac{\left(6\,a^3x^3-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*s
qrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, 1/
24*((6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(c
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

giac [A] time = 0.17, size = 84, normalized size = 0.75

$$\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3x-\frac{8}{a}\right)x+\frac{21}{a^2}\right)x-\frac{32}{a^3}\right)+\frac{7c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{8a^2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x - 8/a)*x + 21/a^2)*x - 32/a^3) + 7/8*c*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

maple [A] time = 0.05, size = 178, normalized size = 1.59

$$-\frac{x\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{4a^2c}+\frac{9x\sqrt{-a^2cx^2+c}}{8a^2}+\frac{9c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{8a^2\sqrt{a^2c}}+\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^3c}-\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c+9/8/a^2*x*(-a^2*c*x^2+c)^(1/2)+9/8/a^2*c/
(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/3/a^3*(-a^2*c*
x^2+c)^(3/2)/c-2/a^3*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)-2/a^2*c/(a^2*c)
^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [A] time = 0.41, size = 93, normalized size = 0.83

$$\frac{9\sqrt{-a^2cx^2+c}\,x}{8a^2}-\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}{4a^2c}-\frac{7\sqrt{c}\arcsin(ax)}{8a^3}-\frac{2\sqrt{-a^2cx^2+c}}{a^3}+\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/8
*sqrt(c)*arcsin(a*x)/a^3 - 2*sqrt(-a^2*c*x^2 + c)/a^3 + 2/3*(-a^2*c*x^2 + c
)^(3/2)/(a^3*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\sqrt{c-a^2cx^2}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}\;(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.714 $\int e^{-2\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2+1/3*x^2*(-a^2*c*x^2+c)
^(1/2)+1/3*(-3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A] time = 0.25, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6167, 6152, 1809, 780, 217, 203}

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) + (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{\int \frac{x\left(-5a^2c+6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 79, normalized size = 0.94

$$\frac{\left(a^2x^2-3ax+5\right)\sqrt{c-a^2cx^2}-3\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((5 - 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] - 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c -
 a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

fricas [A] time = 0.60, size = 150, normalized size = 1.79

$$\left[\frac{2\sqrt{-a^2cx^2+c}\left(a^2x^2-3\,ax+5\right)+3\sqrt{-c}\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{6\,a^2},\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2-3\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*c
*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^2
+ c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c
)*x/(a^2*c*x^2 - c)))/a^2]
```

giac [A] time = 0.15, size = 73, normalized size = 0.87

$$\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x-\frac{3}{a}\right)x+\frac{5}{a^2}\right)-\frac{c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*((x - 3/a)*x + 5/a^2) - c*log(abs(-sqrt(-a^2*c)*x
+ sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

maple [B] time = 0.05, size = 156, normalized size = 1.86

$$-\frac{\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{3 a^{2} c}-\frac{x\sqrt{-a^{2} c\,x^{2}+c}}{a}-\frac{c\arctan\left(\frac{\sqrt{a^{2} c}\,x}{\sqrt{-a^{2} c\,x^{2}+c}}\right)}{a\sqrt{a^{2} c}}+\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^{2} a^{2} c+2\left(x+\frac{1}{a}\right) a c}}{a^{2}}+\frac{2c\arctan\left(\frac{\sqrt{}}{\sqrt{-\left(x+\frac{1}{a}\right)^{2}}}\right)}{a\sqrt{a^{2} c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c-x/a*(-a^2*c*x^2+c)^(1/2)-1/a*c/(a^2*c)^(1/2
)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2*(-(x+1/a)^2*a^2*c+2*(x
+1/a)*a*c)^(1/2)+2/a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2
*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [A] time = 0.42, size = 70, normalized size = 0.83

$$-\frac{\sqrt{-a^{2} c x^{2}+c}\, x}{a}+\frac{\sqrt{c}\,\arcsin\left(a x\right)}{a^{2}}+\frac{2\sqrt{-a^{2} c x^{2}+c}}{a^{2}}-\frac{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}}}{3\,a^{2} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*x/a + sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 + c
)/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax-1\right)}{ax+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.715 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=87

$$-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)^
(1/2)/a-1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6167, 6142, 671, 641, 217, 203}

$$-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\ &= -\left(c\int\frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\ &= -\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\,\text{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2c}}\right.\\ &= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}x}{\sqrt{c-a^2cx^2}}\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 100, normalized size = 1.15

$$\frac{\sqrt{c-a^2cx^2}\left(6\sqrt{1-ax}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)-\sqrt{ax+1}\left(a^2x^2-5ax+4\right)\right)}{2a\sqrt{1-ax}\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 - a
*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.53, size = 134, normalized size = 1.54

$$\left[\frac{2\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{-c}\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}a\sqrt{-c}x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}(ax-4)+3\sqrt{c}\arctan}{2a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x - 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

giac [A] time = 0.17, size = 62, normalized size = 0.71

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3c\log\left(\left|-\sqrt{-a^2c}x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

maple [A] time = 0.05, size = 126, normalized size = 1.45

$$\frac{x\sqrt{-a^{2}c\,x^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{2\sqrt{a^{2}c}}-\frac{2\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2\left(x+\frac{1}{a}\right)ac}}{a}-\frac{2c\arctan\left(\frac{\sqrt{a^{2}c}\,x}{\sqrt{-\left(x+\frac{1}{a}\right)^{2}a^{2}c+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^{2}c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2
*c*x^2+c)^(1/2))-2/a*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)-2*c/(a^2*c)^(1/
2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [A] time = 0.41, size = 47, normalized size = 0.54

$$\frac{1}{2}\sqrt{-a^{2}cx^{2}+c}\,x-\frac{3\sqrt{c}\arcsin\left(ax\right)}{2\,a}-\frac{2\sqrt{-a^{2}cx^{2}+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2
+ c)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^{2}\,c\,x^{2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax-1\right)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.716 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=75

$$\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+arctanh((-a^2*c*x^2+c)^(
1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6152, 1809, 844, 217, 203, 266, 63, 208}

$$\sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \sqrt{c-a^2cx^2}+\frac{\int \frac{-a^2c+2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2}\\
&= \sqrt{c-a^2cx^2}-c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx+(2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \sqrt{c-a^2cx^2}-\frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)+(2ac)\,\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\right.\\
&= \sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right)}{a^2}\\
&= \sqrt{c-a^2cx^2}+2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 97, normalized size = 1.29

$$
\sqrt{c-a^2cx^2}+\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-2\sqrt{c}\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(a^2x^2-1\right)}\right)-\sqrt{c}\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(
-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x
^2]]
```

fricas [A] time = 0.89, size = 191, normalized size = 2.55

$$\left[-2\sqrt{c}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c},\sqrt{-c}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*
sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt
(-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2
- c)) + sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
 + sqrt(-a^2*c*x^2 + c)]
```

giac [A] time = 0.17, size = 95, normalized size = 1.27

$$-\frac{2c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2a\sqrt{-c}\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] -2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) + 2
*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqrt(
-a^2*c*x^2 + c)
```

maple [A] time = 0.05, size = 121, normalized size = 1.61

$$-\sqrt{-a^2c\,x^2+c}+\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)+2\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(x+\frac{1}{a}\right)ac}+\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2c+2\left(\right.}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] -(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2*
(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)+2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [A] time = 0.43, size = 86, normalized size = 1.15

$$a^2\left(\frac{\sqrt{c}\arcsin\left(ax\right)}{a^2}+\frac{\sqrt{-a^2cx^2+c}}{a^2}\right)+a\left(\frac{\sqrt{c}\arcsin\left(ax\right)}{a}+\frac{\sqrt{c}\log\left(\frac{2\sqrt{-a^2cx^2+c}\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] a^2*(sqrt(c)*arcsin(a*x)/a^2 + sqrt(-a^2*c*x^2 + c)/a^2) + a*(sqrt(c)*arcsi
n(a*x)/a + sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x))/
a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)(ax+1)}\,(ax-1)}{x(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x*(a*x + 1)), x)
```

3.717 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)-2*a*arctanh((-a^2*c*x^2
+c)^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.35, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6152, 1807, 844, 217, 203, 266, 63, 208}

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 - 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{\sqrt{c-a^2cx^2}}{x}+\int \frac{2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{x}+(2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx-\left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{\sqrt{c-a^2cx^2}}{x}+(ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right)-\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+a^2c\ldots}\right.\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\ldots\right.}{a}\\
&= \frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 104, normalized size = 1.27

$$
\frac{\sqrt{c-a^2cx^2}}{x}-2a\sqrt{c}\,\log\left(\sqrt{c}\,\sqrt{c-a^2cx^2}+c\right)+a\sqrt{c}\,\tan^{-1}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,\left(a^2x^2-1\right)}\right)+2a\sqrt{c}\,\log(x)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]
*(-1 + a^2*x^2))] + 2*a*Sqrt[c]*Log[x] - 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c
 - a^2*c*x^2]]
```

fricas [A] time = 0.75, size = 210, normalized size = 2.56

$$\left[\frac{a\sqrt{c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+a\sqrt{c}\,x\log\left(-\frac{a^2cx^2+2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x},-\frac{4\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2}}{a^2cx^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a*
sqrt(c)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c))/x, -1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqr
t(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 +
 c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c))/x]
```

giac [A] time = 0.15, size = 134, normalized size = 1.63

$$\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\,\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\,a^2\sqrt{-c}\,c}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] 4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*a^
2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

maple [B] time = 0.05, size = 200, normalized size = 2.44

$$-2\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{c}\;\sqrt{-a^2c\,x^2+c}}{x}\right)a+2\sqrt{-a^2c\,x^2+c}\;a+\frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}+a^2x\sqrt{-a^2c\,x^2+c}+\frac{a^2c\arctan\left(\frac{\sqrt{a^2c}\;x}{\sqrt{-a^2c\,x^2+}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] -2*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a+2*(-a^2*c*x^2+c)^(1
/2)*a+1/c/x*(-a^2*c*x^2+c)^(3/2)+a^2*x*(-a^2*c*x^2+c)^(1/2)+a^2*c/(a^2*c)^(
1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2*a*(-(x+1/a)^2*a^2*c+2*(
x+1/a)*a*c)^(1/2)-2*a^2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*
a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^2cx^2+c}\,(ax-1)}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\,(a\,x-1)}{x^2\,\,(a\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,\,(ax-1)}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

3.718 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a^2*c*x^2+c)^(1
/2)/x^2-2*a*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.35, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6152, 1807, 807, 266, 63, 208}

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) - (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
```

```
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{1}{2}\int \frac{4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{1}{2}\left(3a^2c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}-\frac{1}{4}\left(3a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx,x,x^2\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c-a^2cx^2}\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 76, normalized size = 0.97

$$
\frac{1}{2}\left(\frac{(1-4ax)\sqrt{c-a^2cx^2}}{x^2}+3a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)-3a^2\sqrt{c}\log(x)\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (((1 - 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt[
c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

fricas [A] time = 0.59, size = 149, normalized size = 1.91

$$
\left[\frac{3\,a^2\sqrt{c}\,x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-2\,\sqrt{-a^2cx^2+c}\,(4\,ax-1)}{4\,x^2},\ \frac{3\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a}}{2\,x^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 1))/x^2, 1/2*(3*a^2*sqrt(-c)*x^
2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2 +
 c)*(4*a*x - 1))/x^2]
```

giac [B] time = 0.14, size = 200, normalized size = 2.56

$$-\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^3a^2c+4\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}\,c|a|+\left(\right.}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c + 4*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))*a^2*c^2 - 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^2 - c)^2
```

maple [B] time = 0.05, size = 231, normalized size = 2.96

$$\frac{3\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)\sqrt{c}\,a^2}{2}-\frac{3\sqrt{-a^2c\,x^2+c}\,a^2}{2}-\frac{2a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}-2a^3x\sqrt{-a^2c\,x^2+c}-\frac{2a^3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] 3/2*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*c^(1/2)*a^2-3/2*(-a^2*c*x^2+
c)^(1/2)*a^2-2*a/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^3*x*(-a^2*c*x^2+c)^(1/2)-2*a^
3*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+1/2/c/x^2*(-
a^2*c*x^2+c)^(3/2)+2*a^2*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)+2*a^3*c/(a^
2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax-1)}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x^3\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax-1\right)}{x^{3}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

3.719 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=101

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2)
/x^3-a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.37, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6152, 1807, 835, 807, 266, 63, 208}

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) - (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) - a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{1}{3}\int \frac{6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}-\frac{\int \frac{10a^2c^2-6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c} \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2c}}\right. \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-a\,\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx,x,\sqrt{c}\right. \\
&= \frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.81

$$a^3\sqrt{c}\log(x)+\frac{\left(5a^2x^2-3ax+1\right)\sqrt{c-a^2cx^2}}{3x^3}-a^3\sqrt{c}\log\left(\sqrt{c}\sqrt{c-a^2cx^2}+c\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

[Out] ((1 - 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) + a^3*Sqrt[c]*Log[x] - a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]

fricas [A] time = 0.72, size = 165, normalized size = 1.63

$$\left[\frac{3\,a^3\sqrt{c}\,x^3\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\sqrt{-a^2cx^2+c}\left(5\,a^2x^2-3\,ax+1\right)}{6\,x^3},-\frac{3\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")

[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3, -1/3*(3*a^3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3]

giac [B] time = 0.16, size = 250, normalized size = 2.48

$$\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\left(3\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5a^3c+3\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}\,c|a|\right.}{3\left(\left(\sqrt{-a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")

[Out] 2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) - 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c + 3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) - 12*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^3*c^3 + 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^3

maple [B] time = 0.07, size = 254, normalized size = 2.51

$$-\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{c}\;\sqrt{-a^2c\,x^2+c}}{x}\right)a^3+\sqrt{-a^2c\,x^2+c}\;a^3+\frac{2a^2\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}+2a^4x\sqrt{-a^2c\,x^2+c}+\frac{2a^4c\arctan\left(\right.}{\sqrt{a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x^4,x)

[Out] -c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^3+(-a^2*c*x^2+c)^(1/2)*a^3+2*a^2/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^4*x*(-a^2*c*x^2+c)^(1/2)+2*a^4*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+1/3/c/x^3*(-a^2*c*x^2+c)^(3/2)-a/c/x^2*(-a^2*c*x^2+c)^(3/2)-2*a^3*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)-2*a^4*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax-1)}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x^4\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

3.720 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}$$

```
[Out] 7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a^2*c*x^2+c)^(1
/2)/x^4-2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2-4/3
*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A] time = 0.40, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6152, 1807, 835, 807, 266, 63, 208}

$$-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) - (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{1}{4}\int \frac{8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{\int \frac{21a^2c^2-16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{\int \frac{32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{8}\left(7a^4c\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{16}\left(7a^4c\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{8}\left(7a^2\right)\mathrm{S}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{7}{8}a^4\sqrt{c}\ \mathrm{ta}$$

Mathematica [A] time = 0.16, size = 95, normalized size = 0.73

$$-\frac{7}{8}a^4\sqrt{c}\,\log(x)+\frac{7}{8}a^4\sqrt{c}\,\log\left(\sqrt{c}\,\sqrt{c-a^2cx^2}+c\right)+\frac{\left(-32a^3x^3+21a^2x^2-16ax+6\right)\sqrt{c-a^2cx^2}}{24x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(6 - 16*a*x + 21*a^2*x^2 - 32*a^3*x^3))/(24*x^4) - (7*
a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/8
```

fricas [A] time = 0.54, size = 181, normalized size = 1.39

$$\left[\frac{21\,a^4\sqrt{c}\,x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-2\,\left(32\,a^3x^3-21\,a^2x^2+16\,ax-6\right)\sqrt{-a^2cx^2+c}}{48\,x^4},\frac{21\,a^4\sqrt{-c}\,x^4\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c)
)/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*
c*x^2 - c)) - (32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c))/
x^4]
```

giac [B] time = 0.15, size = 324, normalized size = 2.49

$$-\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}+\frac{21\,\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\,\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2-96\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] -7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 - 96*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^3*a^4*c^3 + 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(
-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 - 32*a^
3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

maple [B] time = 0.07, size = 279, normalized size = 2.15

$$\frac{7\sqrt{c}\ \ln\left(\frac{2c+2\sqrt{c}\ \sqrt{-a^2c\,x^2+c}}{x}\right)a^4}{8}-\frac{7\sqrt{-a^2c\,x^2+c}\ a^4}{8}-\frac{2a^3\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{cx}-2a^5x\sqrt{-a^2c\,x^2+c}-\frac{2a^5c\arctan\left(\frac{\sqrt{a^2c}\ x}{\sqrt{-a^2c\,x^2+\cdots}}\right.}{\sqrt{a^2c}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1)/x^5,x)
```

```
[Out] 7/8*c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)*a^4-7/8*(-a^2*c*x^2+
c)^(1/2)*a^4-2*a^3/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^5*x*(-a^2*c*x^2+c)^(1/2)-2*
a^5*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/3*a/c/x^
3*(-a^2*c*x^2+c)^(3/2)+9/8*a^2/c/x^2*(-a^2*c*x^2+c)^(3/2)+1/4/c/x^4*(-a^2*c
*x^2+c)^(3/2)+2*a^4*(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2)+2*a^5*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-(x+1/a)^2*a^2*c+2*(x+1/a)*a*c)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\,(ax-1)}{(ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x^5\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

3.721 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=227

$$\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)-2*x*(-a^2*c*x^2+c)^(1/2)/a^3
/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3
/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^5/x/(1-1/a^2/x^2
)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) - (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) - (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - a^2*c*
x^2]*Log[1 + a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^4\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^3(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^3}-\frac{4x}{a^2}+\frac{4x^2}{a}-3x^3+ax^4-\frac{4}{a^3(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^4\sqrt{c-}}{5\sqrt{1}}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.38

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{4\log(ax+1)}{a^4}+\frac{4x}{a^3}-\frac{2x^2}{a^2}+\frac{ax^5}{5}+\frac{4x^3}{3a}-\frac{3x^4}{4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 - (2*x^2)/a^2 + (4*x^3)/(3*a) - (3*x^4)/4 +
 (a*x^5)/5 - (4*Log[1 + a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.69, size = 58, normalized size = 0.26

$$\frac{\left(12\,a^5x^5-45\,a^4x^4+80\,a^3x^3-120\,a^2x^2+240\,ax-240\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(12*a^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x - 240*lo
g(a*x + 1))*sqrt(-a^2*c)/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 92, normalized size = 0.41

$$-\frac{\left(-12x^5a^5+45x^4a^4-80x^3a^3+120a^2x^2-240ax+240\ln\left(ax+1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{60a^4\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/60*(-12*x^5*a^5+45*x^4*a^4-80*x^3*a^3+120*a^2*x^2-240*a*x+240*ln(a*x+1))
*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.722 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=186

$$-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^
2/(1-1/a^2/x^2)^(1/2)-x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^
3*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)
/a^4/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*
x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1
/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqr
t[c - a^2*c*x^2]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(-1+ax)^2}{1+ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2c}}{a^4\sqrt{}}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(ax\left(a^3x^3-4a^2x^2+8ax-16\right)+16\log(ax+1)\right)}{4a^4x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-16 + 8*a*x - 4*a^2*x^2 + a^3*x^3) + 16*Log[1 +
a*x]))/(4*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.47, size = 49, normalized size = 0.26

$$\frac{\left(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(-a^2*
c)/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2+c}\,x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 83, normalized size = 0.45

$$\frac{\left(x^4a^4-4x^3a^3+8a^2x^2-16ax+16\ln\left(ax+1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4\left(ax-1\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/4*(x^4*a^4-4*x^3*a^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2
)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\,x^2 \left(\frac{ax - 1}{ax + 1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.723 $\int e^{-3\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\, dx$

Optimal. Leaf size=151

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x*(-a^2*c*x^2+c)^(1/2)/a
/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*ln(
a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^3/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6192, 6193, 77}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
(a^2*x^2)]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\frac{x(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\left(ax\left(2a^2x^2-9ax+24\right)-24\log(ax+1)\right)}{6a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6*a
^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.58, size = 42, normalized size = 0.28

$$\frac{\left(2\,a^3x^3-9\,a^2x^2+24\,ax-24\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fric
as")
[Out] 1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(-a^2*c)/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a^2cx^2+c}\,x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac
")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 76, normalized size = 0.50

$$-\frac{\left(-2x^3a^3+9a^2x^2-24ax+24\ln\left(ax+1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6a^2\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*(-2*x^3*a^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x
+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c}\, x \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.724 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6193, 43}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\left(-3+ax+\frac{4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.50

$$\frac{\sqrt{c-a^2cx^2}\,(ax(ax-6)+8\log(ax+1))}{2a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*x)
```

fricas [A] time = 0.57, size = 33, normalized size = 0.29

$$\frac{\left(a^2x^2-6\,ax+8\,\log\left(ax+1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 67, normalized size = 0.60

$$\frac{\left(a^2x^2-6ax+8\ln\left(ax+1\right)\right)\sqrt{-c\left(a^2x^2-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*
x+1))^(3/2)/a/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c - a^2\,c\,x^2} \left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.725 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=112

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 72}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a*Sqrt[1
 - 1/(a^2*x^2)]*x)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\,(ax-4\log(ax+1)+\log(x))}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x)
```

fricas [A] time = 0.64, size = 26, normalized size = 0.23

$$\frac{\sqrt{-a^2c}\left(ax-4\,\log\left(ax+1\right)+\log(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fric
as")
```

```
[Out] sqrt(-a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

maple [A] time = 0.06, size = 57, normalized size = 0.51

$$\frac{(ax+\ln(x)-4\ln(ax+1))\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] (a*x+ln(x)-4*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3
/2)/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

3.726 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(-a^2*c*x^2+c)^(1/2
)/x/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A] time = 0.23, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) - (3*Sqrt[c - a^2*c*x^
2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^2(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2} - \frac{3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.49

$$\frac{\sqrt{c-a^2cx^2}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[1 -
 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.62, size = 33, normalized size = 0.29

$$\frac{\sqrt{-a^2c}\left(4\,ax\log\left(ax+1\right)-3\,ax\log(x)-1\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

maple [A] time = 0.06, size = 65, normalized size = 0.57

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(3a\ln(x)x-4ax\ln\left(ax+1\right)+1\right)\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(ax-1\right)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(3*a*ln(x)*x-4*a*x*ln(a*x+1)+1)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/(a*x-1)^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

3.727 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=152

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-a^2cx^2}\,\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*(-a^2*c*x^2+c)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3*(-a^2*c*x^2+c)^(1/2)/
x^2/(1-1/a^2/x^2)^(1/2)+4*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2
)-4*a*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [**A**] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-a^2cx^2}\,\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*x^
2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1
- 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^3(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= -\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-a^2cx^2}\log}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 + a
*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.77, size = 88, normalized size = 0.58

$$\frac{8\,a^3\sqrt{-c}\,x^2\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}\,(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+\sqrt{-a^2c}\,(6\,ax-1)}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fr
icas")
```

```
[Out] 1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
+ 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + sqrt(-a^2*c)*(6*a*x - 1))/(a*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

maple [A] time = 0.06, size = 77, normalized size = 0.51

$$\frac{\left(8a^2\ln(x)x^2-8\ln(ax+1)\,x^2a^2+6ax-1\right)\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2x^2\,(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] 1/2*(8*a^2*ln(x)*x^2-8*ln(a*x+1)*x^2*a^2+6*a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a
*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^2/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

3.728 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=193

$$-\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/3*(-a^2*c*x^2+c)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+3/2*(-a^2*c*x^2+c)^(1/2
)/x^3/(1-1/a^2/x^2)^(1/2)-4*a*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-
4*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(a*x+1)*(-a^
2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 88}

$$-\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*x^
2])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(
a^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*
x) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^4(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+ 4a$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.40

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^3\log(x)+4a^3\log(ax+1)-\frac{4a^2}{x}+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[x]
 + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.58, size = 98, normalized size = 0.51

$$\frac{24\,a^4\sqrt{-c}\,x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{-a^2c}\,(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)-\left(24\,a^2x^2-9\,ax+2\right)\sqrt{-a^2c}}{6\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fr
icas")
```

```
[Out] 1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(-a^2*c))
/(a*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

maple [A] time = 0.06, size = 85, normalized size = 0.44

$$-\frac{\left(24a^{3}\ln(x)x^{3}-24a^{3}x^{3}\ln\left(ax+1\right)+24a^{2}x^{2}-9ax+2\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6x^{3}\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*a^3*x^3*ln(a*x+1)+24*a^2*x^2-9*a*x+2)*(-c*(a^2*x^
2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-a^{2}cx^{2}+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a^{2}\,c\,x^{2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

3.729 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=227

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-a^2cx^2}\,\log(ax+}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/4*(-a^2*c*x^2+c)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)/x^
4/(1-1/a^2/x^2)^(1/2)-2*a*(-a^2*c*x^2+c)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+4*a^
2*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(x)*(-a^2*c*x^2+c)^(
1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/
x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6192, 6193, 88}

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-a^2cx^2}\,\log(ax+}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -Sqrt[c - a^2*c*x^2]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2]/
(Sqrt[1 - 1/(a^2*x^2)]*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^
2)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a^3
*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^3*Sqrt[c - a^
2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^5(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^3\sqrt{c}}{\sqrt{}}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.37

$$\frac{\sqrt{c-a^2cx^2}\left(4a^4\log(x)-4a^4\log(ax+1)+\frac{4a^3}{x}-\frac{2a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^4*
Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

fricas [A] time = 0.42, size = 104, normalized size = 0.46

$$\frac{16\,a^5\sqrt{-c}\,x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}\,(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+\left(16\,a^3x^3-8\,a^2x^2+4\,ax-1\right)\sqrt{-a^2c}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fr
icas")
```

```
[Out] 1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*s
qrt(-a^2*c))/(a*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

maple [A] time = 0.06, size = 93, normalized size = 0.41

$$\frac{\left(16a^{4}\ln(x)x^{4}-16\ln\left(ax+1\right)x^{4}a^{4}+16x^{3}a^{3}-8a^{2}x^{2}+4ax-1\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4x^{4}\left(ax-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] 1/4*(16*a^4*ln(x)*x^4-16*ln(a*x+1)*x^4*a^4+16*x^3*a^3-8*a^2*x^2+4*a*x-1)*(-
c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/x^4/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a^{2}\,c\,x^{2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.730 $\int e^{3\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=136

$$-\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+
c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)-4*x^m*hypergeom([1, 1+m],[2+m],a*x)*(-a^
2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6192, 6193, 88, 64}

$$-\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) - (4*x^m*Sqrt[c - a^2*
c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(a*(1 + m)*Sqrt[1 - 1/(a^2*
x^2)])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\frac{x^m(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\left(3x^m+ax^{1+m}+\frac{4x^m}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int\frac{x^m}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2+m;}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.54

$$\frac{x^m\sqrt{c-a^2cx^2}\left(-4(m+2)\,{}_2F_1(1,m+1;m+2;ax)+m(ax+3)+ax+6\right)}{a(m+1)(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(6 + a*x + m*(3 + a*x) - 4*(2 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, a*x]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(a^2x^2+2\,ax+1\right)x^m\sqrt{\frac{ax-1}{ax+1}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x
 + 1))/(a^2*x^2 - 2*a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \sqrt{-a^{2} c \,x^{2}+c}}{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} c x^{2}+c}\, x^{m}}{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

3.731 $\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

```
[Out] -c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^
2+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)-2*a*c*x^(2+m)*hypergeom([1/2, 1
+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)+x^
(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

Rubi [A] time = 0.39, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6151, 1809, 808, 365, 364}

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) - (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6151

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x^m(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{\int \frac{x^m\left(-a^2c(3+2m)-2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-(2ac)\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx-\frac{(c(3+2m))\int \frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{\left(2ac\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}-\frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int}{(2+m)\sqrt{c-a^2c}}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}-\frac{2acx}{}\end{aligned}$$

Mathematica [C] time = 0.23, size = 129, normalized size = 0.75

$$\frac{x^{m+1}\left(\frac{\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-ax}\sqrt{-c(ax+1)}\,F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;ax,-ax\right)}{\sqrt{ax-1}\sqrt{ax+1}}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*((2*Sqrt[1 - a*x]*Sqrt[-(c*(1 + a*x))]*AppellF1[1 + m, 1/2, -1/2
, 2 + m, a*x, -(a*x)])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[c - a^2*c*x^2
]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]
))/(1 + m)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,(ax+1)x^m}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^{m}\sqrt{-a^{2}c\,x^{2}+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int((a*x+1)/(a*x-1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2}cx^{2}+c}\,(ax+1)x^{m}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m}\,\sqrt{c-a^{2}\,c\,x^{2}}\,\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\sqrt{-c\,(ax-1)\,(ax+1)}\,\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

3.732 $\int e^{\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=82

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+c)
^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6192, 6193, 43}

$$\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*Sq
rt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^m(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(x^m+ax^{1+m}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.68

$$\frac{x^m\sqrt{c-a^2cx^2}\,(amx+ax+m+2)}{a(m+1)(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^m*(2 + m + a*x + a*m*x)*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*(2 + m)*Sqrt[1 -
 1/(a^2*x^2)])
```

fricas [A] time = 0.60, size = 74, normalized size = 0.90

$$-\frac{\sqrt{-a^2cx^2+c}\left((am+a)x^2+(m+2)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-\left(am^2+3\,am+2\,a\right)x+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 + (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x +
 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 62, normalized size = 0.76

$$\frac{x^{1+m}\,(amx+ax+m+2)\,\sqrt{-a^2c\,x^2+c}}{(2+m)\,(1+m)\,(ax+1)\,\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+a*x+m+2)*(-a^2*c*x^2+c)^(1/2)/(2+m)/(1+m)/(a*x+1)/((a*x-1)/(
a*x+1))^(1/2)
```

maxima [A] time = 0.34, size = 54, normalized size = 0.66

$$\frac{\left(a\sqrt{-c}\,(m+1)x^2+\sqrt{-c}\,(m+2)x\right)(ax+1)x^m}{\left(m^2+3\,m+2\right)ax+m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] (a*sqrt(-c)*(m + 1)*x^2 + sqrt(-c)*(m + 2)*x)*(a*x + 1)*x^m/((m^2 + 3*m + 2
)*a*x + m^2 + 3*m + 2)
```

mupad [B] time = 1.59, size = 93, normalized size = 1.13

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(m+1\right)}{m^2+3\,m+2}+\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(m+2\right)}{a\,\left(m^2+3\,m+2\right)}\right)}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
+ m^2 + 2) + (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/(x
 - 1/a)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

3.733 $\int e^{-\coth^{-1}(ax)} x^m \sqrt{c - a^2 c x^2} \, dx$

Optimal. Leaf size=83

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+c
)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6192, 6193, 43}

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -((x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)])) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^m(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\left(-x^m+ax^{1+m}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.70

$$\frac{x^m\sqrt{c-a^2cx^2}\,(m(ax-1)+ax-2)}{a(m+1)(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(-2 + a*x + m*(-1 + a*x)))/(a*(1 + m)*(2 + m)*Sqrt
[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.53, size = 75, normalized size = 0.90

$$-\frac{\sqrt{-a^2cx^2+c}\left((am+a)x^2-(m+2)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-\left(am^2+3\,am+2\,a\right)x+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 - (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x +
 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a^2cx^2+c}\,x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 64, normalized size = 0.77

$$\frac{x^{1+m}\,(amx+ax-m-2)\,\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{(2+m)\,(1+m)\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+a*x-m-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(2+m)/
(1+m)/(a*x-1)
```

maxima [A] time = 0.33, size = 57, normalized size = 0.69

$$\frac{\left(a\sqrt{-c}\,(m+1)x^2-\sqrt{-c}\,(m+2)x\right)(ax-1)x^m}{\left(m^2+3\,m+2\right)ax-m^2-3\,m-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

```
[Out] (a*sqrt(-c)*(m + 1)*x^2 - sqrt(-c)*(m + 2)*x)*(a*x - 1)*x^m/((m^2 + 3*m + 2
)*a*x - m^2 - 3*m - 2)
```

mupad [B] time = 1.45, size = 94, normalized size = 1.13

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,(m+1)}{m^2+3\,m+2}-\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,(m+2)}{a\,\left(m^2+3\,m+2\right)}\right)}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
+ m^2 + 2) - (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/(x
 - 1/a)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.734 $\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2}}{m+2}$$

```
[Out] -c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^
2+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)+2*a*c*x^(2+m)*hypergeom([1/2, 1
+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)+x^
(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

Rubi [A] time = 0.38, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6152, 1809, 808, 365, 364}

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) + (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 808

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6152

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[(x^m*(c + d*x^2)^(p + n/2))/(1 - a*x)^n, x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int\frac{x^m(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{\int\frac{x^m\left(-a^2c(3+2m)+2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+(2ac)\int\frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx-\frac{(c(3+2m))\int\frac{x^m}{\sqrt{c-a^2cx^2}}\,dx}{2+m}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{\left(2ac\sqrt{1-a^2x^2}\right)\int\frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}-\frac{\left(c(3+2m)\sqrt{1-a^2x^2}\right)\int\frac{}{}}{(2+m)\sqrt{c-a^2cx}}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}+\frac{2acx^{2+}}{}
\end{aligned}$$

Mathematica [C] time = 0.16, size = 110, normalized size = 0.64

$$\frac{x^{m+1}\left(\frac{\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{\sqrt{1-a^2x^2}}-\frac{2\sqrt{c-acx}\,F_1\left(m+1;\frac{1}{2},-\frac{1}{2};m+2;-ax,ax\right)}{\sqrt{1-ax}}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*((-2*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/2, -1/2, 2 + m, -(a*x), a
*x])/Sqrt[1 - a*x] + (Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2
, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]))/(1 + m)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,(ax-1)x^m}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \sqrt{-a^{2} c\,x^{2}+c}\;\left(a x -1\right)}{a x +1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)/(a*x+1)*(a*x-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^{2} c x^{2} + c} {\left(a x - 1\right)} x^{m}}{a x + 1}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)} \left(a x - 1\right)}{a x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

3.735 $\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;-ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2
+c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)+4*x^m*hypergeom([1, 1+m],[2+m],-a*x)*(-
a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6192, 6193, 88, 64}

$$\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;-ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^m*Sqrt[c - a^2
*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(a*(1 + m)*Sqrt[1 - 1/(
a^2*x^2)])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(-3x^m+ax^{1+m}+\frac{4x^m}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int\frac{x^m}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.55

$$\frac{x^m\sqrt{c-a^2cx^2}\,(4(m+2)\,{}_2F_1(1,m+1;m+2;-ax)+m(ax-3)+ax-6)}{a(m+1)(m+2)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(-6 + a*x + m*(-3 + a*x) + 4*(2 + m)*Hypergeometri
c2F1[1, 1 + m, 2 + m, -(a*x)]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,(ax-1)x^m\sqrt{\frac{ax-1}{ax+1}}}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x
+ 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
```

```
+1)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int x^{m} \sqrt{-a^{2} c \,x^{2}+c}\, \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} c x^{2} + c}\, x^{m} \left(\frac{a x - 1}{a x + 1}\right)^{\frac{3}{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

3.736 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=81

$$-\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\,{}_2F_1\left(8,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[Out] -256*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([8, 4-1/2*n],[5
-1/2*n],(a-1/x)/(a+1/x))/a/(8-n)
```

Rubi [A] time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6191, 6195, 131}

$$-\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\,{}_2F_1\left(8,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-256*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric
2F1[8, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\left(\left(a^6c^3\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{3+\frac{n}{2}}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}{}_2F_1\left(8,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Mathematica [B] time = 2.46, size = 267, normalized size = 3.30

$$-\frac{c^3e^{n\coth^{-1}(ax)}\left(720a^7x^7+120a^6nx^6+24a^5n^2x^5-3024a^5x^5+6a^4n^3x^4-576a^4nx^4+2a^3n^4x^3-152a^3n^2x^3+50\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/5040*(c^3*E^(n*ArcCoth[a*x])*(-912*n + 58*n^3 - n^5 - 5040*a*x + 912*a*n
^2*x - 58*a*n^4*x + a*n^6*x + 1368*a^2*n*x^2 - 64*a^2*n^3*x^2 + a^2*n^5*x^2
 + 5040*a^3*x^3 - 152*a^3*n^2*x^3 + 2*a^3*n^4*x^3 - 576*a^4*n*x^4 + 6*a^4*n
^3*x^4 - 3024*a^5*x^5 + 24*a^5*n^2*x^5 + 120*a^6*n*x^6 + 720*a^7*x^7 + E^(2
*ArcCoth[a*x])*n*(-1152 + 576*n + 104*n^2 - 52*n^3 - 2*n^4 + n^5)*Hypergeom
etric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (-2304 + 784*n^2 - 56*n
^4 + n^6)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/a
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*((a*x - 1)/(a
*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2cx^2-c\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a^2cx^2 - c\right)^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)^3*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c - a^2\,c\,x^2\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^3,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \left(\int 3a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int \left(-3a^4x^4e^{n\,\mathrm{acoth}\,(ax)}\right)dx + \int a^6x^6e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int \left(-e^{n\,\mathrm{acoth}\,(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-3*a**4*x**4*e
xp(n*acoth(a*x)), x) + Integral(a**6*x**6*exp(n*acoth(a*x)), x) + Integral(
-exp(n*acoth(a*x)), x))
```

3.737 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=81

$$\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\,{}_2F_1\left(6,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[Out] 64*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([6, 3-1/2*n],[4-1
/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

Rubi [A] time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6191, 6195, 131}

$$\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\,{}_2F_1\left(6,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (64*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[6, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2dx=\left(a^4c^2\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$
$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{2+\frac{n}{2}}}{x^6}dx,x,\frac{1}{x}\right)\right)$$
$$=\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}{}_2F_1\left(6,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Mathematica [B] time = 1.42, size = 179, normalized size = 2.21

$$\frac{c^2e^{n\coth^{-1}(ax)}\left(24a^5x^5+6a^4nx^4+2a^3n^2x^3-80a^3x^3+a^2n^3x^2-28a^2nx^2+\left(n^4-20n^2+64\right){}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*E^(n*ArcCoth[a*x])*(22*n - n^3 + 120*a*x - 22*a*n^2*x + a*n^4*x - 28*a
^2*n*x^2 + a^2*n^3*x^2 - 80*a^3*x^3 + 2*a^3*n^2*x^3 + 6*a^4*n*x^4 + 24*a^5*
x^5 + E^(2*ArcCoth[a*x])*n*(32 - 16*n - 2*n^2 + n^3)*Hypergeometric2F1[1, 1
 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (64 - 20*n^2 + n^4)*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/(120*a)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x - 1)/(a*x + 1))^(1/2*n),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2cx^2-c\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
```

[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2cx^2 - c\right)^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="maxima")

[Out] integrate((a^2*c*x^2 - c)^2*((a*x - 1)/(a*x + 1))^(1/2*n), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)} \left(c - a^2\,c\,x^2\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^2,x)

[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int \left(-2a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\right) dx + \int a^4x^4e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**2,x)

[Out] c**2*(Integral(-2*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(a**4*x**4*exp(
n*acoth(a*x)), x) + Integral(exp(n*acoth(a*x)), x))

3.738 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx$

Optimal. Leaf size=79

$$-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}{}_2F_1\left(4,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[Out] -16*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([4, 2-1/2*n],[3-1/
2*n],(a-1/x)/(a+1/x))/a/(4-n)
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6191, 6195, 131}

$$-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}{}_2F_1\left(4,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-16*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1
[4, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6191

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)dx=-\left(\left(a^2c\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$
$$=\left(a^2c\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{1+\frac{n}{2}}}{x^4}dx,x,\frac{1}{x}\right)$$
$$=-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}{}_2F_1\left(4,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Mathematica [A] time = 0.77, size = 111, normalized size = 1.41

$$-\frac{ce^{n\coth^{-1}(ax)}\left(2a^3x^3+a^2nx^2+\left(n^2-4\right){}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+(n-2)ne^{2\coth^{-1}(ax)}{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^2\right.\right.}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -1/6*(c*E^(n*ArcCoth[a*x])*(-n - 6*a*x + a*n^2*x + a^2*n*x^2 + 2*a^3*x^3 +
E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*A
rcCoth[a*x])] + (-4 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[
a*x])]))/a
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2cx^2-c\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a^2cx^2-c\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a^2cx^2 - c\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c - a^2\,c\,x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int \left(-e^{n\,\mathrm{acoth}\,(ax)}\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)),
 x))
```

3.739 $\int e^{n\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=78

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\,_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[Out] 4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n]
,(a-1/x)/(a+1/x))/a/(2-n)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6170, 131}

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\,_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6170

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\,_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.05

$$\frac{e^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}\,_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(ax)}\right)+(n+2)\left(\,_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+ax\right)\right)}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arccoth}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x)),x)
```

```
[Out] int(exp(n*arccoth(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x)),x)
```

```
[Out] int(exp(n*acoth(a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{acoth}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x)),x)
```

```
[Out] Integral(exp(n*acoth(a*x)), x)
```

3.740 $\int \frac{e^{n \coth^{-1}(ax)}}{c-a^2cx^2} dx$

Optimal. Leaf size=18

$$\frac{e^{n \coth^{-1}(ax)}}{acn}$$

[Out] exp(n*arccoth(a*x))/a/c/n

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6183}

$$\frac{e^{n \coth^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]

[Out] E^(n*ArcCoth[a*x])/(a*c*n)

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n \coth^{-1}(ax)}}{c - a^2cx^2} dx = \frac{e^{n \coth^{-1}(ax)}}{acn}$$

Mathematica [A] time = 0.05, size = 18, normalized size = 1.00

$$\frac{e^{n \coth^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]

[Out] E^(n*ArcCoth[a*x])/(a*c*n)

fricas [A] time = 0.82, size = 28, normalized size = 1.56

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="fricas")

[Out] -((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2cx^2 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

maple [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{e^{n \operatorname{arccoth}(ax)}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x)
```

```
[Out] exp(n*arccoth(a*x))/a/c/n
```

maxima [A] time = 0.32, size = 31, normalized size = 1.72

$$-\frac{e^{\left(-\frac{1}{2} n \log(ax+1)+\frac{1}{2} n \log(ax-1)\right)}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -e^(-1/2*n*log(a*x + 1) + 1/2*n*log(a*x - 1))/(a*c*n)
```

mupad [B] time = 1.45, size = 39, normalized size = 2.17

$$\frac{\left(\frac{1}{a\,x}+1\right)^{n/2}}{a\,c\,n\,\left(1-\frac{1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2),x)
```

```
[Out] (1/(a*x) + 1)^(n/2)/(a*c*n*(1 - 1/(a*x))^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty} x & \text{for}\: c = 0 \wedge n = 0 \\ \tilde{\infty} \int e^{n \operatorname{acoth}{\left(a x \right)}}\, dx & \text{for}\: c = 0 \\ - \frac{\log{\left(x - \frac{1}{a} \right)}}{2 a c} + \frac{\log{\left(x + \frac{1}{a} \right)}}{2 a c} & \text{for}\: n = 0 \\ \frac{e^{n \operatorname{acoth}{\left(a x \right)}}}{a c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c),x)
```

```
[Out] Piecewise((zoo*x, Eq(c, 0) & Eq(n, 0)), (zoo*Integral(exp(n*acoth(a*x)), x)
, Eq(c, 0)), (-log(x - 1/a)/(2*a*c) + log(x + 1/a)/(2*a*c), Eq(n, 0)), (exp
(n*acoth(a*x))/(a*c*n), True))
```

3.741 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=72

$$\frac{2e^{n\coth^{-1}(ax)}}{ac^2n(4-n^2)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{ac^2(4-n^2)(1-a^2x^2)}$$

```
[Out] 2*exp(n*arccoth(a*x))/a/c^2/n/(-n^2+4)-exp(n*arccoth(a*x))*(-2*a*x+n)/a/c^2
/(-n^2+4)/(-a^2*x^2+1)
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$\frac{2e^{n\coth^{-1}(ax)}}{ac^2n(4-n^2)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{ac^2(4-n^2)(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^(n*ArcCoth[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcCoth[a*x])*(n - 2*a*x
))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2(4-n^2)(1-a^2x^2)}+\frac{2\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c(4-n^2)}\\ &= \frac{2e^{n\coth^{-1}(ax)}}{ac^2n(4-n^2)}-\frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2(4-n^2)(1-a^2x^2)}\end{aligned}$$

Mathematica [A] time = 0.18, size = 55, normalized size = 0.76

$$-\frac{(2a^2x^2-2anx+n^2-2)e^{n\coth^{-1}(ax)}}{ac^2n(n^2-4)(a^2x^2-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(-2 + n^2 - 2*a*n*x + 2*a^2*x^2))/(a*c^2*n*(-4 + n^2)
*(-1 + a^2*x^2)))
```

fricas [A] time = 0.57, size = 80, normalized size = 1.11

$$-\frac{\left(2\,a^{2}x^{2}+2\,anx+n^{2}-2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{ac^{2}n^{3}-4\,ac^{2}n-\left(a^{3}c^{2}n^{3}-4\,a^{3}c^{2}n\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*x^2 + 2*a*n*x + n^2 - 2)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^2*n^3 -
 4*a*c^2*n - (a^3*c^2*n^3 - 4*a^3*c^2*n)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(a^{2}cx^{2}-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

maple [A] time = 0.04, size = 55, normalized size = 0.76

$$-\frac{e^{n\,\mathrm{arccoth}(ax)}\left(2a^{2}x^{2}-2xan+n^{2}-2\right)}{\left(a^{2}x^{2}-1\right)c^{2}an\left(n^{2}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -exp(n*arccoth(a*x))*(2*a^2*x^2-2*a*n*x+n^2-2)/(a^2*x^2-1)/c^2/a/n/(n^2-4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(a^{2}cx^{2}-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

mupad [B] time = 1.59, size = 106, normalized size = 1.47

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,x^2}{a\,c^2\,n\,\left(n^2-4\right)}-\frac{2\,x}{a^2\,c^2\,\left(n^2-4\right)}+\frac{n^2-2}{a^3\,c^2\,n\,\left(n^2-4\right)}\right)}{\left(\frac{1}{a^2}-x^2\right)\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^2,x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((2*x^2)/(a*c^2*n*(n^2 - 4)) - (2*x)/(a^2*c^2*(n^2
 - 4)) + (n^2 - 2)/(a^3*c^2*n*(n^2 - 4))))/((1/a^2 - x^2)*((a*x - 1)/(a*x))
^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty} x e^{-\infty n} \\ \tilde{\infty} x e^{\infty n} \\ \tilde{\infty} \int e^{n \operatorname{acoth}(ax)}\, dx \\ -\frac{a^2x^2 \operatorname{acoth}(ax)}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)} - 4ac^2e^{2\operatorname{acoth}(ax)}} - \frac{2ax \operatorname{acoth}(ax)}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)} - 4ac^2e^{2\operatorname{acoth}(ax)}} + \frac{ax}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)} - 4ac^2e^{2\operatorname{acoth}(ax)}} - \frac{\operatorname{acoth}(ax)}{4a^3c^2x^2e^{2\operatorname{acoth}(ax)} - 4ac \ldots} \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} + \frac{a^2x^2\log\left(x+\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} - \frac{2ax}{4a^3c^2x^2-4ac^2} + \frac{\log\left(x-\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} - \frac{\log\left(x+\frac{1}{a}\right)}{4a^3c^2x^2-4ac^2} \\ \frac{\int \frac{e^{2\operatorname{acoth}(ax)}}{a^4x^4-2a^2x^2+1}\, dx}{c^2} \\ -\frac{2a^2x^2e^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} + \frac{2anxe^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} - \frac{n^2e^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n^3+4ac^2n} + \frac{2e^{n\operatorname{acoth}(ax)}}{a^3c^2n^3x^2-4a^3c^2nx^2-ac^2n \ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((zoo*x*exp(-oo*n), Eq(a, -1/x)), (zoo*x*exp(oo*n), Eq(a, 1/x)), (
zoo*Integral(exp(n*acoth(a*x)), x), Eq(c, 0)), (-a**2*x**2*acoth(a*x)/(4*a*
*3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) - 2*a*x*acoth(
a*x)/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) + a*
x/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) - acoth
(a*x)/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) + 2
/(4*a**3*c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))), Eq(n, -
2)), (-a**2*x**2*log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) + a**2*x**2*log
(x + 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - 2*a*x/(4*a**3*c**2*x**2 - 4*a*c**
2) + log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - log(x + 1/a)/(4*a**3*c**2
*x**2 - 4*a*c**2), Eq(n, 0)), (Integral(exp(2*acoth(a*x))/(a**4*x**4 - 2*a*
*2*x**2 + 1), x)/c**2, Eq(n, 2)), (-2*a**2*x**2*exp(n*acoth(a*x))/(a**3*c**
2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n) + 2*a*n*x*exp(
n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a
*c**2*n) - n**2*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2
 - a*c**2*n**3 + 4*a*c**2*n) + 2*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4
*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n), True))
```

3.742 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3} dx$

Optimal. Leaf size=127

$$-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12(n-2ax)e^{n\coth^{-1}(ax)}}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24e^{n\coth^{-1}(ax)}}{ac^3n\left(n^4-20n^2+64\right)}$$

```
[Out] 24*exp(n*arccoth(a*x))/a/c^3/n/(n^4-20*n^2+64)-exp(n*arccoth(a*x))*(-4*a*x+
n)/a/c^3/(-n^2+16)/(-a^2*x^2+1)^2-12*exp(n*arccoth(a*x))*(-2*a*x+n)/a/c^3/(
n^4-20*n^2+64)/(-a^2*x^2+1)
```

Rubi [A] time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12(n-2ax)e^{n\coth^{-1}(ax)}}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24e^{n\coth^{-1}(ax)}}{ac^3n\left(n^4-20n^2+64\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (24*E^(n*ArcCoth[a*x]))/(a*c^3*n*(64 - 20*n^2 + n^4)) - (E^(n*ArcCoth[a*x])
*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2) - (12*E^(n*ArcCoth[a*x])*(
n - 2*a*x))/(a*c^3*(4 - n^2)*(16 - n^2)*(1 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}+\frac{12\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx}{c\left(16-n^2\right)}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c^2\left(64-20n^2+n^4\right)}\\
&= \frac{24e^{n\coth^{-1}(ax)}}{ac^3n\left(64-20n^2+n^4\right)}-\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 97, normalized size = 0.76

$$\frac{\left(4n^2\left(3a^2x^2-4\right)-8anx\left(3a^2x^2-5\right)+24\left(a^2x^2-1\right)^2-4an^3x+n^4\right)e^{n\coth^{-1}(ax)}}{ac^3n\left(n^2-16\right)\left(n^2-4\right)\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(n^4 - 4*a*n^3*x + 24*(-1 + a^2*x^2)^2 - 8*a*n*x*(-5 +
3*a^2*x^2) + 4*n^2*(-4 + 3*a^2*x^2)))/(a*c^3*n*(-16 + n^2)*(-4 + n^2)*(-1 +
 a^2*x^2)^2)
```

fricas [A] time = 0.60, size = 175, normalized size = 1.38

$$-\frac{\left(24\,a^4x^4+24\,a^3nx^3+n^4+12\left(a^2n^2-4\,a^2\right)x^2-16\,n^2+4\left(an^3-10\,an\right)x+24\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{ac^3n^5-20\,ac^3n^3+64\,ac^3n+\left(a^5c^3n^5-20\,a^5c^3n^3+64\,a^5c^3n\right)x^4-2\left(a^3c^3n^5-20\,a^3c^3n^3+64\,a^3c^3n\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -(24*a^4*x^4 + 24*a^3*n*x^3 + n^4 + 12*(a^2*n^2 - 4*a^2)*x^2 - 16*n^2 + 4*(
a*n^3 - 10*a*n)*x + 24)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^3*n^5 - 20*a*c^3
*n^3 + 64*a*c^3*n + (a^5*c^3*n^5 - 20*a^5*c^3*n^3 + 64*a^5*c^3*n)*x^4 - 2*(
a^3*c^3*n^5 - 20*a^3*c^3*n^3 + 64*a^3*c^3*n)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

maple [A] time = 0.04, size = 101, normalized size = 0.80

$$\frac{\left(24x^4a^4-24x^3a^3n+12a^2n^2x^2-4a\,n^3x-48a^2x^2+n^4+40xan-16n^2+24\right)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\left(a^2x^2-1\right)^2c^3a\left(n^2-16\right)\left(n^2-4\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x)
```

```
[Out] (24*a^4*x^4-24*a^3*n*x^3+12*a^2*n^2*x^2-4*a*n^3*x-48*a^2*x^2+n^4+40*a*n*x-1
6*n^2+24)*exp(n*arccoth(a*x))/(a^2*x^2-1)^2/c^3/a/(n^2-16)/(n^2-4)/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(a^2cx^2-c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

mupad [B] time = 1.73, size = 192, normalized size = 1.51

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{24\,x^4}{a\,c^3\,n\,\left(n^4-20\,n^2+64\right)}-\frac{4\,x\,\left(n^2-10\right)}{a^4\,c^3\,\left(n^4-20\,n^2+64\right)}-\frac{24\,x^3}{a^2\,c^3\,\left(n^4-20\,n^2+64\right)}+\frac{n^4-16\,n^2+24}{a^5\,c^3\,n\,\left(n^4-20\,n^2+64\right)}+\frac{x^2\,\left(12\,n^2-48\right)}{a^3\,c^3\,n\,\left(n^4-20\,n^2+64\right)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^4}+x^4-\frac{2\,x^2}{a^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^3,x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((24*x^4)/(a*c^3*n*(n^4 - 20*n^2 + 64)) - (4*x*(n^
2 - 10))/(a^4*c^3*(n^4 - 20*n^2 + 64)) - (24*x^3)/(a^2*c^3*(n^4 - 20*n^2 +
64)) + (n^4 - 16*n^2 + 24)/(a^5*c^3*n*(n^4 - 20*n^2 + 64)) + (x^2*(12*n^2 -
 48))/(a^3*c^3*n*(n^4 - 20*n^2 + 64))))/(((a*x - 1)/(a*x))^(n/2)*(1/a^4 + x
^4 - (2*x^2)/a^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.743 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=197

$$-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{360(n-2ax)e^{n\coth^{-1}(ax)}}{ac^4\left(4-n^2\right)\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)}-\frac{30(n-4ax)e^{n\coth^{-1}(ax)}}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+$$

```
[Out] 720*exp(n*arccoth(a*x))/a/c^4/n/(-n^2+36)/(n^4-20*n^2+64)-exp(n*arccoth(a*x
))*(-6*a*x+n)/a/c^4/(-n^2+36)/(-a^2*x^2+1)^3-30*exp(n*arccoth(a*x))*(-4*a*x
+n)/a/c^4/(n^4-52*n^2+576)/(-a^2*x^2+1)^2-360*exp(n*arccoth(a*x))*(-2*a*x+n
)/a/c^4/(-n^2+36)/(n^4-20*n^2+64)/(-a^2*x^2+1)
```

Rubi [A] time = 0.18, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6185, 6183}

$$-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{360(n-2ax)e^{n\coth^{-1}(ax)}}{ac^4\left(4-n^2\right)\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)}-\frac{30(n-4ax)e^{n\coth^{-1}(ax)}}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (720*E^(n*ArcCoth[a*x]))/(a*c^4*n*(36 - n^2)*(64 - 20*n^2 + n^4)) - (E^(n*A
rcCoth[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3) - (30*E^(n*Arc
Coth[a*x])*(n - 4*a*x))/(a*c^4*(16 - n^2)*(36 - n^2)*(1 - a^2*x^2)^2) - (36
0*E^(n*ArcCoth[a*x])*(n - 2*a*x))/(a*c^4*(4 - n^2)*(16 - n^2)*(36 - n^2)*(1
 - a^2*x^2))
```

Rule 6183

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} + \frac{30\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{c(36-n^2)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} - \frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2} + \frac{360\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{c^2(576-52n^2+n^4)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} - \frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2} - \frac{360e^{n\coth^{-1}(ax)}(}{ac^4(4-n^2)(576-52n^2}$$

$$= \frac{720e^{n\coth^{-1}(ax)}}{ac^4n(4-n^2)(576-52n^2+n^4)} - \frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} - \frac{30e^{n\coth^{-1}(ax)}(n-4ax}{ac^4(16-n^2)(36-n^2)(1-}$$

Mathematica [A] time = 0.32, size = 152, normalized size = 0.77

$$-\frac{\left(10n^4(3a^2x^2-5)-120an^3x(a^2x^2-2)+720(a^2x^2-1)^3+8n^2(45a^4x^4-105a^2x^2+68)-48anx(15a^4x^4-4\right.}{ac^4n(n^2-36)(n^2-16)(n^2-4)(a^2x^2-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n^6 - 6*a*n^5*x - 120*a*n^3*x*(-2 + a^2*x^2) + 720*(
-1 + a^2*x^2)^3 + 10*n^4*(-5 + 3*a^2*x^2) - 48*a*n*x*(33 - 40*a^2*x^2 + 15*
a^4*x^4) + 8*n^2*(68 - 105*a^2*x^2 + 45*a^4*x^4)))/(a*c^4*n*(-36 + n^2)*(-1
6 + n^2)*(-4 + n^2)*(-1 + a^2*x^2)^3))
```

fricas [A] time = 0.63, size = 310, normalized size = 1.57

$$-\frac{\left(720a^6x^6+720a^5nx^5+n^6+360(a^4n^2-6a^4)x^4-50n^4+120(a^3n^3-16a^3n)x^3+30(a^2n^4\right.}{ac^4n^7-56ac^4n^5+784ac^4n^3-(a^7c^4n^7-56a^7c^4n^5+784a^7c^4n^3-2304a^7c^4n)x^6-2304ac^4n+3(a^5c^4n^7-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -(720*a^6*x^6 + 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 - 6*a^4)*x^4 - 50*n^4 +
120*(a^3*n^3 - 16*a^3*n)*x^3 + 30*(a^2*n^4 - 28*a^2*n^2 + 72*a^2)*x^2 + 544
*n^2 + 6*(a*n^5 - 40*a*n^3 + 264*a*n)*x - 720)*((a*x - 1)/(a*x + 1))^(1/2*n
)/(a*c^4*n^7 - 56*a*c^4*n^5 + 784*a*c^4*n^3 - (a^7*c^4*n^7 - 56*a^7*c^4*n^5
 + 784*a^7*c^4*n^3 - 2304*a^7*c^4*n)*x^6 - 2304*a*c^4*n + 3*(a^5*c^4*n^7 -
56*a^5*c^4*n^5 + 784*a^5*c^4*n^3 - 2304*a^5*c^4*n)*x^4 - 3*(a^3*c^4*n^7 - 5
6*a^3*c^4*n^5 + 784*a^3*c^4*n^3 - 2304*a^3*c^4*n)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{(a^2cx^2-c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

maple [A] time = 0.04, size = 167, normalized size = 0.85

$$-\frac{\left(720x^6a^6 - 720a^5x^5n + 360a^4n^2x^4 - 120a^3n^3x^3 - 2160x^4a^4 + 30a^2n^4x^2 + 1920x^3a^3n - 6a\,n^5x - 840a^2n^2x^2\right.}{\left(a^2x^2-1\right)^3 c^4an\left(n^6 - 56n^4 + 784n^2 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x)
```

```
[Out] -(720*a^6*x^6-720*a^5*n*x^5+360*a^4*n^2*x^4-120*a^3*n^3*x^3-2160*a^4*x^4+30
*a^2*n^4*x^2+1920*a^3*n*x^3-6*a*n^5*x-840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*
a^2*x^2-50*n^4-1584*a*n*x+544*n^2-720)*exp(n*arccoth(a*x))/(a^2*x^2-1)^3/c^
4/a/n/(n^6-56*n^4+784*n^2-2304)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(a^2cx^2-c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

mupad [B] time = 1.76, size = 314, normalized size = 1.59

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n^6-50\,n^4+544\,n^2-720}{a^7\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{720\,x^5}{a^2\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{x^3\,\left(120\,n^2-1920\right)}{a^4\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}+\frac{720\,x^6}{a\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2\right.}\right.}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\left(\frac{1}{a^6}-x^6+\frac{3\,x^4}{a^2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^4,x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((544*n^2 - 50*n^4 + n^6 - 720)/(a^7*c^4*n*(784*n^
2 - 56*n^4 + n^6 - 2304)) - (720*x^5)/(a^2*c^4*(784*n^2 - 56*n^4 + n^6 - 23
04)) - (x^3*(120*n^2 - 1920))/(a^4*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (
720*x^6)/(a*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) - (6*x*(n^4 - 40*n^2 + 2
64))/(a^6*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^2*(30*n^4 - 840*n^2 + 2
160))/(a^5*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^4*(360*n^2 - 2160))/
(a^3*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304))))/(((a*x - 1)/(a*x))^(n/2)*(1/a
^6 - x^6 + (3*x^4)/a^2 - (3*x^2)/a^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

3.744 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{32\left(c-a^2cx^2\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-5}{2}}\,_2F_1\left(5,\frac{5-n}{2};\frac{7-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

```
[Out] 32*(1-1/a/x)^(5/2-1/2*n)*(1+1/a/x)^(-5/2+1/2*n)*(-a^2*c*x^2+c)^(3/2)*hyperg
eom([5, 5/2-1/2*n],[7/2-1/2*n],(a-1/x)/(a+1/x))/a^4/(5-n)/(1-1/a^2/x^2)^(3/
2)/x^3
```

Rubi [A] time = 0.21, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6195, 131}

$$\frac{32\left(c-a^2cx^2\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-5}{2}}\,_2F_1\left(5,\frac{5-n}{2};\frac{7-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (32*(1 - 1/(a*x))^((5 - n)/2)*(1 + 1/(a*x))^((-5 + n)/2)*(c - a^2*c*x^2)^(3
/2)*Hypergeometric2F1[5, (5 - n)/2, (7 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^4*(5 - n)*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}dx=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=-\frac{\left(c-a^2cx^2\right)^{3/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{3}{2}+\frac{n}{2}}}{x^5}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{32\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-5+n)}\left(c-a^2cx^2\right)^{3/2}{}_2F_1\left(5,\frac{5-n}{2};\frac{7-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [B] time = 2.41, size = 280, normalized size = 2.41

$$\frac{c^2\left(96a^3cx^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax+n)e^{n\coth^{-1}(ax)}+2(n-1)e^{(n+1)\coth^{-1}(ax)}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};e^{2\coth^{-1}(ax)}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c^2*(96*a^3*c*(1 - 1/(a^2*x^2))^(3/2)*x^3*(a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1
/(a^2*x^2)]*x*(n + a*x) + 2*E^((1 + n)*ArcCoth[a*x])*(-1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]) - c*(-1 + a^2*x^2)*(2*E^
(n*ArcCoth[a*x])*(-1 + a^2*x^2)^2*(-(a*(-21 + n^2)*x) + 2*n*(1 - n^2 + (3 +
 n^2)*Cosh[2*ArcCoth[a*x]]) + a*(3 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*Ar
cCoth[a*x]]) + 16*a*E^((1 + n)*ArcCoth[a*x])*(-3 + 3*n - n^2 + n^3)*Sqrt[1
- 1/(a^2*x^2)]*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*
x])])))/(192*a*(c - a^2*c*x^2)^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n
), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int e^{n \operatorname{arccoth}(ax)} \left(-a^{2} c\, x^{2}+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} \left(\frac{a x-1}{a x+1}\right)^{\frac{1}{2}\, n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a\, x)} \left(c-a^2\, c\, x^2\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} e^{n \operatorname{acoth}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*exp(n*acoth(a*x)), x)
```

3.745 $\int e^{n \coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=116

$$\frac{8\sqrt{c-a^2cx^2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)x\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 8*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*hypergeom([3, 3/2-1/2*n],[5/
2-1/2*n],(a-1/x)/(a+1/x))*(-a^2*c*x^2+c)^(1/2)/a^2/(3-n)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6195, 131}

$$\frac{8\sqrt{c-a^2cx^2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (8*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*Sqrt[c - a^2*c*x^2]
*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a^
2*(3 - n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=-\frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}}{x^3}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=\frac{8\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}\sqrt{c-a^2cx^2}\,{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.60, size = 101, normalized size = 0.87

$$-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}\,e^{n\coth^{-1}(ax)}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax+n)+2(n-1)e^{\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};e^{2\coth^{-1}(ax)}\right)\right)}{2\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/2*(c*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*(a*Sqrt[1 - 1/(a^2*x^2)]
*x*(n + a*x) + 2*E^ArcCoth[a*x]*(-1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
 + n)/2, E^(2*ArcCoth[a*x])]))/Sqrt[c - a^2*c*x^2]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\sqrt{-a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^2cx^2 + c} \left(\frac{ax - 1}{ax + 1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n \operatorname{acoth}(a x)} \sqrt{c - a^2 c x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c(ax - 1)(ax + 1)} e^{n \operatorname{acoth}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*acoth(a*x)), x)
```

3.746 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=111

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] 2*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1, 1/2-1/2*n],[
3/2-1/2*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6192, 6195, 131}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/
2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])
/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= -\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-a^2cx^2}}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)} x\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.20, size = 81, normalized size = 0.73

$$-\frac{2\sqrt{c-a^2cx^2}\,e^{(n+1)\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};e^{2\coth^{-1}(ax)}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\left(a^2cnx+a^2cx\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*E^((1 + n)*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[1, (1 +
n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(Sqrt[1 - 1/(a^2*x^2)]*(a^2*c*x + a^2
*c*n*x))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.747 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6184}

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.20, size = 43, normalized size = 0.93

$$\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.49, size = 78, normalized size = 1.70

$$-\frac{\sqrt{-a^2cx^2+c}\,(ax+n)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-\left(a^3c^2n^2-a^3c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

[Out] -sqrt(-a^2*c*x^2 + c)*(a*x + n)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^2*n^2 - a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.04, size = 49, normalized size = 1.07

$$\frac{(ax-1)(ax+1)(ax-n)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\left(n^2-1\right)a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 1.49, size = 78, normalized size = 1.70

$$-\frac{\left(\frac{x}{c\left(n^2-1\right)}-\frac{n}{a\,c\left(n^2-1\right)}\right)\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] -((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.748 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*a
rccoth(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6185, 6184}

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c\left(9-n^2\right)}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.63, size = 110, normalized size = 1.08

$$\frac{e^{n\coth^{-1}(ax)}\left(3a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)-3an^2x-2\left(n^2-1\right)n\cosh\left(2\coth^{-1}(ax)\right)+27ax+\cdots\right.}{4ac^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*Co
sh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[
a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.77, size = 163, normalized size = 1.60

$$-\frac{\left(6\,a^3x^3+6\,a^2nx^2+n^3+3\left(an^2-3\,a\right)x-7\,n\right)\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{ac^3n^4-10\,ac^3n^2+\left(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3\right)x^4+9\,ac^3-2\left(a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(6*a^3*x^3 + 6*a^2*n*x^2 + n^3 + 3*(a*n^2 - 3*a)*x - 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.04, size = 84, normalized size = 0.82

$$\frac{(ax-1)\,(ax+1)\left(6x^3a^3-6a^2n\,x^2+3a\,n^2x-n^3-9ax+7n\right){\mathrm e}^{n\,\mathrm{arccoth}(ax)}}{a\left(n^4-10n^2+9\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcco
th(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 1.60, size = 173, normalized size = 1.70

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{6\,x^3}{c^2\,\left(n^4-10\,n^2+9\right)}+\frac{7\,n-n^3}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{3\,x\,\left(n^2-3\right)}{a^2\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{6\,n\,x^2}{a\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -((((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/(
a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9))
- (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x^2*
(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.749 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}} dx$

Optimal. Leaf size=166

$$-\frac{120(n-ax)e^{n\coth^{-1}(ax)}}{ac^3\left(1-n^2\right)\left(9-n^2\right)\left(25-n^2\right)\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\coth^{-1}(ax)}}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-5*a*x+n)/a/c/(-n^2+25)/(-a^2*c*x^2+c)^(5/2)-20*exp(n
*arccoth(a*x))*(-3*a*x+n)/a/c^2/(n^4-34*n^2+225)/(-a^2*c*x^2+c)^(3/2)-120*e
xp(n*arccoth(a*x))*(-a*x+n)/a/c^3/(-n^2+25)/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(
1/2)
```

Rubi [A] time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6185, 6184}

$$-\frac{120(n-ax)e^{n\coth^{-1}(ax)}}{ac^3\left(1-n^2\right)\left(9-n^2\right)\left(25-n^2\right)\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\coth^{-1}(ax)}}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
- (20*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^2*(9 - n^2)*(25 - n^2)*(c - a^2*
c*x^2)^(3/2)) - (120*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^3*(1 - n^2)*(9 - n^
2)*(25 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}+\frac{20\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx}{c\left(25-n^2\right)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{120\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c^2\left(9-n^2\right)\left(25-n^2\right)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\left(25-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\left(9-n^2\right)\left(25-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{120e^{n\coth^{-1}(ax)}(n-ax)}{ac^3\left(1-n^2\right)\left(9-n^2\right)\left(25-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 1.57, size = 299, normalized size = 1.80

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}e^{n\coth^{-1}(ax)}\left(\frac{10n^5}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{340n^3}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{10(n^4-34n^2+225)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2250n}{ax\sqrt{1-\frac{1}{a^2x^2}}}-5n^5\sinh\left(3\coth^{-1}(ax)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/16*(a^2*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))^(3/2)*x^3*((-10*(225 - 34*n
^2 + n^4))/Sqrt[1 - 1/(a^2*x^2)] + (2250*n)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (
340*n^3)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (10*n^5)/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
 + 15*(25 - 26*n^2 + n^4)*Cosh[3*ArcCoth[a*x]] - 45*Cosh[5*ArcCoth[a*x]] +
50*n^2*Cosh[5*ArcCoth[a*x]] - 5*n^4*Cosh[5*ArcCoth[a*x]] - 125*n*Sinh[3*Arc
Coth[a*x]] + 130*n^3*Sinh[3*ArcCoth[a*x]] - 5*n^5*Sinh[3*ArcCoth[a*x]] + 9*
n*Sinh[5*ArcCoth[a*x]] - 10*n^3*Sinh[5*ArcCoth[a*x]] + n^5*Sinh[5*ArcCoth[a
*x]]))/(c^2*(-5 + n)*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(5 + n)*(c - a^2*c*x
^2)^(3/2))
```

fricas [A] time = 0.85, size = 289, normalized size = 1.74

$$-\frac{\left(120\,a^5x^5+120\,a^4nx^4+n^5+60\left(a^3n^2-5\,a^3\right)x^3-30\,n^3+20\left(a^2n^3-13\,a^2n\right)x^2\right.}{ac^4n^6-35\,ac^4n^4+259\,ac^4n^2-\left(a^7c^4n^6-35\,a^7c^4n^4+259\,a^7c^4n^2-225\,a^7c^4\right)x^6-225\,ac^4+3\left(a^5c^4n^6-35\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -(120*a^5*x^5 + 120*a^4*n*x^4 + n^5 + 60*(a^3*n^2 - 5*a^3)*x^3 - 30*n^3 + 2
0*(a^2*n^3 - 13*a^2*n)*x^2 + 5*(a*n^4 - 22*a*n^2 + 45*a)*x + 149*n)*sqrt(-a
^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^4*n^6 - 35*a*c^4*n^4 + 259
*a*c^4*n^2 - (a^7*c^4*n^6 - 35*a^7*c^4*n^4 + 259*a^7*c^4*n^2 - 225*a^7*c^4)
*x^6 - 225*a*c^4 + 3*(a^5*c^4*n^6 - 35*a^5*c^4*n^4 + 259*a^5*c^4*n^2 - 225*
a^5*c^4)*x^4 - 3*(a^3*c^4*n^6 - 35*a^3*c^4*n^4 + 259*a^3*c^4*n^2 - 225*a^3*
c^4)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

maple [A] time = 0.04, size = 140, normalized size = 0.84

$$\frac{(ax-1)(ax+1)\left(120x^5a^5-120n\,a^4x^4+60a^3n^2x^3-20a^2n^3x^2-300x^3a^3+5a\,n^4x+260a^2n\,x^2-n^5-110a\,n\right.}{a\left(n^6-35n^4+259n^2-225\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(120*a^5*x^5-120*a^4*n*x^4+60*a^3*n^2*x^3-20*a^2*n^3*x^2-30
0*a^3*x^3+5*a*n^4*x+260*a^2*n*x^2-n^5-110*a*n^2*x+30*n^3+225*a*x-149*n)*exp
(n*arccoth(a*x))/a/(n^6-35*n^4+259*n^2-225)/(-a^2*c*x^2+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

mupad [B] time = 1.74, size = 289, normalized size = 1.74

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{120\,x^5}{c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{120\,n\,x^4}{a\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}+\frac{x^3\,\left(60\,n^2-300\right)}{a^2\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{n\,\left(n^4-30\,n^2+149\right)}{a^5\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}+\cdots\right.}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^4}+x^4\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^2}\right)\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] -(((a*x + 1)/(a*x))^(n/2)*((120*x^5)/(c^3*(259*n^2 - 35*n^4 + n^6 - 225)) -
 (120*n*x^4)/(a*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) + (x^3*(60*n^2 - 300))/
(a^2*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) - (n*(n^4 - 30*n^2 + 149))/(a^5*c^
3*(259*n^2 - 35*n^4 + n^6 - 225)) + (5*x*(n^4 - 22*n^2 + 45))/(a^4*c^3*(259
*n^2 - 35*n^4 + n^6 - 225)) - (20*n*x^2*(n^2 - 13))/(a^3*c^3*(259*n^2 - 35*
n^4 + n^6 - 225))))/(((c - a^2*c*x^2)^(1/2)/a^4 + x^4*(c - a^2*c*x^2)^(1/2)
 - (2*x^2*(c - a^2*c*x^2)^(1/2))/a^2)*((a*x - 1)/(a*x))^(n/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

3.750 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{5040(n-ax)e^{n\coth^{-1}(ax)}}{ac^4(1-n^2)(9-n^2)(25-n^2)(49-n^2)\sqrt{c-a^2cx^2}}-\frac{840(n-3ax)e^{n\coth^{-1}(ax)}}{ac^3(9-n^2)(25-n^2)(49-n^2)(c-a^2cx^2)^{3/2}}-\frac{}{ac^2(25}$$

```
[Out] -exp(n*arccoth(a*x))*(-7*a*x+n)/a/c/(-n^2+49)/(-a^2*c*x^2+c)^(7/2)-42*exp(n
*arccoth(a*x))*(-5*a*x+n)/a/c^2/(n^4-74*n^2+1225)/(-a^2*c*x^2+c)^(5/2)-840*
exp(n*arccoth(a*x))*(-3*a*x+n)/a/c^3/(-n^2+49)/(n^4-34*n^2+225)/(-a^2*c*x^2
+c)^(3/2)-5040*exp(n*arccoth(a*x))*(-a*x+n)/a/c^4/(n^4-74*n^2+1225)/(n^4-10
*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6185, 6184}

$$-\frac{5040(n-ax)e^{n\coth^{-1}(ax)}}{ac^4(1-n^2)(9-n^2)(25-n^2)(49-n^2)\sqrt{c-a^2cx^2}}-\frac{840(n-3ax)e^{n\coth^{-1}(ax)}}{ac^3(9-n^2)(25-n^2)(49-n^2)(c-a^2cx^2)^{3/2}}-\frac{}{ac^2(25}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 7*a*x))/(a*c*(49 - n^2)*(c - a^2*c*x^2)^(7/2)))
- (42*E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c^2*(25 - n^2)*(49 - n^2)*(c - a^2
*c*x^2)^(5/2)) - (840*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^3*(9 - n^2)*(25
- n^2)*(49 - n^2)*(c - a^2*c*x^2)^(3/2)) - (5040*E^(n*ArcCoth[a*x])*(n - a*
x))/(a*c^4*(1 - n^2)*(9 - n^2)*(25 - n^2)*(49 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac(49-n^2)(c-a^2cx^2)^{7/2}} + \frac{42\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}dx}{c(49-n^2)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac(49-n^2)(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2(25-n^2)(49-n^2)(c-a^2cx^2)^{5/2}} + \frac{840\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,d}{c^2(25-n^2)(49-}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac(49-n^2)(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2(25-n^2)(49-n^2)(c-a^2cx^2)^{5/2}} - \frac{840e^{n\,\mathrm{co}}}{ac^3(9-n^2)(25-}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac(49-n^2)(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2(25-n^2)(49-n^2)(c-a^2cx^2)^{5/2}} - \frac{840e^{n\,\mathrm{co}}}{ac^3(9-n^2)(25-}$$

Mathematica [A] time = 1.68, size = 260, normalized size = 1.09

$$\frac{ax^2\left(1-\frac{1}{a^2x^2}\right)e^{n\coth^{-1}(ax)}\left(-\frac{63ax\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)}{n^2-9}+\frac{35ax\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(5\coth^{-1}(ax)\right)}{n^2-25}-\frac{7ax\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(7\coth^{-1}(ax)\right)}{n^2-49}\right.}{64c^3\left(c-a^2cx\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (a*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))*x^2*((-35*n)/(-1 + n^2) + (35*a*x)/
(-1 + n^2) - (63*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]])/(-9 + n^2)
 + (35*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[5*ArcCoth[a*x]])/(-25 + n^2) - (7*a*S
qrt[1 - 1/(a^2*x^2)]*x*Cosh[7*ArcCoth[a*x]])/(-49 + n^2) + (21*a*n*Sqrt[1 -
 1/(a^2*x^2)]*x*Sinh[3*ArcCoth[a*x]])/(-9 + n^2) - (7*a*n*Sqrt[1 - 1/(a^2*x
^2)]*x*Sinh[5*ArcCoth[a*x]])/(-25 + n^2) + (a*n*Sqrt[1 - 1/(a^2*x^2)]*x*Sin
h[7*ArcCoth[a*x]])/(-49 + n^2)))/(64*c^3*(c - a^2*c*x^2)^(3/2))
```

fricas [A] time = 0.58, size = 451, normalized size = 1.89

$$-\frac{\left(5040\,a^7x^7+5040\,a^6nx^6+n^7+2520\left(a^5n^2-7\,a^5\right)x^5-77\,n^{5}\right.}{ac^5n^8-84\,ac^5n^6+1974\,ac^5n^4+\left(a^9c^5n^8-84\,a^9c^5n^6+1974\,a^9c^5n^4-12916\,a^9c^5n^2+11025\,a^9c^5\right)x^8-12916\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -(5040*a^7*x^7 + 5040*a^6*n*x^6 + n^7 + 2520*(a^5*n^2 - 7*a^5)*x^5 - 77*n^5
 + 840*(a^4*n^3 - 19*a^4*n)*x^4 + 210*(a^3*n^4 - 34*a^3*n^2 + 105*a^3)*x^3
+ 1519*n^3 + 42*(a^2*n^5 - 50*a^2*n^3 + 409*a^2*n)*x^2 + 7*(a*n^6 - 65*a*n^
4 + 919*a*n^2 - 1575*a)*x - 6483*n)*sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x +
1))^(1/2*n)/(a*c^5*n^8 - 84*a*c^5*n^6 + 1974*a*c^5*n^4 + (a^9*c^5*n^8 - 84*
a^9*c^5*n^6 + 1974*a^9*c^5*n^4 - 12916*a^9*c^5*n^2 + 11025*a^9*c^5)*x^8 - 1
2916*a*c^5*n^2 - 4*(a^7*c^5*n^8 - 84*a^7*c^5*n^6 + 1974*a^7*c^5*n^4 - 12916
*a^7*c^5*n^2 + 11025*a^7*c^5)*x^6 + 11025*a*c^5 + 6*(a^5*c^5*n^8 - 84*a^5*c
^5*n^6 + 1974*a^5*c^5*n^4 - 12916*a^5*c^5*n^2 + 11025*a^5*c^5)*x^4 - 4*(a^3
*c^5*n^8 - 84*a^3*c^5*n^6 + 1974*a^3*c^5*n^4 - 12916*a^3*c^5*n^2 + 11025*a^
3*c^5)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

maple [A] time = 0.04, size = 218, normalized size = 0.91

$$\frac{(ax-1)(ax+1)\left(5040a^7x^7-5040n\,a^6x^6+2520a^5n^2x^5-840a^4n^3x^4-17640x^5a^5+210a^3n^4x^3+15960n\,a^4x^4\right.}{a\left(n^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x)
[Out] (a*x-1)*(a*x+1)*(5040*a^7*x^7-5040*a^6*n*x^6+2520*a^5*n^2*x^5-840*a^4*n^3*x
^4-17640*a^5*x^5+210*a^3*n^4*x^3+15960*a^4*n*x^4-42*a^2*n^5*x^2-7140*a^3*n^
2*x^3+7*a*n^6*x+2100*a^2*n^3*x^2-n^7+22050*a^3*x^3-455*a*n^4*x-17178*a^2*n*
x^2+77*n^5+6433*a*n^2*x-1519*n^3-11025*a*x+6483*n)*exp(n*arccoth(a*x))/a/(n
^8-84*n^6+1974*n^4-12916*n^2+11025)/(-a^2*c*x^2+c)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

mupad [B] time = 1.98, size = 441, normalized size = 1.85

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{5040\,x^7}{c^4\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}+\frac{-n^7+77\,n^5-1519\,n^3+6483\,n}{a^7\,c^4\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}-\frac{5040\,n\,x^6}{a\,c^4\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(9/2),x)
[Out] -(((a*x + 1)/(a*x))^(n/2)*((5040*x^7)/(c^4*(1974*n^4 - 12916*n^2 - 84*n^6 +
 n^8 + 11025)) + (6483*n - 1519*n^3 + 77*n^5 - n^7)/(a^7*c^4*(1974*n^4 - 12
916*n^2 - 84*n^6 + n^8 + 11025)) - (5040*n*x^6)/(a*c^4*(1974*n^4 - 12916*n^
2 - 84*n^6 + n^8 + 11025)) + (x^5*(2520*n^2 - 17640))/(a^2*c^4*(1974*n^4 -
12916*n^2 - 84*n^6 + n^8 + 11025)) + (x^3*(210*n^4 - 7140*n^2 + 22050))/(a^
4*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (7*x*(919*n^2 - 65*n
^4 + n^6 - 1575))/(a^6*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) -
 (840*n*x^4*(n^2 - 19))/(a^3*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 110
```

```
25)) - (42*n*x^2*(n^4 - 50*n^2 + 409))/(a^5*c^4*(1974*n^4 - 12916*n^2 - 84*
n^6 + n^8 + 11025))))/(((a*x - 1)/(a*x))^(n/2)*((c - a^2*c*x^2)^(1/2)/a^6 -
 x^6*(c - a^2*c*x^2)^(1/2) + (3*x^4*(c - a^2*c*x^2)^(1/2))/a^2 - (3*x^2*(c
- a^2*c*x^2)^(1/2))/a^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

3.751 $\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=359

$$-\frac{2nx^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a(1-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(n^2+2n+2\right)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(\right.}{a(1-n)(n+1)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -(2+n)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^
3/a/(1+n)/(-a^2*c*x^2+c)^(3/2)+(n^2+2*n+2)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1
/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3/a/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)+(1-1/a
^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^4/(-a^2*c*x^2
+c)^(3/2)-2*n*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2
*n)*x^3*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/a/(1-n)/(-a^
2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.33, antiderivative size = 363, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6192, 6194, 129, 155, 12, 131}

$$\frac{2nx^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(3-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(n^2+2n+2\right)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1\right.}{a(1-n)(n+1)\left(c-a^2cx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(((2 + n)*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x^3)/(a*(1 + n)*(c - a^2*c*x^2)^(3/2))) + ((2 + 2*n + n^2)*(1
 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*
x^3)/(a*(1 - n)*(1 + n)*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*(
1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^4)/(c - a^2*c*x^2)^(
3/2) + (2*n*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))
^((-3 + n)/2)*x^3*Hypergeometric2F1[1, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(
a + x^(-1))])/(a*(3 - n)*(c - a^2*c*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
```

```
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{2x}{a^2}\right)\left(1-\right.}{}\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(\right.}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2+2n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1\right.}{a(1-n)(1+n)}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2+2n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1\right.}{a(1-n)(1+n)}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(2+2n+n^2\right)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1\right.}{a(1-n)(1+n)}$$

Mathematica [A] time = 0.61, size = 133, normalized size = 0.37

$$\frac{\frac{c(anx-1)e^{n\coth^{-1}(ax)}}{n^2-1}-\frac{c\left(a^2x^2-1\right)\left(\frac{2ne^{(n+1)\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};e^{2\coth^{-1}(ax)}\right)}{ax\sqrt{1-\frac{1}{a^2x^2}}}+(n+1)e^{n\coth^{-1}(ax)}\right)}{n+1}}{a^4c^2\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((c*E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(-1 + n^2) - (c*(-1 + a^2*x^2)*(E^(n*A
rcCoth[a*x])*(1 + n) + (2*E^((1 + n)*ArcCoth[a*x])*n*Hypergeometric2F1[1, (
1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(a*Sqrt[1 - 1/(a^2*x^2)]*x)))/(1
+ n))/(a^4*c^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arccoth}(ax)} x^3}{\left(-a^2 c\, x^2 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2 c x^2 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\, e^{n \operatorname{acoth}(a\,x)}}{\left(c - a^2\, c\, x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{n \operatorname{acoth}(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.752 $\int \frac{e^{n \coth^{-1}(ax)} x^2}{\left(c-a^2cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=164

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}} {}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a^3/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)-2*(1-1/a/
x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1, 1/2-1/2*n],[3/2-1/2*n
],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/a^2/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6189, 6192, 6195, 131}

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}} {}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) - (
2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2
)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^2*c*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6189

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a
^3*c*(n^2 - 4*(p + 1)^2)), x] - Dist[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2)), Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^2
 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ
[n])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx}{a^2c}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{a^2c\sqrt{c-a^2cx^2}}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{a^2c\sqrt{c-a^2cx^2}}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.44, size = 127, normalized size = 0.77

$$-\frac{e^{n\coth^{-1}(ax)}\left(2(n-1)\left(a^2x^2-1\right)e^{\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};e^{2\coth^{-1}(ax)}\right)+ax\sqrt{1-\frac{1}{a^2x^2}}\,(ax-n)\right)}{a^4c(n-1)(n+1)x\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-n + a*x) + 2*E^ArcCoth[a
*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2
*ArcCoth[a*x])]))/(a^4*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c -
a^2*c*x^2]))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}x^2}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,e^{n\,\mathrm{acoth}(a\,x)}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
[Out] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.753 $\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{(1-anx)e^{n\coth^{-1}(ax)}}{a^2c(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] exp(n*arccoth(a*x))*(-a*n*x+1)/a^2/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6186}

$$\frac{(1-anx)e^{n\coth^{-1}(ax)}}{a^2c(1-n^2)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6186

```
Int[(E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[((1 - a*n*x)*E^(n*ArcCoth[a*x]))/(a^2*c*(n^2 - 1)*Sqrt[c + d*x
^2]), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(1-anx)}{a^2c(1-n^2)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.20, size = 43, normalized size = 0.93

$$\frac{(anx-1)e^{n\coth^{-1}(ax)}}{a^2c(n^2-1)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(a^2*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 1.20, size = 83, normalized size = 1.80

$$-\frac{\sqrt{-a^2cx^2+c}\,(anx+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2c^2n^2-a^2c^2-(a^4c^2n^2-a^4c^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(a*n*x + 1)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c^2*n^
2 - a^2*c^2 - (a^4*c^2*n^2 - a^4*c^2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.04, size = 49, normalized size = 1.07

$$-\frac{(ax-1)(ax+1)(xan-1)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{a^2\left(n^2-1\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x)
[Out] -(a*x-1)*(a*x+1)*(a*n*x-1)*exp(n*arccoth(a*x))/a^2/(n^2-1)/(-a^2*c*x^2+c)^(
3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [B] time = 1.43, size = 81, normalized size = 1.76

$$-\frac{\left(\frac{1}{a^2\,c\,\left(n^2-1\right)}-\frac{n\,x}{a\,c\,\left(n^2-1\right)}\right)\,{\left(\frac{a\,x+1}{a\,x}\right)}^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x}\right)}^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
[Out] -((1/(a^2*c*(n^2 - 1)) - (n*x)/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((
c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{n\operatorname{acoth}{\left(ax \right)}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.754 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6184}

$$-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]

[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{e^{n\coth^{-1}(ax)}(n-ax)}{ac\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.19, size = 43, normalized size = 0.93

$$\frac{(n-ax)e^{n\coth^{-1}(ax)}}{ac\left(n^2-1\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]

[Out] (E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])

fricas [A] time = 0.55, size = 78, normalized size = 1.70

$$-\frac{\sqrt{-a^2cx^2+c}\,(ax+n)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-\left(a^3c^2n^2-a^3c^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] -sqrt(-a^2*c*x^2 + c)*(a*x + n)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^2*n^2 - a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")

[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)

maple [A] time = 0.04, size = 49, normalized size = 1.07

$$\frac{(ax-1)(ax+1)(ax-n)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\left(n^2-1\right)a\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x)

[Out] (a*x-1)*(a*x+1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)

mupad [B] time = 0.00, size = 78, normalized size = 1.70

$$-\frac{\left(\frac{x}{c\left(n^2-1\right)}-\frac{n}{a\,c\left(n^2-1\right)}\right)\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)

[Out] -((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)

[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)

3.755 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{a^3 2^{\frac{n+1}{2}} x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}} {}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\left(1-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\right.}{}$$

```
[Out] -a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3/
(1+n)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*(1
+1/a/x)^(-1/2+1/2*n)*x^3/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)-2^(1/2+1/2*n)*a^3*(1
-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*x^3*hypergeom([1/2-1/2*n, 1/2-1/2*n
],[3/2-1/2*n],1/2*(a-1/x)/a)/(1-n)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A] time = 0.37, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6192, 6195, 89, 79, 69}

$$-\frac{a^3 2^{\frac{n+1}{2}} x^3 \left(1-\frac{1}{a^2x^2}\right)^{3/2} \left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}} {}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\left(1-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-
1 + n)/2)*x^3)/((1 + n)*(c - a^2*c*x^2)^(3/2))) + (a^3*(1 - 1/(a^2*x^2))^(3
/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^3)/((1 - n^2)*(c
 - a^2*c*x^2)^(3/2)) - (2^((1 + n)/2)*a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a
*x))^((1 - n)/2)*x^3*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (a
- x^(-1))/(2*a)])/((1 - n)*(c - a^2*c*x^2)^(3/2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
```

```
+ 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\,dx, x, \frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int (1\right.}{(1+}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\right.}{\left(1-n^2\right)\left(c-a^2cx^2\right)^{3}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\right.}{\left(1-n^2\right)\left(c-a^2cx^2\right)^{3}}$$

Mathematica [A] time = 0.89, size = 127, normalized size = 0.46

$$\frac{e^{n\coth^{-1}(ax)}\left(ax\sqrt{1-\frac{1}{a^2x^2}}(anx-1)-2(n-1)\left(a^2x^2-1\right)e^{\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-e^{2\coth^{-1}(ax)}\right)\right)}{ac(n-1)(n+1)x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*n*x) - 2*E^ArcCoth[a
*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E^(
2*ArcCoth[a*x])]))/(a*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a
^2*c*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^2x^5-2\,a^2c^2x^3+c^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^2*x^5 -
2*a^2*c^2*x^3 + c^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x\,\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.756 $\int \frac{e^{n\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=463

$$-\frac{2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(n^2+6n+15\right)x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\right.}{(1-n)(n+1)(n+3)\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+n
)/(-a^2*c*x^2+c)^(5/2)-(6+n)*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+
1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+(n^2+6*n+15)*(1-1/
a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2
+n+3)/(-a^2*c*x^2+c)^(5/2)-(-n^3-2*n^2+7*n+18)*(1-1/a^2/x^2)^(5/2)*(1-1/a/x
)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2
)-2*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5*hy
pergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/(1-n)/(-a^2*c*x^2+c)^(
5/2)
```

Rubi [A] time = 0.53, antiderivative size = 467, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6192, 6195, 129, 155, 12, 131}

$$\frac{2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}{}_2F_1\left(1,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(3-n)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(n^2+6n+15\right)x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{a\right.}}{(1-n)(n+1)(n+3)\left(c-a^2cx^2\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3 +
n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - ((6 + n)*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) + ((15 + 6*n + n^2)*(1 - 1/(a^2*x^2))^(5/2)*(1
- 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 - n)*(1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) - ((18 + 7*n - 2*n^2 - n^3)*(1 - 1/(a^2*x^2))^(
5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2
 + n^4)*(c - a^2*c*x^2)^(5/2)) + (2*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^(
(3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5*Hypergeometric2F1[1, (3 - n)/2, (
5 - n)/2, (a - x^(-1))/(a + x^(-1))])/((3 - n)*(c - a^2*c*x^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n \coth^{-1}(ax)} x^4}{\left(c-a^2cx^2\right)^{5/2}} \, dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5\right) \int \frac{e^{n \coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2} x} \, dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5\right) \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}} \left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}}{x} \, dx, x, \frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} + \frac{\left(a\left(1-\frac{1}{a^2x^2}\right)^{5/2} x^5\right) \text{Subst}\left(\int \frac{\left(-\frac{3+n}{a}-\frac{3x}{a^2}\right)(1 \ldots}{\ldots}\right.}{(3+n)\left(c-a^2cx^2 \ldots\right.}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)} \left(1+ \ldots\right.}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/ \ldots}}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)} \left(1+ \ldots\right.}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/ \ldots}}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)} \left(1+ \ldots\right.}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/ \ldots}}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)} \left(1+ \ldots\right.}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/ \ldots}}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)} \left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2} \left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)} \left(1+ \ldots\right.}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/ \ldots}}$$

Mathematica [A] time = 2.17, size = 201, normalized size = 0.43

$$\frac{\left(a^2x^2-1\right)\left(-\frac{8ax\sqrt{1-\frac{1}{a^2x^2}}\, e^{(n+1)\coth^{-1}(ax)}\, {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};e^{2\coth^{-1}(ax)}\right)}{n+1} + \frac{e^{n\coth^{-1}(ax)}\left(-3a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\,\cosh\left(3\coth^{-1}(ax)\right)+3an^2x+2\left(n^2 \ldots\right.\right.}{n^4-10n^2+9} \ldots\right.}{4a^5c\left(c-a^2cx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((-1 + a^2*x^2)*((8*E^(n*ArcCoth[a*x])*(n - a*x))/(-1 + n^2) + (E^(n*ArcCot
h[a*x])*(26*n - 2*n^3 - 27*a*x + 3*a*n^2*x + 2*n*(-1 + n^2)*Cosh[2*ArcCoth[
a*x]] - 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))/(9 -
10*n^2 + n^4) - (8*a*E^((1 + n)*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*Hyper
```

```
geometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(1 + n)))/(4*a^5
*c*(c - a^2*c*x^2)^(3/2))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^4*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^6*c^3*x
^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}x^4}{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^4*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,e^{n\,\mathrm{acoth}(a\,x)}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 e^{n \operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**4/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.757 $\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}+\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)(1-n^2)\left(c-a^2cx^2\right)^{5/2}}-\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\right.}{\left(n^4-10n^2+\right.}$$

```
[Out] -a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3
+n)/(-a^2*c*x^2+c)^(5/2)-3*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+
1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a*(1-1/a^2/x^2)^
(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)/(-a
^2*c*x^2+c)^(5/2)-6*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(
-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A] time = 0.37, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6192, 6195, 45, 37}

$$-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}+\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)(1-n^2)\left(c-a^2cx^2\right)^{5/2}}-\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\right.}{\left(n^4-10n^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3
+ n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a*(1 - 1/(a^2*x^2))^(5/2
)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n + n^
2)*(c - a^2*c*x^2)^(5/2)) + (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((1
- n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x^2)^
(5/2)) - (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x
))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{\frac{1}{2}(}\right.}{(3+n)\left(c-a^2c\right.}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right.}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right.}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right.}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right)^{5/2}}$$

Mathematica [A] time = 0.73, size = 110, normalized size = 0.33

$$-\frac{e^{n\coth^{-1}(ax)}\left(an\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)-6\left(n^2-1\right)\cosh\left(2\coth^{-1}(ax)\right)+3\left(an^3x-9anx-2n\right.\right.}{4a^4c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/4*(E^(n*ArcCoth[a*x])*(3*(10 - 2*n^2 - 9*a*n*x + a*n^3*x) - 6*(-1 + n^2)
*Cosh[2*ArcCoth[a*x]] + a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCo
th[a*x]]))/(a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.54, size = 176, normalized size = 0.53

$$-\frac{\sqrt{-a^2cx^2+c}\left(\left(a^3n^3-7\,a^3n\right)x^3+6\,anx+3\left(a^2n^2-3\,a^2\right)x^2+6\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3+\left(a^8c^3n^4-10\,a^8c^3n^2+9\,a^8c^3\right)x^4-2\left(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a^3*n^3 - 7*a^3*n)*x^3 + 6*a*n*x + 3*(a^2*n^2 - 3*a
^2)*x^2 + 6)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^3*n^4 - 10*a^4*c^3*n^2 +
9*a^4*c^3 + (a^8*c^3*n^4 - 10*a^8*c^3*n^2 + 9*a^8*c^3)*x^4 - 2*(a^6*c^3*n^4
 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 93, normalized size = 0.28

$$-\frac{(ax-1)(ax+1)\left(a^3n^3x^3-7x^3a^3n-3a^2n^2x^2+9a^2x^2+6xan-6\right)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{a^4\left(n^4-10n^2+9\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(a^3*n^3*x^3-7*a^3*n*x^3-3*a^2*n^2*x^2+9*a^2*x^2+6*a*n*x-6
)*exp(n*arccoth(a*x))/a^4/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 1.56, size = 175, normalized size = 0.53

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6}{a^6\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{6\,n\,x}{a^5\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{x^2\,\left(3\,n^2-9\right)}{a^4\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{n\,x^3\,\left(n^2-7\right)}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -(((a*x + 1)/(a*x))^(n/2)*(6/(a^6*c^2*(n^4 - 10*n^2 + 9)) - (6*n*x)/(a^5*c^
2*(n^4 - 10*n^2 + 9)) + (x^2*(3*n^2 - 9))/(a^4*c^2*(n^4 - 10*n^2 + 9)) - (n
```

```
*x^3*(n^2 - 7))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2
- x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{n \operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.758 $\int \frac{e^{n \coth^{-1}(ax)} x^2}{\left(c-a^2 c x^2\right)^{5/2}}\, dx$

Optimal. Leaf size=102

$$\frac{\left(3-n^2\right)(n-ax)e^{n \coth^{-1}(ax)}}{a^3 c^2 \left(n^4-10n^2+9\right)\sqrt{c-a^2 c x^2}}-\frac{(n-3ax)e^{n \coth^{-1}(ax)}}{a^3 c \left(9-n^2\right)\left(c-a^2 c x^2\right)^{3/2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a^3/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+exp(n*a
rccoth(a*x))*(-n^2+3)*(-a*x+n)/a^3/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6189, 6184}

$$\frac{\left(3-n^2\right)(n-ax)e^{n \coth^{-1}(ax)}}{a^3 c^2 \left(n^4-10n^2+9\right)\sqrt{c-a^2 c x^2}}-\frac{(n-3ax)e^{n \coth^{-1}(ax)}}{a^3 c \left(9-n^2\right)\left(c-a^2 c x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
 + (E^(n*ArcCoth[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(9 - 10*n^2 + n^4)*Sqr
t[c - a^2*c*x^2])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6189

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[((n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a
^3*c*(n^2 - 4*(p + 1)^2)), x] - Dist[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2)), Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^2
 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ
[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n \coth^{-1}(ax)} x^2}{\left(c-a^2 c x^2\right)^{5/2}}\, dx &= -\frac{e^{n \coth^{-1}(ax)}(n-3ax)}{a^3 c \left(9-n^2\right)\left(c-a^2 c x^2\right)^{3/2}}-\frac{\left(3-n^2\right)\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-a^2 c x^2\right)^{3/2}}\, dx}{a^2 c \left(9-n^2\right)}\\
&= -\frac{e^{n \coth^{-1}(ax)}(n-3ax)}{a^3 c \left(9-n^2\right)\left(c-a^2 c x^2\right)^{3/2}}+\frac{e^{n \coth^{-1}(ax)}\left(3-n^2\right)(n-ax)}{a^3 c^2 \left(9-10n^2+n^4\right)\sqrt{c-a^2 c x^2}}
\end{aligned}$$

Mathematica [A] time = 0.76, size = 109, normalized size = 1.07

$$\frac{e^{n \coth^{-1}(ax)}\left(3a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)+an^2x-2\left(n^2-1\right)n\cosh\left(2\coth^{-1}(ax)\right)-9ax-2n^3\right.}{4a^3c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(10*n - 2*n^3 - 9*a*x + a*n^2*x - 2*n*(-1 + n^2)*Cosh[2
*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]
]))/(4*a^3*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.57, size = 179, normalized size = 1.75

$$-\frac{\sqrt{-a^2cx^2+c}\left(2\,an^2x+\left(a^3n^2-3\,a^3\right)x^3+\left(a^2n^3-3\,a^2n\right)x^2+2\,n\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3+\left(a^7c^3n^4-10\,a^7c^3n^2+9\,a^7c^3\right)x^4-2\left(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*(2*a*n^2*x + (a^3*n^2 - 3*a^3)*x^3 + (a^2*n^3 - 3*a^2
*n)*x^2 + 2*n)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^3*c^3*n^4 - 10*a^3*c^3*n^2
+ 9*a^3*c^3 + (a^7*c^3*n^4 - 10*a^7*c^3*n^2 + 9*a^7*c^3)*x^4 - 2*(a^5*c^3*n
^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.04, size = 96, normalized size = 0.94

$$\frac{(ax-1)\,(ax+1)\left(a^3n^2x^3-a^2n^3x^2-3x^3a^3+3a^2n\,x^2+2a\,n^2x-2n\right)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\left(n^4-10n^2+9\right)a^3\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(a^3*n^2*x^3-a^2*n^3*x^2-3*a^3*x^3+3*a^2*n*x^2+2*a*n^2*x-2*
n)*exp(n*arccoth(a*x))/(n^4-10*n^2+9)/a^3/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 1.56, size = 175, normalized size = 1.72

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{2\,n}{a^5\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{x^3\,\left(n^2-3\right)}{a^2\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{2\,n^2\,x}{a^4\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{n\,x^2\,\left(n^2-3\right)}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((2*n)/(a^5*c^2*(n^4 - 10*n^2 + 9)) - (x^3*(n^2 -
3))/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n^2*x)/(a^4*c^2*(n^4 - 10*n^2 + 9)) +
 (n*x^2*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a
^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 e^{n \operatorname{acoth}(ax)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.759 $\int \frac{e^{n\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2n(n-ax)e^{n\coth^{-1}(ax)}}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}+\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] exp(n*arccoth(a*x))*(-a*n*x+3)/a^2/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+2*exp(n*
arccoth(a*x))*n*(-a*x+n)/a^2/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6187, 6184}

$$\frac{2n(n-ax)e^{n\coth^{-1}(ax)}}{a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}+\frac{(3-anx)e^{n\coth^{-1}(ax)}}{a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(3 - a*n*x))/(a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) +
(2*E^(n*ArcCoth[a*x])*n*(n - a*x))/(a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2
*c*x^2])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6187

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((2*(p + 1) + a*n*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a^2
*c*(n^2 - 4*(p + 1)^2)), x] - Dist[(n*(2*p + 3))/(a*c*(n^2 - 4*(p + 1)^2)),
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{(2n)\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{ac\left(9-n^2\right)}\\ &= \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{2e^{n\coth^{-1}(ax)}n(n-ax)}{a^2c^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.66, size = 108, normalized size = 1.11

$$\frac{e^{n\coth^{-1}(ax)}\left(-a\left(n^2-1\right)nx\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)+an^3x+6\left(n^2-1\right)\cosh\left(2\coth^{-1}(ax)\right)-9anx+2n^2\right.}{4a^2c^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(6 + 2*n^2 - 9*a*n*x + a*n^3*x + 6*(-1 + n^2)*Cosh[2*Ar
cCoth[a*x]] - a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))
/(4*a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.53, size = 170, normalized size = 1.75

$$-\frac{\left(2\,a^{3}nx^{3}+2\,a^{2}n^{2}x^{2}+n^{2}+\left(an^{3}-3\,an\right)x-3\right)\sqrt{-a^{2}cx^{2}+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{a^{2}c^{3}n^{4}-10\,a^{2}c^{3}n^{2}+9\,a^{2}c^{3}+\left(a^{6}c^{3}n^{4}-10\,a^{6}c^{3}n^{2}+9\,a^{6}c^{3}\right)x^{4}-2\left(a^{4}c^{3}n^{4}-10\,a^{4}c^{3}n^{2}+9\,a^{4}c^{3}\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(2*a^3*n*x^3 + 2*a^2*n^2*x^2 + n^2 + (a*n^3 - 3*a*n)*x - 3)*sqrt(-a^2*c*x^
2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c^3*n^4 - 10*a^2*c^3*n^2 + 9*a^2*
c^3 + (a^6*c^3*n^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^4 - 2*(a^4*c^3*n^4 - 10*
a^4*c^3*n^2 + 9*a^4*c^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.04, size = 86, normalized size = 0.89

$$-\frac{\left(ax-1\right)\left(ax+1\right)\left(2x^{3}a^{3}n-2a^{2}n^{2}x^{2}+a\,n^{3}x-3xan-n^{2}+3\right)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{a^{2}\left(n^{4}-10n^{2}+9\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -(a*x-1)*(a*x+1)*(2*a^3*n*x^3-2*a^2*n^2*x^2+a*n^3*x-3*a*n*x-n^2+3)*exp(n*ar
ccoth(a*x))/a^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 1.55, size = 176, normalized size = 1.81

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{n^2-3}{a^4\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{2\,n^2\,x^2}{a^2\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{2\,n\,x^3}{a\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{n\,x\,\left(n^2-3\right)}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -((((a*x + 1)/(a*x))^(n/2)*((n^2 - 3)/(a^4*c^2*(n^4 - 10*n^2 + 9)) + (2*n^2*
x^2)/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n*x^3)/(a*c^2*(n^4 - 10*n^2 + 9)) -
(n*x*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2
- x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{n\operatorname{acoth}(ax)}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.760 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*a
rccoth(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6185, 6184}

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6184

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n - a*x)*E^(n*ArcCoth[a*x]))/(a*c*(n^2 - 1)*Sqrt[c + d*x^2]), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6185

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[((n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]))/(a*c*(n^2
 - 4*(p + 1)^2)), x] - Dist[(2*(p + 1)*(2*p + 3))/(c*(n^2 - 4*(p + 1)^2)),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}+\frac{6\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx}{c\left(9-n^2\right)}\\ &= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\left(9-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2\left(1-n^2\right)\left(9-n^2\right)\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.24, size = 110, normalized size = 1.08

$$\frac{e^{n\coth^{-1}(ax)}\left(3a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(3\coth^{-1}(ax)\right)-3an^2x-2\left(n^2-1\right)n\cosh\left(2\coth^{-1}(ax)\right)+27ax+2\right.}{4ac^2\left(n^4-10n^2+9\right)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*Co
sh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[
a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

fricas [A] time = 0.50, size = 163, normalized size = 1.60

$$-\frac{\left(6\,a^{3}x^{3}+6\,a^{2}nx^{2}+n^{3}+3\left(an^{2}-3\,a\right)x-7\,n\right)\sqrt{-a^{2}cx^{2}+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{ac^{3}n^{4}-10\,ac^{3}n^{2}+\left(a^{5}c^{3}n^{4}-10\,a^{5}c^{3}n^{2}+9\,a^{5}c^{3}\right)x^{4}+9\,ac^{3}-2\left(a^{3}c^{3}n^{4}-10\,a^{3}c^{3}n^{2}+9\,a^{3}c^{3}\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -(6*a^3*x^3 + 6*a^2*n*x^2 + n^3 + 3*(a*n^2 - 3*a)*x - 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.04, size = 84, normalized size = 0.82

$$\frac{\left(ax-1\right)\left(ax+1\right)\left(6x^{3}a^{3}-6a^{2}n\,x^{2}+3a\,n^{2}x-n^{3}-9ax+7n\right){\mathrm e}^{n\,\mathrm{arccoth}\left(ax\right)}}{a\left(n^{4}-10n^{2}+9\right)\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] (a*x-1)*(a*x+1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcco
th(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}\,n}}{\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [B] time = 0.00, size = 173, normalized size = 1.70

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{6\,x^3}{c^2\,\left(n^4-10\,n^2+9\right)}+\frac{7\,n-n^3}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{3\,x\,\left(n^2-3\right)}{a^2\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{6\,n\,x^2}{a\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -((((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/(
a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9))
- (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x^2*
(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{n\operatorname{acoth}(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.761 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=944

$$-\frac{2^{\frac{n+5}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,{}_2F_1\left(\frac{1}{2}(-n-3),\frac{1}{2}(-n-3);\frac{1}{2}(-n-1);\frac{a-\frac{1}{x}}{2a}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5}{(n+3)\left(c-a^2cx^2\right.}$$

[Out] -a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-3*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)+4*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)+8*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)-8*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+4*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(3/2+1/2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-2^(5/2+1/2*n)*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*x^5*hypergeom([-3/2-1/2*n, -3/2-1/2*n],[-1/2-1/2*n],1/2*(a-1/x)/a)/(3+n)/(-a^2*c*x^2+c)^(5/2)

Rubi [A] time = 0.63, antiderivative size = 944, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6192, 6195, 128, 45, 37, 69}

$$-\frac{2^{\frac{n+5}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,{}_2F_1\left(\frac{1}{2}(-n-3),\frac{1}{2}(-n-3);\frac{1}{2}(-n-1);\frac{a-\frac{1}{x}}{2a}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5}{(n+3)\left(c-a^2cx^2\right.}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]

[Out] -((a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n + n^2)*(c - a^2*c*x^2)^(5/2)) + (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2)) + (4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) + (8*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + 4*n + n^2)*(c - a^2*c*x^2)^(5/2)) - (8*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*x^5)/((3 + 4*n + n^2)*(c - a^2*c*x^2)^(5/2)) + (4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((3 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) - (2^((5 + n)/2)*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-

```
3 - n)/2)*x^5*Hypergeometric2F1[(-3 - n)/2, (-3 - n)/2, (-1 - n)/2, (a - x^
(-1))/(2*a)])/((3 + n)*(c - a^2*c*x^2)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 128

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[
m, 0] && ILtQ[n, 0]))
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6195

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int x^4\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(a^4\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}-4a^4\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-}\right.\right.}{}$$

$$= -\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^{5}\right.}{}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} + \frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1\right.}{(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1\right.}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right.}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1\right.}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right.}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1\right.}{\left(3+4n+n^2\right)\left(c-a^2cx^2\right.}$$

Mathematica [A] time = 1.69, size = 220, normalized size = 0.23

$$\frac{e^{n\coth^{-1}(ax)}\left(6a\left(n^2-1\right)x\sqrt{1-\frac{1}{a^2x^2}}\cosh\left(2\coth^{-1}(ax)\right)-n\left(n^2-1\right)\left(a^2x^2-1\right)\cosh\left(3\coth^{-1}(ax)\right)+ax\sqrt{1-\frac{1}{a^2x}}\right.}{4ac^2(n-1)(n+1)\left(n^2-9\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(42 - 2*n^2 - 45*a*n*x + 5*a
*n^3*x) + 6*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[2*ArcCoth[a*x]] - n*(
-1 + n^2)*(-1 + a^2*x^2)*Cosh[3*ArcCoth[a*x]]) - 8*E^((1 + n)*ArcCoth[a*x])
*(9 - 9*n - n^2 + n^3)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 +
n)/2, -E^(2*ArcCoth[a*x])]])/(4*a*c^2*(-1 + n)*(1 + n)*(-9 + n^2)*Sqrt[1 - 1
/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^6c^3x^7-3\,a^4c^3x^5+3\,a^2c^3x^3-c^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^6*c^3*x^7 -
 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 - c^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x\,{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(5/2)), x)
```

3.762 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=127

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}{}_2F_1\left(-2p-1,\frac{1}{2}(n-2p);-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

[Out] ((a-1/x)/(a+1/x))^(1/2*n-p)*(1-1/a/x)^(-1/2*n+p)*(1+1/a/x)^(1+1/2*n+p)*x*(-a^2*c*x^2+c)^p*hypergeom([-1-2*p, 1/2*n-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2)^p)

Rubi [A] time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-a^2cx^2\right)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}{}_2F_1\left(-2p-1,\frac{1}{2}(n-2p);-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]

[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 - 1/(a*x))^(-n/2 + p)*(1 + 1/(a*x))^(1 + n/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, (n - 2*p)/2, -2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{n}{2}+}\right.\right.$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-\right.}{1+2p}$$

Mathematica [A] time = 0.20, size = 83, normalized size = 0.65

$$-\frac{\left(a^2x^2-1\right)\left(e^{2\coth^{-1}(ax)}-1\right)\left(c-a^2cx^2\right)^p e^{(n-2)\coth^{-1}(ax)}\,{}_2F_1\left(1,-\frac{n}{2}-p;-\frac{n}{2}+p+2;e^{-2\coth^{-1}(ax)}\right)}{a(n-2(p+1))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((E^((-2 + n)*ArcCoth[a*x])*(-1 + E^(2*ArcCoth[a*x]))*(-1 + a^2*x^2)*(c -
a^2*c*x^2)^p*Hypergeometric2F1[1, -1/2*n - p, 2 - n/2 + p, E^(-2*ArcCoth[a*
x])])/(a*(n - 2*(1 + p))))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p,x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(ax-1\right)\left(ax+1\right)\right)^p e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*exp(n*acoth(a*x)), x)
```

3.763 $\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=51

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

```
[Out] (1+1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] time = 0.12, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6192, 6196, 37}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((1 + 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Integ
ersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx\\
&= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1+\frac{x}{a}\right)^{2p}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+2p}x\left(c-a^2cx^2\right)^p}{1+2p}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 0.71

$$\frac{(ax+1)\left(c-a^2cx^2\right)^p e^{2p\coth^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] (E^(2*p*ArcCoth[a*x])*(1 + a*x)*(c - a^2*c*x^2)^p)/(a + 2*a*p)
```

fricas [A] time = 0.75, size = 42, normalized size = 0.82

$$\frac{(ax-1)\left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^p}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] (a*x - 1)*(-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^p/(2*a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^p, x)
```

maple [A] time = 0.03, size = 38, normalized size = 0.75

$$\frac{(ax+1)\,\mathrm{e}^{2p\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^p}{a\left(1+2p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
[Out] (a*x+1)/a/(1+2*p)*exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p
```

maxima [A] time = 0.33, size = 36, normalized size = 0.71

$$\frac{\left(a\,(-c)^p\,x-(-c)^p\right)(ax-1)^{2\,p}}{a\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] (a*(-c)^p*x - (-c)^p)*(a*x - 1)^(2*p)/(a*(2*p + 1))
```

mupad [B] time = 1.34, size = 59, normalized size = 1.16

$$\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)\,\left(\frac{a\,x+1}{a\,x}\right)^p}{a\,\left(2\,p+1\right)\,\left(\frac{a\,x-1}{a\,x}\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

```
[Out] ((c - a^2*c*x^2)^p*(a*x + 1)*((a*x + 1)/(a*x))^p)/(a*(2*p + 1)*((a*x - 1)/(
a*x))^p)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{ix}{\sqrt{c}} & \text{for } a = 0 \wedge p = -\frac{1}{2} \\ c^p x e^{i\pi p} & \text{for } a = 0 \\ \int \frac{e^{-\operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\, dx & \text{for } p = -\frac{1}{2} \\ \frac{ax\left(-a^2cx^2+c\right)^p e^{2p \operatorname{acoth}(ax)}}{2ap+a} + \frac{\left(-a^2cx^2+c\right)^p e^{2p \operatorname{acoth}(ax)}}{2ap+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Piecewise((-I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(I*pi*p), Eq(a
, 0)), (Integral(exp(-acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p, -
1/2)), (a*x*(-a**2*c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a) + (-a**2*
c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a), True))
```

3.764 $\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=52

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

```
[Out] (1-1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] time = 0.12, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6192, 6196, 37}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-a^2cx^2\right)^p}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((1 - 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx\\ &= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{2p}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{1+2p}x\left(c-a^2cx^2\right)^p}{1+2p}\end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 0.69

$$\frac{(ax-1)\left(c-a^2cx^2\right)^p e^{-2p\coth^{-1}(ax)}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
[Out] ((-1 + a*x)*(c - a^2*c*x^2)^p)/(E^(2*p*ArcCoth[a*x])*(a + 2*a*p))
```

fricas [A] time = 0.84, size = 44, normalized size = 0.85

$$\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{\left(2\,ap+a\right)\left(\frac{ax-1}{ax+1}\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
[Out] (a*x + 1)*(-a^2*c*x^2 + c)^p/((2*a*p + a)*((a*x - 1)/(a*x + 1))^p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^p, x)
```

maple [A] time = 0.03, size = 40, normalized size = 0.77

$$\frac{(ax-1)\left(-a^2c\,x^2+c\right)^p \mathrm{e}^{-2p\,\mathrm{arccoth}(ax)}}{a\left(1+2p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x)
[Out] (a*x-1)/a/(1+2*p)*(-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x))
```

maxima [A] time = 0.34, size = 34, normalized size = 0.65

$$\frac{\left(a\,(-c)^p\,x+(-c)^p\right)(ax+1)^{2\,p}}{a\left(2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
[Out] (a*(-c)^p*x + (-c)^p)*(a*x + 1)^(2*p)/(a*(2*p + 1))
```

mupad [B] time = 1.30, size = 59, normalized size = 1.13

$$\frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x-1\right)\,\left(\frac{a\,x-1}{a\,x}\right)^p}{a\,\left(2\,p+1\right)\,\left(\frac{a\,x+1}{a\,x}\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

```
[Out] ((c - a^2*c*x^2)^p*(a*x - 1)*((a*x - 1)/(a*x))^p)/(a*(2*p + 1)*((a*x + 1)/(
a*x))^p)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ix}{\sqrt{c}} & \text{for}\: a = 0 \wedge p = -\frac{1}{2} \\ c^{p} x e^{- i \pi p} & \text{for}\: a = 0 \\ \int \frac{e^{\operatorname{acoth}{(ax)}}}{\sqrt{- c (ax - 1)(ax + 1)}}\, dx & \text{for}\: p = -\frac{1}{2} \\ \frac{ax \left(- a^{2} c x^{2} + c\right)^{p}}{2ape^{2p \operatorname{acoth}{(ax)}} + ae^{2p \operatorname{acoth}{(ax)}}} - \frac{\left(- a^{2} c x^{2} + c\right)^{p}}{2ape^{2p \operatorname{acoth}{(ax)}} + ae^{2p \operatorname{acoth}{(ax)}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/exp(2*p*acoth(a*x)),x)
```

```
[Out] Piecewise((I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(-I*pi*p), Eq(a
, 0)), (Integral(exp(acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p, -1
/2)), (a*x*(-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*p*aco
th(a*x))) - (-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*p*ac
oth(a*x))), True))
```

3.765 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\,{}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

```
[Out] 2^(2+p)*c*(a*x+1)^(1-p)*(-a^2*c*x^2+c)^(-1+p)*hypergeom([-2-p, -1+p],[p],-1
/2*a*x+1/2)/a/(1-p)
```

Rubi [A] time = 0.10, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6141, 678, 69}

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\,{}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[-
2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=c^2\int(1+ax)^4\left(c-a^2cx^2\right)^{-2+p}\,dx$$
$$=\left(c^2(1+ax)^{1-p}(c-acx)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\right)\int(1+ax)^{2+p}(c-acx)^{-2+p}\,dx$$
$$=\frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\,{}_2F_1\left(-2-p,-1+p;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.14

$$-\frac{2^{p+2}(1-ax)^{p-1}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] -((2^(2 + p)*(1 - a*x)^(-1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-2 - p,
 -1 + p, p, (1 - a*x)/2])/(a*(-1 + p)*(1 - a^2*x^2)^p))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)\left(-a^2cx^2+c\right)^p}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^2\left(-a^2cx^2+c\right)^p}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^2\left(-a^2c\,x^2+c\right)^p}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
[Out] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)^2\left(-a^2cx^2+c\right)^p}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,{\left(a\,x+1\right)}^2}{{\left(a\,x-1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{p} \left(a x + 1\right)^{2}}{\left(a x - 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

3.766 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{5}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,\frac{3}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

```
[Out] ((a-1/x)/(a+1/x))^(3/2-p)*(1-1/a/x)^(-3/2+p)*(1+1/a/x)^(5/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, 3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2)
^p)
```

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{5}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,\frac{3}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/(a*x))^(-3/2 + p)*(1 + 1/(a*x
))^(5/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 3/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$
$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{3}{2}+p}\right.\right.$$
$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{5}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-2p,\frac{3}{2}\right.}{1+2p}$$

Mathematica [A] time = 0.28, size = 122, normalized size = 1.03

$$-\frac{4^{p+1}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\right)^{-2p}e^{5\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}\right)^{2p}{}_2F_1\left(p+\frac{5}{2},2p+2;p+\frac{7}{2};e\right.}{2ap+5a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((4^(1 + p)*E^(5*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a
*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[5/
2 + p, 2 + 2*p, 7/2 + p, E^(2*ArcCoth[a*x])])/((5*a + 2*a*p)*(a*Sqrt[1 - 1/
(a^2*x^2)]*x)^(2*p)))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2+2\,ax+1\right)\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}}{a^2x^2-2\,ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))
/(a^2*x^2 - 2*a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

3.767 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=54

$$\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p\ {}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

```
[Out] 2^(1+p)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[1+p],-1/2*a*x+1/2)/a/p/((a*x+
1)^p)
```

Rubi [A] time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6141, 678, 69}

$$\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p\ {}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x)/
2])/(a*p*(1 + a*x)^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6141

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$=-\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\right)$$
$$=-\left(\left(c(1+ax)^{-p}(c-acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1+ax)^{1+p}(c-acx)^{-1+p}\,dx\right)$$
$$=\frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1-ax)\right)}{ap}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.24

$$\frac{2^{p+1}(1-ax)^p\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] (2^(1 + p)*(1 - a*x)^p*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p
, (1 - a*x)/2])/(a*p*(1 - a^2*x^2)^p)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2c\,x^2+c\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^p,x)
[Out] int((a*x+1)/(a*x-1)*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(-a^2cx^2+c\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 19.97, size = 651, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1/
(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/2
 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a - 0**p*log(1/(a**2*x*
*2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))/
a**2 + c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x*
*2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,)
, 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) + Piecewise((0**
p*log(a**2*x**2 - 1)/(2*a) - 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2
 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/
2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
- 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p)
, (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p
)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True)
)
```

3.768 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{3}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,\frac{1}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2-p)*(1-1/a/x)^(-1/2+p)*(1+1/a/x)^(3/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, 1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2)
^p)
```

Rubi [A] time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{3}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,\frac{1}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*x
))^(3/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 1/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^px^{2p}\,dx$$
$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+p}\right.\right.$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{3}{2}+p}x\left(c-a^2cx^2\right)^p{}_2F_1\left(-1-2p,\frac{1}{2}\right.}{1+2p}$$

Mathematica [A] time = 0.22, size = 122, normalized size = 1.03

$$-\frac{4^{p+1}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\right)^{-2p}e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}\right)^{2p}{}_2F_1\left(p+\frac{3}{2},2p+2;p+\frac{5}{2};e\right.}{2ap+3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((4^(1 + p)*E^(3*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a
*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/
2 + p, 2 + 2*p, 5/2 + p, E^(2*ArcCoth[a*x])])/((3*a + 2*a*p)*(a*Sqrt[1 - 1/
(a^2*x^2)]*x)^(2*p)))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax+1)\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}}{ax-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2cx^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2c\,x^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

3.769 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{1}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,-p-\frac{1}{2};-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

```
[Out] ((a-1/x)/(a+1/x))^(-1/2-p)*(1-1/a/x)^(1/2+p)*(1+1/a/x)^(1/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, -1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2
)^p)
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{1}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,-p-\frac{1}{2};-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*(1 - 1/(a*x))^(1/2 + p)*(1 + 1/(a*x
))^(1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -1/2 - p, -2*p
, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$
$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+p}d\right.\right.$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-2p,-\frac{1}{2}\right.}{1+2p}$$

Mathematica [A] time = 0.24, size = 118, normalized size = 1.00

$$-\frac{4^{p+1}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\right)^{-2p}e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}\right)^{2p}{}_2F_1\left(p+\frac{1}{2},2p+2;p+\frac{3}{2};e^{2\coth}\right.}{2ap+a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

```
[Out] -((4^(1 + p)*E^ArcCoth[a*x]*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/
(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2 +
p, 2 + 2*p, 3/2 + p, E^(2*ArcCoth[a*x])])/((a + 2*a*p)*(a*Sqrt[1 - 1/(a^2*x
^2)]*x)^(2*p)))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int\left(-a^2c\,x^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-a^2\,c\,x^2\right)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p, x)
```

3.770 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=55

$$-\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(ax+1)\right)}{ap}$$

```
[Out] -2^(1+p)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[1+p],1/2*a*x+1/2)/a/p/((-a*x
+1)^p)
```

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6142, 678, 69}

$$-\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(ax+1)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*x
)/2])/(a*p*(1 - a*x)^p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 678

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
(d^(m - 1)*(a + c*x^2)^(p + 1))/((1 + (e*x)/d)^(p + 1)*(a/d + (c*x)/e)^(p +
 1)), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6142

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$$
$$= -\left(c\int (1-ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\right)$$
$$= -\left(\left(c(1-ax)^{-p}(c+acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int (1-ax)^{1+p}(c+acx)^{-1+p}\,dx\right)$$
$$= -\frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1+ax)\right)}{ap}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.33

$$\frac{2^{p-1}(1-ax)^{p+2}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1-p,p+2;p+3;\frac{1}{2}(1-ax)\right)}{a(p+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
[Out] (2^(-1 + p)*(1 - a*x)^(2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1 - p, 2
+ p, 3 + p, (1 - a*x)/2])/(a*(2 + p)*(1 - a^2*x^2)^p)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^p(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/(a*x+1)*(a*x-1),x)
[Out] int((-a^2*c*x^2+c)^p/(a*x+1)*(a*x-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(-a^2cx^2+c\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c-a^2\,c\,x^2\right)^p\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1), x)
```

sympy [C] time = 17.80, size = 651, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*(a*x-1)/(a*x+1),x)
```

```
[Out] a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1/
(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/2
 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(1/(a**2*x*
*2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))/
a**2 - c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x*
*2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,)
, 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) - Piecewise((0**
p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) + a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2
 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/
2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
+ 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p)
, (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) + a**(2*p
)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True)
)
```

3.771 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p-\frac{1}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,-p-\frac{3}{2};-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

```
[Out] ((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2+p)*(1+1/a/x)^(-1/2+p)*x*(-a^2*c*x
^2+c)^p*hypergeom([-1-2*p, -3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^
2)^p)
```

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6192, 6196, 132}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p-\frac{1}{2}}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2p-1,-p-\frac{3}{2};-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2 + p)*(1 + 1/(a*x
))^(-1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -3/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6192

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6196

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> -Dist[c^p*x^m*(1/x)^m, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^
(p + n/2))/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] && Eq
Q[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !Integ
ersQ[2*p, p + n/2] && !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$
$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+p}\right.\right.$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{-\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p{}_2F_1\left(-1-2p,-\right.}{1+2p}$$

Mathematica [A] time = 0.28, size = 119, normalized size = 1.01

$$\frac{4^{p+1}\left(ax\sqrt{1-\frac{1}{a^2x^2}}\right)^{-2p}e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{e^{2\coth^{-1}(ax)}-1}\right)^{2p}{}_2F_1\left(p-\frac{1}{2},2p+2;p+\frac{1}{2};e^{2\coth}\right.}{a-2ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4^(1 + p)*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/(-1 + E^(2*ArcCot
h[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2 + p, 2 + 2*p, 1/2
+ p, E^(2*ArcCoth[a*x])])/(E^ArcCoth[a*x]*(a - 2*a*p)*(a*Sqrt[1 - 1/(a^2*x^
2)]*x)^(2*p))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ax-1)\left(-a^2cx^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}}{ax+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a^2cx^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int\left(-a^2c\,x^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^2cx^2+c\right)^p \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-a^2\,c\,x^2\right)}^p\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Timed out
```

3.772 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=342

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{7a}+\frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}{42a}+c^4x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-13\cdots$$

```
[Out] 47/42*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)/a+8/7*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(9/2)/a+c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(9/2)*x+35/16*c^4*arccsc(a*x)/a+c^
4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-67/48*c^4*(1+1/a/x)^(3/2)*(1-1
/a/x)^(1/2)/a-91/120*c^4*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-131/280*c^4*(1+1
/a/x)^(7/2)*(1-1/a/x)^(1/2)/a+61/70*c^4*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a-5
1/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.25, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{7a}+\frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}{42a}+c^4x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-13\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-51*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (67*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(48*a) - (91*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(120*a) - (131*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) +
 (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (47*c^4*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2))/(42*a) + (8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(
a*x))^(9/2))/(7*a) + c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(9/2)*x + (35*c^
4*ArcCsc[a*x])/(16*a) + (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/
a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - c^4\,\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int \frac{\left(\frac{7}{a^2}-\cdots\right.}{\cdots}\right.$$

$$= \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)\cdots$$

$$= \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a}\cdots$$

$$= -\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)\cdots}{42a}$$

$$= -\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a}\cdots$$

$$= -\frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a}\cdots$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \cdots$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \cdots$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \cdots$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \cdots$$

Mathematica [A] time = 0.32, size = 120, normalized size = 0.35

$$\frac{c^4\left(3675a^6\sin^{-1}\left(\frac{1}{ax}\right) + 1680a^6\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right) + \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1680a^7x^7-2816a^6x^6+3045a^5x^5+1952a^4x^4-1330a^3x^3-1056a\cdots\right.}{x^6}\right.}{1680a^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*((Sqrt[1 - 1/(a^2*x^2)]*(240 + 280*a*x - 1056*a^2*x^2 - 1330*a^3*x^3 +
 1952*a^4*x^4 + 3045*a^5*x^5 - 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a^6
*ArcSin[1/(a*x)] + 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a^7)
```

fricas [A] time = 0.63, size = 201, normalized size = 0.59

$$-\frac{7350\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(1680\,a^8c^4x^8\right.}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 - 1136*a^7*c^4*x^7 + 229*a^6*c^4*x^6
+ 4997*a^5*c^4*x^5 + 622*a^4*c^4*x^4 - 2386*a^3*c^4*x^3 - 776*a^2*c^4*x^2 +
 520*a*c^4*x + 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

giac [A] time = 0.18, size = 335, normalized size = 0.98

$$-\frac{1}{840}\,ac^4\left(\frac{3675\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{1680\,\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{1260\,(ax-}{ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -1/840*a*c^4*(3675*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/a^2 + 840*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^
2 + 1680*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (1260*
(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 18921*(a*x - 1)^2*sqrt((a*x
 - 1)/(a*x + 1))/(a*x + 1)^2 + 73152*(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/
(a*x + 1)^3 + 60151*(a*x - 1)^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 + 233
80*(a*x - 1)^5*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^5 + 3675*(a*x - 1)^6*sqr
t((a*x - 1)/(a*x + 1))/(a*x + 1)^6 - 315*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((
a*x - 1)/(a*x + 1) + 1)^7))
```

maple [A] time = 0.07, size = 320, normalized size = 0.94

$$\frac{(ax-1)\,c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^8a^8+1680\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^6a^6+3675a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}+3675a^7x^7\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x)
```

```
[Out] 1/1680*(a*x-1)*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^8*a^8+1680*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6+3675*a^7*x^7*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)+
3675*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+1680*ln((a^2*x+(a^2*x^
2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8-1995*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x^5*a^5-1136*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^4+1050*(a^2*x^2-1)^(3
/2)*(a^2)^(1/2)*x^3*a^3+816*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-280*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)*x*a-240*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a
*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 380, normalized size = 1.11

$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{5355\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+31465\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a
^2 - (5355*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 31465*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 72051*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 71801*c^4*((a*x - 1)/(
a*x + 1))^(9/2) + 4569*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 17619*c^4*((a*x -
1)/(a*x + 1))^(5/2) + 10185*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 1995*c^4*sqrt
((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
 + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)
^8*a^2/(a*x + 1)^8 + a^2))*a
```

mupad [B] time = 1.42, size = 332, normalized size = 0.97

$$\frac{\frac{19\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{97\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8}+\frac{839\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}+\frac{1523\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{280}+\frac{71801\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{840}+\frac{3431\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+\frac{899\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{24}+\cdots}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\frac{a\,(a\,x-1)^8}{(a\,x+1)^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((19*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (97*c^4*((a*x - 1)/(a*x + 1))^(3/
2))/8 + (839*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (1523*c^4*((a*x - 1)/(a*
x + 1))^(7/2))/280 + (71801*c^4*((a*x - 1)/(a*x + 1))^(9/2))/840 + (3431*c^
4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (899*c^4*((a*x - 1)/(a*x + 1))^(13/2))
/24 + (51*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x +
1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (14*
a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x - 1
)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*x - 1)/(
a*x + 1))^(1/2)))/(8*a) + (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{a^8}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^2}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^4}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^6}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] c**4*(Integral(a**8/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a**2/(x**6*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x) + Integral(6*a**4/(x**4*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x) + Integral(-4*a**6/(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)), x))/a**8
```

3.773 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=268

$$\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{5a}+c^3x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{60a}-\cdots$$

```
[Out] 6/5*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)/a+c^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(7/
2)*x+15/8*c^3*arccsc(a*x)/a+c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-
31/24*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-43/60*c^3*(1+1/a/x)^(5/2)*(1-1/
a/x)^(1/2)/a+23/20*c^3*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-23/8*c^3*(1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.19, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{5a}+c^3x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{60a}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-23*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (31*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(24*a) - (43*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2))/(60*a) + (23*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*
c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(5*a) + c^3*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(7/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) + (c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x - c^3\,\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x}dx, x, \frac{1}{x}\right.$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int \right.$$

$$= \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right.$$

$$= -\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right.}{5a}$$

$$= -\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7}}{20a}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}$$

Mathematica [A] time = 0.25, size = 104, normalized size = 0.39

$$\frac{c^3\left(225a^4\sin^{-1}\left(\frac{1}{ax}\right)+120a^4\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(120a^5x^5-184a^4x^4+135a^3x^3+88a^2x^2-30ax-24\right)}{x^4}\right)}{120a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*((Sqrt[1 - 1/(a^2*x^2)]*(-24 - 30*a*x + 88*a^2*x^2 + 135*a^3*x^3 - 184
*a^4*x^4 + 120*a^5*x^5))/x^4 + 225*a^4*ArcSin[1/(a*x)] + 120*a^4*Log[(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5)
```

fricas [A] time = 0.52, size = 179, normalized size = 0.67

$$-\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(120\,a^6c^3x^6-64\,a^5c\right.}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5
*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (120*a^6*c^3*x^6 - 64*a^5*c^3*x^5 - 49*a^4*c^3*x^4 + 223*a^
3*c^3*x^3 + 58*a^2*c^3*x^2 - 54*a*c^3*x - 24*c^3)*sqrt((a*x - 1)/(a*x + 1))
)/(a^6*x^5)
```

giac [A] time = 0.18, size = 273, normalized size = 1.02

$$-\frac{1}{60}ac^3\left(\frac{225\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{120\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{310(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*a*c^3*(225*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 60*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 +
120*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (310*(a*x -
 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 1424*(a*x - 1)^2*sqrt((a*x - 1)/(
a*x + 1))/(a*x + 1)^2 + 970*(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)
^3 + 225*(a*x - 1)^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 + 15*sqrt((a*x -
 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^5))
```

maple [A] time = 0.06, size = 272, normalized size = 1.01

$$\frac{(ax-1)\,c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^6a^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4+225\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5+225\arctan\left(\frac{1}{\sqrt{a^2x^2\cdots}}\right.\right.}{12\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/120*(a*x-1)*c^3*(-120*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^6*a^6+120*(a^2*x^2-
1)^(3/2)*(a^2)^(1/2)*x^4*a^4+225*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+225*
arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)*x^5*a^5+120*ln((a^2*x+(a^2*x^2-1)^(
1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6-105*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^
3*a^3-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+30*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*x*a+24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)
*(a*x+1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

maxima [A] time = 0.41, size = 302, normalized size = 1.13

$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1345\,c^3\left(\frac{ax-\cdots}{ax+\cdots}\right.}{\frac{4(ax-1)a^2}{ax+1}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
 (345*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(9/
2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 86*c^3*((a*x - 1)/(a*x + 1))^(5
/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 105*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1
)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x
+ 1)^6 + a^2))*a
```

mupad [B] time = 1.32, size = 258, normalized size = 0.96

$$\frac{\frac{7\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{61\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{43\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{30}+\frac{827\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{30}+\frac{269\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{12}+\frac{23\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}-\frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((7*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (61*c^3*((a*x - 1)/(a*x + 1))^(3/2
))/12 + (43*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (827*c^3*((a*x - 1)/(a*x
+ 1))^(7/2))/30 + (269*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (23*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/(4*a) + (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{a^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a^2}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] c**3*(Integral(a**6/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*
*6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a**2/(x**4*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**4/(x**2*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))), x))/a**6
```

3.774 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=194

$$c^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}+\frac{3c^2\csc^{-1}(a}{2a}$$

```
[Out] c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x+3/2*c^2*arccsc(a*x)/a+c^2*arctanh((1-
1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/6*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+4
/3*c^2*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a/x)^
(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}+\frac{3c^2\csc^{-1}(a}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (7*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(6*a) + (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(3*a) + c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) + (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\left(c^2\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^2\,\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{4x}{a^2}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)$$

$$= \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int \frac{\left(\frac{3}{a^2}-\frac{7x}{a^3}\right.}{x\sqrt{}}\right.$$

$$= -\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1\right.$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1\right.$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1\right.$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} + \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\left(1\right.$$

Mathematica [A] time = 0.16, size = 94, normalized size = 0.48

$$\frac{c^2\left(6a^2x^2\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+9a^2x^2\sin^{-1}\left(\frac{1}{ax}\right)+\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3-8a^2x^2+3ax+2\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^2
*ArcSin[1/(a*x)] + 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*x^
2)
```

fricas [A] time = 0.52, size = 157, normalized size = 0.81

$$-\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^2x^4-2\,a^3c^2x^3-5\,a^2\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 6*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
```

```
) - 1) - (6*a^4*c^2*x^4 - 2*a^3*c^2*x^3 - 5*a^2*c^2*x^2 + 5*a*c^2*x + 2*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

giac [A] time = 0.17, size = 211, normalized size = 1.09

$$-\frac{1}{3}\,ac^2\left(\frac{9\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{6\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{20\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{9\,(a}{}}{a^2\left(\frac{ax}{ax}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*a*c^2*(9*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*log(sqrt((a*x - 1)/
(a*x + 1)) + 1)/a^2 + 3*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 6*sqr
t((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (20*(a*x - 1)*sqrt
((a*x - 1)/(a*x + 1))/(a*x + 1) + 9*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(
a*x + 1)^2 + 3*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^3)
)
```

maple [A] time = 0.05, size = 224, normalized size = 1.15

$$\frac{(ax-1)\,c^2\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2+9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3+9a^3x^3\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{a}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)\,(ax+1)}\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x)
```

```
[Out] 1/6*(a*x-1)*c^2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+6*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*x^2*a^2+9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3+9*a^3*x^3*(a
^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2))*x^3*a^4-3*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-2*(a^2*x^2-1
)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^4/x^
3/(a^2)^(1/2)
```

maxima [A] time = 0.41, size = 223, normalized size = 1.15

$$-\frac{1}{3}\,a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+29\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (1
5*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2) + c^
2*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1
)^4 + a^2))
```

mupad [B] time = 1.33, size = 183, normalized size = 0.94

$$\frac{c^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+\frac{29\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{3}+5\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{a+\frac{2\,a\,(a\,x-1)}{a\,x+1}-\frac{2\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}-\frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c^2*((a*x - 1)/(a*x + 1))^(1/2) + (c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 + (2
9*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^2*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{2a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a**2/(x**2*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x))/a**4
```

3.775 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=107

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a}+\frac{c\csc^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

```
[Out] c*arccsc(a*x)/a+c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1+1/a/x)^(3
/2)*x*(1-1/a/x)^(1/2)-2*c*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6194, 97, 154, 21, 105, 41, 216, 92, 208}

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a}+\frac{c\csc^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (-2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(3/2)*x + (c*ArcCsc[a*x])/a + (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1
/(a*x)]])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx=-\left(c\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$
$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-c\,\text{Subst}\left(\int\frac{\left(\frac{1}{a}-\frac{2x}{a^2}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$
$$=-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+(ac)\,\text{Subst}\left(\int\frac{-\frac{1}{a^2}+\frac{x}{a^3}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\right.$$
$$=-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{c\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{a}$$
$$=-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\right.}{a^2}$$
$$=-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{a^2}+$$
$$=-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\csc^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-\right.}{a}$$

Mathematica [A] time = 0.12, size = 53, normalized size = 0.50

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(ax-1)+\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x]))/a
```

fricas [A] time = 0.72, size = 104, normalized size = 0.97

$$-\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2cx^2-c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c*x*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1) + a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 - c)*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

giac [A] time = 0.15, size = 123, normalized size = 1.15

$$-ac\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{4(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)a^2\left(\frac{(ax-1)^2}{(ax+1)^2}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -a*c*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - log(sqrt((a*x - 1)/(a*x + 1
)) + 1)/a^2 + log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 4*(a*x - 1)*sqr
t((a*x - 1)/(a*x + 1))/((a*x + 1)*a^2*((a*x - 1)^2/(a*x + 1)^2 - 1)))
```

maple [A] time = 0.05, size = 163, normalized size = 1.52

$$\frac{(ax-1)\,c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\left(\frac{a^2x+\sqrt{}}{}\right.\right.}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x)
```

```
[Out] (a*x-1)*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)
^(1/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/((a*x-
1)/(a*x+1))^(1/2)/((a*x-1)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 117, normalized size = 1.09

$$-\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

mupad [B] time = 0.07, size = 84, normalized size = 0.79

$$\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x))/a**2
```

3.776 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=104

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

```
[Out] arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c-2*(1+1/a/x)^(1/2)/a/c/(1-1/a/x
)^(1/2)+x*(1+1/a/x)^(1/2)/c/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6194, 103, 21, 94, 92, 208}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 + 1/(a*x)])/(a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*S
qrt[1 - 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}}+\frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}}-\frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{ac}\\
&= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{ac}\\
&= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}\\
&= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 56, normalized size = 0.54

$$\frac{\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax-2)}{ax-1}+\frac{\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(-1 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2
*x^2)])*x]/a)/c
```

fricas [A] time = 0.50, size = 93, normalized size = 0.89

$$\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x
 - a*c)
```

giac [A] time = 0.17, size = 127, normalized size = 1.22

$$a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{\frac{3(ax-1)}{ax+1}-1}{a^2c\left(\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}-\sqrt{\frac{ax-1}{ax+1}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(abs(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/(a^2*c) - (3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)*sqrt((
a*x - 1)/(a*x + 1))/(a*x + 1) - sqrt((a*x - 1)/(a*x + 1)))))
```

maple [B] time = 0.06, size = 251, normalized size = 2.41

$$\frac{2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+3\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}\ x^2a^2-4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}}{\sqrt{a^2}}\right)x\,a^2-((ax-1)(ax}{2a\sqrt{a^2}\ (ax-1)\,c\sqrt{(ax-1)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x)
```

```
[Out] 1/2*(2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+
3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2-4*ln((a^2*x+((a*x-1)*(a*x+1))
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-
6*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+2*a*ln((a^2*x+((a*x-1)*(a*x+1))^(
1/2)*(a^2)^(1/2))/(a^2)^(1/2))+3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/a/(a^
2)^(1/2)/(a*x-1)/c/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.32, size = 116, normalized size = 1.12

$$-a\left(\frac{\frac{3(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*c*
sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) + l
og(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

mupad [B] time = 0.08, size = 62, normalized size = 0.60

$$\frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 4)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x)/c
```

3.777 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=180

$$\frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}-\frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}\right.}}{ac^2}$$

```
[Out] arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-4/3/a/c^2/(1-1/a/x)^(3/2)/(1
+1/a/x)^(1/2)+x/c^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)-11/3/a/c^2/(1-1/a/x)^(1
/2)/(1+1/a/x)^(1/2)+8/3*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}-\frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}\right.}}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -4/(3*a*c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) - 11/(3*a*c^2*Sqrt[1 - 1
/(a*x)]*Sqrt[1 + 1/(a*x)]) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*x
)]) + x/(c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a
*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{3}{a^2}+\frac{8x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{a^2\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A] time = 0.22, size = 83, normalized size = 0.46

$$\frac{\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(3a^3x^3-7a^2x^2-5ax+8\right)}{3(ax-1)^2(ax+1)}}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 + a
*x)^2*(1 + a*x)) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

fricas [A] time = 0.52, size = 134, normalized size = 0.74

$$\frac{3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-7\,a^2x^2-5\,ax+8\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^2
- 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

giac [A] time = 0.16, size = 171, normalized size = 0.95

$$\frac{1}{12}\,a\left(\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^2}-\frac{(ax+1)\left(\frac{18\,(ax-1)}{ax+1}+1\right)}{(ax-1)a^2c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}-\frac{24\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*a*(12*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 12*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/(a^2*c^2) - (a*x + 1)*(18*(a*x - 1)/(a*x + 1) + 1
)/((a*x - 1)*a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) + 3*sqrt((a*x - 1)/(a*x + 1
))/(a^2*c^2) - 24*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*((a*x - 1)/(a*x + 1) -
 1)))
```

maple [B] time = 0.07, size = 530, normalized size = 2.94

$$-\frac{-45\sqrt{a^2}\ \sqrt{(ax-1)\,(ax+1)}\ x^5a^5-24\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\ \sqrt{a^2}}{\sqrt{a^2}}\right)x^5a^6+21\sqrt{a^2}\ \left((ax-1)\,(ax+1)\right)^{\frac{3}{2}}x^3a^3+45}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/24*(-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-24*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+21*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^3*a^3+45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+24*ln
((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+11*(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+90*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
)*x^3*a^3+48*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^
3*a^4-5*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-90*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2)*x^2*a^2-48*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)
^(1/2))*x^2*a^3-19*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-45*((a*x-1)*(a*x+1))
^(1/2)*(a^2)^(1/2)*x*a-24*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a
^2)^(1/2))*x*a^2+45*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+24*a*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a/(a*x-1)^2/(a*x+1)^2/(a^2)^
(1/2)/c^2/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.31, size = 160, normalized size = 0.89

$$\frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^2) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

mupad [B] time = 0.07, size = 128, normalized size = 0.71

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2}-\frac{\frac{17\,(a\,x-1)}{3\,(a\,x+1)}-\frac{14\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^2) - ((17*(a*x - 1))/(3*(a*x + 1)) - (14
*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 4*a
*c^2*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/
(a*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))),
x)/c**2
```

3.778 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=254

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right.}$$

```
[Out] -6/5/a/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)-29/15/a/c^3/(1-1/a/x)^(3/2)/(1+1
/a/x)^(3/2)+x/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)+arctanh((1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2))/a/c^3-34/5/a/c^3/(1+1/a/x)^(3/2)/(1-1/a/x)^(1/2)+21/5*(1-1/
a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+16/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -6/(5*a*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)) - 29/(15*a*c^3*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)) - 34/(5*a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(3/2)) + (21*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sq
rt[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(5/2)*(
1 + 1/(a*x))^(3/2)) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{5}{a^2}+\frac{24x}{a^3}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x\right.}{5c^3}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} +$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} +$$

Mathematica [A] time = 0.27, size = 99, normalized size = 0.39

$$\frac{\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(15a^5x^5-38a^4x^4-52a^3x^3+87a^2x^2+33ax-48\right)}{15(ax-1)^3(ax+1)^2}}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-48 + 33*a*x + 87*a^2*x^2 - 52*a^3*x^3 - 38*a^
4*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^3*(1 + a*x)^2) + Log[(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/(a*c^3)
```

fricas [A] time = 0.58, size = 178, normalized size = 0.70

$$\frac{15\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(15\,a^5x^5-38\,a^4\right.}{15\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x - 48)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.18, size = 232, normalized size = 0.91

$$\frac{1}{240}\,a\left(\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{240\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^3}-\frac{(ax+1)^2\left(\frac{40\,(ax-1)}{ax+1}+\frac{450\,(ax-1)^2}{(ax+1)^2}+3\right)}{(ax-1)^2a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{480\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3\left(\frac{ax-1}{ax+1}-1\right)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] 1/240*a*(240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 240*log(abs(sqr
t((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^3) - (a*x + 1)^2*(40*(a*x - 1)/(a*x + 1
) + 450*(a*x - 1)^2/(a*x + 1)^2 + 3)/((a*x - 1)^2*a^2*c^3*sqrt((a*x - 1)/(a
*x + 1))) - 480*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*((a*x - 1)/(a*x + 1) - 1
)) + 5*((a*x - 1)*a^4*c^6*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 24*a^4*c^6*
sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^9))
```

maple [B] time = 0.07, size = 714, normalized size = 2.81

$$-\frac{-525\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}\,x^7a^7-240\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^7a^8+285\left((ax-1)\,(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\,x^5a^5+52}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -1/240*(-525*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^7*a^7-240*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8+285*((a*x-1)*(a*x+1))
^(3/2)*(a^2)^(1/2)*x^5*a^5+525*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^6*a^6+
240*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6*a^7+83*
((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^4*a^4+1575*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(1/2)*x^5*a^5+720*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*x^5*a^6-218*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3-1575*(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-720*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-342*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x
^2*a^2-1575*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-720*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-3*(a^2)^(1/2)*((a*x-1)
```

```
*(a*x+1))^(3/2)*x*a+1575*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+720*ln
((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+243*((a*x
-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+525*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+
240*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-525*(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-240*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2)))/a/(a*x+1)^3/(a^2)^(1/2)/(a*x-1)^3/c^3/((a*x-1)*(
a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.32, size = 194, normalized size = 0.76

$$\frac{1}{240}\,a\left(\frac{\frac{37\,(ax-1)}{ax+1}+\frac{410\,(ax-1)^2}{(ax+1)^2}-\frac{930\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+24\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{240\,\log\left(\sqrt{\right.}}{a^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*a*((37*(a*x - 1)/(a*x + 1) + 410*(a*x - 1)^2/(a*x + 1)^2 - 930*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*x
 - 1)/(a*x + 1))^(5/2)) + 5*(((a*x - 1)/(a*x + 1))^(3/2) + 24*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3)
- 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

mupad [B] time = 0.10, size = 171, normalized size = 0.67

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3}-\frac{\frac{82\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{62\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{37\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^3}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(2*a*c^3) - ((82*(a*x - 1)^2)/(3*(a*x + 1)^2) -
 (62*(a*x - 1)^3)/(a*x + 1)^3 + (37*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(16*a*
c^3*((a*x - 1)/(a*x + 1))^(5/2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + (
(a*x - 1)/(a*x + 1))^(3/2)/(48*a*c^3) - (atan(((a*x - 1)/(a*x + 1))^(1/2)*1
i)*2i)/(a*c^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\int\frac{x^6}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x*
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

3.779 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4} dx$

Optimal. Leaf size=328

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{}{21ac}$$

```
[Out] -8/7/a/c^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(5/2)-11/7/a/c^4/(1-1/a/x)^(5/2)/(1+1/
a/x)^(5/2)-62/21/a/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^(5/2)+x/c^4/(1-1/a/x)^(7/2
)/(1+1/a/x)^(5/2)+arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-269/21/a/c
^4/(1+1/a/x)^(5/2)/(1-1/a/x)^(1/2)+262/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(
5/2)+163/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+128/35*(1-1/a/x)^(1/2)/a/
c^4/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{}{21ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] -8/(7*a*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) - 11/(7*a*c^4*(1 - 1/(
a*x))^(5/2)*(1 + 1/(a*x))^(5/2)) - 62/(21*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/
(a*x))^(5/2)) - 269/(21*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (262
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (163*Sqrt[1 - 1/(a*x)]
)/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) + ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{7}{a^2}+\frac{48x}{a^3}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right.}{7c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^2\,\text{S}}{}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + $$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - $$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - $$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - $$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - $$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - $$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - $$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - $$

Mathematica [A] time = 0.35, size = 115, normalized size = 0.35

$$\frac{\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(105a^7x^7-281a^6x^6-559a^5x^5+965a^4x^4+715a^3x^3-1065a^2x^2-279ax+384\right)}{105(ax-1)^4(ax+1)^3}}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(384 - 279*a*x - 1065*a^2*x^2 + 715*a^3*x^3 + 9
65*a^4*x^4 - 559*a^5*x^5 - 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^4*(1
 + a*x)^3) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

fricas [A] time = 0.68, size = 274, normalized size = 0.84

$$\frac{105\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-\right.}{105\left(a^7c^4x^6-2a^6c^4x^5-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1
)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 +
 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (105
*a^7*x^7 - 281*a^6*x^6 - 559*a^5*x^5 + 965*a^4*x^4 + 715*a^3*x^3 - 1065*a^2
*x^2 - 279*a*x + 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 2*a^6*c^4*x
^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.17, size = 286, normalized size = 0.87

$$\frac{1}{6720}a\left(\frac{6720\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{6720\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^4}-\frac{5(ax+1)^3\left(\frac{42(ax-1)}{ax+1}+\frac{329(ax-1)^2}{(ax+1)^2}+\frac{2940(ax-1)^3}{(ax+1)^3}+3\right.}{(ax-1)^3a^2c^4\sqrt{\frac{ax-1}{ax+1}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/6720*a*(6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 6720*log(abs(
sqrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^4) - 5*(a*x + 1)^3*(42*(a*x - 1)/(a*
x + 1) + 329*(a*x - 1)^2/(a*x + 1)^2 + 2940*(a*x - 1)^3/(a*x + 1)^3 + 3)/((
a*x - 1)^3*a^2*c^4*sqrt((a*x - 1)/(a*x + 1))) - 13440*sqrt((a*x - 1)/(a*x +
 1))/(a^2*c^4*((a*x - 1)/(a*x + 1) - 1)) + 7*(50*(a*x - 1)*a^8*c^16*sqrt((a
*x - 1)/(a*x + 1))/(a*x + 1) + 3*(a*x - 1)^2*a^8*c^16*sqrt((a*x - 1)/(a*x +
 1))/(a*x + 1)^2 + 705*a^8*c^16*sqrt((a*x - 1)/(a*x + 1)))/(a^10*c^20))
```

maple [B] time = 0.08, size = 898, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/13440*(-132300*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+132300*((a*x-
1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^7*a^7-27673*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(
1/2)*x^5*a^5+7705*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3-198450*(a^2)^
```

```
(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5+198450*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(1/2)*x^4*a^4-132300*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^6*a^6-24295*((a*
x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^4*a^4-33075*((a*x-1)*(a*x+1))^(1/2)*(a^2)
^(1/2)*x*a+37095*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+2637*(a^2)^(1/
2)*((a*x-1)*(a*x+1))^(3/2)*x*a+132300*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x
^3*a^3-13440*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*
a^2-16077*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+33075*((a*x-1)*(a*x+1))^(1/2)
*(a^2)^(1/2)+13440*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))+53760*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3
*a^4-53760*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*
a^3-13440*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^9*a
^10+13440*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^8*a
^9-33075*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^9*a^9+19635*((a*x-1)*(a*x+1)
)^(3/2)*(a^2)^(1/2)*x^7*a^7+33075*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^8*a
^8+2893*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^6*a^6+53760*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8-53760*ln((a^2*x+((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6*a^7-80640*ln((a^2*x+((a*x-1)*
(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+80640*ln((a^2*x+((a*x-1)*(
a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5)/a/(a*x+1)^4/(a^2)^(1/2)/(a
*x-1)^4/c^4/((a*x-1)*(a*x+1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [A] time = 0.33, size = 230, normalized size = 0.70

$$\frac{1}{6720}a\left(\frac{5\left(\frac{39\,(ax-1)}{ax+1}+\frac{287\,(ax-1)^2}{(ax+1)^2}+\frac{2611\,(ax-1)^3}{(ax+1)^3}-\frac{5628\,(ax-1)^4}{(ax+1)^4}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{7\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+50\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+705\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{67\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/6720*a*(5*(39*(a*x - 1)/(a*x + 1) + 287*(a*x - 1)^2/(a*x + 1)^2 + 2611*(a
*x - 1)^3/(a*x + 1)^3 - 5628*(a*x - 1)^4/(a*x + 1)^4 + 3)/(a^2*c^4*((a*x -
1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 7*(3*((a*x - 1
)/(a*x + 1))^(5/2) + 50*((a*x - 1)/(a*x + 1))^(3/2) + 705*sqrt((a*x - 1)/(a
*x + 1)))/(a^2*c^4) + 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 6
720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

mupad [B] time = 0.07, size = 210, normalized size = 0.64

$$\frac{47\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{41\,(a\,x-1)^2}{3\,(a\,x+1)^2}+\frac{373\,(a\,x-1)^3}{3\,(a\,x+1)^3}-\frac{268\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{13\,(a\,x-1)}{7\,(a\,x+1)}+\frac{1}{7}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}+\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{96\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (47*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((41*(a*x - 1)^2)/(3*(a*x + 1
)^2) + (373*(a*x - 1)^3)/(3*(a*x + 1)^3) - (268*(a*x - 1)^4)/(a*x + 1)^4 +
(13*(a*x - 1))/(7*(a*x + 1)) + 1/7)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2) -
 64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2)) + (5*((a*x - 1)/(a*x + 1))^(3/2))/(9
6*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - (atan(((a*x - 1)/(a*x
+ 1))^(1/2)*1i)*2i)/(a*c^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^8\int\frac{x^8}{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x*
*6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))), x)/c**4
```

3.780 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal. Leaf size=127

$$-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+\frac{2c^5\log(x)}{a}+c^5x$$

```
[Out] -1/9*c^5/a^10/x^9-1/4*c^5/a^9/x^8+3/7*c^5/a^8/x^7+4/3*c^5/a^7/x^6-2/5*c^5/a
^6/x^5-3*c^5/a^5/x^4-2/3*c^5/a^4/x^3+4*c^5/a^3/x^2+3*c^5/a^2/x+c^5*x+2*c^5*
ln(x)/a
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{4c^5}{a^3x^2}-\frac{2c^5}{3a^4x^3}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{5a^6x^5}+\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}-\frac{c^5}{4a^9x^8}-\frac{c^5}{9a^{10}x^9}+\frac{3c^5}{a^2x}+\frac{2c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] -c^5/(9*a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^7*
x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4*c
^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx$$
$$= \frac{c^5\int \frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^5}{x^{10}}dx}{a^{10}}$$
$$= \frac{c^5\int \frac{(1-ax)^4(1+ax)^6}{x^{10}}dx}{a^{10}}$$
$$= \frac{c^5\int \left(a^{10}+\frac{1}{x^{10}}+\frac{2a}{x^9}-\frac{3a^2}{x^8}-\frac{8a^3}{x^7}+\frac{2a^4}{x^6}+\frac{12a^5}{x^5}+\frac{2a^6}{x^4}-\frac{8a^7}{x^3}-\frac{3a^8}{x^2}+\frac{2a^9}{x}\right)dx}{a^{10}}$$
$$= -\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+$$

Mathematica [A] time = 0.05, size = 127, normalized size = 1.00

$$-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+\frac{2c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] -1/9*c^5/(a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^
7*x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4
*c^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

fricas [A] time = 1.52, size = 122, normalized size = 0.96

$$\frac{1260\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log(x)+3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1\ldots}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/1260*(1260*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) + 3780*a^8*c^5*x^8 + 5
040*a^7*c^5*x^7 - 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 16
80*a^3*c^5*x^3 + 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

giac [A] time = 0.13, size = 115, normalized size = 0.91

$$c^5x+\frac{2\,c^5\log(|x|)}{a}+\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a\ldots}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] c^5*x + 2*c^5*log(abs(x))/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 -
 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 +
540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

maple [A] time = 0.04, size = 116, normalized size = 0.91

$$-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{x^2a^3}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^5,x)
```

```
[Out] -1/9*c^5/a^10/x^9-1/4*c^5/a^9/x^8+3/7*c^5/a^8/x^7+4/3*c^5/a^7/x^6-2/5*c^5/a
^6/x^5-3*c^5/a^5/x^4-2/3*c^5/a^4/x^3+4*c^5/x^2/a^3+3*c^5/a^2/x+c^5*x+2*c^5*
ln(x)/a
```

maxima [A] time = 0.31, size = 114, normalized size = 0.90

$$c^5x+\frac{2\,c^5\log(x)}{a}+\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 2*c^5*log(x)/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 - 840*
a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 + 540*a
^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

mupad [B] time = 0.10, size = 89, normalized size = 0.70

$$\frac{c^5\,\left(\frac{3\,a^2\,x^2}{7}-\frac{a\,x}{4}+\frac{4\,a^3\,x^3}{3}-\frac{2\,a^4\,x^4}{5}-3\,a^5\,x^5-\frac{2\,a^6\,x^6}{3}+4\,a^7\,x^7+3\,a^8\,x^8+a^{10}\,x^{10}+2\,a^9\,x^9\,\ln(x)-\frac{1}{9}\right)}{a^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^5*((3*a^2*x^2)/7 - (a*x)/4 + (4*a^3*x^3)/3 - (2*a^4*x^4)/5 - 3*a^5*x^5 -
 (2*a^6*x^6)/3 + 4*a^7*x^7 + 3*a^8*x^8 + a^10*x^10 + 2*a^9*x^9*log(x) - 1/9
))/(a^10*x^9)
```

sympy [A] time = 0.66, size = 124, normalized size = 0.98

$$\frac{a^{10}c^5x+2a^9c^5\log{(x)}+\frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315ac^5x-140c^5}{1260x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 2*a**9*c**5*log(x) + (3780*a**8*c**5*x**8 + 5040*a**7*c**5*
x**7 - 840*a**6*c**5*x**6 - 3780*a**5*c**5*x**5 - 504*a**4*c**5*x**4 + 1680
*a**3*c**5*x**3 + 540*a**2*c**5*x**2 - 315*a*c**5*x - 140*c**5)/(1260*x**9)
)/a**10
```

3.781 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=90

$$\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4x$$

```
[Out] 1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/a^3/x
^2+2*c^4/a^2/x+c^4*x+2*c^4*ln(x)/a
```

Rubi [A] time = 0.16, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{3c^4}{a^3x^2}-\frac{3c^4}{2a^5x^4}-\frac{2c^4}{5a^6x^5}+\frac{c^4}{3a^7x^6}+\frac{c^4}{7a^8x^7}+\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx$$
$$= -\frac{c^4\int\frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^4}{x^8}dx}{a^8}$$
$$= -\frac{c^4\int\frac{(1-ax)^3(1+ax)^5}{x^8}dx}{a^8}$$
$$= -\frac{c^4\int\left(-a^8+\frac{1}{x^8}+\frac{2a}{x^7}-\frac{2a^2}{x^6}-\frac{6a^3}{x^5}+\frac{6a^5}{x^3}+\frac{2a^6}{x^2}-\frac{2a^7}{x}\right)dx}{a^8}$$
$$= \frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

Mathematica [A] time = 0.04, size = 90, normalized size = 1.00

$$\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

fricas [A] time = 0.41, size = 89, normalized size = 0.99

$$\frac{210\,a^8c^4x^8+420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/210*(210*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 + 630*a^5
*c^4*x^5 - 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7
)
```

giac [A] time = 0.12, size = 82, normalized size = 0.91

$$c^4x+\frac{2\,c^4\log\left(|x|\right)}{a}+\frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] c^4*x + 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 31
5*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

maple [A] time = 0.04, size = 83, normalized size = 0.92

$$\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{x^2a^3}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^4,x)
```

```
[Out] 1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/x^2/a
^3+2*c^4/a^2/x+c^4*x+2*c^4*ln(x)/a
```

maxima [A] time = 0.32, size = 81, normalized size = 0.90

$$c^4x + \frac{2\,c^4\log(x)}{a} + \frac{420\,a^6c^4x^6 + 630\,a^5c^4x^5 - 315\,a^3c^4x^3 - 84\,a^2c^4x^2 + 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^3
*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

mupad [B] time = 1.24, size = 65, normalized size = 0.72

$$\frac{c^4\,\left(\frac{a\,x}{3}-\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5+2\,a^6\,x^6+a^8\,x^8+2\,a^7\,x^7\,\ln(x)+\frac{1}{7}\right)}{a^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^4*((a*x)/3 - (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 + 2*a^6*x^6 + a^8
*x^8 + 2*a^7*x^7*log(x) + 1/7))/(a^8*x^7)
```

sympy [A] time = 0.44, size = 88, normalized size = 0.98

$$\frac{a^8c^4x + 2a^7c^4\log{(x)} + \frac{420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70ac^4x+30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 + 630*a**5*c**4*x**
5 - 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 + 70*a*c**4*x + 30*c**4)/(210*x*
*7))/a**8
```

3.782 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=76

$$-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3x$$

```
[Out] -1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3+2*c^3/a^3/x^2+c^3/a^2/x+c^
3*x+2*c^3*ln(x)/a
```

Rubi [A] time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{2c^3}{a^3x^2}+\frac{c^3}{3a^4x^3}-\frac{c^3}{2a^5x^4}-\frac{c^3}{5a^6x^5}+\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] -c^3/(5*a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2) +
c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$
$$= \frac{c^3\int \frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^3}{x^6}dx}{a^6}$$
$$= \frac{c^3\int \frac{(1-ax)^2(1+ax)^4}{x^6}dx}{a^6}$$
$$= \frac{c^3\int \left(a^6+\frac{1}{x^6}+\frac{2a}{x^5}-\frac{a^2}{x^4}-\frac{4a^3}{x^3}-\frac{a^4}{x^2}+\frac{2a^5}{x}\right)dx}{a^6}$$
$$= -\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 1.00

$$-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] -1/5*c^3/(a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

fricas [A] time = 0.77, size = 78, normalized size = 1.03

$$\frac{30\,a^6c^3x^6+60\,a^5c^3x^5\log(x)+30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(30*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 + 60*a^3*c^3*
x^3 + 10*a^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

giac [A] time = 0.12, size = 71, normalized size = 0.93

$$c^3x+\frac{2\,c^3\log(|x|)}{a}+\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] c^3*x + 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^
2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

maple [A] time = 0.04, size = 71, normalized size = 0.93

$$-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{x^2a^3}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^3,x)
```

```
[Out] -1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3+2*c^3/x^2/a^3+c^3/a^2/x+c^
3*x+2*c^3*ln(x)/a
```

maxima [A] time = 0.31, size = 70, normalized size = 0.92

$$c^3x + \frac{2\,c^3\log(x)}{a} + \frac{30\,a^4c^3x^4 + 60\,a^3c^3x^3 + 10\,a^2c^3x^2 - 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^2*c^3
*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

mupad [B] time = 1.22, size = 56, normalized size = 0.74

$$\frac{c^3\left(\frac{a^2\,x^2}{3} - \frac{a\,x}{2} + 2\,a^3\,x^3 + a^4\,x^4 + a^6\,x^6 + 2\,a^5\,x^5\,\ln(x) - \frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^3*((a^2*x^2)/3 - (a*x)/2 + 2*a^3*x^3 + a^4*x^4 + a^6*x^6 + 2*a^5*x^5*log
(x) - 1/5))/(a^6*x^5)
```

sympy [A] time = 0.33, size = 76, normalized size = 1.00

$$\frac{a^6c^3x + 2a^5c^3\log(x) + \frac{30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15ac^3x-6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 + 60*a**3*c**3*x**3
+ 10*a**2*c**3*x**2 - 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

3.783 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=39

$$\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+\frac{2c^2\log(x)}{a}+c^2x$$

```
[Out] 1/3*c^2/a^4/x^3+c^2/a^3/x^2+c^2*x+2*c^2*ln(x)/a
```

Rubi [A] time = 0.15, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 75}

$$\frac{c^2}{a^3x^2}+\frac{c^2}{3a^4x^3}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx$$
$$=-\frac{c^2\int\frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^2}{x^4}dx}{a^4}$$
$$=-\frac{c^2\int\frac{(1-ax)(1+ax)^3}{x^4}dx}{a^4}$$
$$=-\frac{c^2\int\left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{2a^3}{x}\right)dx}{a^4}$$
$$=\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

fricas [A] time = 0.45, size = 43, normalized size = 1.10

$$\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log(x)+3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
[Out] 1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/(a^4*x^3)
```

giac [A] time = 0.14, size = 36, normalized size = 0.92

$$c^2x+\frac{2\,c^2\log(|x|)}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="giac")
[Out] c^2*x + 2*c^2*log(abs(x))/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

maple [A] time = 0.04, size = 38, normalized size = 0.97

$$\frac{c^2}{3a^4x^3}+\frac{c^2}{x^2a^3}+c^2x+\frac{2c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^2,x)
[Out] 1/3*c^2/a^4/x^3+c^2/x^2/a^3+c^2*x+2*c^2*ln(x)/a
```

maxima [A] time = 0.31, size = 35, normalized size = 0.90

$$c^2x+\frac{2\,c^2\log(x)}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 2*c^2*log(x)/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

mupad [B] time = 0.05, size = 32, normalized size = 0.82

$$\frac{c^2\left(a\,x+a^4\,x^4+2\,a^3\,x^3\,\ln(x)+\frac{1}{3}\right)}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^2*(a*x + a^4*x^4 + 2*a^3*x^3*log(x) + 1/3))/(a^4*x^3)
```

sympy [A] time = 0.18, size = 39, normalized size = 1.00

$$\frac{a^4c^2x+2a^3c^2\log{(x)}+\frac{3ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 2*a**3*c**2*log(x) + (3*a*c**2*x + c**2)/(3*x**3))/a**4
```

3.784 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=21

$$-\frac{c}{a^2x}+\frac{2c\log(x)}{a}+cx$$

```
[Out] -c/a^2/x+c*x+2*c*ln(x)/a
```

Rubi [A] time = 0.09, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6157, 6150, 43}

$$-\frac{c}{a^2x}+\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$
$$=\frac{c\int\frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$
$$=\frac{c\int\frac{(1+ax)^2}{x^2}dx}{a^2}$$
$$=\frac{c\int\left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)dx}{a^2}$$
$$=-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$-\frac{c}{a^2x}+\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

fricas [A] time = 0.50, size = 26, normalized size = 1.24

$$\frac{a^2cx^2+2\,acx\log(x)-c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*c*x^2 + 2*a*c*x*log(x) - c)/(a^2*x)
```

giac [A] time = 0.12, size = 22, normalized size = 1.05

$$cx+\frac{2\,c\log(|x|)}{a}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="giac")
[Out] c*x + 2*c*log(abs(x))/a - c/(a^2*x)
```

maple [A] time = 0.04, size = 22, normalized size = 1.05

$$-\frac{c}{a^2x}+cx+\frac{2c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2),x)
[Out] -c/a^2/x+c*x+2*c*ln(x)/a
```

maxima [A] time = 0.31, size = 21, normalized size = 1.00

$$cx+\frac{2\,c\log(x)}{a}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] c*x + 2*c*log(x)/a - c/(a^2*x)
```

mupad [B] time = 0.04, size = 23, normalized size = 1.10

$$\frac{c\,\left(a^2\,x^2+2\,a\,x\,\ln(x)-1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

sympy [A] time = 0.11, size = 20, normalized size = 0.95

$$\frac{a^2cx+2ac\log(x)-\frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2),x)
```

```
[Out] (a**2*c*x + 2*a*c*log(x) - c/x)/a**2
```

3.785 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=36

$$\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}+\frac{x}{c}$$

[Out] x/c+1/a/c/(-a*x+1)+2*ln(-a*x+1)/a/c

Rubi [A] time = 0.16, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 43}

$$\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]

[Out] x/c + 1/(a*c*(1 - a*x)) + (2*Log[1 - a*x])/(a*c)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx \\
&= \frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c} \\
&= \frac{a^2\int \frac{x^2}{(1-ax)^2}\,dx}{c} \\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(-1+ax)^2}+\frac{2}{a^2(-1+ax)}\right)dx}{c} \\
&= \frac{x}{c}+\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.78

$$
\frac{\frac{1}{a-a^2x}+\frac{2\log(1-ax)}{a}+x}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
[Out] (x + (a - a^2*x)^(-1) + (2*Log[1 - a*x])/a)/c
```

fricas [A] time = 0.61, size = 40, normalized size = 1.11

$$
\frac{a^2x^2-ax+2(ax-1)\log(ax-1)-1}{a^2cx-ac}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^2*c*x - a*c)
```

giac [A] time = 0.13, size = 36, normalized size = 1.00

$$
\frac{x}{c}+\frac{2\log(|ax-1|)}{ac}-\frac{1}{(ax-1)ac}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] x/c + 2*log(abs(a*x - 1))/(a*c) - 1/((a*x - 1)*a*c)
```

maple [A] time = 0.04, size = 36, normalized size = 1.00

$$
\frac{x}{c}+\frac{2\ln(ax-1)}{ac}-\frac{1}{ac(ax-1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2),x)
[Out] x/c+2/a/c*ln(a*x-1)-1/a/c/(a*x-1)
```

maxima [A] time = 0.30, size = 35, normalized size = 0.97

$$\frac{x}{c} - \frac{1}{a^2cx - ac} + \frac{2 \log(ax - 1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] x/c - 1/(a^2*c*x - a*c) + 2*log(a*x - 1)/(a*c)
```

mupad [B] time = 0.05, size = 33, normalized size = 0.92

$$\frac{x}{c} + \frac{1}{a\,(c - a\,c\,x)} + \frac{2\,\ln(a\,x - 1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))*(a*x - 1)),x)
```

```
[Out] x/c + 1/(a*(c - a*c*x)) + (2*log(a*x - 1))/(a*c)
```

sympy [A] time = 0.14, size = 36, normalized size = 1.00

$$a^2 \left(-\frac{1}{a^4cx - a^3c} + \frac{x}{a^2c} + \frac{2\log(ax - 1)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(-1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c))
```

3.786 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=75

$$\frac{7}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2-1/4/a/c^2/(-a*x+1)^2+7/4/a/c^2/(-a*x+1)+17/8*ln(-a*x+1)/a/c^2-1/8*ln(
a*x+1)/a/c^2
```

Rubi [A] time = 0.18, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{7}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= -\frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}$$
$$= -\frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^2}$$
$$= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^2}$$
$$= \frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 1.00

$$\frac{7}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

fricas [A] time = 0.54, size = 93, normalized size = 1.24

$$\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-\left(a^2x^2-2\,ax+1\right)\log\left(ax+1\right)+17\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+12}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*
c^2)
```

giac [A] time = 0.14, size = 57, normalized size = 0.76

$$\frac{x}{c^2}-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{17\,\log\left(|ax-1|\right)}{8\,ac^2}-\frac{7\,ax-6}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 1/8*log(abs(a*x + 1))/(a*c^2) + 17/8*log(abs(a*x - 1))/(a*c^2) - 1/
4*(7*a*x - 6)/((a*x - 1)^2*a*c^2)
```

maple [A] time = 0.04, size = 65, normalized size = 0.87

$$\frac{x}{c^2}-\frac{1}{4a\,c^2\,(ax-1)^2}-\frac{7}{4a\,c^2\,(ax-1)}+\frac{17\ln\left(ax-1\right)}{8a\,c^2}-\frac{\ln\left(ax+1\right)}{8a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^2,x)
```

```
[Out] x/c^2-1/4/a/c^2/(a*x-1)^2-7/4/a/c^2/(a*x-1)+17/8/a/c^2*ln(a*x-1)-1/8*ln(a*x
+1)/a/c^2
```

maxima [A] time = 0.31, size = 69, normalized size = 0.92

$$-\frac{7\,ax-6}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}+\frac{x}{c^2}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{17\,\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x - 6)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 - 1/8*log(a*x
+ 1)/(a*c^2) + 17/8*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 0.09, size = 68, normalized size = 0.91

$$\frac{x}{c^2}-\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}+\frac{17\,\ln\left(a\,x-1\right)}{8\,a\,c^2}-\frac{\ln\left(a\,x+1\right)}{8\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^2*(a*x - 1)),x)
```

```
[Out] x/c^2 - ((7*x)/4 - 3/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (17*log(a*x -
 1))/(8*a*c^2) - log(a*x + 1)/(8*a*c^2)
```

sympy [A] time = 0.37, size = 73, normalized size = 0.97

$$a^4\left(\frac{-7ax+6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-7*a*x + 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) + x/(a*
*4*c**2) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2))
```

3.787 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=110

$$\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}-\frac{5}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3+1/12/a/c^3/(-a*x+1)^3-5/8/a/c^3/(-a*x+1)^2+39/16/a/c^3/(-a*x+1)-1/16/
a/c^3/(a*x+1)+9/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

Rubi [A] time = 0.20, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}-\frac{5}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 + 1/(12*a*c^3*(1 - a*x)^3) - 5/(8*a*c^3*(1 - a*x)^2) + 39/(16*a*c^3*(
1 - a*x)) - 1/(16*a*c^3*(1 + a*x)) + (9*Log[1 - a*x])/(4*a*c^3) - Log[1 + a
*x]/(4*a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

$$= \frac{a^6\int \frac{e^{2\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$

$$= \frac{a^6\int \frac{x^6}{(1-ax)^4(1+ax)^2}\,dx}{c^3}$$

$$= \frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{4a^6(-1+ax)^4}+\frac{5}{4a^6(-1+ax)^3}+\frac{39}{16a^6(-1+ax)^2}+\frac{9}{4a^6(-1+ax)}+\frac{1}{16a^6(1+ax)^2}-\frac{1}{4a^6(1+ax)}\right)dx}{c^3}$$

$$= \frac{x}{c^3}+\frac{1}{12ac^3(1-ax)^3}-\frac{5}{8ac^3(1-ax)^2}+\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(}{4}$$

Mathematica [A] time = 0.08, size = 82, normalized size = 0.75

$$\frac{\frac{2(6a^5x^5-12a^4x^4-15a^3x^3+24a^2x^2+7ax-11)}{(ax-1)^3(ax+1)}+27\log(1-ax)-3\log(ax+1)}{12ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((2*(-11 + 7*a*x + 24*a^2*x^2 - 15*a^3*x^3 - 12*a^4*x^4 + 6*a^5*x^5))/((-1
+ a*x)^3*(1 + a*x)) + 27*Log[1 - a*x] - 3*Log[1 + a*x])/(12*a*c^3)
```

fricas [A] time = 0.46, size = 137, normalized size = 1.25

$$\frac{12\,a^5x^5-24\,a^4x^4-30\,a^3x^3+48\,a^2x^2+14\,ax-3\left(a^4x^4-2\,a^3x^3+2\,ax-1\right)\log\left(ax+1\right)+27\left(a^4x^4-2\,a^3x^3+2\,a\right.}{12\left(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*x^5 - 24*a^4*x^4 - 30*a^3*x^3 + 48*a^2*x^2 + 14*a*x - 3*(a^4*x
^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x + 1) + 27*(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)*log(a*x - 1) - 22)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.13, size = 80, normalized size = 0.73

$$\frac{x}{c^3}-\frac{\log\left(|ax+1|\right)}{4\,ac^3}+\frac{9\,\log\left(|ax-1|\right)}{4\,ac^3}-\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\,(ax+1)(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 9/4*log(abs(a*x - 1))/(a*c^3) - 1/6
*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

maple [A] time = 0.05, size = 95, normalized size = 0.86

$$\frac{x}{c^3}-\frac{1}{12c^3a\,(ax-1)^3}-\frac{5}{8c^3a\,(ax-1)^2}-\frac{39}{16a\,c^3\,(ax-1)}+\frac{9\ln\left(ax-1\right)}{4c^3a}-\frac{1}{16a\,c^3\,(ax+1)}-\frac{\ln\left(ax+1\right)}{4a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^3,x)
```

```
[Out] x/c^3-1/12/c^3/a/(a*x-1)^3-5/8/c^3/a/(a*x-1)^2-39/16/a/c^3/(a*x-1)+9/4/c^3/
a*ln(a*x-1)-1/16/a/c^3/(a*x+1)-1/4*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.31, size = 97, normalized size = 0.88

$$-\frac{15\,a^{3}x^{3}-12\,a^{2}x^{2}-13\,ax+11}{6\left(a^{5}c^{3}x^{4}-2\,a^{4}c^{3}x^{3}+2\,a^{2}c^{3}x-ac^{3}\right)}+\frac{x}{c^{3}}-\frac{\log\left(ax+1\right)}{4\,ac^{3}}+\frac{9\,\log\left(ax-1\right)}{4\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 +
 2*a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 9/4*log(a*x - 1)
/(a*c^3)
```

mupad [B] time = 1.28, size = 94, normalized size = 0.85

$$\frac{x}{c^3}-\frac{\frac{13\,x}{6}+2\,a\,x^2-\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}+\frac{9\,\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^3*(a*x - 1)),x)
```

```
[Out] x/c^3 - ((13*x)/6 + 2*a*x^2 - 11/(6*a) - (5*a^2*x^3)/2)/(c^3 + 2*a^3*c^3*x^
3 - a^4*c^3*x^4 - 2*a*c^3*x) + (9*log(a*x - 1))/(4*a*c^3) - log(a*x + 1)/(4
*a*c^3)
```

sympy [A] time = 0.59, size = 102, normalized size = 0.93

$$a^{6}\left(\frac{-15a^{3}x^{3}+12a^{2}x^{2}+13ax-11}{6a^{11}c^{3}x^{4}-12a^{10}c^{3}x^{3}+12a^{8}c^{3}x-6a^{7}c^{3}}+\frac{x}{a^{6}c^{3}}+\frac{\frac{9\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^{7}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-15*a**3*x**3 + 12*a**2*x**2 + 13*a*x - 11)/(6*a**11*c**3*x**4 - 12*
a**10*c**3*x**3 + 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (9*log(x
- 1/a)/4 - log(x + 1/a)/4)/(a**7*c**3))
```

3.788 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=145

$$\frac{99}{32ac^4(1-ax)}-\frac{11}{64ac^4(ax+1)}-\frac{35}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}+\frac{13}{48ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}+\frac{303\log(1-ax}{128ac^4}$$

```
[Out] x/c^4-1/32/a/c^4/(-a*x+1)^4+13/48/a/c^4/(-a*x+1)^3-35/32/a/c^4/(-a*x+1)^2+9
9/32/a/c^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2-11/64/a/c^4/(a*x+1)+303/128*ln(-a*
x+1)/a/c^4-47/128*ln(a*x+1)/a/c^4
```

Rubi [A] time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{99}{32ac^4(1-ax)}-\frac{11}{64ac^4(ax+1)}-\frac{35}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}+\frac{13}{48ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}+\frac{303\log(1-ax}{128ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 - 1/(32*a*c^4*(1 - a*x)^4) + 13/(48*a*c^4*(1 - a*x)^3) - 35/(32*a*c^4
*(1 - a*x)^2) + 99/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) - 11/(64
*a*c^4*(1 + a*x)) + (303*Log[1 - a*x])/(128*a*c^4) - (47*Log[1 + a*x])/(128
*a*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= -\frac{a^8\int \frac{e^{2\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$
$$= -\frac{a^8\int \frac{x^8}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$
$$= -\frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{8a^8(-1+ax)^5}-\frac{13}{16a^8(-1+ax)^4}-\frac{35}{16a^8(-1+ax)^3}-\frac{99}{32a^8(-1+ax)^2}-\frac{303}{128a^8(-1+ax)}+\frac{1}{32a^8(1+ax}\right.}{c^4}$$
$$= \frac{x}{c^4}-\frac{1}{32ac^4(1-ax)^4}+\frac{13}{48ac^4(1-ax)^3}-\frac{35}{32ac^4(1-ax)^2}+\frac{99}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}$$

Mathematica [A] time = 0.12, size = 98, normalized size = 0.68

$$\frac{\frac{2\left(192a^7x^7-384a^6x^6-819a^5x^5+1254a^4x^4+866a^3x^3-1258a^2x^2-275ax+400\right)}{(ax-1)^4(ax+1)^2}+909\log(1-ax)-141\log(ax+1)}{384ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(400 - 275*a*x - 1258*a^2*x^2 + 866*a^3*x^3 + 1254*a^4*x^4 - 819*a^5*x^
5 - 384*a^6*x^6 + 192*a^7*x^7))/((-1 + a*x)^4*(1 + a*x)^2) + 909*Log[1 - a*
x] - 141*Log[1 + a*x])/(384*a*c^4)
```

fricas [A] time = 0.86, size = 233, normalized size = 1.61

$$\frac{384\,a^7x^7-768\,a^6x^6-1638\,a^5x^5+2508\,a^4x^4+1732\,a^3x^3-2516\,a^2x^2-550\,ax-141\,\left(a^6x^6-2\,a^5x^5-a^4x^4+\right.}{384\,\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/384*(384*a^7*x^7 - 768*a^6*x^6 - 1638*a^5*x^5 + 2508*a^4*x^4 + 1732*a^3*x
^3 - 2516*a^2*x^2 - 550*a*x - 141*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^
3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 909*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4
+ 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 800)/(a^7*c^4*x^6 - 2*a^6
*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.14, size = 96, normalized size = 0.66

$$\frac{x}{c^4}-\frac{47\,\log\left(|ax+1|\right)}{128\,ac^4}+\frac{303\,\log\left(|ax-1|\right)}{128\,ac^4}-\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(ax+1)^2(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 47/128*log(abs(a*x + 1))/(a*c^4) + 303/128*log(abs(a*x - 1))/(a*c^4
) - 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x
 - 400)/((a*x + 1)^2*(a*x - 1)^4*a*c^4)
```

maple [A] time = 0.05, size = 125, normalized size = 0.86

$$\frac{x}{c^4}-\frac{1}{32c^4a\,(ax-1)^4}-\frac{13}{48c^4a\,(ax-1)^3}-\frac{35}{32c^4a\,(ax-1)^2}-\frac{99}{32a\,c^4\,(ax-1)}+\frac{303\ln(ax-1)}{128c^4a}+\frac{1}{64a\,c^4\,(ax+1)^2}-\frac{1}{64a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^4,x)
```

```
[Out] x/c^4-1/32/c^4/a/(a*x-1)^4-13/48/c^4/a/(a*x-1)^3-35/32/c^4/a/(a*x-1)^2-99/3
2/a/c^4/(a*x-1)+303/128/c^4/a*ln(a*x-1)+1/64/a/c^4/(a*x+1)^2-11/64/a/c^4/(a
*x+1)-47/128*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.31, size = 145, normalized size = 1.00

$$-\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\left(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}-\frac{47\,\log(ax+1)}{128\,ac^4}+\frac{303\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x -
400)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x
^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 47/128*log(a*x + 1)/(a*c^4) + 303/128*l
og(a*x - 1)/(a*c^4)
```

mupad [B] time = 1.45, size = 142, normalized size = 0.98

$$\frac{x}{c^4}+\frac{\frac{467\,x}{192}+\frac{437\,a\,x^2}{96}-\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}-\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5+a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+a^2\,c^4\,x^2+2\,a\,c^4\,x-c^4}+\frac{303\,\ln(a\,x-1)}{128\,a\,c^4}-\frac{47\,\ln(a\,x+1)}{128\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^4*(a*x - 1)),x)
```

```
[Out] x/c^4 + ((467*x)/192 + (437*a*x^2)/96 - 25/(12*a) - (529*a^2*x^3)/96 - (81*
a^3*x^4)/32 + (209*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*c^
4*x^4 + 2*a^5*c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) + (303*log(a*x - 1))/(128*
a*c^4) - (47*log(a*x + 1))/(128*a*c^4)
```

sympy [A] time = 0.84, size = 156, normalized size = 1.08

$$a^8\left(\frac{-627a^5x^5+486a^4x^4+1058a^3x^3-874a^2x^2-467ax+400}{192a^{15}c^4x^6-384a^{14}c^4x^5-192a^{13}c^4x^4+768a^{12}c^4x^3-192a^{11}c^4x^2-384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{303\log\left(x-\right.}{128}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-627*a**5*x**5 + 486*a**4*x**4 + 1058*a**3*x**3 - 874*a**2*x**2 - 46
7*a*x + 400)/(192*a**15*c**4*x**6 - 384*a**14*c**4*x**5 - 192*a**13*c**4*x*
*4 + 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 - 384*a**10*c**4*x + 192*a**
9*c**4) + x/(a**8*c**4) + (303*log(x - 1/a)/128 - 47*log(x + 1/a)/128)/(a**
9*c**4))
```

3.789 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=343

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{7a}+c^4x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}}{14a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}$$

```
[Out] 8/7*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)/a+c^4*(1-1/a/x)^(5/2)*(1+1/a/x)^(1
1/2)*x+15/16*c^4*arccsc(a*x)/a+3*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2
))/a-37/16*c^4*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-61/40*c^4*(1+1/a/x)^(5/2)*
(1-1/a/x)^(1/2)/a-303/280*c^4*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-57/70*c^4*(
1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a+15/14*c^4*(1+1/a/x)^(11/2)*(1-1/a/x)^(1/2)
/a-63/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.26, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{7a}+c^4x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}}{14a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-63*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (37*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(40*a) - (303*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) -
(57*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (15*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(11/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*
x))^(11/2))/(7*a) + c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(11/2)*x + (15*c^
4*ArcCsc[a*x])/(16*a) + (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
)/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx = -\left(c^4\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x - c^4\,\text{Subst}\left(\int\frac{\left(\frac{3}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x}dx, x$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x - \frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int\right.$$

$$= \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\right.$$

$$= -\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.}{7a}$$

$$= -\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right.}{14a}$$

$$= -\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right.}{70a}$$

$$= -\frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right.}{280a}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}$$

Mathematica [A] time = 0.27, size = 126, normalized size = 0.37

$$\frac{c^4\left(525a^6x^6\sin^{-1}\left(\frac{1}{ax}\right)+1680a^6x^6\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sqrt{1-\frac{1}{a^2x^2}}\left(560a^7x^7-2496a^6x^6-525a^5x^5+99\right.\right.}{560a^7x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*(Sqrt[1 - 1/(a^2*x^2)]*(-80 - 280*a*x - 96*a^2*x^2 + 770*a^3*x^3 + 992
*a^4*x^4 - 525*a^5*x^5 - 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[1
/(a*x)] + 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)
```

fricas [A] time = 0.49, size = 201, normalized size = 0.59

$$-\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(560\,a^8c^4x^8-19\right.}{560\,a^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*x
^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 - 1936*a^7*c^4*x^7 - 3021*a^6*c^4*x^6 +
 467*a^5*c^4*x^5 + 1762*a^4*c^4*x^4 + 674*a^3*c^4*x^3 - 376*a^2*c^4*x^2 - 3
60*a*c^4*x - 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

giac [A] time = 0.19, size = 335, normalized size = 0.98

$$-\frac{1}{280}\,ac^4\left(\frac{525\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{560\,\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{13300\,(ax-1)\sqrt{}}{ax+1}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -1/280*a*c^4*(525*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 840*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
 + 560*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (13300*(
a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 45871*(a*x - 1)^2*sqrt((a*x
- 1)/(a*x + 1))/(a*x + 1)^2 + 52672*(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(
a*x + 1)^3 + 33201*(a*x - 1)^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 + 1134
0*(a*x - 1)^5*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^5 + 1645*(a*x - 1)^6*sqrt
((a*x - 1)/(a*x + 1))/(a*x + 1)^6 + 1715*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((
a*x - 1)/(a*x + 1) + 1)^7))
```

maple [A] time = 0.07, size = 329, normalized size = 0.96

$$\frac{(ax-1)^2c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^8a^8+1680\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^6a^6+525a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}+525a^7x^7\sqrt{a^2}\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x)
```

```
[Out] 1/560*(a*x-1)^2*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^8*a^8+1680*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6+525*a^7*x^7*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)+
525*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+1680*ln((a^2*x+(a^2*x^2
-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8+35*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
)*x^5*a^5-816*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^4-490*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*x^3*a^3+176*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+280*(a^2*x^2-1
)^(3/2)*(a^2)^(1/2)*x*a+80*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))
^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

maxima [A] time = 0.43, size = 380, normalized size = 1.11

$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2205\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+136\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 - (2205*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 13615*c^4*((a*x - 1)/(a*x + 1)
)^(13/2) + 33621*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 39071*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 12799*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 20811*c^4*((a*x -
1)/(a*x + 1))^(5/2) - 7665*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 1155*c^4*sqrt(
(a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)^
8*a^2/(a*x + 1)^8 + a^2))*a
```

mupad [B] time = 1.38, size = 332, normalized size = 0.97

$$\frac{\frac{12799\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{280}-\frac{219\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8}-\frac{2973\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}-\frac{33\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{39071\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}+\frac{4803\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+\frac{389\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13\cdots}}{8}\cdots}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\frac{a\,(a\,x-1)^8}{(a\,x+1)^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((12799*c^4*((a*x - 1)/(a*x + 1))^(7/2))/280 - (219*c^4*((a*x - 1)/(a*x + 1
))^(3/2))/8 - (2973*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 - (33*c^4*((a*x - 1
)/(a*x + 1))^(1/2))/8 + (39071*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (4803
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (389*c^4*((a*x - 1)/(a*x + 1))^(13/
2))/8 + (63*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x
+ 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (1
4*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x -
 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((a*x - 1)
/(a*x + 1))^(1/2)))/(8*a) + (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\left(-\frac{4a^2}{\frac{ax^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^4}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^6}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)\cdots\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4/(
a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6/(a*x**3*sqrt(a*x/(a*x +
1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x) + Integral(a**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
```

```
 sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**9*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**8*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x))/a**8
```

3.790 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=269

$$c^3x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{5a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{20a}-\frac{17c^3}{}$$

```
[Out] c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x+3/8*c^3*arccsc(a*x)/a+3*c^3*arctanh((
1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-17/8*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/
a-29/20*c^3*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-21/20*c^3*(1+1/a/x)^(7/2)*(1-
1/a/x)^(1/2)/a+6/5*c^3*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a-27/8*c^3*(1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.19, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^3x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{5a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{20a}-\frac{17c^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-27*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (17*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(8*a) - (29*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(20*a) - (21*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*c
^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(5*a) + c^3*(1 - 1/(a*x))^(3/2)*(
1 + 1/(a*x))^(9/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) + (3*c^3*ArcTanh[Sqrt[1 -
1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3dx=-\left(c^3\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$=c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x-c^3\,\text{Subst}\left(\int\frac{\left(\frac{3}{a}-\frac{6x}{a^2}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x}dx,x,\frac{1}{x}\right.$$

$$=\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x-\frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int\frac{\left(\frac{15}{a^2}\right.}{}\right.$$

$$=-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a}+c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right.$$

$$=-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9}}{5a}$$

$$=-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)}{20a}$$

$$=-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}$$

$$=-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}$$

$$=-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}$$

$$=-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}$$

Mathematica [A] time = 0.20, size = 110, normalized size = 0.41

$$\frac{c^3\left(15a^4x^4\sin^{-1}\left(\frac{1}{ax}\right)+120a^4x^4\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sqrt{1-\frac{1}{a^2x^2}}\left(40a^5x^5-152a^4x^4-55a^3x^3+24a^2x^2+\right.\right.}{40a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(8 + 30*a*x + 24*a^2*x^2 - 55*a^3*x^3 - 152*a^4
*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] + 120*a^4*x^4*Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

fricas [A] time = 0.61, size = 179, normalized size = 0.67

$$-\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(40\,a^6c^3x^6-112\,a\right.}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (40*a^6*c^3*x^6 - 112*a^5*c^3*x^5 - 207*a^4*c^3*x^4 - 31*a^3*
c^3*x^3 + 54*a^2*c^3*x^2 + 38*a*c^3*x + 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(
a^6*x^5)
```

giac [A] time = 0.17, size = 273, normalized size = 1.01

$$-\frac{1}{20}\,ac^3\left(\frac{15\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}+\frac{40\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}+\frac{\frac{810\,(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -1/20*a*c^3*(15*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 + 60*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 + 4
0*sqrt((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) + (810*(a*x - 1
)*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 912*(a*x - 1)^2*sqrt((a*x - 1)/(a*x
 + 1))/(a*x + 1)^2 + 470*(a*x - 1)^3*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^3
+ 95*(a*x - 1)^4*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1)^4 + 145*sqrt((a*x - 1)
/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^5))
```

maple [A] time = 0.06, size = 281, normalized size = 1.04

$$\frac{(ax-1)^2\,c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^6a^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4+15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5+15\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\cdots\right.}{40\left(\frac{ax-1}{ax+1}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/40*(a*x-1)^2*c^3*(-120*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^6*a^6+120*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)*x^4*a^4+15*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^5*a^5+15*a
rctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)*x^5*a^5+120*ln((a^2*x+(a^2*x^2-1)^(1
/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+25*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*
a^3-32*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-30*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*x*a-8*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a
*x-1)*(a*x+1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 302, normalized size = 1.12

$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{135\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+575\,c^3\left(\frac{ax-1}{ax+1}\right)\cdots}{\frac{4\,(ax-1)a^2}{ax+1}+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
```

```
(135*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 575*c^3*((a*x - 1)/(a*x + 1))^(9/2)
 + 842*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 298*c^3*((a*x - 1)/(a*x + 1))^(5/2
) - 465*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^
4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x +
1)^6 + a^2))*a
```

mupad [B] time = 1.41, size = 258, normalized size = 0.96

$$\frac{\frac{149\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{10}-\frac{93\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{4}-\frac{21\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{421\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{10}+\frac{115\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}+\frac{27\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{4}}{a+\frac{4\,a\,(a\,x-1)}{a\,x+1}+\frac{5\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{a\,(a\,x-1)^6}{(a\,x+1)^6}}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((149*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/4 - (21*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (421*c^3*((a*x - 1)/(a*x
 + 1))^(7/2))/10 + (115*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 + (27*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2
)))/(4*a) + (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{3a^2}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^4}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\right.}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] c**3*(Integral(3*a**2/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4/(
a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**6/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
Integral(-1/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**6*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**6
```

3.791 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$c^2x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}-\frac{c^2\csc^{-1}(ax)}{2a}-$$

```
[Out] -1/2*c^2*arccsc(a*x)/a+3*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-11/
6*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-4/3*c^2*(1+1/a/x)^(5/2)*(1-1/a/x)^(
1/2)/a+c^2*(1+1/a/x)^(7/2)*x*(1-1/a/x)^(1/2)-5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a
/x)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^2x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}-\frac{c^2\csc^{-1}(ax)}{2a}-$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (11*c^2*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2))/(6*a) - (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(3*a) + c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x - (c^2*ArcCsc[a*x])
/(2*a) + (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2dx = -\left(c^2\,\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - c^2\,\text{Subst}\left(\int\frac{\left(\frac{3}{a}-\frac{4x}{a^2}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$= -\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x + \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int\frac{\left(-\frac{9}{a^2}+\right.}{}\right.$$

$$= -\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2$$

Mathematica [A] time = 0.15, size = 94, normalized size = 0.48

$$\frac{c^2\left(18a^2x^2\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-3a^2x^2\sin^{-1}\left(\frac{1}{ax}\right)+\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3-16a^2x^2-9ax-2\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 - 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x
^2*ArcSin[1/(a*x)] + 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3
*x^2)
```

fricas [A] time = 0.59, size = 156, normalized size = 0.80

$$\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^2x^4-10\,a^3c^2x^3-25\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 18*a^3*c^2*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
```

```
) - 1) + (6*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 25*a^2*c^2*x^2 - 11*a*c^2*x - 2*
c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

giac [A] time = 0.18, size = 212, normalized size = 1.09

$$\frac{1}{3}ac^2\left(\frac{3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{9\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{6\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}-\frac{\frac{28(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1}+\frac{9(ax-}{(}}{a^2\left(\frac{ax-}{ax+}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*a*c^2*(3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 9*log(sqrt((a*x - 1)/(
a*x + 1)) + 1)/a^2 - 9*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 - 6*sqrt
((a*x - 1)/(a*x + 1))/(a^2*((a*x - 1)/(a*x + 1) - 1)) - (28*(a*x - 1)*sqrt(
(a*x - 1)/(a*x + 1))/(a*x + 1) + 9*(a*x - 1)^2*sqrt((a*x - 1)/(a*x + 1))/(a
*x + 1)^2 + 27*sqrt((a*x - 1)/(a*x + 1)))/(a^2*((a*x - 1)/(a*x + 1) + 1)^3)
)
```

maple [A] time = 0.06, size = 233, normalized size = 1.19

$$\frac{(ax-1)^2c^2\left(-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+18\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-3a^3x^3\sqrt{a^2}\,\arctan\left(\right.\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x)
```

```
[Out] 1/6*(a*x-1)^2*c^2*(-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^4*a^4+18*(a^2*x^2-1)
^(3/2)*(a^2)^(1/2)*x^2*a^2-3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^3*a^3-3*a^3*x^
3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a
^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+9*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+2*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1)*(a*x+1))
^(1/2)/a^4/x^3/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 223, normalized size = 1.14

$$\frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{15c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+37c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+1}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 - 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (15
*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 37*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 17*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 21*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
 - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))
```

mupad [B] time = 0.14, size = 183, normalized size = 0.94

$$\frac{\frac{17c^2\left(\frac{ax-1}{ax+1}\right)^{3/2}}{3}-7c^2\sqrt{\frac{ax-1}{ax+1}}+\frac{37c^2\left(\frac{ax-1}{ax+1}\right)^{5/2}}{3}+5c^2\left(\frac{ax-1}{ax+1}\right)^{7/2}}{a+\frac{2a(ax-1)}{ax+1}-\frac{2a(ax-1)^3}{(ax+1)^3}-\frac{a(ax-1)^4}{(ax+1)^4}}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{6c^2\,\mathrm{atanh}\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((17*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(1/2)
 + (37*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6*c^2
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{2a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4/(a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**4
```

3.792 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=76

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

```
[Out] -3*c*arccsc(a*x)/a+3*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1+1/a/
x)^(3/2)*x*(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6194, 98, 12, 105, 41, 216, 92, 208}

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x - (3*c*ArcCsc[a*x])/a + (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
```

```
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\left(c\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^2\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x + c\,\text{Subst}\left(\int -\frac{3\sqrt{1+\frac{x}{a}}}{ax\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{(3c)\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{(3c)\,\text{Subst}\left(\int \frac{}{x\sqrt{1}} \right.}{a}$$

$$= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, \right.}{a^2}$$

$$= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{3c\csc^{-1}(ax)}{a} + \frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] time = 0.10, size = 57, normalized size = 0.75

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}\,(ax+1)+3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-3\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) - 3*ArcSin[1/(a*x)] + 3*Log[(1 + Sqrt[1
 - 1/(a^2*x^2)])*x]))/a
```

fricas [A] time = 0.48, size = 106, normalized size = 1.39

$$\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2+2\,acx+c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 3*a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 + 2*
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

giac [A] time = 0.15, size = 111, normalized size = 1.46

$$ac\left(\frac{6\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2}-\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a^2\left(\frac{(ax-1)^2}{(ax+1)^2}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] a*c*(6*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*log(sqrt((a*x - 1)/(a*x +
1)) + 1)/a^2 - 3*log(abs(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2 - 4*sqrt((a*x
- 1)/(a*x + 1))/(a^2*((a*x - 1)^2/(a*x + 1)^2 - 1)))
```

maple [B] time = 0.06, size = 235, normalized size = 3.09

$$-\frac{(ax-1)^2c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa+3ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\ln\cdots\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax-1)(ax+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x)
```

```
[Out] -(a*x-1)^2*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a+3*a*x*(a^2)^(1/2)*arctan(1/(a^2
*x^2-1)^(1/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-
4*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-4*ln((a^2*x+((a*x-1)*(a*x+1))^(1/
2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x-1
)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 118, normalized size = 1.55

$$-a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 0.09, size = 84, normalized size = 1.11

$$\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int \frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int \left(-\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**3*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a*x + 1)), x))/a**2
```

3.793 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=144

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{\frac{1}{ax}+1}}{3ac\sqrt{1-\frac{1}{ax}}}-\frac{5\sqrt{\frac{1}{ax}+1}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c-5/3*(1+1/a/x)^(1/2)/a/c/(1-1
/a/x)^(3/2)+x*(1+1/a/x)^(1/2)/c/(1-1/a/x)^(3/2)-14/3*(1+1/a/x)^(1/2)/a/c/(1
-1/a/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 99, 152, 12, 92, 208}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{\frac{1}{ax}+1}}{3ac\sqrt{1-\frac{1}{ax}}}-\frac{5\sqrt{\frac{1}{ax}+1}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (-5*Sqrt[1 + 1/(a*x)])/(3*a*c*(1 - 1/(a*x))^(3/2)) - (14*Sqrt[1 + 1/(a*x)])
/(3*a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*(1 - 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{\text{Subst}\left(\int \frac{\frac{3}{a}+\frac{2x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{a\,\text{Subst}\left(\int \frac{-\frac{9}{a^2}-\frac{5x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{3c}\\
&= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{a^2\,\text{Subst}\left(\int \frac{9}{a^3x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{3c}\\
&= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{ac}\\
&= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}\\
&= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 69, normalized size = 0.48

$$\frac{\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(3a^2x^2-19ax+14\right)}{(ax-1)^2}+\frac{9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(14 - 19*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + (9*Log[
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

fricas [A] time = 0.55, size = 128, normalized size = 0.89

$$\frac{9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\left(a^2x^2-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(3\,a^3x^3-16\,a^2x^2-5\,ax+14\right)\sqrt{}}{3\left(a^3cx^2-2\,a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] 1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x^2
 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

giac [A] time = 0.15, size = 148, normalized size = 1.03

$$\frac{1}{3}\,a\left(\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c}-\frac{(ax+1)\left(\frac{12\,(ax-1)}{ax+1}+1\right)}{(ax-1)a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{6\sqrt{\frac{ax-1}{ax+1}}}{a^2c\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] 1/3*a*(9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(abs(sqrt((a*x -
 1)/(a*x + 1)) - 1))/(a^2*c) - (a*x + 1)*(12*(a*x - 1)/(a*x + 1) + 1)/((a*x
 - 1)*a^2*c*sqrt((a*x - 1)/(a*x + 1))) - 6*sqrt((a*x - 1)/(a*x + 1))/(a^2*c
*((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.06, size = 346, normalized size = 2.40

$$9\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^3a^4+9\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^3a^3-27\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3-6\sqrt{a^2}\,\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x)
```

```
[Out] 1/3*(9*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+
9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-27*ln((a^2*x+((a*x-1)*(a*x+1)
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
3/2)*x*a-27*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+27*ln((a^2*x+((a*x-
1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+5*((a*x-1)*(a*x+1))^(3/2)
*(a^2)^(1/2)+27*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-9*a*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-9*((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2))/a/(a*x-1)/(a^2)^(1/2)/c/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(
a*x+1))^(3/2)
```

maxima [A] time = 0.32, size = 133, normalized size = 0.92

$$\frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a
*x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a
^2*c))
```

mupad [B] time = 1.35, size = 100, normalized size = 0.69

$$\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}-\frac{\frac{11\,\left(a\,x-1\right)}{3\,\left(a\,x+1\right)}-\frac{6\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{1}{3}}{a\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-a\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((11*(a*x - 1))/(3*(a*x + 1)
) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) - a
*c*((a*x - 1)/(a*x + 1))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{2}\int \frac{x^{2}}{\frac{a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x)/c
```

3.794 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=181

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}}{5ac^2\sqrt{1-\frac{1}{ax}}}-\frac{9\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{6\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-6/5*(1+1/a/x)^(1/2)/a/c^2/
(1-1/a/x)^(5/2)-9/5*(1+1/a/x)^(1/2)/a/c^2/(1-1/a/x)^(3/2)+x*(1+1/a/x)^(1/2)
/c^2/(1-1/a/x)^(5/2)-24/5*(1+1/a/x)^(1/2)/a/c^2/(1-1/a/x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 103, 21, 99, 152, 12, 92, 208}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}}{5ac^2\sqrt{1-\frac{1}{ax}}}-\frac{9\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{6\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-6*Sqrt[1 + 1/(a*x)])/(5*a*c^2*(1 - 1/(a*x))^(5/2)) - (9*Sqrt[1 + 1/(a*x)]
)/(5*a*c^2*(1 - 1/(a*x))^(3/2)) - (24*Sqrt[1 + 1/(a*x)])/(5*a*c^2*Sqrt[1 -
1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c^2*(1 - 1/(a*x))^(5/2)) + (3*ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{6\,\text{Subst}\left(\int \frac{-\frac{5}{2}-\frac{2x}{a}}{x\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{2\,\text{Subst}\left(\int \frac{\frac{15}{2a}+\frac{9x}{2a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5c^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{(2a)\,\text{Subst}\left(\int -\frac{}{2a^2}\right.}{5}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}}\right.}{ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,d\right.}{}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\right.}{ac^2}$$

Mathematica [A] time = 0.20, size = 78, normalized size = 0.43

$$\frac{3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(5a^3x^3-39a^2x^2+57ax-24\right)}{5(ax-1)^3}}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-24 + 57*a*x - 39*a^2*x^2 + 5*a^3*x^3))/(5*(-1
 + a*x)^3) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

fricas [A] time = 0.71, size = 170, normalized size = 0.94

$$\frac{15\left(a^3x^3-3a^2x^2+3ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^3x^3-3a^2x^2+3ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(5a^4x^4-34a^3x\right.}{5\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
 + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

giac [A] time = 0.16, size = 166, normalized size = 0.92

$$\frac{1}{20}a\left(\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^2}-\frac{(ax+1)^2\left(\frac{10(ax-1)}{ax+1}+\frac{85(ax-1)^2}{(ax+1)^2}+1\right)}{(ax-1)^2a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{40\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/20*a*(60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 60*log(abs(sqrt((
a*x - 1)/(a*x + 1)) - 1))/(a^2*c^2) - (a*x + 1)^2*(10*(a*x - 1)/(a*x + 1) +
 85*(a*x - 1)^2/(a*x + 1)^2 + 1)/((a*x - 1)^2*a^2*c^2*sqrt((a*x - 1)/(a*x +
 1))) - 40*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*((a*x - 1)/(a*x + 1) - 1)))
```

maple [B] time = 0.06, size = 438, normalized size = 2.42

$$-\frac{-125\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^4a^4-120\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^4a^5+85\sqrt{a^2}\,\left((ax-1)(ax+1)\right)^{\frac{3}{2}}x^2a^2+500}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/40*(-125*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-120*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+85*(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(3/2)*x^2*a^2+500*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+48
0*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-148*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-750*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1
/2)*x^2*a^2-720*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*x^2*a^3+67*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+500*((a*x-1)*(a*x+1))^(1/2)
*(a^2)^(1/2)*x*a+480*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*x*a^2-125*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-120*a*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)^2/c^2/((a
*x-1)*(a*x+1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.32, size = 153, normalized size = 0.85

$$\frac{1}{20}a\left(\frac{\frac{9(ax-1)}{ax+1}+\frac{75(ax-1)^2}{(ax+1)^2}-\frac{125(ax-1)^3}{(ax+1)^3}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1)
^3/(a*x + 1)^3 + 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x -
1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 60
*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

mupad [B] time = 0.09, size = 121, normalized size = 0.67

$$\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}-\frac{\frac{15\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{25\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{9\,\left(a\,x-1\right)}{5\,\left(a\,x+1\right)}+\frac{1}{5}}{4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((15*(a*x - 1)^2)/(a*x + 1
)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(4*
a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^2*((a*x - 1)/(a*x + 1))^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

3.795 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=255

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\;}}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-8/7/a/c^3/(1-1/a/x)^(7/2)/
(1+1/a/x)^(1/2)-53/35/a/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)-88/35/a/c^3/(1-
1/a/x)^(3/2)/(1+1/a/x)^(1/2)+x/c^3/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)-281/35/a
/c^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+176/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)
^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\;}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -8/(7*a*c^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - 53/(35*a*c^3*(1 - 1/(a
*x))^(5/2)*Sqrt[1 + 1/(a*x)]) - 88/(35*a*c^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1
/(a*x)]) - 281/(35*a*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (176*Sqrt[1
 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(7/2)*Sqrt
[1 + 1/(a*x)]) + (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{21}{a^2}+\frac{32x}{a^3}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{7c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{a^2\,\text{Subst}}{}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{}{c^3\left(1\right.}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{}{35ac^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{}{35ac^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{}{35ac^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{}{35ac^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{}{35ac^3}$$

Mathematica [A] time = 0.24, size = 101, normalized size = 0.40

$$\frac{3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right) + \frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(35a^5x^5-286a^4x^4+368a^3x^3+125a^2x^2-423ax+176\right)}{35(ax-1)^4(ax+1)}}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(176 - 423*a*x + 125*a^2*x^2 + 368*a^3*x^3 - 28
6*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)^4*(1 + a*x)) + 3*Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x])/(a*c^3)
```

fricas [A] time = 0.55, size = 204, normalized size = 0.80

$$\frac{105\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}\right.}{35\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - 105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) + (35*a^5*x^5 - 286*a^4*x^4 + 368*a^3*x^3 + 125
*a^2*x^2 - 423*a*x + 176)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c
^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.17, size = 205, normalized size = 0.80

$$\frac{1}{560}\,a\left(\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\,\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^3}-\frac{(ax+1)^3\left(\frac{56\,(ax-1)}{ax+1}+\frac{350\,(ax-1)^2}{(ax+1)^2}+\frac{2520\,(ax-1)^3}{(ax+1)^3}+5\right)}{(ax-1)^3a^2c^3\sqrt{\frac{ax-1}{ax+1}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] 1/560*a*(1680*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 1680*log(abs(s
qrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^3) - (a*x + 1)^3*(56*(a*x - 1)/(a*x +
 1) + 350*(a*x - 1)^2/(a*x + 1)^2 + 2520*(a*x - 1)^3/(a*x + 1)^3 + 5)/((a*x
 - 1)^3*a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) + 35*sqrt((a*x - 1)/(a*x + 1))/(
a^2*c^3) - 1120*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*((a*x - 1)/(a*x + 1) - 1
)))
```

maple [B] time = 0.07, size = 714, normalized size = 2.80

$$-\frac{-3675\sqrt{(ax-1)\,(ax+1)}\;\sqrt{a^2}\;x^7a^7-3360\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\;\sqrt{a^2}}{\sqrt{a^2}}\right)x^7a^8+2555\left((ax-1)\,(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\;x^5a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -1/1120*(-3675*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^7*a^7-3360*ln((a^2*x+(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8+2555*((a*x-1)*(a*x
+1))^(3/2)*(a^2)^(1/2)*x^5*a^5+11025*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^
6*a^6+10080*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6
*a^7-1873*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^4*a^4-3675*(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2)*x^5*a^5-3360*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1
/2))/(a^2)^(1/2))*x^5*a^6-4426*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3-
18375*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-16800*ln((a^2*x+((a*x-1)*
(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+3350*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^2*a^2+18375*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+16
```

```
800*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+251
1*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+3675*((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2)*x^2*a^2+3360*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*x^2*a^3-1957*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-11025*((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2)*x*a-10080*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/
2))/(a^2)^(1/2))*x*a^2+3675*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+3360*a*ln((
a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x
-1)^3/c^3/((a*x-1)*(a*x+1))^(1/2)/(a*x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.33, size = 192, normalized size = 0.75

$$\frac{1}{560}a\left(\frac{\frac{51\,(ax-1)}{ax+1}+\frac{294\,(ax-1)^2}{(ax+1)^2}+\frac{2170\,(ax-1)^3}{(ax+1)^3}-\frac{3640\,(ax-1)^4}{(ax+1)^4}+5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{1680\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}+\frac{3}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/560*a*((51*(a*x - 1)/(a*x + 1) + 294*(a*x - 1)^2/(a*x + 1)^2 + 2170*(a*x
- 1)^3/(a*x + 1)^3 - 3640*(a*x - 1)^4/(a*x + 1)^4 + 5)/(a^2*c^3*((a*x - 1)/
(a*x + 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 1680*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c^3) + 35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

mupad [B] time = 1.41, size = 160, normalized size = 0.63

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^3}-\frac{\frac{42\,(a\,x-1)^2}{5\,(a\,x+1)^2}+\frac{62\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{104\,(a\,x-1)^4}{(a\,x+1)^4}+\frac{51\,(a\,x-1)}{35\,(a\,x+1)}+\frac{1}{7}}{16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}-16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}+\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(16*a*c^3) - ((42*(a*x - 1)^2)/(5*(a*x + 1)^2)
+ (62*(a*x - 1)^3)/(a*x + 1)^3 - (104*(a*x - 1)^4)/(a*x + 1)^4 + (51*(a*x -
 1))/(35*(a*x + 1)) + 1/7)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 16*a*c^3
*((a*x - 1)/(a*x + 1))^(9/2)) + (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c
^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\int\frac{x^6}{\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\cdots}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1) - 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a
*x + 1)), x)/c**3
```

3.796 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=329

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/}}$$

```
[Out] -10/9/a/c^4/(1-1/a/x)^(9/2)/(1+1/a/x)^(3/2)-29/21/a/c^4/(1-1/a/x)^(7/2)/(1+
1/a/x)^(3/2)-208/105/a/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)-1147/315/a/c^4/(
1-1/a/x)^(3/2)/(1+1/a/x)^(3/2)+x/c^4/(1-1/a/x)^(9/2)/(1+1/a/x)^(3/2)+3*arct
anh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-1462/105/a/c^4/(1+1/a/x)^(3/2)/(
1-1/a/x)^(1/2)+2609/315*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+1664/315*(1-1
/a/x)^(1/2)/a/c^4/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] -10/(9*a*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2)) - 29/(21*a*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)) - 208/(105*a*c^4*(1 - 1/(a*x))^(5/2)*(1
+ 1/(a*x))^(3/2)) - 1147/(315*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)
) - 1462/(105*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (2609*Sqrt[1 -
 1/(a*x)])/(315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*
a*c^4*Sqrt[1 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{\frac{27}{a^2}+\frac{60x}{a^3}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x\cdots\right.}{9c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \cdots$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3\cdots}}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3\cdots}}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3\cdots}}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3\cdots}}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3\cdots}}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3\cdots}}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3\cdots}}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3\cdots}}$$

Mathematica [A] time = 0.30, size = 117, normalized size = 0.36

$$\frac{3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(315a^7x^7-2669a^6x^6+2967a^5x^5+4029a^4x^4-7399a^3x^3+339a^2x^2+4047ax-1664\right)}{315(ax-1)^5(ax+1)^2}}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1664 + 4047*a*x + 339*a^2*x^2 - 7399*a^3*x^3
+ 4029*a^4*x^4 + 2967*a^5*x^5 - 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*x
)^5*(1 + a*x)^2) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

fricas [A] time = 0.54, size = 248, normalized size = 0.75

$$\frac{945\left(a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-\right.}{315\left(a^7c^4x^6-4a^6c^4x^5+5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(sq
rt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2
*x^2 + 4*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^7*x^7 - 2669*
a^6*x^6 + 2967*a^5*x^5 + 4029*a^4*x^4 - 7399*a^3*x^3 + 339*a^2*x^2 + 4047*a
*x - 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*
c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.17, size = 264, normalized size = 0.80

$$\frac{1}{20160}a\left(\frac{60480\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60480\log\left(\left|\sqrt{\frac{ax-1}{ax+1}}-1\right|\right)}{a^2c^4}-\frac{(ax+1)^4\left(\frac{450(ax-1)}{ax+1}+\frac{2961(ax-1)^2}{(ax+1)^2}+\frac{14700(ax-1)^3}{(ax+1)^3}+\right.}{(ax-1)^4a^2c^4\sqrt{\frac{ax-1}{ax+1}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/20160*a*(60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 60480*log(a
bs(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a^2*c^4) - (a*x + 1)^4*(450*(a*x - 1)/(
a*x + 1) + 2961*(a*x - 1)^2/(a*x + 1)^2 + 14700*(a*x - 1)^3/(a*x + 1)^3 + 9
5445*(a*x - 1)^4/(a*x + 1)^4 + 35)/((a*x - 1)^4*a^2*c^4*sqrt((a*x - 1)/(a*x
 + 1))) - 40320*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4*((a*x - 1)/(a*x + 1) - 1
)) + 105*((a*x - 1)*a^4*c^8*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1) + 30*a^4*c^
8*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^12))
```

maple [B] time = 0.08, size = 766, normalized size = 2.33

$$-\frac{-138915\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^9a^9-120960\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^9a^{10}+98595\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/40320*(-138915*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^9*a^9-120960*ln((a^
2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^9*a^10+98595*((a*x-
1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^7*a^7+416745*((a*x-1)*(a*x+1))^(1/2)*(a^2)^
(1/2)*x^8*a^8+362880*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*x^8*a^9-75113*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^6*a^6-240861*((a*
x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^5*a^5-1111320*((a*x-1)*(a*x+1))^(1/2)*(a^
2)^(1/2)*x^6*a^6-967680*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2
)^(1/2))*x^6*a^7+178863*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^4*a^4+833490*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5+725760*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+252497*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x^3*a^3+833490*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+7257
60*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-1823
07*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-1111320*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2)*x^3*a^3-967680*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*x^3*a^4-101271*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+7407
7*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+416745*((a*x-1)*(a*x+1))^(1/2)*(a^2)^
(1/2)*x*a+362880*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)
)*x*a^2-138915*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-120960*a*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)^4/c^4/(
(a*x-1)*(a*x+1))^(1/2)/(a*x+1)^4/((a*x-1)/(a*x+1))^(3/2)
```

maxima [A] time = 0.32, size = 226, normalized size = 0.69

$$\frac{1}{20160}a\left(\frac{\frac{415\,(ax-1)}{ax+1}+\frac{2511\,(ax-1)^2}{(ax+1)^2}+\frac{11739\,(ax-1)^3}{(ax+1)^3}+\frac{80745\,(ax-1)^4}{(ax+1)^4}-\frac{135765\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{105\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+30\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/20160*a*((415*(a*x - 1)/(a*x + 1) + 2511*(a*x - 1)^2/(a*x + 1)^2 + 11739*
(a*x - 1)^3/(a*x + 1)^3 + 80745*(a*x - 1)^4/(a*x + 1)^4 - 135765*(a*x - 1)^
5/(a*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x -
 1)/(a*x + 1))^(9/2)) + 105*(((a*x - 1)/(a*x + 1))^(3/2) + 30*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4
) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

mupad [B] time = 0.11, size = 203, normalized size = 0.62

$$\frac{5\sqrt{\frac{a\,x-1}{a\,x+1}}}{32\,a\,c^4}-\frac{\frac{279\,(a\,x-1)^2}{35\,(a\,x+1)^2}+\frac{559\,(a\,x-1)^3}{15\,(a\,x+1)^3}+\frac{769\,(a\,x-1)^4}{3\,(a\,x+1)^4}-\frac{431\,(a\,x-1)^5}{(a\,x+1)^5}+\frac{83\,(a\,x-1)}{63\,(a\,x+1)}+\frac{1}{9}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}-64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{192\,a\,c^4}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (5*((a*x - 1)/(a*x + 1))^(1/2))/(32*a*c^4) - ((279*(a*x - 1)^2)/(35*(a*x +
1)^2) + (559*(a*x - 1)^3)/(15*(a*x + 1)^3) + (769*(a*x - 1)^4)/(3*(a*x + 1)
^4) - (431*(a*x - 1)^5)/(a*x + 1)^5 + (83*(a*x - 1))/(63*(a*x + 1)) + 1/9)/
(64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(11/
2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(192*a*c^4) - (atan(((a*x - 1)/(a*x + 1))
^(1/2)*1i)*6i)/(a*c^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

3.797 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx$

Optimal. Leaf size=116

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

```
[Out] 1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/a
^6/x^5+14/3*c^5/a^4/x^3+4*c^5/a^3/x^2-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

Rubi [A] time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{4c^5}{a^3x^2}+\frac{14c^5}{3a^4x^3}-\frac{14c^5}{5a^6x^5}-\frac{4c^5}{3a^7x^6}+\frac{3c^5}{7a^8x^7}+\frac{c^5}{2a^9x^8}+\frac{c^5}{9a^{10}x^9}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx = \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5 dx$$
$$= -\frac{c^5\int \frac{e^{4\tanh^{-1}(ax)}(1-a^2x^2)^5}{x^{10}}dx}{a^{10}}$$
$$= -\frac{c^5\int \frac{(1-ax)^3(1+ax)^7}{x^{10}}dx}{a^{10}}$$
$$= -\frac{c^5\int \left(-a^{10}+\frac{1}{x^{10}}+\frac{4a}{x^9}+\frac{3a^2}{x^8}-\frac{8a^3}{x^7}-\frac{14a^4}{x^6}+\frac{14a^6}{x^4}+\frac{8a^7}{x^3}-\frac{3a^8}{x^2}-\frac{4a^9}{x}\right)dx}{a^{10}}$$
$$= \frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5 \text{lo}}{a}$$

Mathematica [A] time = 0.05, size = 116, normalized size = 1.00

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

fricas [A] time = 0.44, size = 111, normalized size = 0.96

$$\frac{630\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log(x)-1890\,a^8c^5x^8+2520\,a^7c^5x^7+2940\,a^6c^5x^6-1764\,a^4c^5x^4-840\,a^3c^5x^3+270\,}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/630*(630*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) - 1890*a^8*c^5*x^8 + 252
0*a^7*c^5*x^7 + 2940*a^6*c^5*x^6 - 1764*a^4*c^5*x^4 - 840*a^3*c^5*x^3 + 270
*a^2*c^5*x^2 + 315*a*c^5*x + 70*c^5)/(a^10*x^9)
```

giac [A] time = 0.15, size = 184, normalized size = 1.59

$$-\frac{4\,c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^5\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(630\,c^5+\frac{4049\,c^5}{ax-1}+\frac{6201\,c^5}{(ax-1)^2}-\frac{18036\,c^5}{(ax-1)^3}-\frac{89124\,c^5}{(ax-1)^4}-\frac{160146\,c^5}{(ax-1)^5}-\frac{153090\,c^5}{(ax-1)^6}-\right.}{630\,a\left(\frac{1}{ax-1}+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] -4*c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^5*log(abs(-1/(a*x - 1
) - 1))/a + 1/630*(630*c^5 + 4049*c^5/(a*x - 1) + 6201*c^5/(a*x - 1)^2 - 18
036*c^5/(a*x - 1)^3 - 89124*c^5/(a*x - 1)^4 - 160146*c^5/(a*x - 1)^5 - 1530
90*c^5/(a*x - 1)^6 - 80220*c^5/(a*x - 1)^7 - 21420*c^5/(a*x - 1)^8 - 2520*c
^5/(a*x - 1)^9)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^9)
```

maple [A] time = 0.04, size = 105, normalized size = 0.91

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{x^2a^3}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x)
```

```
[Out] 1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/a
^6/x^5+14/3*c^5/a^4/x^3+4*c^5/x^2/a^3-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

maxima [A] time = 0.30, size = 103, normalized size = 0.89

$$c^5x+\frac{4\,c^5\log(x)}{a}-\frac{1890\,a^8c^5x^8-2520\,a^7c^5x^7-2940\,a^6c^5x^6+1764\,a^4c^5x^4+840\,a^3c^5x^3-270\,a^2c^5x^2-315\,ac^5}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 4*c^5*log(x)/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 - 2940*
a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 315*a*
c^5*x - 70*c^5)/(a^10*x^9)
```

mupad [B] time = 0.09, size = 81, normalized size = 0.70

$$\frac{c^5\,\left(\frac{a\,x}{2}+\frac{3\,a^2\,x^2}{7}-\frac{4\,a^3\,x^3}{3}-\frac{14\,a^4\,x^4}{5}+\frac{14\,a^6\,x^6}{3}+4\,a^7\,x^7-3\,a^8\,x^8+a^{10}\,x^{10}+4\,a^9\,x^9\,\ln(x)+\frac{1}{9}\right)}{a^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^5*((a*x)/2 + (3*a^2*x^2)/7 - (4*a^3*x^3)/3 - (14*a^4*x^4)/5 + (14*a^6*x^
6)/3 + 4*a^7*x^7 - 3*a^8*x^8 + a^10*x^10 + 4*a^9*x^9*log(x) + 1/9))/(a^10*x
^9)
```

sympy [A] time = 0.62, size = 112, normalized size = 0.97

$$\frac{a^{10}c^5x+4a^9c^5\log{(x)}+\frac{-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315ac^5x+70c^5}{630x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 4*a**9*c**5*log(x) + (-1890*a**8*c**5*x**8 + 2520*a**7*c**5
*x**7 + 2940*a**6*c**5*x**6 - 1764*a**4*c**5*x**4 - 840*a**3*c**5*x**3 + 27
0*a**2*c**5*x**2 + 315*a*c**5*x + 70*c**5)/(630*x**9))/a**10
```

3.798 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=100

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

```
[Out] -1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/x
^3+2*c^4/a^3/x^2-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

Rubi [A] time = 0.17, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{2c^4}{a^3x^2}+\frac{10c^4}{3a^4x^3}+\frac{c^4}{a^5x^4}-\frac{4c^4}{5a^6x^5}-\frac{2c^4}{3a^7x^6}-\frac{c^4}{7a^8x^7}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -c^4/(7*a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x^4
) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x + (4
*c^4*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx=\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4dx$$
$$=\frac{c^4\int\frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}dx}{a^8}$$
$$=\frac{c^4\int\frac{(1-ax)^2(1+ax)^6}{x^8}dx}{a^8}$$
$$=\frac{c^4\int\left(a^8+\frac{1}{x^8}+\frac{4a}{x^7}+\frac{4a^2}{x^6}-\frac{4a^3}{x^5}-\frac{10a^4}{x^4}-\frac{4a^5}{x^3}+\frac{4a^6}{x^2}+\frac{4a^7}{x}\right)dx}{a^8}$$
$$=-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

Mathematica [A] time = 0.04, size = 100, normalized size = 1.00

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -1/7*c^4/(a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x
^4) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x +
(4*c^4*Log[x])/a
```

fricas [A] time = 0.55, size = 100, normalized size = 1.00

$$\frac{105\,a^8c^4x^8+420\,a^7c^4x^7\log(x)-420\,a^6c^4x^6+210\,a^5c^4x^5+350\,a^4c^4x^4+105\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x-1}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) - 420*a^6*c^4*x^6 + 210*a^5
*c^4*x^5 + 350*a^4*c^4*x^4 + 105*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x
- 15*c^4)/(a^8*x^7)
```

giac [A] time = 0.14, size = 160, normalized size = 1.60

$$-\frac{4\,c^4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^4\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(105\,c^4+\frac{659\,c^4}{ax-1}+\frac{1253\,c^4}{(ax-1)^2}-\frac{231\,c^4}{(ax-1)^3}-\frac{3885\,c^4}{(ax-1)^4}-\frac{5250\,c^4}{(ax-1)^5}-\frac{2730\,c^4}{(ax-1)^6}-\frac{42}{(a}\right.}{105\,a\left(\frac{1}{ax-1}+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -4*c^4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^4*log(abs(-1/(a*x - 1
) - 1))/a + 1/105*(105*c^4 + 659*c^4/(a*x - 1) + 1253*c^4/(a*x - 1)^2 - 231
*c^4/(a*x - 1)^3 - 3885*c^4/(a*x - 1)^4 - 5250*c^4/(a*x - 1)^5 - 2730*c^4/(
a*x - 1)^6 - 420*c^4/(a*x - 1)^7)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^7)
```

maple [A] time = 0.04, size = 93, normalized size = 0.93

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{x^2a^3}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/x
^3+2*c^4/x^2/a^3-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

maxima [A] time = 0.32, size = 92, normalized size = 0.92

$$c^4x+\frac{4\,c^4\log(x)}{a}-\frac{420\,a^6c^4x^6-210\,a^5c^4x^5-350\,a^4c^4x^4-105\,a^3c^4x^3+84\,a^2c^4x^2+70\,ac^4x+15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 4*c^4*log(x)/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 350*a^4
*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^8*x^7
)
```

mupad [B] time = 1.29, size = 72, normalized size = 0.72

$$\frac{c^4\left(a^3\,x^3-\frac{4\,a^2\,x^2}{5}-\frac{2\,a\,x}{3}+\frac{10\,a^4\,x^4}{3}+2\,a^5\,x^5-4\,a^6\,x^6+a^8\,x^8+4\,a^7\,x^7\,\ln(x)-\frac{1}{7}\right)}{a^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^4*(a^3*x^3 - (4*a^2*x^2)/5 - (2*a*x)/3 + (10*a^4*x^4)/3 + 2*a^5*x^5 - 4*
a^6*x^6 + a^8*x^8 + 4*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

sympy [A] time = 0.48, size = 100, normalized size = 1.00

$$\frac{a^8c^4x+4a^7c^4\log{(x)}+\frac{-420a^6c^4x^6+210a^5c^4x^5+350a^4c^4x^4+105a^3c^4x^3-84a^2c^4x^2-70ac^4x-15c^4}{105x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 4*a**7*c**4*log(x) + (-420*a**6*c**4*x**6 + 210*a**5*c**4*x*
*5 + 350*a**4*c**4*x**4 + 105*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**
4*x - 15*c**4)/(105*x**7))/a**8
```

3.799 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=63

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

```
[Out] 1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/a
```

Rubi [A] time = 0.15, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 75}

$$\frac{5c^3}{3a^4x^3}+\frac{c^3}{a^5x^4}+\frac{c^3}{5a^6x^5}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$
$$= -\frac{c^3\int \frac{e^{4\tanh^{-1}(ax)}(1-a^2x^2)^3}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \frac{(1-ax)(1+ax)^5}{x^6}dx}{a^6}$$
$$= -\frac{c^3\int \left(-a^6+\frac{1}{x^6}+\frac{4a}{x^5}+\frac{5a^2}{x^4}-\frac{5a^4}{x^2}-\frac{4a^5}{x}\right)dx}{a^6}$$
$$= \frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

fricas [A] time = 0.47, size = 67, normalized size = 1.06

$$\frac{15\,a^6c^3x^6+60\,a^5c^3x^5\log(x)-75\,a^4c^3x^4+25\,a^2c^3x^2+15\,ac^3x+3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) - 75*a^4*c^3*x^4 + 25*a^2*c^3*
x^2 + 15*a*c^3*x + 3*c^3)/(a^6*x^5)
```

giac [B] time = 0.13, size = 136, normalized size = 2.16

$$-\frac{4\,c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(15\,c^3+\frac{107\,c^3}{ax-1}+\frac{235\,c^3}{(ax-1)^2}+\frac{170\,c^3}{(ax-1)^3}-\frac{30\,c^3}{(ax-1)^4}-\frac{60\,c^3}{(ax-1)^5}\right)(ax-1)}{15\,a\left(\frac{1}{ax-1}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -4*c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^3*log(abs(-1/(a*x - 1
) - 1))/a + 1/15*(15*c^3 + 107*c^3/(a*x - 1) + 235*c^3/(a*x - 1)^2 + 170*c^
3/(a*x - 1)^3 - 30*c^3/(a*x - 1)^4 - 60*c^3/(a*x - 1)^5)*(a*x - 1)/(a*(1/(a
*x - 1) + 1)^5)
```

maple [A] time = 0.04, size = 60, normalized size = 0.95

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x)
```

```
[Out] 1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/a
```

maxima [A] time = 0.31, size = 59, normalized size = 0.94

$$c^3x + \frac{4\,c^3\log(x)}{a} - \frac{75\,a^4c^3x^4 - 25\,a^2c^3x^2 - 15\,ac^3x - 3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 4*c^3*log(x)/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*c^3*x
 - 3*c^3)/(a^6*x^5)
```

mupad [B] time = 0.06, size = 48, normalized size = 0.76

$$\frac{c^3\,\left(a\,x + \frac{5\,a^2\,x^2}{3} - 5\,a^4\,x^4 + a^6\,x^6 + 4\,a^5\,x^5\,\ln(x) + \frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^3*(a*x + (5*a^2*x^2)/3 - 5*a^4*x^4 + a^6*x^6 + 4*a^5*x^5*log(x) + 1/5))/
(a^6*x^5)
```

sympy [A] time = 0.29, size = 65, normalized size = 1.03

$$\frac{a^6c^3x + 4a^5c^3\log{(x)} + \frac{-75a^4c^3x^4+25a^2c^3x^2+15ac^3x+3c^3}{15x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 4*a**5*c**3*log(x) + (-75*a**4*c**3*x**4 + 25*a**2*c**3*x**2
 + 15*a*c**3*x + 3*c**3)/(15*x**5))/a**6
```

3.800 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

```
[Out] -1/3*c^2/a^4/x^3-2*c^2/a^3/x^2-6*c^2/a^2/x+c^2*x+4*c^2*ln(x)/a
```

Rubi [A] time = 0.15, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6167, 6157, 6150, 43}

$$-\frac{2c^2}{a^3x^2}-\frac{c^2}{3a^4x^3}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -c^2/(3*a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*Log
[x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx \\
&= \frac{c^2\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4} \\
&= \frac{c^2\int \frac{(1+ax)^4}{x^4}\,dx}{a^4} \\
&= \frac{c^2\int \left(a^4+\frac{1}{x^4}+\frac{4a}{x^3}+\frac{6a^2}{x^2}+\frac{4a^3}{x}\right)dx}{a^4} \\
&= -\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -1/3*c^2/(a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*L
og[x])/a
```

fricas [A] time = 0.41, size = 56, normalized size = 1.10

$$\frac{3\,a^4c^2x^4+12\,a^3c^2x^3\log(x)-18\,a^2c^2x^2-6\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 + 12*a^3*c^2*x^3*log(x) - 18*a^2*c^2*x^2 - 6*a*c^2*x - c
^2)/(a^4*x^3)
```

giac [B] time = 0.12, size = 112, normalized size = 2.20

$$-\frac{4\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(3\,c^2+\frac{34\,c^2}{ax-1}+\frac{66\,c^2}{(ax-1)^2}+\frac{36\,c^2}{(ax-1)^3}\right)(ax-1)}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^2*log(abs(-1/(a*x - 1
) - 1))/a + 1/3*(3*c^2 + 34*c^2/(a*x - 1) + 66*c^2/(a*x - 1)^2 + 36*c^2/(a*
x - 1)^3)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^3)
```

maple [A] time = 0.04, size = 50, normalized size = 0.98

$$-\frac{c^2}{3a^4x^3}-\frac{2c^2}{x^2a^3}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/3*c^2/a^4/x^3-2*c^2/x^2/a^3-6*c^2/a^2/x+c^2*x+4*c^2*ln(x)/a
```

maxima [A] time = 0.31, size = 46, normalized size = 0.90

$$c^2x + \frac{4\,c^2\log(x)}{a} - \frac{18\,a^2c^2x^2 + 6\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 4*c^2*log(x)/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x^3)
```

mupad [B] time = 0.06, size = 43, normalized size = 0.84

$$-\frac{c^2\,\left(6\,a\,x+18\,a^2\,x^2-3\,a^4\,x^4-12\,a^3\,x^3\,\ln(x)+1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] -(c^2*(6*a*x + 18*a^2*x^2 - 3*a^4*x^4 - 12*a^3*x^3*log(x) + 1))/(3*a^4*x^3)
```

sympy [A] time = 0.21, size = 53, normalized size = 1.04

$$\frac{a^4c^2x + 4a^3c^2\log{(x)} + \frac{-18a^2c^2x^2-6ac^2x-c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 4*a**3*c**2*log(x) + (-18*a**2*c**2*x**2 - 6*a*c**2*x - c**2
)/(3*x**3))/a**4
```

3.801 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=33

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

[Out] c/a^2/x+c*x-4*c*ln(x)/a+8*c*ln(-a*x+1)/a

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6157, 6150, 88}

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]

[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx \\
&= -\frac{c\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)}{x^2}dx}{a^2} \\
&= -\frac{c\int \frac{(1+ax)^3}{x^2(1-ax)}dx}{a^2} \\
&= -\frac{c\int\left(-a^2+\frac{1}{x^2}+\frac{4a}{x}-\frac{8a^2}{-1+ax}\right)dx}{a^2} \\
&= \frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{c}{a^2x}-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

fricas [A] time = 0.59, size = 35, normalized size = 1.06

$$\frac{a^2cx^2+8\,acx\log\left(ax-1\right)-4\,acx\log(x)+c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*c*x^2 + 8*a*c*x*log(a*x - 1) - 4*a*c*x*log(x) + c)/(a^2*x)
```

giac [A] time = 0.14, size = 66, normalized size = 2.00

$$-\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{4\,c\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{(ax-1)c}{a\left(\frac{1}{ax-1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="giac")
[Out] -4*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4*c*log(abs(-1/(a*x - 1) -
1))/a + (a*x - 1)*c/(a*(1/(a*x - 1) + 1))
```

maple [A] time = 0.04, size = 33, normalized size = 1.00

$$cx+\frac{c}{a^2x}-\frac{4c\ln(x)}{a}+\frac{8c\ln\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x)
[Out] c*x+c/a^2/x-4*c*ln(x)/a+8*c/a*ln(a*x-1)
```

maxima [A] time = 0.30, size = 32, normalized size = 0.97

$$cx + \frac{8\,c\log\left(ax - 1\right)}{a} - \frac{4\,c\log(x)}{a} + \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] c*x + 8*c*log(a*x - 1)/a - 4*c*log(x)/a + c/(a^2*x)
```

mupad [B] time = 0.07, size = 32, normalized size = 0.97

$$c\,x + \frac{c}{a^2\,x} - \frac{4\,c\,\ln(x)}{a} + \frac{8\,c\,\ln\left(a\,x - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c*x + c/(a^2*x) - (4*c*log(x))/a + (8*c*log(a*x - 1))/a
```

sympy [A] time = 0.27, size = 26, normalized size = 0.79

$$cx + \frac{4c\left(-\log\left(x\right) + 2\log\left(x - \frac{1}{a}\right)\right)}{a} + \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2),x)
```

```
[Out] c*x + 4*c*(-log(x) + 2*log(x - 1/a))/a + c/(a**2*x)
```

3.802 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

```
[Out] x/c-1/a/c/(-a*x+1)^2+5/a/c/(-a*x+1)+4*ln(-a*x+1)/a/c
```

Rubi [A] time = 0.17, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 77}

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$= -\frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}$$
$$= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c}$$
$$= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c}$$
$$= \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.00

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

fricas [A] time = 0.45, size = 64, normalized size = 1.21

$$\frac{a^3x^3-2\,a^2x^2-4\,ax+4\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+4}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

giac [A] time = 0.12, size = 74, normalized size = 1.40

$$\frac{ax-1}{ac}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{\frac{5\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - (5*a^3*c
/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

maple [A] time = 0.04, size = 51, normalized size = 0.96

$$\frac{x}{c}+\frac{4\ln\left(ax-1\right)}{ac}-\frac{5}{ac\left(ax-1\right)}-\frac{1}{ac\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x)
```

```
[Out] x/c+4/a/c*ln(a*x-1)-5/a/c/(a*x-1)-1/a/c/(a*x-1)^2
```

maxima [A] time = 0.30, size = 49, normalized size = 0.92

$$-\frac{5\,ax-4}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{4\,\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

mupad [B] time = 1.26, size = 48, normalized size = 0.91

$$\frac{x}{c}-\frac{5\,x-\frac{4}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))*(a*x - 1)^2),x)
```

```
[Out] x/c - (5*x - 4/a)/(c + a^2*c*x^2 - 2*a*c*x) + (4*log(a*x - 1))/(a*c)
```

sympy [A] time = 0.23, size = 41, normalized size = 0.77

$$\frac{-5ax+4}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{4\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2),x)
```

```
[Out] (-5*a*x + 4)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

3.803 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2+1/3/a/c^2/(-a*x+1)^3-2/a/c^2/(-a*x+1)^2+6/a/c^2/(-a*x+1)+4*ln(-a*x+1)
/a/c^2
```

Rubi [A] time = 0.17, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 43}

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 + 1/(3*a*c^2*(1 - a*x)^3) - 2/(a*c^2*(1 - a*x)^2) + 6/(a*c^2*(1 - a*x
)) + (4*Log[1 - a*x])/(a*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}$$
$$= \frac{a^4\int \frac{x^4}{(1-ax)^4}\,dx}{c^2}$$
$$= \frac{a^4\int \left(\frac{1}{a^4}+\frac{1}{a^4(-1+ax)^4}+\frac{4}{a^4(-1+ax)^3}+\frac{6}{a^4(-1+ax)^2}+\frac{4}{a^4(-1+ax)}\right)dx}{c^2}$$
$$= \frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.89

$$\frac{3a^4x^4-9a^3x^3-9a^2x^2+27ax+12(ax-1)^3\log(1-ax)-13}{3ac^2(ax-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-13 + 27*a*x - 9*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 12*(-1 + a*x)^3*Log[1 -
 a*x])/(3*a*c^2*(-1 + a*x)^3)
```

fricas [A] time = 0.64, size = 100, normalized size = 1.41

$$\frac{3\,a^4x^4-9\,a^3x^3-9\,a^2x^2+27\,ax+12\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-13}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 9*a^2*x^2 + 27*a*x + 12*(a^3*x^3 - 3*a^2*x^2 +
 3*a*x - 1)*log(a*x - 1) - 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x -
 a*c^2)
```

giac [A] time = 0.13, size = 93, normalized size = 1.31

$$\frac{ax-1}{ac^2}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{18\,a^5c^4}{ax-1}+\frac{6\,a^5c^4}{(ax-1)^2}+\frac{a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^2) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3*
(18*a^5*c^4/(a*x - 1) + 6*a^5*c^4/(a*x - 1)^2 + a^5*c^4/(a*x - 1)^3)/(a^6*c
^6)
```

maple [A] time = 0.04, size = 66, normalized size = 0.93

$$\frac{x}{c^2}-\frac{1}{3c^2a\,(ax-1)^3}+\frac{4\ln\left(ax-1\right)}{a\,c^2}-\frac{6}{a\,c^2\,(ax-1)}-\frac{2}{a\,c^2\,(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x)
```

```
[Out] x/c^2-1/3/c^2/a/(a*x-1)^3+4/a/c^2*ln(a*x-1)-6/a/c^2/(a*x-1)-2/a/c^2/(a*x-1)
^2
```

maxima [A] time = 0.32, size = 75, normalized size = 1.06

$$-\frac{18\,a^2x^2-30\,ax+13}{3\left(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2\right)}+\frac{x}{c^2}+\frac{4\,\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*x^2 - 30*a*x + 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 4*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 1.27, size = 71, normalized size = 1.00

$$\frac{6\,a\,x^2-10\,x+\frac{13}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^2*(a*x - 1)^2),x)
```

```
[Out] (6*a*x^2 - 10*x + 13/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x)
 + x/c^2 + (4*log(a*x - 1))/(a*c^2)
```

sympy [A] time = 0.32, size = 83, normalized size = 1.17

$$a^4\left(\frac{-18a^2x^2+30ax-13}{3a^8c^2x^3-9a^7c^2x^2+9a^6c^2x-3a^5c^2}+\frac{x}{a^4c^2}+\frac{4\log\left(ax-1\right)}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-18*a**2*x**2 + 30*a*x - 13)/(3*a**8*c**2*x**3 - 9*a**7*c**2*x**2 +
9*a**6*c**2*x - 3*a**5*c**2) + x/(a**4*c**2) + 4*log(a*x - 1)/(a**5*c**2))
```

3.804 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=111

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3-1/8/a/c^3/(-a*x+1)^4+11/12/a/c^3/(-a*x+1)^3-49/16/a/c^3/(-a*x+1)^2+11
1/16/a/c^3/(-a*x+1)+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

Rubi [A] time = 0.20, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 - 1/(8*a*c^3*(1 - a*x)^4) + 11/(12*a*c^3*(1 - a*x)^3) - 49/(16*a*c^3*
(1 - a*x)^2) + 111/(16*a*c^3*(1 - a*x)) + (129*Log[1 - a*x])/(32*a*c^3) - L
og[1 + a*x]/(32*a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= -\frac{a^6\int \frac{e^{4\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$
$$= -\frac{a^6\int \frac{x^6}{(1-ax)^5(1+ax)}\,dx}{c^3}$$
$$= -\frac{a^6\int \left(-\frac{1}{a^6}-\frac{1}{2a^6(-1+ax)^5}-\frac{11}{4a^6(-1+ax)^4}-\frac{49}{8a^6(-1+ax)^3}-\frac{111}{16a^6(-1+ax)^2}-\frac{129}{32a^6(-1+ax)}+\frac{1}{32a^6(1+ax)}\right)}{c^3}$$
$$= \frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 0.80

$$\frac{2\left(48a^5x^5-192a^4x^4-45a^3x^3+660a^2x^2-701ax+224\right)+387(ax-1)^4\log(1-ax)-3(ax-1)^4\log(ax+1)}{96ac^3(ax-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (2*(224 - 701*a*x + 660*a^2*x^2 - 45*a^3*x^3 - 192*a^4*x^4 + 48*a^5*x^5) +
387*(-1 + a*x)^4*Log[1 - a*x] - 3*(-1 + a*x)^4*Log[1 + a*x])/(96*a*c^3*(-1
+ a*x)^4)
```

fricas [A] time = 0.53, size = 163, normalized size = 1.47

$$\frac{96\,a^5x^5-384\,a^4x^4-90\,a^3x^3+1320\,a^2x^2-1402\,ax-3\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log(ax+1)+387}{96\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(96*a^5*x^5 - 384*a^4*x^4 - 90*a^3*x^3 + 1320*a^2*x^2 - 1402*a*x - 3*(
a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x + 1) + 387*(a^4*x^4 -
4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 448)/(a^5*c^3*x^4 - 4*a^4
*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

giac [A] time = 0.14, size = 130, normalized size = 1.17

$$\frac{ax-1}{ac^3}-\frac{4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3}-\frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3}-\frac{\frac{333\,a^{11}c^9}{ax-1}+\frac{147\,a^{11}c^9}{(ax-1)^2}+\frac{44\,a^{11}c^9}{(ax-1)^3}+\frac{6\,a^{11}c^9}{(ax-1)^4}}{48\,a^{12}c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^3) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/32
*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(333*a^11*c^9/(a*x - 1) + 147*a^
11*c^9/(a*x - 1)^2 + 44*a^11*c^9/(a*x - 1)^3 + 6*a^11*c^9/(a*x - 1)^4)/(a^1
2*c^12)
```

maple [A] time = 0.04, size = 95, normalized size = 0.86

$$\frac{x}{c^3}-\frac{1}{8a\,c^3\,(ax-1)^4}-\frac{11}{12c^3a\,(ax-1)^3}-\frac{49}{16c^3a\,(ax-1)^2}-\frac{111}{16a\,c^3\,(ax-1)}+\frac{129\ln(ax-1)}{32c^3a}-\frac{\ln(ax+1)}{32a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x)
```

```
[Out] x/c^3-1/8/a/c^3/(a*x-1)^4-11/12/c^3/a/(a*x-1)^3-49/16/c^3/a/(a*x-1)^2-111/1
6/a/c^3/(a*x-1)+129/32/c^3/a*ln(a*x-1)-1/32*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.31, size = 107, normalized size = 0.96

$$-\frac{333\,a^3x^3-852\,a^2x^2+749\,ax-224}{48\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}+\frac{x}{c^3}-\frac{\log(ax+1)}{32\,ac^3}+\frac{129\,\log(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 - 1/32*log(a*x + 1)/(a*c
^3) + 129/32*log(a*x - 1)/(a*c^3)
```

mupad [B] time = 0.11, size = 104, normalized size = 0.94

$$\frac{x}{c^3}-\frac{\frac{749\,x}{48}-\frac{71\,a\,x^2}{4}-\frac{14}{3\,a}+\frac{111\,a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}+\frac{129\,\ln(a\,x-1)}{32\,a\,c^3}-\frac{\ln(a\,x+1)}{32\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^3*(a*x - 1)^2),x)
```

```
[Out] x/c^3 - ((749*x)/48 - (71*a*x^2)/4 - 14/(3*a) + (111*a^2*x^3)/16)/(c^3 + 6*
a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (129*log(a*x - 1))
/(32*a*c^3) - log(a*x + 1)/(32*a*c^3)
```

sympy [A] time = 0.61, size = 114, normalized size = 1.03

$$a^6\left(\frac{-333a^3x^3+852a^2x^2-749ax+224}{48a^{11}c^3x^4-192a^{10}c^3x^3+288a^9c^3x^2-192a^8c^3x+48a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{129\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-333*a**3*x**3 + 852*a**2*x**2 - 749*a*x + 224)/(48*a**11*c**3*x**4
- 192*a**10*c**3*x**3 + 288*a**9*c**3*x**2 - 192*a**8*c**3*x + 48*a**7*c**3
) + x/(a**6*c**3) + (129*log(x - 1/a)/32 - log(x + 1/a)/32)/(a**7*c**3))
```

3.805 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=146

$$\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}-\frac{67}{16ac^4(1-ax)^2}+\frac{83}{48ac^4(1-ax)^3}-\frac{7}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{261\log(1-}{64ac^4}$$

```
[Out] x/c^4+1/20/a/c^4/(-a*x+1)^5-7/16/a/c^4/(-a*x+1)^4+83/48/a/c^4/(-a*x+1)^3-67
/16/a/c^4/(-a*x+1)^2+501/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+261/64*ln(-a*
x+1)/a/c^4-5/64*ln(a*x+1)/a/c^4
```

Rubi [A] time = 0.22, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}-\frac{67}{16ac^4(1-ax)^2}+\frac{83}{48ac^4(1-ax)^3}-\frac{7}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{261\log(1-}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 + 1/(20*a*c^4*(1 - a*x)^5) - 7/(16*a*c^4*(1 - a*x)^4) + 83/(48*a*c^4*
(1 - a*x)^3) - 67/(16*a*c^4*(1 - a*x)^2) + 501/(64*a*c^4*(1 - a*x)) - 1/(64
*a*c^4*(1 + a*x)) + (261*Log[1 - a*x])/(64*a*c^4) - (5*Log[1 + a*x])/(64*a*
c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= \frac{a^8\int \frac{e^{4\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$
$$= \frac{a^8\int \frac{x^8}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$
$$= \frac{a^8\int \left(\frac{1}{a^8}+\frac{1}{4a^8(-1+ax)^6}+\frac{7}{4a^8(-1+ax)^5}+\frac{83}{16a^8(-1+ax)^4}+\frac{67}{8a^8(-1+ax)^3}+\frac{501}{64a^8(-1+ax)^2}+\frac{261}{64a^8(-1+ax)}+\frac{}{64}\right.}{c^4}$$
$$= \frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}+\frac{501}{64ac^4(1-ax)}-$$

Mathematica [A] time = 0.11, size = 98, normalized size = 0.67

$$\frac{\frac{2(480a^7x^7-1920a^6x^6-1365a^5x^5+9300a^4x^4-6800a^3x^3-4900a^2x^2+7541ax-2384)}{(ax-1)^5(ax+1)}+3915\log(1-ax)-75\log(ax+1)}{960ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(-2384 + 7541*a*x - 4900*a^2*x^2 - 6800*a^3*x^3 + 9300*a^4*x^4 - 1365*a
^5*x^5 - 1920*a^6*x^6 + 480*a^7*x^7))/((-1 + a*x)^5*(1 + a*x)) + 3915*Log[1
 - a*x] - 75*Log[1 + a*x])/(960*a*c^4)
```

fricas [A] time = 0.42, size = 207, normalized size = 1.42

$$\frac{960\,a^7x^7-3840\,a^6x^6-2730\,a^5x^5+18600\,a^4x^4-13600\,a^3x^3-9800\,a^2x^2+15082\,ax-75\left(a^6x^6-4\,a^5x^5+5\,a^4x\right.}{960\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/960*(960*a^7*x^7 - 3840*a^6*x^6 - 2730*a^5*x^5 + 18600*a^4*x^4 - 13600*a^
3*x^3 - 9800*a^2*x^2 + 15082*a*x - 75*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*
a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 3915*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 -
 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) - 4768)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 +
 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

giac [A] time = 0.13, size = 170, normalized size = 1.16

$$\frac{(ax-1)\left(\frac{257}{ax-1}+128\right)}{128\,ac^4\left(\frac{2}{ax-1}+1\right)}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4}-\frac{5\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4}-\frac{\frac{7515\,a^{19}c^{16}}{ax-1}+\frac{4020\,a^{19}c^{16}}{(ax-1)^2}+\frac{1660\,a^{19}c^{16}}{(ax-1)^3}+\frac{420\,a^{19}c^{16}}{(ax-1)^4}+\frac{48\,a^{1}}{(ax-}}{960\,a^{20}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/128*(a*x - 1)*(257/(a*x - 1) + 128)/(a*c^4*(2/(a*x - 1) + 1)) - 4*log(abs
(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 5/64*log(abs(-2/(a*x - 1) - 1))/(
a*c^4) - 1/960*(7515*a^19*c^16/(a*x - 1) + 4020*a^19*c^16/(a*x - 1)^2 + 166
```

```
0*a^19*c^16/(a*x - 1)^3 + 420*a^19*c^16/(a*x - 1)^4 + 48*a^19*c^16/(a*x - 1
)^5)/(a^20*c^20)
```

maple [A] time = 0.05, size = 125, normalized size = 0.86

$$\frac{x}{c^4}-\frac{1}{20c^4a(ax-1)^5}-\frac{7}{16c^4a(ax-1)^4}-\frac{83}{48c^4a(ax-1)^3}-\frac{67}{16c^4a(ax-1)^2}-\frac{501}{64ac^4(ax-1)}+\frac{261\ln(ax-1)}{64c^4a}-\frac{}{64ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x)
```

```
[Out] x/c^4-1/20/c^4/a/(a*x-1)^5-7/16/c^4/a/(a*x-1)^4-83/48/c^4/a/(a*x-1)^3-67/16
/c^4/a/(a*x-1)^2-501/64/a/c^4/(a*x-1)+261/64/c^4/a*ln(a*x-1)-1/64/a/c^4/(a*
x+1)-5/64*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.32, size = 135, normalized size = 0.92

$$-\frac{3765\,a^5x^5-9300\,a^4x^4+4400\,a^3x^3+6820\,a^2x^2-8021\,ax+2384}{480\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}+\frac{x}{c^4}-\frac{5\,\log(ax+1)}{64\,ac^4}+\frac{261\,\log(ax-1)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a*
x + 2384)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*
a^2*c^4*x - a*c^4) + x/c^4 - 5/64*log(a*x + 1)/(a*c^4) + 261/64*log(a*x - 1
)/(a*c^4)
```

mupad [B] time = 0.15, size = 131, normalized size = 0.90

$$\frac{\frac{341\,a\,x^2}{24}-\frac{8021\,x}{480}+\frac{149}{30\,a}+\frac{55\,a^2\,x^3}{6}-\frac{155\,a^3\,x^4}{8}+\frac{251\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}+\frac{x}{c^4}+\frac{261\,\ln(a\,x-1)}{64\,a\,c^4}-\frac{5\,\ln(a\,x+1)}{64\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^4*(a*x - 1)^2),x)
```

```
[Out] ((341*a*x^2)/24 - (8021*x)/480 + 149/(30*a) + (55*a^2*x^3)/6 - (155*a^3*x^4
)/8 + (251*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 + 4*a^5*c^4*x^
5 - a^6*c^4*x^6 - 4*a*c^4*x) + x/c^4 + (261*log(a*x - 1))/(64*a*c^4) - (5*l
og(a*x + 1))/(64*a*c^4)
```

sympy [A] time = 0.92, size = 144, normalized size = 0.99

$$a^8\left(\frac{-3765a^5x^5+9300a^4x^4-4400a^3x^3-6820a^2x^2+8021ax-2384}{480a^{15}c^4x^6-1920a^{14}c^4x^5+2400a^{13}c^4x^4-2400a^{11}c^4x^2+1920a^{10}c^4x-480a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{261\log\left(x-\frac{1}{a}\right)}{64}-}{a^9c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-3765*a**5*x**5 + 9300*a**4*x**4 - 4400*a**3*x**3 - 6820*a**2*x**2 +
 8021*a*x - 2384)/(480*a**15*c**4*x**6 - 1920*a**14*c**4*x**5 + 2400*a**13*
c**4*x**4 - 2400*a**11*c**4*x**2 + 1920*a**10*c**4*x - 480*a**9*c**4) + x/(
a**8*c**4) + (261*log(x - 1/a)/64 - 5*log(x + 1/a)/64)/(a**9*c**4))
```

3.806 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=343

$$c^4x\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{7/2}}{7a}+\frac{7c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{5/2}}{6a}+\frac{29c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{3/2}}{30a}+$$

```
[Out] 29/30*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)/a+7/6*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(7/2)/a+8/7*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)/a+c^4*(1-1/a/x)^(9/2)*(1
+1/a/x)^(7/2)*x+35/16*c^4*arccsc(a*x)/a-c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2))/a-1/16*c^4*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+7/40*c^4*(1+1/a/x)^(
5/2)*(1-1/a/x)^(1/2)/a+19/40*c^4*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-19/16*c^
4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.26, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^4x\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{7/2}}{7a}+\frac{7c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{5/2}}{6a}+\frac{29c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{3/2}}{30a}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

```
[Out] (-19*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (c^4*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(16*a) + (7*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(40*a) + (19*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(40*a) + (29*c^
4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(30*a) + (7*c^4*(1 - 1/(a*x))^(5
/2)*(1 + 1/(a*x))^(7/2))/(6*a) + (8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
7/2))/(7*a) + c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(7/2)*x + (35*c^4*ArcCs
c[a*x])/(16*a) - (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x - c^4\,\text{Subst}\left(\int\frac{\left(-\frac{1}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x}dx,x,\frac{1}{x}\right.$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int\right.$$

$$= \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right.$$

$$= \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a} + \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right.}{7a}$$

$$= \frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a} + \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a} + \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)}{6a}$$

$$= \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + \frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a} + \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + \frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a} +$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + 19c$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + 19c$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + 19c$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + 19c$$

Mathematica [A] time = 0.31, size = 120, normalized size = 0.35

$$\frac{c^4\left(3675a^6\sin^{-1}\left(\frac{1}{ax}\right) - 1680a^6\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right) + \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1680a^7x^7+2816a^6x^6+3045a^5x^5-1952a^4x^4-1330a^3x^3+1056a\right.}{x^6}\right.}{1680a^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*((Sqrt[1 - 1/(a^2*x^2)]*(-240 + 280*a*x + 1056*a^2*x^2 - 1330*a^3*x^3
- 1952*a^4*x^4 + 3045*a^5*x^5 + 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a^
6*ArcSin[1/(a*x)] - 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a^7
)
```

fricas [A] time = 0.61, size = 201, normalized size = 0.59

$$-\frac{7350\,a^{7}c^{4}x^{7}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(1680\,a^{8}c^{4}x^{8}\right.}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 + 4496*a^7*c^4*x^7 + 5861*a^6*c^4*x^6
 + 1093*a^5*c^4*x^5 - 3282*a^4*c^4*x^4 - 274*a^3*c^4*x^3 + 1336*a^2*c^4*x^2
 + 40*a*c^4*x - 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

giac [A] time = 0.19, size = 524, normalized size = 1.53

$$-\frac{35\,c^{4}\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)\operatorname{sgn}\left(ax+1\right)}{8\,a}+\frac{c^{4}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\operatorname{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^{2}x^{2}-1}\,c^{4}\operatorname{sgn}\left(\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^4*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^4*sgn(a*x + 1)/a - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*abs(a
)*sgn(a*x + 1) - 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x + 1)
+ 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) - 20160*(x
*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 9065*(x*abs(a) - sqrt(
a^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 -
1))^8*a*c^4*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4*sgn
(a*x + 1) - 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) -
 38976*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) - 6860*(x*abs(a)
 - sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) - 12992*(x*abs(a) - sqrt(a^
2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 3045*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*
abs(a)*sgn(a*x + 1) - 2816*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 -
 1))^2 + 1)^7*a*abs(a))
```

maple [A] time = 0.07, size = 320, normalized size = 0.93

$$-\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^{4}\left(-1680\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}\,x^{8}a^{8}+1680\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}\sqrt{a^{2}}\,x^{6}a^{6}-3675a^{7}x^{7}\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}-36\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/1680*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^
(1/2)*x^8*a^8+1680*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6-3675*a^7*x^7*(a^2)
^(1/2)*(a^2*x^2-1)^(1/2)-3675*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2
```

```
))+1680*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8+1995*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5-1136*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^
4*a^4-1050*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+816*(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)*x^2*a^2+280*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-240*(a^2*x^2-1)^(3/2)
*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 380, normalized size = 1.11

$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{1995\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10185\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a
^2 - (1995*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10185*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 17619*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 4569*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 71801*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 72051*c^4*((a*x -
1)/(a*x + 1))^(5/2) + 31465*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 5355*c^4*sqrt
((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
 + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)
^8*a^2/(a*x + 1)^8 + a^2))*a
```

mupad [B] time = 1.37, size = 332, normalized size = 0.97

$$\frac{\frac{51\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{899\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{24}+\frac{3431\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}+\frac{71801\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{840}+\frac{1523\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}+\frac{839\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+\frac{97\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{8}+\cdots}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\frac{a\,(a\,x-1)^8}{(a\,x+1)^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((51*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (899*c^4*((a*x - 1)/(a*x + 1))^(3
/2))/24 + (3431*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (71801*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/840 + (1523*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (839*
c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (97*c^4*((a*x - 1)/(a*x + 1))^(13/2)
)/8 + (19*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x +
1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (14*
a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x - 1
)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*x - 1)/(
a*x + 1))^(1/2)))/(8*a) - (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int a^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^8}\,dx+\int\left(-\frac{4a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\right)dx+\int\frac{6a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{4a^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)\cdots\right.}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(a**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**8, x) + Integral(-4*a**2*sqrt(a*x/(a*x + 1)
```

```
- 1/(a*x + 1))/x**6, x) + Integral(6*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/x**4, x) + Integral(-4*a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a*
*8
```

3.807 $\int e^{-\coth^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^3 dx$

Optimal. Leaf size=269

$$c^3x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{5a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+\frac{11c^3\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{12a}+\frac{c^3}{}$$

```
[Out] 5/4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)/a+6/5*c^3*(1-1/a/x)^(5/2)*(1+1/a/x)
^(5/2)/a+c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x+15/8*c^3*arccsc(a*x)/a-c^3*a
rctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+1/24*c^3*(1+1/a/x)^(3/2)*(1-1/a/x
)^(1/2)/a+11/12*c^3*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-7/8*c^3*(1-1/a/x)^(1/
2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.19, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^3x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{5a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+\frac{11c^3\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{12a}+\frac{c^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

```
[Out] (-7*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (c^3*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(24*a) + (11*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(12*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(4*a) + (6*c^3*
(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(5*a) + c^3*(1 - 1/(a*x))^(7/2)*(1
 + 1/(a*x))^(5/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) - (c^3*ArcTanh[Sqrt[1 - 1/
(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^3\,\text{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx, x, \frac{1}{x}\right.$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int \right.$$

$$= \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right.$$

$$= \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a}$$

$$= \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} +$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3}{}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3}{}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3}{}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3}{}$$

Mathematica [A] time = 0.21, size = 110, normalized size = 0.41

$$\frac{c^3\left(225a^4x^4\sin^{-1}\left(\frac{1}{ax}\right)-120a^4x^4\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sqrt{1-\frac{1}{a^2x^2}}\left(120a^5x^5+184a^4x^4+135a^3x^3-88a^2x^2-\right.\right.}{120a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(24 - 30*a*x - 88*a^2*x^2 + 135*a^3*x^3 + 184*a
^4*x^4 + 120*a^5*x^5) + 225*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 +
Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5*x^4)
```

fricas [A] time = 0.51, size = 179, normalized size = 0.67

$$-\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(120\,a^6c^3x^6+304\,a^5\right.}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5
*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (120*a^6*c^3*x^6 + 304*a^5*c^3*x^5 + 319*a^4*c^3*x^4 + 47*a
^3*c^3*x^3 - 118*a^2*c^3*x^2 - 6*a*c^3*x + 24*c^3)*sqrt((a*x - 1)/(a*x + 1)
))/(a^6*x^5)
```

giac [A] time = 0.18, size = 394, normalized size = 1.46

$$-\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}\left(ax+1\right)}{4\,a}+\frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^3\mathrm{sgn}\,(}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^3*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^3*sgn(a*x + 1)/a - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a)*s
gn(a*x + 1) - 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) + 150
*(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) - 720*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) - 1120*(x*abs(a) - sqrt(a^2*x^2 -
1))^4*a*c^3*sgn(a*x + 1) - 150*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*
sgn(a*x + 1) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - 13
5*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) - 184*a*c^3*sgn(a*
x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

maple [A] time = 0.06, size = 272, normalized size = 1.01

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^6a^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4-225\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5-225\,\mathrm{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/120*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3*(-120*(a^2*x^2-1)^(1/2)*(a^2)^(1
/2)*x^6*a^6+120*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^4-225*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*x^5*a^5-225*arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)*x^5*a^5+12
0*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+105*(a^2*x^
2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-30*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1
)*(a*x+1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

maxima [A] time = 0.41, size = 302, normalized size = 1.12

$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+305\,c^3\left(\frac{a}{a}\right.}{\frac{4\,(ax-1)}{ax+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
 (105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(9/2
) + 86*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(5/
2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 345*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1
)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x
+ 1)^6 + a^2))*a
```

mupad [B] time = 0.11, size = 258, normalized size = 0.96

$$\frac{\frac{23\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{269\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{827\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{30}+\frac{43\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{30}+\frac{61\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{12}+\frac{7\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}-\frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((23*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (269*c^3*((a*x - 1)/(a*x + 1))^(3
/2))/12 + (827*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (43*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/30 + (61*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (7*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/(4*a) - (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int a^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\right)dx+\int\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{3a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**3*(Integral(a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**6, x) + Integral(3*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**4, x) + Integral(-3*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/x**2, x))/a**6
```

3.808 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$c^2x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}+\frac{4c^2\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}+\frac{3c^2\operatorname{csc}}{2}$$

```
[Out] 4/3*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)/a+c^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/
2)*x+3/2*c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+3
/2*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-1/2*c^2*(1-1/a/x)^(1/2)*(1+1/a/x)^
(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^2x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}+\frac{4c^2\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}+\frac{3c^2\operatorname{csc}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

```
[Out] -(c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) + (3*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(2*a) + (4*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2
))/(3*a) + c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) - (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\left(c^2\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x - c^2\,\text{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{4x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x}dx, x,\right.$$

$$= \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{3}\left(ac^2\right)\text{Subst}\left(\int\right.$$

$$= \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right.$$

$$= -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a} +$$

$$= -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a} +$$

$$= -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a} +$$

$$= -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a} +$$

Mathematica [A] time = 0.15, size = 94, normalized size = 0.48

$$\frac{c^2\left(-6a^2x^2\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+9a^2x^2\sin^{-1}\left(\frac{1}{ax}\right)+\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3+8a^2x^2+3ax-2\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 + 3*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^
2*ArcSin[1/(a*x)] - 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*x
^2)
```

fricas [A] time = 0.64, size = 156, normalized size = 0.80

$$-\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^2x^4+14\,a^3c^2x^3+\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
```

```
) - 1) - (6*a^4*c^2*x^4 + 14*a^3*c^2*x^3 + 11*a^2*c^2*x^2 + a*c^2*x - 2*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

giac [A] time = 0.18, size = 264, normalized size = 1.35

$$-\frac{3\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}\left(ax+1\right)}{a}+\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^2\mathrm{sgn}\left(ax+\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^2*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2*
sgn(a*x + 1)/a - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a*x
 + 1) - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) - 12*(x*abs(
a) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 3*(x*abs(a) - sqrt(a^2*x^2 -
 1))*c^2*abs(a)*sgn(a*x + 1) - 8*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2
*x^2 - 1))^2 + 1)^3*a*abs(a))
```

maple [A] time = 0.05, size = 224, normalized size = 1.15

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)c^2\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+6\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3-9a^3x^3\sqrt{a^2}\,\,\mathrm{arcta}\right.}{6\sqrt{\left(ax-1\right)\left(ax+1\right)}\,a^4x^3\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
x^4*a^4+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2-9*(a^2*x^2-1)^(1/2)*(a^2)^(
1/2)*x^3*a^3-9*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+6*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+3*(a^2*x^2-1)^(3/2)*(a^
2)^(1/2)*x*a-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)*(a*x+1))^(1/2)/a^4/x
^3/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 223, normalized size = 1.14

$$-\frac{1}{3}\,a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+29\,c^2\left(\right.}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(a}{(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3
*c^2*((a*x - 1)/(a*x + 1))^(7/2) + c^2*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^2
*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1
)^4 + a^2))
```

mupad [B] time = 1.30, size = 183, normalized size = 0.94

$$\frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^2*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^2*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-2*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**2, x))/a**4
```

3.809 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=108

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}+\frac{2c\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{a}+\frac{c\csc^{-1}(ax)}{a}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

```
[Out] c*arccsc(a*x)/a-c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1-1/a/x)^(3
/2)*x*(1+1/a/x)^(1/2)+2*c*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6194, 97, 154, 21, 105, 41, 216, 92, 208}

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}+\frac{2c\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{a}+\frac{c\csc^{-1}(ax)}{a}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

```
[Out] (2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*(1 - 1/(a*x))^(3/2)*Sqrt[1
+ 1/(a*x)]*x + (c*ArcCsc[a*x])/a - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\left(c\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)\right)$$
$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x - c\,\text{Subst}\left(\int\frac{\left(-\frac{1}{a}-\frac{2x}{a^2}\right)\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$
$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x - (ac)\,\text{Subst}\left(\int\frac{-\frac{1}{a^2}-\frac{x}{a^3}}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\right.$$
$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x + \frac{c\,\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{a}$$
$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x + \frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{a^2}$$
$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x + \frac{c\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{a^2} - \frac{c\,\text{S}}{}$$
$$= \frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x + \frac{c\csc^{-1}(ax)}{a} - \frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\right.}{a}$$

Mathematica [A] time = 0.09, size = 55, normalized size = 0.51

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}(ax+1)-\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) + ArcSin[1/(a*x)] - Log[(1 + Sqrt[1 - 1
/(a^2*x^2)])*x]))/a
```

fricas [A] time = 0.59, size = 107, normalized size = 0.99

$$-\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2cx^2+2\,acx+c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c*x*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1) - a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 + 2*a*c
*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

giac [A] time = 0.14, size = 121, normalized size = 1.12

$$-\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\text{sgn}\left(ax+1\right)}{a}+\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\text{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c\text{sgn}\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c*log(abs(-x*ab
s(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a*
x + 1)/a + 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a))
```

maple [A] time = 0.05, size = 166, normalized size = 1.54

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,xa-ax\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right.}{\sqrt{(ax-1)(ax+1)}\,a^2x\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2+
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-a*x*(a^2)^(
1/2)*arctan(1/(a^2*x^2-1)^(1/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(
a^2)^(1/2))*x*a^2)/((a*x-1)*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

maxima [A] time = 0.41, size = 117, normalized size = 1.08

$$-a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

mupad [B] time = 1.28, size = 84, normalized size = 0.78

$$\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)^2}{(a\,x+1)^2}}-\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*c*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a**2
```

3.810 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=105

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{\frac{1}{ax}+1}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

```
[Out] -arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c+2*(1-1/a/x)^(1/2)/a/c/(1+1/a/
x)^(1/2)+x*(1-1/a/x)^(1/2)/c/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 21, 94, 92, 208}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{\frac{1}{ax}+1}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)])/(a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*Sq
rt[1 + 1/(a*x)]) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}}+\frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}}+\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}\\
&= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 57, normalized size = 0.54

$$
\frac{\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,(ax+2)}{ax+1}-\frac{\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{c}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x))/(1 + a*x) - Log[(1 + Sqrt[1 - 1/(a^2*x
^2)])*x]/a)/c
```

fricas [A] time = 0.53, size = 67, normalized size = 0.64

$$\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}} - \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] ((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.06, size = 250, normalized size = 2.38

$$-\frac{\left(2\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3 - 3\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^2a^2 + 4\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x\,a^2 + ((ax-1)(}{2a\sqrt{a^2}\,(ax+1)\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x)
```

```
[Out] -1/2*(2*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3
-3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+4*ln((a^2*x+((a*x-1)*(a*x+1)
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)
-6*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+2*a*ln((a^2*x+((a*x-1)*(a*x+1))^
(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-3*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/a*((
a*x-1)/(a*x+1))^(1/2)/(a^2)^(1/2)/(a*x+1)/c/((a*x-1)*(a*x+1))^(1/2)
```

maxima [A] time = 0.32, size = 121, normalized size = 1.15

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1} - a^2c} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^2c} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right)}{a^2c} - \frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

mupad [B] time = 0.05, size = 86, normalized size = 0.82

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c - \frac{a\,c\,(a\,x-1)}{a\,x+1}} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c} - \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + ((a*x -
 1)/(a*x + 1))^(1/2)/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2 \sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}{a^2x^2-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

3.811 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} dx$

Optimal. Leaf size=179

$$\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

```
[Out] -arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-2/a/c^2/(1+1/a/x)^(3/2)/(1-
1/a/x)^(1/2)+x/c^2/(1+1/a/x)^(3/2)/(1-1/a/x)^(1/2)+5/3*(1-1/a/x)^(1/2)/a/c^
2/(1+1/a/x)^(3/2)+8/3*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] -2/(a*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (5*Sqrt[1 - 1/(a*x)])/(3
*a*c^2*(1 + 1/(a*x))^(3/2)) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*
x)]) + x/(c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) - ArcTanh[Sqrt[1 - 1/(
a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{1}{a^2}+\frac{4x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^2\,\text{Subst}\left(\int \frac{-\frac{3}{a^3}+\frac{5}{a}}{x\sqrt{1-\frac{x}{a}}\left(1+\right.}\right.}{3c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^3\,\text{Su}}{}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subs}}{}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subs}}{}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{\text{tanh}}{}$$

Mathematica [A] time = 0.18, size = 85, normalized size = 0.47

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(3a^3x^3+7a^2x^2-5ax-8\right)}{3(ax-1)(ax+1)^2}-\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 5*a*x + 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 +
a*x)*(1 + a*x)^2) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

fricas [A] time = 0.44, size = 119, normalized size = 0.66

$$-\frac{3\left(a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(3\,a^3x^3+7\,a^2x^2-5\,ax-8\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^3c^2x^2-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^2 - 1)*
log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (3*a^3*x^3 + 7*a^2*x^2 - 5*a*x - 8)*sq
rt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - a*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^2, x)
```

maple [B] time = 0.07, size = 530, normalized size = 2.96

$$-\frac{\left(-45\sqrt{a^2}\,\sqrt{(ax-1)(ax+1)}\,x^5a^5+24\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^5a^6+21\sqrt{a^2}\,\left((ax-1)(ax+1)\right)^{\frac{3}{2}}x^3a^3-45$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/24*(-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5+24*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+21*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^3*a^3-45*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+24*ln
((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-11*(a^2)^
(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+90*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2
)*x^3*a^3-48*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^
3*a^4-5*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a+90*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2)*x^2*a^2-48*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)
^(1/2))*x^2*a^3+19*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-45*((a*x-1)*(a*x+1))
^(1/2)*(a^2)^(1/2)*x*a+24*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a
^2)^(1/2))*x*a^2-45*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+24*a*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(1/2)/(a
*x-1)^2/(a*x+1)^2/(a^2)^(1/2)/c^2/((a*x-1)*(a*x+1))^(1/2)
```

maxima [A] time = 0.30, size = 163, normalized size = 0.91

$$-\frac{1}{12}a\left(\frac{3\left(\frac{9(ax-1)}{ax+1}-1\right)}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+18\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{12\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*a*(3*(9*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)
 - a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) - (((a*x - 1)/(a*x + 1))^(3/2) + 18*s
qrt((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 12*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

mupad [B] time = 1.30, size = 137, normalized size = 0.77

$$\frac{\frac{9\,(a\,x-1)}{a\,x+1}-1}{4\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}+\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^2}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12\,a\,c^2}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^2,x)
```

```
[Out] ((9*(a*x - 1))/(a*x + 1) - 1)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(1/2) - 4*a*c^
2*((a*x - 1)/(a*x + 1))^(3/2)) + (3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*c^2)
+ ((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^2) + (atan(((a*x - 1)/(a*x + 1))^(1/2
)*1i)*2i)/(a*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x*
*2 + 1), x)/c**2
```

3.812 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=255

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{\cdots}{3ac^3(\cdots}$$

```
[Out] -4/3/a/c^3/(1-1/a/x)^(3/2)/(1+1/a/x)^(5/2)+x/c^3/(1-1/a/x)^(3/2)/(1+1/a/x)^
(5/2)-arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-13/3/a/c^3/(1+1/a/x)^(
5/2)/(1-1/a/x)^(1/2)+14/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(5/2)+11/5*(1-1/a
/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+16/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{\cdots}{3ac^3(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -4/(3*a*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)) - 13/(3*a*c^3*Sqrt[1 -
 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (14*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*
x))^(5/2)) + (11*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sqr
t[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(3/2)*(1
 + 1/(a*x))^(5/2)) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{3}{a^2}+\frac{16x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x\right.}{3c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{a^2\,\text{S}}{}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{}{c^3\left(1-\frac{1}{ax}\right)^{3/}}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{a}}}{5ac^3\left(1+\frac{1}{ax}\right.}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{a}}}{5ac^3\left(1+\frac{1}{ax}\right.}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{a}}}{5ac^3\left(1+\frac{1}{ax}\right.}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{a}}}{5ac^3\left(1+\frac{1}{ax}\right.}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\sqrt{1-\frac{1}{a}}}{5ac^3\left(1+\frac{1}{ax}\right.}$$

Mathematica [A] time = 0.23, size = 101, normalized size = 0.40

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(15a^5x^5+38a^4x^4-52a^3x^3-87a^2x^2+33ax+48\right)}{15(ax-1)^2(ax+1)^3} - \log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 33*a*x - 87*a^2*x^2 - 52*a^3*x^3 + 38*a^4
*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^2*(1 + a*x)^3) - Log[(1 + Sqrt[1 - 1/(a^
2*x^2)])*x])/(a*c^3)
```

fricas [A] time = 0.79, size = 161, normalized size = 0.63

$$-\frac{15\left(a^4x^4-2\,a^2x^2+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^4x^4-2\,a^2x^2+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(15\,a^5x^5+38\,a^4x^4-52\,a^3x^3\right.}{15\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (15*a^5*x^5
 + 38*a^4*x^4 - 52*a^3*x^3 - 87*a^2*x^2 + 33*a*x + 48)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^3, x)
```

maple [B] time = 0.07, size = 714, normalized size = 2.80

$$-\frac{\left(-525\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^7a^7+240\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^7a^8+285\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\,x^5a^5-5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -1/240*(-525*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^7*a^7+240*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8+285*((a*x-1)*(a*x+1))
^(3/2)*(a^2)^(1/2)*x^5*a^5-525*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^6*a^6+
240*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6*a^7-83*
((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^4*a^4+1575*(a^2)^(1/2)*((a*x-1)*(a*x+
1))^(1/2)*x^5*a^5-720*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*x^5*a^6-218*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+1575*(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-720*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+342*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x
^2*a^2-1575*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+720*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-3*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(3/2)*x*a-1575*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+720*ln
((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3-243*((a*x
-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+525*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a-
240*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+525*(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-240*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^3/(a*x+1)^3/(a
^2)^(1/2)/c^3/((a*x-1)*(a*x+1))^(1/2)
```

maxima [A] time = 0.32, size = 197, normalized size = 0.77

$$\frac{1}{240}\,a\left(\frac{5\left(\frac{23\,(ax-1)}{ax+1}-\frac{120\,(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+450\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}+\frac{240\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*a*(5*(23*(a*x - 1)/(a*x + 1) - 120*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*
c^3*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (3
*((a*x - 1)/(a*x + 1))^(5/2) + 40*((a*x - 1)/(a*x + 1))^(3/2) + 450*sqrt((a
*x - 1)/(a*x + 1)))/(a^2*c^3) - 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2
*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

mupad [B] time = 0.06, size = 178, normalized size = 0.70

$$\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3}-\frac{\frac{23\,(a\,x-1)}{3\,(a\,x+1)}-\frac{40\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6\,a\,c^3}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{80\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,2\mathrm{i}}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^3,x)
```

```
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(8*a*c^3) - ((23*(a*x - 1))/(3*(a*x + 1))
- (40*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/2)
 - 16*a*c^3*((a*x - 1)/(a*x + 1))^(5/2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(6*a
*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) + (atan(((a*x - 1)/(a*x + 1)
)^(1/2)*1i)*2i)/(a*c^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\int\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x*
*4 + 3*a**2*x**2 - 1), x)/c**3
```

3.813 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=329

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -6/5/a/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(7/2)-31/15/a/c^4/(1-1/a/x)^(3/2)/(1+1
/a/x)^(7/2)+x/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(7/2)-arctanh((1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2))/a/c^4-28/3/a/c^4/(1+1/a/x)^(7/2)/(1-1/a/x)^(1/2)+115/21*(1-
1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(7/2)+122/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(
5/2)+93/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+128/35*(1-1/a/x)^(1/2)/a/c
^4/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -6/(5*a*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - 31/(15*a*c^4*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)) - 28/(3*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(7/2)) + (115*Sqrt[1 - 1/(a*x)])/(21*a*c^4*(1 + 1/(a*x))^(7/2)) + (122
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (93*Sqrt[1 - 1/(a*x)])
/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
+ 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{5}{a^2}+\frac{36x}{a^3}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x} \cdots\right)}{5c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{a^2 \cdots}{\cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\cdots}{c^4 \cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{1 \cdots}{21a \cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{1 \cdots}{21a \cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{1 \cdots}{21a \cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{1 \cdots}{21a \cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{1 \cdots}{21a \cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{1 \cdots}{21a \cdots}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{1 \cdots}{21a \cdots}$$

Mathematica [A] time = 0.32, size = 117, normalized size = 0.36

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(105a^7x^7+281a^6x^6-559a^5x^5-965a^4x^4+715a^3x^3+1065a^2x^2-279ax-384\right)}{105(ax-1)^3(ax+1)^4}-\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-384 - 279*a*x + 1065*a^2*x^2 + 715*a^3*x^3 -
965*a^4*x^4 - 559*a^5*x^5 + 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^3*(
1 + a*x)^4) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

fricas [A] time = 0.58, size = 205, normalized size = 0.62

$$-\frac{105\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(105}{105\left(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x +
 1)) + 1) - 105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (105*a^7*x^7 + 281*a^6*x^6 - 559*a^5*x^5 - 965*a^4*x^4 + 71
5*a^3*x^3 + 1065*a^2*x^2 - 279*a*x - 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c
^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^4, x)
```

maple [B] time = 0.08, size = 898, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/13440*(132300*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+132300*((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^7*a^7-27673*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1
/2)*x^5*a^5+7705*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3-198450*(a^2)^(
1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-198450*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(
1/2)*x^4*a^4+132300*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^6*a^6+24295*((a*x
-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^4*a^4-33075*((a*x-1)*(a*x+1))^(1/2)*(a^2)^
(1/2)*x*a-37095*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2+2637*(a^2)^(1/2
)*((a*x-1)*(a*x+1))^(3/2)*x*a+132300*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^
3*a^3+13440*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a
^2+16077*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-33075*((a*x-1)*(a*x+1))^(1/2)*
(a^2)^(1/2)+13440*a*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))-53760*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*
```

```
a^4-53760*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a
^3+13440*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^9*a^
10+13440*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^8*a^
9-33075*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^9*a^9+19635*((a*x-1)*(a*x+1))
^(3/2)*(a^2)^(1/2)*x^7*a^7-33075*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^8*a^
8-2893*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^6*a^6-53760*ln((a^2*x+((a*x-1)
*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8-53760*ln((a^2*x+((a*x-1)*
(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6*a^7+80640*ln((a^2*x+((a*x-1)*(
a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+80640*ln((a^2*x+((a*x-1)*(a
*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5)/a*((a*x-1)/(a*x+1))^(1/2)/(
a*x+1)^4/(a^2)^(1/2)/(a*x-1)^4/c^4/((a*x-1)*(a*x+1))^(1/2)
```

maxima [A] time = 0.33, size = 231, normalized size = 0.70

$$\frac{1}{6720}a\left(\frac{7\left(\frac{47(ax-1)}{ax+1}+\frac{655(ax-1)^2}{(ax+1)^2}-\frac{2625(ax-1)^3}{(ax+1)^3}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+329\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2940\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/6720*a*(7*(47*(a*x - 1)/(a*x + 1) + 655*(a*x - 1)^2/(a*x + 1)^2 - 2625*(a
*x - 1)^3/(a*x + 1)^3 + 3)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*(
(a*x - 1)/(a*x + 1))^(5/2)) + 5*(3*((a*x - 1)/(a*x + 1))^(7/2) + 42*((a*x -
 1)/(a*x + 1))^(5/2) + 329*((a*x - 1)/(a*x + 1))^(3/2) + 2940*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) - 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4)
 + 6720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

mupad [B] time = 1.28, size = 217, normalized size = 0.66

$$\frac{35\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{\frac{131\,(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{175\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{47\,(a\,x-1)}{15\,(a\,x+1)}+\frac{1}{5}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}+\frac{47\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{192\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{32\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{448\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^4,x)
```

```
[Out] (35*((a*x - 1)/(a*x + 1))^(1/2))/(16*a*c^4) - ((131*(a*x - 1)^2)/(3*(a*x +
1)^2) - (175*(a*x - 1)^3)/(a*x + 1)^3 + (47*(a*x - 1))/(15*(a*x + 1)) + 1/5
)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(7
/2)) + (47*((a*x - 1)/(a*x + 1))^(3/2))/(192*a*c^4) + ((a*x - 1)/(a*x + 1))
^(5/2)/(32*a*c^4) + ((a*x - 1)/(a*x + 1))^(7/2)/(448*a*c^4) + (atan(((a*x -
 1)/(a*x + 1))^(1/2)*1i)*2i)/(a*c^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^8\int\frac{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**8*x**8 - 4*a**6*x*
*6 + 6*a**4*x**4 - 4*a**2*x**2 + 1), x)/c**4
```

3.814 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=90

$$\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4x$$

```
[Out] 1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/a^3/x
^2+2*c^4/a^2/x+c^4*x-2*c^4*ln(x)/a
```

Rubi [A] time = 0.15, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$-\frac{3c^4}{a^3x^2}+\frac{3c^4}{2a^5x^4}-\frac{2c^4}{5a^6x^5}-\frac{c^4}{3a^7x^6}+\frac{c^4}{7a^8x^7}+\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$$
$$=-\frac{c^4\int\frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}\,dx}{a^8}$$
$$=-\frac{c^4\int\frac{(1-ax)^5(1+ax)^3}{x^8}\,dx}{a^8}$$
$$=-\frac{c^4\int\left(-a^8+\frac{1}{x^8}-\frac{2a}{x^7}-\frac{2a^2}{x^6}+\frac{6a^3}{x^5}-\frac{6a^5}{x^3}+\frac{2a^6}{x^2}+\frac{2a^7}{x}\right)dx}{a^8}$$
$$=\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

Mathematica [A] time = 0.04, size = 90, normalized size = 1.00

$$\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

fricas [A] time = 1.17, size = 89, normalized size = 0.99

$$\frac{210\,a^8c^4x^8-420\,a^7c^4x^7\log(x)+420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/210*(210*a^8*c^4*x^8 - 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 - 630*a^5
*c^4*x^5 + 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7
)
```

giac [A] time = 0.12, size = 82, normalized size = 0.91

$$c^4x-\frac{2\,c^4\log\left(|x|\right)}{a}+\frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^4*x - 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 31
5*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

maple [A] time = 0.04, size = 83, normalized size = 0.92

$$\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{x^2a^3}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/x^2/a
^3+2*c^4/a^2/x+c^4*x-2*c^4*ln(x)/a
```

maxima [A] time = 0.30, size = 81, normalized size = 0.90

$$c^4x - \frac{2\,c^4\log(x)}{a} + \frac{420\,a^6c^4x^6 - 630\,a^5c^4x^5 + 315\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x + 30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^4*x - 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 315*a^3
*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

mupad [B] time = 1.32, size = 67, normalized size = 0.74

$$-\frac{c^4\left(\frac{a\,x}{3} + \frac{2\,a^2\,x^2}{5} - \frac{3\,a^3\,x^3}{2} + 3\,a^5\,x^5 - 2\,a^6\,x^6 - a^8\,x^8 + 2\,a^7\,x^7\,\ln(x) - \frac{1}{7}\right)}{a^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] -(c^4*((a*x)/3 + (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 - 2*a^6*x^6 - a^
8*x^8 + 2*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

sympy [A] time = 0.46, size = 88, normalized size = 0.98

$$\frac{a^8c^4x - 2a^7c^4\log(x) + \frac{420a^6c^4x^6 - 630a^5c^4x^5 + 315a^3c^4x^3 - 84a^2c^4x^2 - 70ac^4x + 30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**8*c**4*x - 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 - 630*a**5*c**4*x**
5 + 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**4*x + 30*c**4)/(210*x*
*7))/a**8
```

3.815 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$

Optimal. Leaf size=76

$$-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3x$$

```
[Out] -1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3-2*c^3/a^3/x^2+c^3/a^2/x+c^
3*x-2*c^3*ln(x)/a
```

Rubi [A] time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$-\frac{2c^3}{a^3x^2}+\frac{c^3}{3a^4x^3}+\frac{c^3}{2a^5x^4}-\frac{c^3}{5a^6x^5}+\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -c^3/(5*a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2) +
c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx$$
$$= \frac{c^3\int \frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}dx}{a^6}$$
$$= \frac{c^3\int \frac{(1-ax)^4(1+ax)^2}{x^6}dx}{a^6}$$
$$= \frac{c^3\int \left(a^6+\frac{1}{x^6}-\frac{2a}{x^5}-\frac{a^2}{x^4}+\frac{4a^3}{x^3}-\frac{a^4}{x^2}-\frac{2a^5}{x}\right)dx}{a^6}$$
$$= -\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 1.00

$$-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/5*c^3/(a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

fricas [A] time = 0.47, size = 78, normalized size = 1.03

$$\frac{30\,a^6c^3x^6-60\,a^5c^3x^5\log(x)+30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/30*(30*a^6*c^3*x^6 - 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 - 60*a^3*c^3*
x^3 + 10*a^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

giac [A] time = 0.12, size = 71, normalized size = 0.93

$$c^3x-\frac{2\,c^3\log\left(|x|\right)}{a}+\frac{30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^3*x - 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^
2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

maple [A] time = 0.04, size = 71, normalized size = 0.93

$$-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{x^2a^3}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3-2*c^3/x^2/a^3+c^3/a^2/x+c^
3*x-2*c^3*ln(x)/a
```

maxima [A] time = 0.30, size = 70, normalized size = 0.92

$$c^3x-\frac{2\,c^3\log(x)}{a}+\frac{30\,a^4c^3x^4-60\,a^3c^3x^3+10\,a^2c^3x^2+15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^3*x - 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^2*c^3
*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

mupad [B] time = 1.25, size = 56, normalized size = 0.74

$$\frac{c^3\,\left(\frac{a\,x}{2}+\frac{a^2\,x^2}{3}-2\,a^3\,x^3+a^4\,x^4+a^6\,x^6-2\,a^5\,x^5\,\ln(x)-\frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (c^3*((a*x)/2 + (a^2*x^2)/3 - 2*a^3*x^3 + a^4*x^4 + a^6*x^6 - 2*a^5*x^5*log
(x) - 1/5))/(a^6*x^5)
```

sympy [A] time = 0.33, size = 76, normalized size = 1.00

$$\frac{a^6c^3x-2a^5c^3\log\left(x\right)+\frac{30a^4c^3x^4-60a^3c^3x^3+10a^2c^3x^2+15ac^3x-6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**6*c**3*x - 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 - 60*a**3*c**3*x**3
+ 10*a**2*c**3*x**2 + 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

3.816 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$

Optimal. Leaf size=40

$$\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}-\frac{2c^2\log(x)}{a}+c^2x$$

```
[Out] 1/3*c^2/a^4/x^3-c^2/a^3/x^2+c^2*x-2*c^2*ln(x)/a
```

Rubi [A] time = 0.14, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 75}

$$-\frac{c^2}{a^3x^2}+\frac{c^2}{3a^4x^3}-\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$= -\frac{c^2\int \frac{e^{-2\tanh^{-1}(ax)}(1-a^2x^2)^2}{x^4} dx}{a^4}$$
$$= -\frac{c^2\int \frac{(1-ax)^3(1+ax)}{x^4} dx}{a^4}$$
$$= -\frac{c^2\int \left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right) dx}{a^4}$$
$$= \frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.00

$$\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}-\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

fricas [A] time = 0.43, size = 43, normalized size = 1.08

$$\frac{3\,a^4c^2x^4-6\,a^3c^2x^3\log(x)-3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/3*(3*a^4*c^2*x^4 - 6*a^3*c^2*x^3*log(x) - 3*a*c^2*x + c^2)/(a^4*x^3)
```

giac [A] time = 0.13, size = 38, normalized size = 0.95

$$c^2x-\frac{2\,c^2\log(|x|)}{a}-\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c^2*x - 2*c^2*log(abs(x))/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

maple [A] time = 0.04, size = 39, normalized size = 0.98

$$\frac{c^2}{3a^4x^3}-\frac{c^2}{x^2a^3}+c^2x-\frac{2c^2\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2/(a*x+1)*(a*x-1),x)
[Out] 1/3*c^2/a^4/x^3-c^2/x^2/a^3+c^2*x-2*c^2*ln(x)/a
```

maxima [A] time = 0.31, size = 37, normalized size = 0.92

$$c^2x-\frac{2\,c^2\log(x)}{a}-\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c^2*x - 2*c^2*log(x)/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.88

$$-\frac{c^2\,\left(3\,a\,x-3\,a^4\,x^4+6\,a^3\,x^3\,\ln(x)-1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x - 1))/(a*x + 1),x)
[Out] -(c^2*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

sympy [A] time = 0.18, size = 39, normalized size = 0.98

$$\frac{a^4c^2x-2a^3c^2\log{(x)}+\frac{-3ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*(a*x-1)/(a*x+1),x)
[Out] (a**4*c**2*x - 2*a**3*c**2*log(x) + (-3*a*c**2*x + c**2)/(3*x**3))/a**4
```

3.817 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$

Optimal. Leaf size=21

$$-\frac{c}{a^2x}-\frac{2c\log(x)}{a}+cx$$

```
[Out] -c/a^2/x+c*x-2*c*ln(x)/a
```

Rubi [A] time = 0.08, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6167, 6157, 6150, 43}

$$-\frac{c}{a^2x}-\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$
$$= \frac{c\int \frac{e^{-2\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}dx}{a^2}$$
$$= \frac{c\int \frac{(1-ax)^2}{x^2}dx}{a^2}$$
$$= \frac{c\int \left(a^2+\frac{1}{x^2}-\frac{2a}{x}\right)dx}{a^2}$$
$$= -\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$-\frac{c}{a^2x}-\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

fricas [A] time = 0.51, size = 26, normalized size = 1.24

$$\frac{a^2cx^2-2\,acx\log(x)-c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a^2*c*x^2 - 2*a*c*x*log(x) - c)/(a^2*x)
```

giac [A] time = 0.13, size = 22, normalized size = 1.05

$$cx-\frac{2\,c\log(|x|)}{a}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c*x - 2*c*log(abs(x))/a - c/(a^2*x)
```

maple [A] time = 0.04, size = 22, normalized size = 1.05

$$-\frac{c}{a^2x}+cx-\frac{2c\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)/(a*x+1)*(a*x-1),x)
[Out] -c/a^2/x+c*x-2*c*ln(x)/a
```

maxima [A] time = 0.31, size = 21, normalized size = 1.00

$$cx-\frac{2\,c\log(x)}{a}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c*x - 2*c*log(x)/a - c/(a^2*x)
```

mupad [B] time = 1.23, size = 25, normalized size = 1.19

$$-\frac{c\left(2\,a\,x\,\ln(x)-a^2\,x^2+1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x - 1))/(a*x + 1),x)
```

```
[Out] -(c*(2*a*x*log(x) - a^2*x^2 + 1))/(a^2*x)
```

sympy [A] time = 0.11, size = 20, normalized size = 0.95

$$\frac{a^2cx-2ac\log\left(x\right)-\frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**2*c*x - 2*a*c*log(x) - c/x)/a**2
```

3.818 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{ac(ax+1)}-\frac{2\log(ax+1)}{ac}+\frac{x}{c}$$

```
[Out] x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

Rubi [A] time = 0.16, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 43}

$$-\frac{1}{ac(ax+1)}-\frac{2\log(ax+1)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] x/c - 1/(a*c*(1 + a*x)) - (2*Log[1 + a*x])/(a*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx \\
&= \frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c} \\
&= \frac{a^2\int \frac{x^2}{(1+ax)^2}\,dx}{c} \\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(1+ax)^2}-\frac{2}{a^2(1+ax)}\right)dx}{c} \\
&= \frac{x}{c}-\frac{1}{ac(1+ax)}-\frac{2\log(1+ax)}{ac}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.80

$$
\frac{-\frac{1}{a^2x+a}-\frac{2\log(ax+1)}{a}+x}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
[Out] (x - (a + a^2*x)^(-1) - (2*Log[1 + a*x])/a)/c
```

fricas [A] time = 0.56, size = 38, normalized size = 1.09

$$
\frac{a^2x^2+ax-2\,(ax+1)\log\left(ax+1\right)-1}{a^2cx+ac}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*x^2 + a*x - 2*(a*x + 1)*log(a*x + 1) - 1)/(a^2*c*x + a*c)
```

giac [A] time = 0.12, size = 36, normalized size = 1.03

$$
\frac{x}{c}-\frac{2\,\log\left(|ax+1|\right)}{ac}-\frac{1}{(ax+1)ac}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] x/c - 2*log(abs(a*x + 1))/(a*c) - 1/((a*x + 1)*a*c)
```

maple [A] time = 0.04, size = 36, normalized size = 1.03

$$
\frac{x}{c}-\frac{1}{ac\left(ax+1\right)}-\frac{2\ln\left(ax+1\right)}{ac}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2),x)
[Out] x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

maxima [A] time = 0.31, size = 34, normalized size = 0.97

$$\frac{x}{c} - \frac{1}{a^2cx + ac} - \frac{2\log(ax + 1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] x/c - 1/(a^2*c*x + a*c) - 2*log(a*x + 1)/(a*c)
```

mupad [B] time = 0.05, size = 33, normalized size = 0.94

$$\frac{x}{c} - \frac{1}{a\,(c + a\,c\,x)} - \frac{2\,\ln(a\,x + 1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))*(a*x + 1)),x)
[Out] x/c - 1/(a*(c + a*c*x)) - (2*log(a*x + 1))/(a*c)
```

sympy [A] time = 0.15, size = 36, normalized size = 1.03

$$a^2\left(-\frac{1}{a^4cx + a^3c} + \frac{x}{a^2c} - \frac{2\log(ax + 1)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2),x)
[Out] a**2*(-1/(a**4*c*x + a**3*c) + x/(a**2*c) - 2*log(a*x + 1)/(a**3*c))
```

3.819 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{7}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

```
[Out] x/c^2+1/4/a/c^2/(a*x+1)^2-7/4/a/c^2/(a*x+1)+1/8*ln(-a*x+1)/a/c^2-17/8*ln(a*
x+1)/a/c^2
```

Rubi [A] time = 0.17, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$-\frac{7}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] x/c^2 + 1/(4*a*c^2*(1 + a*x)^2) - 7/(4*a*c^2*(1 + a*x)) + Log[1 - a*x]/(8*a
*c^2) - (17*Log[1 + a*x])/(8*a*c^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$
$$= -\frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}$$
$$= -\frac{a^4\int \frac{x^4}{(1-ax)(1+ax)^3}\,dx}{c^2}$$
$$= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{8a^4(-1+ax)}+\frac{1}{2a^4(1+ax)^3}-\frac{7}{4a^4(1+ax)^2}+\frac{17}{8a^4(1+ax)}\right)dx}{c^2}$$
$$= \frac{x}{c^2}+\frac{1}{4ac^2(1+ax)^2}-\frac{7}{4ac^2(1+ax)}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(1+ax)}{8ac^2}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.96

$$\frac{2\left(4a^3x^3+8a^2x^2-3ax-6\right)+(ax+1)^2\log(1-ax)-17(ax+1)^2\log(ax+1)}{8a(acx+c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (2*(-6 - 3*a*x + 8*a^2*x^2 + 4*a^3*x^3) + (1 + a*x)^2*Log[1 - a*x] - 17*(1
+ a*x)^2*Log[1 + a*x])/(8*a*(c + a*c*x)^2)
```

fricas [A] time = 0.54, size = 92, normalized size = 1.26

$$\frac{8\,a^3x^3+16\,a^2x^2-6\,ax-17\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)+\left(a^2x^2+2\,ax+1\right)\log\left(ax-1\right)-12}{8\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 + 16*a^2*x^2 - 6*a*x - 17*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
 + (a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 12)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*
c^2)
```

giac [A] time = 0.13, size = 57, normalized size = 0.78

$$\frac{x}{c^2}-\frac{17\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}-\frac{7\,ax+6}{4\left(ax+1\right)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 17/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) - 1/
4*(7*a*x + 6)/((a*x + 1)^2*a*c^2)
```

maple [A] time = 0.04, size = 65, normalized size = 0.89

$$\frac{x}{c^2}+\frac{\ln\left(ax-1\right)}{8a\,c^2}+\frac{1}{4a\,c^2\left(ax+1\right)^2}-\frac{7}{4a\,c^2\left(ax+1\right)}-\frac{17\ln\left(ax+1\right)}{8a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^2,x)
```

```
[Out] x/c^2+1/8/a/c^2*ln(a*x-1)+1/4/a/c^2/(a*x+1)^2-7/4/a/c^2/(a*x+1)-17/8*ln(a*x
+1)/a/c^2
```

maxima [A] time = 0.31, size = 69, normalized size = 0.95

$$-\frac{7\,ax+6}{4\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}+\frac{x}{c^2}-\frac{17\,\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x + 6)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) + x/c^2 - 17/8*log(a*x
 + 1)/(a*c^2) + 1/8*log(a*x - 1)/(a*c^2)
```

mupad [B] time = 1.31, size = 68, normalized size = 0.93

$$\frac{x}{c^2}-\frac{\frac{7\,x}{4}+\frac{3}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2}+\frac{\ln\left(a\,x-1\right)}{8\,a\,c^2}-\frac{17\,\ln\left(a\,x+1\right)}{8\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^2*(a*x + 1)),x)
```

```
[Out] x/c^2 - ((7*x)/4 + 3/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) + log(a*x - 1)/
(8*a*c^2) - (17*log(a*x + 1))/(8*a*c^2)
```

sympy [A] time = 0.38, size = 75, normalized size = 1.03

$$a^4\left(\frac{-7ax-6}{4a^7c^2x^2+8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{17\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-7*a*x - 6)/(4*a**7*c**2*x**2 + 8*a**6*c**2*x + 4*a**5*c**2) + x/(a*
*4*c**2) + (log(x - 1/a)/8 - 17*log(x + 1/a)/8)/(a**5*c**2))
```

3.820 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{1}{16ac^3(1-ax)}-\frac{39}{16ac^3(ax+1)}+\frac{5}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

```
[Out] x/c^3+1/16/a/c^3/(-a*x+1)-1/12/a/c^3/(a*x+1)^3+5/8/a/c^3/(a*x+1)^2-39/16/a/
c^3/(a*x+1)+1/4*ln(-a*x+1)/a/c^3-9/4*ln(a*x+1)/a/c^3
```

Rubi [A] time = 0.20, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{1}{16ac^3(1-ax)}-\frac{39}{16ac^3(ax+1)}+\frac{5}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] x/c^3 + 1/(16*a*c^3*(1 - a*x)) - 1/(12*a*c^3*(1 + a*x)^3) + 5/(8*a*c^3*(1 +
 a*x)^2) - 39/(16*a*c^3*(1 + a*x)) + Log[1 - a*x]/(4*a*c^3) - (9*Log[1 + a*
x])/(4*a*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$
$$= \frac{a^6\int \frac{e^{-2\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}$$
$$= \frac{a^6\int \frac{x^6}{(1-ax)^2(1+ax)^4}\,dx}{c^3}$$
$$= \frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{16a^6(-1+ax)^2}+\frac{1}{4a^6(-1+ax)}+\frac{1}{4a^6(1+ax)^4}-\frac{5}{4a^6(1+ax)^3}+\frac{39}{16a^6(1+ax)^2}-\frac{9}{4a^6(1+ax)}\right)dx}{c^3}$$
$$= \frac{x}{c^3}+\frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}+\frac{5}{8ac^3(1+ax)^2}-\frac{39}{16ac^3(1+ax)}+\frac{\log(1-ax)}{4ac^3}-\frac{9\,\mathrm{log}}{}$$

Mathematica [A] time = 0.10, size = 104, normalized size = 0.96

$$\frac{2\left(6a^5x^5+12a^4x^4-15a^3x^3-24a^2x^2+7ax+11\right)+3(ax-1)(ax+1)^3\log(1-ax)-27(ax-1)(ax+1)^3\log(ax+}{12a(ax-1)(acx+c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] (2*(11 + 7*a*x - 24*a^2*x^2 - 15*a^3*x^3 + 12*a^4*x^4 + 6*a^5*x^5) + 3*(-1
+ a*x)*(1 + a*x)^3*Log[1 - a*x] - 27*(-1 + a*x)*(1 + a*x)^3*Log[1 + a*x])/(
12*a*(-1 + a*x)*(c + a*c*x)^3)
```

fricas [A] time = 0.57, size = 137, normalized size = 1.27

$$\frac{12\,a^5x^5+24\,a^4x^4-30\,a^3x^3-48\,a^2x^2+14\,ax-27\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log(ax+1)+3\left(a^4x^4+2\,a^3x^3-2\,a\right.}{12\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*x^5 + 24*a^4*x^4 - 30*a^3*x^3 - 48*a^2*x^2 + 14*a*x - 27*(a^4*
x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x
- 1)*log(a*x - 1) + 22)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

giac [A] time = 0.15, size = 80, normalized size = 0.74

$$\frac{x}{c^3}-\frac{9\,\log(|ax+1|)}{4\,ac^3}+\frac{\log(|ax-1|)}{4\,ac^3}-\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(ax+1)^3(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 9/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) - 1/6
*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

maple [A] time = 0.05, size = 95, normalized size = 0.88

$$\frac{x}{c^3}-\frac{1}{16a\,c^3\,(ax-1)}+\frac{\ln(ax-1)}{4c^3a}-\frac{1}{12a\,c^3\,(ax+1)^3}+\frac{5}{8a\,c^3\,(ax+1)^2}-\frac{39}{16a\,c^3\,(ax+1)}-\frac{9\ln(ax+1)}{4a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^3,x)
```

```
[Out] x/c^3-1/16/a/c^3/(a*x-1)+1/4/c^3/a*ln(a*x-1)-1/12/a/c^3/(a*x+1)^3+5/8/a/c^3
/(a*x+1)^2-39/16/a/c^3/(a*x+1)-9/4*ln(a*x+1)/a/c^3
```

maxima [A] time = 0.31, size = 97, normalized size = 0.90

$$-\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}+\frac{x}{c^3}-\frac{9\,\log\left(ax+1\right)}{4\,ac^3}+\frac{\log\left(ax-1\right)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
 2*a^2*c^3*x - a*c^3) + x/c^3 - 9/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)
/(a*c^3)
```

mupad [B] time = 0.12, size = 94, normalized size = 0.87

$$\frac{x}{c^3}-\frac{\frac{13\,x}{6}-2\,a\,x^2+\frac{11}{6\,a}-\frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}+\frac{\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{9\,\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^3*(a*x + 1)),x)
```

```
[Out] x/c^3 - ((13*x)/6 - 2*a*x^2 + 11/(6*a) - (5*a^2*x^3)/2)/(c^3 - 2*a^3*c^3*x^
3 - a^4*c^3*x^4 + 2*a*c^3*x) + log(a*x - 1)/(4*a*c^3) - (9*log(a*x + 1))/(4
*a*c^3)
```

sympy [A] time = 0.61, size = 102, normalized size = 0.94

$$a^6\left(\frac{-15a^3x^3-12a^2x^2+13ax+11}{6a^{11}c^3x^4+12a^{10}c^3x^3-12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{4}-\frac{9\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-15*a**3*x**3 - 12*a**2*x**2 + 13*a*x + 11)/(6*a**11*c**3*x**4 + 12*
a**10*c**3*x**3 - 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (log(x -
1/a)/4 - 9*log(x + 1/a)/4)/(a**7*c**3))
```

3.821 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=143

$$\frac{11}{64ac^4(1-ax)}-\frac{99}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{35}{32ac^4(ax+1)^2}-\frac{13}{48ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}+\frac{47\log(1-ax)}{128ac^4}$$

```
[Out] x/c^4-1/64/a/c^4/(-a*x+1)^2+11/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4-13/48
/a/c^4/(a*x+1)^3+35/32/a/c^4/(a*x+1)^2-99/32/a/c^4/(a*x+1)+47/128*ln(-a*x+1
)/a/c^4-303/128*ln(a*x+1)/a/c^4
```

Rubi [A] time = 0.23, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6167, 6157, 6150, 88}

$$\frac{11}{64ac^4(1-ax)}-\frac{99}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{35}{32ac^4(ax+1)^2}-\frac{13}{48ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}+\frac{47\log(1-ax)}{128ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] x/c^4 - 1/(64*a*c^4*(1 - a*x)^2) + 11/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1
 + a*x)^4) - 13/(48*a*c^4*(1 + a*x)^3) + 35/(32*a*c^4*(1 + a*x)^2) - 99/(32
*a*c^4*(1 + a*x)) + (47*Log[1 - a*x])/(128*a*c^4) - (303*Log[1 + a*x])/(128
*a*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6150

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6157

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

$$= -\frac{a^8\int \frac{e^{-2\tanh^{-1}(ax)}x^8}{\left(1-a^2x^2\right)^4}\,dx}{c^4}$$

$$= -\frac{a^8\int \frac{x^8}{(1-ax)^3(1+ax)^5}\,dx}{c^4}$$

$$= -\frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{32a^8(-1+ax)^3}-\frac{11}{64a^8(-1+ax)^2}-\frac{47}{128a^8(-1+ax)}+\frac{1}{8a^8(1+ax)^5}-\frac{13}{16a^8(1+ax)^4}+\frac{35}{16a^8(1+ax)^3}\right.}{c^4}$$

$$= \frac{x}{c^4}-\frac{1}{64ac^4(1-ax)^2}+\frac{11}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}-\frac{13}{48ac^4(1+ax)^3}+\frac{35}{32ac^4(1+ax)}$$

Mathematica [A] time = 0.13, size = 124, normalized size = 0.87

$$\frac{2\left(192a^7x^7+384a^6x^6-819a^5x^5-1254a^4x^4+866a^3x^3+1258a^2x^2-275ax-400\right)+141(ax-1)^2(ax+1)^4\log}{384a(ax-1)^2(acx+c)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (2*(-400 - 275*a*x + 1258*a^2*x^2 + 866*a^3*x^3 - 1254*a^4*x^4 - 819*a^5*x^
5 + 384*a^6*x^6 + 192*a^7*x^7) + 141*(-1 + a*x)^2*(1 + a*x)^4*Log[1 - a*x]
- 909*(-1 + a*x)^2*(1 + a*x)^4*Log[1 + a*x])/(384*a*(-1 + a*x)^2*(c + a*c*x
)^4)
```

fricas [A] time = 0.59, size = 233, normalized size = 1.63

$$\frac{384\,a^7x^7+768\,a^6x^6-1638\,a^5x^5-2508\,a^4x^4+1732\,a^3x^3+2516\,a^2x^2-550\,ax-909\left(a^6x^6+2\,a^5x^5-a^4x^4-\right.}{384\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/384*(384*a^7*x^7 + 768*a^6*x^6 - 1638*a^5*x^5 - 2508*a^4*x^4 + 1732*a^3*x
^3 + 2516*a^2*x^2 - 550*a*x - 909*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^
3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + 141*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
- 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 800)/(a^7*c^4*x^6 + 2*a^6
*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

giac [A] time = 0.14, size = 96, normalized size = 0.67

$$\frac{x}{c^4}-\frac{303\,\log\left(|ax+1|\right)}{128\,ac^4}+\frac{47\,\log\left(|ax-1|\right)}{128\,ac^4}-\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\,(ax+1)^4(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 303/128*log(abs(a*x + 1))/(a*c^4) + 47/128*log(abs(a*x - 1))/(a*c^4
) - 1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x
 + 400)/((a*x + 1)^4*(a*x - 1)^2*a*c^4)
```

maple [A] time = 0.05, size = 125, normalized size = 0.87

$$\frac{x}{c^4}-\frac{1}{64c^4a\,(ax-1)^2}-\frac{11}{64a\,c^4\,(ax-1)}+\frac{47\ln(ax-1)}{128c^4a}+\frac{1}{32a\,c^4\,(ax+1)^4}-\frac{13}{48a\,c^4\,(ax+1)^3}+\frac{35}{32a\,c^4\,(ax+1)^2}-\frac{9}{32a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^4,x)
```

```
[Out] x/c^4-1/64/c^4/a/(a*x-1)^2-11/64/a/c^4/(a*x-1)+47/128/c^4/a*ln(a*x-1)+1/32/
a/c^4/(a*x+1)^4-13/48/a/c^4/(a*x+1)^3+35/32/a/c^4/(a*x+1)^2-99/32/a/c^4/(a*
x+1)-303/128*ln(a*x+1)/a/c^4
```

maxima [A] time = 0.32, size = 145, normalized size = 1.01

$$-\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}-\frac{303\,\log(ax+1)}{128\,ac^4}+\frac{47\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x +
400)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x
^2 + 2*a^2*c^4*x + a*c^4) + x/c^4 - 303/128*log(a*x + 1)/(a*c^4) + 47/128*l
og(a*x - 1)/(a*c^4)
```

mupad [B] time = 0.15, size = 142, normalized size = 0.99

$$\frac{x}{c^4}-\frac{\frac{467\,x}{192}-\frac{437\,a\,x^2}{96}+\frac{25}{12\,a}-\frac{529\,a^2\,x^3}{96}+\frac{81\,a^3\,x^4}{32}+\frac{209\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4}+\frac{47\,\ln(a\,x-1)}{128\,a\,c^4}-\frac{303\,\ln(a\,x+1)}{128\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^4*(a*x + 1)),x)
```

```
[Out] x/c^4 - ((467*x)/192 - (437*a*x^2)/96 + 25/(12*a) - (529*a^2*x^3)/96 + (81*
a^3*x^4)/32 + (209*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^
4*x^4 + 2*a^5*c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) + (47*log(a*x - 1))/(128*a
*c^4) - (303*log(a*x + 1))/(128*a*c^4)
```

sympy [A] time = 0.85, size = 156, normalized size = 1.09

$$a^8\left(\frac{-627a^5x^5-486a^4x^4+1058a^3x^3+874a^2x^2-467ax-400}{192a^{15}c^4x^6+384a^{14}c^4x^5-192a^{13}c^4x^4-768a^{12}c^4x^3-192a^{11}c^4x^2+384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{47\log\left(x-\frac{1}{a}\right)}{128}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-627*a**5*x**5 - 486*a**4*x**4 + 1058*a**3*x**3 + 874*a**2*x**2 - 46
7*a*x - 400)/(192*a**15*c**4*x**6 + 384*a**14*c**4*x**5 - 192*a**13*c**4*x*
*4 - 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 + 384*a**10*c**4*x + 192*a**
9*c**4) + x/(a**8*c**4) + (47*log(x - 1/a)/128 - 303*log(x + 1/a)/128)/(a**
9*c**4))
```

3.822 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=343

$$c^4x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{11/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/2}}{7a}+\frac{17c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}}{14a}+\frac{11c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)}{10a}$$

```
[Out] 5/8*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)/a+11/10*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(5/2)/a+17/14*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)/a+8/7*c^4*(1-1/a/x)^(9
/2)*(1+1/a/x)^(5/2)/a+c^4*(1-1/a/x)^(11/2)*(1+1/a/x)^(5/2)*x+15/16*c^4*arcc
sc(a*x)/a-3*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+27/16*c^4*(1+1/a
/x)^(3/2)*(1-1/a/x)^(1/2)/a-3/8*c^4*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a+33/16
*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.25, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^4x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{11/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/2}}{7a}+\frac{17c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}}{14a}+\frac{11c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (33*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) + (27*c^4*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(8*a) + (5*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(8*a) + (11*c
^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(10*a) + (17*c^4*(1 - 1/(a*x))^
(7/2)*(1 + 1/(a*x))^(5/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x)
)^(5/2))/(7*a) + c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^(5/2)*x + (15*c^4*A
rcCsc[a*x])/(16*a) - (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^4\,\text{Subst}\left(\int \frac{\left(-\frac{3}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x}dx\right.$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{7}\left(ac^4\right)\text{Subst}\left(\int\right.$$

$$= \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1\right.$$

$$= \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1\right.}{7a}$$

$$= \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1\right.}{14a}$$

$$= -\frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\right.}{10a}$$

$$= \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)}{8a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}$$

Mathematica [A] time = 0.31, size = 126, normalized size = 0.37

$$\frac{c^4\left(525a^6x^6\sin^{-1}\left(\frac{1}{ax}\right) - 1680a^6x^6\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right) + \sqrt{1-\frac{1}{a^2x^2}}\left(560a^7x^7 + 2496a^6x^6 - 525a^5x^5 - 99\right.\right.}{560a^7x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

[Out] (c^4*(Sqrt[1 - 1/(a^2*x^2)]*(80 - 280*a*x + 96*a^2*x^2 + 770*a^3*x^3 - 992*
a^4*x^4 - 525*a^5*x^5 + 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[1/
(a*x)] - 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)

fricas [A] time = 0.49, size = 201, normalized size = 0.59

$$-\frac{1050\,a^7c^4x^7\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^7c^4x^7\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(560\,a^8c^4x^8+30\right.}{560\,a^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] -1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*x
^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 + 3056*a^7*c^4*x^7 + 1971*a^6*c^4*x^6 -
 1517*a^5*c^4*x^5 - 222*a^4*c^4*x^4 + 866*a^3*c^4*x^3 - 184*a^2*c^4*x^2 - 2
00*a*c^4*x + 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)

giac [A] time = 0.25, size = 525, normalized size = 1.53

$$-\frac{15\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{8\,a}+\frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^4\operatorname{sgn}(a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")

[Out] -15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^4*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)
*c^4*sgn(a*x + 1)/a + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*abs(
a)*sgn(a*x + 1) + 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x + 1)
 - 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) + 20160*(x
*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 945*(x*abs(a) - sqrt(a
^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) + 38080*(x*abs(a) - sqrt(a^2*x^2 - 1
))^8*a*c^4*sgn(a*x + 1) + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4*sgn(
a*x + 1) - 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) + 3
2256*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 980*(x*abs(a) -
sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) + 12992*(x*abs(a) - sqrt(a^2*x
^2 - 1))^2*a*c^4*sgn(a*x + 1) - 525*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(
a)*sgn(a*x + 1) + 2496*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))
^2 + 1)^7*a*abs(a))

maple [A] time = 0.07, size = 329, normalized size = 0.96

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^8a^8+1680\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^6a^6-525a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}-525a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x)

[Out] -1/560*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)
^(1/2)*x^8*a^8+1680*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^6*a^6-525*a^7*x^7*(a^2)
^(1/2)*(a^2*x^2-1)^(1/2)-525*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2)
)+1680*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8-35*(a^
2*x^2-1)^(3/2)*(a^2)^(1/2)*x^5*a^5-816*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^

```
4+490*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^3*a^3+176*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*x^2*a^2-280*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a+80*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2))/(a*x-1)/((a*x-1)*(a*x+1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

maxima [A] time = 0.43, size = 379, normalized size = 1.10

$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{1155\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+766}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 + (1155*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 7665*c^4*((a*x - 1)/(a*x + 1))
^(13/2) + 20811*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 12799*c^4*((a*x - 1)/(a*
x + 1))^(9/2) - 39071*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 33621*c^4*((a*x - 1
)/(a*x + 1))^(5/2) - 13615*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 2205*c^4*sqrt(
(a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)^
8*a^2/(a*x + 1)^8 + a^2))*a
```

mupad [B] time = 0.17, size = 332, normalized size = 0.97

$$\frac{\frac{63\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{389\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8}+\frac{4803\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}+\frac{39071\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{280}+\frac{12799\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}-\frac{2973\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}-\frac{219\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13}}{8}}{a+\frac{6\,a\,(a\,x-1)}{a\,x+1}+\frac{14\,a\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{14\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{14\,a\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{14\,a\,(a\,x-1)^6}{(a\,x+1)^6}-\frac{6\,a\,(a\,x-1)^7}{(a\,x+1)^7}-\frac{a\,(a\,x-1)^8}{(a\,x+1)^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((63*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (389*c^4*((a*x - 1)/(a*x + 1))^(3
/2))/8 + (4803*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (39071*c^4*((a*x - 1)/
(a*x + 1))^(7/2))/280 + (12799*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 - (2973
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 - (219*c^4*((a*x - 1)/(a*x + 1))^(13/
2))/8 - (33*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x
+ 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (1
4*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x -
 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((a*x - 1)
/(a*x + 1))^(1/2)))/(8*a) - (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.823 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=269

$$c^3x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}}{5a}+\frac{27c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}}{20a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+$$

```
[Out] 5/4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)/a+27/20*c^3*(1-1/a/x)^(5/2)*(1+1/a/
x)^(3/2)/a+6/5*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)/a+c^3*(1-1/a/x)^(9/2)*(1
+1/a/x)^(3/2)*x+3/8*c^3*arccsc(a*x)/a-3*c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2))/a+3/8*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+21/8*c^3*(1-1/a/x)^(1
/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A] time = 0.18, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^3x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}}{5a}+\frac{27c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}}{20a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (21*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (3*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(8*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3
/2))/(4*a) + (27*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2))/(20*a) + (6*c
^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2))/(5*a) + c^3*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(3/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) - (3*c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - c^3\,\text{Subst}\left(\int \frac{\left(-\frac{3}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x}dx, x, \frac{1}{x}\right.$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{5}\left(ac^3\right)\text{Subst}\left(\int \frac{(-\right.$$

$$= \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{a\right.$$

$$= \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{a\right.}{5a}$$

$$= \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)}{20a}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} +$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} +$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} +$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} +$$

Mathematica [A] time = 0.25, size = 110, normalized size = 0.41

$$\frac{c^3\left(15a^4x^4\sin^{-1}\left(\frac{1}{ax}\right)-120a^4x^4\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)+\sqrt{1-\frac{1}{a^2x^2}}\left(40a^5x^5+152a^4x^4-55a^3x^3-24a^2x^2+30\right.\right.}{40a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(-8 + 30*a*x - 24*a^2*x^2 - 55*a^3*x^3 + 152*a^
4*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

fricas [A] time = 0.44, size = 179, normalized size = 0.67

$$-\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(40\,a^6c^3x^6+192\,a^5c^3\right.}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (40*a^6*c^3*x^6 + 192*a^5*c^3*x^5 + 97*a^4*c^3*x^4 - 79*a^3*c
^3*x^3 + 6*a^2*c^3*x^2 + 22*a*c^3*x - 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^
6*x^5)
```

giac [A] time = 0.20, size = 395, normalized size = 1.47

$$-\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{4\,a}+\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^3\operatorname{sgn}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^3*log(a
bs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*
c^3*sgn(a*x + 1)/a + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a)*s
gn(a*x + 1) + 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) - 10*
(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) + 720*(x*abs(a) -
sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) + 800*(x*abs(a) - sqrt(a^2*x^2 - 1)
)^4*a*c^3*sgn(a*x + 1) + 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn
(a*x + 1) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - 55*(x
*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 152*a*c^3*sgn(a*x +
1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

maple [A] time = 0.06, size = 281, normalized size = 1.04

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(-120\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^6a^6+120\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^4a^4-15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^5a^5-15\,\mathrm{ar}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/40*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^3*(-120*(a^2*x^2-1)^(1/2)*(a^2)^(
1/2)*x^6*a^6+120*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^4*a^4-15*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*x^5*a^5-15*arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)*x^5*a^5+120
*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6-25*(a^2*x^2-
1)^(3/2)*(a^2)^(1/2)*x^3*a^3-32*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+30*(a
^2*x^2-1)^(3/2)*(a^2)^(1/2)*x*a-8*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/(a*x-1)/((
a*x-1)*(a*x+1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 301, normalized size = 1.12

$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+465\,c^3\left(\frac{ax-}{ax-}\right.}{\frac{4\,(ax-1)a^2}{ax+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 465*c^3*((a*x - 1)/(a*x + 1))^(9/2) - 298*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 842*c^3*((a*x - 1)/(a*x + 1))^(5/2)) - 575*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 135*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x + 1)^6 + a^2))*a

mupad [B] time = 1.26, size = 258, normalized size = 0.96

$$\frac{\frac{27\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{115\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{421\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10}+\frac{149\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{10}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}-\frac{21\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{6\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2),x)

[Out] ((27*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (115*c^3*((a*x - 1)/(a*x + 1))^(3/2))/4 + (421*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 + (149*c^3*((a*x - 1)/(a*x + 1))^(7/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 - (21*c^3*((a*x - 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a) - (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^7+x^6}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^6+x^5}\right)dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{3a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{3a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(3/2),x)

[Out] c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**7 + x**6), x) + Integral(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**6 + x**5), x) + Integral(-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Integral(3*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(3*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-3*a**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**7*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**6

3.824 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$c^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{4c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{3a}+\frac{11c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{6a}+\frac{5c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\csc^{-1}(}{2a}$$

```
[Out] -1/2*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+11/
6*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/a+4/3*c^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(
1/2)/a+c^2*(1-1/a/x)^(7/2)*x*(1+1/a/x)^(1/2)+5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a
/x)^(1/2)/a
```

Rubi [A] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 97, 154, 157, 41, 216, 92, 208}

$$c^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{4c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{3a}+\frac{11c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{6a}+\frac{5c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\csc^{-1}(}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) + (11*c^2*(1 - 1/(a*x))^(
3/2)*Sqrt[1 + 1/(a*x)])/(6*a) + (4*c^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)
])/(3*a) + c^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x - (c^2*ArcCsc[a*x])/
(2*a) - (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\left(c^2\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x - c^2\operatorname{Subst}\left(\int \frac{\left(-\frac{3}{a}-\frac{4x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$= \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x - \frac{1}{3}\left(ac^2\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{9}{a^{\cdot}}\right.}{}\right.$$

$$= \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}$$

$$= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}$$

$$= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}$$

$$= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}$$

$$= \frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}$$

Mathematica [A] time = 0.17, size = 94, normalized size = 0.48

$$\frac{c^2\left(-18a^2x^2\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-3a^2x^2\sin^{-1}\left(\frac{1}{ax}\right)+\sqrt{1-\frac{1}{a^2x^2}}\left(6a^3x^3+16a^2x^2-9ax+2\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 - 9*a*x + 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x^
2*ArcSin[1/(a*x)] - 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*
x^2)
```

fricas [A] time = 0.52, size = 156, normalized size = 0.80

$$\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^2x^4+22\,a^3c^2x^3+\right.}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^2*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
```

```
) - 1) + (6*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 7*a^2*c^2*x^2 - 7*a*c^2*x + 2*c^
2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

giac [A] time = 0.18, size = 264, normalized size = 1.35

$$\frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^2\operatorname{sgn}(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2*s
gn(a*x + 1)/a + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a*x
+ 1) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) + 36*(x*abs(a
) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(a^2*x^2 -
1))*c^2*abs(a)*sgn(a*x + 1) + 16*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2
*x^2 - 1))^2 + 1)^3*a*abs(a))
```

maple [A] time = 0.05, size = 233, normalized size = 1.19

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^2\left(-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^4a^4+18\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}\,x^2a^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^3a^3+3a^3x^3\sqrt{a^2}\,a}{6\,(ax-1)\sqrt{(ax-1)(ax+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^2*(-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/
2)*x^4*a^4+18*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*x^2*a^2+3*(a^2*x^2-1)^(1/2)*(a^
2)^(1/2)*x^3*a^3+3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+18*ln((a
^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4-9*(a^2*x^2-1)^(3/2
)*(a^2)^(1/2)*x*a+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/(a*x-1)/((a*x-1)*(a*x+1)
)^(1/2)/a^4/x^3/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 224, normalized size = 1.15

$$\frac{1}{3}a\left(\frac{3\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-37\,c^2}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(a}{(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 + 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21
*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 37*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
 - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))
```

mupad [B] time = 0.09, size = 183, normalized size = 0.94

$$\frac{5\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}+\frac{17\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{3}-7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{a+\frac{2\,a\,(a\,x-1)}{a\,x+1}-\frac{2\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (17*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^2
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{2a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\frac{a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right.$$

$$a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Int
egral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(2*
a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-2*a*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

3.825 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=76

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

```
[Out] -3*c*arccsc(a*x)/a-3*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1-1/a/
x)^(3/2)*x*(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6194, 98, 12, 105, 41, 216, 92, 208}

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] c*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x - (3*c*ArcCsc[a*x])/a - (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
```

```
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\left(c\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x + c\,\text{Subst}\left(\int \frac{3\sqrt{1-\frac{x}{a}}}{ax\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x + \frac{(3c)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2} + \frac{(3c)\,\text{Subst}\left(\int\right.}{}$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\right.}{}$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x - \frac{3c\csc^{-1}(ax)}{a} - \frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

Mathematica [A] time = 0.10, size = 57, normalized size = 0.75

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}\,(ax-1)-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)-3\sin^{-1}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/a
```

fricas [A] time = 1.01, size = 103, normalized size = 1.36

$$\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) - 3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right) + \left(a^2cx^2-c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) + 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 - c)
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

giac [A] time = 0.17, size = 122, normalized size = 1.61

$$\frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a} + \frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}\,c\operatorname{sgn}(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] 6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a
*x + 1)/a - 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)
)
```

maple [B] time = 0.05, size = 234, normalized size = 3.08

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2\,c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x^2a^2+4\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,xa+\left(a^2x^2-1\right)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,x}{(ax-1)\sqrt{(ax-1)(ax+1)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x^2*a^2
+4*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*
(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*x*a-3*a*x*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1
/2))+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2-4*ln((a^2*
x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2)/(a*x-1)/((a*x-1)
*(a*x+1))^(1/2)/a^2/x/(a^2)^(1/2)
```

maxima [A] time = 0.42, size = 118, normalized size = 1.55

$$-\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2} - \frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2} + \frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2} - \frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

mupad [B] time = 0.06, size = 84, normalized size = 1.11

$$\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\frac{a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integra
l(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**2*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

3.826 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=144

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c+5/3*(1-1/a/x)^(1/2)/a/c/(1+
1/a/x)^(3/2)+x*(1-1/a/x)^(1/2)/c/(1+1/a/x)^(3/2)+14/3*(1-1/a/x)^(1/2)/a/c/(
1+1/a/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 99, 152, 12, 92, 208}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (5*Sqrt[1 - 1/(a*x)])/(3*a*c*(1 + 1/(a*x))^(3/2)) + (14*Sqrt[1 - 1/(a*x)])/
(3*a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*(1 + 1/(a*x))^(3/2)) -
 (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x^2\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}+\frac{2x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{9}{a^2}+\frac{5x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^2\,\text{Subst}\left(\int -\frac{9}{a^3x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A] time = 0.15, size = 69, normalized size = 0.48

$$\frac{\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(3a^2x^2+19ax+14\right)}{(ax+1)^2} - \frac{9\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(14 + 19*a*x + 3*a^2*x^2))/(1 + a*x)^2 - (9*Log[(
1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

fricas [A] time = 0.59, size = 96, normalized size = 0.67

$$-\frac{9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(3\,a^2x^2+19\,ax+14\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -1/3*(9*(a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x + 1)*log(sqrt
((a*x - 1)/(a*x + 1)) - 1) - (3*a^2*x^2 + 19*a*x + 14)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^2*c*x + a*c)
```

giac [A] time = 0.16, size = 59, normalized size = 0.41

$$\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\,(ax+1)}{c|a|}+\frac{\sqrt{a^2x^2-1}\,\mathrm{sgn}\,(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a)) + sqrt(a^
2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

maple [B] time = 0.06, size = 346, normalized size = 2.40

$$-\frac{\left(9\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^3a^4-9\sqrt{a^2}\,\sqrt{(ax-1)\,(ax+1)}\,x^3a^3+27\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^2a^3+6\sqrt{a^2}\,((\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x)
```

```
[Out] -1/3*(9*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4
-9*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+27*ln((a^2*x+((a*x-1)*(a*x+1
))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^2*a^3+6*(a^2)^(1/2)*((a*x-1)*(a*x+1))^
(3/2)*x*a-27*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^2*a^2+27*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x*a^2+5*((a*x-1)*(a*x+1))^(3/2
)*(a^2)^(1/2)-27*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x*a+9*a*ln((a^2*x+((a*
x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))-9*((a*x-1)*(a*x+1))^(1/2)*(a^
2)^(1/2))/a*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x+1)/c/((a*x-1)*(a*x+1))
^(1/2)/(a*x-1)
```

maxima [A] time = 0.31, size = 140, normalized size = 0.97

$$-\frac{1}{3}\,a\left(\frac{6\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+12\sqrt{\frac{ax-1}{ax+1}}}{a^2c}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(6*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) - (
((a*x - 1)/(a*x + 1))^(3/2) + 12*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c) + 9*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c))
```

mupad [B] time = 0.07, size = 114, normalized size = 0.79

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3\,a\,c}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (4*((a*
x - 1)/(a*x + 1))^(1/2))/(a*c) + ((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c) + (ata
n(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{2}\left(\int\left(-\frac{x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{3}x^{3}+a^{2}x^{2}-ax-1}\right)dx+\int\frac{ax^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{3}x^{3}+a^{2}x^{2}-ax-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x*
*2 - a*x - 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3
*x**3 + a**2*x**2 - a*x - 1), x))/c
```

3.827 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=181

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c^2\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2+6/5*(1-1/a/x)^(1/2)/a/c^2
/(1+1/a/x)^(5/2)+9/5*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(3/2)+x*(1-1/a/x)^(1/2
)/c^2/(1+1/a/x)^(5/2)+24/5*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6194, 103, 21, 99, 152, 12, 92, 208}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c^2\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(5*a*c^2*(1 + 1/(a*x))^(5/2)) + (9*Sqrt[1 - 1/(a*x)])
/(5*a*c^2*(1 + 1/(a*x))^(3/2)) + (24*Sqrt[1 - 1/(a*x)])/(5*a*c^2*Sqrt[1 + 1
/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c^2*(1 + 1/(a*x))^(5/2)) - (3*ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{3x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{6\,\text{Subst}\left(\int \frac{-\frac{5}{2}+\frac{2x}{a}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{5ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{15}{2a}+\frac{9x}{2a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{5c^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{(2a)\,\text{Subst}\left(\int -\frac{}{2a^2x\sqrt{}}\right.}{5c^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1-}}\right.}{ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x\right.}{a^2c}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{}\right.}{ac^2}$$

Mathematica [A] time = 0.18, size = 78, normalized size = 0.43

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(5a^3x^3+39a^2x^2+57ax+24\right)}{5(ax+1)^3}-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 57*a*x + 39*a^2*x^2 + 5*a^3*x^3))/(5*(1 +
 a*x)^3) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

fricas [A] time = 0.50, size = 135, normalized size = 0.75

$$-\frac{15\left(a^2x^2+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\left(a^2x^2+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(5\,a^3x^3+39\,a^2x^2+57\,ax+\right.}{5\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/5*(15*(a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*(a^2
*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (5*a^3*x^3 + 39*a^2*
x^2 + 57*a*x + 24)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 + 2*a^2*c^2*x +
a*c^2)
```

giac [A] time = 0.20, size = 59, normalized size = 0.33

$$\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{c^2|a|}+\frac{\sqrt{a^2x^2-1}\,\mathrm{sgn}\left(ax+1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^2*abs(a)) + sqrt(
a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^2)
```

maple [B] time = 0.07, size = 438, normalized size = 2.42

$$-\frac{\left(-125\sqrt{a^2}\,\sqrt{(ax-1)\,(ax+1)}\,x^4a^4+120\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^4a^5+85\sqrt{a^2}\,\left((ax-1)\,(ax+1)\right)^{\frac{3}{2}}x^2a^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x)
```

```
[Out] -1/40*(-125*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+120*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+85*(a^2)^(1/2)*((a*x-1
)*(a*x+1))^(3/2)*x^2*a^2-500*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3+48
0*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+148*(
a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-750*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1
/2)*x^2*a^2+720*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*x^2*a^3+67*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-500*((a*x-1)*(a*x+1))^(1/2)
*(a^2)^(1/2)*x*a+480*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*x*a^2-125*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+120*a*ln((a^2*x+((a*x-1
)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(3/2)/(a*x+
1)^2/(a^2)^(1/2)/c^2/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)
```

maxima [A] time = 0.31, size = 161, normalized size = 0.89

$$-\frac{1}{20}\,a\left(\frac{40\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+85\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/20*a*(40*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^
2) - (((a*x - 1)/(a*x + 1))^(5/2) + 10*((a*x - 1)/(a*x + 1))^(3/2) + 85*sqr
t((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(
a^2*c^2) - 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

mupad [B] time = 0.05, size = 141, normalized size = 0.78

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}}+\frac{17\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2\,a\,c^2}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{20\,a\,c^2}+\frac{\operatorname{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^2,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + (17
*((a*x - 1)/(a*x + 1))^(1/2))/(4*a*c^2) + ((a*x - 1)/(a*x + 1))^(3/2)/(2*a*
c^2) + ((a*x - 1)/(a*x + 1))^(5/2)/(20*a*c^2) + (atan(((a*x - 1)/(a*x + 1))
^(1/2)*1i)*6i)/(a*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x*
*4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x) + Integral(a*x**5*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 +
 a*x + 1), x))/c**2
```

3.828 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=253

$$\frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(\right.}$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-2/a/c^3/(1+1/a/x)^(7/2)/(
1-1/a/x)^(1/2)+x/c^3/(1+1/a/x)^(7/2)/(1-1/a/x)^(1/2)+11/7*(1-1/a/x)^(1/2)/a
/c^3/(1+1/a/x)^(7/2)+54/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(5/2)+71/35*(1-1
/a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+176/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1
/2)
```

Rubi [A] time = 0.17, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -2/(a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)) + (11*Sqrt[1 - 1/(a*x)])/(
7*a*c^3*(1 + 1/(a*x))^(7/2)) + (54*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x
))^(5/2)) + (71*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x))^(3/2)) + (176*Sq
rt[1 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*Sqrt[1 - 1/(a*x)]*(1
 + 1/(a*x))^(7/2)) - (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{3}{a^2}+\frac{8x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^2\,\text{Subst}\left(\int \frac{-\ldots}{x\sqrt{1-\ldots}}\right.}{7\ldots}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\ldots}{c^3\ldots}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1\ldots}{35\ldots}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1\ldots}{35\ldots}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1\ldots}{35\ldots}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{1\ldots}{35\ldots}$$

Mathematica [A] time = 0.24, size = 101, normalized size = 0.40

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(35a^5x^5+286a^4x^4+368a^3x^3-125a^2x^2-423ax-176\right)}{35(ax-1)(ax+1)^4} - 3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-176 - 423*a*x - 125*a^2*x^2 + 368*a^3*x^3 + 2
86*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)*(1 + a*x)^4) - 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x])/(a*c^3)
```

fricas [A] time = 0.46, size = 179, normalized size = 0.71

$$-\frac{105 \left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(35\,a^5x^5+2\right.}{35\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) - 105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
- 1) - (35*a^5*x^5 + 286*a^4*x^4 + 368*a^3*x^3 - 125*a^2*x^2 - 423*a*x - 17
6)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x -
a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3, x)
```

maple [B] time = 0.07, size = 714, normalized size = 2.82

$$-\frac{\left(-3675\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}\,x^7a^7+3360\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^7a^8+2555\left((ax-1)(ax+1)\right)^{\frac{3}{2}}\sqrt{a^2}\,x^5a^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x)
```

```
[Out] -1/1120*(-3675*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^7*a^7+3360*ln((a^2*x+(
(a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^7*a^8+2555*((a*x-1)*(a*x
+1))^(3/2)*(a^2)^(1/2)*x^5*a^5-11025*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^
6*a^6+10080*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^6
*a^7+1873*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^4*a^4-3675*(a^2)^(1/2)*((a*
x-1)*(a*x+1))^(1/2)*x^5*a^5+3360*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1
/2))/(a^2)^(1/2))*x^5*a^6-4426*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^3*a^3+
18375*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4-16800*ln((a^2*x+((a*x-1)*
(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5-3350*(a^2)^(1/2)*((a*x-1)*
(a*x+1))^(3/2)*x^2*a^2+18375*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^3*a^3-16
800*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^3*a^4+251
1*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-3675*((a*x-1)*(a*x+1))^(1/2)*(a^2
)^(1/2)*x^2*a^2+3360*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*x^2*a^3+1957*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)-11025*((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2)*x*a+10080*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/
2))/(a^2)^(1/2))*x*a^2-3675*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)+3360*a*ln((
```

```
a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1)
)^(3/2)/(a*x+1)^3/(a^2)^(1/2)/c^3/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)^3
```

maxima [A] time = 0.31, size = 199, normalized size = 0.79

$$-\frac{1}{560}\,a\left(\frac{35\left(\frac{33\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+56\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+350\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2520\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1680\log\left(\sqrt{\frac{a}{a}}\right.}{a^2c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*(35*(33*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3
/2) - a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - (5*((a*x - 1)/(a*x + 1))^(7/2) +
 56*((a*x - 1)/(a*x + 1))^(5/2) + 350*((a*x - 1)/(a*x + 1))^(3/2) + 2520*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 1680*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

mupad [B] time = 0.05, size = 183, normalized size = 0.72

$$\frac{\frac{33\,(a\,x-1)}{a\,x+1}-1}{16\,a\,c^3\sqrt{\frac{a\,x-1}{a\,x+1}}-16\,a\,c^3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}+\frac{9\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3}+\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8\,a\,c^3}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{10\,a\,c^3}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{112\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3,x)
```

```
[Out] ((33*(a*x - 1))/(a*x + 1) - 1)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2) - 16*a
*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (9*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*c^
3) + (5*((a*x - 1)/(a*x + 1))^(3/2))/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2
)/(10*a*c^3) + ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3) + (atan(((a*x - 1)/(
a*x + 1))^(1/2)*1i)*6i)/(a*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] Timed out
```

3.829 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=327

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(\frac{1}{ax}\right.}$$

```
[Out] -4/3/a/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^(9/2)+x/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^
(9/2)-3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-5/a/c^4/(1+1/a/x)^(9
/2)/(1-1/a/x)^(1/2)+28/9*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(9/2)+139/63*(1-1/
a/x)^(1/2)/a/c^4/(1+1/a/x)^(7/2)+202/105*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(5
/2)+719/315*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+1664/315*(1-1/a/x)^(1/2)/
a/c^4/(1+1/a/x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6194, 103, 152, 12, 92, 208}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(\frac{1}{ax}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -4/(3*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - 5/(a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(9/2)) + (28*Sqrt[1 - 1/(a*x)])/(9*a*c^4*(1 + 1/(a*x))
^(9/2)) + (139*Sqrt[1 - 1/(a*x)])/(63*a*c^4*(1 + 1/(a*x))^(7/2)) + (202*Sqr
t[1 - 1/(a*x)])/(105*a*c^4*(1 + 1/(a*x))^(5/2)) + (719*Sqrt[1 - 1/(a*x)])/(
315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - (3*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{a\,\text{Subst}\left(\int \frac{-\frac{9}{a^2}+\frac{24x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \cdots\right.}{3c^4}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{a^2\,\text{Sub}\cdots}{\cdots}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1\cdots\right.}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7\cdots}}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7\cdots}}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7\cdots}}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7\cdots}}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7\cdots}}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7\cdots}}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7\cdots}}$$

Mathematica [A] time = 0.29, size = 117, normalized size = 0.36

$$\frac{\frac{ax\sqrt{1-\frac{1}{a^2x^2}}\left(315a^7x^7+2669a^6x^6+2967a^5x^5-4029a^4x^4-7399a^3x^3-339a^2x^2+4047ax+1664\right)}{315(ax-1)^2(ax+1)^5}-3\log\left(x\left(\sqrt{1-\frac{1}{a^2x^2}}+1\right)\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(1664 + 4047*a*x - 339*a^2*x^2 - 7399*a^3*x^3 -
 4029*a^4*x^4 + 2967*a^5*x^5 + 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*x)
^2*(1 + a*x)^5) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

fricas [A] time = 0.61, size = 275, normalized size = 0.84

$$-\frac{945\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\left(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3\right.}{315\left(a^7c^4x^6+2\,a^6c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/315*(945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
- 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (31
5*a^7*x^7 + 2669*a^6*x^6 + 2967*a^5*x^5 - 4029*a^4*x^4 - 7399*a^3*x^3 - 339
*a^2*x^2 + 4047*a*x + 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 + 2*a^6
*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4, x)
```

maple [B] time = 0.08, size = 766, normalized size = 2.34

$$-\frac{\left(-138915\sqrt{(ax-1)\,(ax+1)}\,\sqrt{a^2}\,x^9a^9+120960\ln\left(\frac{a^2x+\sqrt{(ax-1)(ax+1)}\,\sqrt{a^2}}{\sqrt{a^2}}\right)x^9a^{10}+98595\left((ax-1)\,(ax+1)\right)^{\frac{3}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x)
```

```
[Out] -1/40320*(-138915*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)*x^9*a^9+120960*ln((a^
2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^9*a^10+98595*((a*x-
1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^7*a^7-416745*((a*x-1)*(a*x+1))^(1/2)*(a^2)^
(1/2)*x^8*a^8+362880*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*x^8*a^9+75113*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^6*a^6-240861*((a*
x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^5*a^5+1111320*((a*x-1)*(a*x+1))^(1/2)*(a^
2)^(1/2)*x^6*a^6-967680*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2
```

```
)^(1/2))*x^6*a^7-178863*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)*x^4*a^4+833490*
(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^5*a^5-725760*ln((a^2*x+((a*x-1)*(a*x+
1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^5*a^6+252497*(a^2)^(1/2)*((a*x-1)*(a*
x+1))^(3/2)*x^3*a^3-833490*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(1/2)*x^4*a^4+7257
60*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*x^4*a^5+1823
07*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x^2*a^2-1111320*(a^2)^(1/2)*((a*x-1)
*(a*x+1))^(1/2)*x^3*a^3+967680*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*x^3*a^4-101271*(a^2)^(1/2)*((a*x-1)*(a*x+1))^(3/2)*x*a-7407
7*((a*x-1)*(a*x+1))^(3/2)*(a^2)^(1/2)+416745*((a*x-1)*(a*x+1))^(1/2)*(a^2)^
(1/2)*x*a-362880*ln((a^2*x+((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)
)*x*a^2+138915*((a*x-1)*(a*x+1))^(1/2)*(a^2)^(1/2)-120960*a*ln((a^2*x+((a*x
-1)*(a*x+1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(3/2)/(a*
x+1)^4/(a^2)^(1/2)/c^4/((a*x-1)*(a*x+1))^(1/2)/(a*x-1)^4
```

maxima [A] time = 0.33, size = 231, normalized size = 0.71

$$\frac{1}{20160}a\left(\frac{105\left(\frac{29(ax-1)}{ax+1}-\frac{414(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{35\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+450\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+2961\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+14700\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+95445\right.}{a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/20160*a*(105*(29*(a*x - 1)/(a*x + 1) - 414*(a*x - 1)^2/(a*x + 1)^2 + 1)/(
a^2*c^4*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2))
+ (35*((a*x - 1)/(a*x + 1))^(9/2) + 450*((a*x - 1)/(a*x + 1))^(7/2) + 2961*
((a*x - 1)/(a*x + 1))^(5/2) + 14700*((a*x - 1)/(a*x + 1))^(3/2) + 95445*sqr
t((a*x - 1)/(a*x + 1)))/(a^2*c^4) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

mupad [B] time = 0.06, size = 224, normalized size = 0.69

$$\frac{303\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{29\,(a\,x-1)}{3\,(a\,x+1)}-\frac{138\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-64\,a\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{35\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^4}+\frac{47\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}+\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{576\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4,x)
```

```
[Out] (303*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((29*(a*x - 1))/(3*(a*x + 1)
) - (138*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(3
/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (35*((a*x - 1)/(a*x + 1))^(3/
2))/(48*a*c^4) + (47*((a*x - 1)/(a*x + 1))^(5/2))/(320*a*c^4) + (5*((a*x -
1)/(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*c^4)
+ (atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

3.830 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=321

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3}{a^4x}$$

```
[Out] 1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)+1/5*c^3*(c-c/a^2/x^
2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)-3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1
-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+3/2*c
^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+3*c^3*(c-c/a^2/x^2)^(1/2
)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c
^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c}}{6a^7x^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqrt[c
 - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1
- 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^
2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (c^3*Sqrt[c
- c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3(1+ax)^4}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7-\frac{1}{x^7}-\frac{a}{x^6}+\frac{3a^2}{x^5}+\frac{3a^3}{x^4}-\frac{3a^4}{x^3}-\frac{3a^5}{x^2}+\frac{a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3c^3}{2a^3\sqrt{}}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 0.30

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(a^7x+a^6\log(x)+\frac{3a^5}{x}+\frac{3a^4}{2x^2}-\frac{a^3}{x^3}-\frac{3a^2}{4x^4}+\frac{a}{5x^5}+\frac{1}{6x^6}\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(1/(6*x^6) + a/(5*x^5) - (3*a^2)/(4*x^4) - a^3/x^3
 + (3*a^4)/(2*x^2) + (3*a^5)/x + a^7*x + a^6*Log[x]))/(a^7*(1 - 1/(a^2*x^2)
)^(7/2))
```

fricas [A] time = 0.69, size = 96, normalized size = 0.30

$$\frac{\left(60\,a^7c^3x^7+60\,a^6c^3x^6\log(x)+180\,a^5c^3x^5+90\,a^4c^3x^4-60\,a^3c^3x^3-45\,a^2c^3x^2+12\,ac^3x+10\,c^3\right)\sqrt{a^2c}}{60\,a^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/60*(60*a^7*c^3*x^7 + 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 + 90*a^4*c^3
*x^4 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 12*a*c^3*x + 10*c^3)*sqrt(a^2*c)/(
a^8*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.11, size = 112, normalized size = 0.35

$$\frac{\left(60a^7x^7+60a^6\ln(x)x^6+180x^5a^5+90x^4a^4-60x^3a^3-45a^2x^2+12ax+10\right)\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{7}{2}}x}{60\left(ax+1\right)\left(a^2x^2-1\right)^3\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/60*(60*a^7*x^7+60*a^6*ln(x)*x^6+180*x^5*a^5+90*x^4*a^4-60*x^3*a^3-45*a^2*
x^2+12*a*x+10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)/(a^2*x^2-1)^3/((a*x-
1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

3.831 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=236

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)-1/3*c^2*(c-c/a^2/x
^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/
a^2/x^2)^(1/2)+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c
-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqrt[
c - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^2*
x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2
*Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x
^2)] + (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^2(1+ax)^3}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^5+\frac{1}{x^5}+\frac{a}{x^4}-\frac{2a^2}{x^3}-\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2}{}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.32

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(a^5x+a^4\log(x)+\frac{2a^3}{x}+\frac{a^2}{x^2}-\frac{a}{3x^3}-\frac{1}{4x^4}\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*(-1/4*1/x^4 - a/(3*x^3) + a^2/x^2 + (2*a^3)/x + a^
5*x + a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

fricas [A] time = 0.55, size = 74, normalized size = 0.31

$$\frac{\left(12\,a^5c^2x^5+12\,a^4c^2x^4\log(x)+24\,a^3c^2x^3+12\,a^2c^2x^2-4\,ac^2x-3\,c^2\right)\sqrt{a^2c}}{12\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(12*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 + 12*a^2*c^2*
x^2 - 4*a*c^2*x - 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.07, size = 96, normalized size = 0.41

$$\frac{\left(12x^{5}a^{5}+12a^{4}\ln\left(x\right)x^{4}+24x^{3}a^{3}+12a^{2}x^{2}-4ax-3\right)\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{5}{2}}x}{12\left(ax+1\right)\left(a^{2}x^{2}-1\right)^{2}\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/12*(12*x^5*a^5+12*a^4*ln(x)*x^4+24*x^3*a^3+12*a^2*x^2-4*a*x-3)*(c*(a^2*x^
2-1)/a^2/x^2)^(5/2)*x/(a*x+1)/(a^2*x^2-1)^2/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{5}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(c-\frac{c}{a^{2}\,x^{2}}\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.832 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=146

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2)
/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c*ln
(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 75}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c - c
/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/Sq
rt[1 - 1/(a^2*x^2)] + (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x
^2)])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)(1+ax)^2}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3-\frac{1}{x^3}-\frac{a}{x^2}+\frac{a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.44

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a^3x+a^2\log(x)+\frac{3a^2}{2}+\frac{a}{x}+\frac{1}{2x^2}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(3/2)*((3*a^2)/2 + 1/(2*x^2) + a/x + a^3*x + a^2*Log[x])
)/(a^3*(1 - 1/(a^2*x^2))^(3/2))
```

fricas [A] time = 0.65, size = 42, normalized size = 0.29

$$\frac{\left(2\,a^3cx^3+2\,a^2cx^2\log(x)+2\,acx+c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*c*x^3 + 2*a^2*c*x^2*log(x) + 2*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.06, size = 80, normalized size = 0.55

$$\frac{\left(2x^3a^3+2a^2\ln(x)x^2+2ax+1\right)\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x}{2\,(ax+1)\left(a^2x^2-1\right)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(2*x^3*a^3+2*a^2*ln(x)*x^2+2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a*
x+1)/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c - \frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{a^2\,x^2}\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.833 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=67

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.58

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.48, size = 17, normalized size = 0.25

$$\frac{\sqrt{a^2c}\left(ax+\log(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.06, size = 50, normalized size = 0.75

$$\frac{(ax+\ln(x))\,x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x
)
```

3.834 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=72

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)+ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 - a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x}{-1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{1}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.61

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(ax+\log(1-ax))}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x + Log[1 - a*x]))/(a*Sqrt[c - c/(a^2*x^2)])
```

fricas [A] time = 0.64, size = 24, normalized size = 0.33

$$\frac{\sqrt{a^2c}\left(ax+\log(ax-1)\right)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + log(a*x - 1))/(a^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.05, size = 57, normalized size = 0.79

$$\frac{(ax-1)(ax+\ln(ax-1))}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2*(a*x+
ln(a*x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.835 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(-a
*x+1)/(c-c/a^2/x^2)^(1/2)+5/4*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x
^2)^(1/2)-1/4*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)^2(1+ax)}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.39

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(ax+1)\right)}{4a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1 + a*
x]))/(4*a*(c - c/(a^2*x^2))^(3/2))
```

fricas [A] time = 0.57, size = 68, normalized size = 0.39

$$\frac{\left(4\,a^2x^2-4\,ax-(ax-1)\log\left(ax+1\right)+5\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{a^2c}}{4\left(a^3c^2x-a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)*sqrt(a^2*c)/(a^3*c^2*x - a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.07, size = 101, normalized size = 0.58

$$\frac{(ax-1)\left(4a^{2}x^{2}+5\ln\left(ax-1\right)xa-ax\ln\left(ax+1\right)-4ax-5\ln\left(ax-1\right)+\ln\left(ax+1\right)-2\right)\left(ax+1\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(4*a^2*x^2+5*ln(a*x-1)*x*a-a*x*ln(a*x+1
)-4*a*x-5*ln(a*x-1)+ln(a*x+1)-2)*(a*x+1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.836 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-a}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)^2/(c-c/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)/(c-c/a^2
/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+23/16
*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-7/16*ln(a*x+1)*(1
-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-a}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + Sqrt[1 - 1/(a^2*x^2)]/(a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(8*a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*
c^2*Sqrt[c - c/(a^2*x^2)]) - (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{4a^5(-1+ax)^3}+\frac{1}{a^5(-1+ax)^2}+\frac{23}{16a^5(-1+ax)}+\frac{1}{8a^5(1+ax)^2}-\frac{7}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)} +$$

Mathematica [A] time = 0.12, size = 86, normalized size = 0.33

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(16ax+\frac{16}{1-ax}-\frac{2}{ax+1}-\frac{2}{(ax-1)^2}+23\log(1-ax)-7\log(ax+1)\right)}{16a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(16*a*x + 16/(1 - a*x) - 2/(-1 + a*x)^2 - 2/(1 + a
*x) + 23*Log[1 - a*x] - 7*Log[1 + a*x]))/(16*a*(c - c/(a^2*x^2))^(5/2))
```

fricas [A] time = 0.66, size = 137, normalized size = 0.52

$$\frac{\left(16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-\right.\right.}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
- a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) + 1
2)*sqrt(a^2*c)/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.07, size = 175, normalized size = 0.67

$$\frac{(ax-1)(ax+1)\left(16x^4a^4+23\ln(ax-1)x^3a^3-7a^3x^3\ln(ax+1)-16x^3a^3-23\ln(ax-1)x^2a^2+7\ln(ax+1)\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,a^6x^5\left(\frac{c(a^2x}{a^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(16*x^4*a^4+23*ln(a*x-1)*x^3*a
^3-7*a^3*x^3*ln(a*x+1)-16*x^3*a^3-23*ln(a*x-1)*x^2*a^2+7*ln(a*x+1)*x^2*a^2-
34*a^2*x^2-23*ln(a*x-1)*x*a+7*a*x*ln(a*x+1)+18*a*x+23*ln(a*x-1)-7*ln(a*x+1)
+12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.837 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=359

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)+1/24*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^3/(c-c/a^2/x^2)^(1/2)-11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^2
/(c-c/a^2/x^2)^(1/2)+3/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c-c/a^2/x^2)^(
1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-5/16*(1-1
/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+51/32*ln(-a*x+1)*(1-1/a^2
/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-19/32*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a
/c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(24*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (11*Sqrt[1 - 1/(a^2*x^2)]
)/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/
(2*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (5*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx=\frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$=\frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^4(1+ax)^3}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$=\frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{8a^7(-1+ax)^4}+\frac{11}{16a^7(-1+ax)^3}+\frac{3}{2a^7(-1+ax)^2}+\frac{51}{32a^7(-1+ax)}-\frac{1}{16a^7(1+ax)^3}+\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$=\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1}$$

Mathematica [A] time = 0.16, size = 104, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96ax+\frac{144}{1-ax}-\frac{30}{ax+1}-\frac{33}{(ax-1)^2}+\frac{3}{(ax+1)^2}-\frac{4}{(ax-1)^3}+153\log(1-ax)-57\log(ax+1)\right)}{96a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(96*a*x + 144/(1 - a*x) - 4/(-1 + a*x)^3 - 33/(-1
+ a*x)^2 + 3/(1 + a*x)^2 - 30/(1 + a*x) + 153*Log[1 - a*x] - 57*Log[1 + a*x
]))/(96*a*(c - c/(a^2*x^2))^(7/2))
```

fricas [A] time = 0.60, size = 205, normalized size = 0.57

$$\frac{\left(96\,a^6x^6-96\,a^5x^5-366\,a^4x^4+222\,a^3x^3+338\,a^2x^2-122\,ax-57\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\mathrm{lo}}{96\left(a^7c^4x^5-a^6c^4x^4-2\,a^5c^4x^3+2\,a^4c^4x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/96*(96*a^6*x^6 - 96*a^5*x^5 - 366*a^4*x^4 + 222*a^3*x^3 + 338*a^2*x^2 - 1
22*a*x - 57*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x +
 1) + 153*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x - 1
) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 - a^6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*
x^2 + a^3*c^4*x - a^2*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A] time = 0.07, size = 247, normalized size = 0.69

$$\frac{(ax-1)(ax+1)\left(96x^6a^6+153\ln(ax-1)x^5a^5-57\ln(ax+1)x^5a^5-96x^5a^5-153\ln(ax-1)x^4a^4+57\ln(ax+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/96/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(96*x^6*a^6+153*ln(a*x-1)*x^5*
a^5-57*ln(a*x+1)*x^5*a^5-96*x^5*a^5-153*ln(a*x-1)*x^4*a^4+57*ln(a*x+1)*x^4*
a^4-366*x^4*a^4-306*ln(a*x-1)*x^3*a^3+114*a^3*x^3*ln(a*x+1)+222*x^3*a^3+306
*ln(a*x-1)*x^2*a^2-114*ln(a*x+1)*x^2*a^2+338*a^2*x^2+153*ln(a*x-1)*x*a-57*a
*x*ln(a*x+1)-122*a*x-153*ln(a*x-1)+57*ln(a*x+1)-88)/a^8/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

3.838 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$

Optimal. Leaf size=372

$$\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{15(1-ax)^2}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(1-ax)}-\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{40(1-ax)^3}-\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{2a^6x^7\left(c\right.}{(1-}$$

```
[Out] 11/30*a^3*(c-c/a^2/x^2)^(7/2)*x^4/(-a*x+1)^3-57/16*a^6*(c-c/a^2/x^2)^(7/2)*
x^7/(-a*x+1)^3/(a*x+1)^3+41/24*a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a*x+1)^3/(a*x+
1)^2+57/80*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)-13/40*a^2*(c-c/a^
2/x^2)^(7/2)*x^3*(a*x+1)/(-a*x+1)^3+1/15*a*(c-c/a^2/x^2)^(7/2)*x^2*(a*x+1)/
(-a*x+1)^2+1/6*(c-c/a^2/x^2)^(7/2)*x*(a*x+1)/(-a*x+1)+2*a^6*(c-c/a^2/x^2)^(
7/2)*x^7*arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)+25/16*a^6*(c-c/a^2/x^2)^(
7/2)*x^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2)
```

Rubi [A] time = 0.54, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{41a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{24(1-ax)^3(ax+1)^2}+\frac{57a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{80(1-ax)^3(ax+1)}+\frac{11a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{30(1-ax)^3}-\frac{13a^2x^3(ax+1)\left(c-\frac{}{}\right.}{40(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (11*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(30*(1 - a*x)^3) - (57*a^6*(c - c/(a^2
*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (41*a^5*(c - c/(a^2*x^2))^
(7/2)*x^6)/(24*(1 - a*x)^3*(1 + a*x)^2) + (57*a^4*(c - c/(a^2*x^2))^(7/2)*x
^5)/(80*(1 - a*x)^3*(1 + a*x)) - (13*a^2*(c - c/(a^2*x^2))^(7/2)*x^3*(1 + a
*x))/(40*(1 - a*x)^3) + (a*(c - c/(a^2*x^2))^(7/2)*x^2*(1 + a*x))/(15*(1 -
a*x)^2) + ((c - c/(a^2*x^2))^(7/2)*x*(1 + a*x))/(6*(1 - a*x)) + (2*a^6*(c -
 c/(a^2*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (2
5*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16
*(1 - a*x)^(7/2)*(1 + a*x)^(7/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{5/2}(1+ax)^{9/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}(2a-7a^2x)}{x^6}dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1+ax)}{6(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{\sqrt{1-a}}{}}{30(1-ax)^{7/}}$$

$$= -\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+ax)}{15(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1}{6(1-ax)}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} - \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2(1+a}{15(1-ax)^2}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} + \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)} - \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3(1+ax)}{40(1-ax)^3} +$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} + \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)} - \frac{13a^2(c}{}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} + \frac{57a^4(c}{80(1-}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} + \frac{57a^4(c}{80(1-}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} + \frac{57a^4(c}{80(1-}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2} + \frac{57a^4(c}{80(1-}$$

Mathematica [A] time = 0.18, size = 150, normalized size = 0.40

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(480a^6x^6\log\left(\sqrt{a^2x^2-1}+ax\right)+375a^6x^6\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(240a^6x^6-736a^5x^5+10\right.\right.}{240a^6x^5\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 - 96*a*x + 70*a^2*x^2 +
 352*a^3*x^3 + 105*a^4*x^4 - 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcTa
n[1/Sqrt[-1 + a^2*x^2]] + 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240*
a^6*x^5*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.76, size = 438, normalized size = 1.18

$$\left[-\frac{960\,a^5\sqrt{-c}\,c^3x^5\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-375\,a^5\sqrt{-c}\,c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(240\,a^6c^3x^6-7\right.}{480\,a^6x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2
- 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(240*a^6*c
^3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x
^2 - 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1/240
*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a
^2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 - 736*a^5*c^3*x^5 + 1
05*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 - 96*a*c^3*x - 40*c^3)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

giac [A] time = 113.35, size = 561, normalized size = 1.51

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2cx^2-c}}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*
sgn(x)/a^2 + 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn
(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) + 1440*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^9*c^5*abs(a)*sgn(x) + 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*
c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sg
n(x) + 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) + 6720*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) + 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9
*abs(a)*sgn(x) + 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

maple [B] time = 0.12, size = 795, normalized size = 2.14

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}x\left(-2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,x^7a^9c+2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\,x^5a^9+375\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,x^6a^8c-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(-2016*(c*(a^2*x^2-1)/a^2)^(7/2
)*(-c/a^2)^(1/2)*x^7*a^9*c+2016*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^
5*a^9+375*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*x^6*a^8*c-480*(-c/a^2)^(
1/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^8*c+105*(c*(a^2*x^2-1)/a^2)^(9/2)*
(-c/a^2)^(1/2)*x^4*a^8+2352*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*x^7*a^
7*c^2-560*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7*c^2+224*(c*(
a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^3*a^7-525*(c*(a^2*x^2-1)/a^2)^(5/2)*
(-c/a^2)^(1/2)*x^6*a^6*c^2-2940*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^
7*a^5*c^3+700*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^7*a^5*c^3+630*
(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^2*a^6+875*(c*(a^2*x^2-1)/a^2)^(3
/2)*(-c/a^2)^(1/2)*x^6*a^4*c^3+672*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)
*x*a^5+4410*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^7*a^3*c^4-1050*(-c/a
^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^7*a^3*c^4+280*a^4*(c*(a^2*x^2-1)/
a^2)^(9/2)*(-c/a^2)^(1/2)-2625*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^6
*a^2*c^4-4410*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*c^(9/2
)*x^6*a+1050*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*(-c/a^
2)^(1/2)*c^(9/2)*x^6*a-2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-c)/a^2/x)*x^6*c^5)/(c*(a^2*x^2-1)/a^2)^(7/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 33.43, size = 1059, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**3*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Piecewise(
(I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
```

```
**2 - 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c
), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a
**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 + 2*c**3*Piecewise((2*a**3*sqrt(
c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - s
qrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(
c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3)
 - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 + c**3*Piecewise(
(I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*
x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(
24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**
2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sq
rt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a*
*2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*
x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

3.839 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=294

$$\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-ax)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}+\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{2a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^{5/2}(ax+\cdots}$$

```
[Out] -5/8*a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)^2+25/8*a^4*(c-c/a^2/x^2)^(5/2)*x^
5/(-a*x+1)^2/(a*x+1)^2-17/12*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)
+1/6*a*(c-c/a^2/x^2)^(5/2)*x^2*(a*x+1)/(-a*x+1)^2+1/4*(c-c/a^2/x^2)^(5/2)*x
*(a*x+1)/(-a*x+1)-2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/
(a*x+1)^(5/2)-9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1
)^(1/2))/(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

Rubi [A] time = 0.50, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{17a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{12(1-ax)^2(ax+1)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}+\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/\cdots}}{4(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (-5*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(8*(1 - a*x)^2) + (25*a^4*(c - c/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (17*a^3*(c - c/(a^2*x^2))^(5
/2)*x^4)/(12*(1 - a*x)^2*(1 + a*x)) + (a*(c - c/(a^2*x^2))^(5/2)*x^2*(1 + a
*x))/(6*(1 - a*x)^2) + ((c - c/(a^2*x^2))^(5/2)*x*(1 + a*x))/(4*(1 - a*x))
- (2*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x
)^(5/2)) - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1
+ a*x]])/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$
$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1-ax}(1+ax)^{5/2}(2a-5a^2x)}{x^4}dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1+ax)}{}}{12(1-ax)^{5/2}(1}$$
$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} -$$
$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{(c}{}$$
$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\right.}{}$$
$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\right.}{}$$
$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\right.}{}$$
$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\right.}{}$$

Mathematica [A] time = 0.15, size = 134, normalized size = 0.46

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(48a^4x^4\log\left(\sqrt{a^2x^2-1}+ax\right)+27a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(24a^4x^4-64a^3x^3-3a^2x^2\right.\right.}{24a^4x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 64
*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 48*a^4*x
^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.63, size = 394, normalized size = 1.34

$$\left[-\frac{96\,a^3\sqrt{-c}\,c^2x^3\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-27\,a^3\sqrt{-c}\,c^2x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(24\,a^4c^2x^4-64\right.}{48\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) - 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 - 2
*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*c^2*x
^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(2*a^2
*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c
^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

giac [A] time = 7.33, size = 416, normalized size = 1.41

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-c}\,c^2\mathrm{sgn}}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x)/a^2 + 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)
/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) - 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4
*abs(a)*sgn(x) - 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(
x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) - 160*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) - 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

maple [B] time = 0.06, size = 625, normalized size = 2.13

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}} x\left(-80\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}} x^5a^7c+80\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}} x^3a^7+48\sqrt{-\frac{c}{a^2}}\,\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}} x^4a^6c+27\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1
)/a^2)^(5/2)*x^5*a^7*c+80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^3*a^7+
48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^6*c+27*(-c/a^2)^(1/2)
*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^6*c+60*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a
^2)^(3/2)*x^5*a^5*c^2-75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^6+1
00*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^5*a^5*c^2-80*(-c/a^2)^(1/2)*(
c*(a^2*x^2-1)/a^2)^(7/2)*x*a^5-45*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*
x^4*a^4*c^2-90*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c^3-150
*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c^3-30*a^4*(c*(a^2*x^2-1)
/a^2)^(7/2)*(-c/a^2)^(1/2)+150*(-c/a^2)^(1/2)*c^(7/2)*ln(x*c^(1/2)+(c*(a^2*
x^2-1)/a^2)^(1/2))*x^4*a+90*(-c/a^2)^(1/2)*c^(7/2)*ln((c^(1/2)*((a*x-1)*(a*
x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^4*a+135*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
```

```
)^(1/2)*x^4*a^2*c^3+135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-
c)/a^2/x)*x^4*c^4)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(5/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 20.07, size = 500, normalized size = 1.70

$$c^2\left(\begin{cases}\frac{\sqrt{c}\,\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}+\frac{\sqrt{c}\,\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right|>1\\ \frac{i\sqrt{c}\,\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c^2\left(\begin{cases}-\frac{a\sqrt{c}\,x}{\sqrt{a^2x^2-1}}+\sqrt{c}\,\operatorname{acosh}(ax)+ \\ \frac{ia\sqrt{c}\,x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\,\operatorname{asin}(ax)-\end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**2*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - 2*c**2*Piecewis
e((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 - c**2
*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 +
 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sqr
t(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(
c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**4
```

3.840 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=213

$$\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tan}{2(1-ax)^3}$$

```
[Out] a*(c-c/a^2/x^2)^(3/2)*x^2/(-a*x+1)-5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1)
/(a*x+1)+1/2*(c-c/a^2/x^2)^(3/2)*x*(a*x+1)/(-a*x+1)+2*a^2*(c-c/a^2/x^2)^(3/
2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)+1/2*a^2*(c-c/a^2/x^2)^(3/2)
*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

Rubi [A] time = 0.46, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}+\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\text{ta}}{2(1-ax}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 - a*x) - (5*a^2*(c - c/(a^2*x^2))^(3/2)*
x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 + a*x))/(2*(1
- a*x)) + (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/2)*
(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x]*S
qrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1-ax}(1+ax)^{5/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1+ax)^{3/2}(2a-3a^2x)}{x^2\sqrt{1-ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1+ax}(a^2-5a^3x)}{x\sqrt{1-ax}} d}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}\right.}{2a(1-}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(a^2\left(c-\frac{c}{a^2x^2}\right)\right.}{2(1-}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{\left(a^3\left(c-\frac{c}{a^2x^2}\right)\right.}{}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{2a^2\left(c-\frac{c}{a^2x^2}\right)}{(1-ax)^{3/}}$$

Mathematica [A] time = 0.11, size = 115, normalized size = 0.54

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(2a^2x^2-4ax-1\right)+4a^2x^2\log\left(\sqrt{a^2x^2-1}+ax\right)+a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2a^2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 - 4*a*x + 2*a^2*x^2) + a^2
*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]]
))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.81, size = 317, normalized size = 1.49

$$\left[-\frac{8\,a\sqrt{-c}\,cx\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-a\sqrt{-c}\,cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(2\,a^2cx^2-4\,acx-c\right)\sqrt{\frac{a^2cx^2-}{a^2x^2}}}{4\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqr
```

```
t((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(2*a^2*c*x^2 - 4*a*c*x - c)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x
*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2*a^2
*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*
x^2 - 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

giac [A] time = 0.50, size = 266, normalized size = 1.25

$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}+\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}\,c\operatorname{sgn}(x)}{a^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
 + 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a
)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^3*c^2*abs(a)*sgn(x) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/
2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) - 4*a*c
^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a))
)*abs(a)
```

maple [B] time = 0.06, size = 455, normalized size = 2.14

$$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x\left(12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^3a^5c-12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}x\,a^5+4\sqrt{-\frac{c}{a^2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^2a^4c-\sqrt{-\frac{c}{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a
^2)^(3/2)*x^3*a^5*c-12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^5+4*(-c
/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^2*a^4*c-(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^2*a^4*c+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*
x^3*a^3*c^2-3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)-18*(-c/a^2)^(1/2
)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^3*c^2+18*(-c/a^2)^(1/2)*c^(5/2)*ln(x*c^(1
/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^2*a-6*(-c/a^2)^(1/2)*c^(5/2)*ln((c^(1/2)*(
(a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a+3*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(1/2)*x^2*a^2*c^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/
2)*a^2-c)/a^2/x)*x^2*c^3)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(3/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/a^2/x^2))^(3/2)/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [C] time = 14.14, size = 376, normalized size = 1.77

$$c\left(\begin{cases} \frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log{(ax)}}{a}+\frac{i\sqrt{c}\log{\left(a^2x^2\right)}}{2a}+\frac{\sqrt{c}\operatorname{asin}{\left(\frac{1}{ax}\right)}}{a} & \text{for}\ \left|a^2x^2\right|>1 \\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log{\left(a^2x^2\right)}}{2a}-\frac{i\sqrt{c}\log{\left(\sqrt{-a^2x^2+1}+1\right)}}{a} & \text{otherwise} \end{cases}\right)+\frac{2c\left(\begin{cases} -\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}{(ax)}+\frac{\sqrt{\ldots}}{ax\sqrt{a^{\ldots}}} \\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}{(ax)}-\frac{i\sqrt{\ldots}}{ax\sqrt{-a^{2}\ldots}} \end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(
c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*
sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)
*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c*Piecewise((-a*sqrt(c)*x/sqrt
(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), A
bs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*
x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I*a*sqrt
(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)
/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c)
*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

3.841 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6159, 6129, 102, 157, 41, 216, 92, 208}

$$x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-a-2a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,d}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\right.}{\sqrt{1}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.69

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 0.60, size = 267, normalized size = 2.30

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - 4\,\sqrt{-c}\,\arctan\left(\frac{a^2\,\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,a\,\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - \sqrt{c}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.05, size = 197, normalized size = 1.70

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}-\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-c\ln\left(2\ldots\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.842 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=111

$$-\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-ax}\sqrt{ax+1}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2)^(1/2)-(a*x+1)^2/a^2/x/(c-c/a^2/x^2)
^(1/2)+2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6159, 6129, 78, 50, 41, 216}

$$-\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-ax}\sqrt{ax+1}\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (-2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[c - c/(a^2*x^2)]*x) - (1 + a*x)^2/(a^2*S
qrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*
Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx\\
&= -\frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{x\sqrt{1+ax}}{(1-ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{2\sqrt{1-ax}\,\sqrt{1+ax}\,\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.61

$$\frac{a^2x^2+2\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)-2ax-3}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (-3 - 2*a*x + a^2*x^2 + 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])
/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

fricas [A] time = 0.53, size = 216, normalized size = 1.95

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(a^2x^2-3\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac},-\frac{2\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\cdots}{a^2c\cdots}\right)}{a^2c\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) + (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x - a*c), -(2*(a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^2*c*x - a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a^2*x^2))), x)
```

maple [A] time = 0.05, size = 177, normalized size = 1.59

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\left(\sqrt{c}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x\,a^2+2\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)xac-2a\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}-\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a\sqrt{c}-2\mathrm{l}\cdots\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,c^{\frac{3}{2}}a\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (c*(a^2*x^2-1)/a^2)^(1/2)*(c^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2+2*ln(x*c
^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a*c-2*a*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*c
^(1/2)-(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)-2*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^
2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a^2*x^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)), x)
```

3.843 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] -1/3*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x+2/3*(-2*a*x+5)*(-a*x+1)*(a*x+1)^2/
a^4/(c-c/a^2/x^2)^(3/2)/x^3-2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^4/
(c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.44, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6159, 6129, 98, 143, 41, 216}

$$\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -(1 + a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(5 - 2*a*x)*(1 - a*x)*(
1 + a*x)^2)/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - (2*(1 - a*x)^(3/2)*(1 + a
*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx\\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{(1-ax)^{5/2}\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2+4ax)}{(1-ax)^{3/2}\sqrt{1+ax}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.77

$$\frac{3a^3x^3-11a^2x^2+6(ax-1)\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)-4ax+10}{3a^2cx(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*(-1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[
a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x))
```

fricas [A] time = 0.63, size = 280, normalized size = 2.28

$$\left[\frac{3\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3-14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)},-\frac{6\left(a^2x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(6*
(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

maple [B] time = 0.05, size = 326, normalized size = 2.65

$$\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,x^3a^3-15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+4c^{\frac{3}{2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^2a^2+6\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{\frac{(ax-1)(ax\cdot}{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/3*(3*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^3-15*x^2*a^2*c^(3/2)*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)+4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2+6*ln
(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2
*x^2-1)/a^2)^(1/2)*x*a^2*c-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a-6*ln(x*c
^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(1/2)*a*c+12*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)-2*(c*(a^2*x^2-1
)/a^2)^(1/2)*c^(3/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(1/2)/x^3/(c*(a^2*x^2
-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

3.844 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-ax)^{5/2}\sin^{-1}(ax)}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+$$

```
[Out] -1/5*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x+2/3*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^
2/x^2)^(5/2)/x^2-58/15*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(5/2)/x^3-2/1
5*(-a*x+1)^3*(a*x+1)^2*(43*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5+2*(-a*x+1)^(
5/2)*(a*x+1)^(5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.46, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 150, 143, 41, 216}

$$-\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(1-ax)}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1}{a^6x^5\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -(1 + a*x)^2/(5*a^2*(c - c/(a^2*x^2))^(5/2)*x) + (2*(1 - a*x)*(1 + a*x)^2)/
(3*a^3*(c - c/(a^2*x^2))^(5/2)*x^2) - (58*(1 - a*x)^2*(1 + a*x)^2)/(15*a^4*
(c - c/(a^2*x^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 43*a*x))/(1
5*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*Arc
Sin[a*x])/(a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{7/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+6ax)}{(1-ax)^{5/2}(1+ax)^{3/2}}\,dx}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2(-30a-28a^2x)}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/}\right.}{15a^6\left(c-\right.}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(2}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5}}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(2}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5}}$$

$$= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} - \frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(2}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5}}$$

Mathematica [A] time = 0.11, size = 105, normalized size = 0.52

$$\frac{15a^4x^4-76a^3x^3+32a^2x^2+30(ax-1)^2\sqrt{a^2x^2-1}\log\left(\sqrt{a^2x^2-1}+ax\right)+82ax-56}{15a^2c^2x(ax-1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*(-1 + a*x)^2*Sqrt
[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x
^2)]*x*(-1 + a*x)^2)
```

fricas [A] time = 0.67, size = 352, normalized size = 1.73

$$\left[\frac{15\left(a^4x^4-2a^3x^3+2ax-1\right)\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(15a^5x^5-76a^4x^4+32a^3x^3+82a\right.}{15\left(a^5c^3x^4-2a^4c^3x^3+2a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 - 76*a^4*x^
4 + 32*a^3*x^3 + 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3), -1/15*(30*(a^4*x^4 - 2*a^3
*x^3 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^2*c*x^2 - c)) - (15*a^5*x^5 - 76*a^4*x^4 + 32*a^3*x^3 + 82*a^2*x
^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3
+ 2*a^2*c^3*x - a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

maple [B] time = 0.06, size = 462, normalized size = 2.28

$$\frac{\left(15c^{\frac{5}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^5a^5-45x^4c^{\frac{5}{2}}a^4\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}-16c^{\frac{5}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^4a^4-60c^{\frac{5}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^3a^3+16c^{\frac{5}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/15*(15*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^5*a^5-45*x^4*c^(5/2)*a^4*(
(a*x-1)*(a*x+1)*c/a^2)^(3/2)-16*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4-6
0*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^3*a^3+16*c^(5/2)*(c*(a^2*x^2-1)/a
^2)^(3/2)*x^3*a^3+30*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+
1)*c/a^2)^(3/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^4*c+90*c^(5/2)*((a*x-1)*(a*x+
1)*c/a^2)^(3/2)*x^2*a^2+24*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^2-30*ln(
x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*a^3*c+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x*a-24*c^(
5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a-50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)
-6*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(3/2)
/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

3.845 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{(ax+1)^2}{7a^2x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^{7/2}(1-ax)^{7/2}\sin^{-1}(ax)}{a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+$$

```
[Out] -1/7*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x+2/5*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^
2/x^2)^(7/2)/x^2-124/105*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(7/2)/x^3+7
82/105*(-a*x+1)^3*(a*x+1)^2/a^5/(c-c/a^2/x^2)^(7/2)/x^4+142/35*(-a*x+1)^4*(
a*x+1)^2/a^6/(c-c/a^2/x^2)^(7/2)/x^5+2/35*(-a*x+1)^4*(a*x+1)^3*(107*a*x+72)
/a^8/(c-c/a^2/x^2)^(7/2)/x^7-2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8
/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.50, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 150, 143, 41, 216}

$$\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{782(ax+1)^2(1-ax)^3}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{124(ax+1)^2(1-ax)^2}{105a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+}{5a^3x^2\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -(1 + a*x)^2/(7*a^2*(c - c/(a^2*x^2))^(7/2)*x) + (2*(1 - a*x)*(1 + a*x)^2)/
(5*a^3*(c - c/(a^2*x^2))^(7/2)*x^2) - (124*(1 - a*x)^2*(1 + a*x)^2)/(105*a^
4*(c - c/(a^2*x^2))^(7/2)*x^3) + (782*(1 - a*x)^3*(1 + a*x)^2)/(105*a^5*(c
- c/(a^2*x^2))^(7/2)*x^4) + (142*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a
^2*x^2))^(7/2)*x^5) + (2*(1 - a*x)^4*(1 + a*x)^3*(72 + 107*a*x))/(35*a^8*(c
 - c/(a^2*x^2))^(7/2)*x^7) - (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x]
)/(a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
```

```
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && !(SumSimplerQ[n, 1] && !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{9/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+8ax)}{(1-ax)^{7/2}(1+ax)^{5/2}}\,dx}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4\left(-70a-54a^2x\right)}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{35a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)}{105a^6\left(c-\right.}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} - \frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

Mathematica [A] time = 0.13, size = 133, normalized size = 0.47

$$\frac{105a^6x^6-562a^5x^5+74a^4x^4+1226a^3x^3-636a^2x^2+210(ax-1)^3(ax+1)\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)-65}{105a^2c^3x(ax-1)^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (432 - 654*a*x - 636*a^2*x^2 + 1226*a^3*x^3 + 74*a^4*x^4 - 562*a^5*x^5 + 10
5*a^6*x^6 + 210*(-1 + a*x)^3*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1
 + a^2*x^2]])/(105*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x))
```

fricas [A] time = 0.59, size = 496, normalized size = 1.75

$$\left[\frac{105\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(105a^7x^7-\right.}{105\left(a^7c^4x^6-2a^6c^4x^5-a^5c^4x^4+4a^4c^4x^3-a^3c^4x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) + (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4 - 636*a^3
*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x
+ a*c^4), -1/105*(210*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2
- 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/(a^2*c*x^2 - c)) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^
4 - 636*a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2
*a^2*c^4*x + a*c^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

maple [B] time = 0.08, size = 572, normalized size = 2.02

$$\frac{\left(105c^{\frac{7}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}x^7a^7+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^6a^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}-96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^5a^5-39\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/105*(105*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7+96*(c*(a^2*x^2-1)/
a^2)^(5/2)*c^(7/2)*x^6*a^6-553*x^6*c^(7/2)*a^6*((a*x-1)*(a*x+1)*c/a^2)^(5/2
)-96*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^5*a^5-392*c^(7/2)*((a*x-1)*(a*x+1)
*c/a^2)^(5/2)*x^5*a^5-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^4*a^4+1540*c^
(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^4+210*(c*(a^2*x^2-1)/a^2)^(5/2)*l
n(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x*a^6*
c+240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^3*a^3+350*c^(7/2)*((a*x-1)*(a*x+1
)*c/a^2)^(5/2)*x^3*a^3-210*(c*(a^2*x^2-1)/a^2)^(5/2)*ln(x*c^(1/2)+(c*(a^2*x
^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*a^5*c+180*(c*(a^2*x^2-1)/a^
2)^(5/2)*c^(7/2)*x^2*a^2-1470*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^2*a^2
-180*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x*a-42*c^(7/2)*((a*x-1)*(a*x+1)*c/a^
```

```
2)^(5/2)*x*a-30*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)+462*c^(7/2)*((a*x-1)*(a*x
+1)*c/a^2)^(5/2))*(a*x+1)/((a*x-1)*(a*x+1)*c/a^2)^(5/2)/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)/a^8/c^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

3.846 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=322

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+$$

```
[Out] 1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/x^8/(1-1/a^2/x^2)^(1/2)+3/7*c^4*(c-c/a^2/x^
2)^(1/2)/a^8/x^7/(1-1/a^2/x^2)^(1/2)-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/x^5/(1
-1/a^2/x^2)^(1/2)-3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+2
*c^4*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+4*c^4*(c-c/a^2/x^2)^(1
/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c^4*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/
2)+3*c^4*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^4\sqrt{\ }}{8a^9x^8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a^2*x^2)])/(8*a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*Sqrt
[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c/(a
^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^4*Sqrt[c - c/(a^2*x^2)])
/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^3) + (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^4
*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3(1+ax)^6}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^9-\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}+\frac{6a^4}{x^5}-\frac{6a^5}{x^4}-\frac{8a^6}{x^3}+\frac{3a^8}{x}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2c}{a^4}$$

Mathematica [A] time = 0.09, size = 97, normalized size = 0.30

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(a^9x+3a^8\log(x)+\frac{4a^6}{x^2}+\frac{2a^5}{x^3}-\frac{3a^4}{2x^4}-\frac{8a^3}{5x^5}+\frac{3a}{7x^7}+\frac{1}{8x^8}\right)}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*(1/(8*x^8) + (3*a)/(7*x^7) - (8*a^3)/(5*x^5) - (3*
a^4)/(2*x^4) + (2*a^5)/x^3 + (4*a^6)/x^2 + a^9*x + 3*a^8*Log[x]))/(a^9*(1 -
 1/(a^2*x^2))^(9/2))
```

fricas [A] time = 0.83, size = 96, normalized size = 0.30

$$\frac{\left(280\,a^9c^4x^9+840\,a^8c^4x^8\log(x)+1120\,a^6c^4x^6+560\,a^5c^4x^5-420\,a^4c^4x^4-448\,a^3c^4x^3+120\,ac^4x+35\,c^4\right)\sqrt{a^2c}}{280\,a^{10}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/280*(280*a^9*c^4*x^9 + 840*a^8*c^4*x^8*log(x) + 1120*a^6*c^4*x^6 + 560*a^
5*c^4*x^5 - 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqrt(
a^2*c)/(a^10*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 112, normalized size = 0.35

$$\frac{\left(280a^9x^9+840a^8\ln(x)x^8+1120x^6a^6+560x^5a^5-420x^4a^4-448x^3a^3+120ax+35\right)\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{9}{2}}x}{280\left(ax+1\right)^3\left(a^2x^2-1\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x)
```

```
[Out] 1/280*(280*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6+560*x^5*a^5-420*x^4*a^4-4
48*x^3*a^3+120*a*x+35)*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a*x+1)^3/(a^2*x^2-1
)^3/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{9/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] Timed out
```

3.847 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=324

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\ldots}{3a^4x^3\ldots}$$

```
[Out] -1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)-3/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)-1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(
1-1/a^2/x^2)^(1/2)+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+
5/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(
1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2
)+3*c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\ldots}}{6a^7x^6\sqrt{1\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*Sqr
t[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (c^3*Sqrt[c - c/(a^
2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/
(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sq
rt[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^3*Sq
rt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^2(1+ax)^5}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7+\frac{1}{x^7}+\frac{3a}{x^6}+\frac{a^2}{x^5}-\frac{5a^3}{x^4}-\frac{5a^4}{x^3}+\frac{a^5}{x^2}+\frac{3a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.29

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(3a^6\log(x)-\frac{-60a^7x^7+60a^5x^5-150a^4x^4-100a^3x^3+15a^2x^2+36ax+10}{60x^6}\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(-1/60*(10 + 36*a*x + 15*a^2*x^2 - 100*a^3*x^3 - 1
50*a^4*x^4 + 60*a^5*x^5 - 60*a^7*x^7)/x^6 + 3*a^6*Log[x]))/(a^7*(1 - 1/(a^2
*x^2))^(7/2))
```

fricas [A] time = 0.76, size = 96, normalized size = 0.30

$$\frac{\left(60\,a^7c^3x^7+180\,a^6c^3x^6\log(x)-60\,a^5c^3x^5+150\,a^4c^3x^4+100\,a^3c^3x^3-15\,a^2c^3x^2-36\,ac^3x-10\,c^3\right)\sqrt{a^2c}}{60\,a^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/60*(60*a^7*c^3*x^7 + 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 + 150*a^4*c^
3*x^4 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2 - 36*a*c^3*x - 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 112, normalized size = 0.35

$$\frac{\left(60a^{7}x^{7}+180a^{6}\ln(x)x^{6}-60x^{5}a^{5}+150x^{4}a^{4}+100x^{3}a^{3}-15a^{2}x^{2}-36ax-10\right)\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}x}{60\left(ax+1\right)^{3}\left(a^{2}x^{2}-1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/60*(60*a^7*x^7+180*a^6*ln(x)*x^6-60*x^5*a^5+150*x^4*a^4+100*x^3*a^3-15*a^
2*x^2-36*a*x-10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)^3/(a^2*x^2-1)^2/((
a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

3.848 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=234

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(
1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x
^2)^(1/2)-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-c/a^
2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+3*c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2
/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 75}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqrt[c
 - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^2*x^2
)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*Sq
rt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)
] + (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)(1+ax)^4}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{3a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-}}{\sqrt{1-}}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.37

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{3}{4}a\left(a^4x+4a^3\log(x)-\frac{6a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}\right)+\frac{(ax+1)^5}{4x^4}\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*((1 + a*x)^5/(4*x^4) + (3*a*(-1/3*1/x^3 - (2*a)/x^
2 - (6*a^2)/x + a^4*x + 4*a^3*Log[x]))/4))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

fricas [A] time = 0.60, size = 72, normalized size = 0.31

$$\frac{\left(4\,a^5c^2x^5+12\,a^4c^2x^4\log(x)-8\,a^3c^2x^3+4\,a^2c^2x^2+4\,ac^2x+c^2\right)\sqrt{a^2c}}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 + 4*a^2*c^2*x^2
+ 4*a*c^2*x + c^2)*sqrt(a^2*c)/(a^6*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 96, normalized size = 0.41

$$\frac{\left(4x^5a^5+12a^4\ln(x)x^4-8x^3a^3+4a^2x^2+4ax+1\right)\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}}x}{4\left(ax+1\right)^3\left(a^2x^2-1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/4*(4*x^5*a^5+12*a^4*ln(x)*x^4-8*x^3*a^3+4*a^2*x^2+4*a*x+1)*(c*(a^2*x^2-1)
/a^2/x^2)^(5/2)*x/(a*x+1)^3/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.849 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-3*c*(c-c/a^2/x^2)^(1
/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+3
*c*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[c
- c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)
/Sqrt[1 - 1/(a^2*x^2)] + (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(
a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(1+ax)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3+\frac{1}{x^3}+\frac{3a}{x^2}+\frac{3a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.40

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a^3x+3a^2\log(x)-\frac{3a}{x}-\frac{1}{2x^2}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(3/2)*(-1/2*1/x^2 - (3*a)/x + a^3*x + 3*a^2*Log[x]))/(a^
3*(1 - 1/(a^2*x^2))^(3/2))
```

fricas [A] time = 0.47, size = 44, normalized size = 0.30

$$\frac{\left(2\,a^3cx^3+6\,a^2cx^2\log(x)-6\,acx-c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*c*x^3 + 6*a^2*c*x^2*log(x) - 6*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 69, normalized size = 0.47

$$\frac{\left(2x^3a^3+6a^2\ln(x)x^2-6ax-1\right)\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x}{2\left(ax+1\right)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2*(2*x^3*a^3+6*a^2*ln(x)*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a*
x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.850 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 72}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.46

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax+4\log(1-ax)-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

fricas [A] time = 0.73, size = 27, normalized size = 0.25

$$\frac{\sqrt{a^2c}\left(ax+4\log(ax-1)-\log(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 65, normalized size = 0.60

$$-\frac{(-ax+\ln(x)-4\ln(ax-1))\,x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.851 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=115

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)+2*(1-1/a^2/x^2)^(1/2)/a/(-a*x+1)/
(c-c/a^2/x^2)^(1/2)+3*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 77}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x(1+ax)}{(-1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}} + \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(ax+\frac{2}{1-ax}+3\log(1-ax)\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x + 2/(1 - a*x) + 3*Log[1 - a*x]))/(a*Sqrt[c - c/
(a^2*x^2)])
```

fricas [A] time = 0.50, size = 49, normalized size = 0.43

$$\frac{\left(a^2x^2-ax+3\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{a^2c}}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)*sqrt(a^2*c)/(a^3*c*x - a^2*c
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 85, normalized size = 0.74

$$\frac{(ax-1)\left(a^{2}x^{2}+3\ln\left(ax-1\right)xa-ax-3\ln\left(ax-1\right)-2\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(ax+1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a
^2*(a^2*x^2+3*ln(a*x-1)*x*a-a*x-3*ln(a*x-1)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c-\frac{c}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.852 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(-a
*x+1)^2/(c-c/a^2/x^2)^(1/2)+3*(1-1/a^2/x^2)^(1/2)/a/c/(-a*x+1)/(c-c/a^2/x^2
)^(1/2)+3*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)^3}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{a^3(-1+ax)^3}+\frac{3}{a^3(-1+ax)^2}+\frac{3}{a^3(-1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.08, size = 64, normalized size = 0.37

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2ax+\frac{5-6ax}{(ax-1)^2}+6\log(1-ax)\right)}{2a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(2*a*x + (5 - 6*a*x)/(-1 + a*x)^2 + 6*Log[1 - a*x]
))/(2*a*(c - c/(a^2*x^2))^(3/2))
```

fricas [A] time = 0.50, size = 81, normalized size = 0.47

$$\frac{\left(2\,a^3x^3-4\,a^2x^2-4\,ax+6\left(a^2x^2-2\,ax+1\right)\log\left(ax-1\right)+5\right)\sqrt{a^2c}}{2\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*x^3 - 4*a^2*x^2 - 4*a*x + 6*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) +
 5)*sqrt(a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.07, size = 102, normalized size = 0.60

$$\frac{(ax-1)\left(2x^3a^3+6\ln(ax-1)x^2a^2-4a^2x^2-12\ln(ax-1)xa-4ax+6\ln(ax-1)+5\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^4x^3\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(2*x^3*a^3+6*ln(a*x-1)*x^2*a^2-4*a^2*x^
2-12*ln(a*x-1)*x*a-4*a*x+6*ln(a*x-1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.853 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-a}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)+1/6*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)^3/(c-c/a^2/x^2)^(1/2)-9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)^2/(c
-c/a^2/x^2)^(1/2)+31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)/(c-c/a^2/x^2)^(1/
2)+49/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-1/16*ln(a
*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-a}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^4(1+ax)}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{2a^5(-1+ax)^4}+\frac{9}{4a^5(-1+ax)^3}+\frac{31}{8a^5(-1+ax)^2}+\frac{49}{16a^5(-1+ax)}-\frac{1}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^3}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-ax)^2}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}(1-}$$

Mathematica [A] time = 0.12, size = 86, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48ax+\frac{186}{1-ax}-\frac{54}{(ax-1)^2}-\frac{8}{(ax-1)^3}+147\log(1-ax)-3\log(ax+1)\right)}{48a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(48*a*x + 186/(1 - a*x) - 8/(-1 + a*x)^3 - 54/(-1
+ a*x)^2 + 147*Log[1 - a*x] - 3*Log[1 + a*x]))/(48*a*(c - c/(a^2*x^2))^(5/2
))
```

fricas [A] time = 0.63, size = 138, normalized size = 0.52

$$\frac{\left(48\,a^4x^4-144\,a^3x^3-42\,a^2x^2+270\,ax-3\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax+1\right)+147\left(a^3x^3-3\,a^2x^2+3\,ax\right.}{48\left(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(48*a^4*x^4 - 144*a^3*x^3 - 42*a^2*x^2 + 270*a*x - 3*(a^3*x^3 - 3*a^2*
x^2 + 3*a*x - 1)*log(a*x + 1) + 147*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a
*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2
*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

maple [A] time = 0.07, size = 175, normalized size = 0.66

$$\frac{(ax-1)(ax+1)\left(48x^4a^4+147\ln(ax-1)x^3a^3-3a^3x^3\ln(ax+1)-144x^3a^3-441\ln(ax-1)x^2a^2+9\ln(ax+1\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^6x^5\left(\frac{c(a\right.}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x)

[Out] 1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(48*x^4*a^4+147*ln(a*x-1)*x^3*a^3-3*a^3*x^3*ln(a*x+1)-144*x^3*a^3-441*ln(a*x-1)*x^2*a^2+9*ln(a*x+1)*x^2*a^2-42*a^2*x^2+441*ln(a*x-1)*x*a-9*a*x*ln(a*x+1)+270*a*x-147*ln(a*x-1)+3*ln(a*x+1)-140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)

[Out] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)

[Out] Timed out

3.854 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=360

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)^3\sqrt{c}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)-1/16*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^4/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^3/(
c-c/a^2/x^2)^(1/2)-59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^2/(c-c/a^2/x^2)
^(1/2)+75/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c-c/a^2/x^2)^(1/2)-1/32*(1
-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+201/64*ln(-a*x+1)*(1-1/
a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-9/64*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)
/a/c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)^3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^4) + Sqrt[1 - 1/(a^2*x^2)]/(2*
a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*
c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64
*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^5(1+ax)^2}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{4a^7(-1+ax)^5}+\frac{3}{2a^7(-1+ax)^4}+\frac{59}{16a^7(-1+ax)^3}+\frac{75}{16a^7(-1+ax)^2}+\frac{201}{64a^7(-1+ax)}+\ldots\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^4}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-a\ldots}$$

Mathematica [A] time = 0.14, size = 140, normalized size = 0.39

$$\frac{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{75}{16a^8(1-ax)}-\frac{1}{32a^8(ax+1)}-\frac{59}{32a^8(1-ax)^2}+\frac{1}{2a^8(1-ax)^3}-\frac{1}{16a^8(1-ax)^4}+\frac{201\log(1-ax)}{64a^8}-\frac{9\log(ax+1)}{64a^8}+\frac{x}{a^7}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (a^7*(1 - 1/(a^2*x^2))^(7/2)*(x/a^7 - 1/(16*a^8*(1 - a*x)^4) + 1/(2*a^8*(1
- a*x)^3) - 59/(32*a^8*(1 - a*x)^2) + 75/(16*a^8*(1 - a*x)) - 1/(32*a^8*(1
+ a*x)) + (201*Log[1 - a*x])/(64*a^8) - (9*Log[1 + a*x])/(64*a^8)))/(c - c/
(a^2*x^2))^(7/2)
```

fricas [A] time = 0.52, size = 207, normalized size = 0.58

$$\frac{\left(64\,a^6x^6-192\,a^5x^5-174\,a^4x^4+618\,a^3x^3-118\,a^2x^2-414\,ax-9\left(a^5x^5-3\,a^4x^4+2\,a^3x^3+2\,a^2x^2-3\,ax+1\right)\mathrm{lo}\ldots}{64\left(a^7c^4x^5-3\,a^6c^4x^4+2\,a^5c^4x^3+2\,a^4c^4x^2-\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/64*(64*a^6*x^6 - 192*a^5*x^5 - 174*a^4*x^4 + 618*a^3*x^3 - 118*a^2*x^2 -
414*a*x - 9*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log(a
*x + 1) + 201*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log
(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 +
 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.07, size = 247, normalized size = 0.69

$$\frac{(ax-1)(ax+1)\left(64x^6a^6+201\ln(ax-1)x^5a^5-9\ln(ax+1)x^5a^5-192x^5a^5-603\ln(ax-1)x^4a^4+27\ln(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/64/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(64*x^6*a^6+201*ln(a*x-1)*x^5*
a^5-9*ln(a*x+1)*x^5*a^5-192*x^5*a^5-603*ln(a*x-1)*x^4*a^4+27*ln(a*x+1)*x^4*
a^4-174*x^4*a^4+402*ln(a*x-1)*x^3*a^3-18*a^3*x^3*ln(a*x+1)+618*x^3*a^3+402*
ln(a*x-1)*x^2*a^2-18*ln(a*x+1)*x^2*a^2-118*a^2*x^2-603*ln(a*x-1)*x*a+27*a*x
*ln(a*x+1)-414*a*x+201*ln(a*x-1)-9*ln(a*x+1)+208)/a^8/x^7/(c*(a^2*x^2-1)/a^
2/x^2)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

3.855 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=322

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c}}{a^4x^3\sqrt{}}$$

```
[Out] -1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)+1/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)+3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(
1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-3/2*
c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+3*c^3*(c-c/a^2/x^2)^(1/
2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-
c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-}}{6a^7x^6\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -(c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqrt[
c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^3*Sqrt[c - c/(a^
2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a
^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1
 - 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x
^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (c^3*Sqrt[c
 - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^4(1+ax)^3}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7+\frac{1}{x^7}-\frac{a}{x^6}-\frac{3a^2}{x^5}+\frac{3a^3}{x^4}+\frac{3a^4}{x^3}-\frac{3a^5}{x^2}-\frac{a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-$$

Mathematica [A] time = 0.08, size = 97, normalized size = 0.30

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(a^7x-a^6\log(x)+\frac{3a^5}{x}-\frac{3a^4}{2x^2}-\frac{a^3}{x^3}+\frac{3a^2}{4x^4}+\frac{a}{5x^5}-\frac{1}{6x^6}\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(-1/6*1/x^6 + a/(5*x^5) + (3*a^2)/(4*x^4) - a^3/x^
3 - (3*a^4)/(2*x^2) + (3*a^5)/x + a^7*x - a^6*Log[x]))/(a^7*(1 - 1/(a^2*x^2
))^(7/2))
```

fricas [A] time = 0.52, size = 96, normalized size = 0.30

$$\frac{\left(60\,a^7c^3x^7-60\,a^6c^3x^6\log(x)+180\,a^5c^3x^5-90\,a^4c^3x^4-60\,a^3c^3x^3+45\,a^2c^3x^2+12\,ac^3x-10\,c^3\right)\sqrt{a^2c}}{60\,a^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/60*(60*a^7*c^3*x^7 - 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 - 90*a^4*c^3
*x^4 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 12*a*c^3*x - 10*c^3)*sqrt(a^2*c)/(
a^8*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.06, size = 112, normalized size = 0.35

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}} x\left(-60a^{7}x^{7}+60a^{6}\ln\left(x\right)x^{6}-180x^{5}a^{5}+90x^{4}a^{4}+60x^{3}a^{3}-45a^{2}x^{2}-12ax+10\right)}{60\left(ax-1\right)\left(a^{2}x^{2}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/60*((a*x-1)/(a*x+1))^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x*(-60*a^7*x^7+
60*a^6*ln(x)*x^6-180*x^5*a^5+90*x^4*a^4+60*x^3*a^3-45*a^2*x^2-12*a*x+10)/(a
*x-1)/(a^2*x^2-1)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.856 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=238

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)-1/3*c^2*(c-c/a^2/x^
2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a
^2/x^2)^(1/2)+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-
c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a
^2/x^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqrt[c
 - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*x
^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*
Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^
2)] - (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3(1+ax)^2}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5-\frac{1}{x^5}+\frac{a}{x^4}+\frac{2a^2}{x^3}-\frac{2a^3}{x^2}-\frac{a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-}}{\sqrt{1-}}$$

Mathematica [A] time = 0.07, size = 77, normalized size = 0.32

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(a^5x-a^4\log(x)+\frac{2a^3}{x}-\frac{a^2}{x^2}-\frac{a}{3x^3}+\frac{1}{4x^4}\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*(1/(4*x^4) - a/(3*x^3) - a^2/x^2 + (2*a^3)/x + a^5
*x - a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

fricas [A] time = 1.62, size = 74, normalized size = 0.31

$$\frac{\left(12\,a^5c^2x^5-12\,a^4c^2x^4\log(x)+24\,a^3c^2x^3-12\,a^2c^2x^2-4\,ac^2x+3\,c^2\right)\sqrt{a^2c}}{12\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(12*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 - 12*a^2*c^2*
x^2 - 4*a*c^2*x + 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.06, size = 96, normalized size = 0.40

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}x\left(-12x^5a^5+12a^4\ln(x)x^4-24x^3a^3+12a^2x^2+4ax-3\right)}{12\,(ax-1)\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/12*((a*x-1)/(a*x+1))^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x*(-12*x^5*a^5+
12*a^4*ln(x)*x^4-24*x^3*a^3+12*a^2*x^2+4*a*x-3)/(a*x-1)/(a^2*x^2-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2x^2}\right)^{\frac{5}{2}} \sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.857 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=147

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2
)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-c*l
n(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 75}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] -(c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c -
c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/S
qrt[1 - 1/(a^2*x^2)] - (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^2(1+ax)}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [**A**] time = 0.05, size = 65, normalized size = 0.44

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a^3x-a^2\log(x)-\frac{3a^2}{2}+\frac{a}{x}-\frac{1}{2x^2}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
[Out] ((c - c/(a^2*x^2))^(3/2)*((-3*a^2)/2 - 1/(2*x^2) + a/x + a^3*x - a^2*Log[x]
))/(a^3*(1 - 1/(a^2*x^2))^(3/2))
```

fricas [**A**] time = 0.82, size = 44, normalized size = 0.30

$$\frac{\left(2\,a^3cx^3-2\,a^2cx^2\log(x)+2\,acx-c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
[Out] 1/2*(2*a^3*c*x^3 - 2*a^2*c*x^2*log(x) + 2*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [**A**] time = 0.06, size = 80, normalized size = 0.54

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x\left(-2x^3a^3+2a^2\ln(x)x^2-2ax+1\right)}{2\,(ax-1)\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x*(-2*x^3*a^3+2*
a^2*ln(x)*x^2-2*a*x+1)/(a*x-1)/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2 x^2}\right)^{\frac{3}{2}} \sqrt{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.858 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.60

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.45, size = 19, normalized size = 0.28

$$\frac{\sqrt{a^2c}\left(ax-\log(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
[Out] sqrt(a^2*c)*(a*x - log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.05, size = 52, normalized size = 0.76

$$-\frac{(-ax+\ln(x))\,x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^2x^2}} \sqrt{\frac{ax - 1}{ax + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c - \frac{c}{a^2\,x^2}} \sqrt{\frac{a\,x - 1}{a\,x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.859 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=72

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)-ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(
c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 + a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x}{1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.62

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(ax-\log(ax+1))}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x - Log[1 + a*x]))/(a*Sqrt[c - c/(a^2*x^2)])
```

fricas [A] time = 0.50, size = 26, normalized size = 0.36

$$\frac{\sqrt{a^2c}\left(ax-\log\left(ax+1\right)\right)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(a*x + 1))/(a^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage2
```

maple [A] time = 0.05, size = 59, normalized size = 0.82

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-ax+\ln\left(ax+1\right)\right)}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*x+ln(a*x+1))/(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)/x/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x
)
```

3.860 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(a*
x+1)/(c-c/a^2/x^2)^(1/2)+1/4*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^
2)^(1/2)-5/4*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a
*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)(1+ax)^2}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{4a^3(-1+ax)}+\frac{1}{2a^3(1+ax)^2}-\frac{5}{4a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}} - \frac{5\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.38

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4ax-\frac{2}{ax+1}+\log(1-ax)-5\log(ax+1)\right)}{4a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(4*a*x - 2/(1 + a*x) + Log[1 - a*x] - 5*Log[1 + a*
x]))/(4*a*(c - c/(a^2*x^2))^(3/2))
```

fricas [A] time = 0.82, size = 66, normalized size = 0.38

$$\frac{\left(4\,a^2x^2+4\,ax-5\,(ax+1)\log\left(ax+1\right)+(ax+1)\log\left(ax-1\right)-2\right)\sqrt{a^2c}}{4\left(a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*a^2*x^2 + 4*a*x - 5*(a*x + 1)*log(a*x + 1) + (a*x + 1)*log(a*x - 1)
- 2)*sqrt(a^2*c)/(a^3*c^2*x + a^2*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(const
 gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 100, normalized size = 0.58

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(4a^2x^2+\ln(ax-1)\,xa-5ax\ln(ax+1)+4ax+\ln(ax-1)-5\ln(ax+1)-2\right)(ax-1)}{4a^4x^3\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(4*a^2*x^2+ln(a*x-1)*x*a-5*a*x*ln(a*x+1
)+4*a*x+ln(a*x-1)-5*ln(a*x+1)-2)*(a*x-1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.861 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)^2/(c-c/
a^2/x^2)^(1/2)-(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+7/16*l
n(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-23/16*ln(a*x+1)*(1-
1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(8*a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - Sqrt[1 - 1/(a^2*x^2)]/(a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)]) - (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^2(1+ax)^3}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{8a^5(-1+ax)^2}+\frac{7}{16a^5(-1+ax)}-\frac{1}{4a^5(1+ax)^3}+\frac{1}{a^5(1+ax)^2}-\frac{23}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax}$$

Mathematica [A] time = 0.13, size = 85, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(2\left(8ax+\frac{1}{1-ax}-\frac{8}{ax+1}+\frac{1}{(ax+1)^2}\right)+7\log(1-ax)-23\log(ax+1)\right)}{16a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(2*(8*a*x + (1 - a*x)^(-1) + (1 + a*x)^(-2) - 8/(1
 + a*x)) + 7*Log[1 - a*x] - 23*Log[1 + a*x]))/(16*a*(c - c/(a^2*x^2))^(5/2)
)
```

fricas [A] time = 0.55, size = 135, normalized size = 0.51

$$\frac{\left(16\,a^4x^4+16\,a^3x^3-34\,a^2x^2-18\,ax-23\left(a^3x^3+a^2x^2-ax-1\right)\log\left(ax+1\right)+7\left(a^3x^3+a^2x^2-ax-1\right)\log\left(a\right.\right.}{16\left(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/16*(16*a^4*x^4 + 16*a^3*x^3 - 34*a^2*x^2 - 18*a*x - 23*(a^3*x^3 + a^2*x^2
 - a*x - 1)*log(a*x + 1) + 7*(a^3*x^3 + a^2*x^2 - a*x - 1)*log(a*x - 1) + 1
2)*sqrt(a^2*c)/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
```

```
(correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(const
 gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 175, normalized size = 0.67

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)\left(16x^4a^4+7\ln(ax-1)x^3a^3-23a^3x^3\ln(ax+1)+16x^3a^3+7\ln(ax-1)x^2a^2-23\ln(a\right.}{16a^6x^5\left(\frac{c\left(a^2x^2-1\right.}{a^2x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/16*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(16*x^4*a^4+7*ln(a*x-1)*x^3*a^
3-23*a^3*x^3*ln(a*x+1)+16*x^3*a^3+7*ln(a*x-1)*x^2*a^2-23*ln(a*x+1)*x^2*a^2-
34*a^2*x^2-7*ln(a*x-1)*x*a+23*a*x*ln(a*x+1)-18*a*x-7*ln(a*x-1)+23*ln(a*x+1)
+12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.862 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=358

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)-1/32*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^2/(c-c/a^2/x^2)^(1/2)+5/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c
-c/a^2/x^2)^(1/2)-1/24*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^3/(c-c/a^2/x^2)^(1
/2)+11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-3/2*(1-1/
a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+19/32*ln(-a*x+1)*(1-1/a^2/
x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-51/32*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (5*Sqrt[1 - 1/(a^2*x^2)])
/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(24*a*c
^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (11*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^
3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(2*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^3(1+ax)^4}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{16a^7(-1+ax)^3}+\frac{5}{16a^7(-1+ax)^2}+\frac{19}{32a^7(-1+ax)}+\frac{1}{8a^7(1+ax)^4}-\frac{11}{16a^7(1+ax)^3}+\frac{}{2a^7}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1+a}$$

Mathematica [A] time = 0.17, size = 104, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96ax+\frac{30}{1-ax}-\frac{144}{ax+1}-\frac{3}{(ax-1)^2}+\frac{33}{(ax+1)^2}-\frac{4}{(ax+1)^3}+57\log(1-ax)-153\log(ax+1)\right)}{96a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(96*a*x + 30/(1 - a*x) - 3/(-1 + a*x)^2 - 4/(1 + a
*x)^3 + 33/(1 + a*x)^2 - 144/(1 + a*x) + 57*Log[1 - a*x] - 153*Log[1 + a*x]
))/(96*a*(c - c/(a^2*x^2))^(7/2))
```

fricas [A] time = 0.80, size = 201, normalized size = 0.56

$$\frac{\left(96\,a^6x^6+96\,a^5x^5-366\,a^4x^4-222\,a^3x^3+338\,a^2x^2+122\,ax-153\left(a^5x^5+a^4x^4-2\,a^3x^3-2\,a^2x^2+ax+1\right)\log\right.}{96\left(a^7c^4x^5+a^6c^4x^4-2\,a^5c^4x^3-2\,a^4c^4x^2+a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 1/96*(96*a^6*x^6 + 96*a^5*x^5 - 366*a^4*x^4 - 222*a^3*x^3 + 338*a^2*x^2 + 1
22*a*x - 153*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x
+ 1) + 57*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x - 1
) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 + a^6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*
x^2 + a^3*c^4*x + a^2*c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(const
 gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 247, normalized size = 0.69

$$\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\,(ax-1)\left(96x^6a^6+57\ln(ax-1)\,x^5a^5-153\ln(ax+1)\,x^5a^5+96x^5a^5+57\ln(ax-1)\,x^4a^4-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/96*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(96*x^6*a^6+57*ln(a*x-1)*x^5*a
^5-153*ln(a*x+1)*x^5*a^5+96*x^5*a^5+57*ln(a*x-1)*x^4*a^4-153*ln(a*x+1)*x^4*
a^4-366*x^4*a^4-114*ln(a*x-1)*x^3*a^3+306*a^3*x^3*ln(a*x+1)-222*x^3*a^3-114
*ln(a*x-1)*x^2*a^2+306*ln(a*x+1)*x^2*a^2+338*a^2*x^2+57*ln(a*x-1)*x*a-153*a
*x*ln(a*x+1)+122*a*x+57*ln(a*x-1)-153*ln(a*x+1)-88)/a^8/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

3.863 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=375

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{15(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(ax+1)}-\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}+\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}-\frac{2a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\sin^{-1}(a}{(1-ax)^{7/2}(ax+1)^{7/2}}$$

```
[Out] 7/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(a*x+1)^3+3/8*a^5*(c-c/a^2/x^2)
^(7/2)*x^6/(-a*x+1)^3/(a*x+1)^2-1/15*a*(c-c/a^2/x^2)^(7/2)*x^2/(a*x+1)-19/1
6*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)+2/3*a^3*(c-c/a^2/x^2)^(7/2
)*x^4/(-a*x+1)^2/(a*x+1)-23/120*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a*x+1)/(a*x+1
)+1/6*(c-c/a^2/x^2)^(7/2)*x*(-a*x+1)/(a*x+1)-2*a^6*(c-c/a^2/x^2)^(7/2)*x^7*
arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)+25/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7*
arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2)
```

Rubi [A] time = 0.54, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{8(1-ax)^3(ax+1)^2}-\frac{19a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3(1-ax)^2(ax+1)}-\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}-\frac{ax^2\left(c\right.}{15(}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (7*a^6*(c - c/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (3*a^5*(
c - c/(a^2*x^2))^(7/2)*x^6)/(8*(1 - a*x)^3*(1 + a*x)^2) - (a*(c - c/(a^2*x^
2))^(7/2)*x^2)/(15*(1 + a*x)) - (19*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(16*(1
 - a*x)^3*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a*x)^2*(
1 + a*x)) - (23*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(120*(1 - a*x)*(1 + a*x))
+ ((c - c/(a^2*x^2))^(7/2)*x*(1 - a*x))/(6*(1 + a*x)) - (2*a^6*(c - c/(a^2*
x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (25*a^6*(c
 - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16*(1 - a*
x)^(7/2)*(1 + a*x)^(7/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{9/2}(1+ax)^{5/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{7/2}(1+ax)^{3/2}(-2a-7a^2x)}{x^6}dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int\frac{(1-ax)^{5/2}(1+ax)^{3}}{x}}{30(1-ax)^{7/2}(1+ax}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right.\right.}{}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(1+}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x}\right.}{120(1-ax)(}$$

$$= \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)}{3(1-ax)^2(1+}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right.}{16(1-ax)^3(1}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right.}{16(1-ax)^3(1}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right.}{16(1-ax)^3(1}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right.}{16(1-ax)^3(1}$$

Mathematica [A] time = 0.19, size = 150, normalized size = 0.40

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(-480a^6x^6\log\left(\sqrt{a^2x^2-1}+ax\right)+375a^6x^6\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(240a^6x^6+736a^5x^5+105\right.\right.}{240a^6x^5\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 + 96*a*x + 70*a^2*x^2 -
 352*a^3*x^3 + 105*a^4*x^4 + 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcTa
n[1/Sqrt[-1 + a^2*x^2]] - 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240*
a^6*x^5*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.64, size = 438, normalized size = 1.17

$$\left[\frac{960\,a^5\sqrt{-c}\,c^3x^5\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+375\,a^5\sqrt{-c}\,c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(240\,a^6c^3x^6+\right.}{480\,a^6x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2 -
 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(240*a^6*c^
3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^
2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1/240*
(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 + 736*a^5*c^3*x^5 + 10
5*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40*c^3)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

giac [A] time = 113.47, size = 561, normalized size = 1.50

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2cx^2-}}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*
sgn(x)/a^2 - 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn
(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) - 1440*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^9*c^5*abs(a)*sgn(x) - 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*
c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sg
n(x) - 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) - 6720*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) - 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9
*abs(a)*sgn(x) - 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

maple [B] time = 0.12, size = 795, normalized size = 2.12

$$\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{7}{2}}x\left(-2016\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,x^7a^9c+2016\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\,x^5a^9-375\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,x^6a^8c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(-2016*(c*(a^2*x^2-1)/a^2)^(7/2)
*(-c/a^2)^(1/2)*x^7*a^9*c+2016*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^5
*a^9-375*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*x^6*a^8*c+480*(-c/a^2)^(1
/2)*((a*x-1)*(a*x+1)*c/a^2)^(7/2)*x^6*a^8*c-105*(c*(a^2*x^2-1)/a^2)^(9/2)*(
-c/a^2)^(1/2)*x^4*a^8+2352*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*x^7*a^7
*c^2-560*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7*c^2+224*(c*(a
^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^3*a^7+525*(c*(a^2*x^2-1)/a^2)^(5/2)*(
-c/a^2)^(1/2)*x^6*a^6*c^2-2940*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^7
*a^5*c^3+700*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^7*a^5*c^3-630*(
c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*x^2*a^6-875*(c*(a^2*x^2-1)/a^2)^(3/
2)*(-c/a^2)^(1/2)*x^6*a^4*c^3+672*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*
x*a^5+4410*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^7*a^3*c^4-1050*(-c/a^
2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^7*a^3*c^4-280*a^4*(c*(a^2*x^2-1)/a
^2)^(9/2)*(-c/a^2)^(1/2)+2625*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*x^6*
a^2*c^4-4410*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*c^(9/2)
*x^6*a+1050*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*(-c/a^2
)^(1/2)*c^(9/2)*x^6*a+2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a
^2-c)/a^2/x)*x^6*c^5)/(c*(a^2*x^2-1)/a^2)^(7/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [C] time = 32.68, size = 1059, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**3*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Piecewise(
(I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
```

```
**2 + 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c
), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a
**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 - 2*c**3*Piecewise((2*a**3*sqrt(
c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - s
qrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(
c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3)
 - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 + c**3*Piecewise(
(I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*
x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(
24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**
2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sq
rt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a*
*2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*
x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

3.864 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=293

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}-\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}-\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{2a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\sin^{-1}(ax)}{(1-ax)^{5/2}(ax+1)^{5/2}}$$

```
[Out] -7/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(a*x+1)^2-1/6*a*(c-c/a^2/x^2)^(
5/2)*x^2/(a*x+1)+2*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)-7/24*a^2*
(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)/(a*x+1)+1/4*(c-c/a^2/x^2)^(5/2)*x*(-a*x+1)
/(a*x+1)+2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/(a*x+1)^(
5/2)-9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/
(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

Rubi [A] time = 0.48, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^2(ax+1)}-\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}+\frac{2a^4x^5\left(c\right.}{(1-}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*a^4*(c - c/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (a*(c -
c/(a^2*x^2))^(5/2)*x^2)/(6*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)
/((1 - a*x)^2*(1 + a*x)) - (7*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(24*(1 - a*x
)*(1 + a*x)) + ((c - c/(a^2*x^2))^(5/2)*x*(1 - a*x))/(4*(1 + a*x)) + (2*a^4
*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)^(5/2))
 - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])
/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{7/2}(1+ax)^{3/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}\left(-2a-5a^2x\right)}{x^4}dx}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}\sqrt{1+ax}}{x^3}}{12(1-ax)^{5/2}(1+ax)}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)} - \left(\left(c-\frac{c}{a^2x^2}\right)\right.$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}{4(1+a}$$

$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)}{24(1-ax)(1}$$

$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)}{24(1-ax)(1}$$

$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)}{24(1-ax)(1}$$

$$= -\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)} - \frac{7a^2\left(c-\frac{c}{a^2x^2}\right)}{24(1-ax)(1}$$

Mathematica [A] time = 0.15, size = 134, normalized size = 0.46

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(-48a^4x^4\log\left(\sqrt{a^2x^2-1}+ax\right)+27a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(24a^4x^4+64a^3x^3-3a^2x^2-\right.\right.}{24a^4x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 - 16*a*x - 3*a^2*x^2 + 64
*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 48*a^4*x
^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.52, size = 394, normalized size = 1.34

$$\left[\frac{96\,a^3\sqrt{-c}\,c^2x^3\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+27\,a^3\sqrt{-c}\,c^2x^3\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(24\,a^4c^2x^4+64\,a^3c\right.}{48\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) + 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 - 2*
a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*c^2*x^
4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt(
(a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(2*a^2*
c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c^
2*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

giac [A] time = 7.13, size = 416, normalized size = 1.42

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}-\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{12\sqrt{a^2cx^2-c}\,c^2}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x)/a^2 - 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)
/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) + 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4
*abs(a)*sgn(x) + 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(
x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) + 160*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) + 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

maple [B] time = 0.06, size = 625, normalized size = 2.13

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}x\left(-80\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}x^5a^7c+80\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}x^3a^7-48\sqrt{-\frac{c}{a^2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}x^4a^6c-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(5/2)*x^5*a^7*c+80*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^3*a^7
-48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^6*c-27*(-c/a^2)^(1/2
)*(c*(a^2*x^2-1)/a^2)^(5/2)*x^4*a^6*c+60*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/
a^2)^(3/2)*x^5*a^5*c^2+75*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(7/2)*x^2*a^6+
100*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^5*a^5*c^2-80*(-c/a^2)^(1/2)*
(c*(a^2*x^2-1)/a^2)^(7/2)*x*a^5+45*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)
*x^4*a^4*c^2-90*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c^3-15
0*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c^3+30*a^4*(c*(a^2*x^2-1
)/a^2)^(7/2)*(-c/a^2)^(1/2)+150*(-c/a^2)^(1/2)*c^(7/2)*ln(x*c^(1/2)+(c*(a^2
*x^2-1)/a^2)^(1/2))*x^4*a+90*(-c/a^2)^(1/2)*c^(7/2)*ln((c^(1/2)*((a*x-1)*(a
*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^4*a-135*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^
```

```
2)^(1/2)*x^4*a^2*c^3-135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2
-c)/a^2/x)*x^4*c^4)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(5/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [C] time = 19.52, size = 500, normalized size = 1.71

$$c^2\left(\begin{cases} \frac{\sqrt{c}\sqrt{a^2x^2-1}}{a} - \frac{i\sqrt{c}\log(ax)}{a} + \frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a} + \frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a} + \frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a} - \frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise} \end{cases}\right) - \frac{2c^2\left(\begin{cases} -\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}} + \sqrt{c}\operatorname{acosh}(ax) + \frac{\sqrt{c}}{ax\sqrt{\ldots}} \\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}} - i\sqrt{c}\operatorname{asin}(ax) - \frac{i\ldots}{ax\sqrt{-\ldots}} \end{cases}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**2*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + 2*c**2*Piecewis
e((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 - c**2
*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 +
 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sqr
t(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(
c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**4
```

3.865 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=213

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}-\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-\cdots}$$

```
[Out] -a*(c-c/a^2/x^2)^(3/2)*x^2/(a*x+1)-5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1)
/(a*x+1)+1/2*(c-c/a^2/x^2)^(3/2)*x*(-a*x+1)/(a*x+1)-2*a^2*(c-c/a^2/x^2)^(3/
2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)+1/2*a^2*(c-c/a^2/x^2)^(3/2)
*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

Rubi [A] time = 0.45, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6167, 6159, 6129, 97, 149, 154, 157, 41, 216, 92, 208}

$$-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}-\frac{2a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\sin^{-1}(ax)}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 + a*x)) - (5*a^2*(c - c/(a^2*x^2))^(3/
2)*x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x))/(2*
(1 + a*x)) - (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/
2)*(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x
]*Sqrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$
$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1-ax)^{5/2}\sqrt{1+ax}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1-ax)^{3/2}(-2a-3a^2x)}{x^2\sqrt{1+ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1-ax}\,(a^2+5}{x\sqrt{1+ax}}}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2}\right.\right.}{2a}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{\left(a^2\left(c-\right.\right.}{}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} + \frac{\left(a^3\left(c-\right.\right.}{}$$
$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)} - \frac{2a^2\left(c-\right.}{(1-}$$

Mathematica [A] time = 0.12, size = 115, normalized size = 0.54

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(2a^2x^2+4ax-1\right)-4a^2x^2\log\left(\sqrt{a^2x^2-1}+ax\right)+a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2a^2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 + 4*a*x + 2*a^2*x^2) + a^2
*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]]
))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.51, size = 316, normalized size = 1.48

$$\left[\frac{8\,a\sqrt{-c}\,cx\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+a\sqrt{-c}\,cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(2\,a^2cx^2+4\,acx-c\right)\sqrt{\frac{a^2cx}{a^2x}}}{4\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c)) + a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt
```

```
((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(2*a^2*c*x^2 + 4*a*c*x - c)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2*a^2*
c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*x
^2 + 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

giac [A] time = 0.47, size = 266, normalized size = 1.25

$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}-\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}\,c\,\mathrm{sgn}(x)}{a^2}-\frac{\left(\sqrt{a^2\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
 - 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a
)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^3*c^2*abs(a)*sgn(x) + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/
2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) + 4*a*c
^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a))
)*abs(a)
```

maple [B] time = 0.06, size = 454, normalized size = 2.13

$$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x\left(12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^3a^5c-12\sqrt{-\frac{c}{a^2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}x\,a^5-4\sqrt{-\frac{c}{a^2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^2a^4c+\sqrt{-\frac{c}{a^2}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(3/2)*x^3*a^5*c-12*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(5/2)*x*a^5-4*(-
c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^2*a^4*c+(-c/a^2)^(1/2)*(c*(a^2
*x^2-1)/a^2)^(3/2)*x^2*a^4*c+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)
*x^3*a^3*c^2+3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)-18*(-c/a^2)^(1/
2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^3*c^2+18*(-c/a^2)^(1/2)*c^(5/2)*ln(x*c^(
1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^2*a-6*(-c/a^2)^(1/2)*c^(5/2)*ln((c^(1/2)*
((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a-3*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(1/2)*x^2*a^2*c^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/a^2/x)*x^2*c^3)/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(3/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax-1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [C] time = 13.21, size = 376, normalized size = 1.77

$$c\left(\begin{cases} \frac{\sqrt{c}\sqrt{a^2x^2-1}}{a} - \frac{i\sqrt{c}\log(ax)}{a} + \frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a} + \frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } \left|a^2x^2\right| > 1 \\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a} + \frac{i\sqrt{c}\log\left(a^2x^2\right)}{2a} - \frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise} \end{cases}\right) - \frac{2c\left(\begin{cases} -\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}} + \sqrt{c}\operatorname{acosh}(ax) + \frac{}{ax} \\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}} - i\sqrt{c}\operatorname{asin}(ax) - \frac{}{ax\sqrt{}} \end{cases}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(
c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*
sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)
*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c*Piecewise((-a*sqrt(c)*x/sqrt
(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), A
bs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*
x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I*a*sqrt
(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)
/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c)
*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

3.866 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6159, 6129, 102, 157, 41, 216, 92, 208}

$$x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1-}}}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\left(2a\sqrt{}\right.}{}\\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right.}{\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.69

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}-2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 0.79, size = 267, normalized size = 2.30

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\,\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.05, size = 196, normalized size = 1.69

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}+c\ln\left(\frac{2\sqrt{-}}{}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a^2)^(1/2)+2*c^(1/2)*ln((c^(1/2
)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*a*(-c/a^2)^(1/2)+c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.867 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=112

$$-\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{ax+1}\sqrt{1-ax}\,\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] -(-a*x+1)^2/a^2/x/(c-c/a^2/x^2)^(1/2)-2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2
)^(1/2)-2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/2
)
```

Rubi [A] time = 0.29, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6159, 6129, 78, 50, 41, 216}

$$-\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{ax+1}\sqrt{1-ax}\,\sin^{-1}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] -((1 - a*x)^2/(a^2*Sqrt[c - c/(a^2*x^2)]*x)) - (2*(1 - a*x)*(1 + a*x))/(a^2
*Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^
2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx\\
&= -\frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{x\sqrt{1-ax}}{(1+ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}\\
&= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2\sqrt{1-ax}\,\sqrt{1+ax}\,\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.61

$$
\frac{a^2x^2-2\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax-3}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (-3 + 2*a*x + a^2*x^2 - 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])
/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

fricas [A] time = 0.58, size = 212, normalized size = 1.89

$$\left[\frac{(ax+1)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(a^2x^2+3ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx+ac}, \frac{2(ax+1)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}x^2\sqrt{\frac{a^2c}{a^2}}}{a^2cx^2-c}\right.}{a^2cx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
*c*x + a*c), (2*(a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^2*c*x + a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

maple [A] time = 0.05, size = 179, normalized size = 1.60

$$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2}}\left(-\sqrt{c}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,x\,a^2+2\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)xac-2a\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\sqrt{c}-\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a\sqrt{c}+2\right.}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\,c^{\frac{3}{2}}a\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2)^(1/2)*(-c^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2+2*ln(x
*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a*c-2*a*((a*x-1)*(a*x+1)*c/a^2)^(1/2)
*c^(1/2)-(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)+2*ln(x*c^(1/2)+(c*(a^2*x^2-1)/
a^2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{\sqrt{c-\frac{c}{a^2\,x^2}}\ (a\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\ (ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)), x)
```

3.868 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}} dx$

Optimal. Leaf size=124

$$-\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

```
[Out] -1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x+2/3*(-a*x+1)^2*(a*x+1)*(2*a*x+5)/
a^4/(c-c/a^2/x^2)^(3/2)/x^3+2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^4/
(c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.42, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6167, 6159, 6129, 98, 143, 41, 216}

$$\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\sin^{-1}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -(1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(1 - a*x)^2*(1 + a*x)*(
5 + 2*a*x))/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (2*(1 - a*x)^(3/2)*(1 + a
*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx\\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{\sqrt{1-ax}\,(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2-4ax)}{\sqrt{1-ax}\,(1+ax)^{3/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1-}}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,d}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}\\
&= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.77

$$
\frac{3a^3x^3+11a^2x^2-6(ax+1)\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)-4ax-10}{3a^2cx(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (-10 - 4*a*x + 11*a^2*x^2 + 3*a^3*x^3 - 6*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[
a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
```

fricas [A] time = 0.59, size = 279, normalized size = 2.25

$$\left[\frac{3\left(a^2x^2+2\,ax+1\right)\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3+14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}, \frac{6\left(a^2x^2+2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 + 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2), 1/3*(6*(
a^2*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/
(a^2*x^2))/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

maple [B] time = 0.05, size = 326, normalized size = 2.63

$$\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,x^3a^3+15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}-4c^{\frac{3}{2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^2a^2-6\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\right.}{3\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] 1/3*(3*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^3+15*x^2*a^2*c^(3/2)*((a
*x-1)*(a*x+1)*c/a^2)^(1/2)-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2-6*ln
(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2
*x^2-1)/a^2)^(1/2)*x*a^2*c-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a-6*ln(x*c
^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(1/2)*a*c-12*c^(3/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+2*(c*(a^2*x^2-1
)/a^2)^(1/2)*c^(3/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(1/2)/x^3/(c*(a^2*x^2
-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax - 1}{\left(- c \left(-1 + \frac{1}{ax}\right) \left(1 + \frac{1}{ax}\right)\right)^{\frac{3}{2}} \left(ax + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

3.869 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-ax)^{5/2}\sin^{-1}(ax)}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2}{5a^3x^2}$$

```
[Out] -(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x-2/5*(-a*x+1)^3/a^3/(c-c/a^2/x^2)^(5/2
)/x^2+2/15*(-a*x+1)^3*(a*x+1)/a^4/(c-c/a^2/x^2)^(5/2)/x^3-2/15*(-a*x+1)^3*(
a*x+1)^2*(13*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5-2*(-a*x+1)^(5/2)*(a*x+1)^(
5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A] time = 0.45, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 150, 143, 41, 216}

$$\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^3}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-ax)^{5/2}}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] -((1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x)) - (2*(1 - a*x)^3)/(5*a^3*(c
- c/(a^2*x^2))^(5/2)*x^2) + (2*(1 - a*x)^3*(1 + a*x))/(15*a^4*(c - c/(a^2*x
^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 13*a*x))/(15*a^6*(c - c/
(a^2*x^2))^(5/2)*x^5) - (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcSin[a*x])/(a^
6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{3/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+2ax)}{\sqrt{1-ax}\,(1+ax)^{7/2}}\,dx}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2\left(6a+8a^2x\right)}{\sqrt{1-ax}\,(1+ax)^{5/2}}\,dx}{5a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int}{15a^6\left(c-\frac{c}{a^2x^2}\right)}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+1}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+1}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+ax)^2(28+1}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

Mathematica [A] time = 0.11, size = 105, normalized size = 0.54

$$\frac{15a^4x^4+76a^3x^3+32a^2x^2-30(ax+1)^2\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)-82ax-56}{15a^2c^2x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (-56 - 82*a*x + 32*a^2*x^2 + 76*a^3*x^3 + 15*a^4*x^4 - 30*(1 + a*x)^2*Sqrt[
-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x^
2)]*x*(1 + a*x)^2)
```

fricas [A] time = 1.10, size = 351, normalized size = 1.80

$$\left[\frac{15\left(a^4x^4+2\,a^3x^3-2\,ax-1\right)\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(15\,a^5x^5+76\,a^4x^4+32\,a^3x^3-82\,a^2x^2\right.}{15\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 + 76*a^4*x^
4 + 32*a^3*x^3 - 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3), 1/15*(30*(a^4*x^4 + 2*a^3*
x^3 - 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2
*x^2))/(a^2*c*x^2 - c)) + (15*a^5*x^5 + 76*a^4*x^4 + 32*a^3*x^3 - 82*a^2*x^
2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
 2*a^2*c^3*x - a*c^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

maple [B] time = 0.06, size = 462, normalized size = 2.37

$$\frac{\left(15c^{\frac{5}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^5a^5+45x^4c^{\frac{5}{2}}a^4\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}+16c^{\frac{5}{2}}\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^4a^4-60c^{\frac{5}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{3}{2}}x^3a^3+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/15*(15*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^5*a^5+45*x^4*c^(5/2)*a^4*(
(a*x-1)*(a*x+1)*c/a^2)^(3/2)+16*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^4*a^4-6
0*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x^3*a^3+16*c^(5/2)*(c*(a^2*x^2-1)/a
^2)^(3/2)*x^3*a^3-30*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+
1)*c/a^2)^(3/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^4*c-90*c^(5/2)*((a*x-1)*(a*x+
1)*c/a^2)^(3/2)*x^2*a^2-24*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^2-30*ln(
x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*a^3*c+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)*x*a-24*c^(
5/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a+50*c^(5/2)*((a*x-1)*(a*x+1)*c/a^2)^(3/2)
+6*c^(5/2)*(c*(a^2*x^2-1)/a^2)^(3/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(3/2)
/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

3.870 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^{7/2}(1-ax)^{7/2}\sin^{-1}(ax)}{a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+$$

```
[Out] -1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x+10/3*(-a*x+1)^3/a^3/(c-c/a^2/x^2)
^(7/2)/x^2+12/7*(-a*x+1)^4/a^4/(c-c/a^2/x^2)^(7/2)/x^3+82/105*(-a*x+1)^4*(a
*x+1)/a^5/(c-c/a^2/x^2)^(7/2)/x^4+2/35*(-a*x+1)^4*(a*x+1)^2/a^6/(c-c/a^2/x^
2)^(7/2)/x^5+2/35*(-a*x+1)^4*(a*x+1)^3*(37*a*x+72)/a^8/(c-c/a^2/x^2)^(7/2)/
x^7+2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A] time = 0.49, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 150, 143, 41, 216}

$$\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{82(ax+1)(1-ax)^4}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{12(1-ax)^4}{7a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{10(1-ax}{3a^3x^2\left(c-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] -(1 - a*x)^2/(3*a^2*(c - c/(a^2*x^2))^(7/2)*x) + (10*(1 - a*x)^3)/(3*a^3*(c
 - c/(a^2*x^2))^(7/2)*x^2) + (12*(1 - a*x)^4)/(7*a^4*(c - c/(a^2*x^2))^(7/2
)*x^3) + (82*(1 - a*x)^4*(1 + a*x))/(105*a^5*(c - c/(a^2*x^2))^(7/2)*x^4) +
 (2*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^2*x^2))^(7/2)*x^5) + (2*(1 -
 a*x)^4*(1 + a*x)^3*(72 + 37*a*x))/(35*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) + (
2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])/(a^8*(c - c/(a^2*x^2))^(7/2)
*x^7)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
```

```
] && NeQ[m, -1] && !(SumSimplerQ[n, 1] && !SumSimplerQ[m, 1])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{5/2}(1+ax)^{9/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+4ax)}{(1-ax)^{3/2}(1+ax)^{9/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4(-50a-14a^2x)}{\sqrt{1-ax}\,(1+ax)^{9/2}}\,dx}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)}{21a^6\left(c-\right.}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}$$

Mathematica [A] time = 0.13, size = 131, normalized size = 0.49

$$\frac{105a^6x^6+562a^5x^5+74a^4x^4-1226a^3x^3-636a^2x^2-210(ax-1)(ax+1)^3\sqrt{a^2x^2-1}\,\log\left(\sqrt{a^2x^2-1}+ax\right)+}{105a^2x(ax-1)\sqrt{c-\frac{c}{a^2x^2}}\,(acx+c)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (432 + 654*a*x - 636*a^2*x^2 - 1226*a^3*x^3 + 74*a^4*x^4 + 562*a^5*x^5 + 10
5*a^6*x^6 - 210*(-1 + a*x)*(1 + a*x)^3*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1
 + a^2*x^2]])/(105*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)*(c + a*c*x)^3)
```

fricas [A] time = 0.74, size = 495, normalized size = 1.83

$$\left[\frac{105\left(a^6x^6+2a^5x^5-a^4x^4-4a^3x^3-a^2x^2+2ax+1\right)\sqrt{c}\log\left(2a^2cx^2-2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(105a^7x^7+56\ldots\right.}{105\left(a^7c^4x^6+2a^6c^4x^5-a^5c^4x^4-4a^4c^4x^3-a^3c^4x^2+\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4 - 636*a^3
*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x
+ a*c^4), 1/105*(210*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 +
 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
)/(a^2*c*x^2 - c)) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4
 - 636*a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*
a^2*c^4*x + a*c^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

maple [B] time = 0.08, size = 572, normalized size = 2.12

$$-\frac{\left(-105c^{\frac{7}{2}}\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}x^7a^7+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^6a^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{(ax-1)(ax+1)c}{a^2}\right)^{\frac{5}{2}}+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}x^5a^5+39\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)*(a*x-1)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] -1/105*(-105*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^7*a^7+96*(c*(a^2*x^2-1
)/a^2)^(5/2)*c^(7/2)*x^6*a^6-553*x^6*c^(7/2)*a^6*((a*x-1)*(a*x+1)*c/a^2)^(5
/2)+96*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^5*a^5+392*c^(7/2)*((a*x-1)*(a*x+
1)*c/a^2)^(5/2)*x^5*a^5-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^4*a^4+1540*
c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^4*a^4+210*(c*(a^2*x^2-1)/a^2)^(5/2)
*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x*a^
6*c-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x^3*a^3-350*c^(7/2)*((a*x-1)*(a*x
+1)*c/a^2)^(5/2)*x^3*a^3+210*(c*(a^2*x^2-1)/a^2)^(5/2)*ln(x*c^(1/2)+(c*(a^2
*x^2-1)/a^2)^(1/2))*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*a^5*c+180*(c*(a^2*x^2-1)/
a^2)^(5/2)*c^(7/2)*x^2*a^2-1470*c^(7/2)*((a*x-1)*(a*x+1)*c/a^2)^(5/2)*x^2*a
^2+180*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*x*a+42*c^(7/2)*((a*x-1)*(a*x+1)*c/
```

```
a^2)^(5/2)*x*a-30*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)+462*c^(7/2)*((a*x-1)*(a
*x+1)*c/a^2)^(5/2))*(a*x-1)/((a*x-1)*(a*x+1)*c/a^2)^(5/2)/x^7/(c*(a^2*x^2-1
)/a^2/x^2)^(7/2)/a^8/c^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x-1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

3.871 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=322

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c}{a^4}$$

```
[Out] -1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/x^8/(1-1/a^2/x^2)^(1/2)+3/7*c^4*(c-c/a^2/x
^2)^(1/2)/a^8/x^7/(1-1/a^2/x^2)^(1/2)-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/x^5/(
1-1/a^2/x^2)^(1/2)+3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+
2*c^4*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-4*c^4*(c-c/a^2/x^2)^(
1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c^4*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1
/2)-3*c^4*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^4\sqrt{c-}}{8a^9x^8\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -(c^4*Sqrt[c - c/(a^2*x^2)])/(8*a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*Sqr
t[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c/(
a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^4*Sqrt[c - c/(a^2*x^2)]
)/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*Sq
rt[1 - 1/(a^2*x^2)]*x^3) - (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a
^2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*c^
4*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx = \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^6(1+ax)^3}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^9+\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}-\frac{6a^4}{x^5}-\frac{6a^5}{x^4}+\frac{8a^6}{x^3}-\frac{3a^8}{x}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

Mathematica [A] time = 0.09, size = 97, normalized size = 0.30

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(a^9x-3a^8\log(x)-\frac{4a^6}{x^2}+\frac{2a^5}{x^3}+\frac{3a^4}{2x^4}-\frac{8a^3}{5x^5}+\frac{3a}{7x^7}-\frac{1}{8x^8}\right)}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(9/2)*(-1/8*1/x^8 + (3*a)/(7*x^7) - (8*a^3)/(5*x^5) + (3
*a^4)/(2*x^4) + (2*a^5)/x^3 - (4*a^6)/x^2 + a^9*x - 3*a^8*Log[x]))/(a^9*(1
- 1/(a^2*x^2))^(9/2))
```

fricas [A] time = 0.61, size = 96, normalized size = 0.30

$$\frac{\left(280\,a^9c^4x^9-840\,a^8c^4x^8\log(x)-1120\,a^6c^4x^6+560\,a^5c^4x^5+420\,a^4c^4x^4-448\,a^3c^4x^3+120\,ac^4x-35\,c^4\right)\sqrt{a^2}}{280\,a^{10}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/280*(280*a^9*c^4*x^9 - 840*a^8*c^4*x^8*log(x) - 1120*a^6*c^4*x^6 + 560*a^
5*c^4*x^5 + 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqrt(
a^2*c)/(a^10*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c-\frac{c}{a^2x^2}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.07, size = 112, normalized size = 0.35

$$-\frac{\left(-280a^{9}x^{9}+840a^{8}\ln(x)x^{8}+1120x^{6}a^{6}-560x^{5}a^{5}-420x^{4}a^{4}+448x^{3}a^{3}-120ax+35\right)x\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{280\left(ax-1\right)^{3}\left(a^{2}x^{2}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/280*(-280*a^9*x^9+840*a^8*ln(x)*x^8+1120*x^6*a^6-560*x^5*a^5-420*x^4*a^4
+448*x^3*a^3-120*a*x+35)*x*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^
(3/2)/(a*x-1)^3/(a^2*x^2-1)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{9/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.872 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=324

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\cdots$$

```
[Out] 1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)-3/5*c^3*(c-c/a^2/x^
2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)+1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1
-1/a^2/x^2)^(1/2)+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-5
/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1
/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
-3*c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*Sqrt
[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(
3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqr
t[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*
x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*c^3*Sqr
t[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^5(1+ax)^2}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7-\frac{1}{x^7}+\frac{3a}{x^6}-\frac{a^2}{x^5}-\frac{5a^3}{x^4}+\frac{5a^4}{x^3}+\frac{a^5}{x^2}-\frac{3a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{5}{2a}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.29

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{60a^7x^7-60a^5x^5-150a^4x^4+100a^3x^3+15a^2x^2-36ax+10}{60x^6}-3a^6\log(x)\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*((10 - 36*a*x + 15*a^2*x^2 + 100*a^3*x^3 - 150*a^4
*x^4 - 60*a^5*x^5 + 60*a^7*x^7)/(60*x^6) - 3*a^6*Log[x]))/(a^7*(1 - 1/(a^2*
x^2))^(7/2))
```

fricas [A] time = 0.47, size = 96, normalized size = 0.30

$$\frac{\left(60\,a^7c^3x^7-180\,a^6c^3x^6\log(x)-60\,a^5c^3x^5-150\,a^4c^3x^4+100\,a^3c^3x^3+15\,a^2c^3x^2-36\,ac^3x+10\,c^3\right)\sqrt{a^2c}}{60\,a^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/60*(60*a^7*c^3*x^7 - 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 - 150*a^4*c^
3*x^4 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2 - 36*a*c^3*x + 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.07, size = 112, normalized size = 0.35

$$-\frac{\left(-60a^{7}x^{7}+180a^{6}\ln(x)x^{6}+60x^{5}a^{5}+150x^{4}a^{4}-100x^{3}a^{3}-15a^{2}x^{2}+36ax-10\right)x\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{60\left(ax-1\right)^{3}\left(a^{2}x^{2}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/60*(-60*a^7*x^7+180*a^6*ln(x)*x^6+60*x^5*a^5+150*x^4*a^4-100*x^3*a^3-15*
a^2*x^2+36*a*x-10)*x*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/
(a*x-1)^3/(a^2*x^2-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.873 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=235

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^
(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/
x^2)^(1/2)-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-c/a
^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-3*c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^
2/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 75}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -(c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqrt[
c - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*x^
2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*S
qrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2
)] - (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^4(1+ax)}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+-\frac{c^2}{}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.34

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(a^5x-3a^4\log(x)-\frac{5a^4}{4}-\frac{2a^3}{x}-\frac{a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*((-5*a^4)/4 - 1/(4*x^4) + a/x^3 - a^2/x^2 - (2*a^3
)/x + a^5*x - 3*a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

fricas [A] time = 0.43, size = 74, normalized size = 0.31

$$\frac{\left(4\,a^5c^2x^5-12\,a^4c^2x^4\log(x)-8\,a^3c^2x^3-4\,a^2c^2x^2+4\,ac^2x-c^2\right)\sqrt{a^2c}}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 - 4*a^2*c^2*x^2
+ 4*a*c^2*x - c^2)*sqrt(a^2*c)/(a^6*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 96, normalized size = 0.41

$$-\frac{\left(-4x^5a^5+12a^4\ln(x)x^4+8x^3a^3+4a^2x^2-4ax+1\right)x\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4\left(ax-1\right)^3\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/4*(-4*x^5*a^5+12*a^4*ln(x)*x^4+8*x^3*a^3+4*a^2*x^2-4*a*x+1)*x*(c*(a^2*x^
2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2x^2}\right)^{\frac{5}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.874 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-3*c*(c-c/a^2/x^2)^(1/
2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-3*
c*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[c -
 c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/
Sqrt[1 - 1/(a^2*x^2)] - (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a
^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3-\frac{1}{x^3}+\frac{3a}{x^2}-\frac{3a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.40

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(a^3x-3a^2\log(x)-\frac{3a}{x}+\frac{1}{2x^2}\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
[Out] ((c - c/(a^2*x^2))^(3/2)*(1/(2*x^2) - (3*a)/x + a^3*x - 3*a^2*Log[x]))/(a^3
*(1 - 1/(a^2*x^2))^(3/2))
```

fricas [A] time = 0.63, size = 42, normalized size = 0.28

$$\frac{\left(2\,a^3cx^3-6\,a^2cx^2\log(x)-6\,acx+c\right)\sqrt{a^2c}}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
[Out] 1/2*(2*a^3*c*x^3 - 6*a^2*c*x^2*log(x) - 6*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 69, normalized size = 0.47

$$-\frac{\left(-2x^3a^3+6a^2\ln(x)x^2+6ax-1\right)x\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2\left(ax-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/2*(-2*x^3*a^3+6*a^2*ln(x)*x^2+6*a*x-1)*x*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*(
(a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^2x^2}\right)^{\frac{3}{2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.875 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)-4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 72}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}} - \frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.44

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax-4\log(ax+1)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

fricas [A] time = 0.64, size = 25, normalized size = 0.23

$$\frac{\sqrt{a^2c}\left(ax-4\log(ax+1)+\log(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
[Out] sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.07, size = 63, normalized size = 0.59

$$\frac{(ax+\ln(x)-4\ln(ax+1))\,x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] (a*x+ln(x)-4*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.876 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=113

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)-2*(1-1/a^2/x^2)^(1/2)/a/(a*x+1)/(
c-c/a^2/x^2)^(1/2)-3*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 77}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x(-1+ax)}{(1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{2}{a(1+ax)^2}-\frac{3}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.48

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(ax-\frac{2}{ax+1}-3\log(ax+1)\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x - 2/(1 + a*x) - 3*Log[1 + a*x]))/(a*Sqrt[c - c/
(a^2*x^2)])
```

fricas [A] time = 0.59, size = 47, normalized size = 0.42

$$\frac{\left(a^2x^2+ax-3\,(ax+1)\log\,(ax+1)-2\right)\sqrt{a^2c}}{a^3cx+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (a^2*x^2 + a*x - 3*(a*x + 1)*log(a*x + 1) - 2)*sqrt(a^2*c)/(a^3*c*x + a^2*c
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(const
 gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 87, normalized size = 0.77

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1)\left(-a^{2}x^{2}+3ax\ln(ax+1)-ax+3\ln(ax+1)+2\right)}{(ax-1)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\, x\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)*(-a^2*x^2+3*a*x*ln(a*x+1)-a*x+3*ln
(a*x+1)+2)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{c-\frac{c}{a^{2}x^{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a^2*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.877 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(a*
x+1)^2/(c-c/a^2/x^2)^(1/2)-3*(1-1/a^2/x^2)^(1/2)/a/c/(a*x+1)/(c-c/a^2/x^2)^
(1/2)-3*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(1+ax)^3}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}-\frac{1}{a^3(1+ax)^3}+\frac{3}{a^3(1+ax)^2}-\frac{3}{a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [**A**] time = 0.09, size = 63, normalized size = 0.38

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2ax+\frac{-6ax-5}{(ax+1)^2}-6\log(ax+1)\right)}{2a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(2*a*x + (-5 - 6*a*x)/(1 + a*x)^2 - 6*Log[1 + a*x]
))/(2*a*(c - c/(a^2*x^2))^(3/2))
```

fricas [**A**] time = 0.47, size = 81, normalized size = 0.48

$$\frac{\left(2\,a^3x^3+4\,a^2x^2-4\,ax-6\left(a^2x^2+2\,ax+1\right)\log\left(ax+1\right)-5\right)\sqrt{a^2c}}{2\left(a^4c^2x^2+2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*x^3 + 4*a^2*x^2 - 4*a*x - 6*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1) -
 5)*sqrt(a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(const
 gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 102, normalized size = 0.61

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax+1) \left(-2x^3a^3+6\ln(ax+1)x^2a^2-4a^2x^2+12ax\ln(ax+1)+4ax+6\ln(ax+1)+5\right)}{2a^4x^3\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-2*x^3*a^3+6*ln(a*x+1)*x^2*a^2-4*a^2*
x^2+12*a*x*ln(a*x+1)+4*a*x+6*ln(a*x+1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(
3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

3.878 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=264

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-a}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)-1/6*(1-1/a^2/x^2)^(1/2)/a/c^2
/(a*x+1)^3/(c-c/a^2/x^2)^(1/2)+9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)^2/(c-c
/a^2/x^2)^(1/2)-31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+
1/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-49/16*ln(a*x+
1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-a}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)(1+ax)^4}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{16a^5(-1+ax)}+\frac{1}{2a^5(1+ax)^4}-\frac{9}{4a^5(1+ax)^3}+\frac{31}{8a^5(1+ax)^2}-\frac{49}{16a^5(1+ax)}\right)dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^3} + \frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2} - \frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}$$

Mathematica [A] time = 0.14, size = 85, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48ax-\frac{186}{ax+1}+\frac{54}{(ax+1)^2}-\frac{8}{(ax+1)^3}+3\log(1-ax)-147\log(ax+1)\right)}{48a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(48*a*x - 8/(1 + a*x)^3 + 54/(1 + a*x)^2 - 186/(1
+ a*x) + 3*Log[1 - a*x] - 147*Log[1 + a*x]))/(48*a*(c - c/(a^2*x^2))^(5/2))
```

fricas [A] time = 0.62, size = 137, normalized size = 0.52

$$\frac{\left(48\,a^4x^4+144\,a^3x^3-42\,a^2x^2-270\,ax-147\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\log\left(ax+1\right)+3\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\right.}{48\left(a^5c^3x^3+3\,a^4c^3x^2+3\,a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/48*(48*a^4*x^4 + 144*a^3*x^3 - 42*a^2*x^2 - 270*a*x - 147*(a^3*x^3 + 3*a^
2*x^2 + 3*a*x + 1)*log(a*x + 1) + 3*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*log(a
*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2
*c^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
```

```
(correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(const
 gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 175, normalized size = 0.66

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)(ax-1)\left(48x^4a^4+3\ln(ax-1)x^3a^3-147a^3x^3\ln(ax+1)+144x^3a^3+9\ln(ax-1)x^2a^2-44\right.}{48a^6x^5\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)*(48*x^4*a^4+3*ln(a*x-1)*x^3*a^
3-147*a^3*x^3*ln(a*x+1)+144*x^3*a^3+9*ln(a*x-1)*x^2*a^2-441*ln(a*x+1)*x^2*a
^2-42*a^2*x^2+9*ln(a*x-1)*x*a-441*a*x*ln(a*x+1)-270*a*x+3*ln(a*x-1)-147*ln(
a*x+1)-140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

3.879 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=357

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)/(c-c/a^2/x^2)^(1/2)+1/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^4/(c-
c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^3/(c-c/a^2/x^2)^(1/2
)+59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-75/16*(1-1/
a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+9/64*ln(-a*x+1)*(1-1/a^2/x
^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-201/64*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 88}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(16*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^4) - Sqrt[1 - 1/(a^2*x^2)]/(2*a*c^3*Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^3*Sqr
t[c - c/(a^2*x^2)]*(1 + a*x)^2) - (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64*a
*c^3*Sqrt[c - c/(a^2*x^2)]) - (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^2(1+ax)^5}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{32a^7(-1+ax)^2}+\frac{9}{64a^7(-1+ax)}-\frac{1}{4a^7(1+ax)^5}+\frac{3}{2a^7(1+ax)^4}-\frac{59}{16a^7(1+ax)^3}+\frac{}{16}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^4}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1+}$$

Mathematica [A] time = 0.24, size = 105, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(2\left(32ax+\frac{1}{1-ax}-\frac{150}{ax+1}+\frac{59}{(ax+1)^2}-\frac{16}{(ax+1)^3}+\frac{2}{(ax+1)^4}\right)+9\log(1-ax)-201\log(ax+1)\right)}{64a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(2*(32*a*x + (1 - a*x)^(-1) + 2/(1 + a*x)^4 - 16/(
1 + a*x)^3 + 59/(1 + a*x)^2 - 150/(1 + a*x)) + 9*Log[1 - a*x] - 201*Log[1 +
 a*x]))/(64*a*(c - c/(a^2*x^2))^(7/2))
```

fricas [A] time = 0.63, size = 208, normalized size = 0.58

$$\frac{\left(64\,a^6x^6+192\,a^5x^5-174\,a^4x^4-618\,a^3x^3-118\,a^2x^2+414\,ax-201\left(a^5x^5+3\,a^4x^4+2\,a^3x^3-2\,a^2x^2-3\,ax-\right.\right.}{64\left(a^7c^4x^5+3\,a^6c^4x^4+2\,a^5c^4x^3-2\,a^4c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(64*a^6*x^6 + 192*a^5*x^5 - 174*a^4*x^4 - 618*a^3*x^3 - 118*a^2*x^2 +
414*a*x - 201*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*log
(a*x + 1) + 9*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*log
(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 -
 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(a*x
+1)]Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(const
 gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 247, normalized size = 0.69

$$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)(ax-1)\left(64x^6a^6+9\ln(ax-1)x^5a^5-201\ln(ax+1)x^5a^5+192x^5a^5+27\ln(ax-1)x^4a^4-603\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x)
```

```
[Out] 1/64*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(a*x-1)*(64*x^6*a^6+9*ln(a*x-1)*x^5*a^
5-201*ln(a*x+1)*x^5*a^5+192*x^5*a^5+27*ln(a*x-1)*x^4*a^4-603*ln(a*x+1)*x^4*
a^4-174*x^4*a^4+18*ln(a*x-1)*x^3*a^3-402*a^3*x^3*ln(a*x+1)-618*x^3*a^3-18*l
n(a*x-1)*x^2*a^2+402*ln(a*x+1)*x^2*a^2-118*a^2*x^2-27*ln(a*x-1)*x*a+603*a*x
*ln(a*x+1)+414*a*x-9*ln(a*x-1)+201*ln(a*x+1)+208)/a^8/x^7/(c*(a^2*x^2-1)/a^
2/x^2)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(c-\frac{c}{a^2x^2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

3.880 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx$

Optimal. Leaf size=80

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x^m*(c-c/a^2/x^2)^(1/2)/a/m/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(c-c/a^2/x^2)^(1/2)
/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6197, 6193, 43}

$$\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^m\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{-1+m}(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(x^{-1+m}+ax^m\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [**A**] time = 0.05, size = 53, normalized size = 0.66

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax^{m+1}}{m+1}+\frac{x^m}{m}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^m/m + (a*x^(1 + m))/(1 + m)))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

fricas [**A**] time = 0.42, size = 72, normalized size = 0.90

$$-\frac{\left(amx^2+(m+1)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-\left(am^2+am\right)x+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -(a*m*x^2 + (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.04, size = 63, normalized size = 0.79

$$\frac{x^{1+m}\left(axm+m+1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{\left(1+m\right)m\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x+m+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(1+m)/m/(a*x+1)/((a*x-1)/
(a*x+1))^(1/2)
```

maxima [A] time = 0.38, size = 44, normalized size = 0.55

$$\frac{\left(a\sqrt{c}\,mx+\sqrt{c}\,(m+1)\right)(ax+1)x^m}{\left(m^2+m\right)a^2x+\left(m^2+m\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] (a*sqrt(c)*m*x + sqrt(c)*(m + 1))*(a*x + 1)*x^m/((m^2 + m)*a^2*x + (m^2 + m
)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^m*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.881 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=76

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*x^2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/
2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6197, 6193, 43}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.59

$$\frac{x^2(2ax+3)\sqrt{c-\frac{c}{a^2x^2}}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2*(3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.39, size = 24, normalized size = 0.32

$$\frac{\left(2\,ax^3+3\,x^2\right)\sqrt{a^2c}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(2*a*x^3 + 3*x^2)*sqrt(a^2*c)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 53, normalized size = 0.70

$$\frac{x^3\,(2ax+3)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{6\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(2*a*x+3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\,x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [B] time = 1.44, size = 46, normalized size = 0.61

$$\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x+3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x + 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(a
*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.882 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=71

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6197, 6193}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)
]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx&=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\&=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\&=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.61

$$\frac{\left(\frac{ax^2}{2}+x\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(x + (a*x^2)/2))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.66, size = 21, normalized size = 0.30

$$\frac{\sqrt{a^2c}\left(ax^2+2\,x\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(a^2*c)*(a*x^2 + 2*x)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.03, size = 52, normalized size = 0.73

$$\frac{x^2\left(ax+2\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{2\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*x^2*(a*x+2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [B] time = 1.40, size = 45, normalized size = 0.63

$$\frac{x^2 \sqrt{c - \frac{c}{a^2 x^2}} \, (a x + 2) \sqrt{\frac{a x - 1}{a x + 1}}}{2 \, (a x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*x
 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.883 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=67

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.58

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.54, size = 17, normalized size = 0.25

$$\frac{\sqrt{a^2c}\,\left(ax+\log(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.05, size = 50, normalized size = 0.75

$$\frac{(ax+\ln(x))\,x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x
)
```

3.884 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6197, 6193, 43}

$$\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) + (Sqrt[c - c/(a^2*x^2
)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [**A**] time = 0.04, size = 43, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(a\log(x)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [**A**] time = 0.79, size = 21, normalized size = 0.30

$$\frac{\sqrt{a^2c}\left(ax\log(x)-1\right)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] sqrt(a^2*c)*(a*x*log(x) - 1)/(a^2*x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a*x+1),sign(x)]W
arning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{2,[
0,1,0]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+%%%{1,[
4,0,4]%%%}+%%%{-2,[2,2,2]%%%}+%%%{4,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%%%{1,[0
,2,0]%%%}+%%%{-2,[0,1,0]%%%}+%%%{1,[0,0,0]%%%}] at parameters values [-89,6
3,-49]Warning, choosing root of [1,0,%%%{-2,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+
```

```
%%%{2,[0,1,0]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4]%%%}+%%%{-2,[4,1,4]%%%}+
%%%{1,[4,0,4]%%%}+%%%{-2,[2,2,2]%%%}+%%%{4,[2,1,2]%%%}+%%%{-2,[2,0,2]%%%}+%
%%{1,[0,2,0]%%%}+%%%{-2,[0,1,0]%%%}+%%%{1,[0,0,0]%%%}] at parameters values
 [-86,-64,-30]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur
& l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 50, normalized size = 0.71

$$\frac{(a\ln(x)x-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] (a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

3.885 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6197, 6193, 37}

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x^3}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Mathematica [A]   time = 0.05, size = 47, normalized size = 1.02

$$\frac{\left(-\frac{a}{x}-\frac{1}{2x^2}\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 - a/x))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A]   time = 0.48, size = 21, normalized size = 0.46

$$-\frac{\sqrt{a^2c}\,(2\,ax+1)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="f
ricas")
[Out] -1/2*sqrt(a^2*c)*(2*a*x + 1)/(a^2*x^2)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="g
iac")
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

maple [A]   time = 0.04, size = 53, normalized size = 1.15

$$-\frac{(2ax+1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{2x\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

mupad [B] time = 1.40, size = 63, normalized size = 1.37

$$\frac{\left(x\,\sqrt{c-\frac{c}{a^2\,x^2}}+\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((x*(c - c/(a^2*x^2))^(1/2) + (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a*
x + 1))^(1/2))/(x/a - x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

3.886 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=160

$$\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}+\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] 7/8*x*(c-c/a^2/x^2)^(1/2)/a^3+7/24*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3+1/6*x*
(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3+1/4*x^2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^2
-7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 90, 80, 50, 41, 216}

$$\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}+\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) + (7*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
/(24*a^3) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(6*a^3) + (Sqrt[c - c/(a^
2*x^2)]*x^2*(1 + a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/
(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x^2(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(-1-2ax)(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{4a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{12a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\right.}{8a^2\sqrt{}$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+}{4a^2}$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+}{4a^2}$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+}{4a^2}$$

Mathematica [A] time = 0.12, size = 93, normalized size = 0.58

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(21\log\left(\sqrt{a^2x^2-1}+ax\right)+\sqrt{a^2x^2-1}\left(6a^3x^3+16a^2x^2+21ax+32\right)\right)}{24a^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*
a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.57, size = 222, normalized size = 1.39

$$\left[\frac{2\left(6\,a^4x^4+16\,a^3x^3+21\,a^2x^2+32\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\,\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},\ \frac{\left(6\,a^4x^4+16\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 + 32*a*x)*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 +
32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)*x
^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

giac [A] time = 0.18, size = 128, normalized size = 0.80

$$\frac{1}{48}\left(2\,\sqrt{a^2cx^2-c}\left(\left(2\,x\left(\frac{3\,x\mathrm{sgn}(x)}{a^2}+\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x+\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\,\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2}\right.\right.}{a^4|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 + 8*sgn(x)/a^3) + 21*sgn(
x)/a^4)*x + 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c
*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) - 64*sqrt(-c)*a
bs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

maple [A] time = 0.05, size = 196, normalized size = 1.22

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-6x\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4-16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x\,a^2c+27c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-\right.}{24\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,c\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^3*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4-1
6*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+27*c^(
```

```
3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln((c^(1/2)*((a*x-1
)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-48*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/
(c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.887 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=123

$$\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{a^2\sqrt{1-ax}\,\sqrt{ax+1}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(a*x+
1)^2*(c-c/a^2/x^2)^(1/2)/a^2-x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x+1)
^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.259$, Rules used = {6167, 6159, 6129, 80, 50, 41, 216}

$$\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\sin^{-1}(ax)}{a^2\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(3*a^2) - (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}x\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{3a\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\cdots}{\sqrt{1\cdots}}}{a\sqrt{1-ax}\,\sqrt{1\cdots}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\cdots}{\sqrt{1\cdots}}}{a\sqrt{1-ax}\,\sqrt{1+\cdots}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(\cdots}{a^2\sqrt{1-ax}\,\sqrt{1+a\cdots}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 84, normalized size = 0.68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\,\left(a^2x^2+3ax+5\right)+3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 + 3*a*x + a^2*x^2) + 3*Log[
a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.57, size = 204, normalized size = 1.66

$$\left[\frac{2\left(a^3x^3+3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\frac{\left(a^3x^3+3\,a^2x^2+5\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*s
qrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c))/a^3, 1/3*((a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)))/a^3]
```

giac [A] time = 0.18, size = 116, normalized size = 0.94

$$\frac{1}{6}\left(2\,\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)-\frac{6\,\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}+\frac{\left(3\,a\sqrt{c}\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 + 3*sgn(x)/a^3) + 5*sgn(x)/a^4)
 - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*abs
(a)) + (3*a*sqrt(c)*log(abs(c)) - 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))*
abs(a)
```

maple [A] time = 0.05, size = 174, normalized size = 1.41

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(-\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x\,a^2c+3c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx\cdots}{\sqrt{c}}\right.\right.}{3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(
a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+3*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1
/2))-6*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-6*((
a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/a^3/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.888 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=98

$$\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] 3/2*x*(c-c/a^2/x^2)^(1/2)/a+1/2*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a-3/2*x*arcsi
n(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6167, 6159, 6129, 50, 41, 216}

$$\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(2*
a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a*
x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
```

]

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.79

$$
\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{1-a^2x^2}\,(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a
*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 0.76, size = 188, normalized size = 1.92

$$
\left[\frac{2\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\ \frac{\left(a^2x^2+4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\sqrt{-c}\,\arctan\left(\right.}{2\,a^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2
*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2, 1
/2*((a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a
^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

giac [A] time = 0.16, size = 106, normalized size = 1.08

$$\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{4\,\mathrm{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^2|a|}+\frac{\left(3\,a\sqrt{c}\,\log\left(|c|\right)-8\,\sqrt{-\right.}{a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 + 4*sgn(x)/a^3) - 6*sqrt(c)*log(ab
s(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)
*log(abs(c)) - 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

maple [A] time = 0.04, size = 147, normalized size = 1.50

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\;x\left(-x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\;a^2+\sqrt{c}\;\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)-4\sqrt{c}\;\ln\left(\frac{\sqrt{c}\;\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)-4\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\right.}{2\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2+c^(1
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-4*c^(1/2)*ln((c^(1/2)*((a*x-1)*
(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-4*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a)/(c*(a
^2*x^2-1)/a^2)^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}\,x}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.889 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6159, 6129, 102, 157, 41, 216, 92, 208}

$$x\sqrt{c-\frac{c}{a^2x^2}}-\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{-a-2a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,d}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int\frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{\left(2a\sqrt{c-\right.}{\sqrt{1}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 80, normalized size = 0.69

$$
\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 0.81, size = 267, normalized size = 2.30

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - 4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - \sqrt{c}\,\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.05, size = 197, normalized size = 1.70

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}} + 2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}} - \sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2 - c\ln\left(\frac{2}{}\right.\right.}{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)+2*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

3.890 $\int \frac{e^{2 \coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2x^2}}}{x} \, dx$

Optimal. Leaf size=117

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)-a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x
+1)^(1/2)+2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 157, 41, 216, 92, 208}

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) + (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-2a-a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,d}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\mathrm{Subst}\left(\int \frac{1}{a-ax^2}\,dx,\right.}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 82, normalized size = 0.70

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+ax\log\left(\sqrt{a^2x^2-1}+ax\right)-2ax\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] - 2*a*x*ArcTan[1/Sqrt[-1 + a^2*x
^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 0.50, size = 252, normalized size = 2.15

$$\left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+\sqrt{\frac{a^2cx^2-c}{a^2x^2}},-2\,\sqrt{c}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x
^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), -2*sqrt(c)*arctan(
a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*
log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) +
sqrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

giac [A] time = 0.31, size = 127, normalized size = 1.09

$$\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a}-\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{|a|}+\frac{2\,c^{\frac{3}{2}}\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] (4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
- sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) + 2*
c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs(
a)
```

maple [B] time = 0.06, size = 306, normalized size = 2.62

$$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,x^2a^3c+a^3\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)xa-2\sqrt{-\right.}{a\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*(-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*x^2*a^3*c+a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+(-c/a^2)^(1/2)*c^(3
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a-2*(-c/a^2)^(1/2)*c^(3/2)*ln
```

```
((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x*a-2*(-c/a^2)^(1/2)*
((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x*a^2*c+2*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)*c*x+2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x
)*x*c^2)/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="maxima")
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\;\left(a\,x+1\right)}{x\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x*(a*x - 1)), x)
```

3.891 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=111

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] 3/2*a*(c-c/a^2/x^2)^(1/2)+1/2*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x+3/2*a^2*x*arcta
nh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)
^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6159, 6129, 94, 92, 208}

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(2*x) + (
3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqrt
[1 - a*x]*Sqrt[1 + a*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
```

```
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x} - \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x} - \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x} + \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-\right.}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.70

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((4ax+1)\sqrt{a^2x^2-1}-3a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((1 + 4*a*x)*Sqrt[-1 + a^2*x^2] - 3*a^2*x^2*ArcTan[1
/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.73, size = 177, normalized size = 1.59

$$\left[\frac{3\,a\sqrt{-c}\,x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x}, -\frac{3\,a\sqrt{c}\,x\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(4\,ax+1)\sqrt{\right.}{2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) + 2*(4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
-1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*c*x^2 - c)) - (4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

giac [B] time = 0.33, size = 194, normalized size = 1.75

$$\left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3 ac\mathrm{sgn}(x)-4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) - 4*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))*a*c^2*sgn(x) - 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

maple [B] time = 0.06, size = 347, normalized size = 3.13

$$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,x^3a^3c+4\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}x\,a^3+4\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)x^2a-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(1/2)*x^3*a^3*c+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^3+4*(-c/a^2
)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^2*a-4*(-c/a^2)^(1
/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a-4
*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^2*a^2*c+3*(-c/a^2)^(1/2)*(c
*(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2*c+a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1
/2)+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)*x^2*c^2)
/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x^2\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)),
x)
```

3.892 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=137

$$a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] a^2*(c-c/a^2/x^2)^(1/2)+1/3*a*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x+1/3*(a*x+1)^2*(
c-c/a^2/x^2)^(1/2)/x^2+a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2
/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6159, 6129, 96, 94, 92, 208}

$$a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(3*x^2) + (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^4\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{3\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} - \frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{}{x\sqrt{}}}{\sqrt{1-ax}\,\sqrt{1+}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} + \frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subs}}{\sqrt{}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} + \frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}}{\sqrt{1-ax}}$$

Mathematica [A] time = 0.11, size = 86, normalized size = 0.63

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(5a^2x^2+3ax+1\right)-3a^3x^3\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{3x^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 + 3*a*x + 5*a^2*x^2) - 3*a^3*
x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.67, size = 201, normalized size = 1.47

$$\left[\frac{3\,a^2\sqrt{-c}\,x^2\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(5\,a^2x^2+3\,ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2},-\frac{3\,a^2\sqrt{c}\,x^2\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, -1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/x^2]
```

giac [A] time = 1.88, size = 231, normalized size = 1.69

$$\frac{2}{3}\left(3\,a\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5 ac\,\mathrm{sgn}(x)-3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn
(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) - 3*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) - 12*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - c))*a*c^3*sgn(x) - 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^3)*abs(a)
```

maple [B] time = 0.06, size = 378, normalized size = 2.76

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\;a\left(-6\sqrt{-\frac{c}{a^2}}\;\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\;x^4a^3c+6\sqrt{-\frac{c}{a^2}}\;\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^2a^3+6\sqrt{-\frac{c}{a^2}}\;c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)x^3a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(1/2)*x^4*a^3*c+6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^3+6*(
-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^3*a-6*(-c/a
^2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x
^3*a-6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^2*c+3*(-c/a^2)^(1
/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^2*c+3*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(3/2)*x*a^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x
```

```
)*x^3*c^2+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^
2*x^2-1)/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x^3\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)),
x)
```

3.893 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}+\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}$$

```
[Out] 4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3+2/3*a*(c-c/a^2/x^2)
^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x+7/8*a^4*x*arctanh((-a*x+1)^(1/2)*(
a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 98, 151, 12, 92, 208}

$$\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) + (2*a*Sqrt
[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*S
qrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a*
x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^5\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-8a-7a^2x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2+16a^3x}{x^3\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-32a^3-21a^4x}{x^2\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{24\sqrt{1-a}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{8\sqrt{1-}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x}{8\sqrt{}}$$

Mathematica [A] time = 0.14, size = 94, normalized size = 0.60

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(32a^3x^3+21a^2x^2+16ax+6\right)-21a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{24x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^
3*x^3) - 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*x^
2])
```

fricas [A] time = 0.60, size = 217, normalized size = 1.39

$$\left[\frac{21\,a^3\sqrt{-c}\,x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(32\,a^3x^3+21\,a^2x^2+16\,ax+6\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3}, -\frac{21\,a^3\sqrt{c}\,x^3\,\mathrm{arctan}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (32*a^3*x^3 + 21*a^
2*x^2 + 16*a*x + 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

giac [B] time = 7.40, size = 316, normalized size = 2.03

$$\frac{1}{12}\left(21\,a^2\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7a^2c\,\mathrm{sgn}(x)+45\left(\sqrt{a^2c}\,x-\sqrt{a^2c\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) - 96*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^2*c^3*sgn(x) - 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c^
4*sgn(x) - 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^2 + c)^4)*abs(a)
```

maple [B] time = 0.06, size = 410, normalized size = 2.63

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a^2\left(-48\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^5a^3c+48\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^3a^3+48\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(-c/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(1/2)*x^5*a^3*c+48*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^3*a^
3+48*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^4*a-4
8*(-c/a^2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(
1/2))*x^4*a-48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^4*a^2*c+21*(-
c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^2*c+27*(-c/a^2)^(1/2)*(c*(a^2*
x^2-1)/a^2)^(3/2)*x^2*a^2+21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-c)/a^2/x)*x^4*c^2+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x+6*(c
*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^
(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x^4\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**4,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)),
x)
```

3.894 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}+\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}$$

```
[Out] 6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4+1/2*a*(c-c/a^2/x^2)
^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2+3/4*a^3*(c-c/a^2/x^2)^(1/2)/x+3/
4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^
(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 98, 151, 12, 92, 208}

$$\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) + (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) + (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) + (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^6\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-10a-9a^2x}{x^5\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{36a^2+30a^3x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-90a^3-72a^4x}{x^3\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{144a}{x^2\sqrt{1-}}}{120\sqrt{1-ax}\,\sqrt{1}}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{(\sqrt{}}{}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{(3a}{}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{(3a}{}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} + \frac{3a}{}$$

Mathematica [A] time = 0.12, size = 102, normalized size = 0.56

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(24a^4x^4+15a^3x^3+12a^2x^2+10ax+4\right)-15a^5x^5\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{20x^4\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 + 10*a*x + 12*a^2*x^2 + 15*a^
3*x^3 + 24*a^4*x^4) - 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sqr
t[-1 + a^2*x^2])
```

fricas [A] time = 0.55, size = 233, normalized size = 1.29

$$\left[\frac{15\,a^4\sqrt{-c}\,x^4\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4}, -\frac{15\,a^4\sqrt{c}\,x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, -1/20*(15*a^4*sqrt(c)*x^4*a
rctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (24*a^
4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/x^4]
```

giac [B] time = 8.61, size = 362, normalized size = 2.00

$$\frac{1}{10}\left(15\,a^3\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^9a^3c\mathrm{sgn}(x)+70\left(\sqrt{a^2c}\,x-\sqrt{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) - 40*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) - 200*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^3*a^3*c^4*sgn(x) - 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2
*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*c
^5*sgn(x) - 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^2 + c)^5)*abs(a)
```

maple [B] time = 0.06, size = 447, normalized size = 2.47

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a^2\left(-40\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,x^6a^4c+40\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,x^4a^4+15\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^5a^3c+40\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x-1)*(c-c/a^2/x^2)^(1/2)/x^5,x)
```

```
[Out] -1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(c*(a^2*x^2-1)/a^2)^(1/2)*
(-c/a^2)^(1/2)*x^6*a^4*c+40*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^4*a^
4+15*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c+40*(-c/a^2)^(1/2)*c
^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^5*a^2-40*(-c/a^2)^(1/2)*c^
(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^5*a^2-40*(-
c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c+25*(-c/a^2)^(1/2)*(c*(
a^2*x^2-1)/a^2)^(3/2)*x^3*a^3+15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(
1/2)*a^2-c)/a^2/x)*x^5*a*c^2+16*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^
2*a^2+10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x+4*(c*(a^2*x^2-1)/a^2)
^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax-1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\,\left(a\,x+1\right)}{x^5\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^{5}\,(ax-1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)),
x)
```

3.895 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=186

$$\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x^2*(c-c/a^2/x^2)^(1/2)/a
^2/(1-1/a^2/x^2)^(1/2)+x^3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^
4*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/
a^4/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a^
2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*S
qrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2)
]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x^2(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4}{a^2}+\frac{4x}{a}+3x^2+ax^3+\frac{4}{a^2(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log}{a^4\sqrt{1-\frac{1}{a^2}}}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.38

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4\log(1-ax)}{a^3}+\frac{4x}{a^2}+\frac{ax^4}{4}+\frac{2x^2}{a}+x^3\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((4*x)/a^2 + (2*x^2)/a + x^3 + (a*x^4)/4 + (4*Log[1
- a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.43, size = 48, normalized size = 0.26

$$\frac{\left(a^4x^4+4\,a^3x^3+8\,a^2x^2+16\,ax+16\,\log\left(ax-1\right)\right)\sqrt{a^2c}}{4\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(a^2*c
)/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 89, normalized size = 0.48

$$\frac{\left(x^{4} a^{4}+4 x^{3} a^{3}+8 a^{2} x^{2}+16 a x+16 \ln \left(a x-1\right)\right) x \sqrt{\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x-1\right)}{4 a^{3} \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/4*(x^4*a^4+4*x^3*a^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/
x^2)^(1/2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^{2} x^{2}}}\, x^{3}}{\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.896 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=152

$$\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x^2*(c-c/a^2/x^2)^(1/2)
/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln
(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 77}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
qrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4}{a}+3x+ax^2+\frac{4}{a(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.41

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(ax\left(2a^2x^2+9ax+24\right)+24\log(1-ax)\right)}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6
*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.48, size = 41, normalized size = 0.27

$$\frac{\left(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log\left(ax-1\right)\right)\sqrt{a^2c}}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(a^2*c)/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 82, normalized size = 0.54

$$\frac{\left(2x^3a^3+9a^2x^2+24ax+24\ln\left(ax-1\right)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\left(ax-1\right)}{6a^2\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/6*(2*x^3*a^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/
2)*(a*x-1)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\,x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.897 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=113

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 3*x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(
1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6197, 6193, 43}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^
2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])
/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{-1+ax}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(3+ax+\frac{4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax(ax+6)+8\log(1-ax))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/
(a^2*x^2)])
```

fricas [A] time = 0.69, size = 32, normalized size = 0.28

$$\frac{\left(a^2x^2+6\,ax+8\,\log\left(ax-1\right)\right)\sqrt{a^2c}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(a^2*c)/a^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 73, normalized size = 0.65

$$\frac{\left(a^2x^2+6ax+8\ln\left(ax-1\right)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\left(ax-1\right)}{2a\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/a/(
a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\,x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.898 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 72}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.46

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax+4\log(1-ax)-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

fricas [A] time = 0.71, size = 27, normalized size = 0.25

$$\frac{\sqrt{a^2c}\left(ax+4\log(ax-1)-\log(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 65, normalized size = 0.60

$$-\frac{(-ax+\ln(x)-4\ln(ax-1))\,x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] -(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.899 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (3*Sqrt[c - c/(a^2*x^2)
]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sq
rt[1 - 1/(a^2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^2(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-3a\log(x)+4a\log(1-ax)+\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^(-1) - 3*a*Log[x] + 4*a*Log[1 - a*x]))/(a*Sqrt[1
 - 1/(a^2*x^2)])
```

fricas [A] time = 0.58, size = 32, normalized size = 0.30

$$\frac{\sqrt{a^2c}\left(4\,ax\log\left(ax-1\right)-3\,ax\log(x)+1\right)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] sqrt(a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 67, normalized size = 0.62

$$-\frac{\left(3a\ln(x)x-4\ln\left(ax-1\right)xa-1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\left(ax-1\right)}{\left(ax+1\right)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x)
```

```
[Out] -(3*a*ln(x)*x-4*ln(a*x-1)*x*a-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x
+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

3.900 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=147

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+3*(c-c/a^2/x^2)^(1/2)/x/(
1-1/a^2/x^2)^(1/2)-4*a*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a*ln
(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6197, 6193, 88}

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a^2*
x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[
1 - 1/(a^2*x^2)] + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2
*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^3(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^3}-\frac{3a}{x^2}-\frac{4a^2}{x}+\frac{4a^3}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [**A**] time = 0.05, size = 66, normalized size = 0.45

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^2\log(x)+4a^2\log(1-ax)+\frac{3a}{x}+\frac{1}{2x^2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(2*x^2) + (3*a)/x - 4*a^2*Log[x] + 4*a^2*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [**A**] time = 0.50, size = 85, normalized size = 0.58

$$\frac{8\,a^3\sqrt{c}\,x^2\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}\,(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\sqrt{a^2c}\,(6\,ax+1)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x - 1)*sqrt(c) + a*c)/(a*x^2 - x)) + sqrt(a^2*c)*(6*a*x + 1))/(a^2*x^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 82, normalized size = 0.56

$$-\frac{\left(8a^{2}\ln(x)x^{2}-8\ln\left(ax-1\right)x^{2}a^{2}-6ax-1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{2\left(ax+1\right)^{2}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(8*a^2*ln(x)*x^2-8*ln(a*x-1)*x^2*a^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x-1)/(a*x+1)^2/x/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a^{2}\,x^{2}}}}{x^{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

3.901 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=188

$$\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/3*(c-c/a^2/x^2)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)/x
^2/(1-1/a^2/x^2)^(1/2)+4*a*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^2*
ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(-a*x+1)*(c-c/a^2/x^2
)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a^2*
x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1
- 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^
2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^4(-1+ax)}dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^4}-\frac{3a}{x^3}-\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-}}{}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 0.40

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^3\log(x)+4a^3\log(1-ax)+\frac{4a^2}{x}+\frac{3a}{2x^2}+\frac{1}{3x^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(3*x^3) + (3*a)/(2*x^2) + (4*a^2)/x - 4*a^3*Log[x
] + 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.92, size = 93, normalized size = 0.49

$$\frac{24\,a^4\sqrt{c}\,x^3\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}\,(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\left(24\,a^2x^2+9\,ax+2\right)\sqrt{a^2c}}{6\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] 1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(a^2*c))/(a^
2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 90, normalized size = 0.48

$$-\frac{\left(24a^{3}\ln(x)x^{3}-24\ln\left(ax-1\right)x^{3}a^{3}-24a^{2}x^{2}-9ax-2\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{6\left(ax+1\right)^{2}x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*ln(a*x-1)*x^3*a^3-24*a^2*x^2-9*a*x-2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^2/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

3.902 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=222

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/4*(c-c/a^2/x^2)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)/x^3/(
1-1/a^2/x^2)^(1/2)+2*a*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^2*(c
-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*a^3*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6197, 6193, 88}

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*x^2
)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) + (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (
4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^3*Sqrt[c -
 c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^5(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{4a^2}{x^3}-\frac{4a^3}{x^2}-\frac{4a^4}{x}+\frac{4a^5}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2\ldots}}}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.36

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^4\log(x)+4a^4\log(1-ax)+\frac{4a^3}{x}+\frac{2a^2}{x^2}+\frac{a}{x^3}+\frac{1}{4x^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(4*x^4) + a/x^3 + (2*a^2)/x^2 + (4*a^3)/x - 4*a^4
*Log[x] + 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.42, size = 101, normalized size = 0.45

$$\frac{16\,a^5\sqrt{c}\,x^4\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}\,(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\left(16\,a^3x^3+8\,a^2x^2+4\,ax+1\right)\sqrt{a^2c}}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="f
ricas")
```

```
[Out] 1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*sqrt
(a^2*c))/(a^2*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

maple [A] time = 0.06, size = 98, normalized size = 0.44

$$-\frac{\left(16a^{4}\ln(x)x^{4}-16\ln\left(ax-1\right)x^{4}a^{4}-16x^{3}a^{3}-8a^{2}x^{2}-4ax-1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{4\left(ax+1\right)^{2}x^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x)
```

```
[Out] -1/4*(16*a^4*ln(x)*x^4-16*ln(a*x-1)*x^4*a^4-16*x^3*a^3-8*a^2*x^2-4*a*x-1)*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^3/((a*x-1)/(a*x+1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.903 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=264

$$\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1-}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/5*(c-c/a^2/x^2)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+3/4*(c-c/a^2/x^2)^(1/2)/x
^4/(1-1/a^2/x^2)^(1/2)+4/3*a*(c-c/a^2/x^2)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+2*
a^2*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^3*(c-c/a^2/x^2)^(1/2)/x
/(1-1/a^2/x^2)^(1/2)-4*a^4*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*
a^4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(5*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a^2*
x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sqrt[
1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)
]*x^2) + (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^4*S
qrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(a^2
*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^6(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^6}-\frac{3a}{x^5}-\frac{4a^2}{x^4}-\frac{4a^3}{x^3}-\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 0.34

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^5\log(x)+4a^5\log(1-ax)+\frac{240a^4x^4+120a^3x^3+80a^2x^2+45ax+12}{60x^5}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((12 + 45*a*x + 80*a^2*x^2 + 120*a^3*x^3 + 240*a^4*x
^4)/(60*x^5) - 4*a^5*Log[x] + 4*a^5*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]
)
```

fricas [A] time = 0.43, size = 109, normalized size = 0.41

$$\frac{240\,a^6\sqrt{c}\,x^5\log\left(\frac{2\,a^3cx^2-2\,a^2cx-\sqrt{a^2c}\,(2\,ax-1)\sqrt{c}+ac}{ax^2-x}\right)+\left(240\,a^4x^4+120\,a^3x^3+80\,a^2x^2+45\,ax+12\right)\sqrt{a^2c}}{60\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="f
ricas")
```

```
[Out] 1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
 - 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (240*a^4*x^4 + 120*a^3*x^3 + 80*a^2*x^2
 + 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="g
iac")
```

[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)

maple [A] time = 0.08, size = 106, normalized size = 0.40

$$-\frac{\left(240a^{5}\ln(x)x^{5}-240\ln\left(ax-1\right)x^{5}a^{5}-240x^{4}a^{4}-120x^{3}a^{3}-80a^{2}x^{2}-45ax-12\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{60\left(ax+1\right)^{2}x^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x)

[Out] -1/60*(240*a^5*ln(x)*x^5-240*ln(a*x-1)*x^5*a^5-240*x^4*a^4-120*x^3*a^3-80*a^2*x^2-45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^4/((a*x-1)/(a*x+1))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}}{x^{5}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)

[Out] int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**5,x)

[Out] Timed out

3.904 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx$

Optimal. Leaf size=81

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x^m*(c-c/a^2/x^2)^(1/2)/a/m/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(c-c/a^2/x^2)^(1/2
)/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 43}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a
^2*x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^m\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{-1+m}(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-x^{-1+m}+ax^m\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.62

$$\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax}{m+1}-\frac{1}{m}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m*(-m^(-1) + (a*x)/(1 + m)))/(a*Sqrt[1 - 1/(a^2*x^
2)])
```

fricas [A] time = 0.45, size = 73, normalized size = 0.90

$$-\frac{\left(amx^2-(m+1)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-\left(am^2+am\right)x+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -(a*m*x^2 - (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{a^2x^2}}\,x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 65, normalized size = 0.80

$$\frac{x^{1+m}\left(axm-m-1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{(1+m)\,m\,(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] x^(1+m)*(a*m*x-m-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(
1+m)/m/(a*x-1)
```

maxima [A] time = 0.39, size = 46, normalized size = 0.57

$$\frac{\left(a\sqrt{c}\,mx-\sqrt{c}\,(m+1)\right)(ax-1)x^m}{\left(m^2+m\right)a^2x-\left(m^2+m\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

```
[Out] (a*sqrt(c)*m*x - sqrt(c)*(m + 1))*(a*x - 1)*x^m/((m^2 + m)*a^2*x - (m^2 + m
)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^m*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.905 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=76

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*x^2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 43}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2
*x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-x+ax^2\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.59

$$\frac{x^2(2ax-3)\sqrt{c-\frac{c}{a^2x^2}}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2*(-3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.40, size = 24, normalized size = 0.32

$$\frac{\left(2\,ax^3-3\,x^2\right)\sqrt{a^2c}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*a*x^3 - 3*x^2)*sqrt(a^2*c)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c-\frac{c}{a^2x^2}}\,x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.03, size = 53, normalized size = 0.70

$$\frac{x^3\,(2ax-3)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] 1/6*x^3*(2*a*x-3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*
x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^2x^2}}\, x^2 \sqrt{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [B] time = 1.36, size = 46, normalized size = 0.61

$$\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(a
*x - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.906 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=72

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6197, 6193}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx&=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\&=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\&=-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.58

$$\frac{x(ax-2)\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-2 + a*x))/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.41, size = 21, normalized size = 0.29

$$\frac{\sqrt{a^2c}\left(ax^2-2x\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
[Out] 1/2*sqrt(a^2*c)*(a*x^2 - 2*x)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{a^2x^2}}\,x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.04, size = 52, normalized size = 0.72

$$\frac{x^2(ax-2)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
[Out] 1/2*x^2*(a*x-2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{a^2x^2}}\,x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [B] time = 1.32, size = 45, normalized size = 0.62

$$\frac{x^2 \sqrt{c - \frac{c}{a^2 x^2}} (a x - 2) \sqrt{\frac{a x - 1}{a x + 1}}}{2 (a x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*x
 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.907 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6193, 43}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a-\frac{1}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.60

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.40, size = 19, normalized size = 0.28

$$\frac{\sqrt{a^2c}\left(ax-\log(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

maple [A] time = 0.05, size = 52, normalized size = 0.76

$$-\frac{(-ax+\ln(x))\,x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] -(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c - \frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

3.908 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 43}

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (Sqrt[c - c/(a^2*x^2)]*
Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.59

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(a\log(x)+\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.45, size = 21, normalized size = 0.30

$$\frac{\sqrt{a^2c}\left(ax\log(x)+1\right)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x*log(x) + 1)/(a^2*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(x),sign(a*x+1)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [A] time = 0.06, size = 50, normalized size = 0.72

$$\frac{(a\ln(x)x+1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x)
```

```
[Out] (a*ln(x)*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/x,
 x)
```

3.909 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=47

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 37}

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x^3}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$\frac{\left(\frac{1}{2x^2}-\frac{a}{x}\right)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(2*x^2) - a/x))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.44, size = 21, normalized size = 0.45

$$-\frac{\sqrt{a^2c}\,(2\,ax-1)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fri
cas")
[Out] -1/2*sqrt(a^2*c)*(2*a*x - 1)/(a^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gia
c")
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

maple [A] time = 0.04, size = 53, normalized size = 1.13

$$-\frac{(2ax-1)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax-1)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x)
```

[Out] -1/2*(2*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)/x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="max
ima")

[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)

mupad [B] time = 1.36, size = 63, normalized size = 1.34

$$\frac{\left(x\sqrt{c-\frac{c}{a^2x^2}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a}\right)\sqrt{\frac{ax-1}{ax+1}}}{\frac{x}{a}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)

[Out] ((x*(c - c/(a^2*x^2))^(1/2) - (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a*
x + 1))^(1/2))/(x/a - x^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)

[Out] Timed out

3.910 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=163

$$\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{ax+1}\sqrt{1-ax}}$$

```
[Out] -7/8*x*(c-c/a^2/x^2)^(1/2)/a^3-7/24*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3-1/6*
x*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3+1/4*x^2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)
/a^2-7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 90, 80, 50, 41, 216}

$$\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{8a^3\sqrt{ax+1}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) - (7*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)
)/(24*a^3) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(6*a^3) + (Sqrt[c - c/(a
^2*x^2)]*x^2*(1 - a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])
/(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x^2(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}(-1+2ax)}{\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,d}{12a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2} - \frac{(7}{}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{4a}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{4a}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{4a}$$

Mathematica [A] time = 0.10, size = 93, normalized size = 0.57

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(21\log\left(\sqrt{a^2x^2-1}+ax\right)+\sqrt{a^2x^2-1}\left(6a^3x^3-16a^2x^2+21ax-32\right)\right)}{24a^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6
*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.47, size = 222, normalized size = 1.36

$$\left[\frac{2\left(6a^4x^4-16a^3x^3+21a^2x^2-32ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48a^4}, \frac{\left(6a^4x^4-16a^3x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 - 32*a*x)*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 -
32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)*x
^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

giac [A] time = 0.16, size = 128, normalized size = 0.79

$$\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\operatorname{sgn}(x)}{a^2}-\frac{8\operatorname{sgn}(x)}{a^3}\right)+\frac{21\operatorname{sgn}(x)}{a^4}\right)x-\frac{32\operatorname{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2-c}\right.\right.}{a^4|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 - 8*sgn(x)/a^3) + 21*sgn(
x)/a^4)*x - 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c
*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) + 64*sqrt(-c)*a
bs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

maple [A] time = 0.05, size = 196, normalized size = 1.20

$$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(-6x\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^4+16\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,xa^2c+27c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-48c\right.}{24\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,ca^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4+1
6*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*x*a^2*c+27*c^(
```

```
3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln((c^(1/2)*((a*x-1
)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+48*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/
(c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.911 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=124

$$\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{ax+1}\sqrt{1-ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(-a*
x+1)^2*(c-c/a^2/x^2)^(1/2)/a^2+x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x+
1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6159, 6129, 80, 50, 41, 216}

$$\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{a^2\sqrt{ax+1}\sqrt{1-ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(3*a^2) + (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[p] && IntegerQ[n/2] && !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}x\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2}+\frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{3a\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{a\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)}{a\sqrt{1-ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\sin}{a^2\sqrt{1-ax}\,\sqrt{1}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 84, normalized size = 0.68

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\,\left(a^2x^2-3ax+5\right)-3\log\left(\sqrt{a^2x^2-1}+ax\right)\right)}{3a^2\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 - 3*a*x + a^2*x^2) - 3*Log[
a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.41, size = 204, normalized size = 1.65

$$\left[\frac{2\left(a^{3}x^{3}-3\,a^{2}x^{2}+5\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}+3\,\sqrt{c}\,\log\left(2\,a^{2}cx^{2}-2\,a^{2}\sqrt{c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)}{6\,a^{3}},\ \frac{\left(a^{3}x^{3}-3\,a^{2}x^{2}+5\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*s
qrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c))/a^3, 1/3*((a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)) + 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)))/a^3]
```

giac [A] time = 0.17, size = 117, normalized size = 0.94

$$\frac{1}{6}\left(2\,\sqrt{a^{2}cx^{2}-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^{2}}-\frac{3\,\mathrm{sgn}(x)}{a^{3}}\right)+\frac{5\,\mathrm{sgn}(x)}{a^{4}}\right)+\frac{6\,\sqrt{c}\,\log\left(\left|-\sqrt{a^{2}c}\,x+\sqrt{a^{2}cx^{2}-c}\right|\right)\mathrm{sgn}(x)}{a^{3}|a|}-\frac{\left(3\,a\sqrt{c}\,\log\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 - 3*sgn(x)/a^3) + 5*sgn(x)/a^4)
 + 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*abs
(a)) - (3*a*sqrt(c)*log(abs(c)) + 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))*
abs(a)
```

maple [A] time = 0.05, size = 173, normalized size = 1.40

$$\frac{\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,x\left(\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}}a^{3}-3\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,x\,a^{2}c+3c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^{2}}}+cx}{\sqrt{c}}\right)+6\cdots\right.}{3\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}}}\,a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(a^
2*x^2-1)/a^2)^(1/2)*x*a^2*c+3*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2
))-6*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))+6*((a*
x-1)*(a*x+1)*c/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/a^3/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^{2}x^{2}}}\,x^{2}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \sqrt{c - \frac{c}{a^2 x^2}} \, (a\,x - 1)}{a\,x + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)} \, (ax - 1)}{ax + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.912 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=99

$$-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -3/2*x*(c-c/a^2/x^2)^(1/2)/a-1/2*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a-3/2*x*arc
sin(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6167, 6159, 6129, 50, 41, 216}

$$-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{2a\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(2
*a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a
*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
```

]

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 100, normalized size = 1.01

$$
-\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{ax+1}\left(a^2x^2-5ax+4\right)-6\sqrt{1-ax}\,\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqrt
[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2
])
```

fricas [A] time = 0.48, size = 188, normalized size = 1.90

$$
\left[\frac{2\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\ \frac{\left(a^2x^2-4\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\sqrt{-c}\,\text{arcta}}{2\,a^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2
*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2, 1
/2*((a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a
^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

giac [A] time = 0.18, size = 106, normalized size = 1.07

$$\frac{1}{4}\left(2\sqrt{a^2cx^2-c}\left(\frac{x\operatorname{sgn}(x)}{a^2}-\frac{4\operatorname{sgn}(x)}{a^3}\right)-\frac{6\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^2|a|}+\frac{\left(3\,a\sqrt{c}\log\left(|c|\right)+8\sqrt{-c}\,|a\right.}{a^3|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 - 4*sgn(x)/a^3) - 6*sqrt(c)*log(ab
s(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)
*log(abs(c)) + 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

maple [A] time = 0.05, size = 147, normalized size = 1.48

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,x\left(x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2-\sqrt{c}\,\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)+4\sqrt{c}\,\ln\left(\frac{\sqrt{c}\,\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)-4\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,a\right)}{2\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c^(1/2
)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))+4*c^(1/2)*ln((c^(1/2)*((a*x-1)*(a
*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))-4*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a)/(c*(a^2
*x^2-1)/a^2)^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}\,x}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.913 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6167, 6159, 6129, 102, 157, 41, 216, 92, 208}

$$x\sqrt{c-\frac{c}{a^2x^2}}+\frac{2x\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1-}}}{\sqrt{1-ax}\,\sqrt{1+ax}} \\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\mathrm{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\left(2a\right.}{} \\
&= \sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+a\right.}{\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.69

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}-2\log\left(\sqrt{a^2x^2-1}+ax\right)-\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 0.44, size = 267, normalized size = 2.30

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+4\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-\sqrt{c}\,\text{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.05, size = 196, normalized size = 1.69

$$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2-2\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}+2\sqrt{c}\,\ln\left(\frac{\sqrt{c}\sqrt{\frac{(ax-1)(ax+1)c}{a^2}}+cx}{\sqrt{c}}\right)a\sqrt{-\frac{c}{a^2}}+c\ln\left(\frac{2\sqrt{-}}{}\right.\right.}{\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2\sqrt{-\frac{c}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1),x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-2*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*a^2*(-c/a^2)^(1/2)+2*c^(1/2)*ln((c^(1/2
)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*a*(-c/a^2)^(1/2)+c*ln(2*((-c/
a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/
2)/a^2/(-c/a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

3.914 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=117

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)-a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x
+1)^(1/2)-2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6167, 6159, 6129, 98, 157, 41, 216, 92, 208}

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{ax\sqrt{c-\frac{c}{a^2x^2}}\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{ax+1}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) - (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{2a-a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1\cdots}}}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\cdots\right.}{\sqrt{1-ax}\,\sqrt{1\cdots}}\\
&= \sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+a\cdots}\right.}{\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 82, normalized size = 0.70

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}+ax\log\left(\sqrt{a^2x^2-1}+ax\right)+2ax\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2*x
^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

fricas [A] time = 0.46, size = 252, normalized size = 2.15

$$\left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+\sqrt{\frac{a^2cx^2-c}{a^2x^2}},2\sqrt{c}\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x
^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), 2*sqrt(c)*arctan(a
*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*l
og(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + s
qrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

giac [A] time = 0.28, size = 127, normalized size = 1.09

$$-\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a}+\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{|a|}-\frac{2\,c^{\frac{3}{2}}\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] -(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
 + sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) - 2
*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs
(a)
```

maple [B] time = 0.06, size = 306, normalized size = 2.62

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^2a^3c+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)xa-2\sqrt{-\frac{c}{a^2}}\right.}{a\sqrt{\frac{c(a}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x,x)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*(-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*x^2*a^3*c+a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+(-c/a^2)^(1/2)*c^(3
/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x*a-2*(-c/a^2)^(1/2)*c^(3/2)*ln
```

```
((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x*a+2*(-c/a^2)^(1/2)*
((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x*a^2*c-2*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*(-c/a
^2)^(1/2)*c*x-2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x
)*x*c^2)/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x*(a*x + 1)), x)
```

3.915 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=112

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] -3/2*a*(c-c/a^2/x^2)^(1/2)+1/2*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x+3/2*a^2*x*arc
tanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+
1)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6167, 6159, 6129, 94, 92, 208}

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(2*x) +
(3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqr
t[1 - a*x]*Sqrt[1 + a*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
```

```
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} + \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} - \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} + \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x,\right.}{2\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+\right.}{2\sqrt{1-ax}\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.70

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((4ax-1)\sqrt{a^2x^2-1}+3a^2x^2\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{2x\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*((-1 + 4*a*x)*Sqrt[-1 + a^2*x^2] + 3*a^2*x^2*Ar
cTan[1/Sqrt[-1 + a^2*x^2]]))/(x*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.48, size = 176, normalized size = 1.57

$$\left[\frac{3\,a\sqrt{-c}\,x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x}, -\frac{3\,a\sqrt{c}\,x\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(4\,ax-1)}{2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) - 2*(4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
-1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*c*x^2 - c)) + (4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

giac [B] time = 0.35, size = 194, normalized size = 1.73

$$\left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3 ac\,\mathrm{sgn}(x)+4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2 c^{\frac{3}{2}}|}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] (3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) + 4*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))*a*c^2*sgn(x) + 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

maple [B] time = 0.06, size = 348, normalized size = 3.11

$$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,x^3a^3c+4\sqrt{-\frac{c}{a^2}}\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}x\,a^3+4\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)x^2a-4\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x^2,x)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^
(1/2)*x^3*a^3*c+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x*a^3+4*(-c/a^2)
^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^2*a-4*(-c/a^2)^(1/
2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^2*a+4*
(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^2*a^2*c-3*(-c/a^2)^(1/2)*(c*
(a^2*x^2-1)/a^2)^(1/2)*x^2*a^2*c-a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/
2)-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)*x^2*c^2)/
(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^2\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)),
x)
```

3.916 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=140

$$a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

```
[Out] a^2*(c-c/a^2/x^2)^(1/2)-1/3*a*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x+1/3*(-a*x+1)^2
*(c-c/a^2/x^2)^(1/2)/x^2-a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a
^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6167, 6159, 6129, 96, 94, 92, 208}

$$a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(3*x^2) - (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^4\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2}+\frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{3\sqrt{1-ax}\sqrt{1+ax}} \\
&= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2}-\frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} \\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2}+\frac{\left(a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \cdots}{\sqrt{1-ax}\cdots} \\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2}-\frac{\left(a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\mathrm{S}\cdots}{\cdots} \\
&= a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2}-\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\tan\cdots}{\sqrt{1-\cdots}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 86, normalized size = 0.61

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{a^2x^2-1}\left(5a^2x^2-3ax+1\right)+3a^3x^3\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{3x^2\sqrt{a^2x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 - 3*a*x + 5*a^2*x^2) + 3*a^3*
x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

fricas [A] time = 0.44, size = 200, normalized size = 1.43

$$\left[\frac{3\,a^2\sqrt{-c}\,x^2\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(5\,a^2x^2-3\,ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2},\ \frac{3\,a^2\sqrt{c}\,x^2\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, 1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)))/x^2]
```

giac [A] time = 1.91, size = 231, normalized size = 1.65

$$-\frac{2}{3}\left(3\,a\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5 ac\mathrm{sgn}(x)+3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) + 3*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) + 12*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^3*sgn(x) + 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

maple [B] time = 0.06, size = 378, normalized size = 2.70

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a\left(-6\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^4a^3c+6\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^2a^3+6\sqrt{-\frac{c}{a^2}}\,c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)x^3a-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x^3,x)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/
a^2)^(1/2)*x^4*a^3*c+6*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^2*a^3+6*(
-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^3*a-6*(-c/a
^2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x
^3*a+6*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^3*a^2*c-3*(-c/a^2)^(1
/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^3*a^2*c-3*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2
)^(3/2)*x*a^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x
```

```
)*x^3*c^2+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^
2*x^2-1)/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^3\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)),
x)
```

3.917 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}$$

```
[Out] -4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3-2/3*a*(c-c/a^2/x^2
)^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x+7/8*a^4*x*arctanh((-a*x+1)^(1/2)*
(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 98, 151, 12, 92, 208}

$$-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) - (2*a*Sqr
t[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*
Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a
*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^5\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{8a-7a^2x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2-16a^3x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{12\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{32a^3-21a^4x}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{24\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{24\sqrt{1-a}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{8\sqrt{1-}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(7a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\,t}{8\sqrt{1}}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.60

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(21a^4x^4\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(32a^3x^3-21a^2x^2+16ax-6\right)\right)}{24x^3\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] -1/24*(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-6 + 16*a*x - 21*a^2*x^2
+ 32*a^3*x^3) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(x^3*Sqrt[-1 + a^
2*x^2])
```

fricas [A] time = 0.44, size = 216, normalized size = 1.38

$$\left[\frac{21\,a^3\sqrt{-c}\,x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\left(32\,a^3x^3-21\,a^2x^2+16\,ax-6\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3}, -\frac{21\,a^3\sqrt{c}\,x^3\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 21*a^
2*x^2 + 16*a*x - 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

giac [B] time = 7.73, size = 316, normalized size = 2.03

$$\frac{1}{12}\left(21\,a^2\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7a^2c\,\mathrm{sgn}(x)+45\left(\sqrt{a^2c}\,x-\sqrt{\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) + 96*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^2*c^3*sgn(x) + 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c^
4*sgn(x) + 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^2 + c)^4)*abs(a)
```

maple [B] time = 0.06, size = 410, normalized size = 2.63

$$\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\;a^2\left(-48\sqrt{-\frac{c}{a^2}}\;\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\;x^5a^3c+48\sqrt{-\frac{c}{a^2}}\;\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}x^3a^3+48\sqrt{-\frac{c}{a^2}}\;c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x^4,x)
```

```
[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(-c/a^2)^(1/2)*(c*(a^2*x^2-
1)/a^2)^(1/2)*x^5*a^3*c+48*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^3*a^3
+48*(-c/a^2)^(1/2)*c^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^4*a-48
*(-c/a^2)^(1/2)*c^(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1
/2))*x^4*a+48*(-c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^4*a^2*c-21*(-c
/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^4*a^2*c-27*(-c/a^2)^(1/2)*(c*(a^2*x
^2-1)/a^2)^(3/2)*x^2*a^2-21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-c)/a^2/x)*x^4*c^2+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x-6*(c*
(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(
1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^4\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)),
x)
```

3.918 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}+\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c\cdots}}{4x}$$

```
[Out] 6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4-1/2*a*(c-c/a^2/x^2)
^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2-3/4*a^3*(c-c/a^2/x^2)^(1/2)/x-3/
4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^
(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6167, 6159, 6129, 98, 151, 12, 92, 208}

$$\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{a\cdots}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) - (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) - (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) - (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6129

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[(u*(1 + (d*x)/c)^p*(1 + a*x)^(n/2))/(1 - a*x)^(n/2), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6159

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[(x^(2*p)*(c + d/x^2)^p)/((1 - a*x)^p*(1 + a*x)^p), Int[(u*(1 - a*
x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]))/x^(2*p), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6167

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^6\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{10a-9a^2x}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{36a^2-30a^3x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{90a^3-72a^4x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,dx}{60\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{120\sqrt{1-ax}}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} +$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} +$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} -$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} -$$

Mathematica [A] time = 0.10, size = 102, normalized size = 0.56

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(15a^5x^5\tan^{-1}\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+\sqrt{a^2x^2-1}\left(24a^4x^4-15a^3x^3+12a^2x^2-10ax+4\right)\right)}{20x^4\sqrt{a^2x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 - 10*a*x + 12*a^2*x^2 - 15*a^
3*x^3 + 24*a^4*x^4) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sqr
t[-1 + a^2*x^2])
```

fricas [A] time = 0.45, size = 232, normalized size = 1.28

$$\left[\frac{15\,a^4\sqrt{-c}\,x^4\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4}, \frac{15\,a^4\sqrt{c}\,x \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, 1/20*(15*a^4*sqrt(c)*x^4*ar
ctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (24*a^4
*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
))/x^4]
```

giac [B] time = 9.78, size = 362, normalized size = 2.00

$$-\frac{1}{10}\left(15\,a^3\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^9a^3c\mathrm{sgn}(x)+70\left(\sqrt{a^2c}\,x-\sqrt{a\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] -1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) + 40*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) + 200*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^3*c^4*sgn(x) + 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*
c^5*sgn(x) + 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2 + c)^5)*abs(a)
```

maple [B] time = 0.06, size = 447, normalized size = 2.47

$$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a^2\left(-40\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,x^6a^4c+40\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,x^4a^4-15\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,x^5a^3c+40\sqrt{-\frac{c}{a^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)/(a*x+1)*(a*x-1)/x^5,x)
```

```
[Out] -1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(c*(a^2*x^2-1)/a^2)^(1/2)*
(-c/a^2)^(1/2)*x^6*a^4*c+40*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x^4*a^
4-15*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*x^5*a^3*c+40*(-c/a^2)^(1/2)*c
^(3/2)*ln(x*c^(1/2)+(c*(a^2*x^2-1)/a^2)^(1/2))*x^5*a^2-40*(-c/a^2)^(1/2)*c^
(3/2)*ln((c^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)+c*x)/c^(1/2))*x^5*a^2+40*(-
c/a^2)^(1/2)*((a*x-1)*(a*x+1)*c/a^2)^(1/2)*x^5*a^3*c-25*(-c/a^2)^(1/2)*(c*(
a^2*x^2-1)/a^2)^(3/2)*x^3*a^3-15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(
1/2)*a^2-c)/a^2/x)*x^5*a*c^2+16*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*x^
2*a^2-10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*x+4*(c*(a^2*x^2-1)/a^2)
^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{c-\frac{c}{a^2x^2}}}{(ax+1)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^5\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)),
x)
```

3.919 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=186

$$\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -4*x*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x^2*(c-c/a^2/x^2)^(1/2)/
a^2/(1-1/a^2/x^2)^(1/2)-x^3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x
^4*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/
a^4/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a
^2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*
Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2
)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x^2(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x}}}{a^4\sqrt{}}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 0.37

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(ax\left(a^3x^3-4a^2x^2+8ax-16\right)+16\log(ax+1)\right)}{4a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(-16 + 8*a*x - 4*a^2*x^2 + a^3*x^3) + 16*Log[1
+ a*x]))/(4*a^4*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.43, size = 48, normalized size = 0.26

$$\frac{\left(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log\left(ax+1\right)\right)\sqrt{a^2c}}{4\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(a^2*c
)/a^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{a^2x^2}}\,x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 89, normalized size = 0.48

$$\frac{\left(x^4a^4-4x^3a^3+8a^2x^2-16ax+16\ln\left(ax+1\right)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4\left(ax-1\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/4*(x^4*a^4-4*x^3*a^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/
x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^2 x^2}} x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{c - \frac{c}{a^2 x^2}} \left(\frac{a x - 1}{a x + 1}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.920 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=151

$$-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x^2*(c-c/a^2/x^2)^(1/2)
/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*ln
(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 77}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
qrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.41

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(ax\left(2a^2x^2-9ax+24\right)-24\log(ax+1)\right)}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6
*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.46, size = 41, normalized size = 0.27

$$\frac{\left(2\,a^3x^3-9\,a^2x^2+24\,ax-24\,\log\left(ax+1\right)\right)\sqrt{a^2c}}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(a^2*c)/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{a^2x^2}}\,x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.06, size = 82, normalized size = 0.54

$$-\frac{\left(-2x^3a^3+9a^2x^2-24ax+24\ln\left(ax+1\right)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6a^2\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] -1/6*(-2*x^3*a^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^2x^2}}\, x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\, \sqrt{c-\frac{c}{a^2\,x^2}}\, {\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.921 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=112

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -3*x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/
(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6197, 6193, 43}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x]
)/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-3+ax+\frac{4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [**A**] time = 0.03, size = 53, normalized size = 0.47

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax(ax-6)+8\log(ax+1))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1
/(a^2*x^2)])
```

fricas [**A**] time = 0.41, size = 32, normalized size = 0.29

$$\frac{\left(a^2x^2-6\,ax+8\,\log\left(ax+1\right)\right)\sqrt{a^2c}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(a^2*c)/a^3
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [**A**] time = 0.06, size = 73, normalized size = 0.65

$$\frac{\left(a^2x^2-6ax+8\ln\left(ax+1\right)\right)x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a\,(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a
*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^2x^2}}\, x \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.922 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)-4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6197, 6193, 72}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.44

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax-4\log(ax+1)+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

fricas [A] time = 0.41, size = 25, normalized size = 0.23

$$\frac{\sqrt{a^2c}\left(ax-4\log\left(ax+1\right)+\log(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

maple [A] time = 0.05, size = 63, normalized size = 0.59

$$\frac{(ax+\ln(x)-4\ln(ax+1))\,x\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\;(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] (a*x+ln(x)-4*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a
*x+1))^(3/2)/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^2x^2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c - \frac{c}{a^2\,x^2}} \left(\frac{a\,x-1}{a\,x+1}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

3.923 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) - (3*Sqrt[c - c/(a^2*x
^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])
/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^2(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.49

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-3a\log(x)+4a\log(ax+1)-\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[1
 - 1/(a^2*x^2)])
```

fricas [A] time = 0.45, size = 32, normalized size = 0.30

$$\frac{\sqrt{a^2c}\left(4\,ax\log\left(ax+1\right)-3\,ax\log(x)-1\right)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

maple [A] time = 0.06, size = 67, normalized size = 0.62

$$-\frac{\left(3a\ln(x)x-4ax\ln\left(ax+1\right)+1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\;\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(ax-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x)
```

```
[Out] -(3*a*ln(x)*x-4*a*x*ln(a*x+1)+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*
x-1)/(a*x+1))^(3/2)/(a*x-1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - \frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

3.924 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=146

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+3*(c-c/a^2/x^2)^(1/2)/x/
(1-1/a^2/x^2)^(1/2)+4*a*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*a*l
n(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a^2
*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt
[1 - 1/(a^2*x^2)] - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^
2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^3(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 0.45

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^2\log(x)-4a^2\log(ax+1)+\frac{3a}{x}-\frac{1}{2x^2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 +
 a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.41, size = 83, normalized size = 0.57

$$\frac{8\,a^3\sqrt{c}\,x^2\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{a^2c}\,(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)+\sqrt{a^2c}\,(6\,ax-1)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
1)*sqrt(c) + a*c)/(a*x^2 + x)) + sqrt(a^2*c)*(6*a*x - 1))/(a^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

maple [A] time = 0.06, size = 82, normalized size = 0.56

$$\frac{\left(8a^{2}\ln(x)x^{2}-8\ln\left(ax+1\right)x^{2}a^{2}+6ax-1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2\left(ax-1\right)^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x)
```

```
[Out] 1/2*(8*a^2*ln(x)*x^2-8*ln(a*x+1)*x^2*a^2+6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a^{2}\,x^{2}}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

3.925 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=187

$$-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/3*(c-c/a^2/x^2)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)/
x^2/(1-1/a^2/x^2)^(1/2)-4*a*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^2
*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(a*x+1)*(c-c/a^2/x^2
)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a^2
*x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1
 - 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x
^2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^4(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a}{}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.40

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^3\log(x)+4a^3\log(ax+1)-\frac{4a^2}{x}+\frac{3a}{2x^2}-\frac{1}{3x^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[
x] + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.53, size = 91, normalized size = 0.49

$$\frac{24\,a^4\sqrt{c}\,x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{a^2c}\,(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)-\left(24\,a^2x^2-9\,ax+2\right)\sqrt{a^2c}}{6\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(a^2*c))/(a^
2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

maple [A] time = 0.06, size = 90, normalized size = 0.48

$$-\frac{\left(24a^{3}\ln(x)x^{3}-24a^{3}x^{3}\ln\left(ax+1\right)+24a^{2}x^{2}-9ax+2\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6\left(ax-1\right)^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x)
```

```
[Out] -1/6*(24*a^3*ln(x)*x^3-24*a^3*x^3*ln(a*x+1)+24*a^2*x^2-9*a*x+2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

3.926 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=221

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/4*(c-c/a^2/x^2)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)/x^3/
(1-1/a^2/x^2)^(1/2)-2*a*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^2*(
c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)-4*a^3*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*x^
2)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) - (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(
a^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) +
(4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] - (4*a^3*Sqrt[c
- c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^5(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.36

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(4a^4\log(x)-4a^4\log(ax+1)+\frac{4a^3}{x}-\frac{2a^2}{x^2}+\frac{a}{x^3}-\frac{1}{4x^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^
4*Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

fricas [A] time = 0.54, size = 99, normalized size = 0.45

$$\frac{16\,a^5\sqrt{c}\,x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{a^2c}\,(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)+\left(16\,a^3x^3-8\,a^2x^2+4\,ax-1\right)\sqrt{a^2c}}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*sqrt
(a^2*c))/(a^2*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

maple [A] time = 0.06, size = 98, normalized size = 0.44

$$\frac{\left(16a^{4}\ln(x)x^{4}-16\ln\left(ax+1\right)x^{4}a^{4}+16x^{3}a^{3}-8a^{2}x^{2}+4ax-1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4\left(ax-1\right)^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x)
```

```
[Out] 1/4*(16*a^4*ln(x)*x^4-16*ln(a*x+1)*x^4*a^4+16*x^3*a^3-8*a^2*x^2+4*a*x-1)*(c
*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

3.927 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=263

$$\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/5*(c-c/a^2/x^2)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+3/4*(c-c/a^2/x^2)^(1/2)/
x^4/(1-1/a^2/x^2)^(1/2)-4/3*a*(c-c/a^2/x^2)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+2
*a^2*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4*a^3*(c-c/a^2/x^2)^(1/2)/
x/(1-1/a^2/x^2)^(1/2)-4*a^4*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4
*a^4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6197, 6193, 88}

$$-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -Sqrt[c - c/(a^2*x^2)]/(5*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a^2
*x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sqrt
[1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2
)]*x^2) - (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^4*
Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(a^
2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6193

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2))/x
^(2*p), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^6(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^6}-\frac{3a}{x^5}+\frac{4a^2}{x^4}-\frac{4a^3}{x^3}+\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^3\sqrt{c-}}{\sqrt{1-}}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.34

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-4a^5\log(x)+4a^5\log(ax+1)-\frac{240a^4x^4-120a^3x^3+80a^2x^2-45ax+12}{60x^5}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/60*(12 - 45*a*x + 80*a^2*x^2 - 120*a^3*x^3 + 240
*a^4*x^4)/x^5 - 4*a^5*Log[x] + 4*a^5*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)
])
```

fricas [A] time = 0.57, size = 107, normalized size = 0.41

$$\frac{240\,a^6\sqrt{c}\,x^5\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{a^2c}\,(2\,ax+1)\sqrt{c}+ac}{ax^2+x}\right)-\left(240\,a^4x^4-120\,a^3x^3+80\,a^2x^2-45\,ax+12\right)\sqrt{a^2c}}{60\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fri
cas")
```

```
[Out] 1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x
 + 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (240*a^4*x^4 - 120*a^3*x^3 + 80*a^2*x^2
 - 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

maple [A] time = 0.07, size = 106, normalized size = 0.40

$$-\frac{\left(240a^{5}\ln(x)x^{5}-240\ln\left(ax+1\right)x^{5}a^{5}+240x^{4}a^{4}-120x^{3}a^{3}+80a^{2}x^{2}-45ax+12\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{60\left(ax-1\right)^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x)
```

```
[Out] -1/60*(240*a^5*ln(x)*x^5-240*ln(a*x+1)*x^5*a^5+240*x^4*a^4-120*x^3*a^3+80*a
^2*x^2-45*a*x+12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(
3/2)/(a*x-1)^2/x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-\frac{c}{a^{2}x^{2}}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.928 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=154

$$\frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}-\frac{c2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$

```
[Out] 4*c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*
n],(a-1/x)/(a+1/x))/a/(2-n)-2^(1+1/2*n)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([-1
/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/a/(2-n)
```

Rubi [C] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6194, 136}

$$-\frac{c2^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+4}{2}}F_1\left(\frac{n+4}{2};\frac{n-2}{2},2;\frac{n+6}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+4)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -((2^(2 - n/2)*c*(1 + 1/(a*x))^((4 + n)/2)*AppellF1[(4 + n)/2, (-2 + n)/2,
2, (6 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(4 + n)))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx&=-\left(c\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{1+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\&=-\frac{2^{2-\frac{n}{2}}c\left(1+\frac{1}{ax}\right)^{\frac{4+n}{2}}F_1\left(\frac{4+n}{2};\frac{1}{2}(-2+n),2;\frac{6+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(4+n)}\end{aligned}$$

Mathematica [A] time = 0.26, size = 123, normalized size = 0.80

$$\frac{ce^{n\coth^{-1}(ax)}\left(ne^{2\coth^{-1}(ax)}{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(ax)}\right)+(n+2)\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+4e^{2\coth^{-1}(ax)}\right.}{a(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*E^(n*ArcCoth[a*x])*(2*a*x + a*n*x + E^(2*ArcCoth[a*x])*n*Hypergeometric2
F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcCoth[a*x])] + 4*E^(2*ArcCoth[a*x])*Hypergeometric2F1[
2, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]))/(a*(2 + n))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^{2} c x^{2} - c\right) \left(\frac{a x - 1}{a x + 1}\right)^{\frac{1}{2} \, n}}{a^{2} x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 - c)*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^{2} x^{2}}\right) \left(\frac{a x - 1}{a x + 1}\right)^{\frac{1}{2} \, n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int {\mathrm e}^{n \,\mathrm{arccoth}\left(a x \right)} \left(c -\frac{c}{a^{2} x^{2}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c - \frac{c}{a^{2} x^{2}}\right) \left(\frac{a x - 1}{a x + 1}\right)^{\frac{1}{2} \, n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{e}}^{n\,\mathrm{acoth}\left(a\,x\right)}\,\left(c-\frac{c}{a^2\,x^2}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int a^{2}e^{n\,\operatorname{acoth}(ax)}\,dx+\int\left(-\frac{e^{n\,\operatorname{acoth}(ax)}}{x^{2}}\right)dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x**2,
x))/a**2
```

3.929 $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=150

$$\frac{2\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac}-\frac{(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{acn}+\frac{x\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{c}$$

```
[Out] -(1+n)*(1+1/a/x)^(1/2*n)/a/c/n/((1-1/a/x)^(1/2*n))+(1+1/a/x)^(1/2*n)*x/c/((
1-1/a/x)^(1/2*n))+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n],(a+1/x
)/(a-1/x))/a/c/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6194, 129, 155, 12, 131}

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac(2-n)}-\frac{(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{acn}+\frac{x\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(((1 + n)*(1 + 1/(a*x))^(n/2))/(a*c*n*(1 - 1/(a*x))^(n/2))) + ((1 + 1/(a*x
))^(n/2)*x)/(c*(1 - 1/(a*x))^(n/2)) + (2*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(
a*x))^((-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (a - x^(-1))/(a +
 x^(-1))])/(a*c*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} - \frac{a\,\text{Subst}\left(\int \frac{n^2\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-}}{a^2x}\right.}{cn}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} - \frac{n\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{-1+}}{x}\right.}{ac}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} + \frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-}}{ac(}$$

Mathematica [A] time = 0.27, size = 94, normalized size = 0.63

$$\frac{e^{n\coth^{-1}(ax)}\left(n^2e^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(ax)}\right)+(n+2)\left(n\,{}_2F_1\left(1,\frac{n}{2};\frac{n}{2}+1;e^{2\coth^{-1}(ax)}\right)+anx-1\right)\right)}{acn(n+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + n*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] integral(a^2*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2 e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2),x)
[Out] a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 1), x)/c
```

3.930 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{2\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac^2}-\frac{\left(n^2+4n+6\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}}{ac^2n(n+2)}+\frac{\left(-n^3-n^2+4n+6\right)\Big(}{ac^2(2-n)}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/(2+n)+(-n^3-n^2+4*n+
6)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/n/(-n^2+4)-(n^2+4*n+6)*(1
+1/a/x)^(-1+1/2*n)/a/c^2/n/(2+n)/((1-1/a/x)^(1/2*n))+(1-1/a/x)^(-1-1/2*n)*(
1+1/a/x)^(-1+1/2*n)*x/c^2+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n
],(a+1/x)/(a-1/x))/a/c^2/((1-1/a/x)^(1/2*n))
```

Rubi [A] time = 0.25, antiderivative size = 303, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6194, 129, 155, 12, 131}

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2(2-n)}+\frac{\left(-n^3-n^2+4n+6\right)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{ac^2(2-n)n(n+2)}-\frac{\left(n^2+4n\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(a*c^2*(2 +
 n))) + ((6 + 4*n - n^2 - n^3)*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 +
 n)/2))/(a*c^2*(2 - n)*n*(2 + n)) - ((6 + 4*n + n^2)*(1 + 1/(a*x))^((-2 + n
)/2))/(a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)) + ((1 - 1/(a*x))^(-1 - n/2)*(1
+ 1/(a*x))^((-2 + n)/2)*x)/c^2 + (2*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))
^((-2 + n)/2)*Hypergeometric2F1[1, 1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-
1))])/(a*c^2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{3x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} - \frac{a\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\cdots}}{\cdots}\right.}{\cdots}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} - \frac{\left(6+4n+n^2\right)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)} + \frac{\left(1-\frac{1}{ax}\right.\cdots}{\cdots}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(6+4n-n^2-n^3\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{(6\cdots}{\cdots}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(6+4n-n^2-n^3\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{(6\cdots}{\cdots}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(6+4n-n^2-n^3\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)} - \frac{(6\cdots}{\cdots}$$

Mathematica [A] time = 0.93, size = 180, normalized size = 0.62

$$\frac{e^{n\coth^{-1}(ax)}\left(a^3n^3x^3-4a^3nx^3+(n-2)n^2\left(a^2x^2-1\right)e^{2\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{n}{2}+1;\frac{n}{2}+2;e^{2\coth^{-1}(ax)}\right)+\left(n^2-4\right)n\left(a^2x^2\cdots\right.\right.}{ac^2(n-2)n(n+2)\left(a^2x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-6 + n^2 + 6*a*n*x - a*n^3*x + 6*a^2*x^2 - 2*a^2*n^2*x
^2 - 4*a^3*n*x^3 + a^3*n^3*x^3 + E^(2*ArcCoth[a*x])*(-2 + n)*n^2*(-1 + a^2*
x^2)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + n*(-4 + n
^2)*(-1 + a^2*x^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))
/(a*c^2*(-2 + n)*n*(2 + n)*(-1 + a^2*x^2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{a^4c^2x^4-2\,a^2c^2x^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(a^4*x^4*((a*x - 1)/(a*x + 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2
 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\left(c-\frac{c}{a^2\,x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2,x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4 \int \frac{x^4 e^{n\,\mathrm{acoth}\,(ax)}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*exp(n*acoth(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

3.931 $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=295

$$\frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2^{\frac{n+1}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/
x^2)^(1/2)+2*n*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*hypergeom([1, 1
/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))*(c-c/a^2/x^2)^(1/2)/a/(1-n)/(1-1/a^2
/x^2)^(1/2)-2^(1/2+1/2*n)*(1-1/a/x)^(1/2-1/2*n)*hypergeom([1/2-1/2*n, 1/2-1
/2*n],[3/2-1/2*n],1/2*(a-1/x)/a)*(c-c/a^2/x^2)^(1/2)/a/(1-n)/(1-1/a^2/x^2)^
(1/2)
```

Rubi [C] time = 0.15, antiderivative size = 111, normalized size of antiderivative = 0.38, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6197, 6194, 136}

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+3}{2}}F_1\left(\frac{n+3}{2};\frac{n-1}{2},2;\frac{n+5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+3)\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] -((2^(3/2 - n/2)*Sqrt[c - c/(a^2*x^2)]*(1 + 1/(a*x))^((3 + n)/2)*AppellF1[(
3 + n)/2, (-1 + n)/2, 2, (5 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(3
 + n)*Sqrt[1 - 1/(a^2*x^2)]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}}{x^2}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}F_1\left(\frac{3+n}{2};\frac{1}{2}(-1+n),2;\frac{5+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(3+n)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A] time = 0.52, size = 146, normalized size = 0.49

$$\frac{ax^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}e^{n\coth^{-1}(ax)}\left(a(n+1)x\sqrt{1-\frac{1}{a^2x^2}}+2e^{\coth^{-1}(ax)}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-e^{2\coth^{-1}(ax)}\right)+2ne^{\coth^{-1}(ax}\right.}{(n+1)\left(a^2x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2*(a*(1
 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E^ArcCoth[a*x]*Hypergeometric2F1[1, (1 +
n)/2, (3 + n)/2, -E^(2*ArcCoth[a*x])] + 2*E^ArcCoth[a*x]*n*Hypergeometric2F
1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]))/((1 + n)*(-1 + a^2*x^2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Value
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c - \frac{c}{a^2x^2}} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c - \frac{c}{a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)}e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

3.932 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=183

$$\frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/
x^2)^(1/2)+2*n*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*hypergeom([1, 1
/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/a/(1-n)/(c-c/a^2
/x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6197, 6194, 96, 131}

$$\frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*
x)/Sqrt[c - c/(a^2*x^2)] + (2*n*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n
)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
(a - x^(-1))/(a + x^(-1))])/(a*(1 - n)*Sqrt[c - c/(a^2*x^2)])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\left(n\sqrt{1-\frac{1}{a^2x^2}}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}} + \frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}{}_2F_1\left(1,\frac{1-n}{2}\right.}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A] time = 0.41, size = 112, normalized size = 0.61

$$\frac{\left(a^2x^2-1\right)e^{n\coth^{-1}(ax)}\left(a(n+1)x\sqrt{1-\frac{1}{a^2x^2}}+2ne^{\coth^{-1}(ax)}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};e^{2\coth^{-1}(ax)}\right)\right)}{a^3(n+1)x^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-1 + a^2*x^2)*(a*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E
^ArcCoth[a*x]*n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x
])]))/(a^3*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x - 1)/(a*x + 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x - 1)/(a*x + 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x)
```

3.933 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=116

$$-\frac{2^{-\frac{n}{2}+p+1}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}F_1\left(\frac{n}{2}+p+1;\frac{1}{2}(n-2p),2;\frac{n}{2}+p+2;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2p+2)}$$

```
[Out] -2^(1-1/2*n+p)*(c-c/a^2/x^2)^p*(1+1/a/x)^(1+1/2*n+p)*AppellF1(1+1/2*n+p,1/2
*n-p,2,2+1/2*n+p,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6197, 6194, 136}

$$-\frac{2^{-\frac{n}{2}+p+1}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}F_1\left(\frac{n}{2}+p+1;\frac{1}{2}(n-2p),2;\frac{n}{2}+p+2;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -((2^(1 - n/2 + p)*(c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + n/2 + p)*AppellF1
[1 + n/2 + p, (n - 2*p)/2, 2, 2 + n/2 + p, (a + x^(-1))/(2*a), 1 + 1/(a*x)]
)/(a*(2 + n + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{n}{2}+p}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}F_1\left(1+\frac{n}{2}+p;\frac{1}{2}(n-2p),2;2+\frac{n}{2}\right.}{a(2+n+2p)}$$

Mathematica [F] time = 0.59, size = 0, normalized size = 0.00

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
[Out] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
[Out] integral(((a*x - 1)/(a*x + 1))^(1/2*n)*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int e^{n\,\text{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{2}n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))^p*((a*x - 1)/(a*x + 1))^(1/2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)}\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^p e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

3.934 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(2,2p+1;2(p+1);1-\frac{1}{ax}\right)}{a(2p+1)}$$

```
[Out] (c-c/a^2/x^2)^p*(1-1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2+2*p],1-1/a/x)/a/(
1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = $\{6197, 6194, 65\}$

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(2,2p+1;2(p+1);1-\frac{1}{ax}\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*(1 - 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 - 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p\,dx\\ &= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2p}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1-\frac{1}{ax}\right)^{1+2p}{}_2F_1\left(2,1+2p;2(1+p);1-\frac{1}{ax}\right)}{a(1+2p)}\end{aligned}$$

Mathematica [F] time = 0.63, size = 0, normalized size = 0.00

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
[Out] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]), x]
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a^2*c*x^2 - c)/(a^2*x^2))^p/((a*x - 1)/(a*x + 1))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p/((a*x - 1)/(a*x + 1))^p, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^p e^{-2p\,\mathrm{arccoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
[Out] int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{a^2x^2}\right)^p}{\left(\frac{ax-1}{ax+1}\right)^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p/((a*x - 1)/(a*x + 1))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{-2\,p\,\mathrm{acoth}(a\,x)}\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

```
[Out] int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{p}e^{-2p\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/exp(2*p*acoth(a*x)),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

3.935 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=75

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(2,2p+1;2(p+1);1+\frac{1}{ax}\right)}{a(2p+1)}$$

```
[Out] -(c-c/a^2/x^2)^p*(1+1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2+2*p],1+1/a/x)/a/
(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6197, 6194, 65}

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p {}_2F_1\left(2,2p+1;2(p+1);1+\frac{1}{ax}\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -(((c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p
, 2*(1 + p), 1 + 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 6194

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 -Dist[c^p, Subst[Int[((1 - x/a)^(p - n/2)*(1 + x/a)^(p + n/2))/x^2, x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 6197

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[(c^IntPart[p]*(c + d/x^2)^FracPart[p])/(1 - 1/(a^2*x^2))^FracPar
t[p], Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx &= \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p\,dx\\
&= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{2p}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+2p}{}_2F_1\left(2,1+2p;2(1+p);1+\frac{1}{ax}\right)}{a(1+2p)}
\end{aligned}$$

Mathematica [F] time = 0.60, size = 0, normalized size = 0.00

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
[Out] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax-1}{ax+1}\right)^p\left(\frac{a^2cx^2-c}{a^2x^2}\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
[Out] integral(((a*x - 1)/(a*x + 1))^p*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax-1}{ax+1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x - 1)/(a*x + 1))^p, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int e^{2p\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
[Out] int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{ax-1}{ax+1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x - 1)/(a*x + 1))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{2\,p\,\mathrm{acoth}(a\,x)}\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
```

```
[Out] int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{p} e^{2p\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(c-c/a**2/x**2)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```